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Published in: Composites Science and Technology

Link to article, DOI: 10.1016/j.compscitech.2016.01.026

Publication date: 2016

Document Version Peer reviewed version

Link back to DTU Orbit

Citation (APA):

Joki, R. K., Grytten, F., Hayman, B., & Sørensen, B. F. (2016). Determination of a cohesive law for delamination modelling - Accounting for variation in crack opening and stress state across the test specimen width. *Composites Science and Technology*, *128*, 49-57. https://doi.org/10.1016/j.compscitech.2016.01.026

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Determination of a Cohesive Law for Delamination Modelling 1 Accounting for Variation in Crack Opening and Stress State 2 Across the Test Specimen Width 3

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16 Abstract

17 The cohesive law for Mode I delamination in glass fibre Non-Crimped Fabric reinforced vinylester is determined

18 for use in finite element models. The cohesive law is derived from a delamination test based on DCB specimens

19 loaded with pure bending moments taking into account the presence of large-scale bridging and the multi-axial

20 state of stress in the test specimen. The fracture resistance is calculated from the applied moments, the elastic

21 material properties and the geometry of the test specimen. The cohesive law is then determined in a three step

22 procedure: 1) Obtain the bridging law by differentiating the fracture resistance with respect to opening

23 displacement at the initial location of the crack tip, measured at the specimen edge. 2) Extend the bridging law to

24 a cohesive law by accounting for crack tip fracture energy. 3) Fine-tune the cohesive law through an iterative

25 modelling approach so that the changing state of stress and deformation across the width of the test specimen is

26 taken into account. The changing state of stress and deformation across the specimen width is shown to be

27 significant for small openings (small fracture process zone size). This will also be important for the initial part of

28 the cohesive law with high stress variation for small openings (a few microns), but the effects are expected to be

29 smaller for large-scale-bridging where the stress varies slowly over an increase in crack opening of several

30 millimetres. The accuracy of the proposed approach is assessed by comparing the results of numerical simulation

31 using the cohesive law derived by the above method, with those of physical testing for the standard DCB Mode I

32 delamination test (ASTM D 5528).

33

34 Key words: Polymer-matrix composites (PMCs); Delamination; Finite element analysis (FEA)

35 **1 Introduction**

36 The fracture process zone (FPZ) in a delaminating fibre reinforced polymer laminate is 37 usually long in the plane of fracture in comparison with the smallest specimen dimension, i.e. 38 the thickness of the laminate. The reason for the large FPZ is the development of fibre 39 bridging following in the wake of the crack tip. Fibre bridging is beneficial in the sense that it 40 leads to an increased fracture resistance, and thus increases the damage tolerance. Cohesive 41 zone models (CZM) [1, 2] are well suited for modelling this kind of FPZ. A CZM can be 42 implemented by inserting cohesive elements [3] at interfaces where fracture is expected to 43 propagate. Therefore CZM has become a favoured tool for modelling delamination [4-17]. A 44 cohesive law that relates separation of the fracturing surfaces to the traction transferred 45 between them governs the cohesive elements. Since the law relates tractions to separation, it 46 is often referred to as a traction-separation law. A major challenge in the use of CZM in 47 structural design of engineering structures is to characterise the cohesive law of the real 48 fracture process zone of the material or interface. The existing test standards [18, 19] 49 concerned with interface properties of fibre reinforced polymer composites are designed for 50 determining the critical energy release rate, i.e. under the premises of linear-elastic fracture 51 mechanics (small-scale fracture process zone). Within linear-elastic fracture mechanics, the 52 criterion for crack growth is that the energy release rate is equal to the work per unit area of 53 the cohesive tractions and represents the fracture energy associated with a fully developed FPZ [20, 21]. However, linear-elastic fracture mechanics concepts are not applicable for 54 55 large-scale bridging problems; instead cohesive laws can be used for representing the 56 mechanics of fracture, including the energy dissipation at a crack tip and the work of tractions 57 in a bridging zone behind the crack tip. Crack initiation and arrest, and thus the shape of the 58 delaminated area in a composite structure are governed not only by the overall geometry, the 59 loading and the total fracture resistance but also by underlying details of the traction-60 separation law [22]. More reliable procedures for determining the underlying details of the 61 traction-separation law are needed.

62

63 The path independent J integral [20] has been adopted to determine the cohesive laws from

64 experiments [12-14, 23] for plane problems. This has opened for the possibility of measuring

65 the shape of the cohesive law. For large-scale bridging (LSB) problems, the J integral of the

standard DCB specimen loaded with wedge forces can be determined experimentally by

67 measuring the rotations where the forces are applied [24, 25], i.e. requiring more

2

- instrumentation than for the LEFM delamination test. Although with the *J* integral approach 68 69 more instrumentation is needed in a DCB configuration, there is no need to monitor the crack 70 tip position during the test, which is always difficult. The need to measure rotations can be 71 avoided by applying pure bending moments to the test specimens instead of forces [17, 21], 72 since for the DCB loaded with pure bending moments the J integral is given in closed 73 analytical form, independent of crack length and valid also for LSB-problems. In reality, 3-74 dimensional (3D) effects associated with anticlastic bending [26] of the beams in the cracked 75 region lead to inaccuracy when the crack opening is measured at the edge of the specimen; the 76 anticlastic curvature makes the crack opening at the edge of the specimen smaller than that at 77 mid-width, while restraint of the anticlastic bending in the middle region induces variation of 78 the stress state across the specimen width. These effects also cause the longitudinal position of 79 the crack tip to vary across the width of a fracture mechanics test specimen. Then the resulting 80 cohesive tractions will vary across the specimen, in particular in the representation of the 81 crack tip fracture energy, where the cohesive traction is expected to vary from high to near-82 zero over small openings. 83 84 A remaining challenge is to extend the approach for plane problems to 3-dimensional 85 problems and account for the changing state of stress across the width of the specimen. As 86 will be shown later, both material properties and specimen geometry affect the result. 87 88 The objective of the present study is to demonstrate that a cohesive law for 3D finite element 89 implementation can be fitted from experimental test results taking into account the changing crack opening and state of stress across the width of the test specimen. In the study, DCB tests 90 91 using pure bending moments have been carried out on a set of laminate specimens. Attempts 92 have then been made to derive from these tests a cohesive law for Mode I delamination using 93 a modified iterative modelling approach [27-30]. First, the cohesive law for Mode I 94 delamination is obtained using the J integral approach for plane problems [12], which 95 implicitly assumes that the crack opening is the same across the width of the specimen. A 96 simplified, multi-linear cohesive law is then implemented in a three-dimensional finite 97 element model where the parameters describing the cohesive law are defined as variables. 98 These variables are then optimised for the model result to fit the experimental response. The 99 variation in crack opening across the test specimen is then accounted for. Finally, the 100 accuracy of the approach is tested by numerically simulating a separate test, namely the
 - 3

- 101 standardised ASTM double cantilever beam (DCB) specimen [18], using the derived cohesive
- 102 law, and comparing the calculated load-displacement response to that from a corresponding
- 103 physical test. Note that the standardised DCB specimens and the moment loaded DCB
- 104 specimens had different widths and width to height ratios.
- 105

106 A short theoretical background for the use of the J integral and the effect of changing state of 107 stress is presented in the following section.

108 2 The path independent J integral approach

- 109 The path independent J integral was first applied to crack problems by Rice [20] and can be 110 used to calculate the fracture resistance, $J_{\rm R}$, during crack growth.
- 111

For a homogeneous DCB specimen loaded by pure bending moments, an evaluation of the J
integral along the external boundaries of the DCB specimen in Figure 1 c) gives (assuming
plane stress) [31]

115
$$J_{\rm R,ext} = \frac{12M^2}{B^2 H^3 E_{\rm ext}},$$
 (1)

116 where M is the applied moment, B and H are the beam width and height, respectively and E_{11}

117 is the Young's modulus in the x_1 direction. In the present paper, the composite laminates are

analysed as homogeneous beams. This is assumed to be acceptable provided the correct

- 119 bending stiffness is modelled.
- 120
- 121 Evaluating the *J* integral along the edge of the FPZ in Figure 1 a) gives [20, 31]

122
$$J_{\rm R,FPZ} = \int_{0}^{0} \sigma_{\rm FB}(\delta) d\delta + J_{\rm tip}, \qquad (2)$$

123 where
$$\delta^*$$
 is the end-opening of the FPZ, $\sigma_{FB}(\delta)$ is the traction as a function of separation δ
124 along the FPZ associated with fibre bridging and J_{tip} is the *J* integral evaluated around the
125 crack tip.

126

127 Due to path-independence, $J_{R,ext} = J_{R,FPZ}$. At low load levels when $J_{R,ext}$ (or equivalently, $J_{R,FPZ}$)

- 128 is below a certain value, denoted J_0 , no crack growth takes place and $\delta^* = 0$. When the
- 129 external load is increased so that $J_{\text{R,ext}}$ reaches J_0 , the crack will open ($\delta^* > 0$). J_0 is thus the

- 130 crack tip fracture energy. A bridging zone now forms between the initial and the present crack
- 131 tip. The length of the bridging zone is denoted L. With increasing $J_{R,ext}$, the length of the active
- 132 cohesive zone, L, and the end-opening, δ^* , increase as the crack tip advances. When δ^*
- 133 reaches a critical value, δ_0 , the fracture surfaces are completely separated at the end of the
- 134 crack. The FPZ is then fully developed and the fracture resistance attains a constant value,
- 135 denoted the steady-state fracture resistance, which represents work of separation per unit area
- 136 of the cohesive traction. For steady-state specimens, further crack extension will not cause an
- 137 increase in the active cohesive zone length, L, while for other fracture specimens, the active
- 138 cohesive zone length may continue to change [30].
- 139

In equation (2), the traction-separation law represents a bridging law describing the relation
between traction and separation in the wake of the crack tip. When the FPZ is modelled using
cohesive elements the crack tip energy is included in the traction separation law and equation
(2) becomes [13]

$$J_{\rm R,FPZ} = \int_{0}^{\delta^*} \sigma_{\rm CL}(\delta) d\delta, \qquad (3)$$

145 where $\sigma_{\rm CL}(\delta)$ represents the cohesive law.

The relation between the tractions and the opening separation at the crack end can then be obtained by differentiating the external J integral with respect to the end opening of the cohesive zone [23, 31]. By assuming this is representative for the rest of the interface, the cohesive law for the interfaces is given:

150 $\sigma_{CL}(\delta^*) = \frac{\partial J_{R,ext}}{\partial \delta^*}$ (4)

151 In equation (4), $\sigma_{CL}(\delta^*)$ can be understood as the traction acting at the position of the end

152 opening of the cohesive zone. However, assuming the cohesive law is a material property,

153 independent of position, the general cohesive law is the same as the one found at the end-

154 opening, so that in the functional form for the cohesive law we can replace δ with δ .

155 3 Test specimen, experimental setup, data analysis and results

156 The mechanical properties of the non-crimp fabric glass-fibre vinylester composite material

- and dimensions of the DCB test samples are presented in Table 1 and illustrated in Figure 2.
- 158 The lay-up is $[(90/0)_9]_8$, and the weight distribution within each ply is 95% in the 0° direction
- 159 and 5% in the 90° direction, so that the laminate does not possess bend-twist coupling. The

- 160 end-opening δ^* was measured using an extension extension at tached to the side of the upper and
- 161 lower beams at the initial prefabricated crack tip at $(x_1, x_2) = (0, \pm H/2)$. Mode I
- 162 delamination was promoted by applying equal moments in the opposite directions in a
- 163 progressive manner by increasing the rotations of the specimen beam ends [17]. The fracture
- 164 resistance was calculated from the measured applied moments, equation (1).
- 165

166 An exponential decay function of the following form was fitted to the fracture resistance:

 $J_R(\delta) = J_a \left(1 - e^{-\delta/\delta_a} \right) + J_b \left(1 - e^{-\delta/\delta_b} \right) + J_0$ ⁽⁵⁾

168 where the fitting parameters J_a , δ_a , J_b and δ_b are presented in Table 1. The resulting fracture 169 resistance curves are plotted in Figure 3 a). In Figure 3 b) it can be seen that the fracture 170 resistance increases before any opening displacement is observed. The fracture resistance at 171 which the opening displacement starts is associated with the crack tip fracture energy, J_0 . 172 The crack tip fracture energy is the base for the fitted function plotted in Figure 3 a) and b). A 173 bridging law is obtained by differentiating the fitted function analytically with respect to the 174 end-opening in accordance with equation (4). The result is (see Figure 3 c):

175 $\sigma_{FB} = \delta_a J_a e^{-\delta/\delta_a} + \delta_b J_b e^{-\delta/\delta_b}$ (6) 176 The derived bridging law is highly non-linear. The peak stress is approximately 0.9 MPa and 177 the critical separation, δ_0 is about 3 mm. The bridging law does not include the deformations 178 (separation) associated with the crack tip that gives rise to J_0 , see equation (2). As seen in 179 Figure 3 b), J_0 is dissipated within a small opening displacement (assumed to be in the order 180 of 0.01 mm).

181

182 A cohesive law should prescribe a traction-separation relation that dissipates the total energy 183 associated with both the cracking at the crack tip and fibre bridging in the bridging zone. Figure 3 d) presents a cohesive law that has a cohesive traction that increases to a peak value, 184 $\hat{\sigma}$, stays constant for a small opening, $\hat{\delta}$, and then decays rapidly within small openings 185 186 added to the initial part of the bridging law such that the cohesive law includes work of 187 cohesive tractions corresponding to the crack tip fracture energy. The value of $\hat{\sigma}$ represents the interface strength and $\hat{\delta}$ needs to be fitted so the area under the traction-separation law 188 189 equals the critical crack tip energy. In reality only J_0 , determined by acoustic emission and 190 initiation of crack end opening, can be determined from experiments; the chosen peak stress

- 191 therefore affects the associated separations and vice versa. Accurate determination of the peak
- 192 stress and separation parameters is a key issue in the remainder of this paper.
- 193
- 194 It is well known that for wide specimens the crack front often has a "thumb-nail" shape, i.e.
- 195 the crack is somewhat shorter at the free edges and longest half way across the specimen
- 196 width [32]. The opening displacement at a given x_1 position within the active cohesive zone is
- 197 thus not constant across the specimen width.
- 198
- Therefore, the opening displacement δ^* measured at the side of the specimen does not 199 accurately represent the behaviour for the whole delamination front. Furthermore, partial 200 201 restriction of anticlastic bending leads to deviation from a state of plane stress. Consequently, 202 the interface traction obtained with the plane stress assumption through equation (4) will not 203 be accurate. In the next section it will be shown that the opening tractions are non-uniform 204 across the specimen width and that the non-uniformity is affected by the Poisson's ratio. This 205 is of particular importance for small openings where the value of the cohesive traction is 206 expected to vary significantly, such as in the description of cohesive laws representing the 207 crack tip fracture energy J_0 .
- 208 4 Numerical approach and results

209 A three-dimensional finite element model of the DCB specimen was made using the LS-210 DYNA finite element code. The specimen material and geometric properties are presented in 211 Table 1 and Figure 2. The beams in the moment loaded DCB specimen were modelled using 212 8-node solid elements with an isotropic material description fitted to resemble the flexural 213 stiffness of the composite beams. The beams were modelled using volume elements with all 214 sides having lengths of approximately 0.5 mm. An orthotropic material description with a full 215 lay-up description based on unidirectional plies did not produce significantly different results 216 from an isotropic one as long at the bending stiffness was unchanged. Due to symmetry, only 217 one-half of the width of the specimen was modelled. The model is illustrated in Figure 4. The surfaces at the beam-ends in Figure 4 are modelled as rigid bodies. The simulations were 218 219 executed in a nonlinear dynamic analysis with implicit time integration. The reason for using 220 a dynamic analysis was to introduce the mass-matrix to ease the convergence in each load

221 step. Monotonically increasing equal moments, acting in opposite directions, are prescribed to

- these rigid bodies. The contact area between fixture and beam is the same as in theexperiments [17].
- 224

The fracture interface was modelled with 8-noded finite-thickness cohesive elements with a general cohesive material formulation referred to as *MAT_186 in the LS-DYNA material library [33]. The cohesive elements had the dimensions: 0.5 mm, 0.5 mm, and 0.001 mm in width, length and thickness respectively. The constitutive model is a cohesive law that includes both crack tip and fibre bridging behaviour.

230

231 As a preliminary investigation, to evaluate the effect of the above mentioned stress and crack 232 opening variations across the width [32], a simple bi-linear cohesive law was used as model 233 input. The cohesive law parameters are given in terms of the peak stress (set to 20 MPa) and 234 the critical separation, δ_0 , was set to 0.1 mm. These parameters are chosen for illustration 235 purpose. The bi-linear shape is chosen because this is the shape most commonly used in FEM 236 when delamination is included [4, 7, 34-37]. The value of the critical separation used in these 237 simulations is much smaller than the value determined earlier from the experiments (here J_0 238 was about 3 mm), but is of the order of magnitude corresponding to the parameters describing 239 the crack tip fracture energy. The J integral approach described in section 2 was applied to see 240 if changing the Poisson's ratio or specimen width affected the calculated cohesive law. The 241 aim of this preliminary investigation was to see if the cohesive law used in the model input 242 can be determined from post processing the results of a delamination simulation, and what 243 might affect the outcome. The end-opening displacement was extracted from the simulations for $(x_1, x_2) = (0, \pm H/2)$ and the resulting cohesive law was calculated using equation (4). 244 245 First, a DCB specimen with the width of 30 mm was modelled with four different Poisson's 246 ratios. Then the Poisson's ratio was fixed at 0.30 and the cohesive law was calculated from 247 four models with different widths. 248 249 The calculated cohesive laws are plotted in Figure 5 a) and b). Both figures illustrate that the 250 results are affected by both material properties and specimen geometry. The peak cohesive

251 traction is significantly affected. Critical cohesive traction, as seen from the input cohesive

252 law curve, is 20 MPa. Hence, all tractions above 20 MPa are in error. The reason tractions

- appear to exceed the critical cohesive traction is simply that the tractions are calculated based
- 254 on the assumption that the crack tip opening displacement is equal across the width of the

- sample. Due to the variation of stress state (and deformations) across the width of the beams,
- 256 the crack will start to develop in the centre of the specimen before it is visible at the side of
- 257 the specimen where the crack tip opening displacement is recorded. The calculated cohesive
- 258 law is equal to the law used as model input when the Poisson's ratio is set to zero. The stress
- state is then reduced to plane stress throughout the specimen, with no anticlastic bending, and
- 260 the crack front remains straight.
- 261
- With respect to delamination problems, the effect of Poisson's ratio is thus very important for small separations corresponding to the part of the cohesive law that is associated with the crack tip fracture energy, J_0 , which occurs in the range of separation of the order of tens of microns. The effect of the Poisson's ratio is likely to be much less significant for openings corresponding to the crack bridging regime, where the traction value is much lower and decreases to zero over an increase of 3 mm in the crack opening.

268 4.1 Fitting the cohesive law

269 In the following, we develop a cohesive law that incorporates the crack tip fracture energy, J_0 . The cohesive law is modelled as multi-linear and an iterative approach is applied to 270 271 determine the cohesive law parameters. First, a bridging law is calculated from equation (6). 272 Then an area is added to the bridging law so that it becomes a complete cohesive law that 273 includes the critical crack tip energy. The area added to the bridging law is based on the 274 assumption that the interface behaviour is linearly elastic up to the critical interface strength. 275 The shape presented in Figure 3 d) is chosen. By this approach J_0 will be dissipated within the 276 shortest possible opening displacement without exceeding the assumed interface strength. At some opening displacement, δ_4^* , the crack tip energy is fully dissipated. The cohesive law 277 should here include both the work of the bridging traction and the crack tip fracture energy J_0 278 at the end-opening δ_{A}^{*} . This point can be seen in Figure 3 d) where the Adjusted CL aligns 279 280 with the Calculated BL. The area under the cohesive law at this point is (see equation (2))

281
$$J_{\mathrm{R,FPZ}}\left(\delta_{A}^{*}\right) = \int_{0}^{\delta_{A}^{*}} \sigma_{\mathrm{FB}}\left(\delta^{*}\right) \mathrm{d}\,\delta^{*} + J_{0} \tag{7}$$

For openings larger than δ_A^* , the cohesive law should follow the bridging law, see Figure 3 d). However, in the present paper, the cohesive law is defined as a multi-linear law with linear interpolation between tractions defined for six opening displacements. The reason for using

- such a multi-linear law is not based on a physical interpretation of the actual material
 behaviour, but rather the limitations of the constitutive model used for the numerical
 implementation [33].
- 288

289 Finally, the defined tractions and opening displacements are fitted to the experimental results 290 using the optimisation tool LS-OPT [38]. The optimisation processes in LS-OPT are based on 291 the response surface methodology [39]. The aim is to minimize the residual between a 292 response from the model and a response from experimental test results. The opening 293 displacement at the initial crack tip and the applied moments from the experimental results are 294 used as objective for the optimization process. The response surface is created from a series of FEM simulations where the variables have been given different values. Upper and lower 295 296 limits are defined for each variable, e.g. the opening displacement δ_4 has to have a value 297 higher than δ_3 but lower than δ_5 . The process of setting the values of the variables within the 298 prescribed range is organised by the optimisation scheme used in LS-OPT. Here, an ASA 299 hybrid optimization scheme with a D-optimal sampling procedure of linear order [38] is used. 300 Each sampling point is produced from one FEM simulation of the complete delamination. One iteration includes a minimum number of sampling points defined by i = 1.5(n+1)+1, 301 302 where *i* is the number of sampling points (complete finite element analyses of the entire test) 303 and *n* is the number of variables. Each FEM simulation has a CPU time of 6-8 hours. The 304 initial cohesive stiffness defined by δ_2 and σ_2 is chosen with respect to the finite thickness of 305 the cohesive element and the stiffness of the bulk material. The traction plateau defined by σ_2 306 and σ_3 is kept flat by setting $\sigma_2 = \sigma_3$. To further reduce the number of variables, $\delta_5 = 0.5 \delta_6$. The total number of variables is thus 6 and the number of sampling points therefore becomes 307 308 12. 309

The solution converged after seven iterations. The total CPU time for the optimisation process was approximately 550 hours. Figure 6 shows a selection of iteration results. The cohesive law parameters used as first guess (first sampling point in first iteration) are listed in Table 3, along with the resulting cohesive law after seven iterations. Both of the cohesive laws are plotted together with the measured bridging law in Figure 7. The cohesive law parameters with the greatest changes were δ_3 and σ_5 . 317 A propagating delamination modelled using cohesive elements can have difficulties with 318 convergence if the mesh is coarse [16]. In the present FE model the element dimensions in the 319 plane of delamination were 0.5 x 0.5 mm, i.e. 30 elements across the modelled width of half 320 the specimen. The fully developed failure process zone (FPZ) was more than 40 mm long and 321 thus covered by more than 80 cohesive elements in the direction of crack propagation. The 322 evolution of the crack tip, i.e. the development of J_0 , was covered by approximately 5-10 323 elements as the crack propagated. The actual number of elements that cover the complete FPZ 324 depends on the shape of the cohesive law. It is important to adjust the loading steps in the 325 analysis so that the separation parameters describing the development of J_0 are captured. Figure 8 shows the distribution of normal opening traction in the cohesive elements used in 326 327 the optimisation process as the delamination propagates towards the left. The crack tip does 328 not follow a straight line through the width of the specimen. The plot illustrates that the crack 329 opening displacement observed at the side of the specimen may not relate directly to the 330 observed fracture resistance. It can be seen in Figure 8 that the interface tractions in the centre 331 elements start to decrease from the critical traction level before the elements at the edge reach 332 the critical traction level. In Table 3 it can be seen that the opening displacement (after 333 fitting) is 0.001 mm when the cohesive tractions reaches 20 MPa and 0.013 mm when the 334 tractions start to decrease. This indicates a difference in opening displacement across the 335 width of at least 0.012 mm at a given position x_1 .

336

337 LS-OPT was initially also used to do a sensitivity analysis. It was confirmed that the residual 338 between the model and the experimental results was more sensitive to the changes in δ_3 than in $\sigma_{2,3}$. The traction was then given upper and lower bound values of 28 and 15 MPa, 339 340 respectively. The reason for the choice of upper bound value is that the bulk material has a 341 measured elastic limit at 28 MPa transverse to the fibre orientation [40]. The interface should 342 be the weakest link for normal stresses in the thickness direction of the laminate and should 343 therefore be lower than the damage threshold for the bulk material. The choice of lower limit 344 was set to a low value based on the observed behaviour of the bulk material.

5 Evaluating the fitted cohesive law

346 Experimental results from standardised force-loaded double cantilever beam (DCB)

delamination tests [18] were compared with numerical predictions based on the fitted

348 cohesive law. The standardised test specimens were produced with the same constituents and

349 procedures as the specimens for the moment based delamination tests. Loads were measured

- with a load cell and the opening displacement at the end of the specimen was measured using
 an extensometer. However, an interpretation using linear-elastic fracture mechanics would be
 inappropriate due to large-scale bridging.
- 353

The numerical model had the same element dimensions as those used for modelling the moment based delamination tests. Implicit time integration with adaptive time step control was used. Material and geometrical properties are presented in Table 2. The FEM results based on the two cohesive laws, presented in Figure 7 and Table 3, are compared with the experimental DCB test results in Figure 9. It is clear that the cohesive law that was optimised using LS-OPT gives significantly better results than the multi-linear cohesive law used as a starting point for the optimisation.

361 **6 Discussion**

362 The success of the optimization process depends on the choices made during the optimization 363 setup. The choice of sampling point selection scheme, number of sampling points and 364 possible interaction between variables and optimization algorithm are choices that affect the 365 computational cost of completing the necessary number of iterations. More important are the 366 choices and assumptions made regarding the cohesive law. It is computationally favourable to 367 choose few but well placed variables in the cohesive law and keep as many properties as 368 possible constant. The number of simulations per iteration is governed by the sampling 369 selection scheme used and the number of variables evaluated. Adding an additional variable 370 can cause the number of simulations per iteration to increase significantly. It is therefore 371 important to have an approximate idea of what the actual cohesive law should look like and 372 use as few variables as possible. If the initial value is chosen poorly, it may be difficult for the 373 optimization process to produce acceptable results within reasonable computational costs.

- 374
- The need for adjusting the multi-linear cohesive law based on the *J* integral approach can be seen in Figure 6. The *J* integral approach implicitly assumes the crack opening is the same across the width of the specimen. A 3D FE model will include the anticlastic bending effects and the associated variations in stress state and crack opening across the width. A cohesive law that is determined based on plane assumptions will then fail to capture the response from the experiment when used in a 3D FE model. This is the reason why the blue curve in Figure 6 is very different from the experimental results.

- The choice of using a multi-linear shape for the cohesive law is based on the limitations of the constitutive model used in the finite element implementation. The reason for defining only six points on the multi-linear cohesive law is found in the optimisation process. Every sampling point is based on the result of a complete FEM simulation of the DCB test. Each FEM
- 387 simulation has a CPU-time of approximately 6-8 CPU hours. Increasing the number of
- 388 variables therefore significantly increases the CPU time of the total optimisation procedure.
- The number of sampling points used here was 12 and acceptable results were found after 7
- 390 iterations. The total CPU time for the optimisation process was approximately 550 hours.
- 391
- 392 In Figure 9, the resulting fitted cohesive law produced significantly better results than the
- 393 multi-linear cohesive law used as a starting point for the optimisation procedure. However,
- 394 neither of the simulated models completely captured the stiffness shown by the experimental
- 395 results. The simulated stiffness is in both cases higher than that of the experimental result. The
- 396 discrepancy is partially attributed to the compliance of the test fixture and is considered
- 397 acceptable since the beams are modelled with the same isotropic material model as used for
- 398 the DCB samples. The resin, fabric, sizing, curing procedure and fibre volume fraction are
- applied load at a sequel for both types of DCB samples. In Figure 9 a plateau is observed in the applied load at
- 400 an opening of about 7-8 mm on the FEM results from the fitted cohesive law. This may have
- 401 been caused by the restrictions made on the variables during the optimisation setup. The drop
- 402 in interface stiffness going from the crack-tip dominated region to the fibre bridging
- 403 dominated region of the cohesive law might be too steep. Dissipating the crack tip energy
- 404 within a short opening displacement (i.e. a high peak stress over a small opening), still seems
- 405 to be an appropriate approach without causing numerical instability.
- 406

407 The initial stiffness of the cohesive law should be chosen with respect to the thickness of the 408 cohesive elements so that the traction-separation relation of a finite-thickness element 409 resembles the stress-strain relations of the bulk material. If the stiffness is chosen poorly the 410 overall bending response of the laminate may be affected and become too soft. Another 411 challenge is that rapidly changing stiffness in the cohesive elements may cause numerical 412 instability. Reducing the size of the cohesive elements reduces the rate of change in stiffness 413 in adjacent elements in the direction of the propagating delamination. Reducing the time step 414 will of course also reduce the rate of change in stiffness. If the load step in an implicit model 415 is sufficiently small, an explicit solution might be faster even if the load steps then will be 416 significantly smaller.

418 The varying stress state and crack opening across the width of the specimen (induced by the 419 Poisson effects) is not taken into account in the experimental J integral approach, eq. (4). The 420 effects presented in Figure 5 a) and b) show the tendency of over-estimating the tractions of 421 the cohesive law at small opening displacement, and underestimating them at larger opening 422 displacements. But the total areas of the cohesive laws are all equal to the steady state fracture 423 resistance. The stress variations across the width seem to give the impression of more rapid 424 energy dissipation if evaluated from the opening displacement measured at the side of the 425 specimen. This is also apparent from the fracture resistance curves from the simulations in 426 Figure 6. The cohesive tractions associated with crack bridging vary much more slowly over 427 much larger separations and are not expected to be significantly influenced by the Poisson 428 effects.

429

In the experiments, there were some minor discrepancies between initiation of crack end opening and the first acoustic events. If the crack-end opening displacement first initiates at the half width across the specimen before it initiates at the side of the specimen, the first acoustic events should be detected before any crack-end opening displacement is observed at the side of the specimen.

435

436 The presence of anticlastic bending and the effect it has on the relation between fracture

437 resistance and crack tip opening at small opening displacement might affect the observed

438 value of the critical crack tip energy, J_0 . The findings presented in this paper suggest that

439 beams displaying anticlastic bending might give the appearance of having higher values of J_0

440 than beams with little anticlastic bending. The reason for this apparent higher value of J_0 is

the delayed opening of the crack at the sides of the specimen where the crack tip opening

442 displacement used to evaluate J_0 is measured. This effect could be investigated by checking if

443 the observed value of J_0 changes with increasing specimen width.

444

For materials where the cohesive tractions vary rapidly over small openings it would be convenient to have a correction function that could account for the effect of having states of stress and deformation that change across the width of the specimen. A challenge with such a function is that the difference between measured and actual cohesive law is dependent on the shape of the actual cohesive law. With this in mind, inverse modelling using the three-step optimisation scheme presented here seems currently to be the most promising approach. The 451 effects of Poisson's ratio on the beam deformation (in the form of anticlastic bending) can be 452 reduced by increasing the beam height relative to the beam width (increasing H/B) - going

453 from a plate-like geometry toward a more beam-like geometry. This will not eliminate the

454 presence of anticlastic bending, but it will make the measurements of the opening

- 455 displacements at the side of the specimen closer to the value in the middle of the specimens,
- 456 thus making them more relevant for the overall behaviour.

457 7 Concluding remarks

458 The object of this study was to show that a cohesive law associated with the crack tip fracture

459 energy could be obtained from experimental tests for implementation in 3D finite element models. A procedure to achieve this has been developed and tested. Such a cohesive law for 460 461 large scale bridging problems consists of two distinct energy-dissipating phenomena: crack tip 462 energy and fibre bridging. A bridging law describing the fibre bridging is calculated from the 463 fracture resistance curve by applying the path independent J integral approach for plane 464 stress. An approximate multi-linear cohesive law is then obtained by combining tractions and 465 opening displacements for dissipation of fracture energy within a small opening displacement, 466 corresponding to the crack tip fracture energy, J_0 , and a simplified bridging law that operates 467 over larger openings. The parameters of the multi-linear cohesive law are then fitted to 468 account for the changing stress state through the width of the test specimen by using the 469 optimisation tool LS-OPT. The fitted cohesive law is evaluated by comparing FEM and 470 experimental results for a series of ASTM D 5528 Mode I delamination tests. The FEM result 471 using the fitted cohesive law is found to agree well with the response observed in the 472 experimental tests. The three-step procedure presented here is successfully shown to 473 characterize a Mode I cohesive law. The changing state of stress and deformation across the 474 width of the specimen is affected by both material properties (Poisson's ratio) and the 475 geometry of the test specimen. This three dimensional effect is a significant source of error 476 for small cohesive openings and needs to be taken into account when determining a cohesive 477 law from the fracture resistance, in particular the traction for small separations corresponding 478 to the crack tip fracture energy. The effect is expected to be small for problems where the 479 cohesive tractions represent large-scale bridging for which the tractions are low and decrease 480 slowly to zero over several millimetres.

481 Acknowledgements

- 482 This work is part of the collaborative project "Composite structures under impact loading"
- 483 with the industrial partners Flowtite Technology AS, Nammo Raufoss AS, Ragasco AS and
- 484 the research institutes Norwegian University of Science and Technology (NTNU), University
- 485 of Oslo (UiO), SINTEF Materials and Chemistry and SINTEF Raufoss Manufacturing. The
- 486 authors would like to express their thanks for the financial support by the Norwegian
- 487 Research Council (grant 193238/i40) and the industrial partners. The authors would also like
- 488 to thank all partners in the project for constructive discussions. The fourth author was
- 489 supported by the Danish Centre for Composite Structures and Materials for Wind Turbines
- 490 (DCCSM), grant no. 09-067212 from the Danish Strategic Research Council. The cooperation
- 491 between SINTEF and Technical University of Denmark, Department of Wind Energy was
- 492 partly funded by the Norwegian Research Centre for Offshore Wind Technology
- 493 (NOWITECH).

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- reinforced vinylester composites. Composites Part B: Engineering. 2015;77(0):105-111.

595 Table 1. Material and geometric properties for moment loaded DCB specimen.

В	30.11	mm	Width		
2 <i>H</i>	17.40	mm	Thickness		
l	300	mm	Length		
a_0	59	mm	Initial delamination		
Ε	37	GPa	Flexural modulus		
v_{12}	0.29		Poisson's ratio		
$S_{22} = S_{33}$	28	MPa	Transverse ply strength		
$\sigma_{ m Ic}$	20	MPa	Mode I Critical interface strength		
δ_0	3	mm	Mode I Critical opening displacement		
$J_{ m ss}$	1	kJ/m ²	Mode I Steady state fracture resistance		
J_0	0.21	kJ/m ²	Mode I crack tip fracture energy		
J_a	0.33	kJ/m ²	Fitting parameter for equation (5)		
J_b	0.67	kJ/m ²	Fitting parameter for equation (5)		
δ_a	6.67	mm	Fitting parameter for equation (5)		
δ_b	0.65	mm	Fitting parameter for equation (5)		

596

597 Table 2. Geometric properties for standardised DCB specimen.

W	22	mm	Width
2 <i>H</i>	4.0	mm	Thickness
l	150	mm	Length
a_0	50	mm	Initial delamination

598

599 Table 3. Cohesive law parameters before and after fitting.

Before fitting				After fitting			
[mm]		[MPa]		[mm]		[MPa]	
$\delta_1 =$	0.00	$\sigma_1 =$	0.00	$\delta_1 =$	0.00	$\sigma_1 =$	0.00
$\delta_2 =$	0.0010	$\sigma_2 =$	20.00	$\delta_2 =$	0.0010	$\sigma_2 =$	20.00
$\delta_3 =$	0.0296	$\sigma_3 =$	20.00	$\delta_3 =$	0.0130	$\sigma_3 =$	20.00
$\delta_4 =$	0.0326	$\sigma_4 =$	0.4000	$\delta_4 =$	0.0181	$\sigma_4 =$	0.6200
$\delta_5 =$	1.5088	$\sigma_5 =$	0.0800	$\delta_5 =$	1.5088	$\sigma_5 =$	0.1568
$\delta_6 =$	3.0176	$\sigma_6 =$	0.00	$\delta_6 =$	3.0176	$\sigma_6 =$	0.00

600

601

602 Figure 1. Integration path for the *J* integral: a) integration path locally around the cohesive zone, b)

603 interpretation of traction vs. separation in the FPZ, and c) the integration path along the external

604 boundaries of a DCB specimen loaded with pure bending moments.

605

606 Figure 2. Geometry of DCB specimens.

607

608 Figure 3. Fracture resistance response from a DCB specimen loaded with pure bending moments.

609 Fracture resistance and cohesive traction are shown as a function of normalized separation (normalized

610 by $\delta_{\theta} = 3$ mm). Fitted function on top of experimental scatter a) and b). Details of the calculated bridging

611 law (BL) c) and the adjusted cohesive law (CL) (adjusted for small separations only) - note different scales

- 612 on the axis d).
- 613

614 Figure 4. The FEM model of the DCB specimen for the moment based delamination test.

615

616 Figure 5. Comparing a bi-linear cohesive law used in the FE model and the cohesive law (CL) calculated

617 after post processing the model after a) changing the Poison's ratio, and b) changing the specimen width.

618

619 Figure 6. Fracture resistance during iterative fitting of cohesive law compared to experimental test result620 used as fitting objective.

- 621
- Figure 7. a) The calculated bridging law with the cohesive law before (blue colour) and after fitting (green
 colour). The details of the crack tip relations are presented in b), and the details of the fibre bridging
 region is presented in c).
- 625
- 626 Figure 8. Contour plot illustrating opening tractions in cohesive elements as crack propagates. Several
- 627 cohesive elements were deleted after exceeding critical separation in the lower two plots. The total active
- 628 cohesive zone length, *L*, is indicated in the lower two plots. The indicated zone covered by eight elements
- 629 represents the crack tip zone.
- 630
- 631 Figure 9. FEM result before and after fitting of cohesive law compared to experimental results.









Figure 5







Figure 7 Click here to download high resolution image





