TWO-DIMENSIONAL RANDOM ARRAYS FOR REAL TIME VOLUMETRIC IMAGING

Richard E. Davidsen¹, Jørgen A. Jensen² and Stephen W. Smith^{1,3}

¹Department of Biomedical Engineering
Duke University
Durham, NC 27708

²Electronics Institute, Build. 349
Technical University of Denmark
2800 Lyngby, Denmark

³Department of Radiology
Duke University Medical Center
Durham, NC 27706

Two-dimensional arrays are necessary for a variety of ultrasonic imaging techniques, including elevation focusing, 2-D phase aberration correction, and real time volumetric imaging. In order to reduce system cost and complexity, sparse 2-D arrays have been considered with element geometries selected *ad hoc*, by algorithm, or by random process. Two random sparse array geometries and a sparse array with a Mills cross receive pattern were simulated and compared to a fully sampled aperture with the same overall dimensions. The sparse arrays were designed to the constraints of the Duke University real time volumetric imaging system, which employs a wide transmit beam and receive mode parallel processing to increase image frame rate. Depth-of-field comparisons were made from simulated on-axis and off-axis beamplots at ranges from 30 to 160 mm for both coaxial and offset transmit and receive beams. A random array with Gaussian distribution of transmitters and uniform distribution of receivers was found to have better resolution and depth-of-field than both a Mills cross array and a random array with uniform distribution of both transmit and receive elements. The Gaussian random array was constructed and experimental system response measurements were made at several ranges. Comparisons of B-scan images of a tissue mimicking phantom show improvement in resolution and depth-of-field consistent with simulation results. © 1994 Academic Press, Inc.

Key words: Phased array; random array; sparse array; transducer; two-dimensional; volumetric imaging.

I. INTRODUCTION

Two dimensional array transducers are currently being developed because they offer imaging capabilities which are not possible with conventional linear (1-D) phased arrays, including dynamic focusing in elevation and correction of phase aberrations in two dimensions [1, 2]. Two-dimensional arrays may also be used in conjunction with receive mode parallel processing for real time volumetric imaging [3]. Volumetric data may be

0161-7346/94 \$6.00

Copyright © 1994 by Academic Press, Inc.

All rights of reproduction in any form reserved.

processed and displayed in novel ways, providing more information to the clinician. B-scans oriented in orthogonal planes [4] or several C-scans of multiple ranges [5] may be acquired and displayed simultaneously in real time. Other possible display formats include projection display with perspective cues, 3-D display through stereoscopic pairs [6], and 3-D angle-independent flow imaging [7].

A conventional 1-D array has up to 128 elements with an interelement spacing not exceeding $\lambda/2$ in order to eliminate grating lobes. A square 2-D array with the same interelement spacing and overall dimensions would have $(128)^2 = 16,384$ elements. The complexity of such an array and its supporting electronics makes it impractical. Therefore, it becomes necessary to select a sparse array composed of a subset of the total available elements. Since lateral resolution of an array depends on the overall dimensions of the transducer aperture, the elements of the sparse array should be located throughout the aperture. It is the goal of the designer to improve array performance (i.e. decrease mainlobe width and reduce sidelobes) through the choice of element distribution. Elements of a sparse array may be selected *ad hoc*, by algorithm, or by random process. Since 1986, several *ad hoc* sparse array designs have been used for real time volumetric imaging with parallel processing at Duke University [8]. The arrays have been based on a Mills cross receive pattern with a densely packed central transmit cluster. The Mills cross receive pattern was selected because the parallel processing system hardware was hard wired to accept only *linear arrays* of receive elements. However, this receive constraint has recently been eliminated.

Various 2-D array geometries have been investigated by ultrasound researchers. A synthetic aperture system that employs a sparse annular array was described by Norton [9]. The use of annular arrays to produce nondiffracting beams was described by Lu *et al* [10]. Ebbini has developed an algorithm for adaptive beam synthesis for the compensation of missing array elements that may be applied to the design of 2-D arrays [11].

Recently, Turnbull *et al.* have investigated the use of sparse *random* arrays in ultrasonic imaging [12]. Random array theory was developed for phased array radar systems and is well known [13]. A random array with uniform element density has main beam and first sidelobe characteristics similar to a fully sampled array with the same overall dimensions. Although the interelement spacing of the random array is greater than $\lambda/2$, grating lobes are reduced if there is no periodic pattern of elements. Random arrays have a characteristic sidelobe "pedestal" beyond the first few sidelobes. The average pedestal amplitude is dependent on the number of elements in the array, but is independent of their arrangement. Pedestal sidelobes are also sensitive to the element directivity, pulse bandwidth, and transducer focusing. The sidelobe pedestal places an ultimate limit on contrast resolution. Turnbull has simulated uniformly distributed sparse random arrays with active element densities on the order of 1/8 and 1/32 [14]. The 1/8 order array had 993 active elements out of 7,845 with a sidelobe pedestal approximately 53 to 63 dB below the main beam. The 1/32 order array had 245 active elements in the same aperture with a corresponding sidelobe pedestal approximately 43 to 53 dB below the main beam.

A typical B-scan image is formed by scanning a linear array through a plane perpendicular to the transducer face. Each scan may contain over 100 lines of rf data acquired at 30 frames per second. In the Duke University high speed volumetric imaging system, the ultrasound beam is scanned over a 3-D pyramid, with the apex located at the transducer face. This scan is currently composed of $64^2 = 4,096$ lines in a single volume [5]. To obtain this large volume of data at real time rates, parallel receive processing is employed [15]. As illustrated in figure 1, a widened transmit beam is used to insonify the tissue surrounding the transmit beam center. Echoes are received along 16 parallel, independently steered data paths in the form of a 4 x 4 matrix. Receive lines are processed by parallel hardware and later combined for volumetric display. This increases data acquisition rate by a factor of 16. Thus, a volumetric scan consists of 256 transmitted beams with 4,096 receive lines obtained at 22 frames/second with a maximum range of 13 cm.

The goal of the work described in this paper was to design a 2-D random array for use with the Duke phased array volumetric imaging system that would have improved pulse-echo

Fig. 1 Volumetric scan format. Note wide transmit beam and narrow receive beams. Arrows indicate receive locations (1) $(\theta_r, \phi_r) = (0.0^\circ, 0.0^\circ)$, (2) $(\theta_r, \phi_r) = (1.5^\circ, 1.5^\circ)$, (3) $(\theta_r, \phi_r) = (32.5^\circ, 32.5^\circ)$, (4) $(\theta_r, \phi_r) = (34.0^\circ, 34.0^\circ)$. For (1) and (2), the transmit beam was steered on-axis at $(\theta_t, \phi_t) = (0.0^\circ, 0.0^\circ)$ and for (3) and (4), the transmit beam was steered at $(\theta_t, \phi_t) = (32.5^\circ, 32.5^\circ)$.

beamwidth and reduced sidelobes compared to previously used Mills cross arrays. Other design goals included maintaining good transducer depth-of-field and a transmit beamwidth sufficiently wide for the receive parallel processing system. In this paper, we discuss random array design by computer element selection. Our hypothesis is that a random array with Gaussian spatial tapering of the transmit aperture improves lateral resolution and depth-of-field when compared to arrays previously used with our parallel processing system. Broadband simulations of pulse-echo beamplots are compared for four array geometries. The array geometry with the best overall performance was constructed and system response measurements were made. B-mode images of a tissue mimicking phantom were compared to those of a Mills cross array.

II. METHODS

A. Design

The wide transmit beam requirement has placed an important *constraint* on our 2-D transducer arrays. If the transmitted beam is not sufficiently wide, the pulse-echo beamplot exhibits a double-peaked mainlobe and increased sidelobes. Additionally, insufficient transmit beamwidth results in pulse-echo understeering.

The 2-D transducer design was also limited by several other system and application derived constraints. The array was designed with 192 transmit channels and 64 receive channels for the current Duke phased array system. Any element could be designated as either a transmitter or receiver. However, common transmit and receive elements were not permitted since the receiver preamplifiers were susceptible to a dc offset build-up from the transmit pulse. The transducer center frequency was chosen to be 2.5 MHz, corresponding to a thickness of 0.61 mm for Motorola HD3203 piezoceramic material in bar mode. The element width/thickness ratio was limited to less than 0.6 to reduce the interference of lateral vibration modes. This requirement yielded an element size of 0.35 x 0.35 mm with a 0.1 mm saw kerf.

Cardiac imaging views the heart through an acoustic window corresponding to the intercostal spaces between the ribs, and limits the transducer's usable footprint. The aperture considered in this study was circular with a diameter of 19.4 mm (42 elements) and 1,356 total elements. The sparse arrays we have investigated are subsets of the 1,356 element aperture.

We have considered various means to maintain a wide transmit beam for receive parallel processing. The most obvious choice is to fully pack all the transmit elements in the center of the array. This distribution does not permit receive elements near the array center since hardware limitations prohibit elements common to both transmit and receive apertures. Without receive elements in the center of the array, the receive aperture sidelobes would be unacceptably high. Therefore, this distribution was not selected.

Beamwidth may be increased by either not focusing or by defocusing with a negative focal range in the delay calculations. With these techniques, simulations predicted a significant increase in the near field beamwidth with little change in the far field response. Experiments on our phased array scanner confirmed these predictions. Since good depth-of-field is important for maintaining image quality and is necessary for the proper steering of receive lines, these techniques were not used.

Amplitude apodization is often employed to reduce sidelobe amplitude of linear arrays. In addition, amplitude apodization increases depth-of-field by eliminating the characteristic near field axial minima and maxima [16]. The cost of sidelobe reduction is an increase in mainlobe beamwidth and reduction in array sensitivity since outer elements are attenuated. Since two-dimensional transducer arrays already suffer from poor sensitivity due to high electrical impedance [17], amplitude apodization was not used.

Spatial tapering refers to the variation of element density in a sparse array. This technique may be used to widen the transmit beam while maintaining maximum excitation to all elements so there is no loss in sensitivity. Two random element distributions were considered for the transmit aperture: a uniform random distribution and a Gaussian weighted random distribution. The Gaussian spatial taper increased the transmit beamwidth in a manner similar to amplitude apodization. The peak sidelobe amplitude was slightly reduced by spatial tapering, although the reduction is not as significant as sidelobe reduction using amplitude apodization of fully sampled arrays. Spatially tapered 2-D random arrays with large numbers of elements (2,864) and significantly reduced peak sidelobe height (-30.5 dB) have been reported [18].

Figure 2 shows four 2-D transducer geometries which we have considered. Figure 2A shows an array in which all 1,356 elements within a radius of 21 elements are active in transmit and receive. Although this array does not meet all of our system constraints and its construction is not presently practical, its simulated beamplot is useful as a "gold standard" for comparison to more practical sparse arrays.

Several *ad hoc* transducer arrays have been designed and built for the Duke phased array system [17]. A recent design is shown in figure 2B. This array has 192 transmit elements, illustrated with gray shading, and 64 receive elements shown in black. Transmit elements are concentrated in the center to increase transmit beamwidth. The receive pattern is a combination of two Mills crosses, with one rotated by 45° with respect to the other. The

crosses were designed to maximize the effective receive aperture size, thereby reducing the receive mainlobe width. This array will subsequently be referred to as the Mills cross array.

A random sparse array with *uniformly* distributed transmit and receive elements is shown in figure 2C. This "uniform-uniform" random array has an element sampling density of $1/7$ on transmit and $1/21$ on receive. Element locations for both transmit and receive apertures were selected using a computer pseudorandom number generator. The aperture was divided into 7 concentric annuli, with a portion of the total elements assigned to each annulus. The array was further divided into four equal quadrants, with $1/4$ of the assigned elements placed in each quadrant to avoid heavily loading any one quadrant. Only random element locations meeting all constraints were saved. For the uniformly distributed aperture,

the number of elements placed in each annuli (N_A) was proportional to the area of the annulus ($Area_A$):

$$N_A = N_T \frac{Area_A}{Area_T}. \quad (1)$$

A random sparse array with a *Gaussian* weighted distribution of *transmit* elements and a *uniform* distribution of *receive* elements is shown in figure 2D. This array will subsequently be referred to as the Gaussian-uniform random array. The transmit element density in each annulus was calculated from the area under the Gaussian curve by

$$N_A = N_T \frac{\int_0^{r_o} \exp\left[\frac{-x^2}{2\sigma^2}\right] dx - \int_0^{r_i} \exp\left[\frac{-x^2}{2\sigma^2}\right] dx}{\int_0^{2\sigma} \exp\left[\frac{-x^2}{2\sigma^2}\right] dx}, \quad (2)$$

where x is the distance measured from the center of the array, r_i and r_o are the inner and outer radii of the annulus, respectively, and σ is half the total radius of the array. The Gaussian distribution was truncated at a radius of $2\sigma = 21$ elements. Receive elements were distributed uniformly to maximize receive lateral resolution.

Turnbull has presented a cw derivation of the average pedestal amplitude of a random array [19]. If the array elements are modeled as point sources, the mainlobe pressure amplitude (p_m) at the focus or in the far field of a random array with proper phase alignment of each element is

$$p_m = N \frac{A}{r} e^{j(kr - \omega t)}, \quad (3)$$

where N is the number of elements, A is the excitation amplitude, r is the distance from the array to the field point, and $k = 2\pi/\lambda$ is the wave number. In the sidelobe region, element phases are distributed randomly since the phase of each element is determined by its location. Thus, far field sidelobes of an array with randomly distributed elements and cw excitation have a Rayleigh pdf and an average amplitude of [19]

$$p_s = \frac{\sqrt{(\pi N)}}{2} \frac{A}{r} e^{j(kr - \omega t)}. \quad (4)$$

The relative amplitude of the sidelobe pedestal with respect to the mainlobe amplitude is therefore [19]

$$\frac{p_s}{p_m} = \frac{1}{2} \sqrt{\frac{\pi}{N}} \approx \frac{1}{\sqrt{N}}. \quad (5)$$

However, in the near field away from the transducer focus, the average sidelobe amplitude increases significantly. The average cw pulse-echo pedestal level in the far field of a random array with $N_t = 192$ transmit elements and $N_r = 64$ receive elements may therefore be approximated by

$$20 \text{Log} \frac{1}{\sqrt{N_t N_r}} \approx -41 \text{dB}. \quad (6)$$

Narrowband (cw) beamplot simulations of our random arrays have yielded an average sidelobe pedestal level consistent with Eq. (6). However, a more realistic measure of transducer sidelobes may be obtained using a broadband simulation method.

B. Broadband simulation

The 3-D beamplot of each transducer geometry was calculated using a broadband simulation of the transducer's radiation pattern [20]. The simulation is based on the method of Stepanishen [21] and Tupholme [22] for calculating the pulsed pressure field of a planar piston in an infinite, rigid, planar baffle. The received pulse-echo pressure waveform for a field point at $\vec{r}$ is calculated by [23]

$$p(\vec{r}, t) = \frac{1}{c_0^2} \frac{\partial^2 v_{pe}(t)}{\partial t^2} * h_a^T(\vec{r}, t) * h_a^R(\vec{r}, t) \quad , \quad (7)$$

where c_0 is the propagation velocity, $v_{pe}(t)$ is the pulse-echo electro-mechanical impulse response of the transducer material, h_a^T is the spatial impulse response of the transmit aperture, h_a^R is the spatial impulse response of the receive aperture, and $*$ denotes convolution in time. The pulse excitation waveform described above is included in $v_{pe}(t)$. From Eq. 7, we can see that the calculation of the transducer's pulse-echo response requires only the calculation of the spatial impulse responses of the transmit and receive apertures with a subsequent convolution process. It is possible to rapidly compute the spatial impulse response of the aperture by dividing it into small rectangles and summing the response of each rectangle

$$h_a(\vec{r}, t) \approx \sum_{i=1}^N a(\vec{r}_i) h(\vec{r}_i, t) \quad , \quad (8)$$

where $a(\vec{r}_i)$ represents the spatial source velocity distribution on the aperture, $\vec{r}_i$ denotes the location of each rectangle on the aperture, and $h(\vec{r}_i, t)$ is the farfield response from a particular rectangle. The contribution to the spatial impulse response from each rectangle relies on the farfield approximation

$$|\vec{r}| \gg \frac{d^2}{4c_0 / f_0} \quad (9)$$

where d is the greatest dimension of the rectangle and f_0 is the center frequency of the broadband pulse. This limits the size of the rectangles and therefore determines the number of rectangles necessary to represent the aperture. Phased array steering and focusing may be simulated by adding an appropriate time delay to the response of each element.

A Gaussian pulse with 2.5 MHz center frequency and a 40 percent fractional bandwidth was used in all simulations. The element size was 0.35 x 0.35 mm with a 0.1 mm kerf. The theoretical far field diffraction pattern of a single array element mounted in a soft baffle [24] is calculated by

$$p(\theta) = p_0 \frac{\sin\left(\frac{\pi a \sin \theta}{\lambda}\right)}{\frac{\pi a \sin \theta}{\lambda}} \cos \theta \quad (10)$$

where p is the acoustic pressure, a is the element width, λ is the acoustic wavelength, and θ is the angle between the field point and the center axis perpendicular to the element face. Using Eq. (10), the transmit mode -6 dB beamwidth of an individual array element was *calculated* to be 92.6°. The -6 dB beamwidth of a single element in an array was *measured* in a water

tank to be 52° , substantially more narrow than predictions. A possible explanation for the discrepancy between theoretical and experimental angular response is interelement crosstalk, which may effectively increase element size and therefore decrease beamwidth. The experimental angular response characteristic was implemented in our simulations by applying an angular weighting factor to the spatial impulse response of each rectangle.

Because of the low element density of our arrays, we found significant variations in peak sidelobe height and location for different random apertures with the same distribution constraints. Therefore, 400 independent uniform-uniform and Gaussian-uniform random geometries were generated and the on-axis pulse-echo beamplot of each random array pattern was simulated. Each simulation required approximately 40 minutes on a DECstation model 3100 workstation. The maximum pulse-echo beamwidths at -3, -6, -12, -18, -24, -30, and -36 dB were measured. For both random element distributions, the array pattern having the narrowest overall pulse-echo beamwidth was selected for further study. These patterns are shown in figures 2C and 2D.

Beamplot simulations were made by first selecting a transmit and receive focal point and calculating delay times for each element. Then the pulse-echo response at each field point was calculated. This is analogous to scanning an infinitely small point reflector in the field of a fixed focus transducer and measuring the received signal for each location of the point reflector. Pulse-echo beamplots were generated with a transmit focus at a range of 70 mm and a receive focus which was set to the measurement range to simulate dynamic receive focusing. Beamplots were simulated at 30, 50, 70, 90, 120, and 160 mm ranges to characterize depth-of-field. At each range, the maximum beamwidths were recorded at -3, -6, -12, -18, -24, -30, and -36 dB below the mainlobe peak.

Figure 3 shows a broadband simulated beamplot of the Gaussian random array of figure 2D focused at 70 mm in transmit and receive. Figure 3A shows a 3-D beamplot; figure 3B shows a contour plot with contour lines at -3, -6, -12, -18, -24, -30, and -36 dB; figure 3C shows a lateral beamplot profile, with a peak sidelobe level of -42 dB. The average pedestal amplitude is lower than CW predictions of Eq. (6) because of transducer bandwidth and element directivity.

As previously described, receive parallel processing employs a 4×4 grid of independently steered receive beams for each transmit beam. The intentional misalignment between transmit and receive beams causes degradation of imaging capability in the form of increased sidelobe amplitude. The receive beams steered farthest away from the transmit beam at the four corners of the receive grid are most degraded.

We simulated and compared beamplots of four different steering configurations as illustrated in figure 1. This figure also shows the coordinate system used throughout this paper. Beam steering direction is described by azimuth angle, θ , and elevation angle, ϕ . Although this is not the standard spherical coordinate system, it is more intuitive and useful for this discussion [8]. The transmit beam steering angles are θ_t and ϕ_t while the receive beam steering angles are θ_r and ϕ_r . In the first set of simulations, the transmit and receive beams were steered coaxially at $(\theta_t, \phi_t) = (\theta_r, \phi_r) = (0.0^\circ, 0.0^\circ)$. In the second set of simulations, the transmit beam was again steered at $(\theta_t, \phi_t) = (0.0^\circ, 0.0^\circ)$ but the receive beam was steered at $(\theta_r, \phi_r) = (1.5^\circ, 1.5^\circ)$ in order to simulate the worst case angle for on-axis transmit and receive mode parallel processing. In the third set, the transmit and receive beams were steered off-axis at $(\theta_t, \phi_t) = (\theta_r, \phi_r) = (32.5^\circ, 32.5^\circ)$. Finally, the off-axis receive mode parallel processing case was simulated by steering the receive beam at $(\theta_r, \phi_r) = (34.0^\circ, 34.0^\circ)$ while the transmit beam remained at $(\theta_t, \phi_t) = (32.5^\circ, 32.5^\circ)$. The steering angles for the four sets of simulations are summarized in table I. Comparison of the four sets of beamwidth data as a function of range indicates the amount of degradation in pulse-echo beamwidths and depth-of-field resulting from receive parallel processing.

C. Measurement

Contrast resolution can be quantified by imaging a tissue mimicking phantom with cysts embedded within the tissue parenchyma. Cyst contrast depends on the characteristics

Fig. 3 Simulated on-axis transducer beamplot of random array with Gaussian transmit distribution and uniform receive distribution: (A) 3-D view, (B) Contour plot with levels of -3, -6, -12, -18, -24, -30, and -36 dB, (C) profile view.

of the transducer beamplot [25]. As an alternative, we have chosen to present most of our results in the form of beamplots and measured beamwidths in order to show more details of the transducer response.

Beamplot measurements are typically made in a water tank by physically scanning a transducer with respect to a point target, as was done in our simulations. A 3-D beamplot

Table I. Simulation steering configurations

Steering set	Transmit Angle (θ_t, ϕ_t)	Receive Angle (θ_r, ϕ_r)	Receive Processing
1	(0.0°, 0.0°)	(0.0°, 0.0°)	Normal
2	(0.0°, 0.0°)	(1.5°, 1.5°)	Parallel
3	(32.5°, 32.5°)	(32.5°, 32.5°)	Normal
4	(32.5°, 32.5°)	(34.0°, 34.0°)	Parallel

made in this manner may require thousands of measurement locations and may take several hours to complete, even using automated data acquisition [8]. If the measurements are made in the near field of the transducer, the beam must be electronically focused by the phased array system. Receiver gain instability of phased array ultrasound scanners may cause this type of lengthy measurement to produce erroneous data. An alternative to this measurement method, the far field measurement, does not require phased array focusing and therefore is not dependent on system gain stability. For this measurement, all the transmit elements are tied to a single pulser while all the receive elements are tied to a single receive circuit. A point target is placed in the far field and is mechanically translated relative to the transducer. A disadvantage to using this method is that it does not provide information about the focusing characteristics of the transducer in the near field.

Previously, we have produced high speed experimental 3-D beamplots by a combination of electronic and mechanical steering of the ultrasound beam about a fixed point target [17]. The transducer was mounted in an automated, one axis rotating stage while configured for electronic B-scans oriented perpendicular to the axis of rotation. However, because the beamwidths measured by mechanical and electronic steering were not identical, this hybrid measurement system was replaced.

In our new high speed beamplot measurement system, the phased array scanner electronically steers the beam in both dimensions about a fixed point reflector. This method characterizes both transducer and imaging system. The advantage of this method over others is that both near field and far field characteristics of the transducer may be measured in a fraction of a second, eliminating the deleterious effects of system gain instability. However, this method is not equivalent to the scanned point target approach used in the simulations due to the effects of steering delay quantization, element angular response, and off-axis aperture shading. In addition, the finite size of our point target increases experimental beamwidth by up to 10 percent. For these reasons, beamwidths of measured and simulated beamplots are similar, but not equal.

The system for producing the electronic beamplots is shown in figure 4. The Duke phased array system focused and steered the ultrasound beam from a 2.5 MHz array. The target was a 1.6 mm diameter rod with a hemispherical top surface. For each transmitted beam, the returning rf signal was digitized at 50 MSamples/sec. by a LeCroy 8828D waveform digitizing system with 2 Mbytes of high speed memory. A LeCroy 4222 programmable delay generator was activated by timing signals from the phased array scanner and subsequently enabled the digitizer during a 10 μ sec window centered on the peak arrival time of the return echo. Data was downloaded via an IEEE-488 GPIB interface to a 486DX computer for processing. The system allowed limited rf averaging to reduce the electronic noise floor. The maximum rf peak-to-peak value for each line was plotted on a 3-dimensional graph. The system acquired a 64 x 64 matrix of points for a total of 4,096 points in the beamplot.

Fig. 4 Experimental 3-D system response measurement system.

III. RESULTS

We compared simulated beamplots of four 2-D array geometries with the receive beam steered coaxially with the transmit beam and also steered away from the transmit beam at the worst case direction for receive parallel processing. We evaluated the beam characteristics over a range from 30 mm to 160 mm from the transducer surface to assess depth-of-field.

Transmit beamplot simulations of the uniform-uniform and Gaussian-uniform arrays were made at a range of 70 mm. The -6 dB transmit beamwidth of the uniform-uniform array was 2.3° while the Gaussian-uniform beamwidth was 4.1° . The peak sidelobe amplitudes of the uniform-uniform and Gaussian-uniform arrays were -16.0 and -19.5 dB respectively.

Figure 5 compares -6, -18, -24, and -30 dB simulated pulse-echo beamwidths versus range for the fully sampled aperture (Fig. 2A), Mills cross array (Fig. 2B), uniform-uniform random array (Fig. 2C), and Gaussian-uniform random array (Fig. 2D). In these simulations, the transmit and receive beams were both steered on-axis at $(\theta_t, \phi_t) = (\theta_r, \phi_r) = (0.0^\circ, 0.0^\circ)$. This is the "best case" steering configuration in which the transmit beam and receive beam are coaxial. Figure 5A shows that the -6 dB beamwidth of the Mills cross array is wider ($\sim 3.8^\circ$) than the other arrays, indicating its inferior lateral resolution. The beamwidth of this array is significantly wider than the other three for most simulation steering configurations. The fully sampled aperture and the uniform-uniform random array have nearly identical beamwidths ($\sim 2.5^\circ$) while the Gaussian-uniform random array pulse-echo beamwidth ($\sim 2.8^\circ$) is slightly wider due to its wider transmit beam.

Figure 5B shows similar trends at -18 dB with the exception that peaks of the sidelobe pedestal of the *uniform-uniform* array is above -18 dB at 30 mm. Since beamwidth was calculated as the maximum angle between points in the beamplot of a particular amplitude, peaks in the sidelobe pedestal cause abrupt increases in the beamwidth versus range plot. An arrow is used on these plots to show that the inclusion of a peak of the sidelobe pedestal in the beamwidth calculation has resulted in a beamwidth measurement that is off the scale of the plot. Figures 5C and 5D also show poor depth-of-field for the uniform-uniform array at lower amplitude levels.

The system configuration for figure 6 is identical to that of figure 5 except that the receive beam was steered to $(\theta_r, \phi_r) = (1.5^\circ, 1.5^\circ)$, corresponding to the "worst case" parallel processing steering configuration for on-axis transmit. Figure 6A shows that the sidelobe pedestal of the uniform-uniform array is above -6 dB at 30 mm. Figures 6B to 6D show poor depth-of-field and high sidelobes of the uniform-uniform array when compared to the others, which have good depth-of-field except at 30 mm.

Fig. 5 Simulated beamwidth comparisons for on-axis coaxial transmit and receive beams $(\theta_t, \phi_t) = (\theta_r, \phi_r) = (0.0^\circ, 0.0^\circ)$.

The *off-axis* imaging capabilities of the 2-D arrays is limited by the increase in the amplitude of the sidelobe pedestal when the beam is steered at large angles. Two factors that modify the average amplitude of the sidelobe pedestal are pulse bandwidth and element directivity. At large angles, interference between distant array elements is decreased as pulse bandwidth increases since they do not overlap temporally [26], reducing the average amplitude of the sidelobe pedestal.

Since the element directivity reduces transducer sensitivity at angles away from the axis perpendicular to the array, it also reduces the average amplitude of the sidelobe pedestal at angles most distant from the main beam when the beam is steered *on-axis*. However, when the beam is steered *off-axis*, the relative sensitivity of the main beam is reduced by the lower off-axis element sensitivity while the on-axis sidelobes are now located where the element sensitivity is maximum. This increases the relative amplitude of the sidelobes and therefore reduces the ability of the transducer to detect low contrast targets which are located off-axis. A typical simulated off-axis beamplot for the Gaussian-uniform random array (Fig. 2D) steered at $(\theta_t, \phi_t) = (\theta_r, \phi_r) = (32.5^\circ, 32.5^\circ)$ and measured at a range of 70 mm is shown in figure 7. Note that the sidelobe pedestal has risen to approximately -20 dB compared to a peak of -42 dB for the case of on-axis steering. Also, the mainlobe is elongated in the steering direction due to a reduction of the effective aperture size in this dimension.

Fig. 6 Simulated beamwidth comparisons for on-axis transmit beam and parallel processing mode receive beam $(\theta_t, \phi_t) = (0.0^\circ, 0.0^\circ)$, $(\theta_r, \phi_r) = (1.5^\circ, 1.5^\circ)$.

In order to assess off-axis imaging capabilities of the four arrays, two sets of simulations were done with a transmit angle of $(\theta_t, \phi_t) = (32.5^\circ, 32.5^\circ)$. Figure 8 shows the simulation results when the transmit beam and receive beam were steered coaxially at $(\theta_t, \phi_t) = (\theta_r, \phi_r) = (32.5^\circ, 32.5^\circ)$. Because the sidelobe pedestal is relatively high due to element directivity, the beamwidths compared in figures 8 and 9 are for levels of -3, -6, -12, and -18 dB. Simulations using the theoretical element directivity rather than the measured value revealed similar trends at lower sidelobe levels. Figure 8A shows results similar to the on-axis coaxial configuration. At a range of 30 mm, the uniform-uniform array has on-axis sidelobes above -3 dB as indicated by the precipitous jump in its beamwidth plot. In figures 8B and C, we see that the arrays are well behaved down to -12 dB with the exception of the uniform-uniform array, which has sidelobes above -12 dB at all ranges and is therefore off the scale of the plot. Figure 8D shows that only the fully sampled aperture and the Gaussian-uniform array have all pedestal sidelobes below -18 dB.

In the final set of simulations, the receive beam was steered at $(\theta_r, \phi_r) = (34.0^\circ, 34.0^\circ)$. This steering angle is 1.5° in azimuth and elevation beyond the transmit beam and represents

Fig. 7 Simulated off-axis transducer beamplot of random array with Gaussian transmit distribution and uniform receive distribution: (A) 3-D view, (B) Contour plot with levels of -3, -6, -12, -18, -24, -30, and -36 dB, (C) profile view.

the worst case off-axis parallel processing receive angle. Figures 9A and B show good behavior of all the arrays at -3 and -6 dB with the exception of high on-axis sidelobes for the uniform-uniform array at a range of 30 mm. From figure 9C, we see that the Mills cross array and the uniform-uniform array have pedestal sidelobes above -12 dB. Figure 9D indicates that only the fully sampled array has all pedestal sidelobes below 18 dB.

Fig. 8 Simulated beamwidth comparisons for off-axis coaxial transmit and receive beams ($\theta_t, \phi_t = \theta_r, \phi_r = (32.5^\circ, 32.5^\circ)$).

Simulated beamwidth data of figures 5, 6, 8, and 9 indicate that of the four arrays considered, the fully sampled aperture has the best depth-of-field at levels down to -30 dB for both the coaxial transmit and receive case and the parallel processing receive case. Of the arrays that meet our system constraints, the Gaussian-uniform random array has the best depth-of-field, especially for the receive parallel processing case, while maintaining a main beamwidth only slightly larger than the fully sampled aperture and the uniform-uniform random array. Although the Mills cross array has a good depth-of-field, its mainlobe beamwidth is 50 to 100 percent wider than the other three arrays.

The Gaussian-uniform random array of figure 2D was constructed using previously described techniques [17]. A *measured* system response beamplot is shown in figure 10. The poor sensitivity of the small, high impedance elements in the 2-D array results in a poor signal to noise ratio with an electronic noise floor at -26 dB. The rf signals from twenty volumetric scans were averaged to reduce the electronic noise floor. Further averaging was unsuccessful in reducing the noise floor due to the presence of coherent electronic noise from the system's digital clock signals. System response measurements of this array were made at

Fig. 9 Simulated beamwidth comparisons for off-axis transmit beam and parallel processing mode receive beam ($\theta_t, \phi_t = (32.5^\circ, 32.5^\circ)$, $(\theta_r, \phi_r) = (34.0^\circ, 34.0^\circ)$).

50, 70, 90, and 120 mm and are compared to simulation results in figure 11. In figures 11A and 11B we compare simulated beamplots and experimental system responses by plotting the transducer beamwidth at amplitudes of -6 and -18 dB relative to the mainlobe peak versus range for coaxial transmit and receive beams: $(\theta_t, \phi_t) = (\theta_r, \phi_r) = (0.0^\circ, 0.0^\circ)$. Figure 11C and 11D show a similar comparison for the worst case parallel processing receive angle: $(\theta_t, \phi_t) = (0.0^\circ, 0.0^\circ)$, $(\theta_r, \phi_r) = (1.5^\circ, 1.5^\circ)$. In both cases, the experimental system response is consistently wider than the simulated beamplot. However, as explained in section II, the system response is made by phased array scanning and the simulation mimics mechanical scanning. The experimental system response has good depth-of-field for both coaxial and parallel processing receive angles.

In figure 12, B-mode scans of an RMI tissue mimicking phantom (SN 415-1407, $\alpha = 0.5$ dB/cm MHz) are compared for two arrays. The phantom contains a column of wire targets spaced a 1 cm intervals that are denoted by the letter "W." The phantom also contains a row of three hypoechoic lesions (diameters: 6.5 mm, 4.0 mm, 2.5 mm) at a range of 62.5 mm that are denoted by the letter "C." The image of figure 12A is made with the Mills

Fig. 10 Experimental system response plot of random array with Gaussian transmit distribution and uniform receive distribution: (A) 3-D view, (B) Contour plot with levels of -3, -6, -12, -18, -24, (C) profile view.

cross geometry of figure 1B. Figure 12B shows the same scan made with the Gaussian-uniform random array of figure 1D. Both scans were made over an angle of 65° with a maximum range of 13 cm. The rf signals were logarithmically compressed with a ratio of 30 dB / 20 dB. From these images we can see that the random array has better lateral resolution of the wire targets throughout the range of the scan. The ability to detect the row of cysts is also improved with the random array. This result is consistent with beamplot simulation results.

IV. DISCUSSION

We have described the design and evaluation of random sparse array geometries for use with our real time volumetric imaging system with receive mode parallel processing. This system employs a wide transmit beam with multiple receive beams. Random arrays with and without Gaussian spatial tapering of the transmit element distribution were

Fig. 11 Experimental and simulated beamwidth versus range for random array with Gaussian transmit distribution and uniform receive distribution.

compared with a fully sampled aperture of 1,356 elements and a previously used Mills cross design. The transducer arrays were evaluated by comparison of simulated beamplots with data plotted as beamwidth versus range. Our simulation results show that the random sparse arrays have similar pulse-echo mainlobe dimensions as the fully sampled aperture with the same overall dimensions, while the Mills cross array has a wider pulse-echo mainlobe and therefore poorer lateral resolution. The comparison of lateral resolution for the Gaussian-uniform random array and the Mills cross array was confirmed by B-scan images. The Gaussian-uniform random array had a better depth-of-field than the uniform-uniform random array for both coaxial and parallel processing mode receive angles. In particular, the uniform-uniform random array had a significantly elevated pulse-echo sidelobe amplitude at a range of 30 mm.

Element selection by a computer pseudorandom number generator resulted in sparse arrays with significant variability in sidelobe amplitude and location but with consistent average sidelobe pedestal amplitude. Beamplot comparisons of multiple arrays were necessary to select an array with good overall beam characteristics. Each random array was generated independently of the previous array. Future work in sparse array design may

Fig. 12 B-mode RMI phantom images for (A) Mills cross array and (B) random array with Gaussian transmit distribution and uniform receive distribution.

involve iterative or evolutionary algorithms in which the geometry is iteratively adjusted depending on the simulation results of the previous trial [27].

The off-axis simulations highlight the deleterious effects of interelement cross talk on element directivity. The decrease in the magnitude of off-axis energy emitted and received by the individual elements causes a drastic decrease in the transducer sensitivity and an increase in the relative height of on-axis sidelobes when the beam is steered off-axis. We are investigating new fabrication techniques for our 2-D array transducers which include shielded coax wiring and multilayer ceramic connectors (MLC's) [28] constructed with shielding between layers to reduce electrical cross talk and improve element angular response.

ACKNOWLEDGMENTS

This work was supported in part by HHS Grants CA-43334, CA-37586, CA-56475, and by NSF-ERC CDR-8622201. We would like to acknowledge the assistance of Ned Light and Henry Pavy in the fabrication and testing of these arrays.

REFERENCES

- [1] Flax, S. W. and O'Donnell, M., Phase aberration correction using signals from point reflectors and diffuse scatters: basic principles, *IEEE Trans. Ultrason. Ferroelec. Freq. Contr.* 35, 758-767 (1988).
- [2] Trahey, G. E., Zhao, D., Miglin, J. A. and Smith, S. W., Experimental results with a real-time adaptive ultrasonic imaging system for viewing through distorting media, *IEEE Trans. Ultrason. Ferroelec. Freq. Contr.* 37, 418-427 (1990).
- [3] von Ramm, O. T. and Smith, S. W., Real Time Volumetric Ultrasound Imaging System, in *SPIE Medical Imaging IV: Medical Imaging*, pp. 15-22 (Intn'l Soc. Opt. Eng., Bellingham, WA, 1990).

- [4] Snyder, J. E., Kisslo, J. A. and von Ramm, O. T., Real time orthogonal mode scanning of the heart: A new ultrasonic imaging modality, *J. Am. College Card.* 7, 1279-1285 (1986).
- [5] Pavy, H. G., Smith, S. W. and von Ramm, O. T., An Improved Real Time Volumetric Ultrasonic Imaging System, in *SPIE Medical Imaging V: Image Physics* pp. 54-61, (Intn'l Soc. Opt. Eng., San Jose CA, 1991).
- [6] von Ramm, O. T., Smith, S. W. and Pavy, H. G., High-speed ultrasound volumetric imaging system - Part II: Parallel processing and image display, *IEEE Trans. Ultrason. Ferroelec. Freq. Contr.* 38, 109-115 (1991).
- [7] Bohs, L. N. and Trahey, G. E., A novel method for angle independent ultrasonic imaging of blood flow and tissue motion, *IEEE Trans. Biomed. Engin.* 38, 280-286 (1991).
- [8] Smith, S. W., Pavy, H. J. and von Ramm, O. T., High-speed ultrasound volumetric imaging system - Part I: Transducer design and beam steering, *IEEE Trans. Ultrason. Ferroelec. Freq. Contr.* 38, 100-108 (1991).
- [9] Norton, S. J., Annular array imaging with full-aperture resolution, *J. Acoust. Soc. Am.* 92, 3202-3206 (1992).
- [10] Lu, J.-Y. and Greenleaf, J. F., Steering of Limited Diffraction Beams with a Two Dimensional Array Transducer, in *1992 IEEE Ultrasonics Symposium Proceedings*, pp. 603-607 (IEEE Cat. No. 91CH3079-1, Tuscon, AZ, 1992).
- [11] Ebbini, E. S. and O'Donnell, M., Compensation for Missing Array Elements by Direct Beam Synthesis, in *1991 IEEE Ultrasonics Symposium Proceedings*, pp. 645-649 (IEEE Cat. No. 91CH3079-1, Orlando, FL, 1991).
- [12] Turnbull, D. H., Kerr, A. T. and Foster, F. S., Simulation of B-scan Images from Two-dimensional Transducer Arrays, in *1990 IEEE Ultrasonics Symposium Proceedings*, pp. 769-774 (IEEE Cat. #90CH2938-4, Honolulu, 1990).
- [13] Steinberg, B. D., *Principles of Aperture and Array System Design*, (John Wiley & Sons, New York, 1976).
- [14] Turnbull, D. H. and Foster, F. S., Simulation of B-scan images from two-dimensional transducer arrays: Part I - Methods and quantitative contrast measurements, *Ultrasonic Imaging* 14, 323-343 (1992).
- [15] Shattuck, D. P., Weinshenker, M. D., Smith, S. W. and von Ramm, O. T., Explososcan: a parallel processing technique for high speed ultrasound imaging with linear phased arrays, *J. Acoust. Soc. Am.* 75, 1273-1282 (1984).
- [16] Filipczynski, L. and Etienne, J., Theoretical study and experiments on spherical focusing transducers with Gaussian surface velocity distribution, *Acustica* 28, 121-128 (1973).
- [17] Smith, S. W., Trahey, G. E. and von Ramm, O. T., Two-dimensional array transducers for medical ultrasound, *Ultrasonic Imaging* 14, 213-233 (1992).
- [18] Skolnik, M. I., Sherman, J. W. and Ogg, F. C., Statistically designed density-tapered arrays, *IEEE Trans. Antennas and Prop.* AP-12, 408-417 (1964).
- [19] Turnbull, D. H., Two-Dimensional Transducer Arrays for Medical Ultrasound Imaging, *Ph.D. Dissertation* (University of Toronto, 1991).

- [20] Jensen, J. A. and Svendsen, N. B., Calculation of Pressure Fields from Arbitrarily Shaped, Apodized, and Excited Ultrasound Transducers, *IEEE Trans. Ultrason. Ferroelec. Freq. Contr.* 39, 262-267 (1992).
- [21] Stepanishen, P. R., The time-dependent force and radiation impedance on a piston in a rigid infinite planar baffle, *J. Acoust. Soc. Am.* 49, 841-849 (1971).
- [22] Tupholme, G. E., Generation of acoustic pulses by baffled plane pistons, *Mathematika* 16, 209-224 (1969).
- [23] Jensen, J. A., A model for the propagation and scattering of ultrasound in tissue, *J. Acoust. Soc. Am.* 89, 182-190 (1990).
- [24] Selfridge, A. R., Kino, G. S. and Khuri-Yahub, B. T., A theory for the radiation pattern of a narrow strip acoustic transducer, *Applied Physics Lett.* 37, 35-6 (1980).
- [25] Bly, S. H. P., Lee-Chahal, D., Foster, D. R., Patterson, M. S., Foster, F. S. and Hunt, J. W., Quantitative contrast measurements in B-mode images: Comparison between experiment and theory, *Ultrasound Med. Biol.* 12, 197-208 (1986).
- [26] von Ramm, O. T. and Smith, S. W., Beam steering with linear arrays, *IEEE Trans. Biomed. Engin.* 30, 438-452 (1983).
- [27] Schmidt, R., personal communication, 1993
- [28] Smith, S. W. and Light, E. D., Two-dimensional array transducers using thick film connection technology, *IEEE Trans. Ultrason. Ferroelec. Freq. Contr.* 40, 727-734 (1993).