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Brunskog, Jonas

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The sound transmission of finite ribbed plates using a variational technique

Jonas Brunskog[a)]
Acoustic Technology, DTU Electrical Engineering, Technical University of Denmark,
Kgs. Lyngby DK-2800,
Denmark

Many lightweight structures consist of plates being stiffened by ribs. The rib stiffeners can significantly change the vibration field and the radiation behavior of the structure. These type of structures has thus often been studied in the past. However, there is a lack of simplified expressions for the sound transmission of these structures. Therefore, simplified expressions for the sound transmission of finite single leaf ribbed plates are derived, using a variational technique based on integral equations of the fluid loaded plate.

1 INTRODUCTION

The purpose of the present paper is to derive simple analytical formulas for the airborne sound insulation of a single ribbed wall of finite size, using a variational technique based on the integral-differential equation of the fluid loaded wall. The so derived formulas are valid in the entire audible frequency range. The paper is an extension of a previous work concerned with the same problem of a single wall without rib stiffeners[1,2].

2 THEORY

Small amplitudes and linear theory are assumed. The time dependence is of the form $\mathrm{e}^{i\omega t}$, where $\omega = 2\pi f$ is the angular frequency and t is time, which is suppressed throughout the paper.

2.1 Formulation of the problem

Consider Fig. 1, where a single ribbed wall of finite size is located in an infinite baffle, an acoustically hard plane, at $z = 0$. The ribbed wall consist of a plate with beams attached to it. The beams are located with a distance l inbetween. As a simplification, it is assumed that the presence of the beams in the sound field will not influence the sound field directly. Thus, the beams are approximated to be sound transparent, only affecting the vibration pattern of the wall plate. On the source side, the total acoustic field will consist of one plane incident wave p_i, one

[a)] email: jbr@elektro.dtu.dk

plane geometrically reflected wave p_r and one scattered field p_s that is due to the motion of the finite wall, $p = p_i + p_r + p_s$. On the receiver side, only the transmitted wave is present, $p = p_t$.

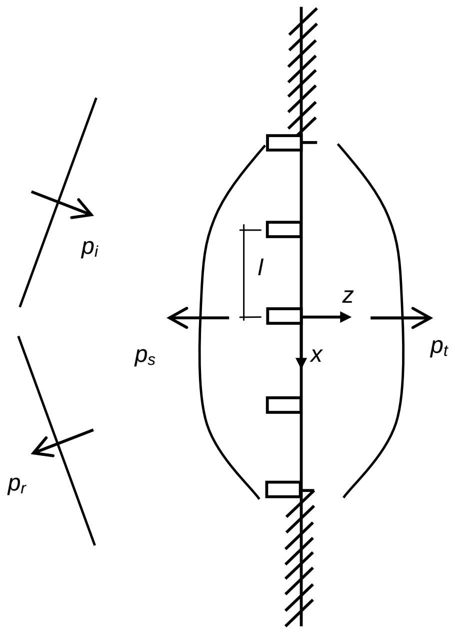

Fig. 1 - The model of a finite wall, located in an infinite rigid baffle in the x-y-plane, at z = 0. The wall consist of a plate and ribbs located with a distanse l apart. On the left side of the wall is the source side (z < 0), the right side is the receiver side (z > 0). On the source side the total acoustic field consists of one plane incident wave, one plane geometrically reflected wave and one scattered field due to the motion of the finite wall. On the receiver side only the radiated field will be present, it being the same as the scattered field but with opposite z-direction propagation.

With the coordinate system defined as the wall being in the x-y-plane and z pointing into the receiver room in Fig. 1, the incident and the reflected plane waves are

$$p_i(x, y, z) = \hat{p}_i e^{-i(k_x x + k_y y + k_z z)},\ p_r(x, y, z) = \hat{p}_i e^{-i(k_x x + k_y y - k_z z)}, \tag{1}$$

where $\hat{p}_i$ is the complex amplitude of the incident wave. The wavenumbers are $k_x = k \sin\theta \cos\varphi$, $k_y = k \sin\theta \sin\varphi$, and $k_z = k \cos\theta$, where $k = \omega/c$. The scattered and transmitted wave will be described by means of the Rayleigh integral, for the scattered wave

$$p_s(x, y, z) = i\omega\rho \int_{S_w} v(S') G(S, z|S', 0) dS' \tag{2}$$

and for the transmitted wave

$$p_t(x, y, z) = -i\omega\rho \int_{S_w} v(S') G(S, z|S', 0) dS', \tag{3}$$

where x', y' and $S' = (x', y')$ refers to the integration point, v is the vibration velocity of the wall and $G(S, z|S', 0)$ is the free field Greens function,

$$G(S, z|S', 0) = -\frac{e^{-ikR}}{2\pi R}. \tag{4}$$

In eqs. (2-3), the integral is a surface integral over the wall area S_w. The distance R between the observation point and the integration point is

$$R = \sqrt{(x - x')^2 + (y - y')^2 + z^2}. \tag{5}$$

It should be noted that due to the symmetry, the transmitted field is the same as the scattered field, but with opposite phase due to the direction of the vibration velocity of the wall v.

The vibrations of the wall can in general terms be describe with a differential operator

$$\mathcal{Z}v(x,y) = p_i(x,y,0) + p_r(x,y,0) + p_s(x,y,0) - p_t(x,y,0) \tag{6}$$

where the wall impedance operator $\mathcal{Z}$, including both the plate and the beams, will be described in more detail below. Using the equations above,

$$\mathcal{Z}v(S) = 2\hat{p}_i e^{-i(k_x x + k_y y)} + 2i\omega\rho \int_{S_w} v(S')G(S,0|S',0)dS' \tag{7}$$

valid for $(x,y) \in S_w$. This is an integral-differential equation with the vibration velocity of the wall v as unknown.

The variational formulation for finite single walls by Brunskog[1,2], based on Morse and Ingard[3], can now directly be adopted,

$$V(v, v_a) = 2\hat{p}_i \int_{S_w} v(S)e^{i(k_x x + k_y y)}\, dS + 2\hat{p}_i \int_{S_w} v_a(S)e^{-i(k_x x + k_y y)}\, dS - \int_{S_w} v_a(S)\mathcal{Z}v(S)\, dS + 2i\omega\rho \int_{S_w}\int_{S_w} v_a(S)v(S')\, G(S,0|S',0)dS'\, dS, \tag{8}$$

where v_a is the vibration velocity of the adjoint problem, with the incident wave in the opposite direction. This is a symmetric functional in v and v_a, if $\mathcal{Z}$ is a symmetric operator (meaning that $v_a\mathcal{Z}v = v\mathcal{Z}v_a$). The functional $V(v, v_a)$ will be used to find approximate variational solutions to the problem by means of minimizing it.

2.2 The impedance operators for the wall plate and the beam stiffeners

The wall impedance operator $\mathcal{Z}$ is used as a general description of wall. If the wall is a thin plate, the Kirchhoff plate equation is the governing equation for the wave motion in the plate,

$$\mathcal{Z}_p = \frac{B'}{i\omega}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 + i\omega m''. \tag{9}$$

To this impedance operator will now be added the contribution from the rib stiffeners. The impedance operator for one Euler beam located in the y-direction is,

$$\mathcal{Z}_f = \frac{B_f}{i\omega}\frac{\partial^4}{\partial y^2} + i\omega m_f'. \tag{10}$$

Assume the beams to be located at $x = nl$, n being an integer and l the distance between the rib stiffeners, see again Fig. 1. Also assume the connection between the beams and the plate to be in the form of a line force and equal displacement of the beam and the plate at $x = nl$. It should be noted that this is a rough simplification of the actual situation, ignoring effects as moment coupling, in-plane wave motion and eccentricity of the beams, and point vice coupling. Equation (7) can with these assumptions be written

$$\left(\mathcal{Z}_p + \sum_{n=-\infty}^{\infty} \mathcal{Z}_f \delta(x - nl)\right) v(x, y) = p(x, y) \tag{11}$$

where $p(x, y)$ is the left hand side of eq. (7). The wall impedance operator $\mathcal{Z}$ can be identified as

$$\mathcal{Z} = \mathcal{Z}_p + \sum_{n=-\infty}^{\infty} \mathcal{Z}_f \delta(x - nl). \tag{12}$$

2.3 Forced variational solution

As a first use of the variation formulation, assume a forced vibration velocity, i.e., the vibration of the wall to be of the same form as the incident wave

$$v(x, y) = \hat{v} e^{-i(k_x x + k_y y)}, \tag{13}$$

and for the adjoint field

$$v_a(x, y) = \hat{v} e^{i(k_x x + k_y y)}. \tag{14}$$

Then, if applying the wall impedance operator to (13),

$$\mathcal{Z} v(x, y) = \hat{v} Z_p e^{-i(k_x x + k_y y)} + \hat{v} Z_f e^{-i(k_x x + k_y y)} \sum_{n=-\infty}^{\infty} \delta(x - nl), \tag{15}$$

where use have been made of

$$Z_p = \frac{B'}{i\omega}\left(k_x^2 + k_y^2\right)^2 + i\omega m'', \; Z_f = \frac{B_f}{i\omega} k_y^4 + i\omega m_f', \tag{16}$$

being the algebraic expressions for the plate and beam impedance (the same apply for v_a). These expressions is inserted in eq. (8). The result is then the same as found by Brunskog[1,2], with the exception for the term including the wall impedance operator

$$\int_{S_w} v_a(S) \mathcal{Z} v(S) \, dS = \hat{v}^2 \int_{S_w} Z_p \, dS + \hat{v}^2 \int_{S_w} \sum_{n=-\infty}^{\infty} Z_f \delta(x - nl) \, dS = \hat{v}^2 \left(Z_p S + n_f Z_f\right), \tag{17}$$

where n_f is the number of beams $\in S_w$. Thus,

$$V(\hat{v}) = 4\hat{p}_i \hat{v} S - \left(Z_p S + n_f Z_f\right) \hat{v}^2 S + 2i\omega\rho \hat{v}^2 \int_{S_w} \int_{S_w} e^{i\left(k_x(x - x') + k_y(y - y')\right)} G(S, 0|S', 0) dS dS'. \tag{18}$$

Differentiate equation (18) with respect to $\hat{v}$ and equal to zero to find the optimum, and then finally

$$\hat{v} = \frac{2\hat{p}_i}{Z_p + n_f Z_f/S + 2\rho c\, z_a}, \tag{21}$$

where the integral over the Green's function in (18) have be identified as the normalized radiation impedance, as defined by Thomasson[4].

$$z_a = -\frac{ik}{S}\int_{S_w}\int_{S_w} e^{i\left(k_x(x-x')+k_y(y-y')\right)} G(S,0|S',0)\,dS\,dS'. \tag{22}$$

The result in (21) corresponds to the corresponding result by Brunskog[1,2], with the addition of the additional contribution $n_f Z_f/S$ to the impedance, being the average beam impedance.

2.4 Variational solution including periodicity

The solution the corresponding problem of the vibration of an infinite plate without radiation load excited by a pressure $\hat{p}e^{-i(k_x x+k_y y)}$ is found by Rumerman[5] (we are here ignoring the moment coupling, for simplicity)

$$v(x,y) = \frac{\hat{p}}{Z_p(k_x,k_y)} e^{-i(k_x x+k_y y)} - \frac{v_0 Z_f(k_y)}{l} e^{-i(k_x x+k_y y)} \sum_{n=-\infty}^{\infty} \frac{e^{2\pi i n x/l}}{Z_p(k_x-2\pi n/l,k_y)} \tag{23}$$

where $v_0 e^{-ik_y y} = v(0,y)$. An adjoint field, with reverse direction of the incident field, would be

$$v_a(x,y) = \frac{\hat{p}}{Z_p(k_x,k_y)} e^{i(k_x x+k_y y)} - \frac{v_0 Z_f(k_y)}{l} e^{i(k_x x+k_y y)} \sum_{n=-\infty}^{\infty} \frac{e^{-2\pi i n x/l}}{Z_p(k_x-2\pi n/l,k_y)} \tag{24}$$

Define a non-dimensional auxiliary function to be

$$T(x) = \frac{Z_f(k_y)}{l}\sum_{n=-\infty}^{\infty} \frac{e^{2\pi i n x/l}}{Z_p(k_x-2\pi n/l,k_y)}, \tag{25}$$

and for the adjoint solution

$$T_a(x) = T^*(x) = \frac{Z_f(k_y)}{l}\sum_{n=-\infty}^{\infty} \frac{e^{-2\pi i n x/l}}{Z_p(k_x-2\pi n/l,k_y)}. \tag{26}$$

The Rumerman solution (23) will be use as a basis for an approximate variational solution of the finite fluid loaded transmission problem. Thus, using two unknown variable $\hat{v}_1$ and $\hat{v}_2$,

$$v(x,y) = \hat{v}_1 e^{-i(k_x x+k_y y)} + \hat{v}_2 T(x) e^{-i(k_x x+k_y y)} \tag{27}$$

$$v_a(x,y) = \hat{v}_1 e^{i(k_x x+k_y y)} + \hat{v}_2 T_a(x) e^{i(k_x x+k_y y)}. \tag{28}$$

Appling the wall operator on equation (27) yields

$$\mathcal{Z}v(x,y) = \hat{v}_1\mathcal{Z}\left\{e^{-i(k_x x + k_y y)}\right\} + \hat{v}_2\mathcal{Z}\left\{T(x)e^{-i(k_x x + k_y y)}\right\}, \tag{29}$$

where the first term is given in eq. (15), and the last term is

$$\mathcal{Z}\left\{T(x)e^{-i(k_x x + k_y y)}\right\} = Z_f(k_y)e^{-i(k_x x + k_y y)}\left(\frac{1}{l}\sum_{n=-\infty}^{\infty} e^{\frac{2\pi i n x}{l}} + T(x)\sum_{m=-\infty}^{\infty}\delta(x - ml)\right) \tag{30}$$

With the use of the Poission's formula

$$\frac{1}{l}\sum_{n=-\infty}^{\infty} e^{\frac{2\pi i n x}{l}} = \sum_{m=-\infty}^{\infty}\delta(x - ml), \tag{31}$$

can (30) be written

$$\mathcal{Z}\left\{T(x)e^{-i(k_x x + k_y y)}\right\} = Z_f(k_y)e^{-i(k_x x + k_y y)}\big(1 + T(x)\big)\sum_{m=-\infty}^{\infty}\delta(x - ml) \tag{32}$$

Inserted in the integral in eq. (8) containing the wall impedance operator, we have

$$\int_{S_w} v_a(S)\mathcal{Z}v(S)\,dS = \hat{v}_1^2\left(Z_p S + Z_f n_f\right) + \hat{v}_2\hat{v}_1\left(Z_p\int_{S_w} T_a(x)dS + Z_f n_f + Z_f\sum_{m\in S} T_a(ml) + Z_f\sum_{m\in S} T(ml)\right) + \hat{v}_2^2 Z_f\left(\sum_{m\in S} T_a(ml) + \sum_{m\in S} T_a(ml)T(ml)\right) \quad . \tag{33}$$

The periodic properties of $T(x)$ can now be made use of

$$T(ml) = \frac{Z_f(k_y)}{l}\sum_{n=-\infty}^{\infty}\frac{e^{2\pi i n m}}{Z_p(k_x - 2\pi n/l, k_y)} = T(0), \tag{34}$$

and therefore

$$\sum_{m=\in S} T(ml) = \sum_{m=\in S} T(0) = T(0)n_f, \tag{35}$$

and the same apply for $T_a(x)$. Moreover,

$$\sum_{m=\in S} T_a(ml)T(ml) = \sum_{m=\in S} T_a(0)T(0) = T_a(0)T(0)n_f. \tag{36}$$

For the radiation term in (8), we have

$$2i\omega\rho\int_{S_w}\int_{S_w} v_a(S)v(S')\,G(S,0|S',0)dS^{'}\,dS = 2i\omega\rho\hat{v}_1^2\int_{S_w}\int_{S_w} e^{i\left(k_x(x-x\prime)+k_y(y-y\prime)\right)}\,G(S,0|S',0)dS^{'}\,dS + 2i\omega\rho\hat{v}_1\hat{v}_2\int_{S_w}\int_{S_w}\big(T(x') + T_a(x)\big)e^{i\left(k_x(x-x\prime)+k_y(y-y\prime)\right)}\,G(S,0|S',0)dS^{'}\,dS + 2i\omega\rho\hat{v}_2^2\int_{S_w}\int_{S_w} T_a(x)T(x')e^{i\left(k_x(x-x\prime)+k_y(y-y\prime)\right)}\,G(S,0|S',0)dS^{'}\,dS \tag{37}$$

The first integral is the same as found in the forced case, eq. (18), related to the forced radiation impedance z_a, equation (22). In a similar manner as for forced radiation impedance,

$$z_a = -\frac{ik}{S}\int_{S_w}\int_{S_w} e^{i\left(k_x(x-x')+k_y(y-y')\right)} G(S,0|S',0)dSdS'$$

$$z_c = -\frac{ik}{S}\int_{S_w}\int_{S_w} \left(T(x)+T_a(x)\right)e^{i\left(k_x(x-x')+k_y(y-y')\right)}\, G(S,0|S',0)\, dS'dS$$
$$z_b = -\frac{ik}{S}\int_{S_w}\int_{S_w} T_a(x)T(x)e^{i\left(k_x(x-x\prime)+k_y(y-y\prime)\right)}\, G(S,0|S',0)\, dS'dS \tag{38}$$

So, using (38) in (37), we have

$$2i\omega\rho \int_{S_w}\int_{S_w} v_a(S)v(S')\, G(S,0|S',0)dS'dS = -2\rho c\hat{v}_1^2 S z_a - 2\rho c\hat{v}_1\hat{v}_2 S z_c - 2\rho c\hat{v}_2^2 S z_b. \tag{39}$$

Moreover, define the following integrals

$$I = \tfrac{1}{S}\int_{S_w} T(x)dS = \tfrac{Z_f(k_y)}{lS}\sum_{n=-\infty}^{\infty} \frac{\int_{S_w} e^{2\pi inx/l}dS}{Z_p(k_x-2\pi n/l, k_y)},$$
$$I_a = \tfrac{1}{S}\int_{S_w} T_a(x)dS = \tfrac{Z_f(k_y)}{lS}\sum_{n=-\infty}^{\infty} \frac{\int_{S_w} e^{-2\pi inx/l}dS}{Z_p(k_x-2\pi n/l, k_y)} \tag{40}$$

Finally, inserting eq. (23-24) in the variational formulation (8), making use of the results above, we have

$$\begin{aligned} V = {} & 2\hat{p}_i(\hat{v}_1 S + \hat{v}_2 IS) + 2\hat{p}_i(\hat{v}_1 S + \hat{v}_2 I_a S) - \hat{v}_1^2\left(Z_p S + Z_f n_f\right) \\ & - \hat{v}_2\hat{v}_1\left(Z_p I_a S + Z_f n_f + Z_f T_a(0) n_f + Z_f T(0) n_f\right) \\ & - \hat{v}_2^2 Z_f\left(T_a(0) + T_a(0)T(0)n_f\right) - 2\rho c\hat{v}_1^2 S z_a - 2\rho c\hat{v}_1\hat{v}_2 S z_c - 2\rho c\hat{v}_2^2 S z_b \end{aligned} \tag{41}$$

Differentiate equation (41) with respect to $\hat{v}_1$ and $\hat{v}_2$, and equal to zero to find the optimum, we finally gets

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}\begin{bmatrix} \hat{v}_1 \\ \hat{v}_2 \end{bmatrix} = \begin{bmatrix} 2\hat{p}_i \\ \hat{p}_i(I_a+I) \end{bmatrix}, \quad \begin{bmatrix} \hat{v}_1 \\ \hat{v}_2 \end{bmatrix} = \tfrac{1}{D}\begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}\begin{bmatrix} 2\hat{p}_i \\ \hat{p}_i(I_a+I) \end{bmatrix} \tag{42}$$

where

$$Z_{11} = Z_p + \frac{Z_f n_f}{S} + \rho c 2 z_a, \qquad Z_{22} = \frac{Z_f n_f}{S} T_a(0)\left(1+T(0)\right) + 2\rho c z_b$$
$$Z_{12} = Z_{21} = \frac{Z_p I_a}{2} + \frac{Z_f n_f}{2S}\left(1 + T_a(0) + T(0)\right) + \rho c z_c$$

and the determinate $D = Z_{11}Z_{22} - Z_{12}^2$.

2.5 Transmission loss

The transmission coefficient is defined as $\tau = \Pi_t / \Pi_i$,which with the present notations becomes

$$\tau = -\frac{\omega \rho^2 c}{S \cos\theta |\hat{p}_i|^2} \Re\left\{ i \int_{S_w} \int_{S_w} v(S') v^*(S) G(S, z|S', 0) dS' \, dS \right\} \tag{43}$$

For a diffuse field result, an integration over al angels of propagation have to be preformed.

For the forced case, section 2.3, the result reads

$$\tau = \frac{\omega \rho^2 c^2 \Re\{z_a\}}{\cos\theta \left| Z_p + n_f Z_f / S + 2\rho c \, z_a \right|^2}. \tag{44}$$

For the solution including periodicity, section 2.4, the result if found by inserting (27) and (42) in (43).

4 DISCUSSION AND CONCLUDING REMARKS

Unfortunate no numerical results are yet ready. However, one can clearly see from the results in sections 2.3 to 2.4 that the present approach can provide a theoretically solid base for the simplified formulas used in building acoustics.

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