Understanding of bridge cable vibrations and the associate flow-field through the full-scale monitoring of vibrations and Wind

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This dissertation investigates the conditions that promote rain-wind-induced vibrations of inclined cable on cable-stayed bridges. Data was collected from the Oresund Bridge. Cable vibrations amplitude and frequencies of vibrations, wind directions and speeds, and rainfall rates are reported. Aerodynamic damping is investigated using the Markov-Block-Hankel matrix. For the first time, back-calculated aerodynamic coefficients were obtained from full scale data and compared to results from wind tunnel tests and literature. Finally, conclusions on the role played by aerodynamic damping for the rain-wind-induced vibrations are given.

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Antonio Acampora

PhD Thesis
Department of Civil Engineering
2013

DTU Civil Engineering Report R-291
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2013
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Understanding of bridge cable vibrations and the associate flow-field through the full-scale monitoring of vibrations and wind

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Preface

This thesis is submitted as a partial fulfillment of the requirements for the Danish Ph.D. degree. The work has been carried out at the Department of Civil Engineering at the Technical University of Denmark and took place in the period between December 2009 to May 2013, with Associate Professor Christos T. Georgakis as main supervisor, Associate Professor Francesco D’assisi Ricciardelli and Mogen Arentoft as co-supervisors. This thesis is based on published/under-review articles in ISI journals and conference articles. The thesis is divided in three parts. The first part introduces the main problem and the motivation for this work, together with a review of the aerodynamics of twin cables, mechanisms of vibrations and the aerodynamic damping for bridge cables. Finally, this part reports also cable vibrations events observed on full-scales cable-supported bridges instrumented with monitoring systems and an insight in the system identification tools used in this work. The second part is made up by separate and unedited reproductions of above mentioned journal/conference papers. Finally, in the third part, the combined work is discussed and conclusions are made. Considerations are given respect to the initially introduced problems together with suggestions for future work.

Kgs. Lyngby, the 31st October 2013
The thesis was defended at a public defence on 31/10/2013.

The official assessment committee consisted of:

Associate Professor Gregor Fischer (chairman), Technical University of Denmark;
Professor Brincker Rune, Århus University, Denmark;
Professor Jakobsen Jasna Bogunovic, University of Stavanger, Norway.

Kgs. Lyngby, the 22nd August 2013

Antonio Acampora
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I would like to thanks my fellow Ph.D students at Technical University of Denmark for their friendship in these years of intense and interesting work.

I gratefully acknowledge the support from the employees and technicians at Denmark Technical University and at FORCE Technology during the experimental work at wind tunnel facility.

Thanks to Rune Brincker to grant the permission to use the material from his book (Brincker, R. and Ventura C.E.: Introduction to operational modal analysis. To be published on Wiley and Sons.) about system identification in the chapter of this thesis.

Thanks to PhD student Anela Bajeric for granting me the permission to use her system identification review.
Thanks to Emanuele Mattiello and Mads Beedholm Eriksen for granting me the permission to use material from their MSc. thesis *Aerodynamic Passive-Dynamic Wind Tunnel Tests of the Å̀resund Bridge Cables.*
Abstract

This dissertation investigates the conditions that promote rain-wind-induced vibrations of inclined cable on cable-stayed bridges. Rain-wind-induced vibrations are known as the most common type of cable vibrations and capable of severe vibrations. The recent increase in the number of cable stayed bridges continuously becoming longer and lighter have resulted in a high number of observations of cable vibrations. A theoretical background for the tool used in this work is presented in terms of cables vibrations mechanisms, aerodynamic damping and system identification techniques. A detailed literature review of reported observations of rain-wind-induced cable vibrations of full-scale bridges is shown. The database of observed events on bridges collects information about the conditions that likely develop the phenomenon, together with the means used to suppress or reduce the occurrence of cable vibrations.

The research starts from data collection of cables vibrations of the Øresund Bridge. A dedicated monitoring system was installed to record full-scale data together with wind field measurements and meteorological data, during cables vibrations.

Results from the monitoring system are reported such as cable vibrations amplitude, cables frequencies involved in the vibrations, wind directions, wind speeds and rainfall rates. Those indications are used to select full-scale cables vibrations for further analyses. In particular, aerodynamic damping is investigated by means of system identification procedures based on the Markov-Block-Hankel matrix.

From identified aerodynamic damping from full-scale data, back-calculated aerodynamic coefficients of Øresund Bridge cable were obtained.

Wind tunnel tests were performed on a scaled model of the actual cables both in dynamic and static tests to investigate aerodynamic coefficients, in parallel to their identification by means of back-calculation from full-scale data.

Finally, conclusions on the role played by aerodynamic damping for the rain-wind-induced vibrations are given, combining the understanding from
the state of the art and the results from full-scale monitoring events.
Special thanks to Rune Brincker to grant permission to use the material of
his newly published book about system identification to form the chapter of
this thesis. A special thanks also to Anela Bajeric for granting the permission
to use her material about system identification review for the needs of this
thesis.

Afhandlingen tager udgangspunkt i indsamling af data over kabelsvingninger på Øresundsbroen. For at kunne optage fuldskala-data sammen med vindmålinger og meteorologisk data under kabelsvingningen blev et overvågningsystem installeret.

Resultaterne fra overvågningssystemet såsom amplituden af kabelsvingninger, kabelfrekvenserne under svingningerne, vindretningerne, vindhastighederne og regnmængderne er afrapporteret. Disse er brugt til at udvælge fuldskala kabelsvingninger til videre analyse. Særligt er aerodynamisk dæmpning undersøgt ved hjælp af procedurer for systemidentifikation baseret på Markov-Block-Handel matricen.

Tilbageberegneede aerodynamiske koefficienter blev fundet for Øresundsbroen ved at identificere aerodynamisk dæmpning fra fuldskala-data.

For at undersøge de aerodynamiske koefficienter blev både statistiske og dynamiske vindtunnelforsøg udført med en skaleret model af de egentlige kabler. Koefficienterne blev sideløbende identificeret ved tilbageberegning fra fuldskala-data.

Endelig er der givet konklusioner på den indflydelse, som aerodynamisk
dæmpning har påregn- og vindinducerede svingninger. Dette er gjort ved at kombinere forståelsen fra state-of-the-art med resultaterne fra fuldskal-overvågningen.
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Part I

Introduction and summary
Chapter 1

Introduction

The continuous development of lighter and more efficient building materials make possible to realize structures that are longer, lighter and slender compared to traditional ones. These developments have increased the number of worlds structures such as long-span bridges, guyed telecommunications masts and stadiums that suffer from some form of wind-induced vibrations. In the field of long-span cable-supported bridges, an increasing interest in the dynamic effects of wind loads on structures has arose throughout the last decades. In particular, cables structures exhibit a reduced stiffness and lower inherent structural damping compared to other civil structures, resulting in structures more susceptible to wind loads and vibrations and their dynamic effects on the overall stability compared to other more massive structures. Extensive researches and full-scales experiences have been undertaken in the field of bridge aerodynamics. Since forties, the Tacoma Narrows Bridge made clear that long and light bridge decks lead to flutter instability. Starting around the seventies, cable stay bridges have become increasingly popular, as improvements in materials and technology have resulted in cable stays bridges becoming an economical way to cross medium to long spans but with the increasing span lengths new aerodynamic challenges arose, forcing engineers and bridge owners to ask for a better understanding of the wind-structure interactions. Cables excitation mechanisms are currently grouped into two forms: those originated by cable-fluid interaction and those which are consequence of some form of end loading of the cables. To the former group belong vortex-shedding, high reduced-velocity vortex-shedding, drag crisis, dry inclined cable galloping, ice or sleet-induced vibrations, wake induced vibrations, rain-wind-induced vibrations. The latter group is represented by deck-cable interaction induced vibrations such as parametric excitations mechanisms, structure-induced vibrations.
Over the past 30 years, information acquired in this field has allowed for the safe design of the current long-span bridges against wind but questions about cables vibrations and fatigue problems still need to be fully investigated and understood. Therefore, a reasonable assessment of wind actions and their effects on the structure in the design phase is becoming more and more important and implementation of monitoring health systems is becoming of interest for engineers, contractors, scientist, public authorities and bridge owners. In 1986 was reported for the first time that cable stays experienced a severe unknown type of vibration: Rain-Wind-Induced Vibration (RWIV). This occurrences were firstly reported by Hikami (1986) as strong vibrations of inclined cable stays of up to 2 meters amplitude occurring under moderate windy and rainy days (4). Adequate knowledge of the phenomenon is required to better understand and to provide adequate means to control vibrations of inclined cables. These information have been obtained by means of full-scale experiences (18) (17) (10), wind tunnel investigations (3) (4) (9) and mathematical description of full-scale observations (8) (6) (7). Most common means used to control cables vibrations include cable-ties, viscous dampers, tuned mass dampers and aerodynamic modification of cables surface. These applications have different negative effects such as an increase in the cost by adding devices to the bridge and increasing maintenance costs (dampers), aesthetics considerations (cross-ties) to name a few and the chance that they result ineffective to fully suppress all range of vibrations. On the other hand, these solutions are becoming less effective in contrasting vibration due to increasing in length of bridge spans.

Rain-Wind-Induced Vibrations (RWIV) are responsible of about 95% of the cable-stay vibration incidences (3), for this reason a major interest in being able to fully understand, predict and control this vibration mechanism has arisen in the last decades. Even though the excessive vibrations rarely lead to ultimate cable failure, both premature fatigue damage, bridge closures and users discomfort are common issues. Characteristic examples of bridges with large amplitude cable vibrations include the Øresund Bridge (DK-SE), Storebælt Bridge (DK), Dongting Bridge (CN), First and Second Severn Bridge (UK), the Humber Bridge and the Fred Hartman Bridge (USA) - to name a few. Many observations have been reported about RWIV from bridges around the world, but collected information are often contradictory or incomplete. To satisfy the need of information, it has been decided to install and operate a monitoring system of cables vibrations on Øresund Bridge to collect continuously in-plane and out-of-plane accelerations together with wind and meteorological conditions. In parallel, static and dynamic wind tunnel tests have been performed at DTU/Force Technology closed circuit Climatic
Wind Tunnel in Kgs. Lyngby, Denmark to investigate RWIV.

1.1 Objectives

Large amplitude wind-induced vibrations of inclined cables are surprisingly common. They are produced by cable-flow interaction classified in various mechanisms, but mainly due to inherently low structural damping and by the additional aerodynamic damping due to aerodynamic properties of the cable surface.

The objective of this thesis is to assess the factors that might contribute to the cable to experience RWIV. In particular the influence to the aerodynamic damping on the overall damping available on site, due to the influence of rainfall. Different conditions of precipitation have been investigated to determine the role of rainfall on RWIV. Aerodynamic damping identified by means of system identification techniques on the full-scale data for both dry and rainy conditions are compared to theoretical aerodynamic damping based on wind tunnel tests as well as drag coefficients from wind tunnel in static and dynamic conditions are compared to back-calculated drag coefficients of actual cable under different weather conditions.

1.2 Methodology

To achieve the aforementioned objectives the present thesis is divided into three parts:

The first part introduces aerodynamic of twin cylinders and a review of vibration mechanisms for bridge cables, in particular the RWIV are presented in a separate chapter. A review of the state of the art and observed cases of RWIV on stay-cables bridges is given together with weather and wind conditions and the means used to control cable vibrations. Theoretical background for output-only methods and aerodynamic damping are introduced. Finally the full-scale monitoring system developed and installed on Øresund Bridge is presented.

In the second part, the results of the research are presented in the form of unedited conference/journal publications. Firstly, it is presented the data collected under simultaneous occurrence of wind, rain and larger displacements. The study determine the range of wind speed, wind direction and rainfall that are associated to RWIV. Consequently, data of interest are used
to identify aerodynamic damping for dry conditions and different level of rainy conditions. Drag coefficients from identified data of Øresund were back-calculated for the different weather conditions. A parallel study was done at climatic wind tunnel to study the aerodynamic properties of a scaled sample of Øresund twin cable for static and dynamic wind tunnel tests. Aerodynamic coefficients were obtained for wind orthogonal wind skewed respect the twin cables. The effect of presence or absence of the double helical fillet on the external surface of the cables was investigated for wind orthogonal to the cable sample. Aerodynamic coefficients were used in the quasi-steady formulation of aerodynamic damping for comparison with aerodynamic damping matrix identified from full-scale data.

The third and final part sums up observations, considerations and conclusions based on the results presented in the preceding section of the research. Conclusions in relation to the aforementioned objectives are presented. Ideas about future work on RWIV and further applications of system identification techniques are given.

1.3 Thesis outline

The present thesis is divided into 10 chapters. This introduction (Chapter 1) is followed by 9 chapters listed below:

Chapter 2 introduces elements about aerodynamic of twin cables, the mechanisms of vibrations for bridge cables and the theoretical approach to aerodynamic damping for one and two degrees of freedom systems.

Chapter 3 presents the state of the art for RWIV phenomenon observed on cable-stayed bridges. Observations of RWIV are listed in a table together with data available and means used to suppress/control those events.

Chapter 4 presents the theoretical framework used to identify dynamic properties from full-scale date acquired by accelerometer mounted on the monitored cables.

Chapter 5 presents a description of the monitoring system installed on Øresund Bridge. Details about both Øresund Bridge and the instrument used on the monitoring system are given.

The four following chapters are made up by separate and unedited repro-
Introduction

ductions of journal and conferences papers either already published or in the process of being published.

Chapter 6 contains several observations from the full-scale monitoring of Øresund Bridge to determine the conditions associated with occurrence of rain, wind and observed larger cables displacements. The monitoring campaign shows that there is a direct correlation between wind-cable angles, wind velocities and the amount of rainfall for larger cables displacements.

Chapter 7 presents results from data identification of full-scale measurements of Øresund Bridge. Using a state-of-the-art method of output-only system identification, the vibration modes of a cable have been detected with sufficient accuracy to identify changes in the total effective damping and stiffness matrices due to the aeroelastic forces acting on the cable. The damping matrices identified from the full-scale measurements have been compared with the theoretical damping matrices based on quasi-steady theory and two different sets of wind tunnel data. Drag coefficients were back-calculated from identifies damping matrices.

Chapter 8 presents the results of the system identification method introduced in chapter 6 for a simple case but here applied to full-scale data for skewed wind and for three different rainy conditions: no rain, light rain and moderate rain. How these conditions influence the aerodynamic damping and stiffness matrices is reported. Back-calculated drag coefficients for each of the three conditions are also presented.

Chapter 9 presents an investigation on a series of wind tunnel tests on a 1:2.3 scale section model of the Øresund Bridge twin stay cable arrangement. Drag, lift and moment coefficients about the longitudinal axis of the twin cable, with and without double helical fillets, are presented for varying wind angles of attacks. Aerodynamic damping of the cable pair is evaluated based on the resulting coefficients and compared with aerodynamic damping identified from full-scale data.

Chapter 10 presents a summary of the main findings and the conclusions as well as suggestions for future work.
1.3 Thesis outline

Introduction
Bibliography


Part II

State of the art and preliminary work
Chapter 2

Aerodynamics of bridge cables

In the present chapter the aerodynamics of twin cables and the vibration mechanisms of bridge cables are described. Total damping of the cable system assure its stability. The total damping is made by different contributions here reported. Each contribution is discussed. Aerodynamic damping theoretical approach for single degree of freedom (DOF) and 2-DOF in steady flow are presented.

The author want to thank Emanuele Mattiello and Mads Beedholm Eriksen for granting me the permission to use material from their MSc. thesis about cables aerodynamic *Aerodynamic Passive-Dynamic Wind Tunnel Tests of the Øresund Bridge Cables.*

2.1 Introduction

The Øresund Bridge cables are made up of a twin cables of outer diameter of 250mm connected to one or two locations along the cable length. The theory for twin cylinders considers the cylinders as parallel, plain, perfectly circular, identical, and the flow approaches the cylinders perpendicularly. In contrast with the theory, the actual cables are not perfectly circular, but may present local deformations due to manufacturing, bridge erection operations and aging. Scratches and labeling may be present on the surface modifying the surface roughness or acting as separation triggers. Moreover the Øresund Bridge cable are manufactured with double helical fillets running along the surface. These fillets change the aerodynamic properties of the actual cable respect the description of ideal twin-cylinders.
2.2 Twin cylinder arrangement

The Øresund Bridge twin-cable configuration imply that the presence of two cylinders interacts with flow differently than the case of single circular cylinder. The flow is often described by the dimensionless quantity Reynolds number, $Re$, which expresses the ratio of inertial forces to viscous forces. For small values of $Re$ the viscous forces are dominating and for larger values the inertial forces become dominating. Reynolds number is defined as:

$$Re = \frac{UD}{\nu}$$

where $D$ is a characteristic dimension of the considered body i.e. the diameter of a cable, $U$ is the flow velocity and $\nu$ is the kinematic viscosity of the fluid, which for air at standard atmospheric pressure and $20^\circ C$ is $1.51 \times 10^5 m^2/s$.

The flow behavior of a twin configuration depends on how the cylinders are positioned relative to each other. Zdravkovich (12) distinguishes between three simple arrangements of the two cylinders: side-by-side, tandem and staggered, which are sketched in Figure 2.1. For two cylinders placed relatively far from each other the flow around and the forces on the cylinders are the same as for the single cylinder. Synchronization of the vortex shedding may occur (11). The flow around two cylinders differs from the behavior of the single cylinder, when the two cylinders come closer. The different interference regions may be identified in:

- Wake interference; the flow around the downstream cylinder is affected by the wake of the upstream cylinder.

- Proximity interference; the cylinders are close to each other whereby the flow around one cylinder affects the other.

- Combined proximity and wake interference; this represents an overlap of the proximity and the wake interference.

Mainly the side-by-side arrangement was considered in the wind tunnel experiences, which means that the proximity interference region is dominating. Therefore three interference flow regimes are presented for this arrangement depending on the ratio between the transverse center-to-center spacing, $S$, and the cylinder diameter, $D$.

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For $1 < S/D < 1.2$ a single vortex street is formed behind both cylinders, which work as a single bluff body. The small gap between the cylinders only allows a weak flow, which suppresses vortex shedding at the gap in the same manner as the near-wall effect. This is the so called single vortex street regime.

For $1.2 < S/D < 2.2$ two wakes are formed behind the cylinders - a narrow and a wide wake. The flow through the gap is directed (biased) towards the narrow wake. The flow is bi-stable and may switch to either side. This is the so called biased flow regime.

For $2.2 < S/D < 5$ the two wakes are equal in size and the vortex shedding is synchronized. The out-of-phase coupling dominates and produces two mirrored vortex streets in the coupled wakes regime.

The interference regimes for the side-by-side arrangement are sketched in Figure 2.2. The center-to-center spacing of the Øresund Bridge cables is
670mm which gives a spacing ratio of \( S/D = 2.68 \), falling within the coupled wakes regime. The gap is sufficiently large for a parallel vortex street to form. Both cylinders undergo vortex shedding at the same frequency, either out-of-phase or in-phase. The behavior of the cylinders is closer to the single cylinder, but the proximity interference effects may lead to various modes of synchronization, anti-phase and in-phase, in the vortex formation and shedding processes. The two vortex streets lead to complex vortex street interaction in the combined wake of the cylinders.

### 2.3 Wind-Induced Cable Vibration Mechanisms

Wind-Induced cable vibrations can occur due to buffeting or vortex-shedding, but in most cases the larger amplitude vibrations are caused by dry galloping or galloping in the presence of rain, sleet, snow or ice. The different types of wind-induced cable vibrations are related to changes in aerodynamic forces. This section contains a description of different vibration mechanisms, which are induced by the wind. At last, the section will focus on rain-wind induced vibrations, which most likely account for 95% of the reported and measured case events on actual bridges, (5), (6).

#### 2.3.1 Buffeting

Wind gusts due to the turbulence in the wind cause buffeting of cables. The amplitudes of vibrations caused by buffeting are generally small compared to other vibration phenomena, but in some case they are sufficiently large to cause structural fatigue damages.

#### 2.3.2 Vortex-Induced vibrations

Vortex-induced vibrations are caused by regular von Kármán vortex shedding on alternating sides of a cable which induce an alternating load perpendicular to the mean flow direction. This results in an oscillating lift force. Vibrations due to vortex shedding are normally characterized by small amplitudes, but they may become large, when the vortex shedding frequency is close or equal to the structural eigenfrequency of the cable - commonly called lock-in or synchronization phenomenon. This resonant excitation can produce large displacements transverse to the wind direction. For a cylinder with diameter \( D \) immersed in a steady flow with flow velocity \( U \), the vortex shedding frequency is described by a dimensionless parameter, the Strouhal number, \( St \) defined as:

\[
St = \frac{f D}{U}
\]
\[ St = \frac{f_v D}{U} \] (2.2)

In the sub-critical regime, the Strouhal number is practically constant about a value of \( St = 0.2 \) for circular cylinders. It is possible to predict the wind speed, at which vortex shedding causes a resonant excitation knowing the eigenfrequencies of the cable.

### 2.3.3 Vortex-Induced Vibrations at High Wind Velocity

Vortex-Induced vibrations of cables at high reduced wind velocity is a three-dimensional phenomenon caused by the presence of an axial flow along the cable and enhanced by the presence of rain (5). The phenomenon has been observed and studied by many, e.g., Main and Jones (8), Matsumoto et al. (9), Cheng et al. (1) and Cheng et al. (2). The two-dimensional von Kármán vortex shedding interacts with the axial flow, and amplified vortices are created. The vortex shedding frequency of the amplified vortices is typically one third of the conventional frequency, meaning that every third vortex is amplified.

### 2.3.4 Galloping

Galloping is a vibration mechanism, which requires an initial transverse motion of the cable with respect to the mean wind direction. The transverse cable motion corresponds to a change in the relative angle of attack \( \alpha \), which induces a transverse load component. Galloping occurs when this load coincides with the motion of the cable and thereby amplifies the response, which may result in large amplitude vibrations. Hartog (1932) was the first to present a stability criterion for cross-flow oscillations. A D-shaped cross-section was used to illustrate the mechanism. A decrease in the drag coefficient and a negative slope of the lift coefficient may lead to galloping instability. Ice-coated cables may also suffer from galloping due to the deformed shape of the cross section, generating a change in the aerodynamic forces.

### 2.3.5 Dry-Inclined Cable Galloping

Galloping-like vibrations of inclined cables on cable-stayed bridges have been observed in dry condition by different authors ((9), (13), (14)). The mechanisms of dry inclined cable galloping have not been fully comprehended yet, and requires further studies.
2.3.6 Wake Induced Vibrations

When the cable is positioned downstream relative to another cable, wake-induced vibrations may be dominating and lead to instability. The turbulence of the wake may be random, but e.g. in case of conventional vortex shedding from the upstream body the wake may induce large vibrations, if the vortex shedding frequency and the eigenfrequency of the cable coincide, hence a vortex street induces buffeting resulting in instability similar to the lock-in due to vortex shedding. In the case of the Øresund Bridge, the distance of 670mm between the twin cables was suggested in the design phase, after wind tunnel testing, to avoid instability due to wake galloping (5).

2.3.7 Rain-Wind-Induced Vibrations

RWIV are believed to be the result of a complex non-linear interaction between the wind load on the cable and axial water rivulets formed by the rain, which run down the cable. RWIV are normally observed to happen in a limited wind speed range as reported in the previous chapter. Lower wind velocities are believed not to produce enough energy to excite the cable, while at higher velocities the water is blown off the cable, so the rivulet is not sustained, (6). Usually two rivulets are formed, one running along the top windward side and a bigger rivulet along the bottom leeward side of the cable leading to an asymmetric cross section and therefore a variation in aerodynamic forces on the cable. The rivulets trigger the flow separation and thereby modify the pressure distribution, compared to the pressure around a normal circular cylinder (2). A decrease on the drag coefficient and a negative slope of the lift coefficient may lead to negative aerodynamic damping and eventually instability, if the total damping becomes negative. Hereinafter the cable has started to vibrate, the rivulets tend to oscillate circumferentially, (5). This may enhance the vibrations. This vibration mechanism is discussed extensively in the next chapter.

![Figure 2.3: Cross section of bridge cable with upper and lower water rivulets](image-url)
2.4 Aerodynamic damping

Damping is one of the key parameters determining dynamic behavior of structures together with mass and stiffness. It depends on mass, stiffness, structural construction and aerodynamic properties. The damping of long cable stays is normally very low and additional measures must be considered to avoid undesired large amplitude vibrations. The total damping of bridge cable consists of different contributions, which can be divided into three categories:

- **Inherent damping** mainly induced by friction between the internal elements of the cable such as the steel wires and the HDPE sheaths covering the strands.

- **Additional mechanical damping** such as external dash-pot dampers, mass tuned dampers, etc.

- **Aerodynamic damping** due to relative cable-wind velocity.

The sum of contributing inherent and mechanical damping is defined as total structural damping. A structure moving in air experiences some forces tending to damp out the vibrations due to the body-air interaction. Those are called aerodynamic forces and in steady air, they mainly depend on the viscosity of air, as Davenport (1962) firstly pointed out. If the body is moving, e.g. due to gusty wind actions, fluctuating components of drag and lift are induced counteracting the motion itself. Aerodynamic damping may be either positive or negative and for light and flexible structures, e.g. line like structures, it may become a significant factor. Especially when negative, it could lead to instability situations if its absolute value overcomes the structural damping.

2.4.1 Aerodynamic damping for 1-DOF motion

The quasi-steady aerodynamic damping for single-degree-of-freedom (1-DOF) vibrations in any plane in a skew wind, including Reynolds number effects, has been studied by Macdonald and Larose (8), (9). Here, it is briefly presented before extension to the two-degree-of-freedom (2-DOF) problem. The assumptions are the basic quasi steady assumptions: the instantaneous forces on the cylinder are determined by the instantaneous relative velocity, any variations of the force coefficients are smooth functions and vibration amplitudes are small, so that the cylinder velocity is much smaller than the wind velocity.
2.4 Aerodynamic damping

Aerodynamics of bridge cables

Figure 2.4: Velocities in the plane of the cylinder axis and the flow velocity vector, cable–wind plane

\[ U_R = \sqrt{U^2 - 2Uv \sin \phi \cos \alpha + v^2} \] (2.3)

The direction of the projection of the relative velocity in the plane normal to the cylinder axis \((U_{NR})\), relative to the normal component of the actual flow velocity \((U_N)\) is given by (Fig. 2(b))

\[ \tan \alpha_d = \frac{v \sin \alpha}{U \sin \phi \cos \alpha} \] (2.4)
Aerodynamics of bridge cables

2.4 Aerodynamic damping

The relative velocity causes drag and lift forces on the cylinder. The angle between the drag force and the direction of cylinder motion is given by:

\[ \alpha_R = \alpha + \alpha_D \] (2.5)

the component of the resultant force, per unit length, acting in the direction of the cylinder velocity, \( \dot{x} \), is given by:

\[ F_x = \frac{\rho U_R^2 D}{2} (C_D \cos \alpha_R - C_L \sin \alpha_R) \] (2.6)

where \( \rho \) is the fluid density, \( U_R \) is the magnitude of the relative velocity, \( D \) is a representative dimension of the cross-section, \( C_D \) and \( C_L \) are the drag and lift coefficients respectively, and \( \alpha_R \) is the relative angle of attack. Based on the quasi-steady approach, this force is a function of the cylinder velocity, \( \dot{x} \), through the relative velocity, the resulting changes in the force coefficients and the angle \( \alpha_R \). It provides a non-linear damping term in the equation of motion of the cylinder. For small amplitude vibrations in a given mode, the equivalent linear aerodynamic damping ratio is then given by:

\[ \zeta_a = \left| \frac{-1}{2m\omega_n} \frac{dF_x}{\dot{x}} \right|_{\dot{x}=0} \] (2.7)

where \( m \) is the cylinder mass per unit length and \( \omega_n \) is the circular natural frequency. Differentiating Eq. (2.6), substituting into Eq. (2.7), and noting that when \( v = 0 \), \( U_R = U \) and \( \alpha_D = 0 \), the following general expression for the small amplitude aerodynamic damping (positive or negative) is obtained:

\[ \zeta_a = -\frac{\rho DU}{4m\omega_n} \left\{ \left( 2 \frac{dU_R}{dv} (C_D \cos \alpha - C_L \sin \alpha) \right) \right. \\
+ \left. U \left[ \frac{dC_D}{dv} \cos \alpha + C_D \frac{d(\cos \alpha_R)}{dv} - \frac{C_L}{dv} \sin \alpha - C_L \frac{d(\sin \alpha_R)}{dv} \right] \right\}_{v=0} \] (2.8)

The detailed evaluation of the definite derivatives is given in Macdonald and Larose (2006). Finally the general expression for aerodynamic damping for
small amplitude vibrations of a cylinder in any given plane can be written as:

\[
ζ_a = \frac{\mu Re}{4mω_n} \cos α \times\n\]

\[
\{ \cos α \left[ C_D \left( 2 \sin φ + \frac{\tan^2 α}{\sin φ} \right) + \frac{∂C_D}{∂Re} Re \sin φ + \frac{∂C_D}{∂φ} \cos φ - \frac{∂C_D}{∂α} \tan α \sin α \right] \}
- \sin α \left[ C_L \left( 2 \sin φ - \frac{1}{\sin φ} \right) + \frac{∂C_L}{∂Re} Re \sin φ + \frac{∂C_L}{∂φ} \cos φ - \frac{∂C_L}{∂α} \tan α \sin α \right] \}
\]

This expression covers the general case of effects due to any function of the static force coefficients, including conventional quasi-steady aerodynamic damping, Den Hartog galloping, and Re effects such as the drag crisis and dry inclined cable galloping.

### 2.4.2 Aerodynamic damping for coupled 2-DOF motion

The above analysis is based on vibrations in 1-DOF in any given plane. However, many structures, in particular cables, can also vibrate in the orthogonal plane. It should be noted that here torsional motion is not take in account on the behavior of the cables. Returning to Figure 2.5 and 2.4, the component of the aerodynamic force in the direction y is given by:

\[
F_x = \frac{ρU^2RD}{2} (C_D \cos α_R - C_L \sin α_R)
\]

and there may also be a component of velocity, \( \dot{y} \), in this direction. Hence, for small amplitude vibrations, the linearised dynamic components of the aerodynamic forces can be described in terms of an aerodynamic damping matrix:

\[
C_a = \begin{bmatrix}
c_{xxa} & c_{xya} \\
c_{yxa} & c_{yya}
\end{bmatrix} = \begin{bmatrix}
\frac{∂F_x}{∂x} & \frac{∂F_x}{∂y} \\
\frac{∂F_y}{∂x} & \frac{∂F_y}{∂y}
\end{bmatrix}_{x=y=0}
\]

Each of the terms can be calculated as in previous section, yielding the following expression for the full 2-DOF aerodynamic damping matrix per unit length of the cylinder:
Aerodynamics of bridge cables

2.4 Aerodynamic damping

\[ C_a = \frac{\mu R}{2} (G B_1 + C_{F\phi} + C'_{F\phi} B_2) \]  \hspace{1cm} (2.12)

where

\[ G = \begin{bmatrix} g(C_D) & -g(C_L) \\ g(C_L) & g(C_D) \end{bmatrix}, \]  \hspace{1cm} (2.13)

\[ C_{F\phi} = \frac{1}{\sin \phi} \begin{bmatrix} C_D & -C_L \\ C_L & C_D \end{bmatrix}, \]  \hspace{1cm} (2.16)

\[ C'_{F\phi} = \frac{1}{\sin \phi} \begin{bmatrix} C't_D & -C'L \\ C'L & C't_D \end{bmatrix}, \]  \hspace{1cm} (2.17)

\[ B_1 = \begin{bmatrix} \cos^2 \alpha & \sin \cos \alpha \\ \sin \cos \alpha & \sin^2 \alpha \end{bmatrix}, \]  \hspace{1cm} (2.18)

\[ B_2 = \begin{bmatrix} -\sin \cos \alpha & \cos^2 \alpha \\ -\sin^2 \alpha & \sin \cos \alpha \end{bmatrix} \]  \hspace{1cm} (2.19)

\[ C_F = C_D \text{or} C_L \]  \hspace{1cm} (2.15)

\[ C'_{F} = \frac{\partial C_F}{\partial \phi}, g(C_F) = C_F \left(2 \sin \phi - \frac{1}{\sin \phi}\right) + \frac{\partial C_F}{\partial Re} Re \sin \phi + \frac{\partial C'_{F}}{\partial \phi} \cos \phi, \]  \hspace{1cm} (2.14)
2.4 Aerodynamic damping
Bibliography


Chapter 3

Rain-Wind-Induced Vibrations

Vibrations due to simultaneous occurrence of wind and rain are able to produce large vibrations of inclined cables, arising concern in the last decades for the integrity of stays of cable-stayed bridges. This chapter gives a literature review of the features observed of rain-wind-induced vibrations and events of vibrations reported by different authors on stay-cables bridges.

3.1 Introduction

In 1986 during the construction of the Meiko-Nishi Bridge in Japan, Hikami and Shiraishi (1988) reported the occurrence of large cable vibrations due to the simultaneous presence of rain and wind on the cables over the Meiko Nishi Bridge in Japan during its construction (4). It was observed that in presence of wind and precipitation from light to moderate rainfall intensity, cables exhibit large oscillation that could induce structural problems due to increased level of stress or to fatigue resulting in failure of connections to the bridge deck, threatening the safety and serviceability of the bridge. More observations from bridges around the world were reported after Hikami pointed out the existence of this mechanism i.e. (10), (11).

RWIV are considered to cause the majority of reported cables vibrations on cable stayed bridges (about 95%) (3), however, the reported information results to be often contradictory or incomplete. Reported events present measurements in terms of amplitude of displacements, frequencies involved, wind conditions and meteorological conditions. Extensive investigations were undertaken on several cable-stayed Japanese bridges, i.e. the Aratsu Bridge, the Tempozan Bridge and the Meiko-Nishi bridge. More recently full-scale measurements have been conducted on the Fred Hartman Bridge in Texas.
3.2 General Features

This section presents the general features of RWIVs from full-scale bridges (3). Observations of RWIVs involve cables with diameters in the range of 80 – 250 mm (2). Some authors believe that vibrations are always accompanied by rain, other support the concept of inherent instability of inclined cables, regardless to the presence of rain. RWIVs occur for limited range of wind directions but observations of wind directions are contradictory. Even if most rain-wind-induced vibration events occur for cable declining in the wind direction (11), observations for cable inclining in the direction of wind were also reported. The wind directions can be expressed as the relative yaw angle, a measure of the direction of the wind with respect to the axis of the stay, that can be written as:

\[ \beta^* = \arcsin(\cos\alpha \sin\beta) \]

where \( \alpha \) is the inclination of the cable and \( \beta \) is the horizontal yaw angle. RWIVs have been observed for relative yaw angle in the range of \( 0^\circ < \beta^* < 45^\circ \) and \( 0^\circ < \beta^* < 55^\circ \). Reported RWIVs were observed to occur within a limited range of wind velocities of \( 6 – 18 m/s \), corresponding to a Reynolds numbers Re of \( 6 \times 10^4 – 2 \times 10^5 \). The most of the reported events were observed for mean wind velocity in the range of \( 6 – 11 m/s \). In presence of rain, RWIV occurs under conditions of light to moderate rain. The rainfall rate upper boundary for the occurrence of high amplitude RWIVs corresponds to \( 8 mm/h \). Rain-wind-induced vibrations may occur in very turbulent wind.
The observed frequency of cable oscillations were in the range of about 1 - 3Hz. Most of the observed vibrations were made up of a single mode, even if seldom compounded vibrations of modes were also observed.

### 3.3 Reported cases of RWIV

In these section reported RWIVs are presented in table. For each bridge, available data are reported together in terms of vibrations features, wind conditions and action undertaken to suppress or limit the vibrations amplitude of cables. The majority of RWIVs occur in a wind speed range between 5 – 18m/s, with amplitude of oscillation up to 2.0m. The most common means to suppress or control RWIV is the use of external dampers (i.e. Øresund Bridge), cross-ties (i.e. Second Severn Bridge). In same case these two methods are used together (i.e. Erasmus bridge).

<table>
<thead>
<tr>
<th>BridgeÂ’s name</th>
<th>Observations</th>
<th>Amplitude of vibrations</th>
<th>Remedies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alzette</td>
<td>Vibration during drizzle and light winds</td>
<td>Not quantified</td>
<td>Neoprene guides inside the guide pipe at deck and petroleum wax fill in the guide pipe</td>
</tr>
<tr>
<td>Aratsu</td>
<td>RWIV during light rain events</td>
<td>Up to 0.3m</td>
<td>Viscous damper</td>
</tr>
<tr>
<td>Ben Ahin</td>
<td>RWIV during light drizzle and wind speed about 10m/s winds</td>
<td>Up to 0.5m</td>
<td>Petroleum wax fill in the guide rope</td>
</tr>
<tr>
<td>Brotonne</td>
<td>Vibrations for wind speed about 15m/s</td>
<td>Up to 0.3m</td>
<td>Viscous damper</td>
</tr>
<tr>
<td>Dongting</td>
<td>RWIV, wind speed 6 – 14m/s, $\beta^*$ in the range 10$^\circ$ – 50$^\circ$, rain-fall rate $\leq 8mm/h$</td>
<td>up to 0.7m</td>
<td>MR dampers</td>
</tr>
</tbody>
</table>
### 3.3 Reported cases of RWIV

#### Rain-Wind-Induced Vibrations

<table>
<thead>
<tr>
<th>Bridge name</th>
<th>Observations</th>
<th>Amplitude of vibrations</th>
<th>Remedies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Erasmus</td>
<td>RWIV for wind speed about 14m/s, modes involved 2nd – 3rd, 0.8 – 1.2Hz)</td>
<td>up to 0.7m/s</td>
<td>Cross-ties, hydraulic damping. Critical damping ratio without damper 0.15%, with dampers is 0.3 (not sufficient to avoid RWIV)</td>
</tr>
<tr>
<td>Faroe</td>
<td>Dry vibrations and RWIV observations</td>
<td>Not quantified</td>
<td>Cross-ties</td>
</tr>
<tr>
<td>Fred Hartman</td>
<td>Dry galloping (wind speed 5 – 15m/s); RWIV (wind speed range 5 – 18m/s, rainfall 10mm/h, Re up to 2 x 10^5, Turb. Intensity 2.5 – 10%), wind direction in the range 0° ≤ β ≤ 30°</td>
<td>up to 1.6m (10D), one of the first three modes involved (single mode displacement)</td>
<td>Viscous dampers and cross-ties</td>
</tr>
<tr>
<td>Helgeland</td>
<td>Parametric excitations</td>
<td>Up to 0.3m</td>
<td>Cross-ties</td>
</tr>
<tr>
<td>Kap Shui Mun</td>
<td>RWIV under severe wind conditions/Large amplitude reported, vertical vibration with single modal shape displacement. wind speed about 12m/s at 90° to deck</td>
<td>Up to 1.5m</td>
<td>Frictional dampers</td>
</tr>
<tr>
<td>Kohlbrandt</td>
<td>RWIV</td>
<td>Up to 0.6m</td>
<td>Viscous damper</td>
</tr>
</tbody>
</table>
### Rain-Wind-Induced Vibrations

#### 3.3 Reported cases of RWIV

<table>
<thead>
<tr>
<th>Bridge name</th>
<th>Observations</th>
<th>Amplitude of vibrations</th>
<th>Remedies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meikonishi</td>
<td>Cables are stable under dry wind condition but really unstable in RW conditions. Vibrations during light rain, wind speeds in the range 5 – 15m/s, cable declining in wind direction</td>
<td>Up to 0.55m</td>
<td>Viscous damper</td>
</tr>
<tr>
<td>Normandy</td>
<td>Steady wind speed about 1 – 2m/s</td>
<td>Up to 1.0m</td>
<td>Viscous damper</td>
</tr>
<tr>
<td>Øresund</td>
<td>RWIV, iced galloping induced vibrations (wind speed up to 14m/s)</td>
<td>Up to 3m</td>
<td>Dash-pot dampers and hydraulic dampers</td>
</tr>
<tr>
<td>Puente Real</td>
<td>Strong vibrations parametric excitation rain/wind</td>
<td>Up to 0.4 m for RWIV; up to 0.02m for parametric excitation from deck</td>
<td>Frictional dampers</td>
</tr>
<tr>
<td>Second Severn</td>
<td>RWIV, Dry induced vibrations</td>
<td>Up to 0.6m</td>
<td>Cross-ties</td>
</tr>
<tr>
<td>Skarnsundet</td>
<td>Low turbulence wind, parametric excitation (Vertical deck frequency 2 cable fn), wind at 13m/s, angle of attack around 60° respect bridge axis</td>
<td>Up to 0.8m for the two longest cables</td>
<td>Not retrofitted</td>
</tr>
</tbody>
</table>

*Department of Civil Engineering - Technical University of Denmark*
### 3.3 Reported cases of RWIV

<table>
<thead>
<tr>
<th>Bridge name</th>
<th>Observations</th>
<th>Amplitude of vibrations</th>
<th>Remedies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steubenville</td>
<td>vibration for winds parallel to deck</td>
<td>Up to 0.3(m)</td>
<td>Viscous damper</td>
</tr>
<tr>
<td>Tjörn</td>
<td>Vibrations during light rain</td>
<td>Not quantified</td>
<td>MR dampers</td>
</tr>
<tr>
<td>Tenpozan</td>
<td>Vibration during rain and wind speed about 10(m/s)</td>
<td>Up to 1.0(m)</td>
<td>Not reported</td>
</tr>
<tr>
<td>Veterans Memorial</td>
<td>RWIV, Dry galloping</td>
<td>Up to 0.3(m)</td>
<td>Viscous damper</td>
</tr>
<tr>
<td>Yobuko</td>
<td>RWIV during light rain events</td>
<td>Up to 0.5(m)</td>
<td>Cross-ties</td>
</tr>
<tr>
<td>Wandre</td>
<td>RWIV during light rain events and wind speed about 10(m/s)</td>
<td>Up to 0.3(m)</td>
<td>Petroleum wax fill in the guide rope</td>
</tr>
</tbody>
</table>
Bibliography


[12] PTI recommendation for stay cables-wind design considerations. Stoyanoff. Appendix A


Chapter 4

System identification background

In the present work, system identification techniques of data from ambient vibrations were used to investigate aerodynamic damping of cable stays for different wind conditions. In particular, to identify the dynamic systems two procedure were used: eigensystem realization algorithm (ERA) using Matlab routines routines written by Jasna Jakobsen (20), University of Stavanger, and stochastic subspace identification (SSI) technique embodied in a commercial software (ARTeMIS). The former was implemented in the Matlab routine giving us the full control of the procedure, the latter was used for an investigation of aerodynamic damping in wet condition (see appendix). The SSI algorithm was implemented in a commercial software (ARTeMIS), but after a preliminary study (reported in the annexes), it was decided to continue to study the behavior of aerodynamic damping having full control of the process (Matlab routines implementing ERA). Theory about SSI algorithm is widely available in the literature, so a short description about SSI is given, while here it has been decided to focus mostly on ERA theory due to its use in the present work.

Following sections were arranged following the structure presented in Brincker, R. and Ventura C.E.: *Introduction to operational modal analysis*, to be published on Wiley and Sons., for which the author, Rune Brincker, granted the permission to use extracts from his book. Finally, examples of data validation of the ERA on simulated data is given in next chapter together with other monitoring issues.

The Author want to thanks also to Anela Bajric, PhD student at University of Denmark, that has reviewed system identification techniques and gave me
4.1 Operation modal analysis

In traditional modal testing, the loading applied to a structure, and its dynamic response is measured. This load can be easily applied to small structures by means of exciters or hammers or other devices, while on large structures such as bridges application of known loads by means of devices is not economically possible. In the cases where a medium-size structure can be excited artificially, modal analysis based on traditional principles can be performed but the background noise from the non-controllable loadings like traffic and wind might become a problem. To extend modal testing to large and massive structure and to investigate those under the real loadings, Operational Modal Analysis (OMA) techniques have been developed. The basic idea of OMA is that instead of considering the environmental loads as an unwanted noise source, the unknown environmental loads are used as the basis of the testing and subsequent modal analysis. The idea is that the structural system is loaded by an unknown force. Most of the theories applied in OMA consider the loading to have a white spectrum, whilst in most of the cases it is colored. For these cases, it is sufficient to assume that the unknown forces acting on the structure is the output from an imaginary system that is loaded by white noise (the so-called excitation system). As result of this hypothesis, while we performing OMA, modal properties for the whole system are found. Structural modes can then be distinguished from those of the excitation system because of their different features - the structural system has a narrow banded response and it shows time invariant properties, whereas the excitation system possesses broad banded response and it is time dependent. Some of further advantages of OMA, compared to other modal techniques to identify modal parameters are the potential of automatically providing modal parameters for SHM; OMA has a potential for estimating unknown environmental loads on structures Structural Health Monitoring (SHM); Combining with a FE model opens a possibility for estimating movements and stresses in arbitrary points.

4.2 OMA assumptions

The OMA technique is based on the following two assumptions: The structural response is random, stationary and zero mean. All physical informa-
tion is concentrated in the second order properties of the stochastic response. These assumptions are rarely fulfilled by random vibrations of real structure, but they serve as a sound basis for the development of the theories behind the identification techniques. The first order properties of the stochastic response is normally not used at all, the reason is that mean is difficult to measure in practice because of larger measurement errors close to DC. The second order properties such as correlation functions and spectral densities comes naturally due to the fact, that according to the central limit theorem, in many cases, responses to natural loads are approximately Gaussian.

4.3 NExT

Natural Excitation Technique (NExT) was proposed by James et al (21). In this method the correlation function (COR) which is obtained from the random response of a structure due to ambient excitation, can be thought as summation of decaying sinusoids. Each of them has a damped natural frequency, damping ratio and mode shape coefficient, for each mode. This technique uses the correlation function to obtain the modal parameters of the dynamic system. OMA methods based on the NExT concept consist of two main steps; the first is obtaining a Time Response Function (TRF) while the second includes identification of modal parameters in time domain. In order to obtain TRF in OMA two methods have been proposed: use of COR; use of time function obtained from Random Decrement technique (RD). RD technique was used for the first time in the modal analysis by Ibrahim (18) and he claimed that the result of RD is the free vibration of the system. In 1982, Vandiver showed that Ibrahim’s claim was incorrect and proved that the result of RD is COR (33). COR can obtain from random response data of the structure and use it to identify modal parameters in OMA based on the NExT.

4.4 Eigenvalue Realization Algorithm ERA

The Eigenvalue Realization Algorithm (ERA) was introduced in 1985 by Juang and Pappa for system identification of aerospace structures (8). ERA is a system identification technique using the time domain response data to generate a system realization. It has become widely applied in civil engineering, especially for structural health monitoring i.e. (12), (26). The basic principle of the algorithm is to identify system matrix $A$ from free-vibration response using minimum realization. Herewith a short mathematical formu-
4.4 Eigenvalue Realization Algorithm ERA  

System identification background

For a viscously damped multiple degree of freedom system, the equation of motion can be described as:

\[
M \ddot{y}(t) + C \dot{y}(t) + K y(t) = x(t) \quad (4.1)
\]

where \(M\), \(C\), and \(K\) are the system mass, damping and stiffness matrices, \(t\) is the time, \(y(t)\) denotes the displacements, \(\dot{y}(t)\) denotes the first derivative of the displacement with respect to time, \(\ddot{y}(t)\) denotes the second derivative of the displacement with respect to time and \(x(t)\) is the forcing of the system.

If we introduce a state vector \(u(t)\) as,

\[
u(t) = \begin{pmatrix} \dot{y}(t) \\ y(t) \end{pmatrix}
\]  

(4.2)

The equation of motion given in (4.1) can be formulated in state-space. The state space formulation is:

\[
\dot{u}(t) = Au(t) + Bx(t) \\
y(t) = Pu(t)
\]  

(4.3)

where \(A\) is the system matrix in continuous time, and \(B\) is the distribution matrix, and \(P\) is the observation matrix. These matrix are:

\[
A = \begin{pmatrix} -M^{-1}C & -M^{-1}K \\ I & \end{pmatrix} \quad B = \begin{pmatrix} M^{-1} \\ [0] \end{pmatrix} \quad P = ([0] \quad I)
\]  

(4.4)

An homogenous solution to the equation of motion (4.1) (free decay problem) is:

\[
u(t) = \varphi e^{\lambda t}
\]  

(4.5)

where \(\varphi\) is the eigenvector, and \(\lambda\) is the eigenvalue. By modal decomposition of (4.3), modal shapes and eigenvalues can be found as:

\[
A = [\varphi_n][\lambda_n][\varphi_n]^{-1}
\]  

(4.6)

The modal shapes are hold as:

\[
\varphi_n^T = (\lambda b_n \quad b_n)^T
\]  

(4.7)

which means that the upper half contains the mode shape velocities, and the lower half contains the \(n\)’th mode displacements; \(b_n\) denotes the mode shapes
obtained experimentally. The eigenvalues are held in the diagonal matrix $\lambda_n$ in (4.6). The eigenvectors of $A$ are the eigenvectors of the system in (4.1), in case of free vibration (homogeneous solution of the problem). In ERA the realization of the system matrix $A$ is sought, as this enables the extraction of modal parameters. The general solution $u(t)$ is derived in Kailath (24), and expressed as:

$$u(t) = e^{A t}u(0) + \int_0^t e^{A(t-\tau)}Bx(\tau) \, d\tau$$  \hspace{1cm} (4.8)$$

where the first term is the transient response obtained from the initial conditions, and the second term is the particular solution associated with the external forcing on the system. The discrete time solution to the homogeneous equation (4.1) is:

$$u(n) = e^{A n \Delta t}u_0$$  \hspace{1cm} (4.9)$$

where $u_0$ are the initial conditions, $n$ is the time step $\Delta t$ is the sampling time. The discrete time system matrix is:

$$D = e^{A \Delta t}$$  \hspace{1cm} (4.10)$$

and the the free decay in discrete time expressed in state-space is:

$$y(k) = PD^k u_0$$  \hspace{1cm} (4.11)$$

where $P$ is the observation matrix given in (4.4) and $t_k = k \Delta t$ with $\Delta t$ being the sampling time. Placing several free decays side-by side, the common free decay matrix is obtained:

$$Y(k) = [y_1(k), y_2(k), ...]$$  \hspace{1cm} (4.12)$$

where $k$ is the time step. The common free decay matrix can be obtained from measurement of free-decay in time domain directly, inverse Fast-Fourier Transform (FFT) of the frequency response functions, cross-correlation functions of random response or inverse FFT of the cross-spectral densities of random responses. The common initial condition matrix is obtained similarly:

$$U_0 = [u_{01}, u_{02}, ...]$$  \hspace{1cm} (4.13)$$

For the free decays the common expression is:

$$Y(k) = PD^k U_0$$  \hspace{1cm} (4.14)$$
From the general expression of the free decay, the block Hankel matrix is:

\[
H(0) = \begin{pmatrix}
Y(0) & Y(1) & \ldots \\
Y(1) & Y(2) & \ldots \\
\vdots & \vdots & \ddots \\
Y(s-1) & Y(s) & \ldots
\end{pmatrix}, \quad H(1) = \begin{pmatrix}
Y(1) & Y(2) & \ldots \\
Y(2) & Y(3) & \ldots \\
\vdots & \vdots & \ddots \\
Y(s) & Y(s+1) & \ldots
\end{pmatrix}
\]

These are constant along its anti diagonals, with \( s \) block rows. \( Y(k) \) are also known as the Markov parameter. The more rows and columns contained in the block Hankel matrix, the more of the vibration modes contributing to the response are covered. A balance is sought to cover the long frequency components of the impulse response function. If we formulate the observability and controllability matrices as,

\[
\Gamma^T = (P \, PD \, PD^2 \, \ldots \, PD^{s-1})^T \quad \Lambda = (U_0 \, DU_0 \, D^2U_0 \, \ldots)
\]

The block Hankel matrices can from (4.14) be expressed as,

\[
H(0) = \Gamma \Lambda \quad H(1) = \Gamma D \Lambda
\]

The idea in ERA is to take the single value decomposition (SVD) of the first block Hankel matrix,

\[
H(0) = USV^T
\]

where \( S \) is a diagonal matrix with real and non-negative singular values in descending order. \( U \) and \( V \) are unitary matrices with singular values as column vectors. This enables the observability and controllability matrices to be expressed as,

\[
\hat{\Gamma} = U\sqrt{S} \quad \hat{\Lambda} = \sqrt{SV^T}
\]

From the second block Hankel matrix \( H(1) \) in (4.17) the discrete time system matrix can be estimated as,

\[
\hat{D} = \hat{\Gamma}^+ H(1)\hat{\Lambda}^+
\]

\( \hat{\Gamma}^+ \) and \( \hat{\Lambda}^+ \) denotes the pseudo inverse. These can be computed by restriction of the diagonal matrix \( S \) to the first \( n \) singular values, i.e. corresponding to \( n/2 \) modes. The unitary matrices are transformed accordingly. This procedure forms the restricted matrices in the SVD denoted \( S_n, U_n \) and \( V_n \). The \( n \)-order realization of the discrete time system matrix follows as,

\[
\hat{D} = S_n^{-1/2}U_n^T H(1) V_n S_n^{-1/2}
\]
From which the eigenvalues and eigenvectors can be estimated as,

$$\hat{D} = [\varphi'_n][\mu_n][\varphi'_n]^{-1}$$  \hspace{1cm} (4.22)

Here the eigenvectors are not directly corresponding to the mode shapes, as the eigenvectors have been transformed through the manipulation in (4.21). From $\mu_n$ the continuous eigenvalues $\lambda_n$ can be found,

$$\mu_n = e^{\lambda_n \Delta t} \Rightarrow \lambda_n = \ln(\mu_n)/\Delta t$$  \hspace{1cm} (4.23)

To transform the eigenvectors $\varphi'_n$ to physical mode shapes, the observation matrix of the realization is estimated as,

$$\hat{P} = U_rS_r^{1/2}$$  \hspace{1cm} (4.24)

And the mode shapes in physical coordinates are then,

$$[\varphi_n] = \hat{P}[\varphi'_n]$$  \hspace{1cm} (4.25)

Knowing the physical mode shapes $\varphi_n$ the system matrix $A$ can be determined by inserting (4.25) into (4.6), and similarly the observation matrix $B$ and distribution matrix $P$ can be found in continuous time. This finalizes the derivation of ERA.

### 4.4.1 Comments on Eigenvalue Realization Algorithm

The ERA technique is advantageous when the number of modes to be estimated is known and the SVD matrix can be adjusted accordingly. It works because the modes with a high participation have large singular values, and only the physical modes will be estimated. The disadvantage of ERA is that when the noise is present (real signals) and the physical modes are not clearly excited, the noise introduces difficulties in determining the rank of the block Hankel matrix. Noise is present in the data, and can be dealt with by including more modes in the model. This will give a better fit of the model to the data. The more noise modes included the better model of noise is obtained. This implies that noise modes is a reason for over sizing the model. Other source of difficulties for ERA are closely spaced modes and uncertainty estimation.

Closely spaced modes are difficult to be detected accurately by ERA because when the modal shapes are sensitive to small changes in the structural system, when the frequency difference between two modes tend to zero, the sensitivity of the modal shapes goes to infinity. The mode shape validation
can be evaluated through the Modal Assurance Criterion (MAC), it measures degree of coherence between an identified mode and its ideal counterpart. Its values range between 1 and 0, where one corresponds to real modes.

Uncertainty estimation on modal parameters are identified by repeating the test and identification. One single tests estimates one single mode shape for each mode, where as one single test can determine the uncertainty on the frequency and damping, by the pole estimates of the data.

A validation of a Matlab routine based on ERA is given in next chapter. The routine has been validated for 1-DOF and 2-DOF with and without noise. All signals have been simulated using the same features of data from monitoring system, i.e. sampling frequency is 30Hz, the time-series are 10-minutes long. 2-DOF simulated signals have been tested for 2 particular cases: well separated and closed-spaced modes, in order to validate the routine for a signal as much as possible similar to the data obtained from monitoring system of bridge cables. All generated signals have been identified without noise and then adding three levels of noise defined as percentage of the standard deviation of the original signal.

4.5 Stochastic Subspace Identification SSI

Stochastic Subspace Identification (SSI) is a modal identification method in time domain. SSI methods are the state of the art methods for modal parameter estimation. It is a computationally efficient methods and can deal with realistic excitation assumptions. The technique was introduced in the '90s, offering a numerically reliable and effective state-space model (27). It was developed for numerical identification of complex dynamic system state-space using direct ambient vibrations data (28). The SSI method was presented using random response of structures, subjected to stochastic excitations by Van Overchee and De Moor 1996, which is well described in (27). Many authors worked to improve this technique, in the bibliography some of the most important works on this technique are reported as in (7) to (10). There are two types of SSI techniques, deterministic and stochastic estimation algorithms. This section explains the steps in SSI technique which are of importance from a civil engineering point of view: the data driven SSI. This system identification technique is considered to be the most powerful class for natural modal analysis in the time domain (9).
4.5.1 SSI Formulation

The classical formulation of a multi-degree of freedom system as in (4.1) is taken from the continues time formulation to the discrete time domain by introducing the state space formulation as in (4.2) - (4.4). By this formulation the general solution (derived in Kailath (24)) is directly given as:

$$u(t) = e^{At}u(0) + \int_0^t e^{A(t-\tau)}Bx(\tau) \, d\tau$$

(4.26)

The discrete time solution to the homogeneous equation (4.1) is:

$$u(n) = e^{A\Delta t}u_0$$

$$D = e^{A\Delta t}$$

$$y(k) = PD^k u_0$$

(4.27)

4.5.2 The Block Hankel Matrix

The common free decay matrix in discrete time is represented by the data matrix,

$$Y = [y(1), y(2), ..., y(N)]$$

(4.28)

where N is the number of data points, y the measured response. The block Hankel matrix is defined in SSI as a of matrices created by shifting the free decay matrix (4.28). It has np number of data point and 2s number of block rows.

$$H = \begin{pmatrix}
    y(1) & y(2) & \cdots & y(np-2s+1) \\
    y(2) & y(3) & \cdots & y(np-2s+2) \\
    \vdots & \vdots & \ddots & \vdots \\
    y(s) & y(s+1) & \cdots & y(np-s) \\
    y(s+1) & y(s+2) & \cdots & y(np-s+1) \\
    y(s+2) & y(s+3) & \cdots & y(np-s+2) \\
    \vdots & \vdots & \ddots & \vdots \\
    y(2s) & y(2s+1) & \cdots & y(np)
\end{pmatrix} = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}$$

(4.29)

$$H_1$$ is the called the past matrix, and $$H_2$$ is the future matrix.

4.5.3 The Projection

The projection matrix is formed from the past and future parts of the block Hankel matrix. When dealing with a stochastic response, the projection is
4.5 Stochastic Subspace Identification SSI System identification background

defined as a conditional mean. Projecting the future into the past, it forms the projection matrix:

\[ \mathbf{O} = \mathbb{E}[\mathbf{H}_2|\mathbf{H}_1] \]  \hspace{1cm} (4.30)

For Gaussian processes this can be described by its covariances, as it is a conditional mean. The projection matrix can therefore be calculated directly from the block Hankel matrices, since it also defines covariances:

\[ \mathbf{O} = \mathbf{H}_2 \mathbf{H}_1^T (\mathbf{H}_1 \mathbf{H}_1^T)^{-1} \mathbf{H}_1 = \mathbf{T}_{21} \mathbf{T}_{11}^+ \mathbf{H}_1 \]  \hspace{1cm} (4.31)

where the block Toeplitz matrices \( \mathbf{T}_{11} \) and \( \mathbf{T}_{21} \) are:

\[ \mathbf{T}_{21} = \mathbf{H}_2 \mathbf{H}_1^T \]
\[ \mathbf{T}_{11} = \mathbf{H}_1 \mathbf{H}_1^T \]  \hspace{1cm} (4.32)

The projection in (4.31) is a conditional mean of the measured random signals which means that it is the same as a random decrement signature. In other words the projection consists of free decays of the system organized to a set of unknown initial conditions.

The projection in SSI is computed using QR decomposition of the transposed block Hankel matrix, as computations of the block Toeplitz matrices is time consuming in practice. QR decomposition is a procedure to calculate the eigenvalues and eigenvectors of a real square matrix by decomposing into a orthogonal matrix \( \mathbf{O} \) and an upper triangular matrix \( \mathbf{R} \). The projection matrix can be expressed as,

\[ \mathbf{O} = \Gamma \mathbf{X} \]  \hspace{1cm} (4.33)

where \( \Gamma \) is the observability matrix, and \( \mathbf{X} \) is the initial conditions for each column in the projection matrix \( \mathbf{O} \), also named the Kalman state matrix. The observability matrix is,

\[ \Gamma^T = (\mathbf{P} \mathbf{P} \mathbf{D} \mathbf{P}^2 \ldots \mathbf{P} \mathbf{D}^{-1})^T \]  \hspace{1cm} (4.34)

Now if the observability matrix was known, the initial conditions \( \mathbf{X} \) could easily be computed, but this is not the case, and therefore these must be found before solving for the observability matrix \( \Gamma \) and later the discrete system matrix \( \mathbf{D} \).
4.5.4 The Kalman States

Kalman states $X$ in (4.33) are the states (initial conditions) for the free responses estimated by the projection $O$. To find the Kalman state $X$ and the observability matrix $\Gamma$, a Singular Value Decomposition (SVD) as in ERA, can be performed,

$$O = USV^T$$  \hspace{1cm} (4.35)

where $S$ is a diagonal matrix with real and non-negative singular values in descending order. $U$ and $V$ are unitary matrices with singular values as column vectors. Similarly to the procedure in ERA the singular values in the diagonal matrix $S$ can be restricted to the first $n$ singular values. This forms the reduced restricted SVD, $S_n$, $V_n$ and $U_n$.

Normally we would like to vary the model order to establish a stabilization diagram. This can of course be done by establishing a series of Block Hankel matrices of different size, but it is somewhat easier, instead of varying the size of the Block Hankel matrix, to vary the number of singular values used in eq. (16). Thus in practice the size of the Block Hankel matrix defines the maximum model order, and the actual model order is varied by varying the number of singular values taken into account when performing the SVD of the projection matrix. The maximum number of eigenvalues must be adjusted to a reasonable level to incorporate the needed range of models.

The observability and Kalman state can then estimated by,

$$\hat{\Gamma} = U_n \sqrt{S_n}$$

$$\hat{X} = \sqrt{S_n} V_n^T$$  \hspace{1cm} (4.36)

The discrete time system matrix $D$ and the observation matrix $P$ can be found from the estimates of the observability matrix $\Gamma$ by solving a least square problem, just as in ERA.
4.5 Stochastic Subspace Identification SSI

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Bibliography


Chapter 5

Monitoring system

The present thesis deals with the analysis of full-scale data collected by a monitoring system installed on the Øresund Bridge cables. The Øresund Bridge was opened in 2000. In 1999, vibrations of stays were reported and attributed to dry galloping or parametric excitation (5). To suppress those vibrations, donut dampers were installed at the ends of the cables. In 2001, donut dampers were replaced due to oil leakage with installation of viscous dampers. In 2004, large amplitude cable vibrations were reported in presence of sleet on the cables. Peak-to-Peak displacements were about 2-3 meters. Tuned mass dampers were added to the longest cables to suppress those vibrations. In 2009, a new long-term monitoring system was installed to have an insight into the reasons for cable vibrations to continue to persist. A description of the Øresund Bridge and the new long term monitoring system are reported in this section.

5.1 Øresund Bridge

The Øresund Bridge, named also Øresundbron, links Denmark and Sweden in the Øresund area. The cable-stayed bridge provides the main navigational span for an 8km bridge link. The bridge has a main span of 490m and an orientation along the direction N-W-W to S-E-E. The deck carries both road and rail traffic and is supported by 4 pylons (204m tall) and by 80 parallel double-stays, all with an inclination of 30 degrees to the horizontal plane. The stays are covered by a black HDPE tube with helical fillets and arranged in a harp-shaped configuration (Figure 5.1). The cables are comprised of seven-wire mono-strands supplied by Freyssinet. All cables have the same outer diameter of 250mm. Each cable pair is connected to each other with brackets at one or two locations. Tuned mass dampers are installed on the first and
5.2 Data-logger system

The data logging system was built programmed using graphical programming language (LabVIEW) of National Instruments (NI). The routine acquires data from the channels using a stand-alone system (Compact Real-time InputOutput device, NI CRI0 9014, with processor performance of 400 MHz). To achieve such a high frequency, the software was based on a field-programmable gate array (FPGA). The RIO FPGA, mounted in the NI chassis, is directly connected to the NI I/O modules for high-performance access, securing very low control latency for system response, enabling programs on the real-time controller to access I/O with less than 500\(\text{ns}\) of jitter between loops and guarantees that all channels are sampled contemporaneously \((6)\). The monitoring system samples at a frequency 30 Hz for all channels. Each 10 minutes a binary file of sampled data is recorded on local hard disk driver, with in-plane and out-of-plane components of accelerations for each cable, wind data for each anemometers and rain gauge data. At same time the mean value of temperature, humidity, pressure at three location in the 10 minutes are introduced in a text file. The slower logging of these information are due to the fact that these physical data changes slowly over time. Data are accessible via internet using a dedicated VPN connection. The data logging system receives synchronized data from three different type of instruments: tri-axial accelerometers, ultrasonic anemometers and rain-gauge. The instruments of the monitoring system and their location on the Øresund Bridge is shown in Figure 5.1. The CRI0 unit is equipped with modules to acquire data from accelerometers (analogical data, NI 9205) and from anemometers and rain-gauges (digital data, NI 9871). Each signal is protected against deterioration by the use of adequate cabling. The main source of deterioration is electrical noise that can be present due to the long distance between acquisition unit and instruments and due to the difficult environment. The data logger unit is provided with independent batteries to provide electricity in case of black-out up to 48 hours, making sure that the system can continue to acquire data during maintenance routines of bridge operator when it is needed to cut the electricity. A scheme of the data-logger system connected to instruments and to DTU is shown in Figure 5.2 and the monitoring hardware is shown in Figure 5.3.
5.2 Data-logger system

5.2.1 Accelerometers

Accelerations were measured on the four longest cables on both sides of the south-eastern pylon. The accelerations are measured using tree-axial accelerometers produced by GeoSIG (GeoSIG AC-53), with a 200 mg/V sensitivity. The accelerometers are oriented to record the components of the cable vibrations in the in-plane and out-of-plane directions. The accelerometers were analogously filtered to avoid aliasing problems.

5.2.2 Ultrasonic anemometers

Three heated ultrasonic anemometers, produced by DeltaOhm HD2003, are installed on the bridge. The HD2003 is guaranteed to function in temperatures as low as $-20^\circ$C. The heating circuit intervenes at temperatures below $+5^\circ$C, preventing the formation of ice, and ensuring that the HD2003 functions correctly even during sleet or snow precipitations. Their position was chosen to give an adequate description of the wind seen by the cables, each of them are positioned at the top of the south-easterly pylon, the deck mid-span (south) and between cable pairs 1 and 2, west of the pylon (see Figure 5.1). At deck level, the anemometers are installed on poles that extend 7m above the deck. On the pylon top, the anemometer is placed on a 4m pole. Wind is measured in the three Cartesian components, whilst an integrated magnetic compass supplies the exact wind incidence. The ultrasonic anemometers, constructed by DeltaOhm (HD2003R), are heated and are capable of measuring wind velocities up to 60m/s with a resolution of 0.01m/s. Further data collected includes atmospheric humidity, temperature, and pressure.
5.2 Data-logger system

**Diagram of data-logger connected to instruments and data transmission to DTU**

**Figure 5.2:** Diagram of data-logger connected to instruments and data transmission to DTU
Figure 5.3: Monitoring hardware of Øresund Bridge
5.3 Data analysis

5.2.3 Rain-gauge

A rain-gauge is located on the pylon top to measure rainfall. The rain-gauge is produced by DeltaOhm (HD2013D). It is heated to ensure accurate measurement even with low temperature climatic conditions or during and after precipitations of snow. The heater is automatically activated around +4°C. It records rainfall with a resolution of 0.2mm and frequency of two seconds.

5.3 Data analysis

Data obtained by the monitoring system were saved on a hard drive disk accessible via internet connection for off-line data processing at DTU. The data were analyzed by means of ad-hoc written Matlab routines and commercial software for operational modal analysis (ARTeMIS). The author would also like to thank Jasna Jakobsen, from University of Stavanger, to provide the system identification routines used in this thesis and applied in (7). This method has been found to be effective for full-scale ambient vibration measurements, having recently been applied to full-scale data from the Clifton Suspension Bridge, identifying clear trends in the flutter derivatives with wind velocity, including aeroelastic coupling between vertical and torsional vibrations of the bridge deck. The method assumes the external loading to be white noise in the proximity of the frequencies of interest and that the system is linear (10).

In this work, all signals have been adequately filtered according the specific analysis and passed through the same processes to maintain consistency among signals.

5.4 Matlab routine: ERA

Data obtained from monitoring system were processed and system identification was carried out using the above mentioned system identification routines (7). In this paragraph, an insight in the way the method is given. The same processing procedure was applied on the Clifton Suspension Bridge in order to identify flutter derivatives from full-scale ambient vibration measurements (10) and for the estimation of flutter derivatives from wind tunnel tests by Jakobsen et al (7). System identification of cables vibrations were obtained using a frequency based curve fitting technique (9); afterward a subspace stochastic identification technique was used to extract modal parameters.
Modal parameters were calculated from the acceleration Power Spectral Densities (PSDs) using the Iterative Windowed Curve-fitting Method (IWCM method) (9), developed for the analysis of ambient vibration data and allowing for the specified loading spectrum (white noise). This method iteratively curve fits in the frequency domain a series of idealized single degree-of-freedom transfer functions, taking into account the loading spectrum, the effect of windowing on the spectra (both from the measured data and equivalently from the idealized transfer functions) and the contributions of multiple modes.

A state space formulation of the dynamic problem was assembled using the Covariance Block-Hankel Matrix method (CBHM method), which is founded on Eigenvalue Realization Algorithm (ERA) described by Juang and Pappa (8). CBHM has similar formulation to ERA, but this method employs covariance estimates of output measured random data instead than the Markov parameters containing Impulse Response Functions (IRF).

The method typically assumes white noise loading. The main forcing on the cables is from wind buffeting, and it is well known that it produces a colored spectrum. However before the application of the system identification method the acceleration records were filtered around each natural frequency, as shown in the system identification related papers. Hence the variation in the magnitude of the loading spectrum over the relevant range of frequencies for each filtered record was small, which make it reasonable to treat the loading locally as white noise. Ordinary filtering technique (low-pass and high pass butter-worth filters) was used. The main concern during the processing of the data was about preserving that after filtering the in-plane and out-of-plane considered modes were not altered by the filtering operations. A final check was achieved, comparing the power spectral density of raw data and filtered data.

The whole method relies on the choice of one parameter since the length of records was fixed to 10 minutes with a sampling frequency of 30 Hz: the number of time delays \( l \) for which the covariance matrix is evaluated. Covariance function length was determined through a sensitivity analysis together with inspection of the time evolution of the auto and cross-covariance functions. Details about the optimum covariance length is given in the papers. The choice of correlation length defines the model order used by the MBH method to extract modal parameters from ambient vibrations. A too short correlation length would restrict the model order and therefore wanted modal parameters would be neglected or biased; a longer correlation length than the
5.5 Validation: 1-DOF

A further analysis was performed adding random white noise to the signal generated previously. Three levels of noise were considered. The noise was
introduces as a random signal with a normal distribution with zero mean and 10%, 50% and 200% of the standard deviation of the 1-DOF without noise. Simulated signals are shown respectively in Figure 5.5, Figure 5.6, and Figure 5.7. The system identification for these signals gave the exact estimates of the given frequency $f = 1.0066$ Hz and a value of damping slightly different than the one in absence of noise, respectively 0.00134, 0.00145, and 0.00153.

**Figure 5.4:** Simulated signal without noise

**Figure 5.5:** Simulated signal with noise (10% of standard deviation of signal simulated signal in Figure 5.4)
5.7 Validation: 2-DOF

The ERA routine was validated on a 2-DOF simulated signals to validate the results of the system identification of the data from the monitoring of the cables accelerations. The signal was made considering a 2 SDOF acting simultaneously with following modal parameters: modal mass $m_1 = 1kg$, $m_2 = 2kg$.
$m_2 = 1\text{kg}$; modal damping of $c_1 = 0.02kN/(m/s)$ and $c_2 = 0.02kN/(m/s)$, modal stiffness of $k_1 = 40kN/m$ and $k_2 = 160kN/m$. The correlation length used was 54s for all simulated signals. The natural frequency of the 2-DOF system and the damping coefficients are therefore:

$$f_1 = \frac{\sqrt{k_1/m_1}}{2\pi} = 1.0066\text{Hz}; \quad f_2 = \frac{\sqrt{k_2/m_2}}{2\pi} = 2.0132\text{Hz} \quad (5.4)$$

$$\zeta_1 = \frac{c_1}{2\sqrt{k_1/m_1}} = 0.0016; \quad \zeta_2 = \frac{c_2}{2\sqrt{k_2/m_2}} = 0.0008 \quad (5.5)$$

In Figure 5.8, the simulated 2-DOFs signal and its bode diagram are shown (Figure 5.9). Three levels of noise signals (10%, 50% and 200% of the standard deviation of the considered channel without noise) were introduced in the original signal. Using the Matlab routine, stiffness and damping matrices were identified. The same correlation length of 30 seconds was applied to all signals. Here are reported the results of the analysis for signal without noise:

- For the simulated signal:
  - $f_{1,id} = 1.0066$; $f_{2,id} = 2.0132$; $\zeta_{1,id} = 0.0015$; $\zeta_{2,id} = 0.0007$;
  - Here are reported the results of the system identification for the simulated signal with level of noise equal to 10:
    - $f_{1,id} = 1.0066$; $f_{2,id} = 2.0132$; $\zeta_{1,id} = 0.0015$; $\zeta_{2,id} = 0.0007$;
  - Here are reported the results of the system identification for the simulated signal with level of noise equal to 50:
    - $f_{1,id} = 1.0066$; $f_{2,id} = 2.0132$; $\zeta_{1,id} = 0.0015$; $\zeta_{2,id} = 0.0007$;

**Figure 5.8: Simulated signal without noise**
5.8 Validation: 2 closed space DOF

Final validation was given for two very close spaced mode, which modal parameters are given here: modal mass $m_1 = 1\,kg$, $m_2 = 1\,kg$; modal damping of $c_1 = 0.02kN/(m/s)$ and $c_2 = 0.04kN/(m/s)$, modal stiffness of $k_1 = 40kN/m$ and $k_2 = 40.02kN/m$. The natural frequency of the 2-DOF system and the damping coefficients are therefore:

$$f_1 = \frac{\sqrt{k_1/m_1}}{2\pi} = 1.0066\,Hz; \quad f_2 = \frac{\sqrt{k_2/m_2}}{2\pi} = 1.0068\,Hz \quad (5.6)$$

$$\zeta_1 = \frac{c_1}{2\sqrt{k_1/m_1}} = 0.0016; \quad \zeta_2 = \frac{c_2}{2\sqrt{k_2/m_2}} = 0.0032 \quad (5.7)$$

In Figure 5.10, the simulated 2-DOFs signal and its bode diagram are shown (Figure 5.11). Three levels of noise signals (10%, 50% and 200% of the standard deviation of the considered channel without noise) were introduced in the original signal. Using the Matlab routine, stiffness and damping matrices were identified. The same correlation length of 30 seconds was applied to all
signals. Here are reported the results of the analysis for signal without noise:

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.10}
\caption{Simulated signal without noise}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.11}
\caption{Simulated signal without noise}
\end{figure}

Here are reported the results of the system identification for the simulated signal: $f_{1,id} = 1.0066; \quad f_{2,id} = 1.0068; \quad \zeta_{1,id} = 0.0015; \quad \zeta_{2,id} = 0.0031;
Here are reported the results of the system identification for the simulated signal with level of noise equal to $10f_{1, id} = 1.0066; f_{2, id} = 1.0069; \zeta_{1, id} = 0.0015; \zeta_{2, id} = 0.0030$.

Here are reported the results of the system identification for the simulated signal with level of noise equal to $50f_{1, id} = 1.0067; f_{2, id} = 1.0068; \zeta_{1, id} = 0.0014; \zeta_{2, id} = 0.0033$.

Here are reported the results of the system identification for the simulated signal with level of noise equal to 200% of standard deviation: $f_{1, id} = 1.0069; f_{2, id} = 1.0068; \zeta_{1, id} = 0.0014; \zeta_{2, id} = 0.0035$.

5.9 ERA applied on a typical time-series

In this section an example of how the system identification routine has been used on data from monitoring system is shown. A typical time history for the in- and out-of-plane accelerations is reported (Figure 5.12) from one of the monitored cable of the Øresund Bridge (cable 10M), being out-of-plane signal channel 2 and in-plane signal channel 1. The raw signal is made of multi-DOF system (Figure 5.13). Filtering the signal with butter-worth filters (band-pass filters on the desired frequency), two single DOF signals were obtained - in-plane and out-of-plane accelerations - (Figure 5.14). In this section, the system identification routine is applied for different values of the lag interval to show its influence on the results (Figure 5.15). It can be observed that for shorter lag values both stiffness and damping matrices presents a variation of their values (lag length smaller than 10 seconds). This is particular true for the values of the damping matrix; the stiffness damping needs a longer lag length than stiffness matrix to show steady values. For lag length too long, the values of the damping matrix increases again due to the contribution of the noise in the considered correlation length. An optimal correlation length to correctly identify the damping matrix for time-series can be evaluated in the range from 35 to 55 seconds. Signals with high level of noise over the signal (noise-signal ratio) were discarded by the following analysis in the present work.
Monitoring system

5.9 ERA applied on a typical time-series

Figure 5.12: In-plane and Out-of-plane time histories of the cable accelerations

Figure 5.13: Time history of the two SDOF
5.9 ERA applied on a typical time-series Monitoring system

**Figure 5.14:** Signals filtering to identify SDOF for in-plane and out-of-plane accelerations

**Figure 5.15:** Stiffness and damping matrices as a function of time lags

66 Department of Civil Engineering - Technical University of Denmark
Bibliography

Part III

Appended Papers
Full-scale observations of RWIV of Øresund Bridge

This chapter presents the results of the observations of RWIVs on the Øresund Bridge in terms of wind speeds, wind-cable angles and rainfall rates. These information are used for following analysis.
Full-scale observations of RWIV of Øresund Bridge
Paper I

"Recent monitoring of the Øresund Bridge: Observations of rain-wind induced cable vibrations"

Antonio Acampora, Christos T. Georgakis

In proceedings: The 13th International Conference on Wind Engineering (ICWE), Amsterdam, the Netherlands, July 2011
Recent monitoring of the Øresund Bridge: Observations of rain-wind induced cable vibrations

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1 INTRODUCTION

In 1999, 2m amplitude cable vibrations were observed on the Øresund Bridge. The vibrations were attributed to a change in cable shape due to sleet accretion. Subsequent monitoring of the bridge, though, revealed other vibration events due to rain-wind induced mechanisms and parametric excitation (Svensson et al., 2004) – albeit of smaller amplitudes. After two significant cable vibration events, inspections of the cable anchorages revealed failures in the damping systems that had been installed to prevent oscillations. Improvements in the damping systems were introduced and additional tuned mass dampers were installed on the longest and second longest cable pairs.

Although the bridge has not suffered from any significant vibration events since the installation of the new damping systems, smaller amplitude vibrations are recorded from time to time. Peak to peak amplitudes of up to three cable diameters have been observed, although the most frequent observations are of smaller amplitude and are almost always in combination with rain.

In this paper, several observations are made, regarding the rain-wind induced vibrations (RWIVs) of the cables, based on a relatively brief full-scale monitoring campaign from January 2010 – December 2010. The monitoring shows that there is a direct correlation between wind-cable angles, wind velocities and the amount of rainfall.

2 PREVIOUS ACCOUNTS

Numerous cable-stayed bridges have experienced rain-wind induced cable oscillations. Hikami (1986) first identified this mechanism when reporting observed oscillations on the Meiko-Nishi Bridge in Japan. During several events, Hikami noticed that the vibrations did not occur for all wind velocities and for all cable frequencies. Instead, the oscillations were limited to a wind velocity range of 5-15 m/s and for cable modes with frequencies between 1-3 Hz. Furthermore, all of the oscillations occurred only in the presence of rain. Similar observations were made by Matsumoto et al. (1995) for the Tenpozan and the Meiko-Nishi Bridges and by Main et al. (2001), Ni et al. (2007) and Zuo et al (2010) for the Fred Hartman and Dongting Lake Bridges, respectively.

Further to Hikami’s observations, Main et al. (2001) showed high amplitude cable responses occurred over a range of relative yaw angles, $\beta^*$, between 0 and 40º. Ni et al. (2007) reported the following meteorological conditions for RWIV: wind velocities between 6-14 m/s under light to moderate rainfall of up to 8 mm/h. The relative yaw angles, $\beta^*$, ranged between approximately 10º and 50º.
3 BRIDGE CONFIGURATION AND MONITORING SYSTEM

The current investigation is based on the monitoring of the Øresund Bridge, between Denmark and Sweden. The cable-stayed bridge provides the main navigational span for an 8km bridge link. The bridge has a main span of 490m and an orientation along the direction N-W-W to S-E-E. The deck carries both road and rail traffic and is supported by 4 pylons (204 meters tall) and by 80 parallel double-stays – all with an inclination of 30 degrees to the horizontal plane. The stays are covered by a black HDPE tube with helical filets and arranged in a harp-shaped configuration (Fig.1). The cables are comprised of seven-wire mono-strands supplied by Freyssinet. All cables have the same outer diameter of 250 mm. Each cable pair is connected to each other with brackets at one or two locations. Tuned mass dampers are installed on the first and second longest cable pairs. The geometry and fundamental natural frequencies of the cables are summarized in the Table 1. The current monitoring system was installed in 2009 and data were collected between January 2010 and December 2010.

Accelerations are measured on the four longest cables on the south-eastern corner of the eastern side span. This was done to limit the measurement of deck-induced cable vibrations, as the deck is restrained through a pier at this location. The lengths of the four cables range from 192-261m. The accelerations are measured using tri-axial accelerometers produced by GeoSIG (AC-53), with a 200 mg/V sensitivity. The accelerometers are oriented to record the components of the cable vibrations in the in- and out-of-plane directions.

Figure 1. Instruments and monitored cables: ● accelerometer, ♦ anemometer and ▼ rain-gauge.

Ultrasonic anemometers are positioned at the top of the south-easterly pylon, the deck mid-span (south) and between cable pairs 1 and 2, west of the pylon (see Fig. 1). At deck level, the anemometers are installed on poles that extend 7m above the deck. On the pylon top, the anemometer is placed on a 4m pole. Wind is measured in the three Cartesian components, whilst an integrated magnetic compass supplies the exact wind incidence. The anemometers, constructed by Delta Ohm (HD2003R), are heated and are capable of measuring wind velocities up to 60 m/s with a resolution of 0.01 m/s.

A rain-gauge is positioned close to the anemometer on the pylon top to measure rainfall. The rain-gauge, also produced by DeltaOhm (HD2013D), is heated and records rainfall with a resolution of 0.2 mm. Further data collected includes atmospheric humidity, temperature, and pressure. The monitoring system samples at a frequency 30 Hz for all channels. The accelerometers were analogue filtered at 40 Hz.

Table 1. Main information about monitored cables.

<table>
<thead>
<tr>
<th>Cable</th>
<th>Diameter [m]</th>
<th>Length [m]</th>
<th>Accel. position a [m]</th>
<th>Damping [%]</th>
<th>1st, 2nd, 3rd, 4th, 5th, 6th Freq.[Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.25</td>
<td>262</td>
<td>43</td>
<td>0.93</td>
<td>0.47, 0.88, 1.35, 1.79, 2.22, 2.71</td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
<td>239</td>
<td>40</td>
<td>0.93</td>
<td>0.57, 1.13, 1.69, 2.24, 2.81, 3.35</td>
</tr>
<tr>
<td>3</td>
<td>0.25</td>
<td>216</td>
<td>37</td>
<td>0.96</td>
<td>0.51, 0.96, 1.47, 1.96, 2.43, 2.93</td>
</tr>
<tr>
<td>4</td>
<td>0.25</td>
<td>192</td>
<td>34</td>
<td>1.04</td>
<td>0.65, 1.27, 1.92, 2.56, 3.19, ~</td>
</tr>
</tbody>
</table>

a along length position of the accelerometer on the cables from the lower end
4 DATA PROCESSING

To identify vibration events of interest, signals are processed using the Hilbert transform to obtain acceleration envelopes. All of the vibration events considered herewith were recorded during rainfall and have envelopes that exceed a set threshold of 0.1 m/s². Acceleration time-histories are passed through a third-order low-pass Butterworth filter to eliminate the high-frequency content mainly due to undesirable noise. The cut-off frequency is set to include the first significant twenty modes for each cable. A modal decomposition is performed on the accelerations using a band-pass filter to identify modal displacements for each stay. After displacements are obtained through numerical integration, a high-pass Butterworth filter is applied to the displacements to eliminate the presence of low frequency noise. The cut-off frequency for the high-pass filter is set to be half of the estimated fundamental frequency of each stay. Meteorological data obtained at the moment of measured cable oscillations is averaged for the same one minute period. The in-plane and out-of-plane displacements are combined to obtain the maximum displacement, \( x_{\text{max}} \), for a one minute window, so that:

\[
x_{\text{max}} = \sqrt{x_{\text{in}}^2 + x_{\text{out}}^2}
\]

5 RESULTS

In Figure 3a, the polar plots of the peak-to-peak mid-span and/or quarter-span cable displacements for all four cables, normalized by the cable diameter, is presented in relation to cable angle of incidence to the bridge axis. A quick examination of these reveals the cable vibrations occur for specific wind directions, symmetrically about the bridge axis (dotted lines in Figure 3 represent the bridge axis). Two sectors are of particular interest, namely wind 30-75° (or 135–180°) degrees in relation to the magnetic north (cable declining in the wind direction) and 270-315° for the same projection (cable inclining in wind direction). In Figure 3b, polar plots of the peak-to-peak cable displacements for each of the four cables are reported. The polar plots show similar behavior for same sectors of wind directions, whilst the larger displacements are recorded for the longer cables.

The results shown in Figure 4 and Figure 5 are for displacements higher than 0.2D, where D is the diameter of the cables. This limit was chosen, so as not to include vibration events that were predominately due to buffeting. Figure 5 shows the distribution of peak-to-peak normalized cables displacements vs. the yaw angle, \( \beta' \), which is a measure of the direction of the wind with respect to the axis of the stay:

\[
\beta' = \arcsin(\sin \beta \cos \alpha)
\]

where \( \beta \) is the horizontal yaw angle and \( \alpha \) is the inclination angle of the stay (the angles of interest are reported in Figure 2). It is found that vibrations occur within two distinct regions, namely between +35° and +55° and between -25° and -50°, where the negative angles signify a cable inclining in the direction of the wind. This is not in direct contradiction to what has been reported before, but it shows that rain-wind induced vibrations on this bridge are not dominated only by flows along declining cables.

On the left side of Figure 5, it is shown the rain-wind induced displacements against rainfall rates evaluated for one minute time-windows. It can be seen that larger displacements were recorded for light to moderate rainfall, between 1 and 10mm/h. On the right side of Figure 5, it is shown the vibration amplitudes in relation to wind velocity. Here, vibrations greater than 0.2D are recorded for wind velocities between 4-18 m/s. This is slightly broader
than the previously reported ranges. For the two longest cable pairs, vibrations occurred predominantly in the third and fourth modes of vibration (1.3-2.2 Hz), whilst for the third and fourth longest pairs, vibrations occurred predominately in the fourth and fifth modes of vibration (1.9-3.2 Hz). This is generally in agreement with what has been reported before.

Figure 6 shows of the normalized power spectral density (PSD) of the in-plane accelerations of cable 1. The first six frequencies, $f_1$-$f_6$, are indicated with six arrows pointing to the peaks.

Figure 2. Coordinate system and directional components in the gravitation reference system for the cable.

Figure 3a. Polar distribution of peak-to-peak normalized mid-span and/or quarter-span cable displacements in cable diameters vs. wind direction for all cables. In the plots the dotted lines represents the bridge axis. Zero indicates magnetic north.
Figure 3b. Polar distribution of peak-to-peak normalized mid-span and/or quarter-span cables displacements in cable diameters vs. wind direction for each cable. In the plots the dotted lines represents the bridge axis. The zero is the direction of magnetic north.
Figure 4. Distribution of peak-to-peak normalized cables displacements in diameter vs. yaw angle $\beta^*$.  

Figure 5. Distribution of cable displacements in cable diameters vs. rainfall (left) and cable displacements relative to wind velocity (right).  

Figure 6. PSD of in-plane acceleration for an event of RWIV for cable 1. The arrows point the first six in-plane frequencies: $f_1$-$f_6$.  

6 CONCLUSION

The rain-wind induced vibrations of the Øresund Bridge cables are presented for the period January 2010 - December 2010. The full-scale monitoring of the bridge revealed maximum cable vibration amplitudes in the order of 0.6 cable diameters for wind velocities in the order of 11-12 m/sec, occurring for a rainfall of about 5 mm/hour. More generally, RWIV of amplitudes greater than 0.2D were recorded for wind velocities between 4-18 m/sec and rainfall rates of between 0-20 mm/hour. The associated relative yaw angles were predominantly between 35º and +55º and between -25º and -50º degrees, indicating that the vibrations are not only occurring for cables declining in the direction of the wind. More interestingly, the aforementioned vibration amplitudes were recorded, even with the relatively large values of cable damping and in the presence of the helical filets, fitted to the HDPE tubing.

7 ACKNOWLEDGEMENTS

The authors would like to thank Femern A/S and Storebælt A/S for their financial support, without which this work would not have been made possible. They would also like to thank Øresundsbron for their help and cooperation during the construction and usage of the monitoring system.

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Full-scale observations of RWIV of Øresund Bridge
Aeroelastic forces and back-calculation of drag coefficients of a twin stay cable of the Øresund Bridge during vibrations in dry conditions

This chapter presents the results of a study on aeroelastic forces acting on an inclined cable of the Øresund Bridge for wind normal to the cable pair. Stiffness and damping matrices have been identified from full-scale measurements using an Eigenvalue Realization Algorithm (ERA).

Static drag coefficients are back-calculated from the full-scale vibration measurements of Øresund Bridge cable with reasonable agreement with direct wind tunnel measurements.
Aeroelastic forces and back-calculation of drag coefficients of a twin stay cable of
the Øresund Bridge during vibrations in dry conditions
"Identification of aeroelastic forces and static drag coefficients of a twin stay bridge cable from full-scale ambient vibration measurements"

Antonio Acampora, John H. G. Macdonald, Christos T. Georgakis, Nikolaos Nikitas

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*Corrected following reviewers comments on 21st September 2013*
Aeroelastic forces and back-calculation of drag coefficients of a twin stay cable of the Øresund Bridge during vibrations in dry conditions
Identification of aeroelastic forces and static drag coefficients of a twin cable bridge stay from full-scale ambient vibration measurements


Abstract

Despite much research in recent years, large amplitude vibrations of inclined cables continue to be of concern for cable-stayed bridges. Various excitation mechanisms have been suggested, including rain–wind excitation, dry inclined cable galloping, high reduced velocity vortex shedding and excitation from the deck and/or towers. Although there have been many observations of large cable vibrations on bridges, there are relatively few cases of direct full-scale cable vibration and wind measurements, and most research has been based on wind tunnel tests and theoretical modelling.

This paper presents results from full-scale measurements on the special arrangement of twin cables adopted for the Øresund Bridge. The monitoring system records wind and weather conditions, as well as accelerations of certain cables and a few locations on the deck and tower. Using the Eigenvalue Realization Algorithm (ERA), the damping and stiffness matrices are identified for different vibration modes of the cables, with sufficient accuracy to identify changes in the total effective damping and stiffness matrices due to the aeroelastic forces acting on the cables. The damping matrices identified from the full-scale measurements are compared with the theoretical damping matrices based on the quasi-steady theory, using three different sets of wind tunnel measurements of static force coefficients on similar shaped twin or single cables, with good agreement. The damping terms are found to be dependent on Reynolds number rather than reduced velocity, indicating that Reynolds number governs the aeroelastic effects in these conditions. There is a significant drop in the aerodynamic damping in the critical Reynolds number range, which is believed to be related to the large amplitude cable vibrations observed on some bridges in dry conditions.

Finally, static drag coefficients are back-calculated from the full-scale vibration measurements, for first time, with reasonable agreement with direct wind tunnel measurements. The remaining discrepancies are believed to be due to the higher turbulence intensity on site than in the wind tunnel.

1. Introduction

Large amplitude wind-induced vibrations of inclined cables are common. Various mechanisms could be responsible, including von Kármán vortex shedding, rain–wind excitations, cable–deck–tower interaction, high reduced velocity vortex shedding and dry inclined cable galloping. The aerodynamic mechanisms acting on inclined cables are complicated by the three-dimensional environment and the fact that typical sized bridge cables stay in moderate to strong winds sit in the critical Reynolds number region, where there is a rapid drop in the drag coefficient and potential changes in the lift coefficient (Larose and Zan, 2001).

Despite many reports of large amplitude vibrations of cables on various bridges, there have been relatively few direct measurements of the behaviour at full scale and most of the research in this area has been based on wind tunnel tests (Matsumoto et al., 1990, Cheng et al., 2003, Flamand and Boujard, 2009; Jakobsen et al., 2012; Nikitas et al., 2012). Excessive wind-induced vibrations of inclined cables were measured with a long-term full-scale monitoring system by Zuo and Jones (2010) on the Fred Hartman Bridge. It was observed that the three-dimensional nature of the cable–wind environment inherently affects the mechanisms associated with the vibrations of inclined cables. Different types of cable vibrations were identified, including von Kármán vortex induced vibration, rain–wind induced vibration and large amplitude dry cable vibrations. Matsumoto et al. (2003) observed various vibrations of a 30 m long inclined test cable which were classified in a similar way. It has, however, been suggested that the mechanisms of rain–wind-induced vibrations and large amplitude dry cable vibrations may be similar (Macdonald and Larose, 2008b; Flamand and Boujard, 2009; Matsumoto et al., 2010).

Macdonald (2002) measured the aerodynamic damping of cables on the Second Severn Crossing, which was found to often be dominant over the structural damping, even in quite light...
winds for low frequency modes. The results from the full-scale measurements agreed well with the theoretical quasi-steady aerodynamic damping derived for single degree-of-freedom (DOF) vibrations of inclined cables in skew winds in the sub-critical Reynolds number range. Later the theoretical framework was generalised to include variations of the static force coefficients with Reynolds number, fully considering the 3-dimensional geometry and allowing for aerelastic coupling in 2 degrees-of-freedom (2DOF) for in-plane and out-of-plane vibrations (Macdonald and Larose 2006, 2008a,b). Boujard and Grillaud (2007) measured both rain-wind induced vibrations and similar vibrations of dry cables during a 3-year measurement campaign on the Iroise Bridge, and for the dry case they compared the results with predicted instability regions based on Macdonald’s 2DOF quasi-steady model and wind tunnel data on a static inclined model in the critical Reynolds number range.

On the Øresund Bridge, where a twin cable arrangement is used for each stay, large amplitude cable vibrations have been reported, both in the presence of ice on the cables, believed to be caused by galloping, and in conditions significantly above freezing (Svensson et al., 2004). More recently a long-term monitoring system has been installed and vibrations under rain–wind conditions have been reported (Acampora and Georgakis, 2011). At the accelerometer positions, 20 m from the lower end of the cables, filtering and numerical integration of the accelerations found cable vibrations generally having amplitudes below 0.1 diameters. The relatively small amplitudes were believed to be due to the presence of different damping systems used to suppress previous events of larger amplitude reported after the opening of the bridge (Svensson et al., 2004). The maximum cable vibration amplitude was around 0.6 diameters, which occurred in association with rainfall, and the maximum amplitude without rain was about 0.4 diameters, which occurred for wind normal to the vertical cable plane with speeds between 5 and 15 m/s.

Most of the measurements of cable vibrations on full-scale bridges in the past have concentrated on the characteristics of the vibrations, the conditions under which they occur, and possibly their comparison with wind tunnel or theoretical predictions. The aim of this paper is to perform system identification on the full-scale cable vibration measurements to identify aerelastic effects in terms of variations in the total stiffness and total damping matrices in relation to the wind velocity and hence Reynolds number. This is the first time that 2DOF system identification has been attempted on such data and that dependence of the behaviour on Reynolds number has been identified on full-scale cables. Measured data were available for 10-min mean wind velocities up to 18 m/s, corresponding to Reynolds numbers up to $3.0 \times 10^5$. For simplicity the analysis is limited to the condition of wind normal to the cables in dry conditions. The results of the identified damping matrix are compared with theoretical aerodynamic values based on the quasi-steady theory, using mean force coefficients measured in wind tunnel tests. Furthermore the static drag coefficient of the full-scale cables was back-calculated from the measured vibrations.

2. Description of the bridge and monitoring system

The current investigation is based on the data collected from monitoring of the Øresund Bridge. The cable-stayed bridge provides the main navigational span for an 8 km bridge link from Denmark to Sweden. The bridge has a main span of 490 m and an orientation of WNW to ESE. The deck carries both road and rail traffic and is supported by four independent pylons, 204 m tall, and 80 twin cable stays, all with an inclination of 30° to the horizontal plane arranged in a harp-shaped configuration (Fig. 1). Each stay is comprised of a pair of cables, arranged vertically with a centre-to-centre distance of 670 mm and with rigid connections between the individual cables in each pair at one or two locations along the length. The actual cable spacing was chosen during the design stage in order to eliminate wake galloping instabilities. The cables, supplied by Freyssinet, comprise multiple seven-wire mono-strands within HDPE tubes, all with the same outer diameter of 250 mm and with double helical fillets on the surface (dimensions 2.1 mm high × 3.0 mm wide with a rounded top and pitch of 550 mm). Similar multi-cable stays can be found in many cable stayed bridges and also in the hangers of suspension and arch bridges (Gimsing and Georgakis, 2012). Tuned mass dampers are installed on the first and second longest cable pairs (10M, 9M, 9S, 10S, Fig. 1).

The current monitoring system was installed in 2009. The instrument locations are shown in Fig. 1. Various different configurations of accelerometers have been installed on the four longest twin cable stays in the main span (cable stays 10M, 9M, 8M, 7M) and side span (cable stays 10S, 9S, 8S, 7S) at the Swedish end of the bridge on the south and north side of the deck respectively. The accelerometers are oriented to record the components of the cable vibrations normal to the cable axis in the in-plane (i.e. in the vertical plane) and out-of-plane (i.e. lateral) directions. Ultrasonic anemometers are positioned at the top of the east pylon (on a 4 m pole) and on the deck (on the south side on poles 7 m above the deck) at mid-span and between cables pairs 1M and 2M, west of the pylon. A rain gauge is positioned close to the anemometer on the top of the pylon. Further data collected include atmospheric temperature, pressure and humidity. The monitoring system samples all channels at a frequency of 30 Hz and saves files 10 min long, chosen to be consistent with the average period for mean wind speeds in Eurocode 1 (British Standards Institution, 2005). The twin cable stay studied in this paper is designated as 8M, and is the third longest stay in the main span (Fig. 1), with length of 216 m, mass per unit length of 99 kg/m and first natural frequency of 0.56 Hz. This cable was chosen since it is the longest one that does not have a tuned mass damper and will subsequently produce for a certain range of wind speeds the highest values of reduced wind speeds.

The data for this paper were collected continuously from October 2010 to August 2012. Records for dry conditions and wind directions within ± 5° of normal to the vertical cable plane were

![Fig. 1. Elevation and plan of Øresund Bridge showing locations of instruments.](image-url)
selected, giving a total of 2101 10-min records (i.e. 4% of the total recording time) that have been analysed herewith. No significant difference was found in the measurements or results between winds from the north or south sides of the bridge, so they have been treated together here. The wind speed was taken to be the mean of the measured wind speeds at the top of the pylons and on the deck at mid-span, which would be representative of the mean wind experienced by the cable (assuming a linear wind speed distribution, which is the only option for the two instruments in place). The largest differences recorded between the two anemometers were of the order of 20%. Records were grouped together in bins at 1 m/s intervals to enable averaging of results at each wind speed, from 0 to 18 m/s, i.e. Reynolds numbers 0–3.0 × 10^5 at intervals of 1.67 × 10^4. The distribution of records in each bin is shown in Fig. 2. Apart from the 0 m/s bin (i.e. 0–0.5 m/s), there were at least 27 records in each wind speed bin for the averaging. For wind speeds above 6 m/s (i.e. Re=1.0 × 10^5) the mean longitudinal turbulence intensity (Iu) was 40% and the mean longitudinal length scale (l_u) was 40 m.

3. System identification

The aim here is to identify the damping and stiffness matrices of the cable vibrations as a function of Reynolds number and to compare them with the theoretical values according to the quasi-steady theory using wind tunnel data of the static force coefficients. The Power Spectral Densities (PSDs) of measured accelerations in the two planes are shown for a typical record in Fig. 3. The auto- and cross-covariance functions of the out-of-plane and in-plane modes of the cable (in the absence of wind) are the external generalised forces in the two planes due to wind (excluding aeroelastic effects) and cable end motion. K is the total generalised stiffness matrix, given by the sum of the structural stiffness matrix, K_s, and the aerodynamic stiffness matrix, K_a:

\[
K = K_s + K_a
\]

where X and Y are the generalised displacements of the out-of-plane and in-plane mode, respectively, dots represent derivatives with respect to time, M is the generalised mass matrix, and \( \omega_i \) are respectively the circular natural frequencies of the ith out-of-plane and in-plane modes of the cable (in the absence of wind), and \( \zeta_i \) are the corresponding modal structural damping ratios.

From output-only measurements alone it is not possible to identify M and the matrices C and K themselves, but it is possible to identify CM and KM in Eq. (1). Also it is not possible to explicitly separate the structural and aerodynamic components from a single record, but the variation of CM and KM with wind conditions can be identified from multiple records in different conditions. Restricting analysis to small amplitude oscillations can effectively discards any implications of non-linear structural damping and stiffness.

For each 10-min record (corresponding to at least 336 vibration cycles), the auto- and cross-covariance functions of the out-of-plane and in-plane filtered accelerations were computed. Examples are plotted in Fig. 4, from a typical record with mean wind speed 7.1 m/s (Re = 1.2 × 10^5) and \( I_p = 3.6\% \). The functions from each record were then analysed with an Eigenvalue Realisation Algorithm (ERA) system identification procedure, based on a state-space realisation.
from the Markov-Block-Hankel matrix, to identify the stiffness matrix $K/M$ and damping matrix $C/M$ (Jakobsen and Hjorth-Hansen, 1995). The results from any one record are subject to significant variance errors, due to the random nature of the vibrations, but the mean results over the large number of records in each bin become statistically reliable. This method has been found to be effective for full-scale ambient vibration measurements, having recently been applied to full-scale data from the Clifton Suspension Bridge, identifying clear trends in the flutter derivatives with wind velocity, including aeroelastic coupling between vertical and torsional vibrations of the bridge deck (Nikitas et al., 2011). The method assumes the external loading to be white noise in the proximity of the frequencies of interest and that the system is linear, including that the aeroelastic forces can be modelled as linear functions of cable displacement and velocity, as in Eq. (1). The white noise assumption seems rational for the case in hand. There is no concentrated harmonic loading as in vortex shedding (i.e. see broad reduced wind speed range, helical fillets acting as vortex suppression measure and turbulent/non-smooth flow) and locally around the cable frequencies any colouring effect of the loading spectrum may be considered small.

An important parameter in the analysis is the choice of the maximum lag length of the covariance functions used (c.f. Fig. 4). If too short, useful information is discarded, but if too long, random fluctuations are included and can increase the variance of the results. In the absence of any theoretical basis for the choice of the maximum lag length, it was aimed to minimise the variance of the outputs from statistical analysis of the results from a representative wind speed bin (6 m/s, i.e. Re = 1.0 × 10^5) for the modes of interest. For a whole series of different maximum lag lengths the system identification procedure was carried out on all records in the bin. For each maximum lag length the coefficient of variation (COV, i.e. standard deviation/mean) of each of the 8 extracted parameters ($K_{XX}/M$, $K_{YY}/M$, etc., $C_{XX}/M$, $C_{YY}/M$, etc.) was computed. The results for the second mode pair are plotted against maximum lag length in Fig. 5. As can be seen, the COVs for the stiffness terms are much lower than for the damping terms, which is consistent with the well-known fact that natural frequencies can be identified quite accurately from ambient vibration measurements but that damping values are more difficult to identify reliably. The stiffness terms are identified relatively inaccurately when using short time lags (Fig. 5(a)), with the accuracy increasing (i.e. COVs decreasing) until about 60 s, after which longer lags make little difference to the accuracy. Considering the damping terms, the COVs of $C_{XX}/M$ and $C_{YY}/M$, which are the outputs of most interest (see Section 6), were a minimum for maximum lag lengths around 54 s, although from about 25–150 s there was little variation in the COV. The COVs of the cross-terms, $C_{XY}/M$ and $C_{YX}/M$, were quite insensitive to the choice of maximum lag length. Considering the accuracy of all the extracted parameters, particularly $C_{XX}/M$ and $C_{YY}/M$, the optimum value of maximum lag length for the analysis was selected as 54 s for the second mode pair. At this lag length the auto-covariance functions had typically decayed to a magnitude of about 0.1–0.2 (see, for example, Fig. 4), which was around the noise level for

![Fig. 4. Typical example of auto- and cross-covariance functions for the combined 2DOF system, for the second mode pair, plotted against time lag (from 10-min record with mean wind speed 7.1 m/s, i.e. Re = 1.2 × 10^5).](image)

![Fig. 5. Coefficient of variance of results as a function of maximum lag length included in the analysis, for the second mode pair. (a) For each element of the stiffness matrix; (b) for each element of the damping matrix.](image)
longer lags, intuitively implying that this was a reasonable maximum lag length. For consistency the same maximum lag length, in terms of number of cycles in the relevant mode pair (i.e. 61 cycles), was used for all of the analysis thereafter.

The system identification was performed on each selected record for the first five pairs of modes, after separately filtering the responses in each pair of modes. The average results for each wind speed bin for each mode are presented and discussed in Section 6.

4. Theoretical aerodynamic damping matrix

For comparison with the results obtained from the full-scale measurements, theoretical values of the aerodynamic damping matrix were calculated. Based on the quasi-steady assumption that the aerodynamic forces on a moving cable are given by the forces on a stationary cable experiencing the same relative wind velocity, the generalised 2DOF aerodynamic damping matrix, allowing for three-dimensional geometry and changes in the aerodynamic forces coefficients with Reynolds number, was given by Macdonald and Larose (2008a) (assuming variations of the force coefficients are smooth functions and linearising for small cable velocity compared to the wind velocity). For a uniform cable in a uniform wind normal to the cable axis, the theoretical quasi-steady aerodynamic matrix normalised by mass simplifies to

\[
C_f = \frac{\rho DU}{2m} \begin{bmatrix}
2C_D + \frac{\partial C_D}{\partial Re} \alpha_D + \frac{\partial C_D}{\partial \alpha} \frac{\alpha_D}{Re} & -C_L + \frac{\partial C_L}{\partial Re} \alpha_D + \frac{\partial C_L}{\partial \alpha} \frac{\alpha_D}{Re} \\
2C_L + \frac{\partial C_L}{\partial Re} \alpha_D + \frac{\partial C_L}{\partial \alpha} \frac{\alpha_D}{Re} & C_D + \frac{\partial C_D}{\partial \alpha} \frac{\alpha_D}{Re}
\end{bmatrix}
\]

where \( \rho \) is the density of air, \( D \) is the reference dimension (cable diameter), \( U \) the mean wind speed, \( m \) the mass per unit length, \( C_D \) and \( C_L \) respectively the drag and lift coefficients, \( \alpha \) the angle of attack about the cable axis, and \( Re \) the Reynolds number (\( = \rho DU/\mu \), where \( \mu \) is the absolute viscosity of air). In the case of angle of attack \( \alpha = 0 \) (i.e. \( X \) along-wind and \( Y \) across-wind and the wind velocity in the plane of symmetry), assuming that \( C_L, \frac{\partial a}{\partial Re} \) and \( \frac{\partial a}{\partial \alpha} \frac{\alpha_D}{Re} \) are zero for a symmetric section, the generalised damping matrix (2) simplifies to

\[
C_f = \frac{\rho DU}{2m} \begin{bmatrix}
2C_D + \frac{\partial C_D}{\partial Re} \alpha_D & 0 \\
0 & C_D + \frac{\partial C_D}{\partial \alpha} \frac{\alpha_D}{Re}
\end{bmatrix}
\]

5. Wind tunnel measurement of static force coefficients

To obtain the drag and lift coefficients to use in the matrix (3), static wind tunnel tests were performed in the DTU/Force Technology 2 m x 2 m cross-section closed circuit Climatic Wind Tunnel in Kgs. Lyngby, Denmark. The wind tunnel has a maximum wind speed of 31 m/s and typical turbulence intensity of 0.64% for smooth flow. The other technical specifications of the wind tunnel were reported by Georgakis et al., (2009).

A sectional cable model was manufactured scaling the dimensions of the twin cables of the Øresund Bridge (full-scale diameter 250 mm) by a factor of 2.2. This scale was chosen as a compromise considering the Reynolds number range of interest and the blockage ratio. The maximum Reynolds number achievable was then \( 2.3 \times 10^5 \) and the blockage ratio was 8.5%, which is in accordance with blockage in previous aerodynamics studies on staggered smooth circular cylinder arrangements (see Table 1, Sumer et al., 2000). The double helical fillet, which is expected to govern the Reynolds transitional behaviour, was reproduced at scale using a steel wire attached with double-sided tape. The length of the model was 1490 mm. The PVC surface was polished to match the scaled target surface roughness of the Øresund cables (made of HDPE). Measured values of the average surface roughness \( Ra \) of the model were in the range of 0.5–1 \( \mu \)m (much below the fillet thickness). The actual roughness of the bridge cables on site is 0.7–1 \( \mu \)m (Matteoni and Georgakis, 2012). The model setup in the wind tunnel is seen in Fig. 6.

The test programme consisted of wind velocity sweeps between 1 and 31 m/s, corresponding to Reynolds numbers of \( 3 \times 10^3 \text{–} 2.3 \times 10^5 \), based on a reference dimension of 1D (single diameter), for angles of attack \( \alpha = 0 \) and \( 30^\circ \) (Acampora and Georgakis, 2011), where \( \alpha = 0 \) is defined as wind normal to the plane containing the two cable axes, as seen in Fig. 6.

The aerodynamic forces were measured with two 6-DOF force transducers (AMTI MC3A-500), mounted at the ends of the twin cable. The force transducers were installed on rigid steel plates between the ends of the cable model and the supporting cardan joints fixed to the wall measuring the global force on the cable ensemble (Fig. 6). The whole cable apparatus lies entirely inside the wind tunnel space, and this configuration was opted to avoid the effects of leakage flow. End effects on a static model normal to the flow, where static force coefficients are measured are not expected to encompass the distorting features that were previously recorded for the dynamic models in Matsumoto et al. (2001) or Yagi et al. (2009). For example in the seminal work by Delany and Sorensen, 1953, despite the leakage end flow the recovered drag evolution with Reynolds number seems undistorted. The mean force coefficients were found by averaging the measurements over 1 min at each wind speed. The drag and lift coefficients are defined here on the basis of the total force on the twin cable setup and the reference dimension of 1D. Therefore, when both cables are exposed to the wind the drag and lift coefficients are expected to be approximately twice the value for a single cylinder. The drag coefficients from the wind tunnel test, corrected for blockage using the Maskell III method (Cooper et al., 1999) are shown in Fig. 7 for \( \alpha = 0 \). Note that the static force coefficients corresponding to a single cylinder were presented and later used, thus the values from the twin cable are plotted in Fig. 7 as half of the total drag coefficient (based on 1D). Also shown for comparison are results from two previous sets of tests related to the Øresund Bridge cables, for single cables with different geometries of helical fillets. Preliminary tests for the Øresund Bridge cables were conducted by Larose and Smitt (1999) to investigate rain–wind induced vibrations, using a twin cable arrangement with double helical fillets and a dynamic rig. Aerodynamic forces were not measured. Static drag coefficients were reported from tests performed by the cable supplier, Freyssinet, during studies.
for the design of the bridge (Øresundsbron, 2003) on a single cable with a single helical fillet. More recently similar tests were conducted by Kleissl and Georgakis (2012) on a single cable with a double helical fillet in the same wind tunnel and using the same blockage correction method as the present tests. However, note that in neither of these two previous cases for which static force coefficients are available was the fillet geometry exactly as on the real bridge nor did they model the actual twin cable arrangement and any resulting flow interaction that might occur impacting the static force measurements. The lift coefficient in the current tests (for $\alpha=0$) and from Kleissl and Georgakis (2012) was small, with a maximum absolute value less than 0.1, whilst it was not reported from the Freyssinet tests and it was considered to be zero.

6. Results and discussion

The results of the system identification from the full-scale data give the modal properties of each pair of cable modes in terms of the stiffness and damping matrices for the coupled 2DOF system. The first five pairs of modes were considered. The outputs are presented as the mean results within each wind speed bin, represented in terms of Reynolds number.

6.1. Identified stiffness matrices and natural frequencies

The identified elements of the total stiffness matrix for each of the first five mode pairs are shown in Fig. 8. The diagonal terms show distinct values for the five modes, with little variation with Reynolds number. In all cases the off-diagonal terms of the total stiffness (or mass) matrix are virtually zero, indicating negligible stiffness coupling between out-of-plane and in-plane vibrations. These results are as expected since, according to the quasi-steady theory, there is no aerodynamic stiffness for translational vibration, which is very different for the modes. For reference, the wind speed range $0–18$ m/s ($Re=0–3.0 \times 10^6$) corresponds to reduced velocities of $0–128$ for the first mode down to $0–26$ for the fifth mode.

The diagonal terms of the damping matrix show linear trends up to about $Re=1 \times 10^5$ (i.e. 6 m/s at full scale). This implies that this can be considered as the end of the sub-critical Reynolds number region. This is in agreement with Simiu and Scanlan (1996) who state that for circular cylinders with a smooth surface if the wind is not turbulent, the sub-critical Reynolds number region corresponds to a range of $0$ to about $2 \times 10^5$, but if the wind is turbulent or if the cable surface is rough, the upper bound of the subcritical region will be lower. Recent investigations indicated that for a smooth-surfaced cylinder, with a wind turbulence intensity of 2.6% in the wind tunnel, the upper bound of the sub-critical region occurs for a Reynolds number of about $1.6 \times 10^5$ (Zasso et al., 2005). Similarly, a critical Reynolds number of $1.5 \times 10^5$ is estimated for a single smooth circular cylinder (i.e. without fillets) with two-dimensional flow, based on the empirical equations provided by ESDU (1986), using the mean values of the stiffness matrix for the bridge (Øresundsbron, 2003) on a single cable with a single helical fillet. More recently similar tests were conducted by Kleissl and Georgakis (2012) on a single cable with a double helical fillet in the same wind tunnel and using the same blockage correction method as the present tests. However, note that in neither of these two previous cases for which static force coefficients are available was the fillet geometry exactly as on the real bridge nor did they model the actual twin cable arrangement and any resulting flow interaction that might occur impacting the static force measurements. The lift coefficient in the current tests (for $\alpha=0$) and from Kleissl and Georgakis (2012) was small, with a maximum absolute value less than 0.1, whilst it was not reported from the Freyssinet tests and it was considered to be zero.

6.2. Identified damping matrix and comparison with theoretical values

The identified elements of the total damping matrix for each of the first five mode pairs are shown as discrete symbols in Fig. 10. Also shown, as points linked with lines, are the theoretical values for the diagonal terms, $C_{xx}/M$ and $C_{yy}/M$, based on Eq. (3) using the static force coefficients obtained from the three different sets of wind tunnel tests. There are clear trends of the identified results with Reynolds number, indicating aerodynamic damping. The identified values are for the total damping matrix, including the structural damping, whereas the theoretical values are for the aerodynamic damping only. However, the trends of the identified values at low wind speeds indicate that the structural damping (i.e. the extrapolated value for zero wind speed) is negligible in relation to the aerodynamic damping.

The cross-terms of the damping matrix, $C_{xy}/M$ and $C_{yx}/M$, are virtually zero in all cases. This indicates no aerodynamic damping coupling between vibrations in the two planes, in agreement with the quasi-steady theory for the chosen wind direction (normal to the cable) (Eq. 3), although for other wind directions coupling is expected (Macdonald and Larose, 2008a). The identified results from the five different modes are remarkably consistent. This not only gives more confidence in the results but also strongly indicates that the aerodynamic damping matrix is a function of the Reynolds number (simply proportional to wind speed) and not of the reduced wind velocity, which is the other commonly-used non-dimensional group based on the wind speed, but is inversely proportional to the frequency of vibration, which is very different for the five modes. For reference, the wind speed range $0–18$ m/s ($Re=0–3.0 \times 10^6$) corresponds to reduced velocities of $0–128$ for the first mode down to $0–26$ for the fifth mode.

The diagonal terms of the damping matrix show linear trends up to about $Re=1 \times 10^5$ (i.e. 6 m/s at full scale). This implies that for the design of the bridge (Øresundsbron, 2003) on a single cable with a single helical fillet. More recently similar tests were conducted by Kleissl and Georgakis (2012) on a single cable with a double helical fillet in the same wind tunnel and using the same blockage correction method as the present tests. However, note that in neither of these two previous cases for which static force coefficients are available was the fillet geometry exactly as on the real bridge nor did they model the actual twin cable arrangement and any resulting flow interaction that might occur impacting the static force measurements. The lift coefficient in the current tests (for $\alpha=0$) and from Kleissl and Georgakis (2012) was small, with a maximum absolute value less than 0.1, whilst it was not reported from the Freyssinet tests and it was considered to be zero.

6. Results and discussion

The results of the system identification from the full-scale data give the modal properties of each pair of cable modes in terms of the stiffness and damping matrices for the coupled 2DOF system. The first five pairs of modes were considered. The outputs are presented as the mean results within each wind speed bin, represented in terms of Reynolds number.
longitudinal turbulence intensity and length scale estimated from the site measurements for wind speeds above 6 m/s (Section 2).

In the sub-critical range, the identified results agree very well with the theoretical results from the quasi-steady theory for all three sets of wind tunnel data used, for vibrations in both planes. This is consistent with the measurements on the cables of the Second Severn Crossing (single smooth circular cables) in the sub-critical Reynolds number range, where the damping (of in-plane modes only) was measured by free-decay tests rather than ambient vibrations, agreeing well with quasi-steady theory (Macdonald, 2002).

For Reynolds numbers above \(1.2 \times 10^5\) the diagonal terms of the damping matrices show a reduction, due to effects in the critical Reynolds number range. The theoretical and experimental values differ in this region, although, as stated, the actual geometry of the cables on the bridge is slightly different than in the previous wind tunnel tests (Øresundsbron, 2003; Kleissl and Georgakis (2012)).

Nevertheless, qualitatively the full-scale and theoretical results show similar features. Based on the twin cable wind tunnel data, the theoretical results for \(C_{xx}/M\) continue a linear trend up to about \(Re = 1.4 \times 10^5\), after which there is a drop in the aerodynamic damping, corresponding with the drop in the measured drag coefficient (Fig. 7). The quantitative difference from the full-scale data could be explained by different turbulence characteristics causing the critical Reynolds number to be lower in the natural wind on site than in the wind tunnel.

The minimum damping from the site data, in both planes, occurs for a Reynolds number around \(2.4 \times 10^5\) (full scale wind speed \(\approx 15\) m/s), but it is still positive, indicating there is not an aerodynamic instability of the cable in these conditions, in line with the observation of no large amplitude vibrations. However, this drop in damping is believed to be related to the large amplitude vibrations of dry cables observed on some bridges in some conditions. Such large
amplitude vibrations have been observed in large scale dynamic wind tunnel tests, in which it was also identified that they are related to effects in the Reynolds number range and that the aerodynamic forcing causing the instability appears to act as negative aerodynamic damping (Jakobsen et al., 2012; Nikitas et al., 2012).

7. Back-calculation of static drag coefficient from full-scale vibration data

Having identified the damping matrix values from the full-scale measurements and having shown that the quasi-steady theory seems to give reasonable results, it was possible to back-calculate the static drag coefficient of the actual twin cable on the bridge from the full-scale vibration data. From Eq. (3), based on quasi-steady theory (Macdonald and Larose, 2008a), and neglecting the structural damping, the relationship between the $C_{XX}/M$ term of the damping matrix and the drag coefficient and Reynolds number can be expressed as:

$$\frac{1}{\text{Re}} \frac{d}{d\text{Re}} (C_D \text{Re}^2) = C_{XX} \frac{2m}{M \mu DU}$$

i.e.

$$\frac{d}{d\text{Re}} (C_D \text{Re}^2) = C_{XX} \frac{2m}{M \mu}$$

(5)

Hence, noting that $C_D \text{Re}^2=0$ for $\text{Re}=0$, the drag coefficient can be back-calculated from the values of $C_{XX}/M$ identified from the full-scale data, as

$$C_D = \frac{2m}{\mu \text{Re}^2} \int_0^{\text{Re}} C_{XX}/M \text{dRe}$$

(6)

Given that the identified values of $C_{XX}/M$ were very consistent for the five different modes, the mean value from the five modes at each Reynolds number was used in the calculations. The results of the back-calculated drag coefficient from the Øresund Bridge cable are shown in Fig. 11, along with the values obtained directly from static wind tunnel tests.

The back-calculated drag coefficient agrees well with the direct measurements from the wind tunnel tests. In particular, the shape of the curve and the maximum value agree very well with the wind tunnel results for the twin cable with the same geometry (Section 5). The curve from the full-scale vibration measurements is shifted to lower Reynolds number relative to the wind tunnel measurements, but this discrepancy could well be accounted for by the greater turbulence intensity on site causing a drop in the critical Reynolds number (ESDU, 1986).

8. Conclusions

The total stiffness and damping matrices for a single cable in a twin cable stay of the Øresund Bridge have been identified for the first five pairs of cable modes from ambient full-scale vibrations for wind
normal to the cables with no rain. It is believed this is the first time such matrices have been identified from full-scale cable vibrations. The stiffness matrix shows near constant values for the range of wind velocities and temperatures experienced, indicating no discernible aerodynamic effect and only minor changes in cable tension, possibly due to variable traffic loading. The damping matrix for this wind direction is diagonal and, based on the very consistent results from all five mode pairs, has been shown to be a function of Reynolds number rather than reduced velocity. This is a significant result that demonstrates that within the critical Reynolds number range, the Reynolds number itself is the dominant non-dimensional parameter in the fluid-structure interaction. Hence, aerodynamic instabilities of dry cables in strong winds are very likely to be governed by critical Reynolds number effects.

The damping values identified from the full-scale data agree well with the theoretical aerodynamic damping matrix based on the quasi-steady theory. In particular, up to about 6 m/s (Re = 1.0 × 10^5), it is believed to be the end of the sub-critical Reynolds number region, there is excellent agreement. For higher Reynolds number the quantitative results differ, but qualitatively they exhibit similar features, with a relative drop in the total damping for both in-plane and out-of-plane vibrations, believed to be due to critical Reynolds number effects. Hence it seems that quasi-steady theory gives a reasonable description of the actual behaviour. The minimum damping in each plane occurs at about Re/C2 = 2.4 × 10^4 (14 m/s) from the site measurements. The total damping remains positive, hence there was no dynamic instability of the cable in the conditions considered, but the large vibrations of some cables on some bridges in strong winds in dry conditions are believed to be related to the drop in the total damping identified.

The quantitative difference in the damping matrix between the identified values and the quasi-steady theoretical values may be because of the higher turbulence intensity on site. The real cable also experiences variable wind velocity along its length.

The identified values of damping matrix have been used to back-calculate the drag coefficient of the Øresund twin cable, assuming the quasi-steady theory applies. It is believed that this is the first time that static force coefficients have been estimated from full-scale vibrations measurements. The back-calculated values show good agreement with the directly measured drag coefficients from wind tunnel tests, except that the full-scale results are shifted to lower Reynolds numbers relative to the results for the twin cable model, which is believed to be likely due to the greater wind turbulence on site. A further similar study on other bridges and more cable stays can unveil more on-site aerodynamic characteristics of this type of stay.

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Amendment C.
Aeroelastic forces and back-calculation of drag coefficients of a twin stay cable of the Øresund Bridge during vibrations in dry conditions
Influence of rain on a cable vibrations for a skewed wind condition

This chapter presents the result of the influence of the precipitation on the damping and the stiffness matrices of a pair cables of the Øresund Bridge for three different conditions of precipitation: no rainfall, light rain and moderate rain.

The system identification, tested in the previous work for dry conditions and wind normal to the cable, is now applied on vibrational data recorded for a skewed wind direction that is reported to be connected with RWIVs. The matrices are identified with sufficient accuracy to recognize changes in the total effective damping and stiffness matrices for the three different conditions of precipitation.

Back-calculation of aerodynamic coefficients were obtained from identified damping matrices of the three conditions of precipitation. Comparison of back-calculated coefficients and aerodynamic coefficients based on wind tunnel tests is shown.
Influence of rain on a cable vibrations for a skewed wind condition
Paper III

"Influence of rain on a cable vibrations and back-calculated aerodynamic force coefficients for a skewed wind condition"

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Influence of rain on a cable vibrations for a skewed wind condition
Influence of rain on a cable vibrations and back-calculated aerodynamic force coefficients for a skewed wind condition

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Abstract

Moderate to large amplitude vibrations of inclined cables continue to be observed and to be of concern for cable-stayed bridges. Different excitation mechanisms have been suggested, including rain-wind-induced excitation, dry inclined cable galloping, high reduced-velocity vortex shedding, von Kármán vortex shedding and excitation from the deck and/or towers. In the case of rain-wind-induced vibrations, which account for the majority of the observed vibrations, research was driven by wind tunnels testing of cable sections with artificially introduced water or solid rivulets and by theoretical works undertaken to determine the propensity of a bridge cable to vibrate when a superficial alteration is introduced. The main contributing factors for those vibrations are the simultaneous occurrence of the low inherent damping of the stays and the presence of water rivulet on the surface of the cable.

Employing the Eigenvalue Realization Algorithm (ERA), damping and stiffness matrices are identified for the first three vibration modes of the twin cables for three different conditions of precipitation (no rainfall, light rain and moderate rain) from the monitoring data collected on the Øresund Bridge. The matrices are identified with sufficient accuracy to recognize changes in the total effective damping and stiffness matrices due to the aeroelastic forces acting on the cables within the three different conditions of precipitation. Back-calculated values of aerodynamic force coefficients based on quasi-steady theory are evaluated from identified damping matrices from the full-force measurements and are compared with the wind tunnel coefficients obtained under dry and rainy conditions. Changes of aerodynamic force coefficients as a function of precipitation are shown.

1. Introduction

Despite many reports of large amplitude vibrations of cables on various bridges due to rain-wind induced vibrations (RWIV), there have been relatively few direct measurements of the behaviour at full scale and most of the research in this area has been based on wind tunnel tests (Matsumoto et al. 1990, Cheng et al. 2003, Flamand & Boujard 2009). Observations of RWIV were reported by Hikami (1988) that first identified this mechanism on the Meiko-Nishi Bridge in Japan. Hikami noticed that the vibrations did not occur for all wind velocities and for all cable frequencies. Instead, the oscillations were limited to a wind velocity range of 5-15m/s and for cable modes with frequencies between 1-3 Hz. Furthermore, all of the oscillations occurred only in the presence of rain. Main et al. (2001) reported high amplitude cable responses occurred over a range of relative yaw angles, between 0° and 40°. Ni et al. (2007) showed the following meteorological conditions for RWIV: wind velocities between 6-14m/s under light to moderate rainfall of up to 8mm/h. The relative yaw angle ranged between approximately 10° and 50°. Zuo & Jones (2010) measured excessive wind-induced vibrations of inclined cables on the Fred Hartman Bridge with a long-term full-scale monitoring system. It was observed that the three-dimensional nature of the cable-wind environment inherently affects the mechanisms associated with the vibrations of inclined cables.

On the Øresund Bridge, large amplitude cable vibrations have been reported, both in the presence of ice on the cables, believed to be caused by galloping, and in conditions significantly above freezing (Larose 1999, Svensson et al. 2004, Øresundsbron 2003). In 2009, a long-term monitoring system was installed and vibrations under rain-wind conditions have been reported (Acampora & Georgakis, 2011). The cable vibrations generally had amplitudes below 0.1 diameters. The maximum cable vibration amplitude was around 0.6 diameters, which occurred in association with rainfall, and the maximum amplitude without rain was about 0.4 diameters, which occurred for skewed wind with respect the vertical cable plane with speeds between 5 and 15 m/s (Acampora & Georgakis 2011). The small amplitudes are believed to be due to the
presence of different damping systems used to suppress previous vibration events of larger amplitude reported after the opening of the bridge (Svensson et al. 2004). Acampora et al. (2013) identified a two-degree-of-freedom system using an Eigenvalue Realization Algorithm (ERA) on data for dry conditions and wind orthogonal to the cable plane showing the dependence of the behaviour on Reynolds number.

The aim of this paper is to perform a system identification on the full-scale cable vibration measurements to identify the effect of different precipitation conditions (no rainfall, light rain and moderate rain) on the total stiffness and total damping matrices for the Reynolds number range up to $1.8 \times 10^5$ (wind velocities up to 13 m/s). Unlike Acampora et al. (2013), this paper deals with the evaluation for skewed winds. The results of the identified damping matrices are used to back calculate aerodynamic coefficients based on quasi-steady theory for the three conditions of precipitation and compared with aerodynamic force coefficients from wind tunnel tests in dry and rainy conditions.

2. Description of the bridge and monitoring system

The current investigation is based on data collected from monitoring of the Øresund Bridge. The cable-stayed bridge provides the main navigational span for an 8 km bridge link from Denmark to Sweden. The bridge has a main span of 490 m and an orientation of WNW to ESE. The deck carries both road and rail traffic and is supported by 4 independent pylons, 204 m tall, and 80 double-stays, all with an inclination of 30° to the horizontal plane arranged in a harp-shaped configuration (Figure 1). Each double-stay is made up of a twin cable, arranged vertically with a centre-to-centre distance of 670 mm and with rigid connections between the two cables in each pair at one or two locations along the length. The cables, supplied by Freyssinet, comprise multiple seven-wire mono-strands within HDPE tubes, all with the same outer diameter $D$ of 250 mm and with double helical fillets on the surface (dimensions 2.1 mm high x 3.0 mm wide with a rounded top and pitch of 550 mm). Tuned mass dampers are installed on the first and second longest cable pairs (10M, 9M, 9S, 10S, Figure 1). The current monitoring system was installed in 2009. The instrument locations are shown in Figure 1. The accelerometers are oriented to record the components of the cable vibrations normal to the cable axis in the in-plane (i.e. in the vertical plane) and out-of-plane (i.e. lateral) directions. Ultrasonic anemometers are positioned at the top of the east pylon (on a 4 m pole) and on the deck (on the south side on poles 7 m above the deck) at mid-span and between cables pairs 1M and 2M, west of the pylon. A rain gauge is positioned close to the anemometer on the top of the pylon. Further data collected includes atmospheric temperature, pressure and humidity. The monitoring system samples all channels at a frequency 30 Hz and saves files of 10 minutes length, chosen to be consistent with the averaging period for mean wind speeds in Eurocode.

The double-stay cable studied in this paper is Cable 8M, the third longest stay on the main span (Figure 1), with a length of 216 m, a mass per unit length of 99 kg/m and first natural frequency of 0.56 Hz. This cable was chosen since it is the longest one that does not have a tuned mass damper. The data for this study was collected continuously from October 2010 to August 2012. 10-minute records for wind directions 155° ± 5° clockwise from geographical north (wind skewed 20° from the normal to the vertical cable plane) for different meteorological conditions were selected. This direction was chosen because of the largest observed vibrations for rainy conditions, which is also in the typical range of wind directions expected for RWIV. A total of 632 records were collected. They were divided in 3 categories of precipitation (no rain; light rain: rainfall rate less than 2.5 mm/hr; moderate rain: rainfall rate in the range of 2.5–5 mm/hr) for a number of 425, 132 and 75 records respectively. The wind velocity is a function of the height over the sea level. The wind speed was taken to be the mean of the measured wind speeds at the top of the pylon and on the deck at mid-span, which would be representative of the mean wind experienced by the cable. The records were grouped together in bins at 0.5 m/s intervals to enable averaging of results at each wind speed, from 2.5 to 13 m/s, i.e. Reynolds numbers range from 0.4 to 1.8 $\times 10^5$ at intervals of 8.5 $\times 10^4$, considering a single diameter $D$. The distribution of records in each bin is shown in Figure 2 for the different precipitation cases. For Reynolds number $Re$ above 1.0 $\times 10^5$ (i.e. wind speeds above 7.5 m/s) the mean longitudinal turbulence intensity ($I_u$) was 3.6% as measured at the top of the tower, away from the influence of the bridge deck and passing traffic,
and the mean longitudinal integral length scale ($L_u$) was 46m, without significant difference for the three precipitation conditions.

**Figure 1.** Elevation view of Øresund Bridge showing locations of instruments.

**Figure 2.** Distribution of records vs. Reynolds number for the three conditions of precipitation: no rain, light rain and moderate rain.

### 3. System identification

The aim here is to identify the damping and stiffness matrices of the cable vibrations as a function of Reynolds number for the three conditions of precipitation. A typical example of the Power Spectral Densities (PSDs) of measured accelerations in the two planes is shown in Figure 3. The cables exhibit vibrations in multiple modes, which are almost a pure harmonic series, in accordance with taut string theory. For simplicity one pair of modes (one mode in each plane with almost identical natural frequencies) is considered at a time. Macdonald & Larose (2008a) showed that, for a linear aerodynamic damping matrix (per unit length of cable), each pair of modes can be considered independently. Therefore the raw accelerations were filtered with 15th order high-pass and low-pass filters to isolate the vibrations of a single
mode in each plane, taking care not to distort the signals for the modes in question themselves (Figure 3). In general the aeroelastic forces can couple the modes with virtually the same natural frequencies in the two planes, so the filtered signals are considered as the response of a 2-DOF system. Since the measurements did not detect any rotation of the twin cables, any influence of this effect is neglected. The equations of motion of the 2-DOF system (assumed to be linear) can be written in the following form:

\[
\begin{bmatrix}
\dot{X} \\
\dot{Y}
\end{bmatrix} + \frac{C}{M} \begin{bmatrix}
X \\
Y
\end{bmatrix} + \frac{K}{M} \begin{bmatrix}
X \\
Y
\end{bmatrix} = \frac{1}{M} \begin{bmatrix}
F_X \\
F_Y
\end{bmatrix}
\]

(1)

where \(X\) and \(Y\) are the generalised displacements of the out-of-plane and in-plane mode, respectively. Dots represent derivatives with respect to time, \(M\) is the generalised mass of the modes (assumed to be the same for vibrations in each plane), and \(F_X\) and \(F_Y\) are the external generalised forces in the two planes due to wind (excluding aeroelastic effects) and cable end motion. \(K\) is the total generalised stiffness matrix, given by the sum of the structural stiffness matrix, \(K_s\), and the aerodynamic stiffness matrix, \(K_a\):

\[
K = \begin{bmatrix}
K_{XX} & K_{XY} \\
K_{YX} & K_{YY}
\end{bmatrix} = K_s + K_a \quad \text{where} \quad K_s = \begin{bmatrix}
M\omega_{X,i}^2 & 0 \\
0 & M\omega_{Y,i}^2
\end{bmatrix}
\]

\(K_s\) can potentially change due to changes in the tension of the cable, for example due to thermal effects or traffic loads. \(C\) is the total generalised damping matrix, given by the sum of the structural, \(C_s\), and aerodynamic part, \(C_a\):

\[
C = \begin{bmatrix}
C_{XX} & C_{XY} \\
C_{YX} & C_{YY}
\end{bmatrix} = C_s + C_a
\]

(2)

\(\omega_{X,i}\) and \(\omega_{Y,i}\) are respectively the circular natural frequencies of the \(i\)th out-of-plane and in-plane modes of the cable (in the absence of wind), and \(\zeta_{X,i}\) and \(\zeta_{Y,i}\) are the corresponding structural damping ratios.

---

**Figure 3.** Typical PSDs of in-plane (top) and out-of-plane (bottom) accelerations for Cable 8M. Vibrations in the first five pairs of modes were isolated by filtering.

From output-only measurements alone it is not possible to identify the generalised mass \(M\) and the matrices \(C\) and \(K\) themselves, but it is possible to identify \(C/M\) and \(K/M\) in Eq. (1). Also it is not possible to explicitly
separate the structural and aerodynamic components from a single record, but the variation of $C/M$ and $K/M$ with wind conditions can be identified from multiple records in different conditions.

For each 10-minute record, the auto- and cross-covariance functions of the out-of-plane and in-plane filtered accelerations were computed. The functions from each record were then analyzed with an Eigenvalue Realization Algorithm (ERA) system identification procedure, based on state-space realization from the Markov-Block-Hankel matrix, to identify the stiffness matrix $K/M$ and damping matrix $C/M$ (Jakobsen & Hjorth-Hansen 1995). This method has been found to be effective for full-scale ambient vibration measurements, having recently been applied to full-scale data from the cables of Øresund Bridge for wind orthogonal to the deck (Acampora et al. 2013) and for Clifton Suspension Bridge, identifying clear trends in the flutter derivatives with wind velocity, including aeroelastic coupling between vertical and torsional vibrations of the bridge deck (Nikitas et al. 2011). The method assumes the external loading to be white noise in the proximity of the frequencies of interest and that the system is linear, including that the aeroelastic forces can be modelled as linear functions of cable displacement and velocity, as in Eq. (1). This hypothesis is valid due to the nature of the forces acting on the cable; in particular the wind turbulence reduces along the cable, loading the twin cables with a non-coherent signal (white noise signal). An important parameter in the analysis is the choice of the maximum lag length of the covariance functions used. If too short, useful information is discarded, but if too long, random fluctuations are included and can increase the variance of the results. In the absence of any theoretical basis for the choice of the maximum lag length, it was aimed to minimise the variance of the outputs from statistical analysis of the results from a representative wind speed bin (6 m/s, i.e. $Re = 1.0\times10^5$) for the modes of interest. For a whole series of different maximum lag lengths the system identification procedure was carried out on all records in the bin. For each maximum lag length the coefficient of variation (COV, i.e. standard deviation / mean) of each of the 8 extracted parameters ($K_{XX}/M$, $K_{XY}/M$, etc., $C_{XX}/M$, $C_{XY}/M$, etc.) was computed. Covariance length was chosen as 45 seconds. More details about the procedure used to calibrate the ERA procedure can be found in Acampora et al. (2013). The system identification was performed on each selected record for the first three pairs of modes, after separately filtering the responses in each pair of modes. The average results for each wind speed bin for each mode are presented and discussed in Section 4.

4. Identified stiffness and damping matrices

The results of the system identification from the full-scale data give the modal properties of each pair of cable modes in terms of the stiffness and damping matrices for the coupled 2-DOF system. The first three pairs of modes were considered. Results are presented as the average values within each wind speed bin, expressed in terms of Reynolds number for three conditions of precipitations (no rain, light rain, moderate rain).

*Identified stiffness matrices and natural frequencies:* The identified total stiffness matrices for each of the first three mode pairs are shown in Figure 4. The diagonal terms show distinct values for the three modes proportional to the square of modal frequencies, with little variation with Reynolds number and no significant difference with precipitation conditions. The small variation can be attributed to marginal differences in the frequency of the cables for different records. Temperature was also considered as a possible cause of variations in the stiffness matrix, potentially due to changes in the cable tension. However, no significant differences in the stiffness matrix were found with temperature, for the measured air temperatures in the range 4 to 17°C. Varying traffic mass could potentially account for some of the variations in the stiffness matrix values, through changes in cable tension, but no direct measurement of the traffic on the bridge was available. In all cases the off-diagonal terms of the total stiffness (or mass) matrices are negligible, compared with the diagonal terms, with non-zero terms, indicating stiffness coupling between out-of-plane and in-plane vibrations.

*Identified damping matrix:* The identified elements of the first three mode pairs of total damping matrix are shown in Figure 5, 6 and 7. In Figure 8, mean values of aerodynamic damping calculated on the three modes for the three categories of precipitation are reported. For the three conditions of precipitation, there are clear
trends of the identified results with Reynolds number, indicating aerodynamic damping. The identified values are for the total damping matrix, including the structural damping. The identified aerodynamic damping results are consistent for the three modes and they are dependent with Reynolds number. The diagonal-terms of the damping matrix, $C_{XX}/M$ and $C_{YY}/M$, are linearly proportional to Reynolds number up to the Reynolds range of $1.4-1.5 \times 10^5$ for the different conditions, after there is a drop of aerodynamic damping that can be considered as the end of the sub-critical Reynolds number region. The drop is higher in presence of rainfall compared to the case of its absence. The drop in damping is in agreement with Simiu & Scanlan (1996) who state that for circular cylinders with a smooth surface if the wind is not turbulent, the sub-critical Reynolds number region corresponds to a range of 0 to about $2 \times 10^5$, but if the wind is turbulent or if the cable surface is rough, the upper bound of the subcritical region will be lower. Similarly, a critical Reynolds number of $1.6 \times 10^5$ is estimated for a single smooth circular cylinder (i.e. without fillets) with two-dimensional flow, based on the empirical equations provided by ESDU (1986), using the mean values of the longitudinal turbulence intensity and length scale estimated from the site measurements for wind speeds above 7.5m/s (Section 2). The cross-terms of the damping matrix, $C_{XY}/M$ and $C_{YX}/M$, are smaller compared to the diagonal terms but not zero, showing an aerodynamic damping coupling between vibrations in the two planes, in agreement with quasi-steady theory for the chosen wind direction (Eq. 2.3 in section 7) (Macdonald & Larose 2008a).

Figure 4. Identified total stiffness matrix for Cable 8M vs. Reynolds number for the first three pairs of modes for three precipitation conditions: no rain, light rain and moderate rain
Figure 5. Damping matrix for cable 8M vs. Reynolds number for the first 3 pairs of modes from the system identification from full-scale data for no rain conditions.

Figure 6. Damping matrix for cable 8M vs. Reynolds number for the first 3 pairs of modes from the system identification from full-scale data for light rain.
5. Wind tunnel measurement of static/dynamic force coefficients

Wind tunnel tests were conducted in dry (static tests) and rainy (dynamic) conditions to obtain drag and lift coefficients (see Figure 10) to compare with back-calculated aerodynamic coefficients evaluated in section 7. The tests were performed at DTU/Force Technology 2m x 2m cross-section closed circuit Climatic Wind.
Tunnel in Kgs. Lyngby, Denmark. The wind tunnel has a maximum wind speed of 31m/s and a smooth flow turbulence intensity of approximately 0.64%. Other technical specifications of the wind tunnel are reported by Georgakis et al. (2009). A sectional cable model was manufactured scaling the dimensions of the twin cables of the Øresund Bridge (full-scale diameter 250mm) by a factor of 2.2. This scale was chosen as a compromise considering the Reynolds number range of interest and the blockage ratio. The actual roughness of the bridge cables on site is in the range 0.7-1μm (Matteoni & Georgakis 2012). The double helical fillet was reproduced at scale using a steel wire attached with double-sided tape (full-scale fillet dimensions are 2.1mm high x 3.0mm wide, helix angle 55°, pitch 550mm). The test programme consisted of wind velocity sweeps between 1 and 31m/s, corresponding to Reynolds numbers of 3x10^4 – 2.3x10^5, based on a reference dimension of 1D. Values of drag and lift coefficients are reported in Figure 11 for the Reynolds number of interest up to 1.8x10^5. Measurements were averaged over 1 minute at each wind speed. Drag and lift coefficients are defined here on the basis of the total force on the twin cable setup and the reference dimension of 1D corrected for blockage using the Maskell III method (Cooper et al. 1999).

**Inclined static test in dry conditions:** The model was 2415mm long. The PVC surface was polished to match as far as possible the target surface roughness of the Øresund cables (made of HDPE). Measured values of the average surface roughness \( R_a \) were in the range of 0.5-1μm. The aerodynamic forces were measured with two 6-DOF force transducers (AMTI MC3A-500), mounted at the ends of the twin cable. The force transducers were installed on rigid steel plates between the ends of the cable model and the supporting cardan joints fixed to the wall.

**Inclined dynamic test in wet condition:** Wet wind tunnel tests were performed and data were provided by Eriksen and Mattiello (2013). The model here was 3050mm long with a blockage ratio of 11% (Figure 9). Average surface roughness \( R_a \) of the model was approximately 0.3-0.5μm. The rain simulation system was not designed to provide enough water to form both upper and lower rivulets on the cables. The upper rivulets started to form at Reynolds number corresponding at 0.25 x 10^5, and both upper and lower rivulets were fully formed at Reynolds number 0.5 x 10^5; for \( \text{Re} > 1.5 \times 10^5 \), the rivulets were no longer continuous but partly blown off the cable surface. The amount of water present on the cable surface can be qualitatively referred as light rain conditions. Displacements time-histories of the twin cable model were recorded by four laser displacement transducers fixed to the end of rig structure (LAS-T-250 for in-plane and LAS-T-500 for out-of-plane vibrations, produced by Waycon Positionsmesstechnik). Detection of the laser spot was guaranteed through Plexiglas plates with a white reflective plastic covering orthogonal to laser bins fixed at the two ends. Total drag and lift forces were evaluated as a product of the mean displacements vector and the model stiffness.
In Figure 10, drag and lift coefficients for dry and rainy conditions obtained from the wind tunnel tests are reported.

![Figure 10. Drag (left) and lift (right) coefficients vs. Reynolds number from wind tunnel tests on inclined-yawed twin cable model with helical fillets in dry conditions (solid lines) and rainy conditions (dashed).](image)

6. Theoretical aerodynamic damping matrix and back calculation of aerodynamic coefficient

A method to evaluate the generalised 2-DOF aerodynamic damping matrix was proposed by Macdonald & Larose (2008a). The method is based on the quasi-steady assumption that the aerodynamic forces on a moving cable are given by the forces on a stationary cable experiencing the same relative wind velocity, allowing for three-dimensional geometry and changes in the aerodynamic force coefficients with Reynolds number. It is also assumed that variations of the force coefficients are smooth functions and linearising for small cable velocity compared to the wind velocity. For a uniform cable in a uniform wind flow, the theoretical quasi-steady aerodynamic damping matrix in (2) can be expressed as:
\[
\frac{C_a}{M} = \frac{\mu \text{Re}}{2m} \begin{bmatrix} C_{a_{XX,a}} & C_{a_{XY,a}} \\ C_{a_{YX,a}} & C_{a_{YY,a}} \end{bmatrix} = \frac{\mu \text{Re}}{2m} (G_B + C_{F\phi} + C_{F\phi} B_2)
\]

where

\[
G = \begin{bmatrix} g(C_D) - g(C_L) \\ g(C_L) - g(C_D) \end{bmatrix}, \quad C_{F\phi} = \frac{1}{\sin(\phi)} \begin{bmatrix} C_D & -C_L \\ C_L & C_D \end{bmatrix}, \quad C_F = \frac{\partial C_F}{\partial \alpha}
\]

\[
B_1 = \begin{bmatrix} \cos^2(\alpha) & \cos(\alpha)\sin(\alpha) \\ \cos(\alpha)\sin(\alpha) & \sin^2(\alpha) \end{bmatrix}, \quad B_2 = \begin{bmatrix} -\cos(\alpha)\sin(\alpha) & \cos^2(\alpha) \\ \sin^2(\alpha) & \cos(\alpha)\sin(\alpha) \end{bmatrix}, \quad C_F = C_D \text{ or } C_L
\]

\[
g(C_F) = C_F \left( 2\sin(\phi) - \frac{1}{\sin(\phi)} \right) + \frac{\partial C_F}{\partial \text{Re}} \text{Re} \sin(\phi) + \frac{\partial C_F}{\partial \phi} \cos(\phi)
\]

\(\mu\) is the absolute viscosity of air, \(C_D\) and \(C_L\) respectively the drag and lift coefficients, \(\alpha\) the angle-of-attack about the cable axis, \(\phi\) is the cable-wind angle, and the Reynolds number (Re = \(\rho DU/\mu\), where \(\mu\) is the absolute viscosity of air). The relative cable-wind angle \(\phi\) and the angle of attack about the cable axis \(\alpha\) are defined as:

\[
\phi = \cos^{-1}(\cos(\beta)\cos(\theta)) \quad \alpha = \cos^{-1}\left(\frac{\tan(\theta)}{\tan(\phi)}\right)
\]

where \(\beta\) is the yaw about the horizontal yaw angle from the normal to the cable plane (20\(^\circ\)) and \(\theta\) is the inclination of the cable (30\(^\circ\)). Figure 11 shows the relationship between angles.

![Figure 11. Geometry of twin cable.](image_url)

**7. Back-calculation of \(C_D\) from the full-vibration data**

The identified damping matrix values for the Øresund Bridge cable were used to back-calculate the drag \((C_{D,\text{back}})\) and lift \((C_{L,\text{back}})\) coefficients and their derivatives with the angle-of-attack about the cable axis \(\alpha\) \((\frac{\partial C_D}{\partial \alpha} \text{ and } \frac{\partial C_D}{\partial \alpha})\) of actual cable 8M of Øresund Bridge for the three conditions of precipitation. From
equation (3), substituting the values of $\beta = 20^\circ$, $\theta = 30^\circ$ and neglecting the terms that have a negligible influence on the final results, the simplified matrix is obtained:

\[
\frac{C_a}{M} = \frac{\mu \text{Re}}{2m} \begin{bmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{bmatrix} \approx \frac{\mu \text{Re}}{2m} \begin{bmatrix} 1.8C_D + 0.9 \frac{\partial C_D}{\partial \text{Re}} \text{Re} & -1.1C_L + 0.9 \frac{\partial C_D}{\partial \alpha} \\ 1.8C_L + 0.9 \frac{\partial C_L}{\partial \text{Re}} & 1.1C_D + 0.9 \frac{\partial C_L}{\partial \alpha} \end{bmatrix}
\]

(4)

The term $C_{xx}/m$ of the damping matrix can be written as:

\[
\frac{0.9}{\text{Re}} \frac{d}{d \text{Re}} \left( C_{D,\text{back}} \text{Re}^2 \right) \approx \frac{2m}{\mu M} \frac{C_{xx}}{\text{Re}}
\]

\[
C_{D,\text{back}} \text{Re}^2 \approx \frac{2m}{0.9\mu} \int_0^{\text{Re}_{\text{max}}} \frac{C_{xx}}{M} d \text{Re}
\]

Hence, noting that $C_{D,\text{back}} \text{Re}^2 = 0$ for $\text{Re} = 0$, the drag coefficient can be back-calculated from the values of $C_{xx}/m$ identified from the full-scale data, as:

\[
C_{D,\text{back}} \approx \frac{2m}{0.9\mu \text{Re}^2} \int_0^{\text{Re}_{\text{max}}} \frac{C_{xx}}{M} d \text{Re}
\]

(5)

Similarly $C_{L,\text{back}}$ can be evaluated as:

\[
C_{L,\text{back}} \approx \frac{2m}{0.9\mu \text{Re}^2} \int_0^{\text{Re}_{\text{max}}} \frac{C_{xy}}{M} d \text{Re}
\]

(6)

The mean value of the $C_{xx}$ and $C_{yy}$ for each Reynolds number of the three identified modes was used to back-calculate, the drag ($C_{D,\text{back}}$) and the lift coefficient ($C_{L,\text{back}}$) of the actual Øresund cable for the three considered conditions. In Figure 12, the back-calculated drag coefficients (markers) are compared with the drag coefficients from wind tunnel tests (lines). Back-calculated drag coefficients for the three conditions of precipitations are slightly different among them and are lower compared to the drag coefficient from wind tunnel tests for both rainy and dry conditions.

Figure 12. Comparison of back-calculated drag coefficient for the precipitation conditions of no rain, light rain and moderate rain with drag coefficient from wind tunnel test in dry and wet conditions.
Figure 13. Comparison of back-calculated lift coefficient for the precipitation conditions of no rain, light rain and moderate rain with lift coefficient from wind tunnel test in dry and wet conditions.

In Figure 13, the back-calculated lift coefficients (markers) are compared with the lift coefficients from wind tunnel tests (lines). The wind tunnel test for the rainy conditions has values of lift coefficient higher than the lift coefficients from dry wind tunnel tests and from back-calculated values due to slight differences in the scaling of helical fillets geometry in the wind tunnel tests. The amount of precipitation on the twin cable of Øresund Bridge affects magnitude and slope of back-calculated lift coefficients. This influence is enhanced for the case of light rain. Once the terms $C_{D,back}$ and $C_{L,back}$ are known, the terms $C_{XY}$ and $C_{YY}$ can be used to back-calculate the derivative of lift and drag coefficient with angles of attack about the cable axis

$$\begin{align*}
\frac{\partial C_L}{\partial \alpha} \bigg|_{\alpha=70^\circ,\backslash} & \text{ and } \frac{\partial C_D}{\partial \alpha} \bigg|_{\alpha=70^\circ,\backslash} \\
(2.5)
\end{align*}$$

Figure 14 shows that the derivative of drag about cable axis $\frac{\partial C_D}{\partial \alpha} \bigg|_{\alpha=70^\circ,\backslash}$ for light rain presents a different behaviour compared the moderate and no rain conditions. Similar results are shown in Figure 15 for the derivative of lift about cable axis $\frac{\partial C_L}{\partial \alpha} \bigg|_{\alpha=70^\circ,\backslash}$.
The derivatives of the drag coefficient about cable axis are consistent for the case of no rain and moderate precipitation, while the light rain condition presents a different behaviour. Similar results are obtained for the derivative of lift about cable axis.
8. Conclusions

The total stiffness and damping matrices have been identified for the first three pairs of cable modes from full-scale vibration monitoring for a selected wind direction skewed respect to the twin cables for three conditions of precipitation.

The identified stiffness matrices present values proportional to the square of frequencies of each mode with no influence of precipitation and small variation with Reynolds number. The diagonal terms show distinct values for the three modes proportional to modal frequencies, with little variation with Reynolds number and no significant difference with the condition of precipitation. Small variation for all terms of stiffness matrices can be addressed to marginal differences in cable tension. The off-diagonal terms of the total stiffness matrix are negligible compared with the diagonal terms but not zero, indicating a certain degree of stiffness coupling between out-of-plane and in-plane vibrations.

The identified total damping show clear trends of the identified results with Reynolds number, indicating aerodynamic damping. The identified aerodynamic damping matrices are consistent for the three modes with a dependency on Reynolds number and not of reduced wind velocity. The diagonal-terms of the damping matrix, present a sub-critical damping matrix $C_{xx}/M$ and $C_{yy}/M$ up to Reynolds number range of $1.4\times10^5$. A dependency of aerodynamic damping with precipitation is observed. This is a noticeable drop in the damping with light rain.

Back-calculated drag and lift coefficients of the Øresund twin cable at the given wind direction and their variations with angle-of-attack about the cable axis are evaluated for three cases of precipitations and compared with the drag and lift coefficients from wind tunnel tests. The back-calculated aerodynamic coefficients for the light rain conditions present a different pattern than the other considered precipitation cases and they all present lower values compared to the wind tunnel tests. The presence of rain on the twin cable produces changes of the back-calculated aerodynamic coefficients for all cases and in particular for the light rain conditions.

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Wind tunnel tests on a scaled Øresund Bridge cable arrangement

This chapter presents wind tunnel test results for varying wind angles-of-attack for a scaled Øresund Bridge cable with and without the presence of helical fillets.
Wind tunnel tests on a scaled Øresund Bridge cable arrangement
"Aerodynamic coefficients of plain and helically filleted twin circular cylinders for varying wind angles-of-attack"

Antonio Acampora, Christos T. Georgakis

Wind tunnel tests on a scaled Øresund Bridge cable arrangement
Aerodynamic coefficients of plain and helically filleted twin circular cylinders for varying wind angles-of-attack

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Abstract

Moderate vibrations continue to be recorded on the Óresund Bridge twin-stay cables. System identification techniques have been applied to investigate the aerodynamic characteristics of the cables based on ambient vibration measurements. As might be expected, the measured aerodynamic damping ratios vary from those estimated through use of aerodynamic coefficients of single circular cylinders, as reported in literature. To address this issue, wind tunnel tests were performed on a 1:2.3 scale section model of the Óresund Bridge cables with and without the presence of helical fillets. The twin cables are circular cylinders with an outer diameter D of 250mm and a centre-to-centre distance of 2.68D. In this paper, the results of those tests are presented for varying wind angles-of-attack.

1 Introduction

Large amplitude cable vibrations have previously been reported on the Óresund Bridge for varying meteorological conditions (Svensson \textit{et al.}, 2004). In 2010, the Technical University of Denmark installed a new monitoring system on the bridge and moderate vibrations under rain-wind conditions have since been reported (Acampora & Georgakis 2011). In 2011, Acampora \textit{et al.} investigated the aerodynamic damping of the bridge’s twin stay cables using system identification techniques applied to the acquired cable vibration data (Acampora \textit{et al.}, 2011). The results showed some deviations of aerodynamic damping of the actual twin stay cables compared to aerodynamic damping based on quasi-steady theory and determined using values from the aerodynamic coefficients of single circular cylinders obtained from previous wind tunnel tests. In an attempt to understand this discrepancy, a new set of wind tunnel tests were performed on a 1:2.3 scale section model of the Óresund Bridge twin stay cable arrangement. The drag, lift and moment coefficients about the longitudinal axis of the twin cable, with and without double helical fillets, are presented herewith for varying wind angles of attacks. The aerodynamic damping of the cable pair is finally re-evaluated based on the resulting coefficients.

2 Model and wind tunnel tests

The Óresund Bridge twin stay cables comprise parallel monostrands covered by a black 250 mm diameter HDPE tube with double helical fillets (2.1 x 3 mm, helix angle 55 °, pitch 550 mm). A 1:2.3 scale twin cylinder section model of the cables was manufactured from PVC piping for the scaled tests. The grey PVC surface was sanded to approximately match the scaled target surface roughness of the full-scale cables. The measured value of the average surface roughness, Ra, of the tubes was in the range of 0.5-1 μm. The distance, d, between stays, from centre to centre, is 670 mm (Larose & Smitt, 1999). Two versions of the scaled section model were tested; one with plain cylinders and the second with double helical fillets attached. The total length, L, of the cylinder pairs was 1615 mm. Static wind tunnel tests were performed at the 2×2 m\textsuperscript{2} cross-section closed-circuit DTU/Force Technology
Climatic Wind Tunnel in Kgs. Lyngby, Denmark. The wind tunnel has a maximum wind speed of 31 m/s and a maximum measured turbulence intensity of approximately 0.62% (Kleissl & Georgakis 2011). The other technical specifications of the wind tunnel are reported by Georgakis et al. (2009). 6 degree-of-freedom (DOF) force transducers (AMTI MC3A-500), placed on either end of the hinged arrangement, measured forces in all directions. The test programme consisted of 3 repetitions of wind velocity sweeps between 4 and 31 m/s, corresponding to Reynolds numbers of $3 \times 10^4 - 2.3 \times 10^5$, for varying wind angles-of-attack, $\alpha = 0^\circ, 30^\circ, 60^\circ, 70^\circ, 80^\circ, 90^\circ, 100^\circ, 110^\circ, 150^\circ$ and $180^\circ$ (Figure 2). Mean values of the aerodynamic coefficient over the repetitions are reported.

![Figure 1: Wind tunnel test setup of twin cylinders without (left) and with (right) double helical fillets for wind angle-of-attack $\alpha = 0^\circ$.](image1.png)

![Figure 2: Setup of the twin cables for the different angles-of-attack.](image2.png)

3 Results and discussion

Reynolds number for the section of the twin cylinders is defined as $Re = \rho V D / \mu$, where $\rho$ is the air density, $V$ is the wind velocity, $D$ is the single diameter of the twin cylinder configurations and $\mu$ is the air viscosity. Drag, lift and moment coefficients, about longitudinal axis of the twin cable, were measured during the tests. The aerodynamic force coefficients $C_F$ are defined as the total measured force $F$ over the frontal surface projection ($2DL$) at $\alpha = 0^\circ$ for all varying angles-of-attack $\alpha$, so that:

$$C_F = \frac{F}{2DLV^2}$$
In Figure 3, the drag, lift and moment coefficients for the twin cable with double helical fillet are plotted versus wind angle-of-attack for $\text{Re} = 0.5, 1.0, 1.5, 2.0 \times 10^5$. With respect to symmetry, the drag coefficient at $\alpha = 70^\circ$ differs from that at $\alpha = 110^\circ$. This is most likely due to the variation in interaction of the cylinders, due to the orientation of the fillets. Lift and moment coefficients about $\alpha = 90^\circ$ are generally antisymmetric in sign, but not in magnitude.
In Figure 4, the drag, lift and moment coefficients for the twin cable with the double helical fillet are plotted versus Reynolds number for $\alpha = 0^\circ, 30^\circ, 60^\circ, 70^\circ, 80^\circ, 90^\circ, 100^\circ, 110^\circ, 120^\circ, 150^\circ$ and $180^\circ$. A gradual reduction in drag coefficient with Reynolds number can be observed, due to the presence of the double helical fillets. The beginning of this drop can be identified between $Re = 1\times10^5$. As might be expected, the drag coefficients reduce with wind angle-of-attack, due to the change of the cross-section seen by the oncoming wind.
In Figure 5, the drag, lift and moment coefficients for the twin cable without double helical fillet are plotted versus wind angle-of-attack for $Re = 0.5, 1.0, 1.5, 2.0 \times 10^5$. Drag coefficients about $\alpha = 90^\circ$ are generally symmetric in sign and magnitude. Lift and moment coefficients about $\alpha = 90^\circ$ are generally antisymmetric.
Figure 6 Drag (top), lift (center) and moment (bottom) coefficients vs Reynolds number for different wind angles-of-attack for the twin cylinders without double helical fillets.

In Figure 6, the drag, lift and moment coefficients for the twin cylinder without the double helical fillet are plotted versus Reynolds number for $\alpha = 0^\circ, 30^\circ, 60^\circ, 70^\circ, 80^\circ, 90^\circ, 100^\circ, 110^\circ, 120^\circ, 130^\circ, 140^\circ, 150^\circ, 160^\circ, 170^\circ, 180^\circ$. The beginning of the drop in drag coefficients can be identified between $Re = 1.6 \times 10^5$. Again, as might be expected, the drag coefficients reduce with wind angle-of-attack, due to the change of the cross-section seen by the oncoming wind. Comparing aerodynamic coefficients from the two setups, it can be observed that the presence of helical fillets reduces the magnitude of aerodynamic coefficients with
respect to the results obtained without helical fillets. The double helical fillets produce a transition to the critical Reynolds number region that is not clearly localized (Figure 4) but prolonged over the entire investigated Reynolds number range. Similar results were obtained by Kleissl and Georgakis (2011), when comparing the results from a plain HDPE cylinder and a HDPE cylinder fitted with helical fillets.

Lift and moment coefficients are reduced in magnitude when helical fillets are fitted on the twin cylinders. From the results it can be observed that lift coefficients are less angle-of-attack dependent in presence of helical fillets compared to the results of twin cylinders without helical fillets. In Figure 3 the lift coefficient for $\alpha = 70^\circ$ (and $110^\circ$) is about 8 times higher than $\alpha = 0^\circ$ while, in absence of helical fillets, the lift coefficients for $\alpha = 70^\circ$ (and $110^\circ$) is about 20 times higher than $\alpha = 0^\circ$ (Figure 5). Moment coefficients exhibit similar patterns with wind angle-of-attack.

4 Aerodynamic damping

Theoretical values of the aerodynamic damping based on quasi-steady theory using aerodynamic coefficients of the twin cylinders for wind angle-of-attack $\alpha = 0^\circ$ are shown in Figure 7. The results are compared with the aerodynamic damping identified from full-scale data from Øresund Bridge for the first five cable vibrations modes (markers) and the theoretical aerodynamic damping based on data from wind tunnel tests on a single cable with single helical fillet, similar to the cables on the Øresund Bridge (Kleissl & Georgakis 2011). Furthermore, the aerodynamic damping based on tests on a single cable with a single helical fillet for a previous study for the bridge carried out by Freyssinet (Øresundsbron 2003) is also presented. Quasi-steady theory assumes that the aerodynamic forces on a moving cable are given by the forces on a stationary cable experiencing the same relative wind velocity; the generalised 2DOF aerodynamic damping matrix is given by Macdonald & Larose (2008a). For a uniform cable in a uniform wind normal to the cable axis, the theoretical quasi-steady in-plane and out-of-plane aerodynamic damping can be expressed as:

$$\frac{C_{a,IP}}{M} = \frac{\rho DU}{2m} \left( 2C_D + \frac{\partial C_D}{\partial Re} Re \right)$$

$$\frac{C_{a,OP}}{M} = \frac{\rho DU}{2m} \left( C_D + \frac{\partial C_L}{\partial \alpha} \right)_{\alpha=0}$$

where $C_{a,IP}$ is the in-plane aerodynamic damping, $C_{a,OP}$ is the out-of-plane aerodynamic damping, $\rho$ is the density of air, $D$ is the reference dimension (cable diameter), $U$ the wind speed, $m$ the mass per unit length, $C_D$ and $C_L$ respectively the drag and lift coefficients, $\alpha$ the angle-of-attack about the cable axis and Re the Reynolds number.

Figure 7. Comparison of theoretically evaluated in-plane (left) and out-of-plane aerodynamic damping (right) for twin cylinders test, for single cylinder tests with identified damping from full-scale data of Øresund Bridge, for angle-of-attack $\alpha = 0^\circ$. 
The aerodynamic damping evaluated with aerodynamic coefficients from twin cylinders test shows a better match with the aerodynamic damping identified from full-scale cable of Øresund Bridge. The differences between the theoretically evaluated aerodynamic damping based on the wind tunnel tests and the aerodynamic damping identified from Øresund Bridge could be explained by possible differences in surface roughness of the cable on the bridge and the tested cylinders.

5 Conclusions

Static wind tunnel tests on a vertical twin circular cylinder configuration – with and without helical fillets - have been carried out. From these, drag, lift and moment coefficients, for Reynolds numbers ranging between \(3 \times 10^4\) – \(2.3 \times 10^5\) and for varying wind angles-of-attack, have been determined and are presented herewith. The results show the importance of the double helical fillets in the change of aerodynamic coefficients, both when comparing the two different configurations and when examining the variation of drag, lift and moment coefficients for varying wind angles-of-attack. The presence of the double helical fillets reduces the magnitude of all aerodynamic coefficients. The aerodynamic damping based on helically filleted twin circular tests using quasi-steady theory gives a reasonable description of the actual behaviour of the cable of Øresund Bridge.

6 Acknowledgments

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References

Part IV

Conclusions
Observations, conclusion and future work

In the present thesis, stay cables vibrations, collected on the Øresund Bridge have been investigated. The data were extensively processed for dry and rainy conditions to identify the role that different factors play on the aerodynamic damping of cables, being the main contribution to the total damping. Drag coefficients were back-calculated from the identified aerodynamic damping from full-scale data. In parallel, wind tunnel tests were performed to investigate aerodynamic coefficients of the twin-cables system and compare them with back-calculated coefficients from cables-stays of Øresund Bridge. In this chapter the afore mentioned research is summarized from the presented journal/conferences papers and used to answer the question that led to this research. Moreover during the present work, new ideas on possible future work have arisen and they are presented at the end of this section.

5.10 Experimental observations

Initially, information available from previous monitoring systems or reports about RWIV have been summarized in the literature review. Together with the literature review, conditions associated with cable vibrations during rainfalls were observed from full-scale monitoring of Øresund bridge in terms of range of rainfall rate, wind speed and angle of attack.

5.10.1 Full-scale data

The most likely observed conditions for RWIV from Øresund Bridge are here recap:

- Wind speed in the range $4 - 18 m/s$, with a majority of cases in the interval $7 - 14 m/s$;
- All cables are inclined to the horizontal at an angles of $30^\circ$;
5.10 Experimental observations Observations, conclusion and future work

- Vibrations are associated for cable declining with wind directions in the range of relative yaw angles between $35^\circ$ and $+55^\circ$ and between $-25^\circ$ and $-50^\circ$ (cable inclining in the wind direction);

- Precipitation for light (less or equal to $2.5\text{mm/hr}$), moderate (in the range from $2.5\text{mm/hr}$ to $7.5\text{mm/hr}$) and heavy (more than $7.5\text{mm/hr}$) rainfall;

- All pair cable outer diameters are $250\text{mm}$;

while from literature review of RWIV it was found the following conditions:

- Wind speed in the range $5 - 20\text{m/s}$, with a majority of reported cases lying in the interval $8 - 12\text{m/s}$;

- Cables are inclined to the horizontal at angles of $20^\circ$ to $45^\circ$;

- Wind directions for cable declining with wind direction in the range $20^\circ - 60^\circ$ to the longitudinal axis;

- Rainfall rates of light, drizzle, moderate and heavy are reported;

- The cable diameter is in the range $80 - 200\text{mm}$.

Comparing the results obtained from Øresund Bridge monitoring system with those reported in literature it is apparent that there is a substantial agreement of the collected data on the Øresund Bridge with the ones reported in the past, in particular for what concern wind speed, wind direction (for declining cables) and rainfall intensities. It can be observed that the majority of vibrations that potentially can lead to large displacements belongs to a restricted wind speed and wind direction range. The upper rainfall rate can be fixed about $8\text{mm/hr}$.

5.10.2 System identification

Data within the conditions of RWIVs were analyzed to obtain information about stiffness and aerodynamic damping matrices in dry and rainy conditions. In this section, the results of these two cases for cable 8M, the longest one that does not have a tuned mass damper, are recap.
Dry condition

Initially, ambient vibrations collected in dry conditions and for wind orthogonal to the cables were analyzed to calibrate and validate the system identification method for a simple case. The results show that the total stiffness and damping matrices for the first five pairs of cable modes are not dependent on temperatures and only minor dependency with cable tension is observed, possibly due to variable traffic loading.

Stiffness matrices have nearly constant values with wind speed. Stiffness matrices are diagonal, implying that for each of the considered modes, the in-plane and out-of-plane modes are uncoupled as expected for the chosen wind direction. The diagonal terms of stiffness matrices are proportional to the square of each of the considered modes.

The damping matrices are diagonal too. They are a function of Reynolds number rather than reduced velocity. The damping values identified from the full-scale data agree well with the theoretical aerodynamic damping matrix based on quasi-steady theory; up to about $6 \text{m/s}(Re = 1.0 \times 10^5)$, which is believed to be the end of the sub-critical Reynolds number region for the actual cable for considered wind direction. For higher Reynolds numbers the quantitative results differ, but qualitatively they exhibit similar features. The minimum damping in each plane occurs at about $Re = 2.4 \times 10^5(14 \text{m/s})$ from the site measurements. The total damping remains positive. This huge reduction of aerodynamic damping, that is the main contribution to the total damping for the cable stability is believed to be related to large observed vibrations of some cables on some bridges in strong winds in dry condition.

Subsequently, system identification was applied to data within the conditions of RWIVs. For this wind condition, it can be observed that the drop in the aerodynamic damping is observed for higher Reynolds number at the end of the investigated range, that begins about $Re = 1.6 - 1.8 \times 10^5$).

Rainy condition

System identification was applied to data within the conditions of RWIVs, focusing on the influence of different levels of precipitations on the aerodynamic damping. It was decided to investigate the first three pairs of cable modes from full-scale vibrations for a selected wind direction skewed respect to the pair cables. Three conditions of precipitation are investigated such as: dry, light-rain and moderate-rain. The dry condition was chosen to compare
the effects that the presence or absence of rain on the cables might have on the identified matrices.

The identified stiffness matrices present values proportional to the square of frequencies of each mode with no influence of precipitation and small variation with Reynolds number. The diagonal terms show distinct values for the three modes, with little variation with Reynolds number and no significant difference with or without rainfall. Small fluctuations of all terms of stiffness matrices can be addressed to marginal differences in cable tension. The off-diagonal terms of the total stiffness matrix are negligible compared with the diagonal terms but not zero, indicating a certain degree of stiffness coupling between out-of-plane and in-plane vibrations detected by the system identification procedure.

The identified total damping shows clear trends with Reynolds number, indicating aerodynamic damping. Aerodynamic damping matrices are consistent for the three modes with a dependency on Reynolds number and not of reduced wind velocity. The diagonal-terms of the damping matrix, present a sub-critical damping matrix up to Reynolds number range of $1.4 - 1.5 \times 10^5$. A dependency of aerodynamic damping with precipitation is observed. In particular aerodynamic damping presents a lower values for moderate rainfall intensity. Off-diagonal terms are negligible, but not zero.

### 5.10.3 Drag coefficients

**Back-calculated drag coefficients**

Mean identified values of damping matrices have been used to back-calculate the drag coefficients of the Øresund Bridge twin cable, based on quasi-steady theory. The back-calculated values are evaluated for two cases of rainfall intensity at the given wind direction and in dry conditions for two wind directions orthogonal to the twin cables. The back-calculated aerodynamic coefficients are dependent on wind direction and on rainfall intensity.

**Drag coefficients from wind tunnel tests**

Static wind tunnel tests and dynamic wind tunnel tests were performed to compare aerodynamic coefficients of the Øresund twin-cable configuration with those back-calculated from full-scale ambient vibrations. These tests were performed at the DTU/Force Technology $2m \times 2m$ cross-section closed circuit Climatic Wind Tunnel in Kgs. Lyngby, Denmark, with a maximum
wind speed of 31 m/s.

Sectional cable models were manufactured scaling the dimensions of the twin cables of the Øresund Bridge (full-scale outer diameter 250 mm) by a factor of 2.2. The double helical fillet of the cable was reproduced at scale using a steel wire attached with double-sided tape (full-scale fillet 2.1 mm high x 3.0 mm wide, helix angle 55°, pitch 550 mm). Surface of the cables was polished to match the scaled target surface roughness of the Øresund cables (made of HDPE). The measured values of the average surface roughness Ra of the model were in the range of 0.5 – 1 µm. The actual roughness of the bridge cables on site was measured in the range of 0.7 – 1 µm.

Moreover, results for static tests in the case of wind orthogonal to the twin cylinders for varying wind angles-of-attack show that double helical fillets change aerodynamic coefficients, both when comparing the two different configurations and when examining the variation of drag, lift and moment coefficients for varying wind angles-of-attack. The aerodynamic damping based on helically filleted twin circular tests using quasi-steady theory gives a reasonable description of the actual behavior of the cable of Øresund Bridge.

### 5.11 Conclusions

Based on the reported observations in the previous section, it can be stated that small amplitude cable vibrations were observed on the Øresund Bridge for a specific wind speed range within well defined ranges of wind directions and rainfall intensities. The observed vibrations exhibit small amplitude due to presence of different dampers systems on the cables, installed for events preceding this research.

System identification techniques were used to identify aerodynamic damping of the cables under different conditions. It is observed that the aerodynamic damping is a function of wind direction and rainfall intensity. In particular the role of wind direction on aerodynamic damping is observed in the change of the Reynolds number whereat the reduction of aerodynamic damping is observed and in a reduction of maximum contribution to the total damping. The rainfall intensities plays a role on the maximum level of aerodynamic damping contributing to cable stability. In particular it is observed that there is a limited range corresponding to moderate rainfall intensities where the observed aerodynamic damping presents lower values than for the condition of dry cables. Therefore, It is believed that the combination of this two
5.12 Future work

Factors can lead to a dynamic system that is prone to exhibit large amplitude vibrations in an unfavorable conditions or not adequate level of structural damping.

The observations made for the dry condition orthogonal to the cables show also that inclined cables, if structural damping is not sufficiently provided are prone to vibrate due to the drop in the aerodynamic damping. The Reynolds number where this minimum is reached is a function of the wind direction.

5.12 Future work

During the present research further possible investigations came in mind of the author as basis for future work. In this section, the most significant ones are presented.

System identification back-calculation of aerodynamic coefficient from ambient vibrations

The Matlab routines used during this work, proved to be able to identify the aerodynamic damping of the twin cables from accelerations acquired on bridge cables with enough accuracy. The method revealed to be interesting to obtain back-calculated aerodynamic coefficients from ambient vibrations of real structure without the need to study a model of the structure in wind tunnel tests. This would be an interesting and economical approach to investigate existing structures for early investigations, before to proceed to more expensive tests.

System identification of iced-cable and back-calculation of aerodynamic coefficient

A further application of the system identification method used in the research would be to identify aerodynamic damping from data obtained during ice-driven cables vibrations. In particular, it would be possible to describe the aerodynamic coefficients while ice grows on the external surface of the cable with installation of cameras connected to the monitoring system.
Back-calculation of aerodynamic coefficients for a rose of wind directions under different meteorological conditions

A full description of back-calculated aerodynamic coefficients (drag, lift, and their derivative with angle-of-attack, Reynolds number) would be an interesting topic for further research. This investigation can be extended to virtually all wind directions, choosing an adequate range of over-lapping bins of records. Aerodynamic coefficients can be studied for different level of turbulence and wind velocities.

Development of structural health monitoring system

During the monitoring of Øresund Bridge, the enhancement of the monitoring system had to deal with international laws and strict rules due to the presence of Copenhagen international airport few kilometers away. Future developments that can improve the structural health monitoring system are herewith listed.

- **Video cameras**: The system would benefit from the introduction of cameras at the deck and pylon level. These cameras can be used also to collect information on the distribution of traffic on the deck and to record vibrations of all cable.

- **Wind profile**: A precise description of the wind profile could be achieved using lidar and sodar systems together with anemometers. To avoid a waste of processing capacity and storage space on the hard drive disk, these additional device can be activated only when a trigger events happen (i.e. certain level of accelerations of the cables are reached; wind speed higher than a certain values or coming from certain direction and so on).

- **Axial flow of cables** Axial flow on a specific cable could be measured using an arrangement of Pitot tubes.
5.12 Future work

Observations, conclusion and future work
Appended Conference Papers

This appendix contains conference papers prepared and presented during this work in chronological order for easy access.
Paper V

"Identification of aeroelastic forces on bridge cables from full-scale measurements"

Antonio Acampora, John.H. Macdonald, Christos T. Georgakis

In proceedings: EVACES 2011 Experimental Vibration Analysis for Civil Engineering Structures, Varenna, Italy, October, 2011
Identification of aeroelastic forces on bridge cables from full-scale measurements

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ABSTRACT: Despite much research in recent years, large amplitude vibrations of inclined cables continue to be of concern for cable-stayed bridges. Various excitation mechanisms have been suggested, including rain-wind excitation, dry inclined cable galloping, high reduced velocity vortex shedding and excitation from the deck and/or towers. Although there have been many observations of large cable vibrations on bridges, there are relatively few cases of direct full-scale cable and wind measurements, and most research has been based on wind tunnel tests and theoretical modelling.

This paper presents results from full-scale measurements on the cables of the Øresund Bridge. The system records wind and weather conditions, as well as accelerations of certain cables and a few locations on the deck and tower. Using a state-of-the-art method of output-only system identification, the vibration modes of the cables have been detected with sufficient accuracy to identify changes in the total effective damping and stiffness matrices due to the aeroelastic forces acting on the cables. The damping matrices identified from the full-scale measurements have been compared with the theoretical damping matrices based on quasi-steady theory and two different sets of wind tunnel data.

1 INTRODUCTION

Large amplitude wind-induced vibrations of inclined cables are surprisingly common. Various mechanisms could be responsible, including von Kármán vortex shedding, rain-wind excitation, cable-deck-tower interaction, high reduced velocity vortex shedding and dry inclined cable galloping. The aerodynamic mechanisms acting on inclined cables are complicated by the three-dimensional environment and the fact that typical sized bridge cable stays in moderate to strong winds sit in the critical Reynolds number region, where there is a rapid drop in the drag coefficient (Larose & Zan 2001).

Excessive wind-induced vibrations of inclined cables have been observed with a long-term full-scale measurement system by Zuo & Jones (2010) on the Fred Hartman Bridge. It was observed that the three-dimensional nature of the cable-wind environment inherently affects the mechanisms associated with the vibrations of inclined cables. Different types of cable vibrations were identified, including von Kármán vortex induced vibration, rain-wind induced vibration and large amplitude dry cable vibrations. Matsumoto et al. (2003) observed various vibrations of a 30 m long inclined test cable which were classified in a similar way.
Macdonald (2002) measured the aerodynamic damping of cables on the Second Severn Crossing, which was found to often be dominant over the structural damping, even in quite light winds for low frequency modes. The results from the full-scale measurements agreed well with the theoretical quasi-steady aerodynamic damping derived for single degree-of-freedom (1DOF) vibrations of inclined cables in skew winds in the sub-critical Reynolds number range. Later the theoretical framework was generalised to include variations of the static force coefficients with Reynolds number, fully considering the 3-dimensional geometry and allowing for aeroelastic coupling in 2 degrees-of-freedom (2DOF) for in-plane and out-of-plane vibrations (Macdonald & Larose 2008). Boujard & Grillaud (2007) measured both rain-wind induced vibrations and similar vibrations of dry cables during a 3-year measurement campaign on the Iroise Bridge, and for the dry case they compared the results with predicted instability regions based on the same 2DOF quasi-steady model and wind tunnel data on a static inclined model in the critical Reynolds number range.

On the Øresund Bridge, large amplitude cable vibrations have been reported, both in the presence of ice on the cables, believed to caused by galloping, and in warmer conditions (Svensson et al. 2004). More recently a long-term monitoring system has been installed and vibrations under rain-wind conditions have been reported (Acampora & Georgakis 2011). In this paper, the cable vibration measurements have been used to estimate the total stiffness and total damping matrices for different pairs of modes for a range of wind velocities up to 19 m/s in the simple condition of wind normal to the cables in dry conditions, with the particular aim of evaluating the actual aerodynamic damping as a function of wind speed.

2 BRIDGE DESCRIPTION AND MONITORING SYSTEM

The current investigation is based on the monitoring of the Øresund Bridge. The cable-stayed bridge provides the main navigational span for an 8 km bridge link from Denmark to Sweden. The bridge has a main span of 490 m and an orientation of WNW to ESE. The deck carries both road and rail traffic and is supported by 4 independent pylons, 204 m tall, and by 80 parallel double-stays, all with an inclination of 30° to the horizontal plane arranged in a harp-shaped configuration (Fig.1). Each double-stay is made up of a pair of cables, arranged vertically with a gap of three diameters and with rigid connections between the two cables in each pair at one or two locations along the length. The cables, supplied by Freyssinet, comprise multiple seven-wire mono-strands within HDPE tubes, all with the same outer diameter of 250 mm and with double helical fillets on the surface. Additional tuned mass dampers are installed on the first and second longest cable pairs.

The current monitoring system was installed in 2009. Since then different configurations of accelerometers have been installed on the four longest double-stays in the main span and side span at the Swedish end of the bridge on the South and North side of the deck respectively (Fig.1). The accelerometers are oriented to record the components of the cable vibrations normal to the cable axis in the in-plane (i.e. in the vertical plane) and out-of-plane (i.e. lateral) directions.

Figure 1. Instruments and monitored cables: • accelerometer, ◆ anemometer and ▼ rain-gauge.
Ultrasonic anemometers are positioned at the top of the east pylon (on a 4 m pole) and on the deck (on the south side on poles 7 m above the deck) at mid-span and between cables pairs 1M and 2M, west of the pylon (Fig. 1). A rain gauge is positioned close to the anemometer on the top of the pylon. Further data collected include atmospheric humidity, temperature, and pressure. The monitoring system samples all channels at a frequency 30 Hz for all channels and saves files of 10 minutes length.

The double-stay studied in this paper is Cable 8M, the third longest stay in the main span (Fig. 1), with a length of 216m and a mass per unit length of 99kg/m. The data for this paper were collected continuously between October 2010 and April 2011.

3 SYSTEM IDENTIFICATION

The aim here is to identify the damping and stiffness matrices of the cable vibrations as a function of wind velocity and to compare them with the theoretical values according to quasi-steady theory using wind tunnel data for the static aerodynamic force coefficients. The cables exhibit vibrations in multiple modes, which are almost a pure harmonic series in accordance with taut string theory and the accelerations measured on the cables also contain components of global vibration modes of the whole bridge. The focus here is on the local cable vibrations and for simplicity one mode in each plane is considered at a time. Therefore the raw accelerations were filtered with 15th order high-pass and low-pass filters to isolate the vibrations of a single mode in each plane, taking care not to distort the signals for the modes in question themselves. In general the aeroelastic forces can couple the modes with virtually the same natural frequencies in the two planes, so the filtered signals are considered as the response of a 2 translation degree-of-freedom system (2DOF). It is assumed that there is no coupling with the other cable modes at different frequencies (Macdonald & Larose 2008). The equations of motion of the 2DOF system (assumed linear) can be written in the form:

\[
\begin{bmatrix}
\dot{X} \\
\dot{Y}
\end{bmatrix} + \frac{C}{M} \begin{bmatrix}
X \\
Y
\end{bmatrix} + \frac{K}{M} \begin{bmatrix}
X \\
Y
\end{bmatrix} = \frac{1}{M} \begin{bmatrix}
F_X \\
F_Y
\end{bmatrix}
\]

where \( M \) is the generalised mass (assumed to be the same for vibrations in each plane), \( X \) and \( Y \) are respectively the out-of-plane and in-plane generalised displacements, dots represent derivatives with respect to time, and \( F_X \) and \( F_Y \) are the external generalised forces in the two planes due to wind and cable end motion. \( \mathbf{K} \) is the total stiffness matrix, given by the sum of the structural stiffness matrix, \( \mathbf{K}_s \), and the aerodynamic stiffness matrix, \( \mathbf{K}_a \):

\[
\mathbf{K} = \begin{bmatrix}
K_{XX} & K_{XY} \\
K_{YX} & K_{YY}
\end{bmatrix} = \mathbf{K}_s + \mathbf{K}_a,
\]

where \( \mathbf{K}_s = \begin{bmatrix}
M \omega_{X,i}^2 & 0 \\
0 & M \omega_{Y,i}^2
\end{bmatrix} \).

\( \mathbf{C} \) is the total damping matrix, given by the sum of the structural, \( \mathbf{C}_s \), and aerodynamic part, \( \mathbf{C}_a \):

\[
\mathbf{C} = \begin{bmatrix}
C_{XX} & C_{XY} \\
C_{YX} & C_{YY}
\end{bmatrix} = \mathbf{C}_s + \mathbf{C}_a,
\]

where \( \mathbf{C}_s = \begin{bmatrix}
2M \omega_{X,i} \zeta_{X,i} & 0 \\
0 & 2M \omega_{Y,i} \zeta_{Y,i}
\end{bmatrix} \).

\( \omega_{X,i} \) and \( \omega_{Y,i} \) are respectively the circular natural frequencies of the \( i \)th out-of-plane and in-plane modes of the cable (in the absence of wind), and \( \zeta_{X,i} \) and \( \zeta_{Y,i} \) are the corresponding structural damping ratios.
From output-only measurements alone it is not possible to identify $M$ and the matrices $C$ and $K$ themselves, but it is possible to identify $C/M$ and $K/M$. Also it is not possible to explicitly separate the structural and aerodynamic components from a single record, but the variation of $C/M$ and $K/M$ with wind conditions can be identified from multiple records in different conditions.

For each record, the auto- and cross-covariance functions of the out-of-plane and in-plane filtered accelerations were computed. Examples are plotted against time lag in Fig 2. These were then analyzed with a system identification procedure based on state-space realisation from the Markov-Block-Hankel matrix to identify the stiffness matrix $K/M$ and damping matrix $C/M$ (Jakobsen & Hjorth-Hansen 1995). This method was recently applied to full-scale ambient vibration data from the Clifton Suspension Bridge, identifying clear trends in the flutter derivatives with wind velocity, including aeroelastic coupling between vertical and torsional vibrations (Nikitas et al. 2011). The method assumes the external loading to be white noise in the proximity of the frequencies of interest and that the system is linear (including that the aeroelastic forces can be modelled as linear functions of cable displacement and velocity).

The present study focussed on the aerodynamic behaviour of dry cables in winds normal to the cable axis (i.e. normal to the bridge axis), using Cable 8M as an example, being the longest cable monitored without a tuned mass damper. All records in dry conditions and with the mean wind direction within ±5° of normal to the bridge deck were selected for analysis. This gave 378 records of 10-minutes length, distributed across wind speeds from 2 to 19 m/s (using the mean of the measured wind speeds at the top of the pylon and on the deck at mid-span). An important parameter in the analysis is the choice of the maximum lag length of the covariance functions used. This was selected as 54 s on the basis of the minimum value of the standard deviation of the results from multiple records at comparable wind speeds. From Fig. 2 it is seen visually that this is when the auto-covariance functions have typically decayed to a magnitude of about 0.1 to 0.2. The analysis was performed on each selected record for the first three pairs of modes, after separately filtering the responses in each pair of modes.

Figure 2. Auto- and cross-covariance functions for the combined 2DOF system plotted against time lag.
4 THEORETICAL AERODYNAMIC DAMPING MATRIX

Based on the quasi-steady assumption that the aerodynamic forces on a moving cable are given by the forces on a stationary cable experiencing the same relative wind velocity, the generalised 2DOF aerodynamic damping matrix, allowing for three-dimensional geometry and changes in the aerodynamic force coefficients with Reynolds number, was given by Macdonald & Larose (2008) (assuming variations of the force coefficients are smooth functions and the cable velocity is small compared to the wind velocity). Hence, for a uniform cable in a uniform wind normal to the cable axis, the theoretical quasi-steady aerodynamic matrix, $C_a/M$, is given by:

$$
\begin{bmatrix}
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \\
\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \\
\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}
\end{bmatrix}
\begin{bmatrix}
2C_D + \frac{\partial C_D}{\partial Re} \frac{Re}{\partial Re} - C_L + \frac{\partial C_D}{\partial \alpha} \\
2C_L + \frac{\partial C_L}{\partial Re} \frac{Re}{\partial Re} C_D + \frac{\partial C_L}{\partial \alpha} \\
\end{bmatrix}
\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}
$$

where $\rho$ is the air density, $D$ the cable diameter, $U$ the wind speed, $m$ the mass per unit length, $C_D$ and $C_L$ respectively the drag and lift coefficients, $\alpha$ the angle of attack about the cable axis, and $Re$ the Reynolds number ($= \rho DU/\mu$, where $\mu$ is the absolute viscosity of air).

For a single cable with helical fillets in flow normal to its axis, averaged over the cable length, $\partial C_D/\partial \alpha$ and the lift coefficient and its derivatives are all expected to be zero. Hence in this case $C_a/M$ reduces to:

$$
\begin{bmatrix}
2C_D + \frac{\partial C_D}{\partial Re} \frac{Re}{\partial Re} 0 \\
0 2C_L + \frac{\partial C_L}{\partial Re} \frac{Re}{\partial Re} C_D \\
\end{bmatrix}
\begin{bmatrix}
\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}
\end{bmatrix}
\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}
$$

Furthermore, in the sub-critical Reynolds number range, $\partial C_D/\partial Re = 0$, which gives diagonal terms of the matrix equal to the conventional quasi-steady aerodynamic damping expressions for along and across-wind vibrations of a circular cylinder (Virlogeux 1998). Note however that for the actual twin cable arrangement used on the Oresund Bridge the force coefficients may be different and the above assumptions for a single cable with a helical fillet may not hold.

5 RESULTS AND DISCUSSION

Figures 3 and 4 present the results of the system identification as mean values of the terms of the total stiffness and for the total damping matrices divided by the modal mass, $K/M$ and $C/M$ respectively, for wind speeds in the range 2 to 19m/s. Each point represents the mean of the results from all records within a 1m/s range. The results are shown for the first, second and third modes.

The total stiffness matrices (Fig. 3) show well defined values for the three modes in each plane, with little variation with wind speed. This is as expected since according to quasi-steady theory there is no aerodynamic stiffness for translational vibrations, although there can be changes in cable tension, and hence the stiffness matrices, due to the static wind load on the structure. In all cases the off-diagonal terms of the total stiffness matrix are virtually zero, indicating negligible stiffness coupling between out-of-plane and in-plane vibrations, despite them being at virtually the same frequencies. Based on the diagonal terms, the mean natural frequencies of the identified modes are given in Table 1. The natural frequencies in the two planes are very similar and they are close to a harmonic series, as expected for a taut cable.
The damping matrices (Fig. 4) show more interesting features. There are clear trends of the identified results with wind speed, indicating aerodynamic damping. Also shown are the theoretical values based on Eq. (3), using the static force coefficients obtained from two different sets of wind tunnel tests with flow normal to the cable. The solid lines represent the results using data from recent wind tunnel tests on a single cable with single helical fillets, similar to the cables on the Øresund Bridge (Kleissl & Georgakis 2011), while the dotted line uses data from tests on a single cable with a single helical fillet by Freyssinet, from a previous study for the bridge (Øresundsbron 2003). However, note that in neither case was the exact fillet geometry the same as on the actual bridge nor did they model the actual double cable arrangement. The Kleissl & Georgakis tests confirmed the lift coefficient was small (maximum absolute value less than 0.1), while the lift coefficient from the Freyssinet tests is not available and is assumed to be zero.

The identified values are for the total damping matrix whereas the theoretical values are for the aerodynamic damping only. However, the trends of the identified values at low wind speeds indicate that the structural damping (i.e. the extrapolated value for zero wind speed) is negligible in relation to the aerodynamic damping.

The off-diagonal terms of the damping matrix are virtually zero in all cases. This indicates no coupling between vibrations in the two planes, as expected for the chosen wind direction (normal to the cable) (Eq. 3), although for different wind directions coupling is expected (Macdonald & Larose 2008).

The identified results from the three different modes are remarkably consistent. This not only gives more confidence in the results but also strongly indicates that the aerodynamic damping matrix is a function of the Reynolds number (simply proportional to wind speed) and not of the reduced wind velocity, which is the other commonly-used non-dimensional group based on...
the wind speed, but is inversely proportional to the frequency of vibration, which is very different for the three modes.

Figure 4. Total damping matrix for cable 8M vs. wind speed for 1st mode (diamonds), 2nd mode (triangles) and 3rd mode (circles). Lines show theoretical values based on quasi-steady theory and wind tunnel tests on a single cable with single helical fillets by Kleissl & Georgakis (2011) (solid lines) and on single cable with single helical fillet by Freyssinet (Øresundsbron 2003) (dotted lines).

The diagonal terms of the damping matrix show linear trends up to 7 m/s, i.e. $Re = 1.2 \times 10^5$. This implies that this can be considered as the end of the sub-critical Reynolds number region. This is in agreement with Simiu & Scanlan (1996) who state that for circular cylinders with a smooth surface if the wind is not turbulent, the sub-critical Reynolds number region corresponds to a range of 0 to about $2 \times 10^5$, but if the wind is turbulent or if the cable surface is rough, the upper bound of the subcritical region will be lower. Recent investigations indicated that for a smooth-surfaced cylinder, with a wind turbulence intensity of 2.6%, the upper bound of the sub-critical region occurs for a Reynolds number of about $1.6 \times 10^5$ (Zasso et al. 2005).

In the sub-critical range, the identified results agree well with the theoretical results from quasi-steady theory. This is consistent with the measurements on the cables of the Second Severn Crossing in the sub-critical Reynolds number range, where the damping (of in-plane modes only) was measured by free-decay tests rather than ambient vibrations, agreeing well with quasi-steady theory (Macdonald 2002).

For wind speeds above 7 m/s the diagonal terms of the damping matrices show a reduction, due to effects in the critical Reynolds number regime. The theoretical and experimental values differ in this region, although, as stated, the actual geometry of the cables on the bridge is different than in the wind tunnel tests, and using the Freyssinet data (Øresundsbron 2003) there is a slight drop in $C_{xx}/M$ starting at 5m/s, showing similar features to the measured damping, due to a drop in the drag coefficient in the wind tunnel tests. The minimum aerodynamic damping in both planes occurs at around 15m/s ($Re = 2.5 \times 10^5$), but it is still positive, indicating there is not an aerodynamic instability of the cable in these conditions, in line with the observation of no large amplitude vibrations.
6 CONCLUSION

The total stiffness and damping matrices have been identified for three pairs of cable modes from full-scale vibrations for wind normal to the cables with no rain. The matrices for this wind direction are diagonal and appear to be a function of Reynolds number rather than reduced velocity. The experimental data agree well with the theoretical aerodynamic damping matrix based on quasi-steady theory up to 7 m/s (Re = 1.2 x 10^5), which is believed to be the end of the sub-critical Reynolds number region. For better comparison, wind tunnel tests are required for the double cable configuration of the Øresund Bridge in normal and skewed wind. Ongoing work aims to identify the effective aerodynamic damping matrix with rain and for different wind directions, expected to cause aeroelastic coupling between modes in the two planes.

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8 REFERENCES


"Determination of the aerodynamic damping of dry and wet bridge cables from full-scale monitoring"

Christos T. Georgakis, Antonio Acampora

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Introduction

Despite the considerable research of the topic in recent years, large amplitude vibrations of inclined cables continue to be observed and to be of concern for cable-stayed bridges. Various excitation mechanisms have been suggested, including rain-wind-induced excitation, dry inclined cable galloping, high reduced-velocity vortex shedding and excitation from the deck and/or towers. Although there have been many observations of large cable vibrations on bridges, there are relatively few cases of direct and simultaneous full-scale cable and meteorological measurements. Most investigative research has been based on wind tunnel testing and theoretical modelling. In the case of rain-wind-induced vibrations (RWIV), which account for 95% of the observed vibrations, research has focused on the wind tunnels testing of cable sections with artificially introduced water or solid rivulets. Some theoretical work has been undertaken to determine the propensity of a bridge cable to vibrate when an artificial rivulet has been added to it. In all cases, the debate surrounding the exact excitation mechanism(s) often focuses on the generation of negative aerodynamic damping at specific Reynolds numbers.

In this paper, a comparison of the quantified aerodynamic damping of the Øresund Bridge cables, in both dry and wet conditions, is presented. The aerodynamic damping is determined from full-scale measurements of the cable vibrations and the simultaneous meteorological conditions employing a well-known operation modal analysis (OMA) technique. The OMA is able to determine the overall mean damping of the cable for a series of vibration records on a mode-by-mode basis and for specific wind velocities and wind directions. The aerodynamic damping is then determined by subtracting the known structural damping. A comparison is made with theoretical estimates of the aerodynamic damping for the bridge cable’s first mode of vibration, based on measured force coefficients from wind-tunnel tests.

BACKGROUND

Although often unreported, large amplitude wind-induced vibrations of inclined bridge cables are common. Mechanisms that are often quoted as being responsible include von Kármán vortex-shedding, rain-wind excitation, dry inclined cable galloping, high reduced-velocity vortex shedding and parametric or indirect excitation. Zuo and Jones [1] reported excessive wind-induced vibrations of the inclined cables of the Fred Hartman Bridge, as measured employing a long-term full-scale monitoring system. They observed that the three-dimensional nature of the cable-wind environment inherently affects the mechanisms associated with the vibrations of inclined cables. Several of the aforementioned excitation mechanisms were identified, including von Kármán vortex-induced excitation, RWIV and dry inclined cable galloping. Matsumoto et al. [2] observed similar mechanisms when monitoring an experimental 30 m long inclined cable section exposed to ambient meteorological conditions.

Naturally, the aerodynamics of inclined bridge cables are complicated by the three-dimensional nature of the flow around them and the fact that typical sized smooth bridge stay cables find themselves in in the critical Reynolds number region for moderate to strong winds, corresponding to wind velocities between 5 and 15 m/s. Larose and Zan [3] showed the effect of the drop in the drag coefficient in this region on the lift coefficient. Matteoni and Georgakis [4] also showed the effect of varying surface
roughness and shape distortion on these coefficients. This simultaneous drop in drag and increase in lift can be theoretically shown to generate negative aerodynamic damping. Several wind tunnel tests have been carried out to investigate the influence of the rain on the dynamic behaviour of bridge cables and to replicate observed RWIVs, by both spraying water onto the cable model using a shower system [5-6] or by fixing stationary artificial rivulets on the surface of the cable model [7]. The former approach showed that the movement of the rivulet over the cable surface affects the RWIVs, whilst the latter approach was used to show that a rivulet at a certain position on the cable could generate galloping type instability, as a result of cable shape change.

In the past decade, efforts have been made to suppress RWIVs of bridge cables through the introduction of surface helical filets or dimples. In 1999, Larose and Smitt [8] reported some of the first findings from dynamic wind tunnel tests of cables with double helical filets. More specifically, the parallel stay cables in tandem arrangement of the Øresund Bridge were tested under both dry and wet conditions. They reported that for the given wind velocity of 12 m/s, an important contribution to the total cable damping could be attributed to positive aerodynamic damping in dry conditions. Nevertheless, in wet conditions, the total damping showed the presence of a negative aerodynamic damping. Interestingly, this apparently occurred predominantly without the presence of a water rivulet and whilst the cable was drying. More recently, several bridges in China have been reported to exhibit large amplitude cable vibrations in wet conditions, even though they have been fitted with dimpled cables.

In 2004, large amplitude cable vibrations were reported on the Øresund Bridge, in the presence of sleet on the cables [9]. Due to these vibrations, additional vibration suppression measures were implemented, including the installation of tuned mass dampers on the longest cable. However, recent long-term monitoring has revealed moderate vibrations of the cables under wet conditions [10]. To aid in the understanding of why these cable vibrations continue to persist, an attempt is made herewith to quantify the aerodynamic damping of helically filleted cables in both dry and wet conditions, based on the full-scale monitoring of the Øresund Bridge, between October 2010 and June 2011.

BRIDGE AND MONITORING SYSTEM

The Øresund Bridge provides the main navigational span for an 8 km road and rail link between Denmark to Sweden. The bridge has a main span of 490 m and an orientation of WNW to ESE. The deck is supported by 4 independent pylons, 204 m tall, and by 80 stays, all with an inclination of 30° to the horizontal plane and arranged in a harp-shaped configuration (Fig.1). Each stay is comprised of a pair of parallel seven wire monostrand cables, supplied by Freyssinet and arranged in vertical tandem. The gap between the cables is three diameters, with rigid connections between the two cables in each pair - at one or two locations along the length. The cables are covered with HDPE tubes, all with the same outer diameter of 250 mm and with double helical filets on the surface (Fig. 2). Additional tuned mass dampers are installed on the first and second longest cable pairs.

![Figure 1. Instruments and monitored stays: • accelerometer, ◆ anemometer, and □ rain-gauge.](image-url)
The current full-scale monitoring system was installed in 2009 [10]. Six tri-axial accelerometers are activated at any one time. The accelerometers are oriented to record the components of the cable vibrations normal to the cable axis in the in-plane (i.e. in the vertical plane) and out-of-plane (i.e. horizontal) directions. Accelerations of the adjacent deck and pylon are also measured. Ultrasonic anemometers are positioned at the top of the east pylon on a 4 m pole and on the south side of the deck at mid-span and between cables pairs 1M and 2M, west of the pylon (Fig. 1). The deck anemometers are placed on the south side on poles 7 m above the deck. A rain gauge is positioned close to the anemometer on the top of the pylon. Further data collected includes atmospheric humidity, temperature, and pressure. The monitoring system samples all channels at a frequency of 30 Hz and saves files in lengths of 10 minutes.

Since 2009, several different accelerometer measurement configurations have been employed on the four longest stays in the main span and eastern (Swedish) side span, on the south and north side of the deck, respectively (Fig.1). The aerodynamic damping of the third longest stay in the main span (Fig. 1), a.k.a. Stay 8M, is presented herewith. Each individual cable of the stay has a length of approximately 216 m, with a mass per unit length of 99 kg/m. The mode 1 out-of-plane frequency of the stay is 0.568 Hz. The structural modal damping in Mode 1 has been measured to be 0.68 % of critical [11]. Applying a linear regression to the quantified damping in this study reveals mode 1 structural damping to be 0.56% of critical, leading to a Scruton number of 14.48 for each cable in still air.

MEASUREMENTS

Initially, all of the wind records for the measurement period were sorted according to wind direction. Wind from the south, with a 10-min mean direction perpendicular to bridge axis, ±5°, were then extracted. Along- and across-wind cable accelerations were then divided in bins according the acting 10-min mean wind velocities, ranging from 0 to 14 m/s (assuming the mean of the measured wind velocities from the top of the pylon and the deck at mid-span). Accelerations were binned in 1 m/s wind intervals. Finally, data sets were split according to whether the cable was deemed to be dry or wet, as quantified from the rain-gauge measurements. Figure 3 shows the distribution of wind records used for the subsequent analyses in both dry and wet cable conditions. The total number of records for the dry condition is 850; whilst for wet condition there are 526.
Accelerations were analogue filtered to reduce noise and avoid aliasing. Sorting did not account for variations in the wind turbulence intensities, external temperatures or rainfall. The mean (+/- one standard deviation) turbulence intensities relating to the binned wind velocities can be viewed in Figure 4.

![Histogram of wind distribution for wind normal to Stay 8M for dry conditions (left) and for wet conditions (right).](image1)

![Mean (+/- one standard deviation) turbulence intensities versus Reynolds number for the binned wind velocities of Fig. 3.](image2)

**ANALYSIS**

To determine the overall damping of the stay for each selected wind record, an operational modal analysis (OMA) of the measured accelerogram was undertaken using the commercially available OMA package ARTeMIS Extractor Pro [12]. In this, a convergence of the eigenvalues employing the stochastic subspace identification (SSI) method leads to the determination of the stay frequency and an estimate of the overall damping for that record. In Fig. 5, a stabilisation diagram of principle components using estimated state-space models, as extracted from ARTeMIS, is seen. The results from each record are then binned according to Fig. 3, so that a system identification could be undertaken to obtain the total averaged stay damping from the binned set of accelerations. As the total damping was evaluated as a mean value for the bin at each wind velocity, the aerodynamic damping is obtained by subtracting the structural damping (for zero wind velocity) from the total damping. An averaging diagram, as extracted from ATReMIS, can be viewed in Fig. 6, below.
RESULTS AND DISCUSSION

The resulting quantified aerodynamic damping ratio of the cable versus Reynolds number, for both dry and wet conditions can be seen in Fig. 7. The solid line with solid triangles represents the damping for dry conditions, whilst the dash-dotted line with solid circles represents the damping in wet conditions. Two important observations should be noted. Firstly, the steep drop in aerodynamic damping for the dry stay, starting at Re=1.5×10^5 and secondly, the drop of aerodynamic damping into
negative values for the wet stay. For the dry cable, the steep drop in aerodynamic damping is unexpected, as the wind tunnel tests of individual helically filleted cables have shown that the drag coefficient drops gradually with increase in wind velocity, leading to the theoretically determined values of aerodynamic damping that increase near linearly with wind velocity. This can be understood by viewing the dotted line and the line with stricken crosses. The dotted line represents the theoretically determined values of aerodynamic damping using drag coefficients derived from wind tunnel tests by Kleissl and Georgakis [13] on sections of double helically filleted cables, with a slightly different specification to those on the Øresund Bridge and for a turbulence intensity of 1.1%. The line with stricken crosses represents the theoretically determined values of the aerodynamic damping using drag coefficients derived from wind tunnel tests by Freyssinet [11] on a sample cable with similar specification to that on the Øresund Bridge, but with a single fillet. No test results are available for the twin cable configuration. The expected along-wind aerodynamic damping of the cable can be determined with knowledge of the drag coefficient as [14]:

$$\zeta_{Ai} = \frac{\rho C_D U}{4\pi f_i \mu}$$

where $C_D$ is the drag coefficient, $\rho$ is the density of air, $U$ is the wind velocity, $f_i$ is the modal frequency and $\mu$ is the mass per unit length of each individual cable. The observed sudden drop in aerodynamic damping for the dry stay is more reminiscent of what is often observed for smooth cylinders. There is no explanation for this currently, but it should be pointed out that the twin cable configuration might be responsible for this. Wind tunnel tests are currently being undertaken to determine the effect of the twin cable arrangement on the stays force coefficients. In any case, the measured aerodynamic damping, when averaged, broadly follows the theoretically anticipated values.

Figure 7. Quantified aerodynamic damping ratio vs. Reynolds number, based on along wind accelerations of Stay 8M in dry and wet conditions. Line with stricken crosses is the theoretically calculated aerodynamic damping ratio from the drag coefficients, as reported in [11]. The dotted line represents the theoretically calculated aerodynamic damping ratio from the drag coefficients, as reported by Kleissl and Georgakis [13].
In wet conditions, the drop in aerodynamic damping from $Re=1\times10^5$ is steep, leading to negative values at about $1.3\times10^5$. The negative aerodynamic damping continues until $Re=2\times10^5$. For the Øresund Bridge, these values correspond to wind velocities of between 7.3 m/s and 18.9 m/s, which are comparable to the values at which rain-wind induced vibrations might generally be expected to occur, as reported by Acampora and Georgakis [10]. In the specific case of flow perpendicular to the stay, no significant stay vibrations have been reported, though. The reason for this might be found in the fact that, although the stay exhibits negative aerodynamic damping when wet, its overall damping remains positive, as witnesses by examining the cable Scruton number in Figure 6. In no case does the Scruton number fall below zero. The values are from approximately 6 to 33. Further work is currently being undertaken to determine the aerodynamic damping for other cable-wind angles of attack, particularly the sensitive relative cable-wind angles of $+35^\circ$ to $+55^\circ$ and $-25^\circ$ to $-50^\circ$ [10].

CONCLUSIONS

The aerodynamic damping of one of the Øresund Bridge stays has been determined through use of multiple vibration acceleration records and OMA. When wind is perpendicular to the stay, the measured aerodynamic damping in dry conditions stays positive and is broadly in line with what might be expected when theoretically determining the damping using experimentally derived cable force coefficients. This is not the case in wet conditions, though, where a large drop in aerodynamic damping is observed for a wind velocity of about 5 m/s. At approximately 7m/s, the aerodynamic damping becomes negative and remains so up to 19 m/s. Nevertheless, the overall damping of the stay remains positive, which might explain the absence of any moderate to large amplitude stay vibrations, for the particular cable-wind angle. The presence of negative aerodynamic damping in wet conditions for other cable-wind angles though, could be the likely cause of moderate but persistent oscillations on the stays on the bridge.

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Identification of aeroelastic forces on bridge twin cables from full-scale measurements for skewed wind angles of attack

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Identification of aeroelastic forces on bridge twin cables from full-scale measurements for skewed wind angles of attack

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1 ABSTRACT

Despite much research in recent years, large amplitude vibrations of inclined bridge cables continue to be of concern. Various mechanisms for the excitation have been suggested, including rain-wind excitation, dry inclined cable galloping, high reduced-velocity vortex shedding and excitation from the deck and/or towers.

Since 2010, the Technical University of Denmark has been monitoring the vibrations of the twin inclined cables of the Øresund Bridge. From the acquired data, Georgakis and Acampora [1] showed that the cable aerodynamic damping can be determined for wind orthogonal to the twin cables, in dry and wet conditions. In parallel, Acampora et al. [2] showed for the same cables that both coupled and uncoupled aeroelastic forces can be determined from the monitoring data, again when the wind is orthogonal to the cable and in dry conditions. In an expansion of the previous work, the aim of this paper is to identify the aeroelastic forces for in-plane and out-of-plane vibrations of bridge cables in dry conditions as in [2], but now for skewed winds. To achieve this, an output-only system identification employing the Eigenvalue Realisation Algorithm (ERA) [3] has been applied to selected vibration events. From this, the effective stiffness and damping matrices (including aeroelastic effects) have been identified from the cable vibrations.

2 INTRODUCTION

Large amplitude wind-induced vibrations of inclined cables are common. Various mechanisms could be responsible, including von Kármán vortex shedding, rain-wind excitation, cable-deck-tower interaction and dry inclined cable galloping. The aerodynamic mechanisms acting on inclined cables are complicated by the three-dimensional environment and the fact that typical sized bridge cable stays in moderate to strong winds sit in the critical Reynolds number region, where there is a rapid drop in the drag coefficient and potentially changes in the lift coefficient. Large wind-induced vibrations of inclined cables have been observed with a long-term full-scale measurement system on the Fred Hartman Bridge. It was observed that the three-dimensional nature of the cable-wind environment affects the mechanisms associated with the vibrations of inclined cables. Macdonald [4] measured the aerodynamic damping of cables on the Second Severn Crossing, which was found to be dominant over the structural damping, even in light winds for low frequency modes. On the Øresund Bridge, large amplitude cable vibrations have been reported, both in the presence of ice on the cables and in warmer conditions [5]. Recently a long-term monitoring system has been installed and vibrations under rain-wind conditions have been reported [1]. In the present paper, the measurements obtained from the Øresund Bridge monitoring system have been used to estimate the total stiffness and damping matrices for the first and second mode of Cable 8M, the longest cable monitored without a damper, for a range of wind velocities from 2 to 15m/s in dry conditions for relative wind to cable plane angles $\beta$ of $0^\circ$, $10^\circ$, $30^\circ$, $45^\circ$ and $70^\circ$.

3 BRIDGE AND MONITORING SYSTEM

The current investigation is based on data obtained from the monitoring system on the Øresund Bridge. The bridge has a main span of 490 m and an orientation of WNW to ESE. The deck carries both road and rail traffic and is supported by 4 independent 204 m tall pylons, and by 80 parallel double-stays. All of the stays have an inclination of $30^\circ$ to the horizontal plane and are arranged in a harp-shaped configuration (Fig.2). Each double-stay is made up of a pair of cables, arranged vertically with a centre-to-centre distance of three diameters and with rigid connections between the two cables in each pair at one or two locations along the length. The cables comprise multiple seven-wire mono-strands within HDPE tubes, all with the same outer diameter of 250 mm and with double helical fillets on the surface. Radial dampers are installed in the anchorages of the
longest cables. Accelerometers are oriented to record the components of the cable vibrations normal to the cable axis in the in-plane (i.e. in the vertical plane) and out-of-plane (i.e. lateral) directions. Ultrasonic anemometers are positioned at the top of the east pylon (on a 4 m pole) and on the deck (on the south side on poles 7 m above the deck) at mid-span and between the shortest two pairs of cables west of the east pylon (Fig. 1). A rain gauge is positioned on the top of the pylon. Further data collected include atmospheric humidity, temperature, and pressure. All channels are acquired at a frequency of 30 Hz and saved in files of 10 minutes length. The double-stay studied in this paper is Cable 8M, on the windward side of the deck, the third longest stay in the main span with a length of 216m and a mass per unit length of 99kg/m. The data were collected continuously between January and December 2011.

Figure 2. Instruments and monitored cables: ◆ accelerometer, ◇ anemometer and ▼ rain-gauge.

4 SYSTEM IDENTIFICATION

The aim is to identify the damping and stiffness matrices of the cable as a function of wind velocity for different relative wind angles \( \beta \) (10°±5°, 30°±5°, 45°±5°, 70°±5°, see Fig. 1) and to compare them with the values obtained for relative wind angles \( \beta = 0°\pm 5° \) (normal to cable pairs) [2] in dry conditions. This results in 378, 205, 235 and 291 records of 10-minutes length, respectively, distributed over wind speeds from 2-15m/s (using the mean of the measured wind speeds at the top of the pylon and on the deck at mid-span). The cable exhibits vibrations in multiple modes. The focus here is on the local cable vibrations and, for simplicity, one mode in each plane is considered at a time. Therefore the raw accelerations were filtered with 15th order high-pass and low-pass filters to isolate the single mode in each plane, taking care not to distort the signal. The filtered signals are considered as the response of a translational 2DOF system. It is assumed that there is no coupling with the other cable modes at different frequencies [6].

The equations of motion of the 2-DOF system (assumed linear) can be written in the form:

\[
\begin{bmatrix}
X \\
Y
\end{bmatrix} + \frac{1}{M} \begin{bmatrix}
X \\
Y
\end{bmatrix} + \begin{bmatrix}
K_{XX} & K_{XY} \\
K_{YX} & K_{YY}
\end{bmatrix} \begin{bmatrix}
X \\
Y
\end{bmatrix} = \begin{bmatrix}
K_{X} \\
K_{Y}
\end{bmatrix} = K_s + K_a \quad (1)
\]

where, \( M \) is the generalised mass (assumed to be the same for vibrations in each plane), \( X, Y \) are respectively the out-of-plane and in-plane generalised displacements, \( K_s \) represent derivatives with respect to time, and \( F_X \) and \( F_Y \) are the external generalised forces in the two planes due to wind and cable end motion. \( K \) is the total stiffness matrix, given by the sum of the structural, \( K_s \), and the aerodynamic stiffness matrix, \( K_a \). \( C \) is the total damping matrix, given by the sum of the structural, \( C_s \), and aerodynamic part, \( C_a \):

\[
C = \begin{bmatrix}
C_{XX} & C_{XY} \\
C_{YX} & C_{YY}
\end{bmatrix} = C_s + C_a \quad (4)
\]

where, \( \omega_{0,i} \) and \( \omega_{y,i} \) are respectively the circular natural frequencies of the \( i \)th out-of-plane and in-plane modes of the cable (in the absence of wind), and \( \zeta_{X,i} \) and \( \zeta_{Y,i} \) are the corresponding structural damping ratios. From output-only measurements alone it is possible to identify \( C/M \) and \( K/M \). The accelerations were analyzed with a system identification procedure based on ERA. The method assumes the external loading to be white noise and that the system is linear (including the aeroelastic forces, modelled as linear functions of cable displacement and velocity). The maximum lag length of the covariance functions used for the ERA method was optimised as 54s, based on the minimum value of the standard deviation of the results for multiple records at comparable wind speeds.

5 RESULTS AND DISCUSSION

Figures 3, 4 and 5 present the results of the system identification as mean values of the terms of the total stiffness and aerodynamic damping matrices divided by the modal mass, \( K/M \) and \( C/M \) respectively, for the relative cable-wind angles \( \beta = 0°, 10°, 30°, 45° \) and 70° for the first and second mode of Cable 8M. Each point represents the mean of the results from all records within a 1m/s range. The terms of the
stiffness matrices (Fig. 3) show well defined values with little variation with wind speed and no significant variation with wind to cable direction \( \beta \) for the first and second mode. The natural frequencies in the two planes (from the square roots of the diagonal terms) are near identical and they are close to a harmonic series (shown by the factor of \( 4 \) in the diagonal terms between the first and second modes), as expected for a taut cable. The off-diagonal terms of the stiffness matrix have near zero values for all wind directions. The terms of the aerodynamic damping matrices are shown in Figures 4 and 5 for the first and second mode, respectively. There are clear trends with wind speed, indicating aerodynamic damping for the diagonal terms of the two matrices. The diagonal terms of the aerodynamic damping matrices show linear trends, compatible with conventional quasi-steady theory [6], up to around \( 7 \) m/s, i.e. \( \text{Re} = 1.2 \times 10^5 \), which can be considered as the end of the sub-critical Reynolds number region. From static wind tunnel tests, Kleissl and Georgakis [7] found that the critical Reynolds number for a single inclined cable with double helical fillet corresponds to a range of about \( 0.8-1.3 \times 10^5 \). In Figs. 4 and 5 the maximum value of \( C_{XY}/M \) corresponds to damping ratios of approximately \( 0.34\% \) for mode 1 and \( 0.17\% \) for mode 2. In Figs. 4 and 5 there appears to be little influence of the wind direction, \( \beta \). The minimum aerodynamic damping in both planes occurs at around \( 13-15 \) m/s (Re = 2.2-2.5 \( \times 10^5 \)), but within the range of wind speeds with data available it is still positive, indicating there is not an aerodynamic instability of the cable in these conditions, in agreement with the observation of no large amplitude vibrations. The off-diagonal terms of the damping matrix are virtually zero for all the considered wind directions. It is notable that the first and second modes give very similar results, despite their different natural frequencies, hence reduced velocities. This is clear evidence that the drop in aerodynamic damping is governed by the Reynolds number rather than the reduced velocity.

6 CONCLUSIONS

The analysis shows that cable aerodynamic force coefficients for skewed wind directions can be well identified for in-plane and out-of-plane vibrations. There is negligible effect of aeroelastic stiffness. However there are clear trends in the diagonal terms of the damping matrix indicating aerodynamic damping. These trends with wind speed are very similar for the first and second modes, with different frequencies hence reduced velocities. This gives clear evidence that the behaviour is governed by Reynolds number rather than the reduced velocity. The aerodynamic damping is linearly related to the wind speed in the sub-critical Reynolds number region, but shows a drop (though not becoming negative) in the critical Reynolds number range. The off-diagonal terms of the stiffness and damping matrices appear to show no significant aeroelastic coupling between the two planes for the cases investigated.

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Figure 4. The aerodynamic damping matrices of first mode pair for relative cable-wind angle $\beta = 0^\circ$ (blue circle), 10° (green square), 30° (black triangle), 45° (blue star) and 70° (red cross).

Figure 5. The aerodynamic damping matrices of second mode pair for relative cable-wind angle $\beta = 0^\circ$ (blue circle), 10° (green square), 30° (black triangle), 45° (blue star) and 70° (red cross).

8 REFERENCES


This dissertation investigates the conditions that promote rain-wind-induced vibrations of inclined cable on cable-stayed bridges. Data was collected from the Oresund Bridge. Cable vibrations amplitude and frequencies of vibrations, wind directions and speeds, and rainfall rates are reported. Aerodynamic damping is investigated using the Markov-Block-Hankel matrix. For the first time, back-calculated aerodynamic coefficients were obtained from full scale data and compared to results from wind tunnel tests and literature. Finally, conclusions on the role played by aerodynamic damping for the rain-wind-induced vibrations are given.