Comment on “Temporal Correlations of the Running Maximum of a Brownian Trajectory”

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Bénichou et al. [1] use the running maximum (RM) position in a single experimental trajectory of a particle exhibiting 1D Brownian motion (BM) to estimate its diffusion coefficient. This is unreliable: While the estimator’s precision (reproducibility) increases with the suggested parameter tuning, so does its inaccuracy (bias), as increasing emphasis is put on the RM’s maximum value.

In the mathematical idealization for BM used in Ref. [1], \( B_t = \sqrt{2D} \), where \( W_t \) is the Wiener process. In this model, BM is a scale-free process.

Experimentally, one samples positions \( x_i \) at time points \( t_i \), typically constant time lapse \( \Delta t \) is used, such that \( t_i = i \Delta t \) and \( T = N \Delta t \). For BM, measured positions relate as \( x_{i+1} = x_i + \sqrt{2D} \Delta t \), where \( \eta_t = W_{t+1} - W_t \) is a Gaussian white noise with \( \langle \eta_t \rangle = 0 \) and \( \langle \eta_t \eta_j \rangle = \Delta t \delta_{ij} \) for all \( i, j \). Each of the \( N-1 \) displacements \( \Delta x_i = x_{i+1} - x_i \) contains information about \( D \); hence, variances of estimators in this discrete case are limited by \( N \), not \( T \), due to the scale invariance of BM.

A reasonable estimator \( \hat{D} \) for \( D \) should (i) be unbiased, i.e., \( \langle \hat{D} \rangle = D \), and (ii) have a variance that decreases as \( N \) increases. For sufficiently large \( N \), \( \hat{D} \) is unbiased, which is indicated (full line). The theoretical variance indeed decreases [Fig. 1(d)]. The bias of \( \hat{D}_{\text{es}}^{(N,k)} \) vanishes too slowly with \( N \) to ensure any practical relevance of \( \hat{D}_{\text{es}}^{(N,k)} \) relative to \( \hat{D}_{\text{es}}^{(N)} \) [Figs. 1(c) and 1(d)]. In summary, the estimator suggested by Bénichou et al. [1] unfortunately yields biased values for the diffusion coefficient, while optimal, plug-and-play alternatives already exist [2,3].

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