



## A Bayesian Approach for Uncertainty Quantification of Extreme Precipitation Projections Including Climate Model Interdependency and Nonstationary Bias

Sunyer Pinya, Maria Antonia; Madsen, Henrik; Rosbjerg, Dan; Arnbjerg-Nielsen, Karsten

*Published in:*  
Journal of Climate

*Link to article, DOI:*  
[10.1175/JCLI-D-13-00589.1](https://doi.org/10.1175/JCLI-D-13-00589.1)

*Publication date:*  
2014

*Document Version*  
Peer reviewed version

[Link back to DTU Orbit](#)

*Citation (APA):*  
Sunyer Pinya, M. A., Madsen, H., Rosbjerg, D., & Arnbjerg-Nielsen, K. (2014). A Bayesian Approach for Uncertainty Quantification of Extreme Precipitation Projections Including Climate Model Interdependency and Nonstationary Bias. *Journal of Climate*, 27(18), 7113-7132. <https://doi.org/10.1175/JCLI-D-13-00589.1>

---

### General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

1

2

3 **A Bayesian approach for uncertainty quantification of extreme**  
4 **precipitation projections including climate model interdependency**  
5 **and non-stationary bias**

6

7 Maria Antonia Sunyer

8 *Department of Environmental Engineering, Technical University of Denmark*

9 *Miljøvej building 113, Kgs. Lyngby 2800, Denmark.*

10 *masu@env.dtu.dk*

11

12 Henrik Madsen

13 *DHI*

14

15 Dan Rosbjerg

16 *Department of Environmental Engineering, Technical University of Denmark*

17

18 Karsten Arnbjerg-Nielsen

19 *Department of Environmental Engineering, Technical University of Denmark*

1 **Abstract**

2 Climate change impact studies are subject to numerous uncertainties and assumptions. One of the  
3 main sources of uncertainty arises from the interpretation of climate model projections.  
4 Probabilistic procedures based on multi-model ensembles have been suggested in the literature to  
5 quantify this source of uncertainty. However, the interpretation of multi-model ensembles remains  
6 challenging. Several assumptions are often required in the uncertainty quantification of climate  
7 model projections. For example, most methods often assume that the climate models are  
8 independent or/and that changes in climate model biases are negligible. This study develops a  
9 Bayesian framework that accounts for model dependencies and changes in model biases and  
10 compares it to estimates calculated based on a frequentist approach. The Bayesian framework is  
11 used to investigate the effects of the two assumptions on the uncertainty quantification of extreme  
12 precipitation projections over Denmark. An ensemble of regional climate models from the  
13 ENSEMBLES project is used for this purpose.

14 The results confirm that the climate models cannot be considered independent and show that the  
15 bias depends on the value of precipitation. This has an influence on the results of the uncertainty  
16 quantification. Both the mean and spread of the change in extreme precipitation depends on both  
17 assumptions. If the models are assumed independent and the bias constant, the results will be  
18 overconfident and may be treated as more precise than they really are. This study highlights the  
19 importance of investigating the underlying assumptions in climate change impact studies, as these  
20 may have serious consequences for the design of climate change adaptation strategies.

21

## 1 **1. Introduction**

2 Information on the expected changes in extreme precipitation under climate change conditions is  
3 needed for the design of infrastructure such as dams, bridges and urban drainage. However, these  
4 changes are subject to numerous uncertainties. Uncertainties are introduced at the different steps in  
5 climate change impact studies, i.e. the choice of emission scenario and Global Circulation Model  
6 (GCM) as well as dynamical and/or statistical downscaling method. It is generally recognized that  
7 robust decision-making on climate change adaptation should be based on probabilistic climate  
8 projections that include these sources of uncertainty (Fowler et al. 2007; Tebaldi and Knutti 2007).

9 Recent studies have addressed and compared the contribution of the various sources of uncertainty  
10 (e.g. Wilby and Harris 2006; Déqué et al. 2007; Dessai and Hulme 2007; Hawkins and Sutton 2011;  
11 Räisänen and Rätty 2012). In most cases the authors found, that the inherent uncertainty in climate  
12 models exceeds the uncertainty due to the natural variability, emission scenario, and statistical  
13 downscaling. Nonetheless, the relevance of each uncertainty source varies depending on the  
14 projection horizon. Natural variability and emission scenario become more important when  
15 analysing, respectively, short and long projection horizons (Hawkins and Sutton 2011).

16 Uncertainty in climate models is often assessed based on a multi-model framework. Large multi-  
17 model ensemble datasets have been produced in recent years. One of the largest datasets of GCMs  
18 is the ensemble recently created in the 5<sup>th</sup> Coupled Model Intercomparison Project (CMIP5) (Taylor  
19 et al. 2012). In Europe, the PRUDENCE project (Christensen et al. 2007) aimed at addressing the  
20 uncertainties in a multi-model ensemble of Regional Climate Models (RCMs). This work was  
21 continued in the ENSEMBLES project (van der Linden and Mitchell 2009). As part of the  
22 ENSEMBLES project a large dataset of state-of-the-art RCMs was created and made freely  
23 available. Currently, in the CORDEX project (<http://wcrp-cordex.ipsl.jussieu.fr/>) several ensembles  
24 of RCMs covering the majority of populated areas in the world are being created.

1 Multi-model ensembles provide valuable information for estimating the uncertainty in climate  
2 model projections. However, there are still remaining challenges in their interpretation (Knutti et al.  
3 2010). Some of the main challenges are: the limited number of climate models in the ensemble, the  
4 interdependency of climate models, and the lack of consensus on how to evaluate model  
5 performance. Therefore, several assumptions are often needed in climate change studies (Fischer et  
6 al. 2012). For example, most studies assume that the climate models in the ensemble are  
7 independent. This assumption may not be valid since some models share similar or even identical  
8 parameterization schemes as well as submodels with identical code (Masson and Knutti 2011).  
9 Additionally, in the case of ensembles of RCMs, some models are driven by the same GCM  
10 boundary conditions. Another important assumption in climate change studies is the assumption of  
11 constant bias in the climate models. The bias is assumed to be stationary in time, i.e. its change is  
12 considered negligible. This assumption is often used in statistical downscaling and bias correction  
13 methods. However, a recent study by Boberg and Christensen (2012) showed that the bias of  
14 temperature is dependent on the value of the temperature.

15 In recent years, several probabilistic methods based on multi-model ensembles have been suggested  
16 in the literature, e.g. the Reliability Ensemble Averaging method suggested by Giorgi and Mearns  
17 (2002) and the Bayesian methods suggested by Tebaldi et al. (2004; 2005), Leith and Chandler  
18 (2010), and Buser et al. (2009; 2010). These methods account for some of the challenges in  
19 interpreting multi-model ensembles. The assumption of model independency is discussed in several  
20 studies, e.g. Furrer et al. (2007), Tebaldi and Knutti (2007), Buser et al. (2009; 2010), Knutti et al.  
21 (2010) and Maulet et al. (2012). Although most of these studies discussed the possible invalidity of  
22 this assumption, none of the methods include the model interdependency in the uncertainty  
23 quantification approach. Buser et al. (2009; 2010) addressed the effects of assuming constant bias.  
24 They tested the assumption of constant bias versus the assumption of temperature dependent bias

1 and found that the choice of constant or dependent bias assumption largely influenced the results  
2 obtained for future summer mean temperatures.

3 This study develops a Bayesian framework that accounts for the interdependency and the changes in  
4 the biases of the climate models. The main goal of the study is to develop and analyse a procedure  
5 for the incorporation of these two factors in the uncertainty quantification of climate model  
6 projections. The study addresses the uncertainty in extreme precipitation projections over Denmark  
7 using an ensemble of RCMs. The focus is on extreme precipitation as it is likely to be one of the  
8 most important impacts of climate change in cities (Fowler and Hennessy 1995; IPCC 2012).  
9 Hence, a quantitative assessment of the uncertainty in these projections is needed for the design of  
10 climate change adaptation in cities. Most studies have focused on the uncertainty in mean  
11 precipitation projections at rather large scales (e.g. Giorgi and Mearns, 2002; Fischer et al. 2012),  
12 and only a few studies have assessed the uncertainties in extreme precipitation projections at a  
13 regional scale (e.g. Frei et al. 2006; Fowler and Ekström, 2009).

14 The next section describes the case study and the data used, followed by the methodology section.  
15 Section 4 presents and discusses the results, and section 5 summarizes the findings and conclusions  
16 of the study.

## 17 **2. Data and case study**

18 The index selected to represent extreme precipitation in this study is the 95<sup>th</sup> percentile of  
19 precipitation amount on wet days, hereafter referred to as  $RR_{wn95}$ . Wet days are defined as days  
20 with precipitation higher or equal to 1 mm (Peterson 2005).  $RR_{wn95}$  is included in the list of  
21 extreme precipitation indices defined by STARDEX (Haylock and Goodess 2004). Additionally, it  
22 is often used in climate change impact studies focusing on extreme precipitation (e.g. Beldring et al.

1 2008; Benestad 2010). In Denmark, under present climate conditions,  $RR_{wn95}$  is approximately 13  
2  $mm\ day^{-1}$  in winter and  $19\ mm\ day^{-1}$  in summer.

3 The observational dataset used in this study is the gridded precipitation product created by the  
4 Danish Meteorological Institute known as Climate Grid Denmark (CGD). This gridded dataset has a  
5 spatial resolution of  $10\ x\ 10\ km$  and it is based on approximately 300 stations (Scharling 1999).  
6 Daily precipitation data is available for the period 1989-2010.

7 The climate model output used is daily precipitation from an ensemble of 15 RCMs from the  
8 ENSEMBLES project (van der Linden and Mitchell 2009). These 15 RCMs are all the models from  
9 ENSEMBLES available at a temporal resolution of  $0.22^\circ$  (approximately 25 km) and covering the  
10 time period 1950-2100. This ensemble contains 11 different RCMs driven by 6 different GCMs (see  
11 Table 1). Thirteen RCMs use the same rotated pole grid system, while two models (RM5.1 and  
12 RegCM) use a Lambert conformal grid system. Daily precipitation data from the RCMs for the time  
13 periods 1989-2010 and 2081-2100 for all the 66 land grid points covering Denmark are used in this  
14 study. The future period considered here is the same as the one used in the Special Report on  
15 Managing the Risks of Extreme Events and Disasters to Advance Climate Change Adaptation  
16 (IPCC 2012).

17 In order to be able to compare the indices obtained from the observational dataset and the RCMs,  
18 the values of  $RR_{wn95}$  estimated for each of the grids in the CGD have been re-interpolated to the  
19  $0.22^\circ$  rotated pole grid system. The method used for this purpose is the natural neighbour  
20 interpolation method suggested by Sibson (1980; 1981). Additionally, and for the same reason, the  
21 two models using the Lambert conformal grid system have been re-interpolated to the rotated pole  
22 grid system using the same interpolation method.

23 TABLE 1 SHOULD GO HERE

### 1 **3. Methodology**

2 The methodology followed in this study is a Bayesian framework based on the approach presented  
3 by Tebaldi et al. (2004; 2005). They suggested a Bayesian statistical model that combines  
4 information from a multi-model ensemble and observations. The statistical model was used to  
5 determine probability distributions of future temperature change for 22 land regions. Here, the  
6 approach has been modified to include the interdependency and the change in bias of the climate  
7 models. A frequentist approach could have also been used (after a re-parameterization of the model)  
8 to estimate the uncertainty in the parameters of the statistical model defined here. The discussion on  
9 frequentist versus Bayesian frameworks is generally recognized and on-going (e.g. Wilks 2006;  
10 Beven 2010). One important difference between the two frameworks is in the interpretation of  
11 probability. In the frequentist view, probability represents the likelihood of an event that would be  
12 found if it was possible to take a large number of samples (Beven 2010). In the Bayesian view,  
13 probability represents the degree of belief on the occurrence of an uncertain event (Wilks 2006).  
14 According to Tebaldi et al. (2005) Bayesian methods are a natural way to represent uncertainty in  
15 the context of climate change projections. We agree with this view, and we interpret the  
16 probabilities found here as a degree of belief given the limited amount of data. Therefore, and  
17 because our statistical model is mainly inspired by Tebaldi et al. (2004; 2005), we apply a Bayesian  
18 approach to estimate the uncertainty in the model parameters. Nonetheless, we compare the results  
19 of the Bayesian analysis with a frequentist approach. The estimation of the parameters using the  
20 frequentist approach is described in Appendix B.

21 This section introduces the Bayesian statistical model as well as the methods used to estimate the  
22 interdependency and change in bias of the climate models. The methods described are applied to  
23 the index  $RR_{wn95}$  separately for winter (December to February) and summer (June to August)  
24 season. The analysis is also carried out considering the whole year; this is referred to as annual



1 period in the results section. For clarity, the subscript to indicate seasons is not included in the  
2 mathematical expressions.

3 *a. Bayesian statistical model*

4 In a Bayesian framework probability distributions are explicitly used for quantifying the uncertainty  
5 in parameters. In this study we apply Bayesian inference, where statistical interpretations about a set  
6 of parameters,  $\Theta$ , are made in terms of the conditional probability on the data,  $\mathbf{D}$ . Following Bayes'  
7 rule the conditional probability of  $\Theta$  depending on  $\mathbf{D}$  can be expressed as:

8 
$$p(\Theta|\mathbf{D}) \propto p(\Theta)p(\mathbf{D}|\Theta) \quad (1)$$

9 where  $p(\Theta|\mathbf{D})$  is the posterior distribution,  $p(\Theta)$  is the prior distribution and  $p(\mathbf{D}|\Theta)$  is the likelihood  
10 function (or sampling distribution).

11 For the purpose of our study here,  $\Theta$  is the set of parameters of a statistical model. As in Tebaldi et  
12 al. (2004; 2005), the statistical model is constructed based on the assumption that the values of  
13  $RR_{wn95}$  estimated from the observations and the climate models are normally distributed. The main  
14 difference between the approach suggested here and the approach by Tebaldi et al. (2004; 2005) is  
15 that the climate models are assumed to follow a multivariate normal distribution instead of  
16 independent univariate normal distributions. The use of a multivariate normal distribution allows us  
17 to take into account the interdependency between the climate models.

18 A multivariate normal distribution is defined for the climate models for both present and future time  
19 periods. The mean of the multivariate normal distributions is considered to be the “true mean” plus  
20 the common bias of the climate models. Additionally, a univariate distribution is defined for the  
21 observations with mean equal to the “true mean” and a known variance. Then, the statistical model  
22 for  $RR_{wn95}$  can be expressed as:

$$\begin{aligned}
\mathbf{X} &\sim N_M((\mu + \beta)\mathbf{1}, \lambda^{-1}\mathbf{R}) \\
\mathbf{Y} &\sim N_M((\nu + \alpha\beta)\mathbf{1}, (\theta\lambda)^{-1}\mathbf{R}) \\
X_{Obs} &\sim N(\mu, \sigma_{Obs}^2)
\end{aligned} \tag{2}$$

where  $M$  is the total number of climate models;  $\mathbf{X}$  and  $\mathbf{Y}$  are vectors of length  $M$  containing the values of the RR<sub>wn95</sub>  $X_m$  and  $Y_m$  for all the climate models  $m = 1, \dots, M$  for present and future conditions, respectively;  $X_{Obs}$  is the value of RR<sub>wn95</sub> estimated from the observations;  $\mu$  and  $\nu$  are scalars that represent the “true mean” of RR<sub>wn95</sub> for present and future conditions;  $\beta$  and  $\lambda^{-1}$  are scalars representing, respectively, the common bias and variance of the climate models for present conditions;  $\alpha$  and  $\theta$  are introduced to account for the fact that the bias and the variance might change from present to future;  $\mathbf{R}$  is the correlation matrix of the climate models assumed to be constant from present to future;  $\sigma_{Obs}^2$  is the variance of the observations.  $X_m$ ,  $Y_m$  and  $X_{Obs}$  are estimated as the average of RR<sub>wn95</sub> over all grid points in the study area. In the following,  $\mathbf{R}$ ,  $\alpha$ , and  $\sigma_{Obs}$  are considered as known. Procedures for assessing these three terms will be given in the following sections. From Eq. (2) it follows that the likelihood of the data given a set of parameter values can be estimated as:

$$p(\mathbf{D}|\Theta) = f_M(\mathbf{X}; (\mu + \beta)\mathbf{1}, \lambda^{-1}\mathbf{R}) f_M(\mathbf{Y}; (\nu + \alpha\beta)\mathbf{1}, (\theta\lambda)^{-1}\mathbf{R}) f(X_{Obs}; \mu, \sigma_{Obs}^2) \tag{3}$$

where  $f_M$  is a multivariate normal probability density function with dimension  $M$ , and  $f$  is a univariate normal probability density function;.

The prior distribution,  $p(\Theta)$ , can be expressed as the product of the marginal prior distributions of all the parameters. For the parameters related to the mean values ( $\mu$ ,  $\nu$ , and  $\beta$ ) the prior distribution is assumed to be a univariate normal distribution, while a gamma distribution is assumed for the parameters related to the variances ( $\lambda$  and  $\theta$ ). This selection of prior distributions is based on the natural conjugate prior families, i.e. the distributional forms for the likelihood and the prior are

1 conjugate. Table 2 summarizes the distribution and distribution parameters (hyperparameters)  
2 chosen for each of the parameters. The values of the hyperparameters have been determined to  
3 ensure that marginal prior distributions carry little information. This implies that the data available  
4 are the main factor defining the posterior distribution. Uninformative priors are chosen because we  
5 assume that “a priori” we do not have any knowledge on the distribution of the parameters. Hence,  
6 the parameters chosen for the priors should not affect the results, as long as the prior distributions  
7 are uninformative. The selection of the distributions and their parameters is inspired by the prior  
8 distributions chosen by Tebaldi et al. (2004; 2005) and Buser et al. (2009; 2010).

9 TABLE 2 SHOULD GO HERE

10 A Markov Chain Monte Carlo (MCMC) algorithm is applied using Gibbs sampling to infer the  
11 posterior distribution (Gelman et al. 2003). In the Gibbs sampler the value of each parameter is  
12 estimated conditional on the value of all the other parameters using the marginal conditional  
13 probabilities. The conditional probability distributions are shown in Appendix A. A *burn-in* period  
14 has been used in the MCMC to account for the effect of the starting distribution. As suggested in  
15 Gelman et al. (2003) the first half of the iterations has been discarded. Additionally, the sequence  
16 has been *thinned* by keeping only one in 15 iterations to avoid dependency within the MCMC. The  
17 independency of the sequence and the convergence of the algorithm have been assessed following  
18 the guidelines in Gelman et al. (2003). For this purpose, 5 sequences of length 100,000 iterations  
19 (after the *burn-in* period) have been considered.

20 The approach defined here necessarily relies on a number of subjective assumptions. Two  
21 simplifications considered here contrast with the approaches suggested in the literature, e.g. Tebaldi  
22 et al. (2004; 2005) and Buser et al. (2009; 2010). These are: the use of the same variance and bias  
23 for all the climate models.

1 The use of the same variance has been applied for two main reasons. Firstly, the focus of this study  
2 is not to estimate the reliability or weighting of the individual climate models, which would require  
3 considering different variances as suggested by Tebaldi et al. (2004; 2005). Currently, there is no  
4 consensus on the best approach to weight climate models (Knutti et al. 2010; Weigel et al. 2010),  
5 and some of the approaches suggested lead to virtually equal weights, e.g. the approach suggested  
6 in the ENSEMBLES project (Christensen et al. 2010). Secondly, by considering the same variance  
7 for all climate models it is possible to derive the conditional probability for each parameter, which  
8 allows us to use Gibbs sampling.

9 The bias in the climate models is assumed to be the sum of a common bias and an individual bias,  
10 i.e. for the present period the common bias is  $\beta$  and the individual bias of the model  $m$  is  $X_m - \beta - \mu$ .  
11 Along with the common bias, the same change in bias  $\alpha$  is used for all the climate models. This  
12 differs from the individual biases considered in Buser et al. (2009; 2010). Several studies have  
13 shown that climate models have common biases in the simulation of precipitation (e.g. Boberg, et  
14 al. 2010; Sunyer et al. 2013a). A drawback of using a common bias is that the individual biases for  
15 the present and future period are assumed independent. This assumption is questionable, but  
16 comparable to the simple model defined in Tebaldi et al. (2004). Here we use this simple model to  
17 illustrate the effects of the interdependency of RCMs and change in bias in the uncertainty  
18 quantification, but further work could focus on identifying such a bias and testing its importance.

19 The use of common variance and bias reduces the number of parameters significantly. This allows  
20 better identification of model parameters considering the available data. If different parameters  
21 were used for each climate model, the number of parameters would be  $2(M-1)$  higher than in the  
22 statistical model described above, i.e. the statistical model would have 33 parameters instead of 5  
23 parameters. It is not clear how these simplifications affect the results. A possible effect of using a  
24 common bias is that if the RCM biases are very different, the uncertainty in  $\beta$  will be large. This

1 will affect the uncertainty in  $\mu$  and  $\nu$ , probably by increasing their uncertainty. A similar effect  
2 could occur when applying the same variance in all the models.

3 *b. Variables considered known in the Bayesian statistical model*

4 1) Observed variance

5 The variance of  $RR_{wn95}$  estimated from the observations,  $\sigma_{Obs}^2$ , is considered known in the  
6 Bayesian model. Since  $RR_{wn95}$  is given by the regional average of the indices calculated for each  
7 grid point, the variance  $\sigma_{Obs}^2$  can be estimated as the average of the variances for the individual grid  
8 points divided by the total number of independent grid points in the region. For each grid point  $i$ ,  $\sigma_i^2$   
9 is calculated by applying the jackknife resampling technique (Miller, 1974; Wilks, 2006), where the  
10 index is recomputed  $n$  times using a sub-set of the data which excludes one year of data each time  
11 ( $n$  is the number of years in the sample).

12 As described in Bretherton et al. (1999), the number of independent grid points in the region can be  
13 estimated by dividing the total number of grid points with the decorrelation length defined as the  
14 distance between independent observations. It can be estimated using semivariograms as the  
15 distance at which the semivariance levels off. Here, we use the semivariograms and the  
16 methodology used in Sunyer et al. (2013a,b) to estimate the decorrelation length of the  
17 observational dataset.

18 It must be noted that here the observed variance,  $\sigma_i^2$ , is considered to represent the sampling error in  
19 the estimation of  $RR_{wn95}$  rather than a measure of natural variability as in Tebaldi et al. (2004;  
20 2005). Natural variability is not explicitly included in the model as done in Buser et al. (2009;  
21 2010). The model defined in Eq. (2) considers two stationary time slices, control and future period,  
22 i.e. the value of  $RR_{wn95}$  in one time slice does not depend on time. Decadal variability is also not  
23 included in the analysis and cannot be estimated with the data available. Longer observation records

1 and a reformulation of the model in Eq. (2) would be required to be able to introduce decadal  
2 variability in the analysis. Different ways to define the observed variance exist and one must be  
3 aware that the choice of model may influence the results.

#### 4 2) Interdependency of RCMs

5 As described in the previous section, the interdependency of the climate models is included in the  
6 Bayesian approach through the correlation matrix of the climate models,  $\mathbf{R}$ . This is estimated using  
7 the approach suggested by Pennell and Reichler (2011) to assess the amount of independent  
8 information in an ensemble of climate models. The approach is based on the idea that two models  
9 are not independent because they lead to different results but because they follow different paths to  
10 reach the results. Hence, according to Pennell and Reichler (2011), a suitable approach for assessing  
11 the independency of the climate models is to statistically analyse their errors.

12 A metric,  $d$ , that represents the error of the climate models for present conditions is used to estimate  
13  $\mathbf{R}$ . The metric is calculated separately for all the grid points of each climate model. It is estimated  
14 from the values of the individual model error and the ensemble average error, which again are  
15 estimated from the value of  $RR_{wn}95$  obtained from the climate models and the observations for the  
16 present time period. The individual model error is estimated by subtracting the observed values  
17 from the climate model values, and dividing this difference by the standard deviation of the  
18 observations. The standard deviation of the observations at each grid point is estimated as the  
19 interannual variability. The ensemble average error is estimated as the average of the individual  
20 model errors. The metric is then calculated by removing from the individual model error the part  
21 present in the ensemble average error. This is done using the correlation,  $r$ , between the individual  
22 model errors and the ensemble average error. For each grid point,  $i$ , and climate model,  $m$ , the  
23 metric,  $d_{i,m}$ , is estimated as:  $d_{i,m} = e_{i,m}^* - r_m \bar{e}_i^*$ , where  $e_{i,m}^*$  and  $\bar{e}_i^*$  are the standardized values of the

1 individual model error and the ensemble average error, respectively. More details on the method  
2 used to estimate  $d$  can be found in Pennell and Reichler (2011) and Sunyer et al. (2013a).

3 The matrix  $\mathbf{R}$  is then estimated from the values of  $d$  obtained for each climate model. The elements  
4 of  $\mathbf{R}$  are the correlation coefficients  $\text{corr}(\mathbf{d}_{\cdot,m}, \mathbf{d}_{\cdot,n})$ , where  $\mathbf{d}_{\cdot,m}$  and  $\mathbf{d}_{\cdot,n}$  are the vectors containing the  
5 value for all the grid points of the models  $m$  and  $n$ , respectively. In the Bayesian approach the  
6 absolute values of the matrix  $\mathbf{R}$  are used in the multivariate normal distribution for both present and  
7 future. Hence, it is assumed that a negative correlation also implies that the models are  
8 interdependent.

### 9 3) Change in bias

10 In the statistical model defined in Eq. (2), the possible change in bias is accounted for by the term  $\alpha$ .  
11 Boberg and Christensen (2012) suggested a procedure that uses the area-averaged monthly bias  
12 from the RCMs driven by re-analysis data to study biases of RCM simulations. Here we suggest to  
13 modify the procedure and use the value of  $\text{RR}_{\text{wn}95}$  estimated by the RCMs driven by GCMs and  
14 compare with observations at each grid point to calculate the bias, because the main objective is to  
15 study the biases of the projected values of  $\text{RR}_{\text{wn}95}$ . The procedure is based on two main steps. First,  
16 for each grid point,  $i$ , and RCM,  $m$ , the bias,  $\text{ABias}_{m,i}$ , is estimated as  $X_{m,i} - X_{\text{Obs},i}$ . Then, a linear  
17 regression is estimated between  $\mathbf{ABias}_{\cdot,}$  and  $\mathbf{X}_{\text{Obs},}$ , i.e. all the climate models are used to estimate  
18 the linear regression. It must be noted that this approach ignores the fact that the grid points are  
19 correlated. This is expected to have minor influence on the linear regression.

20 The slope parameter of the linear regression,  $A$ , is subsequently used to estimate the change in the  
21 bias,  $\alpha$ . Linear regressions are derived for the two seasons and annual data, i.e. a different value of  $\alpha$   
22 is used for winter, summer and annual period.  $\alpha$  depends on the constant  $A$  and the parameters  $\beta$ ,  $\mu$ ,  
23 and  $\nu$ .  $\alpha$  is estimated from the linear regression and Eq. (2) as:

$$\alpha = 1 + \frac{A}{\beta}(\nu - \mu) \quad (4)$$

Eq. (4) is used in the MCMC to estimate  $\alpha$  in each iteration. It is assumed that the estimated structure of the bias over the region and the linear regression approach can be used to describe the change in bias in the future. More elaborate procedures could be developed to assess the possible change in bias under future conditions. The simple procedure presented here is considered appropriate to illustrate the effects of change in bias on the uncertainty in climate projections.

#### 4. Results

The results of this study are presented in three parts. First, the main results regarding the interdependency and the change in bias of the RCMs are described. Second, the results from the uncertainty quantification are presented and discussed. In the last part, the effects of considering the interdependency and the bias are investigated. All the results are presented for RR<sub>wn</sub>95 for winter, summer and annual period.

##### *a. Interdependency and change in bias of RCMs*

The interdependency of the RCMs has been analysed using the correlation matrix,  $\mathbf{R}$ . The average of all the elements in  $\mathbf{R}$  (without considering the diagonal) is 0.25 in winter, 0.20 in summer and 0.27 in the annual period. The minimum and maximum correlations found are 0 and 0.68 in winter, 0 and 0.74 in summer, and 0 and 0.80 in the annual period, respectively. The average values for the three periods are similar, with the average for the summer period slightly lower. The maximum value found is slightly higher in the annual period. The maximum correlations found for each period correspond to different RCM-GCM combinations. In winter the highest correlation is found for two RCM-GCMs with different RCM and GCM (RCA-HadCM3 and RACMO-ECHAM5); in summer for two RCM-GCMs driven by the same GCM (RegCM-ECHAM5 and HIRHAM-ECHAM5); and



1 in the annual period for two RCM-GCMs from the Hadley centre (HadRM3Q16-HadCM3Q16 and  
2 HadRM3Q0-HadCM3Q0).

3 The pairs of RCM-GCMs leading to correlations higher than 0.6 are compared for the three periods  
4 to assess whether high correlation between the RCM-GCMs arises from the same RCM driven by  
5 different GCMs, or vice versa. In the winter period, high correlations are obtained for five pairs of  
6 RCM-GCMs, three pairs of RCM-GCMs with the same RCM but different GCMs, and two pairs of  
7 RCM-GCMs with different RCMs and GCMs. In the summer period, high correlations are found  
8 for two pairs of RCMs driven by the same GCM and for three pairs of RCM-GCMs with different  
9 RCMs and GCMs. In the annual period, high correlations are found for four pairs of RCM-GCMs  
10 with the same RCM and different GCMs, two pairs with different RCMs and GCMs, and one pair  
11 of two RCM-GCMs from the Hadley centre. In general, there are more combinations with the same  
12 RCM but different GCMs leading to high correlations than different RCMs driven by the same  
13 GCM. In addition, some RCM-GCMs using different RCMs and GCMs also lead to high  
14 correlations. This indicates that other factors than the RCM or GCM used have a large influence on  
15 the correlation between RCM-GCMs. A detailed analysis of these factors is outside the scope of this  
16 study and is not addressed further here.

17 The results show that the RCMs are interdependent and that there are no significant differences  
18 between the degree of interdependency for the summer, winter and annual period. It must also be  
19 noticed that in the three periods some RCMs are found to be uncorrelated. A more detailed analysis  
20 of the interdependency of this ensemble of RCMs over Denmark can be found in Sunyer et al.  
21 (2013a).

22 The change in bias is estimated using the bias of the RCMs over the region. Figure 1 shows the  
23 linear regression found between the bias and the observations for winter, summer and annual

1 period. The figure shows both the linear regression fitted for each of the RCMs individually and the  
2 linear regression fitted considering all the RCMs together. For all three periods, all the individual  
3 linear regressions show a decrease and a shift in the sign of the bias for increasing precipitation. The  
4 negative slope from the linear regression considering all the RCM-GCMs together is more  
5 pronounced in the summer period. The values of the slope,  $A$ , from the linear regression that will be  
6 used in the MCMC to estimate  $\alpha$  are -0.54, -0.78 and -0.57 for winter, summer and annual period,  
7 respectively.

8 **FIGURE 1 SHOULD GO HERE**

9 For both seasons and the annual period, the correlation found between the RCMs and the bias  
10 depending on the precipitation value point towards the invalidity of assuming independency and  
11 constant bias. Hence, it is important to include these two results in the uncertainty quantification.  
12 The results show that there is a similar interdependency of the RCMs for the three periods, but that  
13 the change in bias is more pronounced for the summer period.

#### 14 *b. Uncertainty quantification*

15 This section presents the probability distributions estimated for the parameters of the statistical  
16 model described in the methodology section. The convergence and autocorrelation of the outputs  
17 from the Gibbs sampling have been analysed, leading to the conclusion that convergence was  
18 reached and that the outputs are not autocorrelated (results not shown). Hence, the results from the  
19 sampling algorithm can be used to estimate the marginal and joint posterior distributions of the  
20 parameters.

21 First, the assumption of normal distributions in Eq. (2) is analysed. Figure 2 compares the values of  
22  $\mathbf{X}$  and  $\mathbf{Y}$  with theoretical values from a normal distribution. For simplicity the interdependency of

1 the RCMs is not considered here. All the values of  $\mathbf{X}$  and  $\mathbf{Y}$  fall within the 95% confidence limits,  
2 and hence they can be considered to approximately follow a normal distribution.

3 FIGURE 2 SHOULD GO HERE

4 Figure 3 shows the marginal posterior and prior distribution for each of the parameters for winter,  
5 summer, and annual period. The values of  $RR_{\text{wn}95}$  found for each of the RCMs and the  
6 observations are also shown. For all the parameters and for all three periods, the prior and the  
7 posterior distributions are noticeably different. This shows that the data inputs to the model ( $\mathbf{X}$ ,  $\mathbf{Y}$ ,  
8  $X_{\text{Obs}}$ ,  $\sigma_{\text{Obs}}$ ,  $\mathbf{R}$ , and  $A$ ) exert the main influence on the posterior distribution.

9 For the winter period,  $X_{\text{Obs}}$  lies well within the range of the values from the RCMs, while for the  
10 summer and annual periods most of the RCMs have smaller values than the observations. The larger  
11 difference between RCMs and observations found for the summer period is likely to be due to the  
12 nature of extreme events in summer, which are mainly caused by convective precipitation. RCMs  
13 are better at representing extreme events caused by frontal precipitation than convective  
14 precipitation (e.g. Herrera et al. 2010; Fowler et al. 2005). In agreement with the results found here,  
15 several studies have shown that RCMs have larger biases and inter-model differences in the summer  
16 period (Frei et al., 2006; Kendon et al. 2008; Fowler and Ekström, 2009).

17 For the three periods, the posterior distributions of  $\mu$  are sharp compared to the individual values  
18 from the RCMs. This is because the standard deviation estimated for the observations is  
19 considerably smaller than the spread of the values from the RCMs. For the summer and annual  
20 period this is also due to the influence of the bias in some RCMs.  $\sigma_{\text{Obs}}$  is 0.20, 0.30 and 0.15 mm  
21  $\text{day}^{-1}$  for winter, summer and annual period, respectively. Small values of  $\sigma_{\text{Obs}}$  lead to less  
22 uncertainty in the parameter  $\mu$ . Conversely, all the values of the RCMs are encompassed in the  
23 posterior distribution of  $\nu$ . This is a combined effect of the fact that the value of  $\sigma_{\text{Obs}}$  does not affect

1 this parameter and that all the RCMs have the same weight. It must also be noted that the median of  
2 the distributions increases from present to future for both winter and annual periods, but decreases  
3 for the summer period.

4 The bias found for the RCMs is centred on approximately  $0 \text{ mm day}^{-1}$  for the winter period (the  
5 median is  $-0.01 \text{ mm day}^{-1}$ ). For the summer and annual period, the medians of the bias are  $-2.25$  and  
6  $-1.86 \text{ mm day}^{-1}$ , respectively. The uncertainty in this parameter is larger for the summer and annual  
7 period than for the winter period. The values of the 5<sup>th</sup> and 95<sup>th</sup> percentiles are  $-1.28$  and  $1.14 \text{ mm}$   
8  $\text{day}^{-1}$  for the winter period,  $-4.68$  and  $0.40 \text{ mm day}^{-1}$  for the summer period, and  $-3.74$  and  $0.16 \text{ mm}$   
9  $\text{day}^{-1}$  for the annual period. This shows that the RCMs have a tendency towards underestimation of  
10 extreme precipitation for both the summer and the annual period. This is likely due to the  
11 underestimation of extreme events caused by convective precipitation as discussed above. The  
12 distributions for these two periods resemble bimodal distributions, where one mode corresponds to  
13 a negative bias and the other is approximately at  $0 \text{ mm day}^{-1}$ . This is likely due to the fact that the  
14 distribution of the bias is dependent on both the values found for present and future climate. For the  
15 present climate the bias found is negative, while in the future climate due to the lack of observations  
16 the bias tends to  $0 \text{ mm day}^{-1}$ .

17 The distribution of  $\lambda$ , which represents the reliability (the inverse of the variance) of the RCMs for  
18 present climate, varies depending on the period. Both the median and the spread are higher for  
19 winter than for summer and annual period. This reflects a lower reliability of the RCMs for these  
20 two periods. The lowest values are found for the summer period. This is due to the larger  
21 differences between the RCMs for this period. The median of the posterior distribution of  $\lambda$  for the  
22 winter period is  $0.63 \text{ mm}^{-2} \text{ day}^2$ . This corresponds to a standard deviation of  $1.27 \text{ mm day}^{-1}$ , which  
23 is considerably higher than the estimated standard deviation of the observational dataset. The same  
24 is observed for the summer and annual period, where the median of  $\lambda$  corresponds to a standard

1 deviation of  $3.03 \text{ mm day}^{-1}$  and  $2.32 \text{ mm day}^{-1}$ , respectively. These results are in agreement with the  
2 general perception that the reliability of the observations should be higher than for the RCMs.

3 The distribution of  $\theta$ , which represents the change in the reliability from present to future climate,  
4 also varies depending on the period considered. The median for the winter period is 1.02, pointing  
5 to a small change in the reliability. However, the values range from a decrease to an increase in the  
6 reliability (5<sup>th</sup> percentile of 0.41 and a 95<sup>th</sup> percentile of 2.65). As in the case of  $\lambda$ , lower values and  
7 lower uncertainty are found for the summer and annual period. The medians are 0.41 and 0.44, the  
8 5<sup>th</sup> percentiles 0.17 and 0.18, and the 95<sup>th</sup> percentiles 1.03 and 1.10 for the summer and the annual  
9 period, respectively. For these two periods, most of the values are lower than one. This is caused by  
10 the larger inter-model differences in the future period and shows the decrease in the reliability of the  
11 RCMs.

12 Figure 3 also shows the mean of the parameters obtained using the Bayesian and frequentist  
13 approach; the values for all the parameters are virtually the same except for  $\beta$  in winter and  $\theta$ . A  
14 more detailed comparison and description of the differences between the two approaches can be  
15 found in Appendix B.

16 **FIGURE 3 SHOULD GO HERE**

17 The marginal posterior distributions of the parameters provide information about the uncertainty  
18 and the expected value of the parameters. Moreover, the joint posterior distribution of the  
19 parameters provides information about the relation between the parameters. Figure 4 shows the  
20 histogram for each of the parameters and the scatter plot of all the combinations of pairs of  
21 parameters for the summer period. It also shows the correlation coefficient for each pair of  
22 parameters.

1 The mean values for present and future conditions are practically uncorrelated. This is due to the  
2 small contribution of the RCMs in the distribution of  $\mu$  compared to observations, and differs from  
3 the correlation between the values for the present and future obtained from the RCMs. These two  
4 parameters ( $\mu$  and  $\nu$ ) have a negative correlation with  $\beta$ . This result follows both from Eq. (2) and  
5 from the analysis of the conditional probability of  $\beta$  (see Appendix A). The conditional probability  
6 shows that for small values of  $\mu$  and  $\nu$  the bias will be positive (i.e. RCMs overestimate), while for  
7 large values the bias will be negative (i.e. RCMs underestimate). This is also in agreement with the  
8 values found from the analysis of change in bias.

9 The three parameters  $\mu$ ,  $\nu$ , and  $\beta$  are not correlated with the parameters  $\lambda$  and  $\theta$ . However, it can be  
10 observed that for high values of  $\lambda$  the range of  $\mu$ ,  $\nu$ , and  $\beta$  decreases. The same relation is observed  
11 between these parameters and  $\theta$ . This can be interpreted as: the uncertainty in  $\mu$ ,  $\nu$ , and  $\beta$  is low  
12 when the reliability in the RCMs is high (high values of  $\lambda$  and  $\theta$ ). The same interpretation can be  
13 drawn from the analysis of the conditional probabilities. The parameters  $\lambda$  and  $\theta$  also show the  
14 expected relationship. For higher values of  $\lambda$ , smaller values of  $\theta$  and vice versa. Similar  
15 correlations were obtained for the winter and annual period (results not shown).

16 When using the priors shown in Figure 3 some posterior values of  $\nu$  are negative, which cannot be  
17 interpreted physically. The probability of obtaining a negative value is very low (approximately  
18 0.2%) and could be avoided by using slightly more informative priors.

19 **FIGURE 4 SHOULD GO HERE**

20 *c. Effect of interdependency and change in bias*

21 The previous section analyses the uncertainty and joint distribution of the parameters in the  
22 statistical model. This section focuses on the effects of taking into account the interdependency and  
23 change in bias of the RCMs. A set of tests using different assumptions have been defined for this

1 purpose. First, three tests considering either independency or interdependency and change in bias or  
2 constant bias are carried out. Then, a total of 45 tests are run using different levels of  
3 interdependency and change in bias of the RCMs. The main output analysed from these tests is the  
4 change in  $RR_{wn95}$  from present to future. This change, referred to as change factor (CF), is defined  
5 as the relative change between  $\mu$  and  $\nu$ , i.e.  $CF = \nu/\mu$ . The value of CF has been estimated in each of  
6 the iterations of the MCMC runs from which the distribution of CF has been derived.

7 Figure 5 shows the results from the first three tests: (1) the RCMs are independent and the bias  
8 constant, (2) the RCMs are interdependent and the bias constant, (3) the RCMs are interdependent  
9 and the bias changes from present to future. This last test is the one for which the results have been  
10 analysed in detail in the previous section. In the test assuming that the RCMs are independent,  $\mathbf{R}$  is  
11 set equal to the identity matrix. Similarly, in the tests where the bias is assumed constant  $A$  is set to  
12 0 (i.e.  $\alpha$  equal to 1). Figure 5 shows the marginal posterior distributions for  $\mu$ ,  $\nu$  and, CF for the  
13 three different tests for winter, summer, and annual period.

14 For  $\mu$ , the posterior distributions obtained for the three tests are similar. The median as well as the  
15 5<sup>th</sup> and 95<sup>th</sup> percentiles for the three tests differ less than  $0.1 \text{ mm day}^{-1}$  for all the periods. Larger  
16 differences are found in the distributions of  $\nu$ . The uncertainty in  $\nu$  found for the test assuming  
17 independency is smaller than for the other tests. This is due to the fact that less independent  
18 information is considered in the tests assuming interdependency. This leads to a lower reliability in  
19 the RCMs, which in turn leads to higher uncertainty in  $\nu$ . The effects of assuming constant or  
20 changing bias are also noticed in the distribution of  $\nu$ . The change in bias has different effects  
21 depending on the season. In winter and annual periods, the median of  $\nu$  increases due to a more  
22 negative bias. The median of  $\alpha\beta$  in winter and annual periods is  $-0.56$  and  $-3.74 \text{ mm day}^{-1}$ ,  
23 respectively. In the summer period, the bias in the future period (the median of  $\alpha\beta$  is  $-2.35 \text{ mm day}^{-1}$ )  
24 <sup>1</sup>) is approximately equal to the bias in the present period. This leads to values of  $\nu$  similar to when

1 the bias is considered constant. Additionally, for the winter summer and annual period, the  
2 uncertainty in  $\nu$  increases when the change in bias is introduced. This is due to an increase in the  
3 uncertainty of the bias for the future period.

4 A similar difference as the one obtained for  $\nu$  between the three tests is observed in the results for  
5 CF. The uncertainty of this variable is higher for the tests accounting for the interdependency of the  
6 RCMs for all the periods. In addition, the uncertainty is also higher for the test accounting for  
7 change in bias. The median of CF depends on the test. In all seasons, the median of CF in the test  
8 assuming independency is similar to the median estimated directly from the RCMs (for climate  
9 model  $m$ ,  $CF_m$  is estimated as  $Y_m/X_m$ ). In the winter period, a larger median is obtained when the  
10 change in bias and interdependency are taken into account. For this season, large differences are  
11 also observed for the 5<sup>th</sup> and 95<sup>th</sup> percentiles being: 1.11 and 1.21 for test (1); 1.02 and 1.26 for test  
12 (2); and 1.01 and 1.47 for test (3). Even though the uncertainty in the tests where interdependency is  
13 taken into account is larger, all tests show an increase of extreme precipitation for winter at a 5%  
14 level of significance.

15 For the summer period, similar differences as in  $\nu$  are obtained in CF. As in the winter period, the  
16 comparison of the 5<sup>th</sup> and 95<sup>th</sup> percentiles points out the differences in the uncertainty of CF being:  
17 0.98 and 1.13 for test (1); 0.78 and 1.25 for test (2); and 0.60 and 1.40 for test (3). For this season,  
18 the tests do not agree on the direction of the change. Similar results are obtained for the annual  
19 period regarding the uncertainty in CF. The values of the median found for this period are 1.09,  
20 1.10 and 1.22 for test (1), (2), and (3), respectively. The 5<sup>th</sup> and 95<sup>th</sup> percentile for tests (2) and (3)  
21 vary between a decrease and an increase of extreme precipitation, while the values for test (1) point  
22 to an increase of extreme precipitation. In summary, for the summer and annual period, even though  
23 most of the RCMs point towards an increase in extreme precipitation, the interdependency between



1 the RCMs means that (given the information available) a hypothesis of a decrease in extreme  
2 precipitation cannot be rejected on the 5% significance level.

3 FIGURE 5 SHOULD GO HERE

4 Appendix B compares the standard errors of  $\mu$ ,  $\nu$ , and CF and describes the differences between the  
5 two approaches.

6 The difference obtained for the three tests can be further analysed by considering the joint  
7 distributions of  $\mu$  and  $\nu$ . Figure 6 compares the scatter plots obtained for each of the tests. The  
8 correlation between the two parameters is higher for the test where the RCMs are assumed  
9 independent. For example, for the summer the correlation coefficient is 0.32 for test (1) and 0.1 for  
10 test (2) and (3). In other words, for a specific value of  $\mu$  the range of possible values of  $\nu$  is larger in  
11 the tests where the interdependency of the RCMs is taken into account. This contributes to the  
12 higher uncertainty of CF found for these tests. Figure 6 also highlights the differences in the  
13 uncertainty for winter, summer and annual period. As shown in Figure 5, for all three tests, the  
14 range of values of  $\nu$  in the winter period is considerably smaller than the range of values obtained  
15 for summer and annual period. In the case of  $\mu$ , the range of values obtained for the annual period is  
16 smaller than for the winter and summer period. This is due to the lower variance of the observations  
17 for this period. Additionally, the correlation between  $\mu$  and  $\nu$  is higher for the three tests for the  
18 winter period. This shows that, in agreement with the results found in Figure 3 and Figure 5, there is  
19 a lower uncertainty in the winter of CF.

20 FIGURE 6 SHOULD GO HERE

21 In order to study in more detail the influence of the interdependency and the changes in the bias, 45  
22 MCMC simulations have been run using different values of  $\mathbf{R}$  and  $A$ . In these simulations, artificial  
23 values of  $\mathbf{R}$  and  $A$  are used. In  $\mathbf{R}$ , the same correlation coefficients are used for all the pairs of

1 RCMs ranging from 0 to 0.8 (all the values in the diagonal are kept equal to 1). The values of  $A$  are  
2 set to range between -0.8 to 0.8. The mean and the standard deviation of CF are analysed for each of  
3 these simulations. Due to the use of artificial values of  $\mathbf{R}$  and  $A$ , the results of these 45 simulations  
4 cannot be directly compared to the results of the three tests described above, except in the case of  
5 independency and constant bias (test 1).

6 Figure 7 summarizes the results of the 45 MCMC simulations for the annual period. The y- and x-  
7 axes display the values used to define  $\mathbf{R}$  and  $A$ , respectively. The results show that the uncertainty  
8 in CF is influenced by both  $\mathbf{R}$  and  $A$ . The standard deviation of CF increases for higher values of  $\mathbf{R}$   
9 and it decreases for smaller absolute values of  $A$ . This is in agreement with the results from the  
10 three tests. The mean CF is also influenced by both  $\mathbf{R}$  and  $A$ , but in this case the main influence is  
11 the value of  $A$ . The mean CF decreases for increasing values of  $A$  ranging from -0.8 to  
12 approximately 0.4, and it increases for increasing values of  $A$  higher than 0.4. The mean values of  
13 CF range from virtually no increase (mean CF equal to 1.03) to an increase of 40%, and the  
14 standard deviation varies from 0.03 to 0.86. The two smallest standard deviations are obtained for  
15 the cases where the models are considered independent and  $A$  equal to 0.2 and 0 (test (1)), while the  
16 largest standard deviation is obtained for correlations of 0.8 and  $A$  equal to -0.8.

17 **FIGURE 7 SHOULD GO HERE**

## 18 **5. Conclusions**

19 A Bayesian approach based on a multi-model ensemble of RCMs has been developed to assess the  
20 uncertainty in extreme precipitation projections over Denmark. This approach accounts for both  
21 interdependency and linear changes in the bias of the RCMs. The results of the Bayesian approach  
22 are compared to estimates calculated based on a frequentist approach.

1 The analyses of the outputs from the RCMs show that, for the region of study, both the assumption  
2 of independency and constant bias are unlikely to be valid. Hence, interdependency and change in  
3 bias are included in the uncertainty estimation. The uncertainty estimated for  $RR_{wn95}$  is higher for  
4 summer than for winter both for the present and future time period. This is due to the higher  
5 reliability in the values of the RCMs for winter and a lower observed variance. The lower reliability  
6 in the summer period is explained by the larger differences between the RCMs and between the  
7 RCMs and the observations. For the annual period, the uncertainty for the present is lower than for  
8 the winter period, but it is higher for the future period. The change estimated from present to future  
9 is more uncertain for the summer and annual period than for the winter. The change in the  
10 distribution median points towards an increase in extreme precipitation for winter and annual  
11 period, and to approximately no change in extremes for the summer period.

12 The influence of accounting for the interdependency and the change in bias of the RCMs has been  
13 tested. These two assumptions do not affect significantly the distribution of the mean of  $RR_{wn95}$  for  
14 the present. However, they largely affect the distribution for the future period, which again  
15 influences the distribution of the change factor. A higher uncertainty in the change factor is  
16 obtained when the interdependency and change in bias of the RCMs is taken into account.

17 The methods used to estimate the interdependency and the change in bias are subject to several  
18 assumptions and uncertainties. Therefore, a total of 45 MCMC simulations have been run using  
19 different (artificial) levels of interdependency and change in the bias of the RCMs. The uncertainty  
20 of the change factor increases for higher RCM correlations and it decreases for smaller absolute  
21 values of the change in bias. The mean CF is also influenced by both the interdependency and  
22 change in bias, but mainly by the change in bias.

1 The results of this study highlight the importance of the assumptions taken in uncertainty estimation  
2 methods for climate change impact studies. In this case study, the assumption of independency and  
3 constant bias of the climate models has led to lower uncertainty estimates of the change in extreme  
4 precipitation in comparison to the more realistic assumption of interdependency and change in bias.  
5 This highlights the risk of reaching overconfident results, which may lead to overconfident  
6 decisions regarding adaptation to climate change.

7 The approach presented here addresses some of the assumptions often taken in uncertainty  
8 estimation methods for climate change projections. It suggests a way in which the interdependency  
9 and change in bias can be introduced in the uncertainty quantification of climate model projections.  
10 However, there are still remaining challenges and further work is needed to address some of the  
11 limitations of this approach. The approach could be further extended by relaxing some of the  
12 assumptions such as the constant interdependency of the RCMs from present to future,  
13 independency between individual errors from present to future, the assumption of a common bias of  
14 the RCMs, or the definition of observed variance. Hence, the specific results found here should be  
15 treated with care. The main message of this study is the need to include the interdependency of  
16 climate models and change in bias in the uncertainty quantification of climate change projections.

## 17 **Acknowledgments**

18 This work was carried out with the support of the Danish Council for Strategic Research as part of  
19 the project RiskChange, contract no. 10-093894 (<http://riskchange.dhigroup.com>). The Climate  
20 Grid Denmark dataset is a product of the Danish Meteorological Institute. The data from the RCMs  
21 used in this work was funded by the EU FP6 Integrated Project ENSEMBLES contract no. 05539  
22 (<http://ensembles-eu.metoffice.com>), whose support is gratefully acknowledged. We would like to

- 1 thank the three reviewers for their constructive comments, which have significantly contributed to
- 2 improve the quality of this paper.
- 3

1 **APPENDIX A**

2 **Full conditional probabilities**

3 The full conditional probabilities for each of the parameters are:

4 
$$\mu | \dots \sim N\left(\tilde{\mu}, \left(\lambda \sum_{p=1}^M \sum_{m=1}^M s_{m,p} + \sigma_{Obs}^{-2} + \sigma_{\mu}^{-2}\right)^{-1}\right);$$

5 
$$\nu | \dots \sim N\left(\tilde{\nu}, \left(\theta \lambda \sum_{p=1}^M \sum_{m=1}^M s_{m,p} + \sigma_{\nu}^{-2}\right)^{-1}\right);$$

6 
$$\beta | \dots \sim N\left(\tilde{\beta}, \left((\lambda + \theta \lambda \alpha^2) \sum_{p=1}^M \sum_{m=1}^M s_{m,p} + \sigma_{\beta}^{-2}\right)^{-1}\right);$$

7 
$$\lambda | \dots \sim Ga\left(a_{\lambda} + M, \left(\frac{1}{2}(\mathbf{X} - \mu - \beta)\mathbf{R}^{-1}(\mathbf{X} - \mu - \beta) + \frac{1}{2}\theta(\mathbf{Y} - \nu - \alpha\beta)\mathbf{R}^{-1}(\mathbf{Y} - \nu - \alpha\beta) + \frac{1}{b_{\lambda}}\right)^{-1}\right);$$

8 
$$\theta | \dots \sim Ga\left(a_{\theta} + \frac{M}{2}, \left(\frac{1}{2}\lambda(\mathbf{Y} - \nu - \alpha\beta)\mathbf{R}^{-1}(\mathbf{Y} - \nu - \alpha\beta) + \frac{1}{b_{\theta}}\right)^{-1}\right);$$

9 
$$\tilde{\mu} = \frac{\frac{1}{2}\lambda \sum_{p=1}^M \left\{ \sum_{m=1}^M [(X_m - \beta)s_{m,p}] + (X_p - \beta) \sum_{m=1}^M s_{m,p} \right\} + \frac{X_{Obs}}{\sigma_{Obs}^2} + \frac{\mu_0}{\sigma_{\mu}^2}}{\lambda \sum_{p=1}^M \sum_{m=1}^M s_{m,p} + \sigma_{Obs}^{-2} + \sigma_{\mu}^{-2}};$$

10 
$$\tilde{\nu} = \frac{\frac{1}{2}\theta \lambda \sum_{p=1}^M \left\{ \sum_{m=1}^M [(Y_m - \alpha\beta)s_{m,p}] + (Y_p - \alpha\beta) \sum_{m=1}^M s_{m,p} \right\} + \frac{\nu_0}{\sigma_{\nu}^2}}{\theta \lambda \sum_{p=1}^M \sum_{m=1}^M s_{m,p} + \sigma_{\nu}^{-2}};$$

$$\tilde{\beta} = \frac{\frac{1}{2}\lambda \sum_{p=1}^M \left\{ \sum_{m=1}^M [(X_m - \mu)s_{m,p}] + (X_p - \mu) \sum_{m=1}^M s_{m,p} \right\}}{(\lambda + \theta\lambda\alpha^2) \sum_{p=1}^M \sum_{m=1}^M s_{m,p} + \sigma_\beta^{-2}} +$$

$$+ \frac{\frac{1}{2}\theta\lambda\alpha \sum_{p=1}^M \left\{ \sum_{m=1}^M [(Y_m - \nu)s_{m,p}] + (Y_p - \nu) \sum_{m=1}^M s_{m,p} \right\} + \frac{\beta_0}{\sigma_\beta^2}}{(\lambda + \theta\lambda\alpha^2) \sum_{p=1}^M \sum_{m=1}^M s_{m,p} + \sigma_\beta^{-2}}$$

1

2 where  $a_\lambda$  and  $a_\theta$  are the shape parameters of the prior distribution of  $\lambda$  and  $\theta$ , respectively;  $b_\lambda$  and  $b_\theta$   
3 are the scale parameters of the prior distribution of  $\lambda$  and  $\theta$ , respectively;  $\mu_0$  and  $\sigma_\mu^2$  are the mean  
4 and variance of the prior distribution of  $\mu$ ;  $\nu_0$  and  $\sigma_\nu^2$  are the mean and variance of the prior  
5 distribution of  $\nu$ ;  $\beta_0$  and  $\sigma_\beta^2$  are the mean and variance of the prior distribution of  $\beta$ ;  $s_{m,p}$  is the  
6 element  $[m,p]$  of the inverse of the correlation matrix of the RCMs,  $\mathbf{R}^{-1}$ .  $\text{Ga}(a,b)$  denotes a gamma  
7 distribution with shape  $a$  and scale  $b$  and  $\text{N}(c,d^2)$  denotes a normal distribution with mean  $c$  and  
8 variance  $d^2$ .

9

10

11

## 1 APPENDIX B

### 2 Frequentist approach

3 To estimate the parameters of the model defined in Eq. (2) using the frequentist approach a re-  
4 parameterization of the model is needed. The re-parameterized model considers five parameters:  $\mu$ ,  
5  $\mu_X, \mu_Y, \sigma_X^2$ , and  $\sigma_Y^2$  and can be expressed as:

$$\begin{aligned} X_{obs} &\sim N(\mu, \sigma_{obs}^2) \\ \mathbf{X} &\sim N(\mu_X \mathbf{1}, \sigma_X^2 \mathbf{R}) \\ \mathbf{Y} &\sim N(\mu_Y \mathbf{1}, \sigma_Y^2 \mathbf{R}) \end{aligned}$$

7 The new set of parameters is defined using the parameters in Eq. (2) as:

$$\begin{aligned} \mu_X &= \mu + \beta \\ \mu_Y &= \nu + \alpha\beta = \mu + \beta + (A+1)(\nu - \mu) \\ \sigma_X^2 &= \lambda^{-1} \\ \sigma_Y^2 &= (\theta\lambda)^{-1} \end{aligned}$$

9 Following the frequentist approach the parameters in the re-parameterized model can be estimated  
10 as:

$$\begin{aligned} \hat{\mu} &= X_{obs}; \quad \hat{\mu}_X = \frac{\mathbf{1}^T \mathbf{R}^{-1} \mathbf{X}}{\mathbf{1}^T \mathbf{R}^{-1} \mathbf{1}}; \quad \hat{\mu}_Y = \frac{\mathbf{1}^T \mathbf{R}^{-1} \mathbf{Y}}{\mathbf{1}^T \mathbf{R}^{-1} \mathbf{1}} \\ \hat{\sigma}_X^2 &= \frac{(\mathbf{X} - \hat{\mu}_X \mathbf{1})^T \mathbf{R}^{-1} (\mathbf{X} - \hat{\mu}_X \mathbf{1})}{M-1}; \quad \hat{\sigma}_Y^2 = \frac{(\mathbf{Y} - \hat{\mu}_Y \mathbf{1})^T \mathbf{R}^{-1} (\mathbf{Y} - \hat{\mu}_Y \mathbf{1})}{M-1}; \end{aligned}$$

12 and the variance of the main estimated parameters ( $\hat{\mu}$ ,  $\hat{\mu}_X$  and  $\hat{\mu}_Y$ ) is:

$$\text{var}(\hat{\mu}) = \hat{\sigma}_{obs}^2; \quad \text{var}(\hat{\mu}_X) = \frac{\hat{\sigma}_X^2}{\mathbf{1}^T \mathbf{R}^{-1} \mathbf{1}}; \quad \text{var}(\hat{\mu}_Y) = \frac{\hat{\sigma}_Y^2}{\mathbf{1}^T \mathbf{R}^{-1} \mathbf{1}}$$

14 Then, the estimates of the parameters in Eq. (2) can be obtained as:



1 
$$\hat{\mu} = X_{obs}; \quad \hat{\nu} = \frac{\hat{\mu}_Y - \hat{\mu}_X}{A+1} + \hat{\mu}; \quad \hat{\beta} = \hat{\mu}_X - \hat{\mu}; \quad \hat{\lambda} = (\hat{\sigma}_X^2)^{-1}; \quad \hat{\theta} = \frac{\hat{\sigma}_X^2}{\hat{\sigma}_Y^2};$$

2 and the standard error of  $\hat{\mu}$ ,  $\hat{\nu}$ , and  $\hat{\beta}$ :

3 
$$SE_{\hat{\mu}} = \sqrt{\sigma_{Obs}^2}; \quad SE_{\hat{\nu}} = \sqrt{\frac{\sigma_X^2 + \sigma_Y^2}{(A+1)^2 \mathbf{1}^T \mathbf{R}^{-1} \mathbf{1}} + \sigma_{Obs}^2}; \quad SE_{\hat{\beta}} = \sqrt{\frac{\sigma_X^2}{\mathbf{1}^T \mathbf{R}^{-1} \mathbf{1}} + \sigma_{Obs}^2}$$

4 It must be noted that the standard error of  $\hat{\nu}$  is estimated assuming independency between  $\hat{\mu}$ ,  $\hat{\mu}_X$   
 5 and  $\hat{\mu}_Y$ , and that the standard error of  $\hat{\beta}$  assumes independency between  $\hat{\mu}$  and  $\hat{\mu}_X$ . The standard  
 6 errors of the parameter estimates  $\hat{\lambda}$  and  $\hat{\theta}$  can be estimated from the inverse of  $\hat{\sigma}_X^2$  and  $\hat{\sigma}_Y^2$ . From  
 7 the re-parameterised model it can be found that the inverse of  $\hat{\sigma}_X^2$ , i.e.  $\hat{\lambda}$ , follows a gamma  
 8 distribution with variance:

9 
$$SE(\hat{\lambda}) = \sqrt{2(M+2)[(X - \mu_X \mathbf{1})^T \mathbf{R}^{-1} (X - \mu_X \mathbf{1})]^{-2}}$$

10 The variance of the inverse of  $\hat{\sigma}_Y^2$  can be estimated in a similar way. The variance of  $\hat{\theta}$  can be  
 11 estimated using independent draws from these distributions.

12 Finally, the mean of CF can be found from  $\hat{\nu} / \hat{\mu}$ . The standard error can be obtained using  
 13 independent draws from the distributions of  $\hat{\mu}$  and  $\hat{\nu}$ , which follow a normal distribution with  
 14 mean  $\hat{\mu}$  and  $\hat{\nu}$  and variance  $(SE_{\hat{\mu}})^2$  and  $(SE_{\hat{\nu}})^2$ , respectively.

15 In addition to the difference in the interpretation of probability in the Bayesian and frequentist  
 16 approach, there are two other differences to be noted. First, it is not possible to estimate the  
 17 probability density function of the parameters from the frequentist approach. Secondly, the

1 frequentist approach does not require MCMC or the use of priors which might have an influence on  
2 the results. These differences should be kept in mind when comparing the results of the two  
3 approaches.

#### 4 **Comparison of Bayesian and frequentist approach**

5 The comparison of the results obtained using the frequentist and Bayesian approach helps to address  
6 the influence of the priors and MCMC in the results of the Bayesian approach. Figure B1 shows the  
7 mean and standard error estimated using the frequentist approach and the Bayesian approach for the  
8 three tests (see description of the tests in section 4.c).

9 **FIGURE B1 SHOULD GO HERE**

10 The mean and standard error of  $\mu$  for the three tests and for the three periods are virtually the same  
11 in the frequentist and Bayesian approach. For the other parameters, the results of the mean and  
12 standard error are also similar, but slight differences are seen between the two approaches.

13 The largest differences between the mean of  $\nu$  obtained using the Bayesian and the frequentist  
14 approach is 0.02, 0.4 and 0.03 mm day<sup>-1</sup> in winter, summer, and annual period, respectively. In the  
15 case of the mean of  $\beta$ , the largest differences are 0.16, 0.1, and 0.05 mm day<sup>-1</sup>, in winter, summer,  
16 and annual period, respectively. The largest differences in the mean of  $\nu$  and  $\beta$  are obtained for test  
17 (3). A possible reason for the differences in  $\beta$  is the fact that in the re-parameterized model  $\beta$  is  
18 estimated from  $X_{Obs}$  and  $\mathbf{X}$ , while in the Bayesian model in Eq. (2)  $\beta$  is also influenced by  $\mathbf{Y}$ . The  
19 standard errors of  $\nu$  and  $\beta$  are similar for the two approaches. In some cases it is slightly larger for  
20 the Bayesian approach, and in others it is larger in the frequentist approach (e.g.  $\nu$  in test (3) in  
21 winter and annual period).

1 The mean and standard errors of  $\lambda$  are also very similar in the two approaches. The largest  
2 differences are seen in the mean of  $\theta$ , which is smaller in the frequentist approach. The largest  
3 difference is obtained for the winter period, in this case the Bayesian approach leads to 1.22 and the  
4 frequentist approach to 1.01 (which is approximately equal to the median of  $\theta$  in the Bayesian  
5 approach). For this parameter similar differences between the frequentist and Bayesian approach are  
6 obtained for the three tests.

7 The mean and standard error of CF are similar in the two approaches. As in the case of  $\nu$ , the largest  
8 difference between the two approaches is found for the mean CF for summer and test (3). In this  
9 case, the Bayesian approach leads to a mean CF of 1.02 and the frequentist approach leads to 1.04.  
10 The differences in the standard errors are similar to the ones found for  $\nu$  and  $\beta$ .

11 The influence of the re-parameterization has been assessed by applying the Bayesian inference  
12 technique (use of Gibbs sampling and priors shown in Table 2) defined in section 3 to the re-  
13 parameterized model. In this case, the largest difference between the Bayesian and frequentist  
14 approaches in the mean of  $\nu$ ,  $\beta$ , and  $\theta$  is 0.07 mm day<sup>-1</sup>, 0.02 mm day<sup>-1</sup>, and 0.004, respectively.

15 There are several reasons for which differences in the parameters arise between the two approaches.  
16 These include: the priors selected in the Bayesian approach, the convergence of the MCMC in the  
17 Bayesian approach, and the re-parameterization of the model used in the frequentist approach.  
18 Nonetheless, Figure B1 shows that the differences between the Bayesian and frequentist approach  
19 are small and do not affect the conclusions of this study.

20

## 1 **References**

- 2 Beldring, S., T. Engen-Skaugen, E. J. Forland, and L. A. Roald, 2008: Climate change impacts on  
3 hydrological processes in Norway based on two methods for transferring regional climate model  
4 results to meteorological station sites, *Tellus A*, **60A**(3), 439–450, doi:10.1111/j.1600-  
5 0870.2008.00306.x.
- 6 Benestad, R. E., 2010: Downscaling precipitation extremes, *Theor. Appl. Climatol.*, **100**(1-2), 1–21,  
7 doi:10.1007/s00704-009-0158-1.
- 8 Beven, K., 2009: *Environmental Modelling: An Uncertain Future?*. CRC Press, 328 pp.
- 9 Boberg, F., P. Berg, P. Thejll, W. Gutowski, and J. H. Christensen, 2010: Improved confidence in  
10 climate change projections of precipitation further evaluated using daily statistics from ensembles  
11 models, *Clim. Dynam.*, **35**, 1509–1520, doi:10.1007/s00382-009-0683-8.
- 12 Boberg, F., and J. H. Christensen, 2012: Overestimation of Mediterranean summer temperature  
13 projections due to model deficiencies, *Nature Climate Change*, **2**(6), 433–436,  
14 doi:10.1038/NCLIMATE1454.
- 15 Bretherton, C. S., M. Widmann, V. P. Dymnikov, J. M. Wallace, and I. Bladé, 1999: The effective  
16 number of spatial degrees of freedom of a time-varying field, *J. Climate*, **12**(7), 1990–2009.
- 17 Buser, C. M., H. R. Künsch, D. Lüthi, M. Wild, and C. Schär, 2009: Bayesian multi-model  
18 projection of climate: bias assumptions and interannual variability, *Clim. Dynam.*, **33**, 849–868, doi:  
19 10.1007/s00382-009-0588-6.
- 20 Buser, C. M., H. R. Künsch., and C. Schär, 2010: Bayesian multi-model projections of climate:  
21 generalization and application to ENSEMBLES results, *Clim. Res.*, **44**, 227–241, doi:  
22 10.3354/cr00895.

- 1 Christensen J. H., T. R. Carter, M. Rummukainen, G. Amanatidis, 2007: Evaluating the  
2 performance and utility of regional climate models: the PRUDENCE project, *Climatic Change*, **81**,  
3 1–6, doi: 10.1007/s10584-006-9211-6.
- 4 Christensen, J. H., E. Kjellström, F. Giorgi, G. Lenderink, and M. Rummukainen, 2010: Weight  
5 assignment in regional climate models, *Clim. Res.*, **44**(2-3), 179–194, doi:10.3354/cr00916.
- 6 Déqué, M., D. Rowell, D. Lüthi, F. Giorgi, J. Christensen, B. Rockel, D. Jacob, E. Kjellström, M.  
7 de Castro, and B. van den Hurk, 2007: An intercomparison of regional climate simulations for  
8 europe: assessing uncertainties in model projections, *Climatic Change*, **81**, 53–70.
- 9 Dessai, S., and M. Hulme, 2007: Assessing the robustness of adaptation decisions to climate  
10 change uncertainties: A case study on water resources management in the east of England, *Global*  
11 *Environmental Change*, **17**(1), 59–72.
- 12 Fischer, A.M., Weigel, A.P., Buser, C.M., Knutti, R., Künsch, H.R., Liniger, M.A., Schär, C., and  
13 C. Appenzeller. 2012: Climate change projections for Switzerland based on a Bayesian multi-model  
14 approach. *Int. J. Climatol.*, **32**, 2348–2371, doi: 10.1002/joc.3396
- 15 Fowler, A. M., K. J. Hennessy, 1995: Potential impacts of global warming on the frequency and  
16 magnitude of heavy precipitation. *Nat. Hazards*, **11**, 283–303.
- 17 Fowler, H. J., S. Blenkinsop, and C. Tebaldi, 2007: Linking climate change modelling to impacts  
18 studies: recent advances in downscaling techniques for hydrological modelling, *Int. J. Climatol.*,  
19 **27**(12), 1547–1578.
- 20 Fowler, H. J. and M. Ekström, 2009: Multi-model ensemble estimates of climate change impacts on  
21 UK seasonal precipitation extremes, *Int. J. Climatol.*, **29**(3), 385–416.

- 1 Fowler, H.J., M. Ekström, C.G. Kilsby, P.D Jones, 2005: New estimates of future changes in  
2 extreme rainfall across the UK using regional climate model integrations 1. Assessment of control  
3 climate, *Journal of Hydrology*, **300**(1), 212–233.
- 4 Frei, C, R. Schöll, S. Fukutome, J. Schmidli, and P.J. Vidale, 2006: Future change of precipitation  
5 extremes in Europe: Intercomparison of scenarios from regional climate models, *Journal of*  
6 *Geophysical Research*, **111**, D06105.
- 7 Furrer, R., R. Knutti, S. R. Sain, D. W. Nychka, and G. A. Meeh, 2007: Spatial patterns of  
8 probabilistic temperature change projections from a multivariate Bayesian analysis, *Geophys. Res.*  
9 *Lett.*, **34**, L06711.
- 10 Gelman, A., J. B. Carlin, H. S. Stern and D. B. Rubin, 2003: *Bayesian Data Analysis*, Chapman &  
11 Hall/CRC, 668pp.
- 12 Giorgi, F., and L. O. Mearns, 2002: Calculation of average, uncertainty range, and reliability of  
13 regional climate changes from AOGCM simulations via the “reliability ensemble averaging” (rea)  
14 method, *J. Climate*, **15**, 1141–1158, doi:10.1175/1520-0442.
- 15 Hawkins, E., and R. Sutton, 2011: The potential to narrow uncertainty in projections of regional  
16 precipitation change, *Clim. Dynam.*, **37**, 407–418.
- 17 Haylock, M. R., and C. M. Goodess, 2004: Interannual variability of European extreme winter  
18 rainfall and links with mean large-scale circulation, *Int. J. Climatol.*, **24**, 759–776,  
19 doi:10.1002/joc.1033.
- 20 Herrera, S., L. Fita, J. Fernández, J. M. Gutiérrez, 2010: Evaluation of the mean and extreme  
21 precipitation regimes from the ENSEMBLES regional climate multimodel simulations over Spain,  
22 *Journal of Geophysical Research*, **115**(D21), D21117.

- 1 IPCC, 2012: Managing the Risks of Extreme Events and Disasters to Advance Climate Change  
2 Adaptation. A Special Report of Working Groups I and II of the Intergovernmental Panel on  
3 Climate Change [Field, C.B., V. Barros, T.F. Stocker, D. Qin, D.J. Dokken, K.L. Ebi, M.D.  
4 Mastrandrea, K.J. Mach, G.-K. Plattner, S.K. Allen, M. Tignor, and P.M. Midgley (eds.)].  
5 Cambridge University Press, Cambridge, UK, and New York, NY, USA, 582 pp.
- 6 Kendon, E. J., D. P. Rowell, R. G. Jones, E. Buonomo, 2008: Robustness of future changes in local  
7 precipitation extremes, *J. Climate*, **21**(17), 4280–4297.
- 8 Knutti, R., R. Furrer, C. Tebaldi, J. Cermak, and G. A. Meehl, 2010: Challenges in combining  
9 projections from multiple climate models, *J. Climate*, **23**, 2739–2758,  
10 doi:10.1175/2009JCLI3361.1.
- 11 Leith, N. A., and R. E. Chandler, 2010: A framework for interpreting climate model outputs, *Appl.*  
12 *Statist.*, **59**(2), 279–296, doi:10.1111/j.1467-9876.2009.00694.x.
- 13 Masson, D., R. Knutti, 2011: Climate model genealogy, *Geophys. Res. Lett.*, **38**, L08703,  
14 doi:10.1029/2011GL046864.
- 15 Maule, C., P. Thejll, J. H. Christensen, S. Svendsen, and J. Hannaford, 2012: Improved confidence  
16 in regional climate model simulations of precipitation evaluated using drought statistics from the  
17 ensembles models, *Clim. Dynam.*, **40**, 155–173.
- 18 Miller R.G., 1974: The jackknife - a review, *Biometrika*, **61**, 1–15
- 19 Pennell, C., and T. Reichler, 2011: On the effective number of climate models, *J. Climate*,  
20 **24**, 2358–2367, doi:10.1175/2010JCLI3814.1.
- 21 Peterson, T. C., 2005: Climate Change Indices, *WMO Bulletin*, **54**(2), 83–86.

1 Räisänen, J., and O. Räty, 2012: Projections of daily mean temperature variability in the future:  
2 cross-validation tests with ENSEMBLES regional climate simulations, *Clim. Dynam.*,  
3 doi:10.1007/s00382-012-1515-9.

4 Scharling, M., 1999: Klimagrid Danmark nedbør 10\*10 km (ver.2), Technical Report number 99-15  
5 available in Danish at [www.dmi.dk](http://www.dmi.dk), Danish Metereological Institute, Denmark.

6 Sibson, R., 1980: A vector identity for the dirichlet tessellation, *Math. Proc. Cambridge*, **87**(1),  
7 151–155, doi:10.1017/S0305004100056589.

8 Sibson R., 1981: *Interpreting Multivariate Data*, Wiley, 374.

9 Sunyer, M. A., H. Madsen, D. Rosbjerg, and K. Arnbjerg-Nielsen, 2013a: Regional  
10 interdependency of precipitation indices across Denmark in two ensembles of high resolution  
11 RCMs, *J. Climate*, **26**, 7912–7928, doi:10.1175/JCLI-D-12-00707.1.

12 Sunyer, M. A., H. J. D. Sørup, O. B. Christensen, H. Madsen, D. Rosbjerg, P. S. Mikkelsen, and K.  
13 Arnbjerg-Nielsen, 2013b: On the importance of observational data properties when assessing  
14 regional climate model performance of extreme precipitation, *Hydrol. Earth Syst. Sci. Discuss.*, **10**,  
15 1–41, doi:10.5194/hessd-10-1-2013.

16 Taylor, K. E., J. R. Stouffer, and G. A. Meehl, 2012: An Overview of CMIP5 and the Experiment  
17 Design. *Bull. Amer. Meteor. Soc.*, **93**, 485–498, doi: [http://dx.doi.org/10.1175/BAMS-D-11-](http://dx.doi.org/10.1175/BAMS-D-11-00094.1)  
18 00094.1

19 Tebaldi, C., L. O. Mearns, D. Nychka, and R. L. Smith, 2004: Regional probabilities of  
20 precipitation change: a bayesian analysis of multimodel simulations, *Geophys. Res. Lett.*, **31**,  
21 L24213.



- 1 Tebaldi, C., R. Smith, D. Nychka, and L. O. Mearns, 2005: Quantifying uncertainty in projections  
2 of regional climate change: a Bayesian approach to the analysis of multi-model ensembles, *J.*  
3 *Climate*, **18**, 1524–1540, doi:10.1175/JCLI3363.1.
- 4 Tebaldi, C., and R. Knutti, 2007: The use of the multi-model ensemble in probabilistic climate  
5 projections, *Philos. Trans. Roy. Soc. A*, **365**, 2053–2075.
- 6 van der Linden, P. and J. F. Mitchell, 2009: Ensembles: Climate change and its impacts: Summary  
7 of research and results from the ensembles project, Technical Report, Met Office Hadley Centre,  
8 Exeter, UK.
- 9 Weigel, A.P., R. Knutti, M.A. Liniger, and C. Appenzeller 2010: Risks of model-weighting in  
10 multimodel climate projections, *J. Clim*, **23**, 4175-4191.
- 11 Wilby, R. L., and I. Harris, 2006: A framework for assessing uncertainties in climate change  
12 impacts: Low-flow scenarios for the river thames, UK, *Water Resour. Res.*, **42**(2), 1–10.
- 13 Wilks, D. S., 2006: *Statistical Methods in the Atmospheric Sciences, second edition*. Academic  
14 Press, 627 pp.

1

Table 1 - List of RCMs used in this study, driving GCMs, and source of the RCMs.

<b>RCM</b>	<b>GCM</b>	<b>Institute</b>
HIRHAM5	ARPEGE	Danish Meteorological Institute
HIRHAM5	ECHAM5	
HIRHAM5	BCM	
REMO	ECHAM5	Max Planck Institute for Meteorology
RACMO2	ECHAM5	Royal Netherlands Meteorological Institute
RCA	ECHAM5	Swedish Meteorological and Hydrological Institute
RCA	BCM	
RCA	HadCM3Q3	
CLM	HadCM3Q0	Swiss Federal Institute of Technology, Zürich
HadRM3Q0	HadCM3Q0	UK Met Office
HadRM3Q3	HadCM3Q3	
HadRM3Q16	HadCM3Q16	
RCA3	HadCM3Q16	Community Climate Change Consortium for Ireland
RM5.1	ARPEGE	National Centre for Meteorological Research in France
RegCM3	ECHAM5	International Centre for Theoretical Physics

2

3

- 1 Table 2 Prior distributions and hyperparameters. In the case of normal distribution the parameters
- 2 are the mean and variance. In the case of gamma distribution the parameters are the shape and scale.

<b>Parameter</b>	<b>Distribution</b>	<b>Mean/Shape</b>	<b>Variance/Scale</b>
$\mu$	Normal	$X_{Obs}$	1000
$\nu$	Normal	$X_{obs} + (\bar{Y} - \bar{X})/(A + 1)$	1000
$\beta$	Normal	$\bar{X} - X_{obs}$	1000
$\lambda$	Gamma	0.001	1/0.001
$\theta$	Gamma	0.001	1/0.001

3

1 **Figure caption list**

2 Figure 1 – Linear regressions between the bias,  $ABias_{...}$ , and observations,  $X_{Obs.,}$  for winter, summer  
3 and annual period. The grey lines show the linear regressions fitted to each RCM individually, the  
4 black lines show the linear regressions fitted considering the biases of all the RCMs together. The  
5 dots show the values estimated for all grid points and all RCMs. The figure illustrates how  $\alpha$  is  
6 estimated from the values of  $\mu$ ,  $\nu$ , and  $\beta$ .

7 Figure 2 – **X** (grey points) and **Y** (black points) vs. theoretical values of a normal distribution for  
8 winter, summer and annual period. The thick lines show the identity line. The thin lines represent  
9 the 95% confidence limits.

10 Figure 3 – Figure 3 –Posterior (black line in primary y-axis) and prior (grey shaded area in  
11 secondary y-axis) probability density functions of the five parameters in the model ( $\mu$ ,  $\nu$ ,  $\beta$ ,  $\lambda$ , and  $\theta$ )  
12 for winter, summer and annual period. The grey dots are the outputs from each of the RCMs and the  
13 black dots are the values estimated from the observations. The vertical thick dashed lines represent  
14 the median, while the thin lines represent the 5<sup>th</sup> and 95<sup>th</sup> percentiles. The black and grey solid lines  
15 represent the mean obtained using the Bayesian and frequentist approach, respectively.

16 Figure 4 - Scatter plots for all the pairs of parameters and histograms of the marginal posterior  
17 distributions of each parameter for the summer period. In the diagonal from top left to bottom right:  
18  $\mu$  [mm day<sup>-1</sup>],  $\nu$  [mm day<sup>-1</sup>],  $\beta$  [mm day<sup>-1</sup>],  $\lambda$  [mm<sup>-2</sup> day<sup>2</sup>], and  $\theta$  [-]. The values of RRwn95 obtained  
19 from the RCMs for present and future period are also shown in the scatter plot for the pair of  
20 parameters [ $\mu$ ,  $\nu$ ].

21 Figure 5 – Marginal posterior distributions of  $\mu$ ,  $\nu$ , and CF for three different tests. The grey dots are  
22 the outputs from each of the RCMs and the black dots are the values estimated from the  
23 observations. The vertical thick line represents the median, while the thin lines represent the 5<sup>th</sup> and

1 95<sup>th</sup> percentiles. The dotted line indicates CF equal to 1, while the dash-dotted line shows the  
2 median of the CFs estimated from the RCMs in the ensemble.

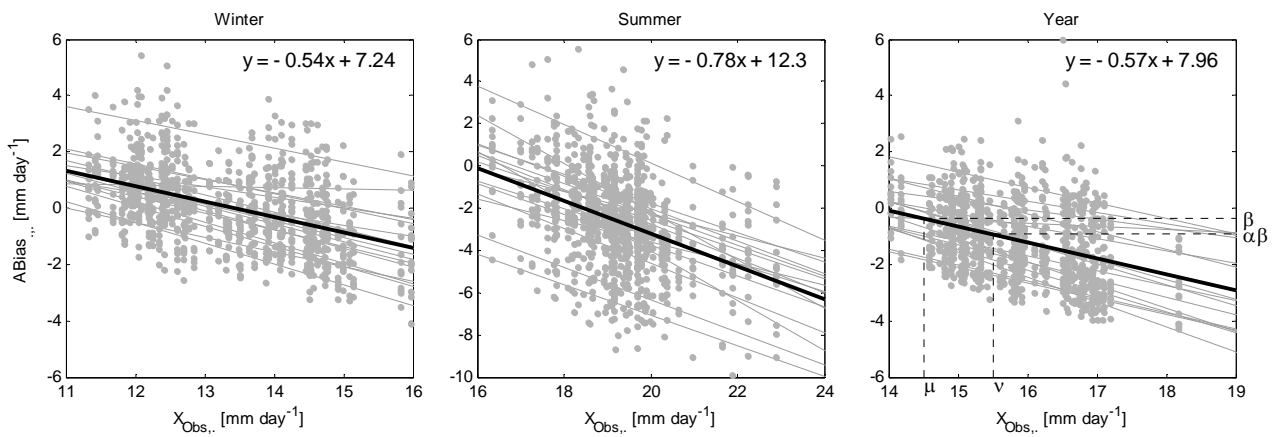
3 Figure 6 – Scatter plot for  $\mu$  and  $\nu$  for three different tests for the winter, summer, and annual  
4 period.

5 Figure 7 – Median and standard deviation of CF for a set of tests. The y- and x-axes show the  
6 factors that are multiplied to  $\mathbf{R}$  and  $A$ , respectively. The colour of the circles indicates the  
7 distribution median and the size of the circles indicates the standard deviation. Larger circle sizes  
8 correspond to higher standard deviation. The number 1, 2, and 3 indicate the three tests analysed in  
9 detail

10 Figure B1 – Mean and standard error of the parameters in Eq. (2) and CF estimated using a  
11 Bayesian (black) and frequentist approach (grey) for the three tests described in section 4.c. The  
12 circles indicate the mean and the lines represent the standard error (the total length of the line is  
13 equal to the standard error).

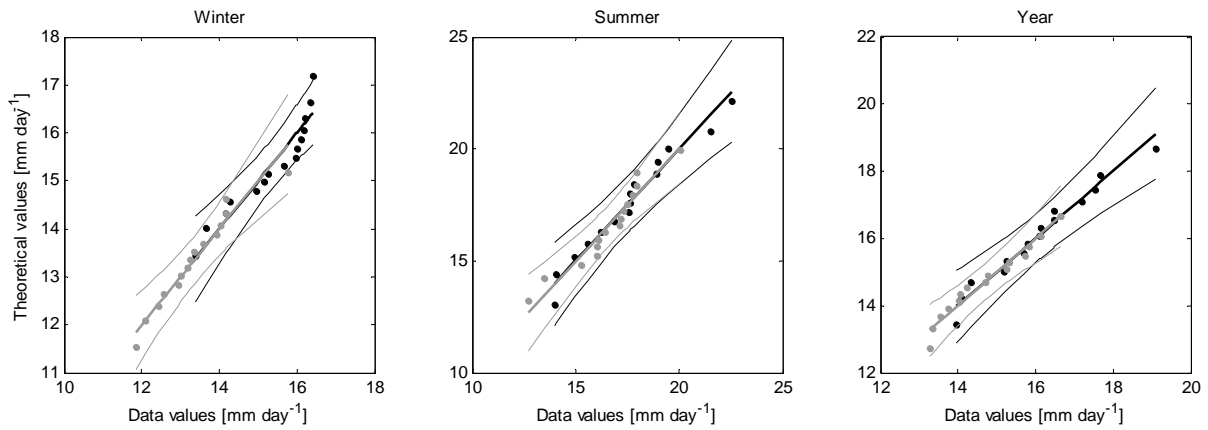
14

15



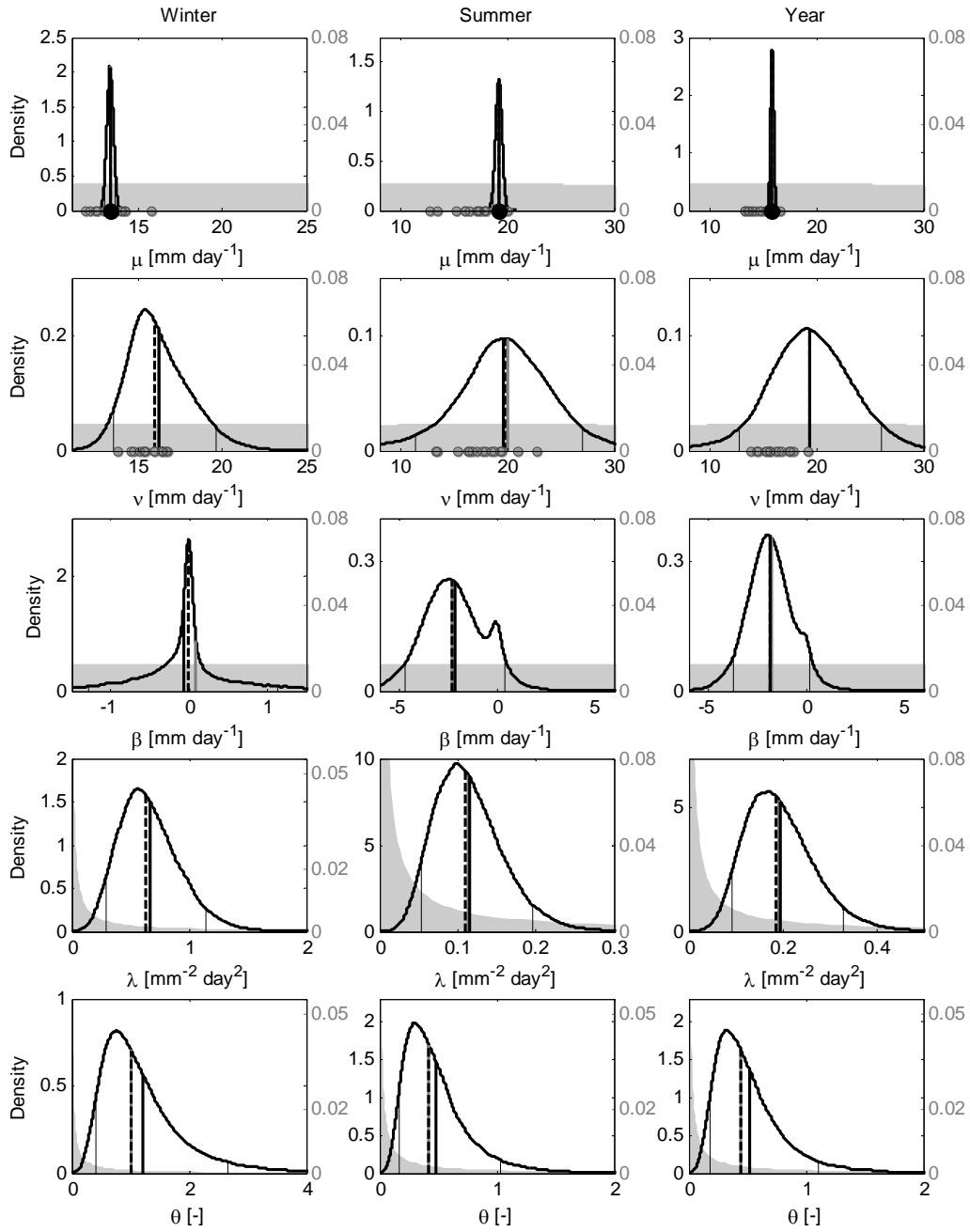
1

2 Figure 1 – Linear regressions between the bias,  $ABias_{i,j}$ , and observations,  $X_{Obs,i,j}$ , for winter, summer  
 3 and annual period. The grey lines show the linear regressions fitted to each RCM individually, the  
 4 black lines show the linear regressions fitted considering the biases of all the RCMs together. The  
 5 dots show the values estimated for all grid points and all RCMs. The figure illustrates how  $\alpha$  is  
 6 estimated from the values of  $\mu$ ,  $\nu$ , and  $\beta$ .



1

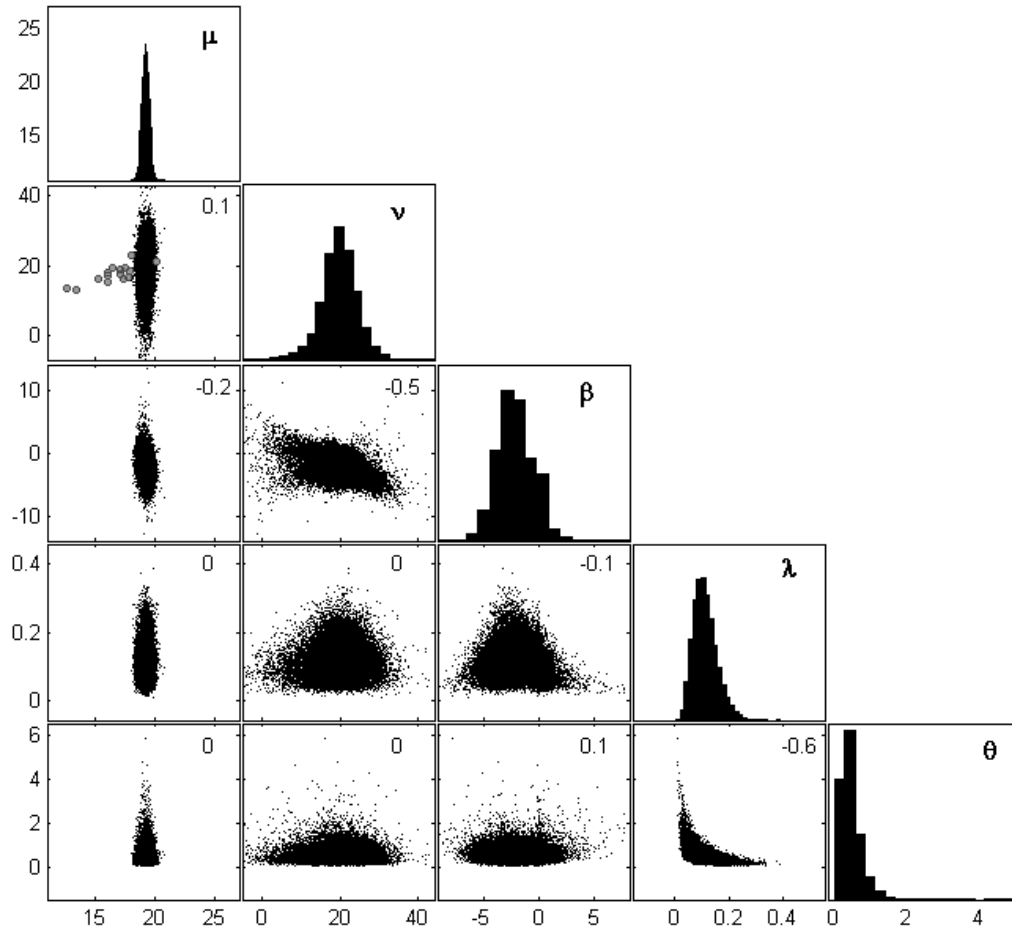
2 Figure 2 – X (grey points) and Y (black points) vs. theoretical values of a normal distribution for  
 3 winter, summer and annual period. The thick lines show the identity line. The thin lines represent  
 4 the 95% confidence limits.



1

2 Figure 3 –Posterior (black line in primary y-axis) and prior (grey shaded area in secondary y-axis)  
 3 probability density functions of the five parameters in the model ( $\mu$ ,  $v$ ,  $\beta$ ,  $\lambda$ , and  $\theta$ ) for winter,  
 4 summer and annual period. The grey dots are the outputs from each of the RCMs and the black dots  
 5 are the values estimated from the observations. The vertical thick dashed lines represent the median,  
 6 while the thin lines represent the 5<sup>th</sup> and 95<sup>th</sup> percentiles. The black and grey solid lines represent  
 7 the mean obtained using the Bayesian and frequentist approach, respectively.

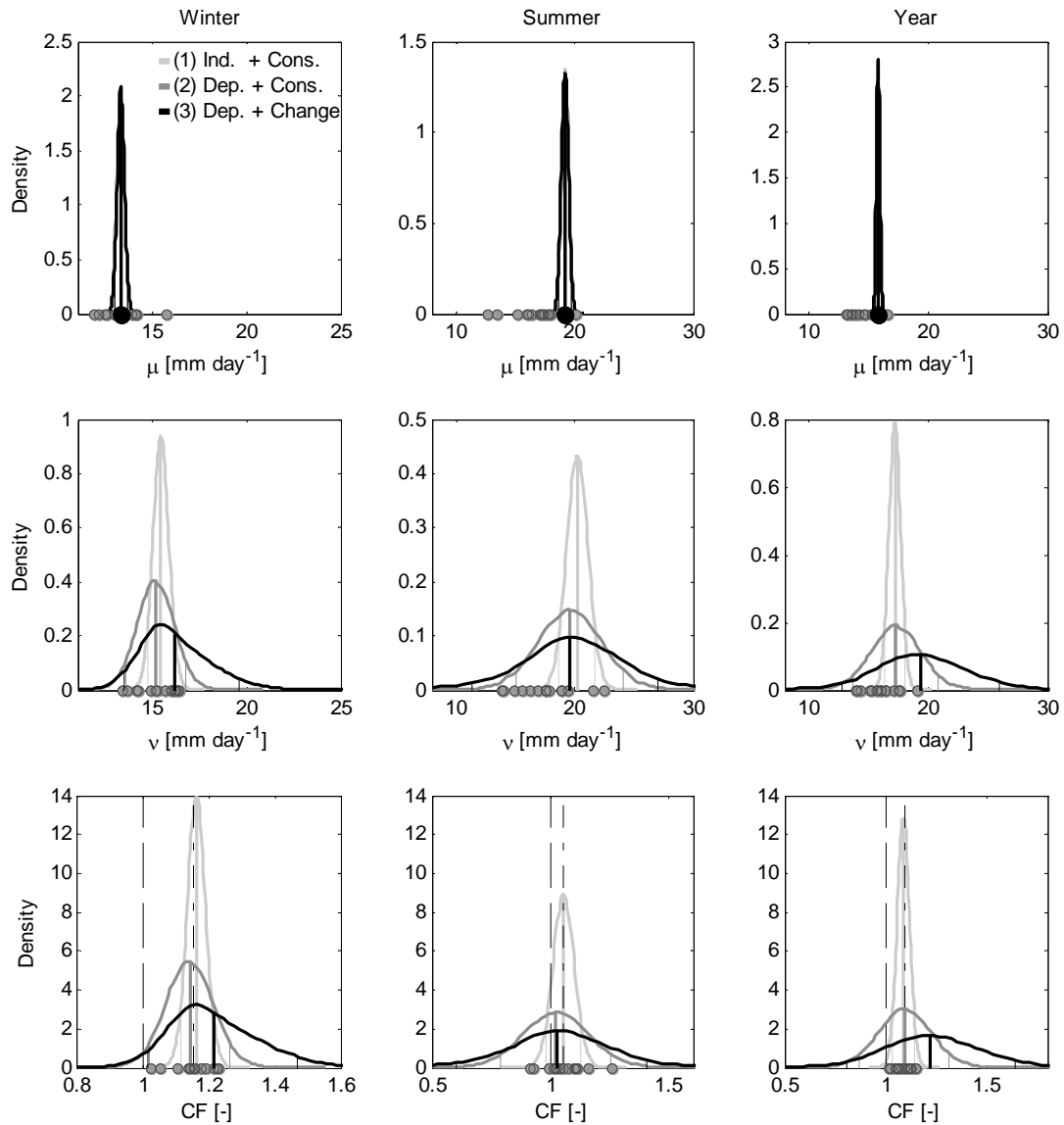




1

2 Figure 4 - Scatter plots for all the pairs of parameters and histograms of the marginal posterior  
 3 distributions of each parameter for the summer period. In the diagonal from top left to bottom right:  
 4  $\mu$  [ $\text{mm day}^{-1}$ ],  $\nu$  [ $\text{mm day}^{-1}$ ],  $\beta$  [ $\text{mm day}^{-1}$ ],  $\lambda$  [ $\text{mm}^{-2} \text{day}^2$ ], and  $\theta$  [-]. The values of RRwn95 obtained  
 5 from the RCMs for present and future period are also shown in the scatter plot for the pair of  
 6 parameters [ $\mu, \nu$ ]. The number shown in the plots is the linear correlation coefficient.

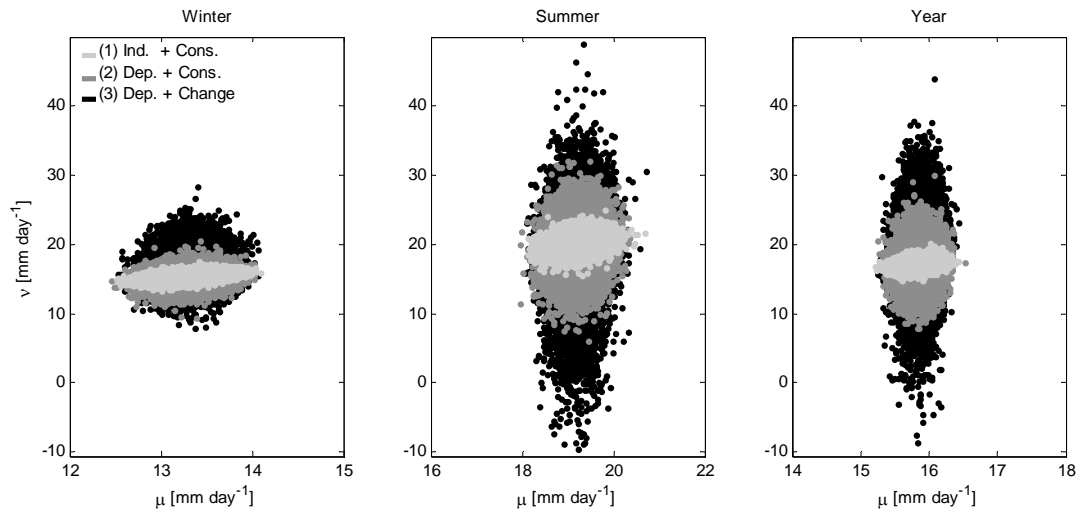
7



1

2 Figure 5 – Marginal posterior distributions of  $\mu$ ,  $\nu$ , and CF for three different tests. The grey dots are  
 3 the outputs from each of the RCMs and the black dots are the values estimated from the  
 4 observations. The vertical thick line represents the mean, while the thin lines represent the 5<sup>th</sup> and  
 5 95<sup>th</sup> percentiles. The dashed line indicates CF equal to 1, while the dash-dotted line shows the  
 6 median of the CFs estimated from the RCMs in the ensemble.

1



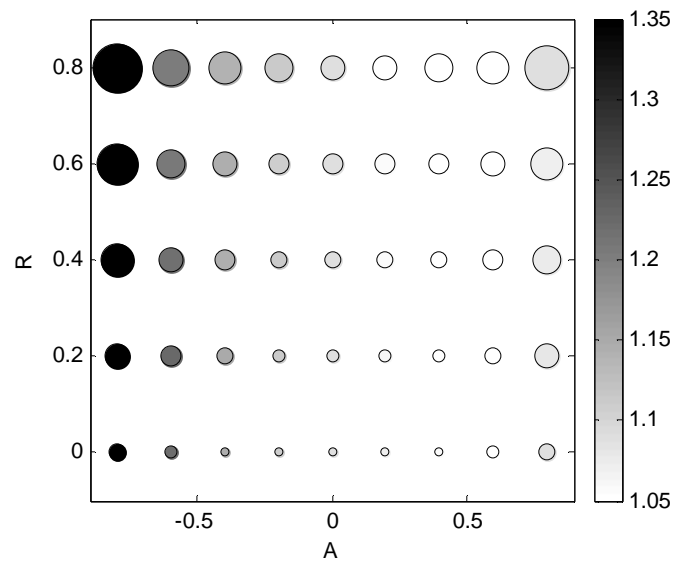
2

3

Figure 6 – Scatter plot for  $\mu$  and  $v$  for three different tests for the winter, summer, and annual period.

4

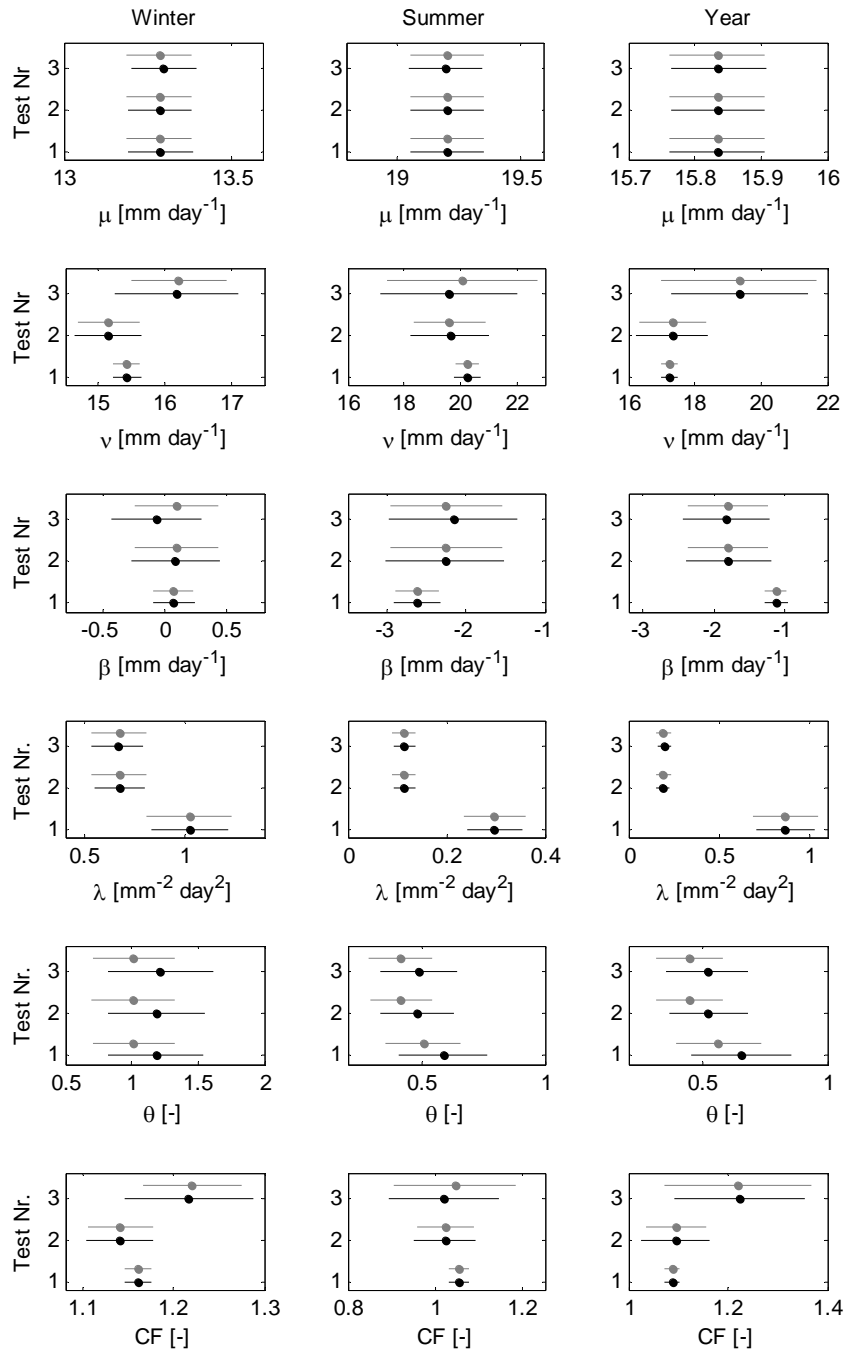
5



1

2 Figure 7 – Mean and standard deviation of CF for a set of tests. The y- and x-axes show the values  
 3 used to define the correlation coefficients in  $\mathbf{R}$  and  $A$ , respectively. The colour of the circles  
 4 indicates the distribution mean and the size of the circles indicates the standard deviation. Larger  
 5 circle sizes correspond to higher standard deviation.

6



1

2

3

4

5

6

Figure B1 – Mean and standard error of the parameters in Eq. (2) and CF estimated using a Bayesian (black) and frequentist approach (grey) for the three tests described in section 4.c. The circles indicate the mean and the lines represent the standard error (the total length of the line is equal to the standard error).