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Convex Relaxation of Optimal Power Flow in Distribution Feeders with Embedded Solar Power

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Abstract

There is an increasing interest in using Distributed Energy Resources (DER) directly coupled to end user distribution feeders. This poses an array of challenges because most of today's distribution feeders are designed for unidirectional power flow. Therefore when installing DERs such as solar panels with uncontrolled inverters, the upper limit of installable capacity is quickly reached in many of today's distribution feeders. This problem can often be mitigated by optimally controlling the voltage angles of inverters. However, the optimal power flow problem in its standard form is a large scale non-convex optimization problem, and thus can't be solved precisely and also is computationally heavy and intractable for large systems. This paper examines the use of a convex relaxation using Semi-definite programming to optimally control solar power inverters in a distribution grid in order to minimize the global line losses of the feeder. The mathematical model is presented in details. Further, case studies are completed with simulations involving a 15-bus radial distribution system. These simulations are run for 24 hour periods, with actual solar data and demand data.

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Keywords: Semi-definite programming; Optimal Power Flow; Convex Relaxation; 15 bus test feeder

1. Nomenclature

p_i	Injected active power at bus *i*
q_i	Injected reactive power at bus *i*
p_{ij}	active power flow from node *i* to node *j*
q_{ij}	Reactive power flow from node *i* to node *j*
b_{ij}	Imaginary part of complex admittance matrix
g_{ij}	Real part of complex admittance matrix

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$\underline{p_i}$	Lower limit of active power injection
$\overline{p_i}$	Upper limit of active power injection
$\lvert v_i \rvert$	Voltage magnitude at bus *i*
θ_i	Bus voltage angle at bus *i*
$v_i = \lvert v_i \rvert \angle \theta_i$	Bus voltage at bus *i*
$\boldsymbol{Y}$	Nodal Bus admittance matrix
Y_{ij}	The *ij*th entry of the Nodal bus admittance matrix
$Tr(\cdot)$	The trace operator
$diag(\cdot)$	The diagonal operator, returns diagonal of a matrix
$P_{d,i}$	Active power demand at bus *i*
$Q_{d,i}$	Reactive power demand at bus *i*
$\boldsymbol{I}_f$	Incidence Matrix from nodes
$\boldsymbol{I}_t$	Incidence Matrix to nodes

2. Introduction

There is going to be a radical change in the future distribution systems, due to upcoming needs and possibilities that are created by European and US initiatives such as EU FP7 Ideal Grid for all and US DOE Smart Grid[1]. This transformation from classical line drop compensated distribution grids to modern “Smart Grids” is enabled by the expected integration of modern communications systems, variable renewable energy sources and storage-capable loads such as batteries and grid connected electric vehicles.

Optimal control of the active/reactive power balance in Distributed Energy Resources (DER) and Demand Response Resources (DRR) such as battery storage [2], has become increasingly interesting with the possibility of them being installed and connected directly to the distribution grid(see e.g. [3]–[5]). In the past large arrangements of Wind Turbines (WTs) and Photo Voltaic panels (PVs) have been connected on a feeder of their own in order to ease the issues of over and under voltage and congestion. If DERs are connected directly to the Distribution network (DN), the Distribution System Operator (DSO) will most times require them to run at unity power factor or a fixed power factor, since they can interfere with DSO control measures such as on-load tap changing of the main feeder HV/MV transformers [6].

Also massive penetration of DERs in distribution networks is likely to have an effect on the Voltage limits in the feeder because resistance-to-reactance ratios r/x are such that voltage bus levels are quite sensitive to the active power injections [7], [8]. Therefore, hard limits on the amount of installable DERs are quickly reached due to over- and under-voltage concerns.

A DER on a feeder of a distribution system may even worsen the voltage levels in the system even if it is running at unity power factor [9]. Now, with the decreasing cost of PVs and other DERs such as battery storage, there is increasing interest in connecting them in large amounts to the end consumer distribution network. One of the most important issues of connecting large amounts of DERs to the DN are over- and under-voltage concerns [6], [10].

If DERs and DRRs are connected to the grid through controllable power converters, the opportunity arises to use to provide ancillary services to the grid [11]–[13]. One possibility is to use them to provide reactive power in order to aid the primary voltage control that is usually provided by tap-changing transformers. This will also help improve the lifetime of the on-load tap changing mechanisms, since they are designed for slow intra-hour load changes, and thus only operate a few times a day. When large amounts of renewable based DERs are connected to a feeder, the variations in load are much faster, and therefore a tap changer would have to operate several times an hour, which could severely limit the life span of this expensive hardware. If inverter based DERs can aid the control of fast voltage variations, the cost of tap-changing maintenance could also be reduced. However, in order to make meaningful contributions to the control of the grid, the DERs have to be coordinated in a complex way.

As a way to mitigate the issues of over and under voltage, and at the same time to optimize the power flow of the distribution grid, the system can be modelled with optimal power flow (OPF) equations.

The general description of the optimal power flow equations is a large scale non-linear non-convex problem that can be difficult to solve, especially when considering large systems (see e.g. [14]). Therefore many papers introduce linearized models that can be easier to solve and provide less computational burden, however the solutions provided will be suboptimal in regards to the real system (see e.g. [15]). Semi-definite programming (SDP) has been studied for quite some time as a possible convex relaxation of the OPF equations, and recently it has been proved that convex SPD problems can be used to find global solutions to real distribution systems under specific conditions (see e.g. [7], [16]–[21]).

It has recently been noted in [17] that under certain conditions, the dual problem to the OPF problem which is represented by the SDP problem, achieves a zero duality gap. This has been noted for several of the IEEE standardized test feeders, and for some randomly generated test feeders. Some later works have presented some answers, as to when this convex relaxation has zero duality gap: In [20], [22], [23] it is noted that this is achieved when the network topology is tree-like and some constraints on the bus power injections are satisfied.

This paper presents the use of the convex SDP relaxation for a distribution test feeder and compares the results to the standard non-convex model that is solved by Newton-Raphson method. The paper is structured as following. Section 3 presents the non-convex standard OPF model and the SPD relaxed model. Section 4 presents the study of the test feeder with solar power connected to selected busses and section 5 presents some concluding remarks.

3. Mathematical model

This paper mainly focuses on the modelling aspect of the OPF problem in distribution networks. This section begins with the non-convex non-linear model, that was first used to solve the OPF problem [24]. The NLP OPF model is an NP-Hard model, which means that it cannot be solved in polynomial time by today's solvers. Therefore iterative heuristic methods are mainly applied, which do not guarantee globally optimal solutions to the problem. The model is traditionally solved by newton type solvers which was first applied in [25]. Since many other techniques have surfaced but most of them still rely on a newton-type interior point method, however this won't be discussed any further since the modelling and not solving techniques is center in this paper.

The second part of this section focuses on the convex relaxation. The convex OPF problem is modeled as an SDP problem, where voltage and power flows are modelled via a positive semidefinite matrix.

The distribution networks found today are mainly radial, with a feeder as the single point source of power injection. To get from any point a to point b in radial feeders, there is only one possible way for the power to flow. Therefore the topology of the network can be described as a connected tree, with the edge set denoted by κ. The notation $(i,j) \in \kappa$ denotes the edge that connects note i to node j. If a two nodes are connected the notation $i \sim j$ is used and $i \nsim j$ otherwise.

The standard form of a constrained optimal power flow problem can be formulated as in equations (16) to (7). Equation (16) is some function that is chosen to minimize a meaningful quantity for the system, such as losses or generating costs. For this paper the objective function will simply be total active power loss minimization.

$$\min_{v,p,q} f(v,p,q) \tag{1}$$

$$s.t. \sum_j p_{ij} = p_i \;\; \forall i \tag{2}$$

$$\sum_j q_{ij} = q_i \;\; ,\forall i \tag{3}$$

$$\underline{p_i} \le p_i \le \overline{p_i} \;\; ,\forall i \tag{4}$$

$$\underline{q_i} \le q_i \le \overline{q_i} \;,\forall i \tag{5}$$

$$p_{ij}^2 + q_{ij}^2 \le \overline{s_{ij}^2} \;\; ,\forall (i,j) \in \kappa \tag{6}$$

$$\underline{v_i} \le |v_i| \le \overline{v_i} \;\; ,\forall i \tag{7}$$

3.1. Standard non-convex OPF model

The non-convex OPF model is modeled by calculating the nodal power flows from the angle differences on the busses. The nodal power flows are found as shown in equation (8) - (9)

$$p_{ij} = g_{ij}|v_i|^2 - |v_i||v_j|\left(g_{ij}\cos(\theta_i - \theta_j) - b_{ij}\sin(\theta_i - \theta_j) \right) \tag{8}$$

$$q_{ij} = b_{ij}|v_i|^2 - |v_i||v_j|\left(g_{ij}\sin(\theta_i - \theta_j) + b_{ij}\cos(\theta_i - \theta_j) \right) \tag{9}$$

These equations are non-convex due to the sinusoidal relation of voltage angle variations. An advantage with this formulation is that both bus voltage magnitude and angles are immediately meaningful quantities. This is in contrast to the convex SDP relaxation that will be shown in the next section, where the bus voltage angles are not explicitly present. This can be troublesome if the optimization has to include constraints on the stability of the system.

3.2. Convex relaxation

The convex relaxation in this paper is achieved by the means of semi-definite programming.

Let $\boldsymbol{v} = [v_1\ v_2 \ldots]$ be the vector of bus voltages. Then introduce the Hermitian matrix $\boldsymbol{W} = \boldsymbol{v}\boldsymbol{v}^*$ where $\boldsymbol{v}^*$ denotes the conjugate transpose of $\boldsymbol{v}$. The notation $\boldsymbol{W} \succcurlyeq 0$ implies that $\boldsymbol{W}$ is positive semi-definite.

$$\min_{\boldsymbol{W}\succcurlyeq \boldsymbol{0}} \sum_i Re\{diag(\boldsymbol{Y}^*\boldsymbol{W})\} \tag{10}$$

$$s.t.\ diag(\boldsymbol{W}) = \boldsymbol{v}\boldsymbol{v}^*, \forall i \tag{11}$$

$$\underline{p_i} \leq Re\{diag(\boldsymbol{Y}^*\boldsymbol{W})\} + P_{d.i} \leq \overline{p_i} \quad ,\forall i \tag{12}$$

$$\underline{q_i} \leq Im\{diag(\boldsymbol{Y}^*\boldsymbol{W})\} + Q_{d,i} \leq \overline{q_i} \quad ,\forall i \tag{13}$$

$$abs\left(diag\left(\boldsymbol{Y}_f^*\boldsymbol{W}\boldsymbol{I}_f\right)\right) \leq \overline{s_{ij}} \quad \forall (i,j) \in \kappa \tag{14}$$

$$abs\left(diag\left(\boldsymbol{Y}_t^*\boldsymbol{W}\boldsymbol{I}_t\right)\right) \leq \overline{s_{ij}} \quad \forall (i,j) \in \kappa \tag{15}$$

$$\underline{v_i} \leq |v_i| \leq \overline{v_i} \quad ,\forall i \tag{16}$$

The voltage magnitude $|v_i|$ can be directly extracted from the semi-definite matrix $|v_i|^2 = diag(\boldsymbol{W})$**.** Note that the formulation (10)-(16) is not convex due to equation (11), which defines $\boldsymbol{W}$ as a semi-definite rank-1 matrix. The rank-1 constraint is of non-convex nature. It has been widely argued that this constraint can be dropped and that a necessary and sufficient condition can replace this to guarantee zero duality gap [17]. The objective function in equation (10) is minimizing the active power losses in the system.

4. Case Studies

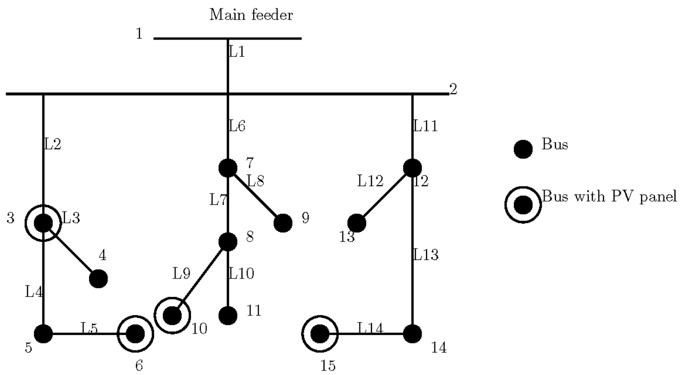

Figure 1: The test feeder used in the case studies, with 15 buses, and 4 PV panels at buses 3,6, 10 and 15

The radial feeder used for the case studies is displayed in Figure 1. It is a radial 15 bus network with solar power at 4 points in the network. The main feeder has not received a limit in the simulations in order to guarantee convergence for the optimization algorithms, i.e. the whole grid can for peak loads be supplied by the main feeder.

The main goal of this case study is to show the results achieved when simulating the system for a 24 hour period with the SDP relaxed model and compare it to the classical NLP model.

In Table 1 the data for the test feeder is summarized for both the lines and bus power peak power demands. The simulations of the convex optimization are run for 24 hour periods with 1 minute resolution; hence the data for PV production and bus loads are also generated with 1 minute resolution. The PV panels at the four selected buses have a nominal power output of 0.1MW each, and the inverters are assumed lossless. In Figure 2 the output power of one of the solar panels can be seen. The solar data is taken from a day in March at the University of Nevada, USA. It is for simplicity assumed that all four PV panels have the same output power. The active power demand curves for some selected buses are shown in Figure 3. The data is available with one minute resolution.

The voltage limits are set to a lower bound of 0.9 p.u. and 1.1 p.u. for the upper bound.

Table 1: The bus load and line data for the test feeder in Figure 1. $S_B = 100MVA$, $V_B = 12.66kV$. Data and test feeder topology from [26].

To bus	*From bus*	*P(MW)*	*Q(Mvar)*	*R(ohms)*	*X(ohms)*	I_{max}
2	1	0	0	1.0e-8	1.0e-8	2500
3	2	2	1.6	0.1202	0.1603	1400
4	3	3	0.4	0.1282	0.1763	500
5	3	2	-0.4	0.1442	0.2885	500
6	5	1.5	1.2	0.0641	0.0641	500
7	2	4	2.7	0.1763	0.1763	1400
8	7	5	1.8	0.1282	0.1763	1000
9	7	1	0.9	0.1763	0.1763	500
10	8	0.6	-0.5	0.1763	0.1763	500
11	8	4.5	-1.7	0.1282	0.1763	500
12	2	1	0.9	0.1763	0.1763	1400
13	12	1	-1.1	0.1442	0.1923	500
14	12	1	0.9	0.1282	0.1763	500
15	14	2.1	-0.8	0.0641	0.0641	500

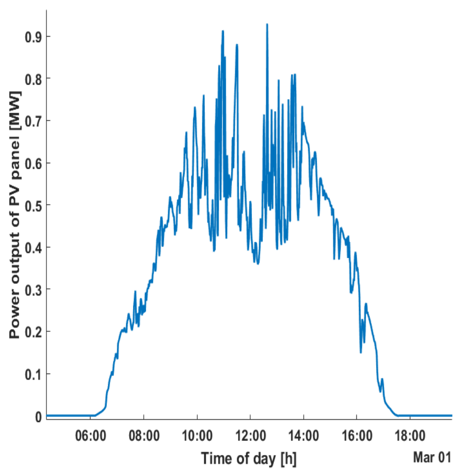

Figure 2: The output power of each of the solar arrays

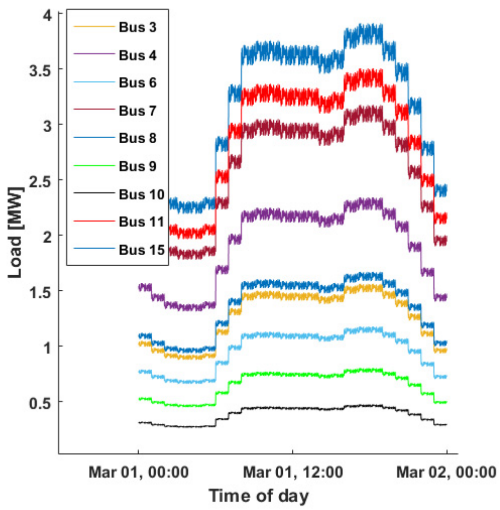

Figure 3: The active power demand of some selected buses during the day

4.1. OPF simulations and comparisons of non-convex and SDP-model

The simulations are all run on a laptop with an Intel Core i5-4300 CPU rated at 1.9GHz and 2.5GHz. The reactive power capabilities of the solar power inverters are assumed to be ideal; i.e. they can absorb and produce reactive power at the full rated power which is 0.1 MVA, and this without losses. The initial conditions for both the NLP and the SDP simulations are set to 0, for both voltages and angles at all busses. As a solver for the NLP model, the Matlab interior point solver is chosen. The SDP model is solved by the commercial solver package Mosek.

As an objective function for both the NLP and SDP model, total active power losses in the lines are being minimized. There are many other measurements, that are more meaningful in reality, such as total generating cost minimization, or social welfare maximization. However, this paper is meant to demonstrate the exactness of the SDP model approach and hence the choice of objective function is of minor importance.

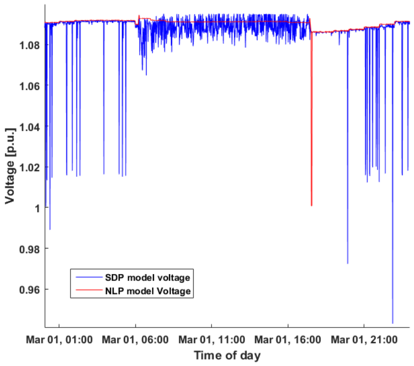

Figure 4: Average voltages at the buses taken from simulation, the voltage limits have been set to 1.1 p.u.

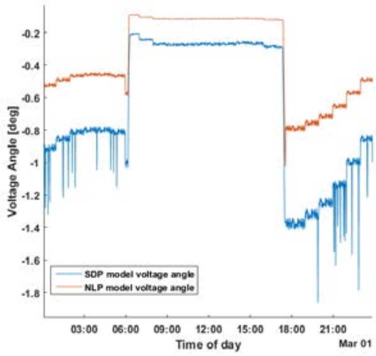

Figure 5: Average voltage angle at the busses.

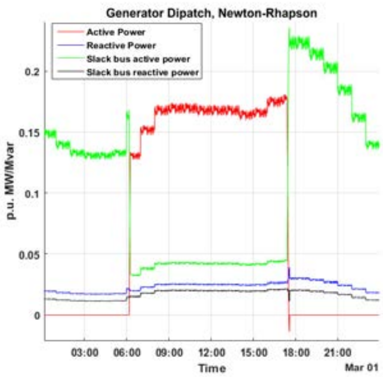

Figure 6: Power injection on slack bus and generator buses when solved by the SDP model

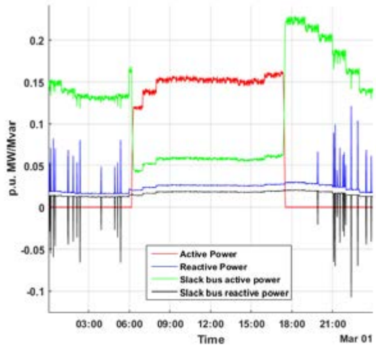

Figure 7: Power injection on slack bus and generator buses when solved with the NLP model

The average voltages at the busses are shown in Figure 4, when solved by the SDP and the NLP models, while the voltage angles found from the two different models are displayed in Figure 5. The angles and voltages have been averaged over all the busses in the feeder. The active/reactive power dispatch of the sum of all the PV-panels and the slack bus, are shown in Figure 6 and Figure 7 for the SDP and NLP models respectively. It can be noticed for all the cases that results in general are quite similar. However there are a lot more minor fluctuations for voltage magnitudes and angles, from the SDP model, and this can be attributed to the choice of solver. Mosek is generally a fast solver for SDP type problems, but can suffer from numerical errors compared to other solvers. It is chosen here because of its general robustness and speed.

The total power produced in a 24 hour period calculated from the NLP model is $11.03MWh$ and the SDP model calculates this number to $10.99MWh$.

5. Discussion and conclusion

The general advantages of convex optimization are the ability to create a process that is tractable even for large systems. The NLP problem formulated in section 3.1 can for larger networks quickly become unmanageable to solve. The convex program achieved by SDP programming, is here shown to agree well with the solutions found from the NLP model. This however should not always be expected, since the solutions found from the NLP model cannot be guaranteed to be globally optimal. In this case the network topology and general problem structure is rather simple such that the NLP model achieved a solution that is very similar to the solution from the SDP model.

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