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THERMAL MODELS FOR INTELLIGENT HEATING OF BUILDINGS

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ABSTRACT

The Danish government has set the ambitious goal that the share of the total Danish electricity consumption, covered by wind energy, should be increased to 50% by year 2020. This asks for radical changes in how we utilize and transmit electricity in the future power grid. To fully utilize the high share of renewable power generation, which is in general intermittent and non-controllable, the consumption side has to be much more flexible than today. To achieve such flexibility, methods for moving power consumption in time, within the hourly timescale, have to be developed. One approach currently being pursued is to use the heat capacity of the thermal mass in buildings to temporarily store excess power production by increasing the electrical heating. Likewise can the electrical heating be postponed in periods with lack of production. To exploit the potential in thermal storage and to ensure the comfort of residents, proper prediction models for indoor temperature have to be developed. This paper presents a model for prediction of indoor temperature and power consumption from electrical space heating in an office building, using stochastic differential equations. The heat dynamic model is build using a grey box approach, i.e. by formulating the model using physical knowledge about heat flow, while the parameters in the model are estimated using collected data and statistics. The physical parameters in the model, e.g. heat capacities and resistances to transfer heat, have been estimated for an actual office building using a maximum likelihood technique.

Keywords: Heat dynamic modelling, demand side management, flexible load, smart grid, power system services

1. INTRODUCTION

A part of the solution to the growing global interest in reducing \(\text{CO}_2\) emissions is an increased integration of renewable energy into our power systems. However, coping with an increasing multitude of small, highly fluctuating, non-controllable and often distributed energy sources puts a high demand on the future power systems. As an example the newly elected government in Denmark has set the goal that the power consumption in the national grid, covered by wind energy, should be increased
to 50% by year 2020, from an already world high of 22% in 2010, [1]. Integration of that amount of intermittent generation, with a non-flexible consumption, would lead to great power fluctuations in the power system with only little means of control. Such a situation is depicted in Figure 1 were the projected power consumption and wind power production is plotted, if such a scenario was implemented in Denmark in year 2025.

The figure shows that the power production from wind turbines exceeds the consumptions several times during the second quarter of 2025. To ensure the stability of the power system, some of the wind turbines would have to be shut down, but this is both undesirable from a socio-economic point of view and also expensive for wind turbine owners. Therefore, if such an ambitious goal is to be reached we have to rethink how electric energy is distributed, utilized and especially the role of consumers has to be reconsidered.

If excess power generation could somehow be stored temporarily it would greatly benefit to the stability of the power system, without wasting energy. There are many different technologies offering energy storage, but these are usually expensive and associated with energy losses. Another option is to store the energy locally on the consumer side, whenever there is such a capacity. The thermal mass of buildings offers such a capacity and can therefore be used as a unidirectional storage for electrical power, wherever electrical space heating is used. In practice this can be done by increasing electrical space heating in periods with excess power production and postpone consumption in periods with low production. Such a contribution might seem modest, but many units aggregated intelligently in the distribution grid can have a huge impact on power balancing, within the hourly timescale. Varying space heating with power generation of course affects the indoor temperature in a building. If comfort should not be compromised for the residents of the building, the indoor temperature should not be allowed to vary more than a few degrees Celsius. But for well insulated buildings with a high thermal mass, even letting the indoor temperature vary within a single degree, means that heating can be precipitated or postponed up to an hour, which would benefit the power system.

To ensure the comfort of the residents of the house, models for prediction of indoor temperature, based on heating decisions, are needed. This paper presents a heat dynamic model for prediction of the indoor temperature and future energy needs for electrical heating in an office building. The model has successfully been used in applications for providing power system services in the small distributed power system, SYSLAB.

Several studies have been carried out in modelling the heat dynamics of buildings, for example [3] and [4], but these have been conducted either for small buildings with few rooms or for test cells. The research in this paper uses principal component analysis to estimate one representing parameter for the temperature of an office building, with eight rooms. Also pseudo-random sequences have been used to design a heat input from electrical space heaters that ensures optimal conditions for parameter estimation. The model is derived for a light office building placed at the DTU Risø campus in Denmark. For heat dynamic models of other types of buildings, see [5].

For a more detailed description of the model, see [6].

2. MATERIALS AND METHODS

This section initially describes the distributed power system, SYSLAB, which the intelligent office building, FlexHouse is a part of. The thermal model has been derived specific for FlexHouse, which has also formed the basis for data gathering. Section 2.2 describes the general theory behind heat transfer and how models of the mechanisms
behind it can be used as sub-models in an overall model of
heat exchange in an office building. In Section 2.3 a
description of how the heat input was designed for optimal
parameter estimation is given. This section also describes
how the collected temperature data was processed to
reduce dimensionality from eight measurements to one
parameter representing an aggregated temperature in the
building. Finally in Section 2.4 a technique for parameter
estimation is briefly described, along with the software
CTSM\(^1\) that can be used to find parameter estimates.

2.1 SYSLAB and FlexHouse

The DTU Risø campus is housing an experimental research
facility for distributed power systems called SYSLAB. SYSLAB
is a small power system with a high share of generation
from renewable energy sources. The power system consists
of a number of small distributed power generating and
consuming units that can be coupled together in any given
configuration. Also part of the system can be decoupled
from the local distribution grid, thus allowing parts of the
system to be run in island mode. Among the components of
interest are:

- Two wind turbines, 11kW and 10kW [Generation]
- Photovoltaics, 7kW [Generation]
- Diesel generator, 48kW [Generation]
- Vanadium battery, 15kW/120kWh
  [Storage/Generation/Consumption]
- Plug-in hybrid vehicle, 9kWh [Consumption/Storage]
- Electric vehicle, 16kWh [Consumption/Storage]
- FlexHouse, 20kW [Consumption/Storage]

Most of the power generating units in SYSLAB are
renewable and SYSLAB is therefore an ideal laboratory for
research in power systems with a high penetration of
renewable energy. Each unit in SYSLAB is controlled locally
by a small computer on which custom build control
software can be implemented and tested. This gives a very
flexible set-up with emphasis on control of the distributed
energy resources.

On the consumption side there is an intelligent office
building, called FlexHouse, with a peak consumption of
20kW. The building has been build from light materials and
consists of seven small office rooms, a main hall with a
kitchenette and a toilet. The layout of the building can be
seen in Figure 2.

From the layout it can be seen that the house has a big
window facade in the southwards direction, why solar
radiation has a big impact on the heat dynamics of the
building.

FlexHouse has been equipped with wireless sensors and
actuators to enable the building to be easily monitored
and controlled. Each of the eight rooms has been equipped with

\(^1\) Continuous Time Stochastic Modelling (http://www2.imm.dtu.dk/~ctsm/)

a temperature sensor, mounted centrally on the wall and a
motion sensor for monitoring of presence in the room. All
windows and doors have been equipped with sensors, such
that the state of these can be monitored as well.

![Figure 2: FlexHouse layout](image)

The main power consumption of the house comes from ten
electrical space heaters, with a total consumption of 10kW.
Also a water heater, a refrigerator and heat pumps have
been installed in the building. Each of the units is controlled
individually by an actuator that communicates wirelessly
with a house controller.

A software platform has been developed for control of the
building. It gathers sensor input, logs the building state in a
database and sends commands to the actuators. The
platform makes implantation of custom control algorithms
simple, allowing different control concepts to be tested
easily.

The software platform is also gathering local weather data
from a weather mast placed outside FlexHouse. The
weather data can be used to estimate future needs for
electric heating.

2.2 Thermal Model Formulation

The approach to develop the heat dynamic model for
FlexHouse is to use a grey-box modelling technique, thus
using prior physical knowledge about heat dynamics of
buildings to establish the model structure together with
collected data and statistics to estimate the model
parameters.

Modelling is typically an iterative process that involves
model formulation, parameter estimation and model
validation. Due to the complexity of the heat flow in
FlexHouse, several models structures were proposed and
investigated before a final model was accepted. It is only
the model that has shown the best performance that will
be derived in this paper. For further information about the
iterative process of formulation heat dynamic models of
building see [5].
Heat dynamic models are usually composed of sub models for the mechanisms behind heat transfer. The most general of these are:

**Conduction:** Heat transfer through a medium  
**Convection:** Heat transfer between two different media  
**Ventilation:** Heat transfer due to mass transfer, e.g. heated air through an open window  
**Radiation:** Heat transfer between objects that are in optical contact, e.g. solar radiation

Heat transfer between two media by conduction, convection or ventilation is proportional to the temperature difference between the two media involved. In the following also heat exchange by ventilation is assumed to be linear with the temperature difference between the indoor and outdoor air. This approximation holds for low wind speed, however, it is well known that for high wind speed, the natural ventilation of buildings becomes nonlinear with both wind speed and direction. To keep the model in a linear framework the linear approximation is accepted to be good enough. The heat transfer from conduction, convection and ventilation can therefore be modeled using thermal resistance,

\[
\dot{Q} = \frac{dQ}{dt} = \frac{1}{R} (T_1 - T_2) \tag{2.1}
\]

where \(T_1\) and \(T_2\) are the temperatures of each media involved and \(R\) is the resistance to transfer thermal energy.

Another important aspect of heat dynamics is the heat capacity that describes a material's ability to accumulate heat. The temperature development in a medium is given by the differential equation,

\[
\dot{Q} = C(T) \frac{dT}{dt} \approx C \frac{dT}{dt} \tag{2.2}
\]

As seen in the equation the heat capacity is dependent on temperature, however, the temperature interval in which the house is operated is very small, thus the heat capacity can be assumed constant.

The heat dynamics of FlexHouse is modeled as one large room with some interior walls and a house envelope. The air temperature inside the room, house envelope and interior walls and floors is assumed to be homogenous distributed. Thereby the model can be formulated as a lumped model with three temperature states; the temperature of the indoor air \((T_i)\), the temperature in the house envelope \((T_{om})\) and in the interior walls and floors \((T_{im})\).

The modeled heat flow in the one room model is depicted in Figure 3. It can be seen that the interior wall exchanges heat with the indoor air by convection, (1). Likewise does the indoor air exchange heat with the inside of the house envelope (2) and directly to the outside by ventilation, (3). Moreover the outside of the house envelope exchanges heat with the ambient environment, (4). Energy is feed into the building from the controllable space heaters (5) and by solar radiation through the windows, (6). The solar radiation is partly absorbed when it passes through the indoor air before hitting the interior walls. It is assumed that the fraction of solar radiation absorbed by the air, is given by \((1-p)\), thus only the fraction \(p\) reaches the interior walls. Heat conduction between interior walls and the house envelope is neglected due to the small surface being shared between these two media.

**Figure 3:** Exchange of heat in a building

Due to similarities between Equation 2.1 and 2.2 and those of electrical resistance and capacitance, the heat dynamics of a building is usually modeled as an electrical network, consisting of resistors and capacitors, with temperature equivalent to voltage and heat flow equivalent to flow of electrical charge, i.e. current. The equivalent electrical diagram to the heat flow in Figure 3 can be seen in Figure 4.

**Figure 4:** Equivalent RC-diagram

It should be noted that \(T_{om}\) and \(T_{im}\) are not directly observable in FlexHouse but they can be estimated, e.g. using a Kalman filter.
From Figure 4 the three coupled differential equation describing the temperature states can be derived as

\[
\frac{dT_i}{dt} = \frac{1}{R_a}(T_a - T_i) + \frac{1}{R_i}(T_{im} - T_i) + \frac{1}{R_{im}}(T_{om} - T_i) + A(1-p)\Phi_s + \Phi_n
\]  
(2.3)

\[
\frac{dT_{im}}{dt} = \frac{1}{R_i}(T_i - T_{im}) + Ap\Phi_s
\]

\[
\frac{dT_{om}}{dt} = \frac{1}{R_{im}}(T_i - T_{om}) + \frac{1}{R_{am}}(T_a - T_{om})
\]

where \(C_i, C_{im}\) and \(C_{om}\) are the heat capacities of the indoor air, interior walls and house envelope, respectively. \(A\) is the effective window area and \(p\) is the fraction of solar radiation not being absorbed by the indoor air before hitting a interior wall.

Equation 2.3 can be reformulated on matrix form

\[
\frac{d\mathbf{T}}{dt} = \mathbf{A}\mathbf{T} + \mathbf{B}\mathbf{U}
\]  
(2.4)

where \(\mathbf{T} = [T_i, T_{im}, T_{om}]^T\) is a state vector containing the temperature states and \(\mathbf{U} = [T_a, \Phi_s, \Phi_n]^T\) is the exogenous inputs to the system; ambient temperature, solar radiation and heater input. The 3x3 matrices, \(\mathbf{A}\) and \(\mathbf{B}\) defines how the current state and exogenous input affects future states, respectively.

Equation 2.4 describes a purely deterministic system. To account for deviations from this ideal world a stochastic process is usually added to the differential equation, which leads to the stochastic differential equation given by

\[
d\mathbf{T} = \mathbf{A}\mathbf{T}dt + \mathbf{B}\mathbf{U}dt + d\mathbf{\omega}(t)
\]  
(2.5)

where \(d\mathbf{\omega}(t)\) is a stochastic process which in the following is assumed to be a process with independent increments, i.e. a Wiener process, with incremental covariance \(\mathbf{R}_1\).

For predictions at time \(\tau\), Equation 2.5 can be integrated over a given prediction horizon, \(\tau\). The solution can analytically be found to be

\[
\mathbf{T}(t + \tau) = \mathbf{T}(t)e^{\mathbf{A}\tau} + \int_t^{t+\tau} e^{\mathbf{A}(t+s)\mathbf{B}\mathbf{U}(s)}ds + \int_t^{t+\tau} e^{\mathbf{A}(t+s)\mathbf{R}_1}\mathbf{\omega}(s)
\]  
(2.6)

Assuming that the exogenous input, \(\mathbf{U}\), is constant over the prediction horizon, Equation 2.6 can be reduced to the linear stochastic state space model.

\[
\mathbf{T}(t + \tau) = \mathbf{\Phi}(\tau)\mathbf{T}(t) + \mathbf{\Gamma}(\tau)\mathbf{U}(\tau) + \mathbf{\nu}(t, \tau)
\]  
(2.7)

where

\[
\mathbf{\Phi}(\tau) = e^{\mathbf{A}\tau}
\]

\[
\mathbf{\Gamma}(\tau) = \int_0^\tau e^{\mathbf{A}\tau\mathbf{B}}d\mathbf{\omega}(s)
\]

\[
\mathbf{\nu}(t, \tau) = \int_t^{t+\tau} e^{\mathbf{A}(t+s)\mathbf{R}_1}\mathbf{\omega}(s)
\]

Assuming that \(\mathbf{\omega}(t)\) is a Wiener process, \(\mathbf{\nu}(t, \tau)\) becomes normal distributed white noise with zero mean and covariance,

\[
\mathbf{R}_1(\tau) = \left[\begin{array}{ccc}
R_{11} & 0 & 0 \\
0 & R_{22} & 0 \\
0 & 0 & R_{33}
\end{array}\right]
\]

Only one of the three states in \(\mathbf{T}\) is observable, therefore an observation equation is introduced,

\[
T_i(\tau) = \mathbf{C}\mathbf{T}(\tau) + e(\tau)
\]  
(2.9)

where \(e(\tau)\) is the measurement error of the temperature. The error is assumed to be normal distributed white noise with zero mean and variance \(R_e\). In this case only indoor air temperature is measured, i.e. the first state in \(\mathbf{T}\), thus \(\mathbf{C} = [1,0,0]^T\).

### 2.3 Data processing

Four time series of the room temperatures and ambient parameters were collected in FlexHouse during four experiments in February and March 2008. The data in the time series was collected with a sampling time of 5 minutes. The heaters in FlexHouse were controlled in a binary synchronous pattern, i.e. all ten heaters on or off, in any given time step. To ensure optimal conditions for parameter estimation, the binary control signals were generated as pseudo random binary sequences, which is a deterministic signal with white-noise properties. This was done to ensure independence from all other exogenous input, e.g. solar radiation. Also the signal can be generated such that the system is excited in the right area of the time domain where the time constants are expected to be. All interior doors were open during the measurements, to allow the heat from the heaters to dissipate freely between rooms.

Since the indoor temperature is modeled as a single state, the dimension of the temperature measurements has to be reduced from eight to one. An obvious approach would be simply to use the average temperature, but since the rooms are not equal in size, the measurements should not be weighted equally. Therefore principal component analysis has been used to decrease the dimension of measurements to a single temperature representing parameter. It was found that the first principal component accounts for approximately 90% of the variance in data, why the first principal component gives a sufficiently good estimate of an indoor temperature to be used as the temperature representing parameter.
A plot of the average temperature development, the first principal component and the heat input can be seen in Figure 5.

The plot clearly shows how the heat input excites the indoor temperature. Also the affect of solar radiation can be seen in the plot. It is especially clear around measurement number 500, where the temperature raises despite that all the heaters are off.

### 2.4 Parameter Estimation

The parameters in Equation 2.3 have been estimated using CTSM; a parameter estimation tool developed at the Department of Informatics and Mathematical Modelling at the Technical University of Denmark (DTU). CTSM can be used to find maximum likelihood estimates which are given by

\[
\hat{\theta} = \arg\max_{\theta} \{ L(T(n); \theta) \}
\]

where \( \theta \) is a vector containing the unknown model parameters in Equation 2.3, \( T(n) \) is a vector with all the observations up to and including \( n \) and \( L \) is the joint probability function. Given the model structure and observed data, a routine in CTSM maximizes the joint probability function, i.e. determines the most likely parameter set, using a Kalman filter, and returns the estimates including the standard deviation of the estimates.

CTSM is a great tool for estimation of model parameters in stochastic linear state space models. The system and measurement equation are easily implemented in CTSM, along with the assumption about the Wiener process and the normal distributed measurement error.

### 3. RESULTS

This section initially states the maximum likelihood estimates and then the performance of the model is presented. Finally a description of the model validation is given.

Based on the four time series collected in FlexHouse model, parameters have been successfully estimated using CTSM. The parameters were first estimated, allowing the fraction of the solar radiation absorbed by the interior walls, \( p \), to be free. The estimate was found to be 0.995, i.e. 99.5%, which imply that none of the incoming solar radiation is absorbed by the indoor air, but is absorbed entirely by the interior walls and floors. The parameters were re-estimated assuming that none of the solar radiation is absorbed by the indoor air, i.e. \( p = 1 \). The final parameter estimates can be seen in Table 1.

The estimates shows that the heat capacity of the house envelope is quite large compared to the other heat capacities, which is what was expected, since the interior walls are made from light weight materials. It is also noted that the heat capacity of the interior wall is very small compared to that of the indoor air. This is not what would be expected since the interior walls should be able of store more heat than the indoor air. The very low heat capacity can partly be explained by the estimate of the variance of the temperature in the interior walls, \( R_{22,1} \), which is quite large, indicating that there is a very large uncertainty on the temperature in the interior walls. Additional collected data could be used to reduce this uncertainty.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>ML estimate</th>
<th>Standard Error of estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_i )</td>
<td>46.848°C/kW</td>
<td>0.134</td>
</tr>
<tr>
<td>( C_i )</td>
<td>8.120kWh/°C</td>
<td>0.294</td>
</tr>
<tr>
<td>( R_{im} )</td>
<td>1.809°C/kW</td>
<td>0.147</td>
</tr>
<tr>
<td>( R_a )</td>
<td>8.021°C/kW</td>
<td>0.501</td>
</tr>
<tr>
<td>( C_{im} )</td>
<td>0.00553kWh/°C</td>
<td>0.000611</td>
</tr>
<tr>
<td>( C_{am} )</td>
<td>292.950kWh/°C</td>
<td>7.675</td>
</tr>
<tr>
<td>( R_{am} )</td>
<td>0.0846°C/kW</td>
<td>0.0294</td>
</tr>
<tr>
<td>( A )</td>
<td>20.212m²</td>
<td>1.491</td>
</tr>
<tr>
<td>( \bar{p} )</td>
<td>1.000</td>
<td>-</td>
</tr>
<tr>
<td>( \log(R_{31,1}) )</td>
<td>-2.754°C²</td>
<td>0.299</td>
</tr>
<tr>
<td>( \log(R_{22,1}) )</td>
<td>6.368°C²</td>
<td>0.0586</td>
</tr>
<tr>
<td>( \log(R_{33,1}) )</td>
<td>-0.943°C²</td>
<td>0.312</td>
</tr>
<tr>
<td>( \log(R_2) )</td>
<td>-7.066°C²</td>
<td>0.0755</td>
</tr>
</tbody>
</table>

Table 1: Maximum likelihood estimates with \( p = 1 \)
Unfortunately there is no building data available, for FlexHouse, to compare with the found estimates. However, the total window area can be measured and compared to the estimate. Assuming that the effective area is approximately 60% of the physical area, which has been measured to be 30m², the estimate is very close to the expected.

From the estimates in Table 1 and by use of Equation 2.7 with $\tau = 5$ minutes, a discrete model has been derived for 5 minute predictions. The one hour predicted temperature can be found by recursively calculate the 5 minute prediction. This way the one hour predictions in Figure 6 were found, assuming constant solar radiation, ambient temperature and heat input in the following hour. The plot also depicts the actual temperature development, along with the prediction error in the bottom plot. The plot shows that the indoor temperature can be predicted within 1°C over an hour, which is acceptable when considering the natural variation of the weather which is assumed constant when the predictions are made.

The model has been validated using the 5 minute prediction residuals. If these resemble white noise, the model is correct. The autocorrelation function for the residuals in Figure 7 shows some autocorrelation at lag 1-25 outside the 95% confidence interval. This indicates that the residuals are not fully described by white noise. However the autocorrelations are in general small and drops quite fast, why the model can be used as a good approximation.

Another test has been used to validate the model in the frequency domain. If the residuals were white noise the spectrum of these should be equally distributed over all frequencies. This can be shown by a cumulated periodogram which should form a straight line, if all frequencies are equally represented. Figure 9 depicts the cumulated periodogram for the residuals which almost stays within the 95% confidence interval. This shows that almost all frequencies are equally represented, thus the residuals show white noise properties in the frequency domain.
4. DISCUSSION

This section describes some of the applications of the heat dynamic model.

4.1 Applications

The heat dynamic model has been used in several applications for letting FlexHouse act as a flexible load, thus providing system services within SYSLAB. These applications have been used to demonstrate both direct and indirect control of power consumption.

FlexHouse has been integrated in a virtual power plant, run by the Danish electricity company DONG Energy. The virtual power plant sends a setpoint for the power consumption to the house controller which then modifies its consumption if not violating constraints for user comfort. The setpoint is based on an hourly consumption forecast for the house generated by the heat dynamic model. The forecast is generated every day at 10 a.m. for the following 38 hours, i.e. until midnight the following day. A local weather forecast, with predicted solar radiation and outdoor temperature, is used as model input. Given the forecast the virtual power plant optimizes the power consumption over a given day, asking the house deviate from the forecast if necessary.

The model has also been implemented in a model predictive controller for indirect control of power consumption. The controller downloads a price signal for the following 12 house, which is then used to optimize the heating of FlexHouse, such that cost of heating is minimized. Figure 9 shows the optimized power consumption such that heating during peak load hours is avoided. The figure clearly illustrates how the controller avoids heating the building during periods of high prices, postponing the heating to periods with low prices.

For more information about indirect control using price signals and model predictive control, see [7]
CONCLUSIONS

This paper demonstrates that a heat dynamic model can be used in applications for intelligent control of electrical heating. By controlling the electrical space heating in an office building intelligently, power consumption can usually be increased or postponed within the hourly timescale, thus allowing the building to provide ancillary services in a power system.

The model has shown good results in predicting indoor temperatures and future power consumption needs. Despite the good performance of the model, much can be done to improve the performance of the model. Due to high amount of natural ventilation in FlexHouse especially the nonlinear properties of wind conditions should be integrated in the model, due to its influence on the indoor temperature.

Unfortunate FlexHouse is a very light building that does not allow a lot of thermal energy to be stored in the building. Brick buildings with thick concrete floors offers larger thermal capacities and would therefore be preferred as flexible loads when using thermal storage. However for demonstration and proof of concept the applications have show good results and demonstrated that FlexHouse can be used as a flexible load in SYSLAB.
REFERENCES


