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1. Statement of contribution/potential impact

The Economic Speed of an Oceangoing Vessel in a Dynamic Setting

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Statement of Contribution

Under the combined pressure of low freight rates and increased fuel prices observed since 2008, increased attention has been devoted in the maritime community to the determination of a cost effective speed of oceangoing vessels. The recent (early 2015) drop in fuel prices coupled with a further drop in freight rates has enhanced the importance of the problem. Significant work has appeared in the literature on the joint selection of speed and voyages in a deterministic environment, mainly in the context of the operation of lines. However there have been few results for the case of vessels operating in a tramp mode, by which we mean chartering on a voyage to voyage basis at random rates, these voyages being long, oceangoing ones. In fact, most formulations in the literature assume implicitly repetitions of the same sequence of voyages at the same known rates, abstracting from the uncertainty in the charter market.

In this paper we extend the existing models in several directions. First, we generalize the problem of finding the "optimal cycle" on a graph of origins – destinations and known freight rates (as introduced by Dantzig and his coworkers almost fifty years ago) to the case where speed is an additional parameter of choice. Second we consider the same problem when freight rates between origins – destinations are random variables of known distributions. The choice of voyage and speed is a joint decision in these models. In the case of charter rates that are independent from one voyage to the next, the optimal speed is constant in the sense that it depends on the average freight rate and the fuel cost but is independent of the particular voyage freight rate. Different speeds are optimal for voyages with differing fuel costs. When rates depend on an overall market process, speed does depend on the market state and indirectly on the voyage freight.

The dynamic programming equations in our models differ from the ones that appear in Markovian decision processes, since in our setting the choice of voyage is made after the realization of the random charter rates and thus there is an inversion of the expectation and maximization operations. We develop variations of existing solution methods to solve the modified dynamic programming equations – stochastic approximation, value iteration, policy iteration.

Our results lead to "rules of thumb" which might be of use to practitioners in the maritime industry who are not expected to go into the technical detail of the literature. The suggested voyage selection rule consists of comparing the available charters to an ideal voyage, and choosing to undertake the voyage that is best compared to the ideal one. Once the voyage is selected, the optimal speed is determined by a rule close to the standard economic speed formulae. For reasonable parameter values uniform changes in fuel prices do not affect the voyage choice, although non-uniform prices in bunker will affect the voyage selection through a change in the solution of the dynamic programming equations. The optimal voyage selection will tend to avoid ports where bunker price is high, unless the freight rate realization is sufficient to compensate for the high bunker cost. The bunkering problem has recently been examined in the literature, and we indicate how it might be incorporated in our dynamic programming formulation.

Response to Editor and Reviewers' comments

Comments to the editor

We would like to thank you and the reviewers for the comments which, hopefully, enhanced our paper.

The topics of the revision are as follows

- We made a more accurate reference to the early works on the minimum cost to time ratio cycle, which is due not only to Dantzig et al. (1967) but more accurately to E. Lawler, as stated in his Combinatorial Optimization textbook (p.9, para. 2)
- We included the very important reference by Besbes and Savin that was pointed out by Reviewer No. 1. It is actually a nice continuation of Dantzig, Blattner, Rao and Lawler work in the field of Transportation Research. We showed how to include in principle their routing - refueling problem within a routing - speed selection context (p. 30 para. 1)
- We incorporated time charter selection within the dynamic programming framework by slightly extending the choice of charters (p. 30 para. 2)
- We incorporated and solved the very interesting illustrative example suggested by Reviewer no. 1 (p. 12, last para.)
- We incorporated the views of a high ranking executive on the managerial applicability of our models, as a response to state managerial insights (Section 7.2)
- We enriched our references (included some recent work appearing in Trans. Res. B..)

Response to Reviewer #1:

(a) "the ballast and laden voyages should be traversed at the same speed" in Line 54-54, Page 6. In my opinion, the "kj" in Eq. (2) should be different ballast and laden voyages, and hence the speeds should be different.	We meant exactly the same, but our syntax was unfortunate. We rephrased this sentence see p. 6, last paragraph, last 6 lines.
(b) "The expression for (letter) a" should be "… for alpha", Line 9, Page 7. Also: Line 30 Page 8 "target (letter) a" and Line 3 Page 26 "rate (letter) a".	Ok
(c) Line 37, Page 7. "increase the speed by half the percentage". Check the accuracy of the word "half" (e.g., the square root of 144% is 120%, not 122%).	We meant 'by approximately half the percentage', and revised… p. 7 last para. of section 2.1.
(d) This research is mainly based on Dantzig et al. (1967). Why was there no relevant research on this topic in the past half century?	Except for Besbes and Savin there was indeed little use of this work in transportation. We cited a survey on its application to CAD p. 8, last lines of para. 1
(e) Line 30, Page 9. The reference "Dantzig et al. (1969)" is missing.	Mistaken reference, meant Dantzig et al. (1967)
(f) Line 28-34, Page 9. A brief introduction of how to find a cycle and the computational complexity could be added.	We referenced the relevant section in Lawler's textbook and added the complexity estimate. We thought that outlining of say the Floyd Warshall algorithm would inordinately lengthen the paper, we could add an appendix if you insist.. p.9, first 2 paragraphs.
(g) For stochastic parts, you might contrast your research with "Besbes, O., Savin, S., 2009. Going bunkers: the joint route selection and refueling problem. Manufacturing & Service Operations Management".	We are grateful for bringing to our attention this paper. We made extensive mention of it and showed how their viewpoint could be incorporated in our problem – see the Conclusions section 7.1
(h) The review paper "Psaraftis, H.N., Kontovas, C.A., 2013. Speed models for energy-efficient maritime transportation: a taxonomy and survey. Transportation Research Part C" could be cited as it provides extensive information on relevant topics.	We had mistakenly omitted it, confusing the 2013 and 2014 papers by the same authors. We corrected this omission in this revision.
(i) Line 1 Page 12. You should define "tau_w" before Eq. (10).	We rephrased the exposition, so that the τ_w parameter appears early on.
(j) Line 29, Page 22. "constrains" should be "constraints".	OK
(k) Line 27, Page 23. "in the Table 3" should be "in Table 3".	OK

(l) Line 31, Page 23. "as expected from equation (9)". How? (note that the expected profit alpha appears in equation (9), and alpha is related to fuel price).	Since eq. (9) involves only distance and freight parameters its solution (ζ and the h's) are independent of the fuel price and so is the optimal cycle. However the optimal speed will depend on alpha which will depend on both ζ and the fuel price. We tried to rephrase the section as well as the comments following equ. (9). See p. 10, para. following equ. (5')
(m)Line 7, Page 31. Delete ".".	OK
(n)Eq. (10), Page 11. I could not follow this equation. My questions are as follows. First, Is tau_w a constant or a decision variable for the shipping company? It seems that it is a constant according to "we assume a minimum wait time tau_w and we arbitrarily set tau_w=10 days" in Lines 39-41 on Page 22. Second, what does the phrase "a minimum wait time tau_w" mean? Do you mean if the revenue is low, then the ship should wait at a port for at least tau_w days? What if the revenue is very high in the next day? Third, how to determine when the ship should wait (still related to the definition of tau_w)? Fourth, how often is the revenue of a voyage updated? Every day? In sum, I am sure that Eq. (10) is one of the major contributions of the study. Therefore, I suggest using the following example to demonstrate this equation: There are two ports 1 and 2. The revenue of a voyage from port 1 to 2 is always 0, the revenue of a voyage from port 2 to 1 has equal probability of \$1 and \$2. Therefore, the ship will never wait at port 1, but may (I am not sure) wait at port 2 for the higher revenue of \$2. The speed can be considered fixed such that it takes one day from port 1 to port 2 and one day from 2 to 1, and the fuel cost can be assumed 0. I would like to see the optimal policy for this example, and how the optimal policy is derived.	Adding the possibility to wait for a time quantum τw is indeed a form of optimal stopping. In this formulation we might have to wait for an integral number of τ_w intervals until a satisfactory charter is observed, i.e. some destination value is greater than the value of waiting at the same port for a τw quantum. The restrictive assumption here is that freight rate observations a quantum distance apart are independent. A more satisfactory optimal stopping formulation would entail introducing stochastic processes for all rates, requiring a huge number of states, so we settled for the compromise model stated in the paper. See the rephrasing in p 10 last paragraph , p. 11 first lines. We thank the reviewer for the example. We include it as well as its solution which is quite interesting and is a nice application of dynamic programming methods. We could add a more complicated example with 2 ports and speed variation, but it would take too much space. See p. 12 last paragraph

Response to Reviewer #2

- There are some sentences with typing errors (for example line 4-5 in second paragraph in Introduction).	Ok!
- The introduction is quite long.	We tried to shorten it, actually splitting it in subsections 1.1 and 1.2 for better readability
- One section for the structure of the paper.	We added a subsection, splitting the introductory section. See section 1.2
- Consistent use of indexes (for example dj or dij, vj or v in Section 2.1).	We would like, if possible, to keep the letter d for distance and v for speed. The indices are usually double, except in section 2. We warn the reader when we move from one index to two indices. See p. 8 line 3.
- I would like to see some managerial insight in the conclusion.	We added a subsection (7.2) at the end with a report on conversations with an experienced maritime industry executive. His comments were indeed revealing

Highlights

The Economic Speed of an Oceangoing Vessel in a Dynamic Setting

Evangelos Magirou, Harilaos Psaraftis, Theodore Bouritas

- Examines the simultaneous selection of charters and speed of tramp vessels in an infinite horizon setting, for deterministic of stochastic rates
- For a known voyage ensemble the optimal speed on each voyage depends on its fuel cost and the average and not the individual freight rate
- For a voyage graph it is shown how to determine the optimal cycle of voyages and their optimal speeds. Again optimal speed on each voyage depends on its fuel cost and the optimal average profit rate
- For stochastic rates, independent for each voyage we determine the optimal choice of voyages and speeds. Again optimal speed depends on the average profit rate
- For stochastic rates and a Markovian description of freight rates, the optimal speed depends on the state as well, favourable states corresponding to higher speeds
- Solutions to the relevant dynamic programming equations are obtained through novel algorithms.

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Abstract

The optimal (economic) speed of oceangoing vessels has become of increased importance due to the combined effect of low freight rates and volatile bunker prices. We examine the problem for vessels operating in the spot market in a tramp mode. In the case of known freight rates between origin destination combinations, a dynamic programming formulation can be applied to determine both the optimal speed and the optimal voyage sequence. Analogous results are derived for random freight rates of known distributions. In the case of independent rates the economic speed depends on fuel price and the expected freight rate, but is independent of the revenue of the particular voyage. For freight rates that depend on a state of the market Markovian random variable, economic speed depends on the market state as well, with increased speed corresponding to good states of the market. The dynamic programming equations in our models differ from those of Markovian decision processes so we develop modifications of standard solution methods, and apply them to small examples.

Keywords

Economic speed, Dynamic programming, Markov and Semi Markov Decision Processes, Policy Iteration, Value Iteration, Stochastic Approximation

1. Introduction

1.1 The economic speed problem

From 2008 and until the middle of 2014 a combination of low freight rates and high fuel prices led to a widespread practice of low speed (slow steaming) in oceangoing vessels. The desire to reduce CO_2 emissions in view of environmental regulations also contributed to the use of lower speed; see Kontovas and Psaraftis (2011). At the time of writing, we are witnessing a precipitous drop in bunker prices – see Figure 1 - coupled with a further drop in freight rates, and the overall effect on speed is ambiguous. [1]These developments have led to significant research on how speed is to be incorporated in fleet and line management models; see for instance the survey of speed models in maritime transportation by Psaraftis and Kontovas (2013). By contrast, as stated by Ronen (2011), Christiansen et al. (2007), in the years following the 1970's oil crises and up to 2008 the literature on the topic of optimal speed for a tramp vessel was limited, and models did not change significantly from the approach presented in Ronen (1982). To our knowledge, the extent to which economic speed models have been used by practitioners has not been documented – see the mention of this problem by the authors' previous work in Magirou, Psaraftis and Christodoulakis (1992).

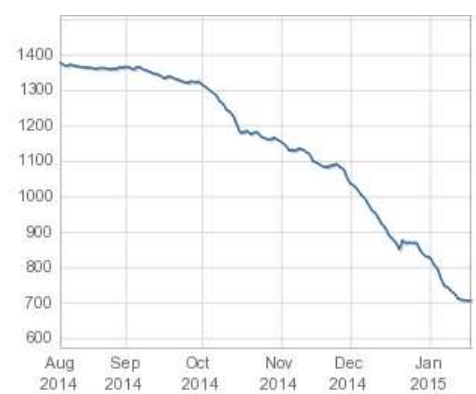

Figure 1. BunkerWorld Bunker Index, January 2015
http://www.bunkerworld.com/prices?tag=1-149695-173914849-0-BW

Most economic speed models optimize from the point of view of the ship-owner, on a voyage to voyage basis, assuming thus tramp operation. Attention has also been paid to the optimal speed from the point of view of the vessel's charterer. In general, the viewpoint of charterer and owner are different, although as shown in Devanney (2010) and outlined in Psaraftis and Kontovas (2013), their speed optimization problems turn out to be equivalent under the assumption that the charterer will

[1] In the formulations in Psaraftis and Kontovas (2013), the optimal speed is a function of the ratio of fuel price divided by the market spot rate, so if both drop their ratio may increase, decrease or stay the same.

have to charter additional tramp ship capacity if needed or charter out excess capacity at prevailing rates. In this paper we take the point of view of the vessel's operator. We consider the operation of a single vessel, ignoring interactions that may occur when managing a fleet.

In situations where a vessel has to undertake a sequence of voyages at known loads, freight rates, time windows etc., the speed of each voyage might be one of the decision variables in a mathematical programming formulation. Such models have been developed for several situations of practical importance, as in Norstad et al. (2011), Fagerholt et al. (2010), and Christiansen et al. (2013). The objective in these works is the total benefit and not the profit per time unit; accordingly the allowed speed variation is limited, speed being an operational rather than a strategic parameter. Speed selection is also important when scheduling a fleet of liners, their number being a decision parameter, as in Ronen (2011) and Noteboom and Vernimmen (2009). In liner management applications, when striving to maintain an acceptable level of service at minimum cost speed selection can take into account voyage and port uncertainties; this has been modelled as a stochastic programming problem in Wang and Meng (2012).

The effect of speed on voyage selection has been examined in Psaraftis and Kontovas (2014) using accurate expressions for fuel consumption as a function of speed and load. They show that voyage choice and speed will depend on the variations of the ship's hydrodynamic resistance, fuel cost and of course freight rates. In this paper we also examine the integrated problem of selecting voyage and speed but in infinite horizon problems. We first consider speed selection in the case where the operator knows all future freight rates. In this way we sidestep the difficulty of classifying voyages as either income generating or positioning legs (in the latter case charging an opportunity cost to the voyage days) as in Ronen (1982). Fuel price is considered known, and might vary from port to port. We assume there is refueling at every port and ignore the possibility of fuel stockpiling strategies as in Besbes and Savin (2009), Meng et al. (2015). In the deterministic models optimal speed depends on the average revenue, the ship's hydrodynamic resistance and the fuel cost variations from port to port but not on the freight of the particular voyage. We then consider the situation where voyages are to be selected on a graph, and show how to determine the optimal voyage cycle, the speed of the various voyages on it, again assuming just sufficient refueling at every port. Changes in fuel cost might influence the selection of voyages, in order for example to avoid voyages to destinations where fuel is expensive and are followed by long legs in ballast. However, when fuel prices are the same in all locations, we show that uniform fuel price changes do not affect the voyage selection.

Fluctuations in freight rates are of paramount importance in the operation of a tramp vessel. To our knowledge, speed models with stochastic rates have not appeared before except in our previous work, Magirou et al. (2013). In case freight rates of different voyages are independent random variables with time invariant distributions, we show that the optimal speed depends on the expected average daily profit but not on the freight rate of the particular voyage, a result which contrasts with the standard economic speed formulae where profitable voyages should be traversed at a higher speed. If one introduces a state space description of the overall freight market, optimal speed is higher in a favorable market state and conversely in bad ones. Speed variation should be interpreted from this viewpoint as an effort to take advantage of good times while they last and vice versa.

1.2 Structure of the paper

The structure of the paper is as follows. In Section 2 we examine speed selection when facing a known, repeating sequence of voyages. This is extended to voyage selection on a graph of ports, again with speed selection an option. In Section 3 we consider stochastic freight rates, rates being random variables which are independent from one voyage to the next and whose distributions are the same for each port of origin. Chartering decisions are then shown to be independent of speed selection, and if fuel price is the same in all locations, the choice of voyages will not depend on the overall fuel price level. In Section 4 we extend the model to include a state of the charter market which behaves as a finite state continuous time Markov Chain. When voyage times are either small or large with respect to the average duration of a charter market state, the optimal speed and voyage selection results simplify and have an intuitively plausible interpretation. Since the dynamic programming equations used differ from the standard ones for Markovian Decision Processes, we develop alternative solution methods first by stochastic approximation and then by a quasi-value iteration procedure. In Section 5 we examine models with a discounted net revenue criterion and compare them to the average undiscounted profit ones. Computational results are presented in Section 6 while Conclusions, managerial relevance and the authors' plans for further work are in Section 7. Several proofs and other details appear in the Appendices.

2. Deterministic Charter Rates and Fuel Prices

2.1 Speed considerations for a sequence of voyages

Consider a vessel that will undertake a sequence voyages indexed by $i,j=1,2,..,N$ which for voyage j have revenues P_j, distances d_j and port times t_{pj}. The fuel consumption for voyage j will depend on the voyage distance, speed v_j and the nature of the voyage, be it laden or in ballast. The daily fuel consumption is given by a function of the form $k_j v^3_j$, the parameter k_j incorporating the vessel's loading and thus depending on the particular voyage. Clearly, similar results can be obtained for different consumption function exponents. The total voyage fuel cost f_j will depend on the given fuel price $p_{F;j}$ which will depend on the voyage itself, as for instance when refueling is done in the port of origin. We do not consider fuel stockpiling, as in Besbes and Savin (2009) where extra fuel can be bought at locations where it is inexpensive. With these assumptions, the fuel cost for voyage j is $f_j=p_{F;j}k_jv^3(d_j/v)=p_{F;j}k_jd_{ij}v^2$. We ignore fuel consumption at port, and thus the net average daily revenue of the vessel owner for the sequence of voyages j is given by the expression

$$G(v_1,..,v_N) = \frac{\sum_{i=1}^{N}[P_i - f_i(v_i)]}{T(v_1,..,v_N)} = \frac{\sum_{i=1}^{N}[P_i - f_i(v_i)]}{\sum_{i=1}^{N}\left[t_{P_i} + \frac{d_i}{v_i}\right]} \qquad (1)$$

We tacitly assumed that all other operating costs are constant on an average daily basis, and we denoted by $T=T(v_1,..,v_N)$ the total time for all voyages, including port times.

Let us for simplicity assume that we can freely choose speeds for each voyage. In practice charter party obligations, engine specifications, weather conditions etc. impose constraints on speed, but these can be handled with standard techniques while obscuring the larger picture, so we will assume that the optimizing speed is within the allowed range of all the above constraints. We show how to deal with simple and upper or lower bounds on speed in Appendix C, but ignore any speed constraints in the main part of the paper.

To obtain the optimal speed we calculate the partial derivatives of (1) with respect to the voyage speeds and get

$$\frac{\partial G}{\partial v_j} = \frac{\sum_{i=1}^{N}\left(P_i - f_i(v_i)\right)}{T^2} \cdot \frac{d_j}{{v_j}^2} - \frac{2p_{Fj}d_jk_jv_j}{T}$$

Equating the derivatives to zero we obtain the following expression for v_j:

$$v_j = \left(\frac{\sum_{i=1}^{N}\left(P_i - f_i(v_i)\right)}{2p_{Fj}k_jT}\right)^{1/3} = \left(\frac{\alpha}{2p_{Fj}k_j}\right)^{1/3} \qquad (2)$$

The term alpha α is defined as the average net profit for the collection of voyages

$$\alpha = \frac{\sum_{i=1}^{N}\left(P_i - f_i(v_i)\right)}{T} \qquad (3)$$

Thus alpha (α) is the optimal net revenue for the entire trip sequence. It is independent of the particular voyage j, depending on the entire set of voyages. The dependence of the speed on the voyage is strictly through the term $p_{Fj}k_j$, the product of the fuel price by the specific daily consumption. It follows that there should be variations in the speed as a function of the daily fuel cost of the voyages. On the other hand the revenue of the particular voyage is of no importance for the determination of the voyage's speed, this revenue affecting speed only through its contribution to the average net profit alpha - α. In this sense, ballast and laden voyages have equal contributions to the average daily profit and thus differences in their optimal speeds are due to differences in hydraulic resistance and fuel price. The slow speed recommendation on expensive fuel can be interpreted as an effort to buy less fuel at locations where its price is high.

In order to compute the value of the average profit α in terms of the original parameters we substitute the values of v from (2) in (3) to get

$$\alpha = \frac{P - \sum_{j=1}^{N} \gamma_j d_j \left(\frac{\alpha}{2\gamma_j}\right)^{2/3}}{T_P + \sum_{j=1}^{N} d_j \left(\frac{\alpha}{2\gamma_j}\right)^{-1/3}}$$

Here T_P is the sum of all port times, P is the sum of all freights and we set $\gamma_j=p_{F,j}k_j$. We will refer to γ as the daily fuel cost at unit speed, or as the specific daily fuel cost. The above expression for alpha (α) in conjunction with the expression for v_j in (2), $\alpha=2\gamma_j v_j^3$ reduces to the following equation for v_j, the optimal speed on voyage j

$$2\gamma_j T_P v_j^3 + 3\gamma_j^{2/3} v_j^2 \left(\sum_{k=1}^{N} \gamma_k^{1/3} d_k\right) - P = 0 \tag{4}$$

Equation (4) implies that a different speed should be used in every voyage. The optimal speed depends on the voyage daily fuel cost γ_j as well as on the voyage ensemble characteristics, i.e. the total revenue P, total port time T_P and the weighted voyage distances, but not on the particular voyage revenue P_j. In case the γ_j's are the same for all voyages, the optimal speed is constant $v_j=v$ and satisfies the equation

$$2\gamma T_P v^3+3\gamma v^2 D-P=0$$

This is the equation appearing in Ronen (1980) that applies to a single laden voyage with T_P the port time, P the freight, D the distance. It generalizes provided the fuel price – consumption characteristics are independent of the voyage. Furthermore, if total port time is negligible the economic speed is given by the expression

$$v = \left(\frac{P/D}{3\gamma}\right)^{1/2} \tag{5}$$

Interpreting (5) gives us the following rule of thumb: increase the speed by about half the percentage increase in rates, decrease it by about half the percentage increase in fuel. Comparing the optimal speed in (5) with the original expression (2), one might observe an inconsistency in the exponents; but this discrepancy is deceptive since the α term in (2) is the net profit rate while the nominator in (5) is the gross profit rate, which is of course larger and hence the higher root in the latter expression is justified.

2.2 Simultaneous speed and voyage selection

The operator of a tramp vessel does not know beforehand the sequence of voyages his vessel will undertake, and it will actually depend on the freight rates that will prevail. One can generalize the tramp scheduling problem posed by Dantzig et al. (1967) to include speed selection: In that work, voyages are considered as transitions on the nodes of a graph which correspond to ports. The revenue for a voyage between nodes - ports i, j is known and constant P_{ij}, while the distance d_{ij} and a port time

$t^p_{i,j}$ is also known. We assume that on any voyage i,j the speed v_{ij} can be selected without constraints. Note that from this section on we use d_{ij}, v_{ij} to denote the distance between ports i,j as well as the speed of the corresponding voyage; in the previous subsection d_j, v_j were the distance and the speed of voyage j, and the voyages were specified exogenously. The fuel cost per unit time is a known function depending on the voyage i, j – we assume the form $p^F_{i,j}k_{i,j}v^3_{i,j}$ and the total fuel cost for the voyage is $\gamma_{ij}v^2_{ij}d_{ij}$, as before. The voyage length is $T_{i,j}=t^p_{i,j}+\gamma_{ij}d_{ij}v_{ij}^{-1}$. In Dantzig's formulation, the objective is to select voyages so as to maximize revenue per unit time for an infinite horizon, and we extend the problem by selecting both voyages and the corresponding speeds to achieve the same goal. Besbes and Savin (2009) consider the same problem with the possibility of the vessel loading added fuel at ports where prices are low. The authors derive methods to solve the joint refueling – voyage selection problem using dynamic programming, although at exogenously given speed. Their formulation can be combined with ours as we will show in the concluding section. The optimal refueling problem has recently been formulated using mathematical programming, see Meng et al. (2015). Note that the applicability of the minimum cost to time ratio problem of Dantzig et al. op. cit. is not limited to the transportation domain, but has applications to several Computer Aided Design problems. See the survey by Dasdan et al. (1999). In fact, Dantzig and his coworkers consider a transportation problem much more complicated than identifying the minimum cost to time ratio cycle; this cycle problem is just a part of an efficient column generation solution technique for their full problem.

We model the above situation in the form of a semi Markovian decision problem where states correspond to ports. The decision to undertake a certain voyage determines the next state and the dwell time with certainty. A straightforward dynamic programming argument as in Ross (1970) Ch. 7 Theorem 7.6 shows that the optimal policy is characterized by a per unit time profit parameter alpha - α and port values h_j, $j=1, 2,.., N$ and $h_1=0$ which satisfy the equations

$$h_i = \max_{j\neq i}\max_v \left\{P_{i,j} - \gamma_{i,j}d_{i,j}v^2 - \alpha\left(t^p_{i,j} + \frac{d_{i,j}}{v}\right) + h_j\right\} \quad i = 1,2,..,N \text{ and } h_1 = 0 \qquad (6)$$

The interpretation of (6) is as follows. The voyage selection policy consists of determining a profit target α and a port "profit factor" h_j; charter selection is based on comparing for each possible destination j its differential profit net of fuel with respect to an ideal voyage, i.e. $P_{i,j} - \gamma_{i,j}d_{i,j}v^2 - \alpha\left(t^p_{i,j} + \frac{d_{i,j}}{v}\right)$ and then adding the destination port factor h_j. This is optimized for speed and the destination is then selected that maximizes the profits

If destination j is selected from origin i the optimal speed is determined by setting the partial derivative of the expression in brackets in (6) to zero, obtaining an expression similar to the case of a known voyage sequence (2), namely

$$v_{i,j} = \left(\frac{\alpha}{2\gamma_{i,j}}\right)^{1/3} \qquad (2')$$

This shows that the optimal speed depends on the specific daily fuel cost γ_{ij} but not explicitly on the voyage freight rate, although the overall rates influence the nominator alpha (α) and the destination j is determined by the fact that it corresponds to the relatively highest rate. If the fuel-speed characteristics are approximately the same for all voyages, speed should be the almost constant regardless of the freight rate of the particular voyage. However, if the vessel's hydrodynamic characteristics differ – as for instance for a ship in ballast in contrast with a laden one, for a voyage in rough seas in contrast with one in predictably calm seas, there should be variations in speed. In practice these variations can be considerable – up to 30% for VLCC's as stated in Psaraftis and Kontovas (2013) and (2014). Variations in fuel consumption parameters should lead to differences in voyage selection. We show however that in important special cases the choice of voyages is independent of the fuel cost parameters.

Determining the values of α, h can be done through several algorithms and we will show some in the following Sections. It is interesting though to consider the following bisection algorithm in the spirit of the work of Lawler (1976) Chapter 3.13 and secondarily of Dantzig et al. (1967). It is based on the following observation

> Lawler – Dantzig observation extended: Consider an arbitrary α and select on the voyage graph speeds as given by (2'). On every edge consider weights
>
> $$w_{ij}(\alpha) = P_{i,j} - \gamma_{i,j} d_{i,j} v_{i,j}^2(\alpha) - \alpha\left(t_{i,j}^p + \frac{d_{i,j}}{v_{i,j}(\alpha)}\right)$$
>
> If there is a cycle of nonnegative total w value then there is a sequence of voyages that has a net average profit greater than α. Conversely, if all cycles have negative total weight, the value of α provides an upper bound on the net average profit.

The proof is immediate by summing the w's over the cycle.

The existence or nonexistence of a cycle of negative total value in an n vertex graph can be determined in polynomial complexity $O(n^3)$ by several algorithms (Bellman-Ford, Floyd-Warshall) – see for instance Lawler's textbook, Chapter 3.11. Based on this observation Lawler (1976) Chapter 3.13, constructs a bisection type algorithm whose computational complexity is $O(n^3 \log n)$, n being the number of vertices. Since this algorithm does not obviously generalize to stochastic rates we show in the next section algorithms that do not use the bisection principle.

Equation (2') determines the speed as a function of the value of the profit rate α obtained from the dynamic programming equation (6), which is stated in terms of the problem's parameters. It is thus not clear what is the direct dependence of speed on freight rates and fuel parameters, as in the case of a sequence of trips. We would like to obtain the analog of equation (4) where the optimal speed was determined on the basis of the problem parameters directly. We proceed with an analysis similar to the one that led to (4): For ease of exposition we first assume that port times are negligible. Substituting the optimal speed (2′) in (6) with zero port times we have after some algebra

$$h_i = \max_{j \neq i}\left\{P_{i,j} - \frac{3}{2}\left(2\gamma_{i,j}\right)^{1/3}\alpha^{2/3}d_{i,j} + h_j\right\} \quad i = 1,2,..,N \text{ and } h_1 = 0 \tag{7}$$

Setting $\beta = \alpha^{2/3}$, $d'_{ij} = \frac{3}{2}\left(2\gamma_{i,j}\right)^{1/3}d_{i,j}$ the previous equation (7) becomes

$$h_i = \max_j\left\{P_{i,j} - \beta d'_{ij} + h_j\right\} \qquad i = 1,2,..,N \text{ and } h_1 = 0 \tag{8}$$

These equations correspond to a minimum cost to time ratio cycle problem i.e. voyage selection without speed considerations. Solving it for β we obtain the analog of the optimal speed formula (2′), namely

$$v_{i,j} = \beta^{1/2}\left(\frac{1}{2\gamma_{i,j}}\right)^{1/3} \tag{2″}$$

In case the daily fuel cost fuel parameters γ_{ij} are the same for all voyages and equal to γ equation (7) becomes

$$h_i = \max_j\left\{P_{i,j} - \zeta d_{ij} + h_j\right\} \quad i = 1,2,..,N \text{ and } h_1 = 0 \tag{9}$$

Here $\zeta = \frac{3}{2}(2\gamma)^{1/3}\alpha^{2/3}$. The dynamic programming equation (9) is identical to that in the original minimum cycle problem, so its solution in ζ, h is the same as before, and does not depend on speed or fuel considerations. Furthermore, the selection of charters is the same as in the problem with the same distances d_{ij} (although expressed in time units) and freight rates P_{ij}. The optimal speed can then be expressed in terms of ζ by substituting in (2′) the expression of α in terms of ζ i.e. $\alpha = \left(\frac{2\zeta}{3}\right)^{3/2}(2\gamma)^{-1/2}$, to obtain

$$v = \left(\frac{\zeta}{3\gamma}\right)^{1/2} \tag{5′}$$

Optimal speed is indeed a function of the fuel cost parameter γ and the average profit rate ζ, and the latter does not depend on fuel cost. It is the analog of equation (5) in the problem with a known voyage sequence. We thus have a separation of voyage choice from optimal speed selection, and the rule proposed for the known voyage sequence, i.e. increase speed by about half the percentage of a freight rate increase etc. is still valid.

We can obtain similar results when port times are explicitly taken into account. Substituting the optimal speed (2′) in (6) we get the equation

$$h_i = \max_j\left\{P_{i,j} - \frac{3}{2}\left(2\gamma_{i,j}\right)^{1/3}\alpha^{2/3}d_{i,j} - t^p_{i,j}\alpha + h_j\right\} \quad i = 1,2,..,N \text{ and } h_1 = 0$$

Setting $\beta = \alpha^{2/3}$, $d'_{ij} = \frac{3}{2}\left(2\gamma_{i,j}\right)^{1/3} d_{i,j}$ the previous equation becomes

$$h_i = \max_j \left\{P_{i,j} - \beta d'_{ij} - t^p_{i,j}\beta^{3/2} + h_j\right\} \quad i = 1,2,..,N \text{ and } h_1 = 0 \qquad (9')$$

This last equation can be solved by the same methods (a bisection method will work since the right hand side is decreasing in β) to determine β and h, α and the optimal speed by (2′) and (2″). In this case there does not seem to be any straightforward relationship between optimal speed and the various voyage parameters or fuel cost. In Section 6 we present computations illustrating the above results.

3. Stochastic Freight Rates – Independence

3.1 Model formulation

The models presented in Sections 2.1, 2.2 have the obvious drawback that they assume known freight rates. We will show in this and the following Sections that we can preserve the same voyage - speed selection principle even for random freight rates. We will do so by extending the dynamic programming approach to the stochastic case. Consider first a simple model with stochastic freight rates. The rates to all destination ports j from origin i become known to the vessel's operator upon arriving at port i (unavailability of charters to some specific destination would correspond to a null freight rate). We also assume that the operator can freely select his voyage speed and knows the fuel costs. As stated earlier, we will not address in the main part of the paper constraints on speed, but we will show in Appendix C how to incorporate upper and lower bounds on speed.

The problem for the vessel's operator is to select the optimal destination-speed combination, j and v_{ij}, after having observed the prevailing rates P_{ij}. Upon arriving at the destination j the process is repeated i.e. the operator observes new rates P_{jk} to all destinations k. We assume in this Section that the rates P are independent between any voyage and the future ones. A more satisfactory modelling approach would be to consider at each port a set of available charters. Each charter would be characterized by its destination, its freight rate if it is laden, the vessel's payload etc. We could include more complicated charters, i.e. travel from i to load at port i' and unload at j, or even time charters. The vessel's hydrodynamic resistance coefficient k and other voyage parameters will then depend on the charter choice and not simply the destination. We do not include such considerations here but present these alternative formulations in the Concluding section. We assume that the resistance coefficient will depend on the voyage, thus implicitly categorizing voyages ij as laden, ballast, or partial load ones. The owner's objective is to maximize the expected value of the net revenue per unit time for an infinite horizon, and assume that freight rates are independent from one voyage to the next. In other Sections we will consider discounted objectives and also allow implicit rate dependence between voyages. Finally, in case the rates observed at some port are unsatisfactory, we allow the vessel to wait for a better freight rate for a fixed, exogenous wait time interval τ_w; after that interval a different realization of rates can be observed, independent of the initial one, and if even these are unsatisfactory to wait once more for the interval τ_w, etc. A more satisfactory – but intractable - approach would be to model the freight rates as stochastic processes and consider the corresponding continuous time optimal stopping problems.

We again employ the theory of Semi Markov Decision Processes for infinite horizon, time average profit, as stated in Ross (1970). States correspond to ports. The dynamic programming equations are written in terms of an optimal average profitability α and port values h_j, relative to say port *1* for which $h_1=0$. The optimal α, h_j satisfy equations similar to the Markovian Decision Process ones, namely

$$h_i = Emax\left(\left[\max_{j\neq i} \max_v(\tilde{P}_{i,j} - f_{i,j}(v) - \alpha\tau_{ij}(v) + h_j)\right]; -\alpha\tau_w + h_i\right) \quad i = 1,2,..,n \text{ and } h_1 = 0 \qquad (10)$$

In (10), $\tilde{P}_{i,j}$ is the freight random variable, $f_{ij}(v)$ the fuel cost at speed v and $\tau_{ij}(v)$ the total voyage time at speed at sea v. The random variables are independent from on voyage to the next, but there can be dependence between them for the destinations j. We allow a stay at the same port awaiting a better charter for a fixed time interval τ_w, incorporating thus in the formulation a rudimentary optimal stopping problem.

Equation (10) is analogous to those in semi Markovian decision problems, as in Ross (1970) Section 7.4, with deterministic state transition times. It can be justified as follows: h_i is the expected value of location i before the rates are observed, hence the expectation operator precedes the maximization ones. After observing the rates we either select to sail to destination j at the optimal speed or decide to wait for a period τ_w. In the first case we subtract from the net voyage profit the implicit cost $\alpha\tau_{ij}$ and add h_j, the value of the destination j. In the second case wait we incur the implicit cost $\alpha\tau_w$ and add h_i, the benefit of remaining at i. The correctness of equation (10) can be proven using the same methods as in Ross op. cit., namely by proving that if (10) has a solution in α, h there is an optimal policy and conversely.

A simple example could clarify the above model, as well as the dynamic programming formulation incorporated in (10). Consider a world with two ports 1,2 and a vessel which moves at given speed, the voyages between 1 and 2 being of unit time length. There are no charters from 1 to 2, i.e. $P_{12}=0$. On the other hand P_{21} can be either 1 or 2 with probability 50%. There is a wait τ_w between observations of the freight rate at 2. Naturally for zero wait time the vessel will wait until a rate of 2 occurs, giving a 1-2 cycle with an average profit of 1. On the other hand if τ_w is large, the ship upon arriving at port 2 will accept the first freight observed. Thus the average profit will be ¾ (1 time unit obtaining no revenue and the other time unit obtaining an average revenue of 3/2). To determine the critical value of τ_w we use equation (10) which gives for port 1 $h_1=0=0-\alpha+h_2$ and hence $h_2=\alpha=3/2$. For the no wait policy to be optimal at port 2, the condition is $-\alpha\tau_w+h_2<1-\alpha$ or $\tau_w>2/3$. In case $\tau_w<2/3$, equation (10) for h_2 gives $\alpha = [(-\alpha\tau_w+\alpha)+(2-\alpha)]/2$ and hence $\alpha=(1+ \tau_w/2)^{-1}$. Thus the optimal mean profit is a decreasing function of the wait time. One can extend the model it to include speed, which will then depend on the profit rate. A decreased wait time will increase profits and thus indirectly speed!

As far as speed selection is concerned, equation (10) has the same structure as the deterministic problem treated previously. For fuel cost functions and voyage times as in the previous Section, once a voyage j is selected it should be carried out at speed $v_{i,j} = \left(\frac{\alpha}{2\gamma_{i,j}}\right)^{1/3}$ showing thus that optimal speed does not depend on the observed freight rate but on the destination selected. Of course the realization of the freight rate random variables will affect the voyage selection and thus indirectly the relevant speed, but speed will not necessarily be an increasing function of the freight rate. If the fuel cost – speed parameters are uniform, the speed is independent of individual charter rates and voyage selection.

Substituting the expression for the optimal speed in (10) we get the analog of the voyage selection equation (8) in the deterministic case: Setting again $\beta = \alpha^{2/3}, d'_{ij} = \frac{3}{2}\left(2\gamma_{i,j}\right)^{1/3} d_{i,j}$ equation (10) specializes to

$$h_i = E\left(max\left[\max_j\{\tilde{P}_{i,j} - \beta d'_{ij} + h_j\};\ -\beta^{3/2}\tau_w + h_i\right]\right) \quad i = 1,2,..,N \text{ and } h_1 = 0 \tag{8'}$$

For negligible port times and uniform fuel-speed parameters $\gamma_{ij}=\gamma$ the dynamic programming equation reduces to the following one, similar to (9) in the deterministic rate case

$$h_i = E\left(max\left[\max_j\{\tilde{P}_{i,j} - \zeta d_{ij} + h_j\};\ -q\zeta^{3/2}\tau_w + h_i\right]\right) \quad i = 1,2,..,N \text{ and } h_1 = 0 \tag{11}$$

Equation (11) follows directly from (8') by setting $\zeta = \frac{3}{2}(2\gamma)^{1/3}\alpha^{2/3}$, keeping the distances d_{ij} unaffected; q stands for the expression $(2/3)^{3/2}(2\gamma)^{-1/2}$ which results from setting $\alpha = q\zeta^{3/2}$. The daily profit rate α follows once (11) has been solved for ζ.

Equation (11) is the stochastic analog of the minimum cost to time ratio cycle problem and has been studied by the authors; see Magirou and Bouritas (2010). One must select voyages on a graph where rates are random and speed is exogenous; the freight rates become known upon arriving at the destination of the previous voyage and are independent from one voyage to the next. The profit rates for the voyages are net of fuel costs, and hence ζ, the optimal profit rate, does not explicitly depend on fuel cost which is included in the rates P_{ij}. The optimal speed is as in (5′) $v = \left(\frac{\zeta}{3\gamma}\right)^{1/2}$. Therefore a uniform multiplicative change in rates or in fuel parameters affects the optimal speed in a square root law fashion, and does not influence the choice of voyage.

The equation for the case where fuel consumption depends on the voyage and port times are not negligible is similar to that in the deterministic case – equation (9') (for convenience we do not include the wait option) namely

$$h_i = E\left\{\max_{j\neq i}\left[\tilde{P}_{i,j} - \beta d'_{ij} - t^p_{i,j}\beta^{3/2} + h_j\right]\right\} \quad i = 1,2,..,n \text{ and } h_1 = 0 \qquad (12)$$

As before $\beta = \alpha^{2/3}$, $d'_{ij} = \frac{3}{2}\left(2\gamma_{i,j}\right)^{1/3} d_{i,j}$ and $v_{i,j} = \beta^{1/2}\left(\frac{1}{2\gamma_{i,j}}\right)^{1/3}$. If port times are negligible and freights change uniformly by a multiplicative parameter, namely $\widetilde{P'}_{i,j} = \lambda\tilde{P}_{i,j}$ for a constant λ, it is clear that equation (12) has a solution in which the β, h parameters have been multiplied by λ, and hence speed increases by the square root of λ. Similarly if all fuel prices increase by λ, optimal speed decreases by the square root of λ. We thus get an extension of the previously stated speed selection rule.

3.2 Solution methods

The equations in the previous Section differ from the standard dynamic programming ones because of the reversal of the order of the expectation and maximization operators; therefore modified solution methods must be used. We examine the following two methods: The first is an application of the stochastic approximation of Robbins and Munro (1951) applied to the multidimensional case, as analyzed by Blum (1954). The second was developed in the authors' previous work and is to some extent related to Lawler's bisection argument as well as value iteration, and we refer to it as Quasi Value Iteration.

a. Stochastic approximation

For ease of exposition, we describe the method as applied to equation (8') without the wait option, its application to the other equations being similar.

We form the random sequence of h_i*'s* indexed by n:

$$h_i^{n+1} = h_i^n + \eta_n\left[max_j\left\{\tilde{P}_{i,j} - \beta^n d'_{ij} + h_j^n\right\} - h_i^n\right] \quad i = 2,..N$$

$$\beta^{n+1} = \beta^n + \eta_n\left[max_j\left\{\tilde{P}_{1,j} - \beta^n d'_{1j} + h_j^n\right\} - \beta^n\right]$$

The $\tilde{P}_{i,j}$ are realizations of the freight rate random variables. The second relation which is used to update β relies on the normalization $h_1=0$. The parameters η_n must satisfy the conditions of the stochastic approximation algorithms, namely $\eta_n \propto n^{-1}$. We have not examined theoretically the properties of these procedures in the spirit of Blum (1954), and have used in our computations ad hoc methods to achieve convergence by empirically adjusting the η parameters. To rectify this, upon apparent convergence to some values for β, h we performed a verification step by substituting the candidate values to the right hand side of the dynamic programming equation and obtaining the sample mean. We then checked that the sample means thus generated were sufficiently close to the values of β, h being tested.

This verification method can in principle be analyzed as follows: Take for instance equation (10) without the wait option, namely

$$h_i = E\left\{\max_{j\neq i,v}\left[\tilde{P}_{i,j} - f_{i,j}(v) - \alpha\tau_{ij}(v) + h_j\right]\right\} \quad i = 1,2,..,N \text{ and } h_1 = 0$$

Assume that some particular values for α, h have been determined by stochastic approximation or any other method. We can estimate the right hand side by taking a large sample of the random variable $\max_{j,v}\left[\tilde{P}_{i,j} - f_{i,j}(v) - \alpha\tau_{ij}(v) + h_j\right]$. Assuming that the standard deviation of the sample mean has been computed, one can assign (under a normality assumption) a probability β that the equalities are satisfied up to ε namely

$$h_i \leq \max_{j,v}\left[\tilde{P}_{i,j} - f_{i,j}(v) - \alpha\tau_{ij}(v) + h_j\right] \leq h_i + \varepsilon$$

Then one can claim using Lemma 2 of Appendix A that the policy implied by the parameters α, h is ε close to the optimal with probability β.

b. Quasi value iteration

Various forms of this algorithm were applied in the previous work of the authors Magirou et al. (1997), Magirou and Bouritas (2010), Magirou (2012), Magirou et al. (2013) in infinite horizon Markovian decision problems with the average value criterion. The algorithm can be used in the obvious way when a speed choice is involved as well. For ease of exposition we exhibit the method as applied to equation (11) without the option of waiting, namely

$$h_i = E\left[\max_j\left\{P_{i,j} - \zeta d_{ij} + h_j\right\}\right] \qquad i = 1,2,..,N \text{ and } h_1 = 0 \tag{11'}$$

We informally describe the procedure in the following steps

Step 0. Start with arbitrary location values h_{in}, and say $n=1$

Step 1. Find a "good" average rate b by solving the stochastic programming problem

max b

Such that

$$h_{in} \leq E\left(\max_j\left\{P_{i,j} - bd_{ij} + h_{jn}\right\}\right) \quad i = 1,2,..,N \text{ and } h_{1n} = 0$$

Call the maximum b found α_n.

A practical method to solve the above problem is by bisection as in the Lawler algorithm already cited, based on the observation that the right hand side is monotonic in *b*: start with a large value of *b* so that the constraints are not satisfied, keep halving *b* until the constraints are satisfied, and proceed until the desired accuracy is obtained. The satisfaction of the constraints with a desired probability is verified by computing the sample mean of $\max_j\{P_{i,j} - bd_{ij} + h_j\}$. It is shown in Appendix A that if we follow a policy based on α_n and h_{in} we obtain a return greater than α_n

Step 2: Update the *h*'s by setting

$$h_{i,n+1} = E\left(\max_j\{P_{i,j} - a_n d_{ij} + h_{jn}\}\right) \quad i = 2,..,N \text{ and } h_{1n} = 0$$

It follows by the definition of α_n that $h_{in+1} \geq h_{in}$.

Step 3. If $E\left(\max_j\{P_{i,j} - a_n d_{ij} + h_{jn}\}\right) - h_{in} \leq \varepsilon_o$ for all *i* stop.

(ε_o is the desired accuracy)

Otherwise return to Step 1 with $n=n+1$ using the updated h_{in+1}'s

The justification of the Algorithm is as follows: By Lemma 2 of Appendix A we know that if the gap in the inequalities in Step 1 is less than ε, the policy implied by a_n, h_{in} is ε close to the optimal value. If we are not satisfied with the current approximation ε, repeating Step 1, will improve the policy: Indeed, if we perform Step 1 we will get an improved value α_{n+1}, in the sense that $\alpha_{n+1} \geq \alpha_n$. This is due to the inequality

$$h_{i,n+1} = E\left(\max_j\{P_{i,j} - a_n d_{ij} + h_{jn}\}\right) \leq E\left(\max_j\{P_{i,j} - a_n d_{ij} + h_{jn+1}\}\right)$$

which follows from the inequality $h_{in+1} \geq h_{in}$ of the updating Step 2. Thus α_n is feasible for the problem in Step 1; hence its solution, α_{n+1}, provides an equal or higher value than α_n. In the case of equality of a_n and a_{n+1}, it can be shown that using for *h* the average of h_{in} and h_{in+1} will give a strict increase in *a*, see Appendix A. In Appendix A we also provide several other results needed to establish the correctness of the algorithm and the validity of the verification procedure in the stochastic approximation method.

4. Stochastic Freight Rates – Markovian Freight Market States

An extensive theory exists about the maritime freight market. In particular several continuous state Markovian - stochastic differential equation models have been used, see Dixit and Pindyck (1994), and the appealing geometric mean recurrent process models in Tvedt (1997), (2003). However, the freight market's explosive rise up to 2008 followed by its equally dramatic fall might provide grounds for a consideration of simpler models. In this vein, we assume that the charter market (or for a particular sector, say bulk carriers of a certain type) can be in one of a small number of states indexed by *k* or *l*.

The freight rates observed between various ports are assumed to be independent random variables whose distributions depend on the prevailing market state k, i.e. we observe rates $\tilde{P}_{i,j}^{k}$ which have a density $g_{i,j;k}$ but are otherwise independent. We model the freight market as a continuous time Markov chain model. The freight market will thus remain in state k for a random time interval governed by an negative exponential distribution of parameter λ_k and will then move to a different state l with probability p_{kl}, with $p_{kk}=0$. Thus the expected dwell time in state k is $1/\lambda_k$. The transition probabilities $P\{Z(t)=l|Z(0)=k\}=P_{kl}(t)$, $Z(t)$ being the state of the process at time t, follow the Chapman Kolmogorov equations, again see Ross (1970)

$$\frac{dP_{kl}}{dt} = \lambda_k\left(\sum_{m\neq k} p_{km} P_{ml}(t) - P_{kl}(t)\right) \quad (13)$$

For constant p, λ's these are linear and can be explicitly solved.

Let us return to the problem of economic speed. In an infinite horizon and using the average profit criterion, the dynamic programming equation is again in terms of the average profit per unit time alpha. This time though, the port parameters h should include the market state. We consider port – market state parameters $h_{i,k}$ and say $h_{1,1}=0$ but not necessarily $h_{1,k}=0$ for k different from 1. The dynamic programming equation is

$$h_{i,k} = E\left\{max\left(\max_{j\neq i,v}\left[\tilde{P}_{i,j}^{k} - f_{i,j}(v) - \alpha\tau_{ij}(v) + \sum_l P_{kl}\left(\tau_{ij}(v)\right)h_{j,l}\right]; -\alpha\tau_w + \sum_l P_{kl}(\tau_w)\, h_{i,l}\right)\right\}$$

$$for\ \ i = 1,2,..,N\ , k = 1,2,..,M \text{ and } h_{1,1} = 0 \quad (14)$$

The interpretation of (14) is straightforward. At port i, state k one observes the rates $\tilde{P}_{i,j}^{k}$ and selects destination j and speed v_{ij}. Upon arriving at destination j after time $\tau_{ij}(v)$ the process has moved to state l with probability $P_{kl}\left(\tau_{ij}(v)\right)$ and thus at location i the transition to j has an expected locational benefit equal to $\sum_l P_{kl}\left(\tau_{ij}(v)\right)h_{j,l}$. The vessel can opt to wait at i for τ_w time units, after which there is a new observation of the rates, at possibly a new market state. A proof for (14) follows the standard dynamic programming arguments and is similar to Theorem 7.6 in Ross (1970).

To determine the optimal speed once destination j has been selected we differentiate the expression inside brackets in (14). For ease of exposition, if we neglect port times the optimality condition becomes

$$-f'_{i,j}(v) - \alpha\tau'_{ij}(v) + \sum_l \frac{dP_{kl}(\tau)}{d\tau}\tau'_{ij}(v)\, h_{j,l} = 0$$

Primes denote differentiation by v in the case of f, τ. The derivatives of the transition probabilities are determined by the Chapman Kolmogorov equations, and hence in principle the speed equations can be solved. These equations do not provide an explicit expression for the optimal speed, since the speed appears as an argument in the derivative of P_{kl}, and hence of the P_{kl} themselves. Even when we have solved the Chapman Kolmogorov equations, as we will do in a simple example, the P's will be of an exponential form, leading to an implicit expression for the speed in contrast to the usual explicit economic speed formulae.

We consider two important and common market situations, and refer to them as the **Steady Market Case** and as the **Volatile Market Case**. The **Steady Market Case** is when the market state is unlikely to change during any particular voyage. This will happen provided $\tau_{ij} \ll 1/\lambda_k$ for any reasonable speed and all market states k. The **Volatile Market Case** is when the opposite holds, $\tau_{ij} \gg 1/\lambda_k$.

Consider the **Volatile Market** case first. The transition probabilities are then essentially independent of the voyage speed and equal the steady state transition probabilities $P_{kl}(\infty)=P_{kl}$; thus their time derivatives vanish. The optimal speed is independent of the state and is, as before, equal to $v_{i,j} = \left(\frac{\alpha}{2\gamma_{i,j}}\right)^{1/3}$. Ignoring the possibility of waiting at the same port, the dynamic programming equation simplifies to

$$h_{i,k} = E\left\{\max_j\left[\tilde{P}_{i,j}^k - 3d_{ij}\gamma_{ij}v_{ij}^2 + \sum_{l\neq k} P_{kl}h_{jl}\right]\right\}$$

$$for\ \ i = 1,2,..,N\ , k = 1,2,..,M \text{ and } h_{1,1} = 0 \qquad (15)$$

The speed is constant in case the daily fuel cost is the same for all voyages. This lack of dependence of speed on the market state is reasonable if the market is so volatile that it is expected to change radically by the end of the voyage.

Consider now the **Stable Market Case**. From the theory of continuous time Markov chains it is known that, see Ross (1970)

$$\frac{dP_{kl}}{d\tau}(0) = \lambda_k p_{kl} \quad for\ k \neq l$$

$$\frac{dP_{kk}}{d\tau}(0) = -\lambda_k \quad for\ all\ k$$

For voyage times which are small relative to the state dwell times we can take the above expressions as approximations for the derivatives at the voyage time τ and hence the speed optimality conditions, assuming $f_{ij}(v)=\gamma_{ij}d_{ij}v^2$, $\tau_{ij}=d_{ij}/v$ become

$$-2\gamma_{ij}v + \frac{1}{v^2}\left[\alpha - \lambda_k\left(\sum_{l\neq k} p_{kl}h_{jl} - h_{jk}\right]=0\right.$$

The optimal speed is thus of the form encountered before, namely $v_{i,j;k} = \left(\frac{\alpha_{j;k}}{2\gamma_{i,j}}\right)^{1/3}$, where

$$\alpha_{j,k} = \alpha - \lambda_k \left(\sum_{l \neq k} p_{kl} h_{jl} - h_{jk} \right).$$

The voyage speed depends on the market state, the origin and the destination. Even in the case of uniform daily specific fuel cost there will be a dependence of speed on the state and also on the destination, unlike the previous cases where the speed was constant for all voyages. We expect this dependence to be slight for the usual parameters in maritime applications, since it is caused by the variation in the likelihood of a change of market state in voyages of different lengths which is indeed negligible in practice.

The expression for the optimal speed and in particular its nominator can be interpreted as follows. In a good state k, the sum $\sum_{l \neq k} p_{kl} \left(h_{jl} - h_{jk} \right)$ is negative and thus $a_{j,k}$ is greater than the average net profit rate a, the difference being more marked for large λ_k, i.e. small dwell times; consequently the economic speed is higher than average. We might interpret this increased speed in a good state as an effort to take advantage of the good times while they last. Conversely, when the state is bad, one tends to slow down so that the transition to a better market becomes more likely.

To derive the approximate dynamic programming equation for stable markets, we express the transition probabilities as $P_{kl}(\tau)=\lambda_k p_k \tau$ for $k \neq l$ and $P_{kk}(\tau)=1-\lambda_k \tau$, since $P_{kl}(\tau) \approx P_{kl}(0)+P'_{kl}(0)\tau$ and $P_{kl}(0)=\delta_{kl}$. Then equation (14) specializes to

$$h_{i,k} = E\left\{ max \left(\max_{j \neq i.v} \left[\tilde{P}_{i,j}^{k} - d_{ij}\gamma_{ij}v^2 - \left[\alpha - \lambda_k \sum_{l \neq k} p_{kl}\left(h_{jl} - h_{jk}\right) \right] \frac{d_{ij}}{v} + h_{j,k} \right], - \left(\alpha - \lambda_k \sum_{l \neq k} p_{kl}\left(h_{il} - h_{ik}\right) \right) \tau_w + h_{i,k} \right) \right\}$$

$$for\ \ i = 1,2,..,N\ , k = 1,2,..,M \text{ and } h_{1,1} = 0 \qquad (14')$$

Using the expression for the optimal speed v_{jk} and simplifying we get from (14′)

$$h_{i,k} = E\left\{ max\left(max_{j \neq i}\left[\tilde{P}_{i,j}^{k} - 3d_{ij}\gamma_{ij}v_{jk}^{2} + h_{j,k} \right], - \alpha_{i,k}\tau_w + h_{i,k} \right) \right\}$$

$$for\ \ i = 1,2,..,N\ , k = 1,2,..,M \text{ and } h_{1,1} = 0 \qquad (16)$$

Alternatively, this can be written in terms of the a_{jk} as

$$h_{i,k} = E\left\{ max\left(max_{j \neq i}\left[\tilde{P}_{i,j}^{k} - \tfrac{3}{2}d_{ij}(2\gamma_{ij})^{\frac{1}{3}} a_{jk}^{\frac{2}{3}} + h_{j,k} \right], - \alpha_{i,k}\tau_w + h_{i,k} \right) \right\}$$

$$for\ \ i = 1,2,..,N\ , k = 1,2,..,M \text{ and } h_{1,1} = 0 \qquad (16')$$

The solution of these expressions characterizes the optimal speeds. They can be solved by a modification of the methods stated earlier, stochastic approximation and quasi value iteration. We will show numerical results in Section 6.

In the case where the state dwell times are large, it is shown in Appendix B that the solution can be approximated by the solution to M problems with independent freight rates, one for each state. These dynamic programming equations are

$$h_{i,k} = E\left\{\max_{j,v}\left[\tilde{P}_{i,j}^{k} - f_{i,j}(v) - \alpha_k \tau_{ij}(v) + h_{j,k}\right]\right\}$$

$$for\ \ i = 1,2,..,N\ , k = 1,2,..,M \text{ and } h_{1,k} = 0$$

They are decoupled in the sense that the a_k, h_{ik} values do not depend on a_m, h_{im} for $k\neq m$, and we need to solve M problems in N variables each rather than one in MN variables. Consequently, the speed depends on the state; for uniform fuel cost it is the same for the voyages in the same market state.

We will need for the numerical examples in Section 6 the analytic solution of a two market state model, consisting of a good and a bad state, indexed by g and b respectively. The parameters are the state dwell times λ_g^{-1}, λ_b^{-1}, the transition probabilities p_{gb}, p_{bg} being unity. The solution of the Kolmogorov equation gives

$$P_{gb}(t) = \frac{\lambda_g}{\lambda_g+\lambda_b}\left(1 - e^{-(\lambda_g+\lambda_b)t}\right) \quad P_{bg}(t) = \frac{\lambda_b}{\lambda_g+\lambda_b}\left(1 - e^{-(\lambda_g+\lambda_b)t}\right)$$

The optimal speed is determined by the condition

$$-f'_{i,j}(v) - \alpha\tau'_{ij}(v) + \sum_l \frac{dP_{kl}(\tau)}{d\tau}\tau'_{ij}(v)\, h_{j,l}=0$$

This specializes to the equations – with $\lambda=\lambda_g+\lambda_b$

$$2\gamma_{ij}\left(\bar{v}_{i,j}^{g}\right)^3 = \alpha - \lambda_g\left(h_{jb} - h_{jg}\right)e^{-\lambda d_{ij}/\bar{v}_{i,j}^{g}} \quad \text{for the good state and}$$

$$2\gamma_{ij}\left(\bar{v}_{i,j}^{b}\right)^3 = \alpha - \lambda_b\left(h_{jb} - h_{jg}\right)e^{-\lambda d_{ij}/\bar{v}_{i,j}^{b}} \quad \text{for the bad state}$$

This is an implicit expression for the optimal speed. In contrast to the approximate expressions, speed depends on the distance parameter as well. The numerical determination of the optimal speed is not difficult since the expressions are already of a form $v=f(v)$ and a simple iterative scheme of the form $v_{n+1}=f(v_n)$ is effective. To solve the problem completely for the a, h values, the expressions of the optimal speed must be substituted in the dynamic programming equations. As we show in Section 6, the stochastic approximation scheme succeeds in obtaining numerical answers, and these answers are in agreement with the approximation results derived earlier.

5. Discounted Profit Models

In this section we state the previous models in a discounted profit framework. The resulting equations are in the same spirit as before, and give the same qualitative results for small discount rates, although the equations are more complicated and the optimal speed expressions are not as easy to interpret. A discounted deterministic problem of a graph traversal with speed selection like the one presented in Section 2 can be solved using dynamic programming as in Section 2.2. Using the same terminology, the dynamic programming equation can be written in terms of the optimal infinite horizon discounted net profit starting at port i, V_i, r being the discount rate:

$$V_i = \max_{j \neq i} \max_{v} \left\{ P_{i,j} - \gamma_{i,j} d_{i,j} v^2 + e^{-r d_{ij}/v} V_j \right\} \qquad i = 1,2,..,n \qquad (17)$$

The optimal speed satisfies the implicit relation $v^3 = \frac{r e^{-r d_{ij}/v} V_j}{2\gamma_{i,j}}$.

For small rates of interest we expect the above equation to reduce to the ones we derived for the average profit criterion. Indeed, one can verify by direct computation that if α, h are the solutions of the average profit problem as stated in (6), the expression $\bar{V}_i = h_i + \frac{\alpha}{r}$ approximately satisfies (17), the approximation improving as r decreases.

For independent stochastic rates as in Section 3 and a discounted profit criterion, the dynamic programming equation corresponding to (10) is

$$V_i = \ Emax \left[\max_{j \neq i} \max_{v} \left\{ \tilde{P}_{i,j} - \gamma_{i,j} d_{i,j} v^2 + e^{-r d_{ij}/v} V_j \right\};\ e^{-r t_w} V_i \right] \quad i = 1,2,..,N \qquad (17')$$

Again for small discount rates r the solution of (17') can be approximated by the form $\bar{V}_i = h_i + \frac{\alpha}{r}$ with h, a the solution of (10). The solution of (17') can be obtained by stochastic approximation, policy or value iteration. Calculations are presented in Section 6 which confirm the above statements.

The optimal speed satisfies the implicit expression shown earlier from which the current freight rate is absent, although it influences the speed indirectly through the choice of destination. This leads to the following paradox: Assume that a high freight rate is observed for a destination j of small value V_j, and thus this destination is selected, the voyage being however implemented at a low speed, in contrast to the principle that profitable voyages should be traversed at high speeds. Conversely, a small freight rate might lead to the selection of a high valued destination and thus the voyage is carried at high speed.

For the discounted profit version of the market state models in Section 4, one can write by inspection an optimality condition analogous to (14). Ignoring for simplicity the possibility of waiting, the equation becomes

$$V_{i,k} = E \left\{ \max_{j,v} \left[\tilde{P}_{i,j}^k - f_{i,j}(v) + e^{-r d_{ij}/v} \sum_l P_{kl}\left(d_{ij}/v\right) V_{j,l} \right] \right\}$$

$$for\ \ i = 1,2,..,N\ , k = 1,2,..,M \qquad (18)$$

The optimal speed corresponding to (18) from origin *i*, destination *j* and state *k* is given by the expression

$$v^3 = \frac{e^{-rd_{ij}/v}}{2\gamma_{i,j}} \sum_l \left(rP_{kl}\left({d_{ij}}/{v} \right) - P'_{kl}\left({d_{ij}}/{v} \right) \right) V_{jl}$$

In the calculations shown in Section 5, a two state example was explicitly solved for the transition probabilities and thus the optimal speed could be computed exactly. For small r it can be shown by standard methods that (18) reduces to the average profit model equation (14) with approximate solutions $\bar{V}_{ik} = h_{ik} + \frac{\alpha}{r}$. An approximation to the optimal speed for small voyage times can be obtained using the Kolmogorov equations by solving the equation

$$v^3 = \frac{1}{2\gamma_{ij}} \left[\alpha - e^{-rd_{ij}/v} \lambda_k \left(\sum_{l \neq k} p_{kl} V_{jl} - V_{jk} \right) \right]$$

The parameter α is the daily profit introduced in the average profit model while the p_{kl} are the transition probabilities of the continuous time Markov chain.

To solve the discounted problem equation (18) one can use a stochastic approximation algorithm:

$$V_{ik}^{n+1} = V_{ik}^n + \eta_n \left\{ \max_{j,v} \left[\tilde{P}_{i,j}^k - f_{i,j}(v) + e^{-rd_{ij}/v} \sum_l P_{kl}\left(d_{ij}/v\right) V_{il}^n \right] - V_{ik}^n \right\} \quad for \;\; i = 1, .., n$$

In the discounted case there is no special treatment of any particular state as was necessary in the undiscounted case where we arbitrarily set $h_1=0$. Upon convergence of the stochastic approximation algorithm to say U_{jk}, a verification step can be performed by computing the sample mean of the random variable

$$\max_{j,v} \left[\tilde{P}_{i,j}^k - f_{i,j}(v) + e^{-rd_{ij}/v} \sum_l P_{kl}\left(d_{ij}/v\right) U_{j,l} \right],$$

The result should then be compared to the candidate solution $U_{j,k}$.

In a sense, the stochastic approximation algorithm is analogous to value iteration; the analog of a policy iteration algorithm could be carried out as follows. First start with arbitrary values V^n_{ik}, $n=0$. Then find by stochastic approximation the value of the policy that is based on selecting voyages and speed by the expression:

$$(j^*, v^*) = arg \left\{ \max_{j,v} \left[\tilde{P}_{i,j}^k - f_{i,j}(v) + e^{-rd_{ij}/v} \sum_l P_{kl}\left(d_{ij}/v\right) V_{il}^n \right] \right\}$$

The stochastic approximation algorithm used to find the value of the above policy is of the form

$$W_{ik}^{n+1} = W_{ik}^n + \eta_n \left\{ \left[\tilde{P}_{i,j*}^k - f_{i,j*}(v^*) + e^{-rd_{ij}/v^*} \sum_l P_{kl}\left(d_{ij*}/v^*\right) W_{il}^n \right] - W_{ik}^n \right\} \quad for \;\; i = 1, .., n$$

Upon convergence of W^n_{ik} to say W^{∞}_{ik} we set $V^{n+1}_{ik}=W^{\infty}_{ik}$ and repeat. It can be shown with standard policy iteration arguments that we obtain thus an increasing sequence of value functions. A similar policy iteration algorithm can be used in the undiscounted case.

6. Computational results

The computations presented in this Section are meant as a proof of concept rather than as efficient calculations suitable for large scale models. Vessel parameters are inspired by the HandyMax type of bulk carriers; see the description of a HandyMax Bulk Carrier that appears in the site of the shipyard Brodosplit Inc. http://www.brodosplit.hr/Portals/17/Bulk.pdf. Nominal speed for this type of vessels is about 15 knots, with a daily consumption of 30 tons. At mid-2014 prices of 600 USD for marine fuel this is about 18 thousand USD daily. These fuel prices were way above historical averages, so we took the γ parameter to be 12, 15 or 18 and 20 thousand USD per day. However, most of our examples are at the currently reasonable value of 12. Speed will increase substantially in the future following a drop in fuel price provided rates improve, and this will incidentally act as an increase in the supply of shipping as pointed out by maritime economists – see Stopford (2008). We count voyage time in days at nominal speed, most voyages being of the order of 10 days. In 10 days a vessel will cover about 3500 miles, the distance from Australia to Japan. Speed is presented as a fraction u of 14 knots, i.e. a speed of $u=0.85$ means *11.2* knots. Operation constrains on speed, upper or lower, will not be taken into account.

We consider a four origin destination world. The distances between them expressed in days at sea at nominal speed are given in Table 1, and are used in all examples. These distances are not symmetric. Non symmetric distances might be caused by ocean currents, prevailing weather or other voyage conditions. We ignore port time. As stated in Sections 3 and 4 when rates are stochastic, we allow a vessel to wait at any port for an interval τ_w to get a new freight rate observation which again might be accepted or turned down to wait for another τ_w interval, and so on. We arbitrarily set τ_w to 10 days.

Table 1

Distances in days at nominal speed

Destination / Origin	1	2	3	4
1	10	10	10	6
2	8	10	6	10
3	10	7	10	9
4	7	8	8	10

6.1 A deterministic example

The following example uses the parameters in a previous presentation by one of the authors, Magirou (2012). Rates are deterministic, constant and are shown in Table 2. We want to determine an optimal cycle of voyages, where choice of speed is possible.

Table 2

Rates in thousand USD/Day

Destination / Origin	1	2	3	4
1	-	35	26	17
2	15	-	15	40
3	20	25	-	30
4	20	0	20	-

We solved the example by the iterative methods outlined in Section 2.2 and 3.2 for various fuel costs, fuel price being the same at all ports. The optimal speed as a function of fuel price is shown in Table 3. The optimal voyage cycle is 1-2-4-1 (in case a vessel is at port 3 it should go to port 2 and follow the cycle thereafter) and is independent of fuel price, as expected from the comments following equation (9). The results verify the inverse square root dependence of speed on fuel cost. Similar results have been obtained for changes in freight rates.

Table 3

Optimal Speed -Nominal speed for $u=1$ is *14* knots

Case No.	1	2	3	4
Fuel cost parameter ’000 USD/Day	10	12	15	20
Optimal Daily Net Profit ’000 USD/Day	23.03	21.03	18.81	16.28
Relative speed u	1.05	0.96	0.86	0.74
Absolute speed in knots v	14.7	13.4	12.0	10.4

The numerical calculations for the h’s (which we do not show) confirm equation (9): the h values are $h_1=0$, $h_2=-20.4$ $h_3=-76.1$ and $h_4=-90.7$ and are independent of the fuel cost parameter.

We also calculated the optimal routes in an infinite horizon discounted profit cost model as in Section 5, equation (17). The results of the computations are shown in Table 4 and are consistent with those of the average profit models, since the discounted profit value functions are well approximated by the expression $V_i = a/r + h_i$, for reasonable values of the interest rate r. In Table 4 we show the location values for various interest rates, using the same voyage values as before and fuel cost at 12 thousand USD per day. The calculations confirm the results of Section 2. The optimal speed depends in principle on the origin - destination pair, the interest rate and other parameters. However, for the parameter values in the example this dependence is insignificant and the optimal speed is almost identical to the one in the average profit models.

Table 4 - Discounted Profit Model Location Values

r	10%	5%	1%
V_1	76,894.59	153,646.5	767,662.5
V_2	76,874.26	153,626.2	767,642.2
V_3	76,818.42	153,570.4	767,586.4
V_4	76,803.72	153,555.7	767,571.8
V_2- V_1 *i.e.* h_2	-20.33	-20.35	-20.36
V_3- V_1 *i.e.* h_3	-76.16	-76.13	-76.11
V_4- V_1 *i.e.* h_4	-90.87	-90.80	-90.75
rV (approximate)	21.1	21.0	21.0
Approx. Optimal speed – all voyages	96%	96%	96%

6.2 Stochastic models – Independent Rates

We ran the same four origin-destination example allowing a stochastic variation in the rates. The rates observed from origin i to destination j are the ones given in the tables of the previous Section, multiplied by a zero mean random variable whose realizations are independent among voyages – although including a dependence among the R_{ij}'s from the same origin for different destinations j would have been more realistic. The actual form used in our examples is the random daily rate $R_{ij} = R_{ij}^{nom} \cdot (1 + \mu_{ij}\tilde{\epsilon})$. The term μ_{ij} is a variability parameter while $\tilde{\epsilon}$ is a random variable uniform in [-1,1]. The total freight P_{ij} is R_{ij} multiplied by the nominal distance d_{ij}. The corresponding dynamic programming equations are (10), (11) for the average profit case and (17') for the discounted profit case. These equations were solved by the methods outlined in Section 3.2, i.e. stochastic approximation, policy iteration, quasi value iteration. The solutions obtained were then verified by simulation in two ways: in the first verification method we substituted the proposed solution in the right hand side of the corresponding equation, estimated the expected value by simulation and then compared it to the left hand side. In the second verification, we simulated the voyage policy implied by the solution and then verified that the profits obtained were indeed those corresponding to the proposed solution.

For a uniform rate variability parameter $\mu=\mu_{ij}$ (same for all voyages) equal to 50%, the results are as follows

Table 5 Optimal Speed - Stochastic, Independent Freight Rates, Variability 50%				
Case No.	1	2	3	4
Fuel cost parameter (th.USD/Day)	10	12	15	20
Optimal Daily Net Profit (th.USD/Day)	24.04	21.95	19.63	17.00
Relative speed u	1.06	0.97	0.87	0.75
Absolute speed v in knots	14.9	13.6	12.2	10.5

Although the freight rate variability is high, the results are close to that of the deterministic case as shown in Table 3. The average daily profits are slightly higher, and so is the speed. This higher expected profit rate is due to the possibility to choose the best of the observed freight rates, and the option to wait, choices that were absent in the deterministic case. For smaller variability in the rates the profit improvement is negligible. Note that the relative port values are the same regardless of the fuel costs, a somewhat counterintuitive conclusion that is due to the assumption of uniformity in fuel costs. The location values are close to the deterministic case, as shown in Table 6.

Table 6 Location values				
	h_1	h_2	h_3	h_4
Deterministic Freight Rates	0	-20.4	-76.1	-90.7.
Stochastic Freight Rates	0	-22.5	-57.9	-94.1

We also solved the discounted profit models corresponding to a daily fuel cost of 12 thousand USD, and various interest rates. The results are consistent with the deterministic and the average profit case as seen in Table 7. The policy parameters (the daily profits rate alpha and the location values h) are approximately the same. Of course the voyage selection process is quite different in the stochastic case, since it is the observed freight rates that determine the voyage to be undertaken. The overall conclusion, probably important for applications, is that the parameters obtained by a simple deterministic model with average profit optimization might not change significantly when uncertainty is introduced.

In all models in this Section the computed optimal speed was to a good approximation independent of the actual observed freight, but depending of the overall freight rate level and the fuel cost. The dependence is an inverse square root one on fuel cost and also a square root dependence on the overall freight rate level.

Table 7 Discounted Profit Model Location Values Stochastic, Independent Freight Rates			
r	10%	5%	1%
V_1	80,484.3	160,930.8	804,821.9
V_2	80,462.5	160,911.0	804,800.7
V_3	80,426.3	160,873.8	804,764.2
V_4	80,390.3	160,837.1	804,728.3
V_2- V_1 *i.e.* h_2	-21.7	-19.8	-21.1
V_3- V_1 *i.e.* h_3	-58.0	-57.0	-57.7
V_4- V_1 *i.e.* h_4	-94.0	-93.7	-93.6
rV – same for all locations	22.05	22.03	22.05
Optimal speed – all voyages	97%	97%	97%

6.3 Stochastic models – Markov Process Freight Rates

The model developed in Section 4 was that of a Continuous Time Markov chain freight market. We showed in Section 4 a two state model, with a good and a bad market state, for which the transition probabilities were computed explicitly; we now present some computations for that model. The nominal daily freight rates for the bad market state are those used in the deterministic example and shown in Table 2. In a good market the rates are assumed twice those of the bad state rates. The stochastic variations are those of the uniform market case.

The computations for the average time criterion are shown in Table 8. We varied the expected dwell time at states "bad", $T_b=\lambda_b^{-1}$, and "good", $T_g=\lambda_g^{-1}$, keeping the fuel cost at $\gamma=12$ thousand USD daily. The calculated average profit *a* depends on the relative lengths of stay in the two states. The *h* value differences among ports are roughly the same for a given state, while there is a jump in the *h* values corresponding to a state change. The speed differs significantly with the state, approximately by a factor of √2, reflecting the uniform doubling of rates from the bad to the good market state.

Table 8 Average time criterion												
Case	T_b	T_g	A	h_{2b}	h_{3b}	h_{4b}	h_{1g}	h_{2g}	h_{3g}	h_{4g}	v_b %	v_g %
1	10	1	25.6	-21.0	-56.7	-93.1	13,497	13,456	13,383	13,310	97.2	136.8
2	5	1	28.7	-21.7	-57.2	-93.1	12,227	12,183	12,109	12,037	97.2	137.3
3	2	1	35.3	-21.9	-57.6	-93.9	9,841	9,833	9,736	9,669	97.0	137.4

The calculations were done by the stochastic approximation method, implemented in a simple spreadsheet. The method required manual intervention to converge. Once convergence was achieved, we verified the computations as stated earlier: First by simulating the right hand side of the dynamic programming equation (14) for the given values of α and h, and verifying that it is close to the corresponding h value. Second, we generated realizations of the rates, choose voyages by the policy implied by the h, α parameters and computed the average net profit for a "long sequence" of voyages. These simulations confirmed the values obtained to a reasonable accuracy.

Similar results were obtained for the Markovian market state discounted profit models of Section 5. For the same vessel, freight, distance etc. parameters we solved the relevant equation (18) by a stochastic approximation method, and verified the results obtained by the same methods. The results are in Table 9 and are consistent with those of the average time model. Even at the high interest rate of 10%, the approximate value $V=a/r+h$ is valid. The speed again is in principle dependent on origin, destination and distance but for the parameters of the example it practically depends on the state only, just as in the average time criterion models.

Table 9 Discounted Profit Criterion r=10%												
Case	T_b	T_g	V_{1b}	V_{2b}	V_{3b}	V_{4b}	V_{1g}	V_{2g}	V_{3g}	V_{4g}	v_b %	v_g %
1	10	1	92,415	92395	92,359	92,321	104,750	104,706	104,639	104,565	97.1	137.0
2	5	1	102,646	102,635	102,588	102,550	114,040	113,999	113,930	113,854	97.2	136.8
3	2	1	125,818	125,793	125,755	125,721	135,140	135,115	135,031	139,950	97.2	136.8

7. Conclusions: Model Extensions, Managerial insights

7.1 Extensions

Using a continuous time Markov Chain to model the charter market index is a plausible approach but which has not been statistically examined. There is extensive literature on modeling the overall charter market using time series methods. There is also extensive use of stochastic differential equation models, assuming that charter market indices are a diffusion, namely $dx_t=f(x,t)dt+s(x,t)dw_t$, with x_t a market index and w_t a Wiener process. For example, Dixit and Pindyck (1994) use such a model to evaluate a ship, taking explicit account of the layup possibility, while Tvedt (1997), (2003) assumes that rates follow a Geometric Mean Reversion process and uses it in a real option evaluation of alternative ship designs. One might consider describing individual freight rates by stochastic differential equations, but such a model would become totally intractable even for a small number of ports.

In the context of this paper, we might assume that the rates P_{ij} are random variables whose density includes a parameter x_t which in turn is a diffusion process. In a dynamic programming formulation for such a problem, the value functions in the discounted profit and the alpha (α),h parameters in the average profit cases will be functions of the continuous state variable x_t and of the location j. Solving them would require computing transition probabilities by the forward Kolmogorov partial differential equations to obtain the transition probabilities $P(y|x,\tau)$ of being at y having been at state x at the beginning of the voyage τ time units earlier, see for instance the PDE for Finance notes by R. Kohn (2011) for a succinct exposition. The dynamic programming equation for discounted profits is stated in terms of $V_i(x)$, the optimal discounted profit when being at location i while the market is at state x:

$$V_i(x) = E\left\{\max_{j,v}\left[\tilde{P}_{i,j}^{x} - f_{i,j}(v) + e^{-r\tau_{ij}(v)}\int P\big(y\big|x,\tau_{ij}(v)\big)\,V_j(y)dy\right]\right\}$$

For the case of average, infinite horizon profit maximization the dynamic programming equation is

$$h_i(x) = E\left\{\max_{j,v}\left[\tilde{P}_{i,j}^{x} - f_{i,j}(v) - a\tau_{ij}(v) + \int P\big(y\big|x,\tau_{ij}(v)\big)\,h_j(y)dy\right]\right\}$$

To solve these equations a stochastic approximation method with a finite number of basis functions, as in Tsitsiklis and Van Roy (1999) could be used.

The formulations in this paper do not address the issue of seasonality, which affects charter rates in superposition with the overall charter market effects. Modelling seasonality would require the introduction of an additional variable to indicate the time of the year, as in Magirou et al. (1997). In the discounted case we would introduce of a function $V_{i,\tau}(x)$, the optimal discounted profit when being at location i at instance τ while the market is at state x. The dynamic programming equation would then be

$$V_{i,\tau}(x) = E\left\{\max_{j,v}\left[\tilde{P}_{i,j}^{x,\tau} - f_{i,j}(v) + e^{-r\tau_{ij}(v)}\int P\big(y\big|x,\tau_{ij}(v)\big)\,V_{j,\tau'}(y)dy\right]\right\}$$

The term τ' in the above equation incorporates seasonality; it stands for $(\tau+d_{ij})\ modT$ where T is the period and τ takes values up to T. We might take discrete values of τ from 0 to T-1, but need a suitable discretization of speed so that voyage lengths are consistent with the discretization.

A further extension would involve introducing a model for fuel prices, so that future prices are a function of the current ones. The state space would have to increase further since now a state consists of a freight market state coupled with a fuel market state. The bunker procurement problem of Besbes and Savin (2009) can also be included in our model as follows (for simplicity consider independent freight rates, no market state, known bunker prices differing at ports): Assume that a vessel of loading capacity C is available for charter at port i with a quantity Q in its fuel hold, and decides to load an extra fuel quantity q at price $p_{F;i}$. Then the payload of the vessel is at most C-Q-q and the revenue is $P_{ij}(C\text{-}Q\text{-}q)$ which is stochastic. If a voyage is undertaken to j at speed v the required fuel is $d_{ij}k_jv^2$. Thus, if the fuel hold's capacity is say H, the refueling quantity q satisfies $d_{ij}k_jv^2 \leq Q+q \leq H$. The value of port i, h_i, should depend on the fuel quantity upon arrival Q, hence $h_i=h_i(Q)$. Then the dynamic programming equation is

$$h_i(Q)=E\max_{j,q,v}[P_{ij}(C\text{-}Q\text{-}q)\text{-}p_{F;i}q\text{-}\alpha d_{ij}/v^2+h_j(Q+q\text{-}k_{ij}d_{ij}v^2)].$$

This is a stochastic problem even in the case of known fuel prices since the stochastic freight is added to the fuel price when deciding about fuel procurement, good freight rates inducing the procurement of just enough fuel for the current voyage. The maximization in this problem is done subject to the constraints on q, and there is no obvious separation of say voyage from speed selection. Still, the optimal speed satisfies $v_{ij}^3=\alpha/(2h_j^{'}k_{ij})$ and is a function of the derivative h'_j of the port value h_j with respect to Q, which is the implicit value of fuel when arriving at the destination port. The fuel procurement policy seems difficult to characterize – it could be an all or nothing policy if the h functions are essentially linear or of the *(s,S)* type if they are piecewise linear. Such procurement problems with stochastic prices and constraints on storage have been dealt in Kalymon (1973), Magirou (1985) (1992), and Golabi (1983) where answers are derived for specific situations.

As stated in Section 3, a richer formulation would be to have the vessel choose from a set of available charters indexed by say c. For each charter we have a profit P_c, a destination $j(c)$ and a payload leading to fuel consumption coefficient k_c. If c is a time charter the fuel cost is undertaken by the charterer and hence k_c is zero. The analog of equation (10) is then

$$h_i = E\max_{c,v}(\tilde{P}_c - p_i^F d_{ij(c)} k_c v^2 - \alpha\tau_{ij(c)}(v) + h_{j(c)}) \quad i = 1,2,..,n \text{ and } h_1 = 0$$

7.2 Managerial Insights

We have had several conversations with maritime industry senior managers; most of them consider our approach interesting, although they feel their activities are too much dependent on details to benefit from analyses that omit even minute aspects. Indeed tramp vessel management is complicated, most of the relevant tradeoffs being difficult to quantify, while decisions are numerous and taken under time pressure by a very small number of operators. A very important aspect in charter selection is taking proper care of seasonality; having the vessel unload at a time and place when nice charters are available nearby, i.e. good vessel positioning, is a key to profitability. As such speed is an important but secondary goal, being subordinate to chartering agreements which stipulate loading and unloading times and other chartering modalities. As shown in the previous subsection, seasonality can be included in our models, at the expense of increased dimensionality.

Other features of our model could be of value to professionals, such as the possibility of analyzing relocation voyages (going in ballast from a port of unloading to a different one); relocation voyages are a common practice, e.g. often a vessel unloads in the Black Sea where back hauls are scarce, then sails unchartered to West Mediterranean expecting a better charter. Time chartering decisions are also important and it is convenient to have an analytic way to assess them. Time charters have an element of risk avoidance; they protect the ship owner from market fluctuations but at the same time do not give him the opportunity to profit from expert positioning. Incorporating the risk features (risk averseness or risk proneness) of the owners would be important in a satisfactory decision support tool.

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9. Appendices

Appendix A

We present indicative proofs of the results stated in Section 3.2

Lemma 1. Assume that there are α, h satisfying for $i=1,2,..,N$ the inequality

$$h_i \geq E\left(\max_j\{P_{i,j} - \zeta d_{ij} + h_j\}\right)$$

Then the average value of any infinite horizon policy is bounded by ζ.

Sketch of Proof: Consider a path $i_1, i_2, .., i_k, ., i_{N-1}$ resulting from any policy *p*. Sum along the path the expression

$$h_{i_{n+1}} - h_{i_n} + P_{i_n, i_{n+1}} - \zeta d_{i_n i_{n+1}}$$

Thus we obtain

$$h_{i_N} - h_{i_1} + \sum_{k=1}^{N} \left(P_{i_k, i_{k+1}} - \zeta d_{i_k, i_{k+1}} \right) = D_N \left[\frac{h_{i_N} - h_{i_1}}{D_N} + P_N - \zeta \right]$$

Here D_N is the total time for all the voyages and P_N the time average revenue. Each summand of the original expression is less than its maximum with respect to the destination and hence its expectation is nonpositive. By the strong law of large numbers (and well behaved random variables *P*) the above sum divided by D_N is also nonpositive, and hence $P_N \leq \zeta$ with probability 1.

Lemma 2. Assume that there are *b, h* and *ε* satisfying for all *i* the inequalities

$$h_i \leq E\left(\max_j \{ P_{i,j} - \zeta d_{ij} + h_j \} \right) \leq h_i + \varepsilon$$

Then the average value of the policy exceeds ζ while the optimal rate is less than $\zeta + \varepsilon / min\{d_{ij}\}$.

Sketch of proof: The proof follows the same line of argument as Lemma 1. We form the same sum along the path implied by the *ζ*, *h* policy. Considering the locations visited infinitely often (the other locations do not count in the limit) and using again the law of large numbers the conclusion follows.

Combining Lemmas 1 and 2 we get:

Proposition: Let the following equation A.1 (equation 9 in the paper) have a solution ζ, h_i

$$h_i = \max_j \{ P_{i,j} - \zeta d_{ij} + h_j \} \quad i = 1,2,..,N \text{ and } h_1 = 0 \qquad \text{(A.1)}$$

For any policy, the limit of the average profit of any policy for an infinite horizon is bounded by *ζ* with probability 1. Conversely, the policy implied by (A.1) attains *α*.

We state a property of the solutions of the equation that leads to a good initial guess for the *h*'s. Consider the deterministic version of the problem and use the expected value of $\tilde{P}_{ij}$, $\bar{P}_{ij}$ as a deterministic rate. The dynamic programming equation for the average revenue problem is $\bar{h}_i = \max_J \{ (\bar{P}_{ij} - \bar{\zeta} d_{ij} + \bar{h}_j \}$ whose solution provides thus an "average" profit $\bar{\zeta}$. This value of $\bar{\zeta}$ is feasible in the stochastic programming problem in Step 1 of the Quasi Value Iteration Algorithm of Section 3.2 since

$$E \max_j \{ \tilde{P}_{ij} - \bar{\zeta} d_{ij} + \bar{h}_j \} \geq \max_j E\{ \tilde{P}_{ij} - \bar{\zeta} d_{ij} + \bar{h}_j \} \geq \max_j \{ \bar{P}_{ij} - \bar{\zeta} d_{ij} + \bar{h}_j \} = \bar{h}_i$$

Therefore we can start the proposed algorithm with the certainty equivalent values, and be certain that there will be an improvement in the average rate. Trivially, using the result mentioned in the previous paragraph, it follows that the optimal *ζ* is greater than $\bar{\zeta}$.

Finally, consider the improvement Step 3 in the Quasi Value Iteration algorithm. Consider a set of h_{in} corresponding to some α_n. The improvement step will provide a set of h_{in+1} and $h_{in+1} \geq h_{in}$ for all i. Let $I_=$ be the i's with $h_{in+1} = h_{in}$ and I_+ those with $h_{in+1} > h_{in}$. If we perform Step 1 with h_{in+1} and there is no improvement in a, this must be due to an equality for some i in I_+. We claim that if we use instead of h_{in+1} the h values $(h_{in}+ h_{in+1})/2$ there will be strict improvement for both i's in $I_=$ and i's in $I+$. For the i, h_i's in $I_=$ this is valid because there is a strict increase in $E\left(\max_j\left\{P_{i,j} - a_n d_{ij} + h_{jn}\right\}\right)$ and the h's in $I_=$ do not change. For the i, h_i's in I_+ we have strict inequality for $h_{i,n}$ in $h_{i,n} \leq E\left(\max_j\left\{P_{i,j} - a_n d_{ij} + h_{jn}\right\}\right)$ and equality in the corresponding relation for $h_{i,n+1}$, so taking the average of the h's gives a strict inequality.

Appendix B

Consider markets described by a continuous time Markov Chain where state dwell times are exponential random variables with parameters λ_k that are small, corresponding to large expected dwell times. Specifically we examine parameters $\lambda_k=\lambda\cdot\mu_k$ with μ_k constant and λ progressively smaller. We also consider the M "decoupled equations" (ignoring the option to wait at port for a better charter) each corresponding to a state k in isolation, namely

$$h_{i,k} = E\left\{\max_{j,v}\left[\tilde{P}_{i,j}^k - f_{i,j}(v) - \alpha_k \tau_{ij}(v) + h_{j,k}\right]\right\}$$

$$for \;\; i = 1,2,..,N\,, k = 1,2,..,M \text{ and } h_{1,k} = 0 \qquad \text{(B.1)}$$

The random variables P_{ij}^k have the same distributions as in the original problem when the market state is k. These equations are of the type considered in Section 3.1, and represent situations where the rates and hence the optimal speeds differ. For every state k, a different optimal speed is valid, depending on α_k and given by the formula (2), $v_{i,j}^k = \left(\frac{\alpha_k}{2\gamma_{i,j}}\right)^{1/3}$. Based on the solution of (B.1) – which is easier to solve than the original MN variable ones – we will construct approximate solutions of the original dynamic programming equations (14′).

Consider a solution of (B.1) $a_k, \bar{h}_{jk}$. Then consider (14') cast in terms of the h's relative to port 1, namely $\Delta h_{jk}=h_{jk}-h_{1k}$. Then $h_{jk}-h_{jl}= \Delta h_{jk}-\Delta h_{jl} + h_{1k}-h_{1\lambda}$ and (14') becomes (ignoring the possibility of waiting at the same port)

$$h_{i,k} = E\left\{\max_j \max_v\left[\tilde{P}_{i,j}^k - d_{ij}\gamma_{ij}v^2 - \left[\alpha - \lambda\cdot\mu_k \sum_{l\neq k} p_{kl}\left[\left(\Delta h_{jl} - \Delta h_{jk}\right) + \left(h_{1l} - h_{1k}\right)\right]\right]\frac{d_{ij}}{v} + h_{j,k}\right]\right\}$$

$$for \;\; i = 1,2,..,N\,, k = 1,2,..,M \text{ and } h_{1,1} = 0 \qquad \text{(B.2)}$$

We consider the solutions h'_{1k} of the linear equations

$$\alpha^k = \alpha - \mu_k \sum_{l\neq k} p_{kl}(h'_{1l} - h'_{1k})$$

These have a unique solution in α, h's since the μ_k are nonzero and provided the transition matrix p_{kl} is nonsingular.

We construct an approximate solution of (B.2) by taking $\Delta h_{jk}=\bar{h}_{jk}$, and setting $h_{1k}= h'_{1k}/\lambda$, resulting in the candidate solution $h_{ik}=\Delta h_{ik}+h_{1k}=\bar{h}_{jk}+h'_{1k}/\lambda$. To verify that it is indeed an approximate solution of (B.2), consider the expression

$$\alpha-\lambda\cdot\mu_k\sum_{l\neq k}p_{kl}\left[\left(\Delta h_{jl}-\Delta h_{jk}\right)+\left(h_{1l}-h_{1k}\right)\right]$$

As the value of λ takes progressively smaller values, its product with the first expression within the sum (which is a constant) will tend to zero, but its product with the second term will equal a^k by virtue of the definition of the h_{1k}'s. Thus we have constructed an ε-approximate solution of (B.2) which, by an argument analogous to that of Lemma 2 in Appendix A gives a policy which is an ε- approximation of the optimal. This construction shows that for large state dwell times the optimal speeds are determined by the decoupled equations (B.1). These conclusions are borne out in our numerical examples.

Appendix C

Consider first a known sequence of voyages $j=1,..,N$ as in Section 2.1, with the speed constraints $v_{mj}\leq v_j\leq v_{Mj}$, the v_{mj}, v_{Mj} being upper and lower speed bounds. We want to maximize the daily net profit in equation (1), repeated here for convenience

$$G(v_1,..,v_N)=\frac{\sum_{j=1}^{N}[P_j-f_j(v_j)]}{T(v_1,..,v_N)}=\frac{\sum_{j=1}^{N}[P_j-f_j(v_j)]}{\sum_{j=1}^{N}\left[t_{Pj}+\frac{d_j}{v_j}\right]} \tag{C.1}$$

We can easily verify by Kuhn Tucker analysis that the optimal speed is given by the expression

$$v_j(a)=\begin{cases} v_{Mj} & \text{for } v_j^*>v_{Mj} \\ v_j^*=\left(\frac{\alpha}{2\gamma_j}\right)^{1/3} & \text{for } v_{mj}<v_j^*<v_{Mj} \\ v_{mj} & \text{for } v_j^*<v_{mj} \end{cases} \tag{C.2}$$

As in Section 2.1, the parameter a is the optimal profit rate. To determine a we use its definition in (1), to obtain the equation $a=G(v_1(a),..,v_N(a))$ which can be solved by say a bisection procedure.

Considering now the more general problem of an optimal cycle on a graph as in Section 2.2 with bounds on speed $v_{m,ij}\leq v_{ij}\leq v_{M,ij}$. One can consider again edge weights

$$w_{ij}(\alpha) = P_{i,j} - \gamma_{i,j} d_{i,j} v_{i,j}^2(\alpha) - \alpha\left(t_{i,j}^p + \frac{d_{i,j}}{v_{i,j}(\alpha)}\right)$$

In this expression $v_{ij}(a)$ is the analog of (C.2) with *i,j* in place of *j*. It can be verified that $w_{ij}(a)$ is decreasing in *a*, and thus the negative cycle – bisection algorithm of Section 2.2 is applicable.

For a dynamic programming point of view, we modify equation (6) of Section 2.2 by introducing the analog of the optimal speed expression (C.2) with *ij* in place of *j*

$$h_i = \max_{j\neq i}\left\{P_{i,j} - \gamma_{i,j} d_{i,j} v_{ij}^2(\alpha) - \alpha\left(t_{i,j}^p + \frac{d_{i,j}}{v_{ij}(\alpha)}\right) + h_j\right\} \quad i = 1,2,..,N \text{ and } h_1 = 0 \qquad \text{(C.3)}$$

Again this equation can be solved for *a, h* by using the procedures in Section 3.2.

A similar approach is valid for the stochastic rate models. In the model with independent rates, equation (10), introducing speed bounds $v_{m,ij} \le v_{ij} \le v_{M,ij}$ will lead to the dynamic programming equation, with $v_{ij}(a)$ as before:

$$h_i = E\left(\left[\max_{j\neq i}(\tilde{P}_{i,j} - f_{i,j}(v_{ij}(\alpha)) - \alpha\tau_{ij}\big(v_{ij}(\alpha)\big) + h_j)\right]\right) \quad i = 1,2,..,n \text{ and } h_1 = 0 \qquad \text{(C.4)}$$

The right hand sides of w_{ij}, (C.3) and (C.4) is nonlinear but decreasing in *a*, and thus the steps of the Quasi Value Iteration Algorithm can be implemented exactly as in the case with a linear parameter. Indeed if $v_{ij}(a)$ is within the allowed bounds, the negative term depends on $a^{2/3}$, and when $v_{ij}(a)$ is outside the bounds the dependence of the negative terms is linear on *a*. Hence the negative term is decreasing with *a*.

We repeated the computations of the example in Section 6.2 - random but independent freight rates. The results without speed bounds were shown in Table 5. For a Fuel Cost Parameter equal to 20 Th. USD/Day the optimal speed was 0.75 of the nominal; for Fuel at 30 Th. USD/Day the optimal speed is 0.62 while the daily net profit is 13.91 Th. USD/Day. However, if the minimum speed is say 0.70 the methodology in this Appendix gives a lower daily profit at 13.51 with speed at 0.70 of the nominal. On the other extreme, a super low fuel cost of 5 Th. USD/Day gives a daily profit of 34.1 Th. USD/Day and speed at 1.50 of nominal. If speed were limited to say 1.30, the daily profit will be only 33.1 Th. USD/Day.