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## **Auto-Calibration for Data Assimilation in Linear Reservoir Models Used in Flow Forecasting of Urban Runoff**

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### **ABSTRACT**

The production of quality flow forecasts is useful for the management of urban drainage systems and the models that produce the forecasts need to be kept up to date, e.g. through assimilation of observations in real-time. In this study, flow forecasts of 0–4 hours had been made by applying a lumped linear reservoir model with three cascading reservoirs to a catchment located in Ballerup, Denmark. In order to improve the forecast abilities of the model, data had continuously been assimilated into the model through auto-calibration of the model parameters with maximum *a Posteriori* estimation. Here, three parameters were evaluated: the effective area, the transport time through the system and the mean value of the dry weather flow. Maximum *a Posteriori* estimation requires probability distributions to be assigned to the three parameters. The mean values were kept constant while the standard deviations were varied. It was found that the parameters tend to drop and increase suddenly to fit the present flow conditions but often this leads to poor flow forecasts because of altered flow conditions in the future. The benefit of this type of data assimilation diminished with the forecasting length and is completely gone for forecasts longer than 2 hours.

### **KEYWORDS**

Real-time flow forecasting, data assimilation, auto-calibration, maximum *a Posteriori* estimation, linear reservoir models

### **INTRODUCTION**

Forecast models can be an essential part of managing urban drainage systems in real-time. They can be used to e.g. control the inlet flow to upstream storage basins, thus distributing water more efficiently and avoiding combined sewer overflow, or in the activation of wet weather operation of aeration tank systems at wastewater treatment plants. In order to ensure the quality of the forecasts, it is essential to keep the model up to date with the most recent flow conditions before the forecasts are made. In places where catchment characteristics such as effective surface area vary significantly over time, it is necessary for forecasting models to include flexible parameters that can handle these variations. In the current work, a method is presented that continuously auto-calibrates parameters of a model by applying *Maximum a Posteriori* estimation. This method is currently in operation at two locations in Denmark, i.e. Copenhagen (Lynetten) and Aalborg, where simple, lumped linear reservoir models with three cascading reservoirs are used for producing radar based flow

forecasts (Thorndahl *et al.*, 2013). Despite the application of this method in the two mentioned operational systems, it has not yet been documented in the open literature. Since most urban hydrologists are familiar with auto-calibration methods, the described data assimilation method is easy to use for practitioners. When used on a model with a simple structure, this method provides an easy and relatively transparent approach. This makes it possible to investigate the forecasting abilities of the model and how the auto-calibrated parameters vary over time, which will be examined in this study.

## THEORY & METHOD

### Principle of flow forecasting

Flow forecasts of urban runoff are in the current work produced by extrapolating a model, which has been calibrated to present conditions, into the future. This procedure requires three overall steps: initialisation, auto-calibration and forecasting. The auto-calibration step is where prior knowledge about the parameters and observed data are incorporated into the model. The purpose of the auto-calibration step is to calibrate the model as well as possible to present conditions through optimization of an objective function. The initial conditions for the auto-calibration step are produced by performing an ordinary model simulation of the initialisation step that precedes the auto-calibration step. The auto-calibrated set of parameters is afterwards used as parameters during the flow forecasting step. All three steps are illustrated in Figure 1.

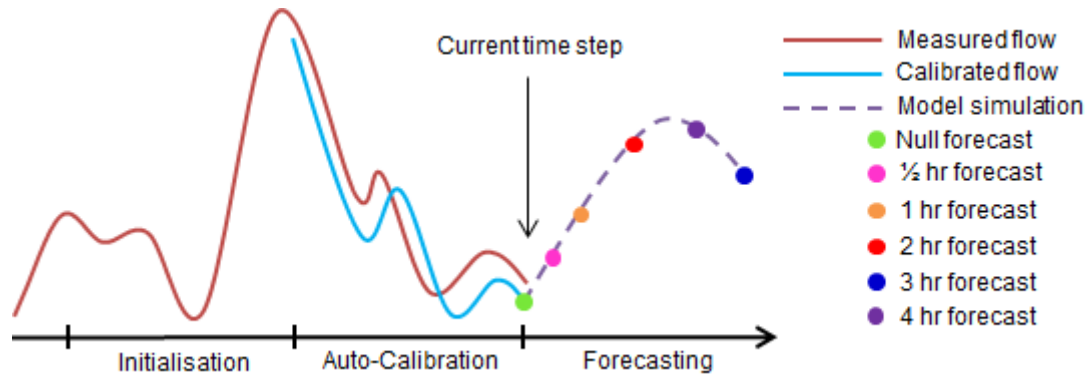
### Data assimilation with auto-calibration of parameters

The calibration procedure used in this study is based on *Maximum a Posteriori* estimation (MAP). MAP is a statistical method used to make parameter estimates based on empirical data. It is related to Maximum Likelihood but also includes statistical distributions which describe parameter uncertainties. MAP seeks to maximize the following likelihood function:

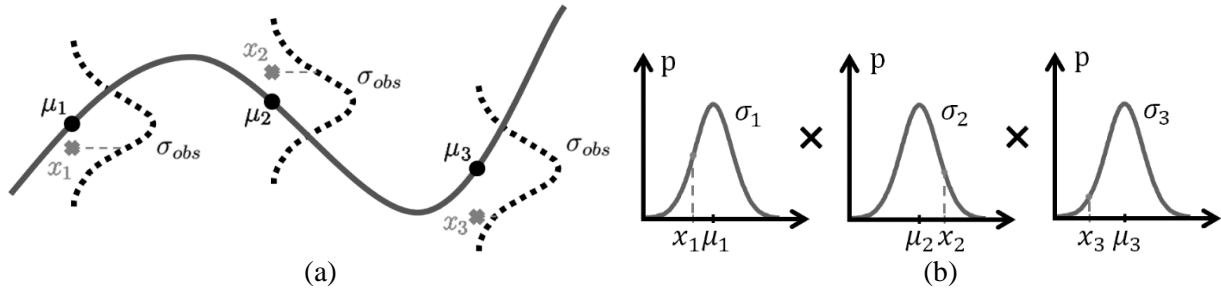
$$\hat{\theta}_{\text{MAP}} = \text{argmax}_{\theta} f(x|\theta)g(\theta) \quad (1)$$

Here  $f(x|\theta)$  is the probability of observing certain measurements,  $x$ , with a model containing a specific set of parameters,  $\theta$ , while the total likelihood of the parameter values is denoted  $g(\theta)$ .

$f(x|\theta)$  implies that the observations carry uncertainties, which here are described by a Gaussian distribution with mean,  $\mu_j$ , corresponding to the model value, and a standard deviation,  $\sigma_{\text{obs}}$ , containing information about the deviations between model and observations. This is shown in Figure 2a. The total likelihood is found by multiplying the likelihood of all observations.



**Figure 1.** Illustration of the three steps that constitute the applied flow forecasting procedure.



**Figure 2.** Depiction of Maximum a Posteriori estimation. (a) How likely the observation are to be an outcome of the model,  $f(x|\theta)$  (b) Total parameter uncertainty,  $g(\theta)$ .

$g(\theta)$  assigns an uncertainty to the model parameters and relies on the accuracy of prior information related to these parameters such as mean and standard deviation. This is visualized in Figure 2b. The total likelihood is obtained by multiplying the likelihood of all parameters.

### Modelling tool and application of *Maximum a Posteriori* estimation

The models in this study are set up and auto-calibrated in the generic water modelling tool WaterAspects, which is open source software maintained by Krüger A/S - Veolia Water (Grum *et al.*, 2004).

In WaterAspects, the likelihood function is not maximized as in Equation 1. Instead the negative log likelihood function is minimized, and Equation 1 becomes

$$\hat{\theta}_{\text{MAP}} = \text{argmin}_{\theta} -(\ln(f(x|\theta)) + \ln(g(\theta)))$$

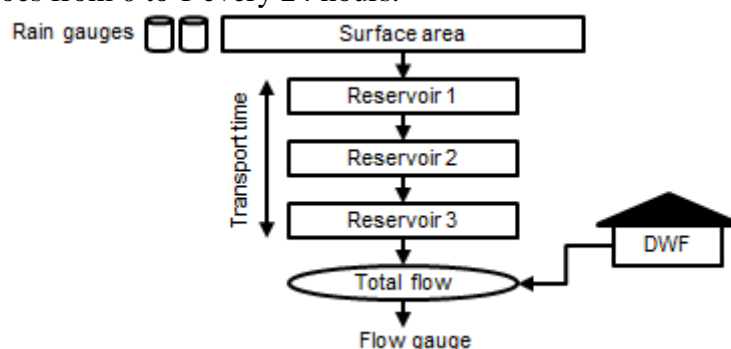
The minimization of the negative log likelihood function in WaterAspects is done with the Broyden-Fletcher-Goldfarb-Shanno approximation of Newton's method. Within the field of urban hydrology, this optimization algorithm has among others been used by Thorndahl and Rasmussen (2013). In the current work, the algorithm is allowed to iterate for 90 seconds at most, whereafter the set of parameters that produce the highest likelihood is chosen. After the best parameters have been chosen,  $\sigma_{\text{obs}}$  is re-estimated based on how well the model fits the observations, thus determining a posterior probability distribution for the observations. A good fit results in a low  $\sigma_{\text{obs}}$  and vice versa. The new  $\sigma_{\text{obs}}$  is then applied in the next time step.

### Modelling approach

Flow measurements in drainage systems are often a combination of rainfall-runoff and dry weather flow (DWF) and these two components are modelled in different ways. In this study, a lumped, linear reservoir model is used in order to emulate the relationship between rainfall and runoff in a given catchment area. Linear reservoir models are described in detail by Chow *et al.* (1988). The applied model is relatively simple with three cascading reservoirs which allows for more focus on the data assimilation method than on the model composition. The DWF originates from the use of water in households and industry which vary both during the day and over the course of the year. Generally, two peaks appear every day which in this study is imitated by a double sine curve as seen in Equation 2.

$$\text{DWF} = \mu_{\text{DWF}} + a_1 \sin(2\pi T + \omega_1) + a_2 \sin(4\pi T + \omega_2) \quad (2)$$

Here  $\mu_{\text{DWF}}$  is the mean of the DWF,  $a_i$  controls the amplitude,  $\omega_i$  controls the frequency and  $T$  is a function that goes from 0 to 1 every 24 hours.



**Figure 3.** The setup of the cascading linear reservoir model which also takes DWF into account.

The model contains three parameters which are auto-calibrated: a surface area, a transport time and a mean dry weather flow. The surface area is the effective area of the catchment while the transport time is the average time it takes for runoff to flow through the system. This transport time thus have to account for both fast and slow runoff at the same time. It has been observed that the mean DWF,  $\mu_{\text{DWF}}$ , changes seasonally. The other variables in Equation 2, which describe daily fluctuations, are kept constant for the sake of simplicity. How all three parameters relate to the model setup can be seen in Figure 3.

In this study, an initialisation step and an auto-calibration step of each 16 hours have been used. In order to perform the auto-calibration, it is necessary to assign statistical uncertainty to the three parameters, i.e. a mean and standard deviation needs to be determined for each parameter. The mean values of the parameters are determined by auto-calibrating a single set of parameters that creates the best model fit to the data, which also means that no validation period is applied. This calibration is carried out by using Maximum Likelihood. The variables in the DWF equation (Equation 2) are determined on a dry period in the middle of the considered data set while the mean effective area and transport time are determined on the entire period. The standard deviations will determine how much the model can deviate from the mean values and are therefore based on observed parameter variations. The standard deviations are determined based on information from both dry periods and single extreme events in the data set, as these are seen as a guide to how much the parameters should be allowed to vary over time.

## CASE STUDY

The catchment considered in this study has an area of  $1.32 \times 10^7 \text{ m}^2$  and is situated in Ballerup outside of Copenhagen, Denmark. This area is part of a larger complex of catchment areas connected to the Wastewater treatment plant in Avedøre, and is already well known from several research studies (Breinholt *et al.*, 2013; Löwe *et al.*, 2014).

The considered time period in this study is four months from July 1st 2010 to October 30th 2010. Rain data is obtained from two rain gauges situated outside of the catchment area. The two tipping bucket gauges have a volumetric resolution of 0.2 mm and are about 12 km apart (Jørgensen *et al.*, 1998). The applied rain data in this study is an average of the measurements from the two gauges, which relies on the assumption that 50% of the rainfall on the catchment can be described by one gauge while the other 50% can be described by the other gauge.

## RESULTS AND DISCUSSION

### Parameter variability and determination of priors

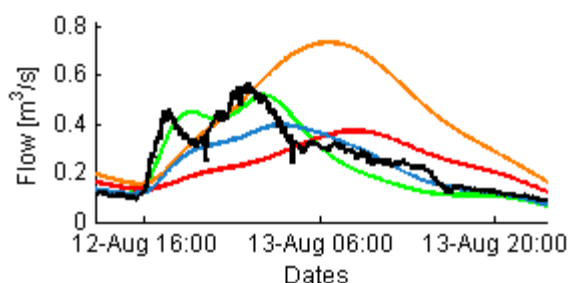
The calibrated mean values for the Gaussian distributions are determined to  $0.096 \text{ m}^3/\text{s}$  for the mean DWF, an effective area of  $547,699 \text{ m}^2$  and a transport time of 8.31 hours. To provide an overview of how much the properties of the catchment vary from these mean values from event to event, the area and transport time have been calibrated for numerous single rain events with a constant mean DWF. This is illustrated in Figure 4 where the calibrated parameters from three sampled events are applied on a completely separate rain event. Figure 4 illustrates how different the catchment properties can be from event to event, and that parameters calibrated for single events cannot simply be extrapolated to others. A single parameter set will thus not be able to model the runoff well for the entire period and continuous auto-calibration is needed.

The most extreme value from the single events shown in Figure 4 is for the area found to be  $1,460,919 \text{ m}^2$  while it is 14.78 hours for the transport time. The most extreme value for the mean DWF is obtained by calibration on a dry weather period in the very end of the data set which gives a value of  $0.116 \text{ m}^3/\text{s}$ . These three extreme parameter values are each used in three different distributions: one where they correspond to  $1\sigma$ , one where they correspond to  $2\sigma$  and one where they correspond to  $3\sigma$ . This is visualized in Figure 5 where it is also clarified that  $1\sigma$  is the most variable while  $3\sigma$  is the most constrained of the three distributions. The application of these three uncertainty measures will hereafter be referred to as the  $1\sigma$ -,  $2\sigma$ - and  $3\sigma$ -scenarios.

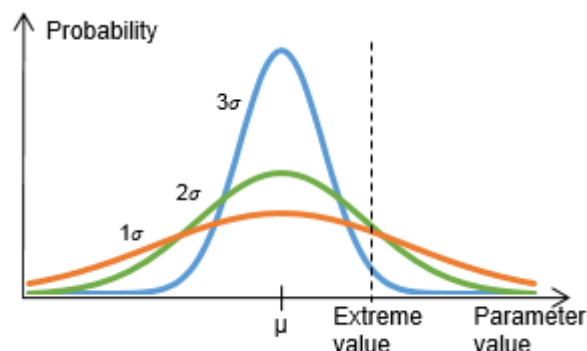
### Effects of applying different $\sigma$ -scenarios

The effects of applying the different  $\sigma$ -scenarios on the overall model performance are shown in Figure 6, where the Nash-Sutcliffe Efficiency (NSE) of all scenarios for 0–4 hour forecasts are compared. Figure 6 shows that the most constrained scenario,  $3\sigma$ , produces the best forecasts for all time lengths. It is also noticeable that the  $3\sigma$ -scenario's advantage over the less constrained scenarios increase over time. This indicates that the parameters in the more variable scenarios are over-tuned to present conditions and thus not suitable for extrapolation far into the future. Figure 6 also shows the “forecasting” performance of a model without continuous data assimilation, hence a simulation model that applies the prior

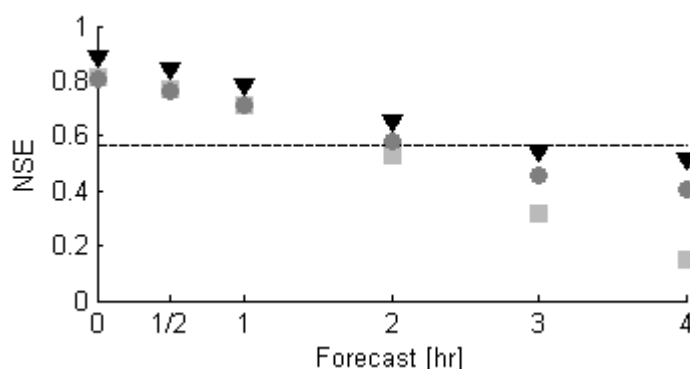
determined mean parameters. When the forecast models are compared to the simulation model, it is seen that the addition of data assimilation result in better 0-2 hour forecasts but worse 3-4 hour forecast. Thus, the benefit of data assimilation is gone for forecasts of more than two hours.



**Figure 4.** The observed flow compared to models containing parameters auto-calibrated on different time periods. —Measured flow —Event 1 —Event 2 —Event 3 —Entire Period



**Figure 5.** Visualisation of the connection between extreme parameter values and applied standard deviations.



**Figure 6.** Forecast performance for different  $\sigma$ -scenarios with a fixed auto-calibration step of: (left) 8 hours (center) 12 hours (right) 16 hours ■ 1 $\sigma$  ● 2 $\sigma$  ▼ 3 $\sigma$  --- No data assimilation

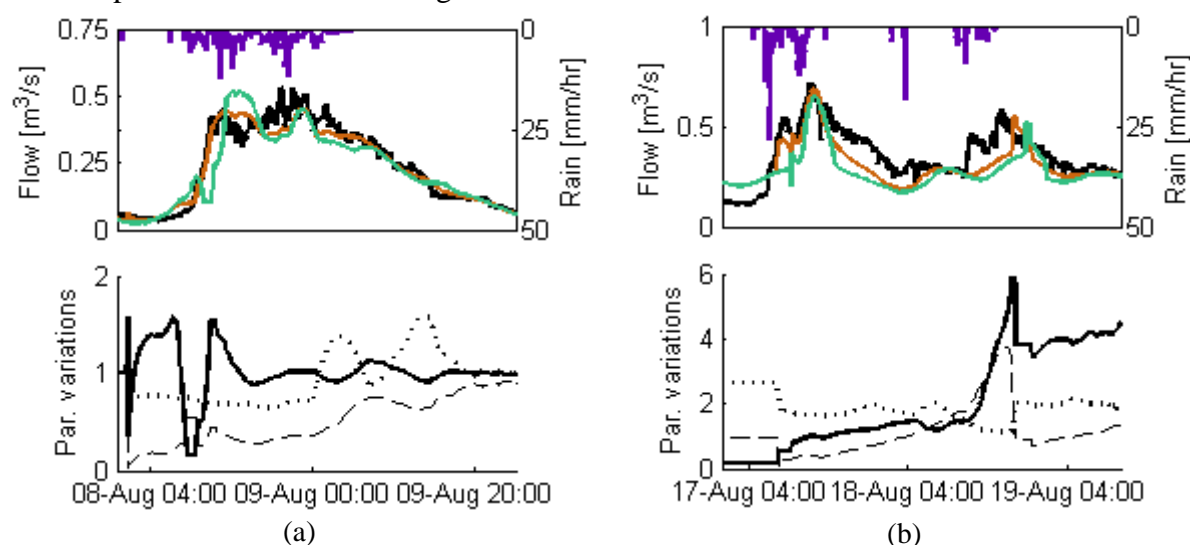
In this section, two different examples of 2 hour flow forecasts will be given in order to visualize how auto-calibration with MAP produces flow forecasts through continuous changes in the underlying model parameters. The provided examples are from the 3 $\sigma$ -scenario since this was seen to perform the best in Figure 6.

**Figure 7a** shows an event which is forecasted fairly well. A sudden decrease in the size of the effective area and an increased transport time in the beginning of the event makes the model able to fit the flow well within the auto-calibration step. The good fit in the auto-calibrated step does however result in a forecast which is delayed by approximately two hours. When the flow decreases again around midnight, the area becomes more or less stable while an increasing transport time together with a fluctuating mean DWF ensures that the tail is modelled, and forecasted, well.

**Figure 7b** illustrates a situation with a couple of consecutive rainfalls where the forecast does not imitate the runoff very well. These rain events appear after a week of heavy rainfall and it is worth noting that the model already from the beginning of the investigated events is

deviating from the observed flow and the first flow peak is captured poorly as a result. The first of the shown rainfalls is itself very large, and the corresponding hydrograph has a very long tail, possibly due to high soil saturation caused by previous events. These conditions remain in the catchment area at the time of the last rainfall and the flow level is thus still affected by the prior rainfalls. This results in larger amounts of runoff and a higher downstream flow peak than what would be expected considering the size of the last rainfall. The model tries to compensate for that by increasing the area sixfold but the response is not fast enough and the forecast imitates the flow peak poorly.

By comparing the parameter variations in Figure 7a and Figure 7b, it is seen that the parameters, especially the area, can vary a lot in order to make the simple model fit the complex reality. These variations span from parameter values near zero to several times higher than the expected mean value. The model should be able to adjust to reality, but it is seen that the weighing of the likelihood of the parameters and the fit between the null forecast and observed flow does not always give precise forecasts because of altered catchment conditions. This results in changes in the individual parameter values that occur as sudden increases or drops, as seen in both shown examples. These erratic properties most likely stem from the MAP auto-calibration but is probably also affected by the quality of the rain input. The fact that the two gauges provide point estimates of rainfall and that they are placed outside the catchment will generate uncertainty in the forecasts. Changes in the effective area and transport time can result in large



**Figure 7.** Example of two rain events modelled with the  $3\sigma$ -scenario together with an auto-calibration step of 16 hours. (Upper) Measured flow together with the null forecast and the 2 hr forecast. (Lower) Variations of parameters. Flow: —Measured flow — Null forecast — 2 hr forecast — Rain. Parameter variations: —Area ---Transport time ...Mean DWF

changes in the amount of water in each reservoir of the model from one time step to another. Conceptually, updating of the parameters can thus be seen as indirect state updates which in some cases translate directly into sudden, unrealistic changes in the downstream flow forecasts. The sudden changes could potentially give good results if they happen when the model deviates from the measured flow, however they are seen mostly to worsen the flow forecasts.

## CONCLUSIONS

The use of MAP based auto-calibration as a data assimilation method for runoff forecasting purposes is interesting due to its potential enhancement of the flow forecast capabilities, especially in catchments where properties such as the effective surface area vary greatly from event to event. This is done by continuously weighing prior knowledge about the model parameters against the model fit to incoming flow observations in real time.

Two aspects are of main importance for the quality of the produced flow forecasts with MAP auto-calibration: the mean of the parameters and the assigned uncertainty to each parameter. The mean of the parameters functions as the most likely value at any given time and is thus a basis point to which the parameters return. The uncertainty assigned to the parameters dictates how much they can deviate from the mean values. These parameter variations solely build on observed extreme parameter values and are not bound by physical conditions. At times, this results in over-tuning of the model to present flow conditions which does not fit well with the future. This can be an explanation why it, in this study, has been found that the most constrained scenario produced the best forecasts. No investigations were made of further constrained parameter variations, and it is therefore not known whether such scenarios would perform better. Additionally, the results indicate that the benefit of this data assimilation only improves forecasts for 0-2 hours into the future. Longer forecasts than this have not been observed to improve with MAP data assimilation.

The examples shown in this study have revealed that auto-calibration with MAP for urban runoff forecasting is a sensitive method. It is seen to perform poorly in cases where the prior flow conditions are no longer valid in the forecast period due to altered catchment conditions, which is e.g. evident in situations with consecutive rain events. Along with the quality of the rain input, this leads to abrupt changes in the parameter values and these erratic properties are translated directly into the forecasts.

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