Exact Constraint Design and its potential for Robust Embodiment

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Abstract

The design of exact, also referred to as minimal, constraints means applying just enough constraints between the various components of a mechanical assembly, in order to unambiguously define their positions in six degrees of freedom (3 translations, 3 rotations), their desired motions respectively. To ensure a predictable and reliable product performance, a systematic design of the corresponding elementary mechanical interfaces between components is of utmost importance. Overconstraints, i. e. part-to-part connections with redundant interfaces which constrain one single degree of freedom, are largely susceptible to variation and therefore result in design solutions which frequently experience production/assembly issues, reduced performance, excessive and non-predictable wear-rates, etc.

Being a basic rule of embodiment design, literature provides various well-known and widely applied approaches for Exact Constraint Design. Examples are the calculation of a mechanism’s mobility using the Grübler-Kutzbach criterion, the analysis of statically determinate assemblies by means of the screw theory or so called Schlussartenmatrizen, as well as the analysis of engaging surfaces in terms of location schemes or interface ambiguity. However, despite the various existing approaches, workshops with practitioners and academics have shown that the systematic design of optimal constraints appears to be cumbersome for many engineers. Based on an overview of the most relevant approaches for Exact Constraint design, this contribution therefore reviews the challenges experienced by the workshop participants, discusses the necessity of kinematically correct constraints for robustness, and derives an initial prescriptive procedure for a coherent design of constraints throughout the embodiment design phase, which, despite a variety of available approaches, seems to be still missing.

Keywords: Robust Design; Kinematic Design; Design processes; Quality by Design

1. Introduction

Given the ever-increasing quality requirements towards more and more complex products, a systematic and purposeful quality management strategy is of vital importance for manufacturing companies. At the same time, many quality assurance activities focus almost exclusively on the control and the continuous improvement of manufacturing processes. The relevance of less visible upstream costs for quality assurance are in contrast largely neglected [1]. Despite indisputable achievements of quality initiatives, such as Total Quality Management, Lean Manufacturing, or Six Sigma, high safety factors, late and frequent design changes, or excessive inspection activities are consequently still prevalent in industrial practice [2], leading to the impression that:

“Quality issues are frequently mitigated by inefficient products and processes leading to quality at excessive costs.”

In light of the above, there is a wide consensus that the widely implemented, production-focused quality management strategies have to be complemented by upstream quality management strategies focusing on the design of products and processes. The design of such solutions is often based on the systematic and purposeful design of the corresponding interfaces between the different components of a product, which is the subject of this contribution.

Nomenclature

| EC | Exact Constraint |
| RD | Robust Design |
| RDM | Robust Design Methodology |
| Rx, Ry, Rz | rotational constraints |
| x, y, z | translational constraints |
efforts [3, 4]. Instead of controlling the compliance of partsand systems only in production, quality has to be systematically designed into products and processes and continuously monitored and optimised based on suitable verification and validation activities during development and ramp up.

Among other quality-oriented design methodologies, this insight has led to the emergence of various Robust Design (RD) principles, methods and tools over the last decades. Originating from the seminal work of Genichi Taguchi in the late 1950s [5], the term robustness describes the insensitivity of products or processes to the various sources of variation. Examples are production and assembly tolerances, load scenarios, ambient use conditions, or deterioration of components over time. A corresponding Robust Design Methodology (RDM) consequently aims at designing robust products and processes which perform consistently in spite of these noise factors, and thus an acknowledged way to avoid the inefficiencies of products and processes otherwise necessary to mitigate the resulting variation effects [5, 6].

Despite the potential benefits, the acceptance of RDM in industrial practice is limited though [7]. Traditionally focusing on an improvement of robustness via (computational) expensive virtual/physical experiments and the corresponding statistical analyses, the methodology has been often criticized for not offering enough guidance and support in early design stages [8, 9]. As a result, several contributions have sought to address concept generation aiming at attaining robustness, see for example the overview provided by Jugulum and Frey (2007) [9].

However, in the opinion of the authors, these predominant focal points of RD research, on either the earliest design phases or experiments with fully specified solutions/prototypes at the end of the design process, largely disregard one of the essential facets of Robust Design. Robustness is essentially dependent on the early embodiment of a chosen principle solutions, i.e. on the determination of the general arrangement as well as preliminary shapes and materials of components. Overconstrained designs, ambiguous interfaces between components, unfavourable material combinations, etc. (1) are largely susceptible to variation and therefore frequently experience production/assembly issues, reduced performance and excessive wear-rates. Due to over-complex structures, redundant interfaces between components, these variation effects are furthermore difficult to predict, resulting in a (2) time and cost intensive, as well as inherently inaccurate variation analysis during subsequent design verification and robustness optimisation activities.

Building on previous research [10], this contribution therefore aims at creating awareness for this fundamental phase of early embodiment. For this purpose, it discusses the potential of different approaches in the field of Exact Constraint (EC) design for a successful RDM, reviews available EC methods and reflects on their applicability based on a series of conducted RD Workshops.

2. Research Methodology and Outline

While this paper aims at fostering the use of EC design methods and tools for RD purposes, it has to be noted that the importance of optimally constrained mechanical assemblies is hardly new. On the contrary, the systematic design of unambiguously constrained mechanical connections is a basic rule of embodiment design [11], considered an essential task in precision engineering for well over a century [12], and has even been already classified as an essential RD activity by several authors [10, 13, 14]. For this reason, the research approach is twofold. Based on an overview of some of the most relevant contributions in the field of EC design (section 3), the corresponding approaches are put into an initial method sequence based on the underlying model representations (section 4). For a first qualitative evaluation of this basic hypothesis, the method sequence is then used during a RD workshop build around the embodiment of a simple consumer product (section 5), i.e. a hand-held glue gun for thermoplastic adhesives. Concluding, the challenges faced by the industry delegates during the workshop, the results of this study as well as its implications for future research are summarised (section 6).

3. Theoretical Background

3.1. Engineering Design Methodology

Literature on Engineering Design Methodology provides a vast amount of design process models, prescribing a structured procedure for the systematic design of technical systems and products. Referring exemplarily to the explanations in [11], their purpose is to decompose the challenging and usually highly iterative development process into a series of subsequent design phases (e.g. Task clarification, Conceptual Design, Embodiment Design, Detail Design), subdivided into single working steps, as well as the corresponding intermediate results in order to reduce the complexity of the development tasks.

While being inherently generic, and hence requiring adaptation to the specific industry branch or development task [11], the corresponding design process models offer a fundamental understanding of the importance of engineering models during development. Basically, engineering models summarise the information about the developed product available in different design phases, and thus provide the basis for an evaluation its characteristics, its behavior, etc. In a coherent methodical design process, they consequently represent intermediate results on different levels of abstraction, which are gradually concretised and detailed towards a full design solution.

At the same time, it is the author’s impression that, in spite of various well-described development processes and model
toolbox for the Conceptual Design phase, there is no well-
documented or commonly accepted embodiment design
approach. In a first attempt to bridge this gap, this paper
consequently focusses on the above laid out importance of a
coherent model toolbox. Referring to the idea of a gradual
corrélationisation of design solutions in design process models as
well as the decomposition of an overall product into its
structure and the interfaces between components, well-known
and commonly applied tools for the design of optimally
constrained mechanical assemblies are mapped out in an
initial procedure for early embodiment considerations for
robustness.

3.2. Exact Constraint Design

The design of exact constraints, also referred to as minimal
constraints, means applying just enough constraints between
the various components of a mechanical assembly, in order to
unambiguously define their positions or desired motions in six
degrees of freedom (3 translations, 3 rotations). While
designing the corresponding elementary mechanical interfaces
is, first of all, essential to ensure the assembly’s basic
functionality, the below summarised approaches f are
moreover particularly interesting from a RD perspective as
they allow for a simple analysis of constraint patterns based
on abstract models or exclusively nominal dimensions, see
also [13], hence a straightforward robustness analysis in early
phases of embodiment design.

A well-known and widely applied approach for the
evaluation of overconstraints in mechanical assemblies is for
example the calculation of a mechanism’s mobility by means of
the Kutzbach criterion [10, 13]:

\[ M = 6(n-1) - \sum U - \sum F_{id} \]  

(1)

By counting the number of parts \( n \), the number of
constraints \( U \), as well as the number of identical degrees of
freedom \( F_{id} \), equation (1) evaluates the degree of mobility \( M \)
of the mechanism under consideration. The calculated
mobility indicates either the number of remaining degrees of
freedom which need to be controlled \( M > 0 \), or the existing
overconstraints \( M < 0 \) potentially leading to excessive
internal forces, vibrations and/or wear.

Although the Kutzbach-criterion basically also applies to
static structures \( M = 0 \), it is mainly useful for the assessment
of moving mechanisms. As the mobility calculation is
inherently dimensionless, i.e. only accounts for the number of
bodies and the type of joints between them, it does not offer
any insight on the underlying constraint pattern causing the
assembly to be under- or overconstrained. For this reason,
Whitney [13] suggests the so-called Screw Theory for
analysing the overall constraint status of a mechanical system.
Based on the description of potential motions between two
engaging components using twist matrices, the equivalent
constraints with the reciprocal wrench matrix respectively, the
theory allows for a calculation of the overall constraint status
of an assembly based on matrix transformations. Fig. 1
illustrates the corresponding characterisation of a pin-slot
connection expressed in global coordinates, where \( \omega_i \) captures
potential angular velocities and \( v_i \) describes the corresponding
translational velocities.

![Fig. 1: Description of interfaces using twist matrices, examples adopted from [13]](image1)

Particularly with respect to an application in an early
embodiment phase, two aspects of the screw theory should
be noted though. Providing a theoretically comprehensive
approach for a constraint pattern analysis, it relies on a
specification of constraints based on the connecting features,
or feature combinations. Furthermore, all engaging surface
contacts are considered as inherently frictionless.

As a counterexample, the authors’ therefore like to point to
the so-called Schlussartenmatrizen (joint closure matrices)
suggested by Roth [15]. The corresponding approach is
intentionally kept non-quantified, and instead characterises
the fit between two components by an abstract description of
the closure type. Gradually derived from the translation
constraints between two components \( a \) and \( b \), the joint closure
matrix \( S_{ab} \) summarises the overall constraints of an interface
and characterises them as form or force closure in a positive
and a negative direction of movement, e.g. \( x \) and \( \bar{x} \), for all six
degrees of freedom, see Fig. 2.

However, in the same way as the screw theory, the joint
closure matrix focuses exclusively on the constraints between
two parts imposed by the overall contact features. For this
reason, the above laid out approaches are complemented by a
short overview of methods for the detailed analysis as well as
the design of interfaces based on a consideration of location
schemes and nesting forces in the following.

![Fig. 2: Description of interfaces based on joint closures matrices, example adopted from [15]](image2)

Fundamentally, the term location scheme [16] refers to a
positioning system, assembly fixtures or jigs in most cases,
which appropriately constrains all six degrees of a part, and

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1Planar mechanisms with 3 degrees of freedom are evaluated in a similar
manner by means of the Grübler criterion, see also [10, 13].
thus ensures its reliable and repeatable position. An example of the most commonly used orthogonal 3–2–1 location scheme is shown in Fig. 3, including a plane contact A (three contact points constraining the z-transation and the $R_z$, $R_y$-rotations), a line contact B (two contact points constraining the x-translation and $R_z$-rotation), as well as a point contact C (constraining the remaining y-translation). While the design of a location scheme consequently appears to be straight forward at first glance, it is worth noting its immense impact on how variation is propagated between components [16]. Particularly in case of fixture layouts in assembly systems or for car bodies, the detailed analysis of contact points on mating surfaces, previously only considered in their entirety, is an important step to ensure an overall suitable constraint pattern, in other words the resulting robustness. Corresponding guidelines and working procedures for the analysis of location schemes are summarised in [16].

![Fig. 3: Design of exactly constrained mechanical connections based on location schemes (15) (Image)](image)

For general design tasks [12, 17] and the design of smart assemblies [18], literature furthermore provides guidelines and approaches to appropriately define the position as well as the magnitude of so-called nesting forces. Interpreting the contact points of a location scheme as solely unidirectional constraints, the part-to-part connection in Fig. 3 requires an accompanying force to hold the two mating bodies in contact. As illustrated by the example in Fig. 4, this so-called nesting force cannot be replaced by a fourth static contact point, as this would lead to an overconstrained and non-predictable mechanical connection [17].

![Fig. 4: Design of exactly constrained mechanical connections based on nesting forces, adopted from ([15, 16]) (Image)](image)

Finally, the author’s would like to point to an aspect which is rarely detailed in literature. The ambiguity of mechanical connections refers to the risk that even in case of a kinematically correct interface between components, there might be still a large potential for variation effects. Available approaches essentially rely on interpretation of the entire surface (joint closures matrices), the contact features (Screw Theory), or the contact points (location schemes and nesting forces) as an ideal point-contact geometry between the engaging components. In reality, these sharp contact points can neither be used for load transmission nor be produced by commonly available manufacturing processes, eventually leading to unforeseen, additional constraints [13]. On the other hand, carelessly designed interfaces might even lead to an abrupt change of the engaged surfaces, which obviously affects the functional performance. In order to avoid corresponding risk, literature offers a number of principles which directly point designers towards critical interfaces or the underlying surface shapes in form of catalogues, see for example [10].

4. Early embodiment of robust products

In the following, the method overview in the previous section is used to derive an initial procedure for the consideration of constraints in early phases of embodiment design, see Fig. 5. Referring to the idea of a gradual concretisation of design solutions throughout the development process, the prescribed sequence of EC design methods is thereby based on the underlying representation of the mechanism or product under consideration.

![Fig. 5: Procedure for ensuring exactly constrained mechanical assemblies in early phases of embodiment design (Image)](image)

Premise for a consideration of constraints in mechanical assemblies is a given preliminary/basic layout of the product under development in form of a structural graph. Illustrating not more than the arrangement of different components as well as the existing connection points between them, the graph can be easily extended to a simple kinematic diagram by a characterization of the joints, that is either allowing for relative motion in form of remaining degrees of freedom, or force transmission in form of applied constraints. In case of moving parts or mechanisms, this inherently non-geometric and dimensionless representation already enables a first assessment of the system’s constraint status by a calculation of the mechanisms mobility (Mobility Analysis).

Furthermore, the extended structural graph can also be used for the identification of static support structures within the system, which are subsequently analysed using screw theory or joint closure matrices (System Clarity Analysis). While offering insight about the number of overconstraints in different degrees of freedom as well as for different load paths in the system, it should be noted though that both approaches
require additional information, e.g. an initial idea of the shapes of engaging surfaces.

In a last step, the analysis is then further detailed towards an in-depth consideration of contact points and nesting forces on the single surfaces, which in addition to general shapes requires a fully specified position of contact points on the corresponding surfaces (Interface Clarity). However, in light of a usually complex system and the corresponding effort for an analysis, the procedure also foresees a systematic application of ambiguity principles. In form of a screening, they allow to spot critical, variation-relevant interfaces before a full specification of contact points, hence the shape and dimensions of two different components, is available.

5. Robust Design Workshop

5.1. Workshop set up

For a first evaluation of this research, the derived Robust Embodiment procedure was used as basis for the conceptualisation of a RD workshop. Conducted with around 60 delegates (~75% from industry) in the scope of a symposium as well as with ca. 100 students during lectures, the workshop did not only allow for a first impression on the applicability of the chosen method sequence, but also helped to identify challenges, which the participants experienced while using the chosen EC design approaches.

For full engagement of the participants, the workshop was built around the example of a consumer product. The corresponding mini trigger glue gun, shown in Fig. 6 (a), allows for a manual application of thermoplastics adhesives for flexible use in various areas, particularly for hobby and handcraft projects. It is designed as a hand held device, which contains a continuous duty heating element and a mechanical, five-bar trigger mechanism pushing the melted glue through the nozzle, both positioned relative to each other by two housing shells.

Accordingly, the workshop focusses on the design of an optimally constrained (1) mechanical trigger mechanism as well as the constraint pattern of the (2) static support structure. Essential premise for all participants is to systematically ensure a consistent performance, i.e. accurate dosing and application of glue in spite of the existing variation. The workshop material consisted of a CAD model and a physical sample of the case product, as well as of a workbook with the necessary design files and templates for the application of methods, as seen in Fig. 6 (b) and (c).

5.2. Workshop Results

Given its design, the glue gun proved to be the ideal example case for this research. On the hand, it provides the opportunity to create awareness for the importance of exact constraints in, both moving as well as static, mechanical assemblies. On the other hand, it also offers the possibility to stimulate a discussion among the workshop participants about potential challenges of common embodiment design tasks as well as of the application of EC design approaches. At the same time, the task templates, collected for evaluation purposes, did unfortunately only allow for a qualitative assessment of the group results due to the limited prior knowledge of many workshop participants. In line with previous research, e.g. in [13, 17], the workshop consequently showed that:

- Although being a basic rule of embodiment design, many engineers are not familiar with the available EC approaches, so that a systematic design of optimal constraints seems to be cumbersome for them.

As first important result of the conducted workshop, the authors would therefore like to emphasize the importance of this fundamental early embodiment design phase for a coherent RDM, hereinafter referred to as Increase Predictability. The corresponding objective is to systematically bridge the still existing gap between early-stage consideration of conceptual robustness and late-stage robustness optimization, referred to as Decrease Sensitivity, see Fig. 6.

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The workshop material is also available open source. For more information, please visit www.robustdesign.org.
At the same time, the workshop did however not allow for a concluding assessment of the suggested method sequence for the consideration of constraint patterns in early embodiment design. While its’ evaluation is therefore still subject to ongoing research, some interesting insights, identified during the so far conducted workshops, are summarised in the following:

- A coherent analysis of constraints in mechanical assemblies requires a comprehensive description of all different states in an assembly’s used cycle.
- The calculation of a mechanism’s mobility requires further information on acting forces/geometries if it has (at least) two remaining degrees of freedom, which are controlled through force equilibriums.
- The assessment of constraint patterns by means of screw theory seems to be too complex and time-consuming, even for the rather simple example case.
- The reviewed EC approaches do not allow for a seamless transfer of information between them, as the interpretation of constraints differ significantly.

Exemplarily, the dependency of the trigger mechanism’s mobility on the acting forces/its geometries is clarified in the following. As seen in Fig. 8 (a), the planar mechanism has two remaining degrees of freedom ($M = 2$). Before the user applies force on the trigger, the mechanism’s position is however only controlled by a single input given by the preloaded spring. The second required constraint is instead given by the (varying) dimensions of the mechanism’s components and the housing shells, or the diameter and material of the glue stick, which acts as new end-stop when inserted.

![Fig. 8: Control of link positions in the glue gun’s trigger mechanism](image)

**Discussion and Conclusion**

In an attempt to create a baseline for future research on a systematic design of optimally constrained, i.e. robust, mechanical assemblies, this paper provides an overview about available EC design approaches and suggest a first prescriptive procedure for their application in early stages of embodiment design. Derived from a consideration of the underlying model representation, the procedure is used for the conceptualisation of a RD workshop in order to allow for a first evaluation and to identify the most common challenges experienced by practitioners and academics.

In conclusion, the authors hope to have contributed to establish a deeper understanding of the relevance of EC design methods for the development of robust products and processes. At the same time, workshop participants experienced numerous challenges when following the suggested procedure as well as during the application of the single methods. Particularly the analysis of constraint patterns in (complex) assemblies appears to be cumbersome, leading to the necessity of additional approaches for a first manual screening of the system’s structure regarding potential constraint mistakes and robustness issues. Overall it is obvious that further research is necessary to close the gap between early-stage consideration of conceptual robustness and late-stage robustness optimisation.

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**References**


