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# A Numerical Analysis of Phononic-Assisted Control of Ultrasound Waves in Acoustofluidic Device

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## Abstract

The ability to precisely sort individual microparticles/cells/droplets in suspension is important for various chemical and biological applications such as cancer cell detection, drug screening etc. The past decade, label-free particle handling of particle suspensions by ultrasonic radiation forces and streaming has received much attention, since it relies solely on mechanical properties such as particle size and contrast in density and compressibility. We present a theoretical study of phononic-assisted control of ultrasound waves in acoustofluidic devices. We propose the use of phononic crystal diffractors, which can be introduced in acoustofluidic structures. These diffractors can be applied in the design of efficient resonant cavities, directional sound waves for new types of particle sorting methods, or acoustically controlled deterministic lateral displacement. The PnC-diffractor-based devices can be made configurable, by embedding the diffractors, all working at the same excitation frequency but with different resulting diffraction patterns, in exchangeable membranes on top of the device.

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**Keywords:** particle sorting; acoustofluidics; phononic crystal;

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## 1. Introduction

Currently, many biologists employ optical, magnetic, electric tweezers for single-cell manipulation due to their excellent precision and versatility Grier (2003). However, they have their disadvantages. Optic tweezers may damage cells by heating and the system itself is large and expensive. Recently developed acoustophoresis techniques have enabled the separation and manipulation of microparticles of different sizes and densities in microfluidic channels. This method is notably advantageous because it requires no pretreatment of the particles, label free and can be applied to virtually all kinds of particles, regardless of optical or charge properties. At first this approach uses bulky transducers to generate bulk acoustic waves (BAWs), which are then coupled into a silicon-based microchannel with a width equal to half the BAW wavelength. The resonance of the BAWs inside the channel results in a standing BAW field with a pressure node at the channel center. Particles injected along the sidewalls of the channel experience axial acoustic forces whose magnitudes depend on the particle size, density, and compressibility. These differing forces reposition

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the particles with different lateral displacements, thus achieving particle separation Wiklund (2012). However, the formation of the standing BAW requires the channel material to possess excellent acoustic reflection properties (e.g., silicon, glass, etc.), that the soft polymer materials commonly used in microfluidic applications, such as polydimethylsiloxane (PDMS), do not have. Other solution is to introduce phononic crystals in systems. Phononic crystals (PC) are two- or three-dimensional periodic structures that are made of at least two materials with different mechanical properties. Phononic crystals are the acoustic analogs of photonic crystals for optical waves. They can exhibit complete band gaps, i.e., finite continuous frequency regions where energy propagation is forbidden for all possible wave directions Kushwaha (1993), or conversely where only evanescent waves are allowed Laude (2009). Band-gap locations and widths mainly depend on the materials employed in the crystal construction, the lattice geometry, and the size and shape of any inclusions Reinke (2011). In this paper, we introduce deeply corrugated one-dimensional phononic crystal grating in PDMS for the energy localization in the PDMS walls near the microchannel with respective protection of other parts of the device.

The paper is organized as it follows. At first, governing equations set up then finite size phononic crystal grating is investigated for energy localization.

## 2. Theory. Governing equations.

Eulers equation and continuity equation when the wave amplitude is small are of the form Landau (1987)

$$\rho \frac{\partial v}{\partial t} = -\nabla p, \quad \frac{\partial p}{\partial t} = -(1/\kappa_s)\nabla \cdot v. \quad (1)$$

where  $p$  defines pressure and  $v$  velocity vector.  $\kappa_s = 1/(\rho_f c_f^2)$  is the compressibility.

Solid for small deformations is modeled by Hooke's law and by the equation of second Newton's law in the absence of exterior forces Royer (1999)

$$T_{ij} = c_{ijkl} S_{kl}, \quad \rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial T_{ji}}{\partial x_j}, \quad (2)$$

where Einstein's summation convention on repeated indices is implied. Elastic stress tensor  $T_{ij} = T_{ji}$  and deformation tensor  $S_{kl} = S_{lk}$ , elastic material constants  $c_{ijkl}$  satisfy  $c_{ijkl} = c_{jikl} = c_{ijlk} = c_{ijlk}$ .  $\rho$  denotes the mass density.

Acoustic-elastic wave coupling is achieved by boundary conditions applied on fluid/solid boundary. First, at interface the normal acceleration is continuous across both domains and is related to the normal derivative of the pressure  $p$  in the fluid,

$$\rho_f \omega^2 u_n = \frac{\partial p}{\partial n}, \quad (3)$$

with  $u_n$  the normal component of the displacement and  $\rho_f$  the mass density of the fluid. Second, the normal traction must be continuous across the interface, i.e.,

$$T_{ij} n_j = -p n_i, \quad (4)$$

## 3. Numerical model

Channel geometry is represented on Figure 1. It is characterized by inner width  $w$  and height  $h$  and outer width  $w_2$  and height  $h_2$ , where  $w$  is fixed to be  $400 \mu\text{m}$  and  $h = 150 \mu\text{m}$ . Height is chosen to be lower than the width in order to avoid resonances in y-direction. Elastic waves inside the channel and acoustic pressure waves in water are accounted for through a fluid-structure weak form FEM implementation discussed in Moiseyenko (2013) using Eq.1-4. At the interface between fluid and solid, a boundary condition relating the pressure in the fluid to the normal acceleration of the solid boundary is imposed. Conversely, the traction acting on the solid boundary is taken as the pressure exerted by the fluid. Excitation is modeled as a source of pressure waves through implementation of displacement parallel x-axis (red line) border in order to excite half wave-length resonance. Other boundaries are left free of traction.

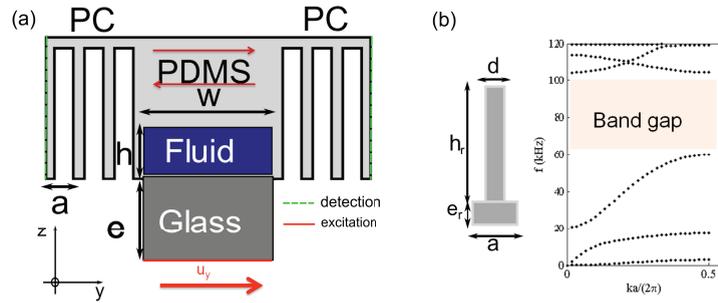


Fig. 1. (a) PDMS ordinary channel and with phononic crystal (b) Unit-cell geometry. Band structure.

Material properties are given in Table 1, where  $c_s$  and  $c_l$  correspond to shear and longitudinal sound velocities respectively,  $\rho$  is the material density. Acoustic impedance  $z$  is introduced as product of material density to longitudinal velocity. Sixth column represents  $z/z_w$  the ratio of material impedance with respect to acoustic impedance in water.

We show how phononic crystal can be used for energy localization, when a protection of part of the device is needed. Phononic crystal of periodically distributed ridges is introduced into PDMS. Thanks to PDMS material properties and ridges geometry we can get low frequency band gap.  $n$  is the number of periodically spaced holes. Figure 2 represents (a) finite-size ( $n = 3, 4$  and  $5$  ridges from each sides are considered) and (b) unit-cell of infinite-size phononic crystal and respective band structure. Geometric values are  $a = 200 \mu\text{m}$  for periodicity,  $d = 180 \mu\text{m}$  is a ridge width,  $h_r = 270 \mu\text{m}$  is a ridge height,  $e_r = 30 \mu\text{m}$  is for the PDMS plate.

Figure 2 (a) shows transmission through finite-size structure from 60 kHz to 100 kHz. Detection is made at the left and right sides of the structure (green dashed line). Figure 2 (b) shows acoustic energy density inside the channel as integral over water area  $V$ ,  $E_{ac} = \frac{1}{hw} \int_{V_{water}} \kappa_s p^2 dV$ . Channel becomes the defect in periodic structure. Thus its local resonance can be put inside the gap as can be seen from displacement and pressure field. One can notice that  $z$ -component of displacement field (for  $n = 5$ ,  $f = 68.3 \text{ kHz}$ ) at the resonance shows energy localization around the channel.

Table 1. Material properties.

	$c_s$ (m/s)	$c_l$ (m/s)	$\rho$ (kg/m <sup>3</sup> )	$z = \rho c_l$	$z/z_w$
Water		1450	1000	1450000	1
PDMS	32.2	1019	1108	1038361	0.7161
Glass	3280	5640	2320	13084800	9.024

#### 4. Conclusions and perspectives

Theoretical study of phononic-assisted control of ultrasound waves in acoustofluidic devices has been presented. We propose the use of phononic crystal diffractors, which can be introduced in acoustofluidic structures. These diffractors can be applied in the design of efficient resonant cavities, directional sound waves for new types of particle sorting methods, or acoustically controlled deterministic lateral displacement. The PnC-diffractor-based devices can be made configurable, by embedding the diffractors, all working at the same excitation frequency but with different resulting diffraction patterns, in exchangeable membranes on top of the device. Phononic crystal can be used if localization of acoustic energy is required or if parts of the device need to be shielded from elastic waves

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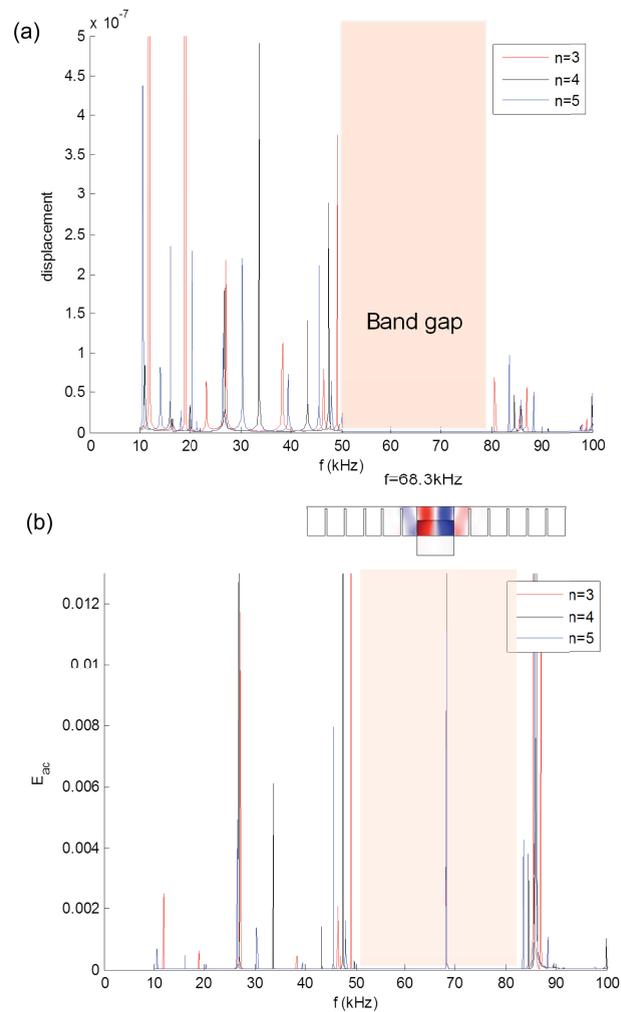


Fig. 2. (a) Transmission through the phononic crystal (b) Acoustic energy density  $E_{ac}$  in the channel.  $n$  is the number of periodically spaced holes. Pressure field and  $z$ -component of displacement field for  $n = 5$ ,  $f = 68.3$  kHz.

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