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Response predictions using the observed autocorrelation function

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Abstract

This article studies a procedure that facilitates short-*time*, deterministic predictions of the wave-induced motion of a marine vessel, where it is understood that the *future* motion of the vessel is calculated ahead of time. Such predictions are valuable to assist in the execution of many marine operations (crane lifts, helicopter landings, etc.), as a specific prediction can be used to inform whether it is safe, or not, to carry out the particular operation within the nearest time horizon. The examined prediction procedure relies on observations of the correlation structure of the wave-induced response in study. Thus, predicted (future) values ahead of time for a given time history recording are computed through a mathematical combination of the sample autocorrelation function and previous measurements recorded just prior to the moment of action. Importantly, the procedure does *not* need input about the exciting wave system, and neither does it rely on off-line training. In the article, the prediction procedure is applied to experimental data obtained through model-scale tests, and the procedure's predictive performance is investigated for various irregular wave scenarios. The presented results show that predictions can be successfully made in a time horizon corresponding to about 8-9 wave periods ahead of current time (the moment of action).

Keywords:

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1. Introduction

Most marine operations require a high level of safety. This is also the case when concern is for here-and-now operations such as lifts by floating cranes, helicopter landings on (smaller) ships, tow of drilling and production vessels/platforms, and various ship-to-ship actions. The execution of these operations can be made safer if the particular vessels wave-induced motions can be predicted ahead of current time. Thus, the ability to calculate accurately, in a deterministic sense, the future wave-induced behaviour of the vessel can reduce significantly the probability of failure of the actual operation. Some of the before mentioned operations involve dynamically positioned (DP) vessels and one means to apply the predicted response/motion ahead of current time can, in this case, be used directly in proactive control strategies for the DP system. Examples of strategies may be to adjust the controller gains, change the set-point of smaller vessels, and for larger vessels accelerate the vessel into the waves to avoid drift-off, or, if worst comes to worst, have sufficient time to emergency-abort the operation safely. Other practices, where the prediction of vessel motions ahead of current time is valuable, occur for general heave compensation systems, and for robotic manipulators on ships and other seaborne platforms, since efficient operation of the manipulators requires precise motion planning and control algorithms. As a practical remark, it should be noted that *current time*, in the following, relates to the very instant from when a prediction is made, meaning that measurements have been recorded (are known) only until the current time. Equivalently, this specific time could be defined as the *moment of action* from when the future (hydrodynamic) behaviour of the vessel is predicted.

1.1. Previous work

In the past, a number of studies has been conducted to investigate procedures for the prediction of the wave-induced motion of a marine vessel. Some of

28 the initial studies, e.g., Dalzell [1], Triantafyllou et al. [2, 3] and Sidar and
 29 Doolin [4], were concerned about the landing of aircrafts on naval destroyers.
 30 Since then, several of other works have followed both with naval and merchant
 31 applications; for instance, Broome [5], Broome and Hall [6], Chung et al. [7],
 32 Duan et al. [8], From et al. [9], Khan et al. [10, 11, 12], Naaijen et al. [13], Peng
 33 et al. [14], Woodacre et al. [15], Zhao et al. [16]. Most works in the existing
 34 literature belong to one of two main categories. Either the established prediction
 35 procedure relies on; a) a combined knowledge of the exciting wave system and
 36 the hydrodynamic behaviour of the ship, e.g. in terms of the ships transfer
 37 function, or b) the procedure relies on some sort of offline training which is
 38 necessary for 'standard' autoregressive (AR) models and Neural Networks that,
 39 on the other hand, not necessarily require input about the waves/sea state.
 40 Obviously, independence of (information about) the sea state is beneficial, as
 41 real-time ocean surface and sea state estimation, at a ship's exact location,
 42 in itself can be a difficult problem to handle in practice, not to mention the
 43 uncertainty associated to the actual estimate produced by whatever estimating
 44 means [17, 18, 19].

45 It is possible to formulate a prediction procedure, see Andersen et al. [20],
 46 which neither requires information about the wave conditions, nor does it re-
 47 quire offline training. In the particular procedure - for any considered motion
 48 component - the sample autocorrelation function (ACF) for a *recent* time win-
 49 dow needs to be obtained. The (sampled) ACF must represent a stationary
 50 situation which, in time and properties, is so close to the current time that
 51 the statistics and the correlation structure in the *dynamical system* have not
 52 changed significantly. Thus, leaving the basic details for later, the prediction
 53 procedure relies on a linear model based on the correlation structure, in terms
 54 of the autocorrelation function, of the physical process in question together with
 55 the most recent - past - measurement points. In this connection, it is important
 56 to realise that the autocorrelation function is a direct measure of the physical
 57 process' underlying memory effect; here due to the free surface oscillations of
 58 the sea surface. Another property to keep in mind, when discussing a process's

memory and the autocorrelation function, is the fact that, for a stationary process, "... *the autocorrelation function and the spectrum are transforms of each other, (hence) they are mathematically equivalent*" [21]. This fact is made directly use of later, but, as a qualitative interpretation of the property, it means that an infinitely narrow-banded process has infinitely long prediction horizon; since the process has, in the extreme case, one single frequency component and, hence, is described by a sine wave. The opposite is true for an infinitely broad-banded process (i.e. white noise), where the deterministic prediction horizon is zero.

In a recent study, Nielsen and Jensen [22] investigated the procedure, [20], to predict vessel responses up to 50 seconds ahead of current time. The study [22] was focused on simulated time histories of a ship's wave-induced vertical acceleration at the centreline at a longitudinal position forward of the COG. In total, 20×60 minutes of measurements data were simulated, and predictions, looking 50 seconds ahead, were made every 10 seconds within the single 60-minutes time strips. Hence, 7,200 ($= 3,600\text{s}/10\text{s} \times 20$) sets of {predictions vs. measurements} were analysed and statistically evaluated. The study showed that predictions of the acceleration level could be successfully made up to 20 seconds ahead of time for most of the sets (about 85-90%); however, with prediction accuracy reducing beyond this time to a success rate of 10-20% at the end of the prediction intervals (spanning 50 seconds). Various metrics were derived to establish the statistical comparison between the predictions and the (simulated) measurements but, obviously, there is no unique way of doing the comparison of individual time history strips; a fact which also will be addressed later in the present study.

1.2. Content of the study

The investigated procedure by Nielsen and Jensen [22] is also examined in the present study but, herein, the measurements data consist of motion recordings obtained from model-scale experiments rather than numerically simulated time histories. Some of the findings made in [22] are directly applied in the present

89 work and, as as such, the study herein is a continuation of the former one,
90 including the recommended further work.

91 In most studies on stochastic wave-structure interactions, the statistical con-
92 cept of a *stationary process* is important. Indeed, this is so herein and through-
93 out it is a fundamental assumption that conditions are stationary. In principle,
94 this calls for a discussion on requirements for a process to be stationary, or
95 maybe rather a discussion of the theoretical/mathematical consequences if the
96 process is not strictly (nor weakly) stationary. However, this particular dis-
97 cussion is not touched upon, although some remarks are given. Overall, the
98 importance is that stationarity will be assumed; without necessarily stating
99 this.

100 It should also be mentioned that the interest in this study concerns 'standard
101 marine crafts', such as ships or other ship-like structures and floating platforms,
102 and *not* tethered marine structures. On the other hand, the theoretical formu-
103 lations might apply to the latter type of structures; *if* the particular response
104 is characterised by a (Gaussian) stationary process.

105 1.3. Composition of paper

106 The paper has been organised into five main sections, and the remaining
107 four are as follows: In Section 2, the theoretical formulations are outlined with
108 mentioning also about general properties about the (sample) autocorrelation
109 function of a stationary process. The experimental facility, including descrip-
110 tions of the test cases, and pre-analyses of the recorded model-scale data are
111 described in Section 3. All predictions, and associated results and comparisons
112 with measurements, follow in Section 4. Finally, a short summary and an ex-
113 traction of main findings and conclusions are given in Section 5.

114 2. Theoretical formulation

115 The prediction procedure addressed herein is established from the fact that
116 any stationary wave-induced response has some memory in its behaviour. The

117 reminiscence arises due to a memory effect in the exciting force which is gov-
 118 erned by the wave oscillations of the sea surface. The ability of a wave-induced
 119 process to "remember its past" may conversely be expressed by saying that the
 120 future values (= outcomes) of the particular process will be conditional on its
 121 prior outcomes. Thus, it makes sense to introduce the (statistical) concept of
 122 *conditional processes*, and the prediction procedure makes directly use of results
 123 which can be derived from the definition of the joint probability density function
 124 of time-dependent normal distributed variables. Indeed, Lindgren [23] studied
 125 properties of a normal process, and results and general findings were outlined
 126 for the conditional behaviour of a normal process. The mentioned study [23]
 127 is at a somewhat high-level of mathematical abstraction/notation, and some of
 128 the findings have been concretised by Jensen [24] and Andersen et al. [20]. The
 129 following section outlines the expressions that have been derived for a normal
 130 process, conditional on a set of known, i.e., (prior) measured values.

131 The main focus in the article is on application; rather than going through
 132 all the details of the theory. That said, all relevant and necessary theory is
 133 included in the following, but some of the algebra, derived from the original
 134 work by Lindgren [23], has been left out. In order to assist the reader, the
 135 relevant theory has been compressed down to a set of bullet points specified in
 136 subsection 2.2, and the reader may jump directly to this subsection.

137 2.1. *Conditional process based on a set of known values*

138 The measurement $x(t_0) = x_0$ of an arbitrary wave-induced response at an
 139 instant t_0 is considered, and measured values $x(t_1) = x_1, x(t_2) = x_2, \dots, x(t_n) =$
 140 x_n exist at a set of times $t_1 > t_2 > \dots > t_n$ prior to t_0 . Mathematically, the
 141 measurements are described by the stochastic normal process $X(t)$, and the
 142 interest is concerned with the expected mean variation $\widehat{X(t)}$ of $X(t)$ ahead of
 143 current time (i.e. $t > t_0$). By definition, the expected mean variation of the

144 conditional process, conditioning $X(t)$ on its prior values, is given by

$$\begin{aligned}\widehat{X(t)} &\equiv E[X(t)|X(t_0) = x_0, X(t_1) = x_1, \dots, X(t_n) = x_n] \\ &= \int_{-\infty}^{\infty} u \cdot p(u|x_0, x_1, \dots, x_n) du\end{aligned}\quad (1)$$

145 where $p(x|x_0, x_1, \dots, x_n)$ is the conditional probability density function of $X(t)$
 146 given $X(t_0) = x_0, X(t_1) = x_1, \dots, X(t_n) = x_n$. Since the probability den-
 147 sity function of $X(t)$ is normal distributed, the conditional probability density
 148 function will also be (multivariate) normal distributed,

$$p(x|x_0, x_1, \dots, x_n) = \varphi(x(t); \mu_n(t), \sigma_n(t)) \quad (2)$$

149 where $\varphi(x; \mu, \sigma)$ is the probability density function of a normal distributed vari-
 150 able

$$\varphi(x; \mu, \sigma) \equiv \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) \quad (3)$$

151 with mean value μ and standard deviation σ . In the particular case, Eq. (2),
 152 the mean value and the standard deviation are themselves *processes* rather
 153 than variables. Notably, the 'mean value' will be identical to the expected
 154 mean variation, i.e. $\mu_n(t) \equiv \widehat{X(t)}$, which is the very solution to the prediction
 155 problem. The derivation of the explicit formula for $\widehat{X(t)}$, generally expressed
 156 in terms of n prior measured values of $X(t)$, requires some algebra. Below, the
 157 solution will be indicated only for the special case $n = 1$.

158 The conditional probability density function of the process $X(t)$, given $X(t_0) =$
 159 x_0 and $X(t_1) = x_1$, can be written, cf. Jensen [24]

$$p(x(t)|x_0, x_1) = \frac{p(x(t), x_0, x_1)}{p(x_0, x_1)} \quad (4)$$

160 Thus, the interest is in the marginal probability density functions, $p(x_0, x_1)$ and
 161 $p(x(t), x_0, x_1)$, which both are multivariate versions of the normal distribution.
 162 For the k -variate case, with $\mathbf{x}$ being a vector of k elements, the expression reads

$$p(\mathbf{x}) = \frac{1}{\sqrt{|(2\pi)\mathbf{\Sigma}|}} \exp\left(-\frac{1}{2}(\mathbf{x}^T \mathbf{\Sigma}^{-1} \mathbf{x})\right) \quad (5)$$

163 where $\mathbf{\Sigma}$ is the (auto)covariance matrix of $p(\mathbf{x})$, and $|\cdot|$ denotes determinant.
 164 In Eqs. (4)-(5), the autocovariance matrices $\mathbf{\Sigma}_2$ and $\mathbf{\Sigma}_3$ for $p(x_0, x_1)$ and

165 $p(x(t), x_0, x_1)$, respectively, are defined by

$$\Sigma_2 = \begin{bmatrix} E[X(0)^2] & E[X(0)X(t_1)] \\ E[X(t_1)X(0)] & E[X(t_1)^2] \end{bmatrix} \quad (6)$$

166

$$\Sigma_3 = \begin{bmatrix} E[X(t)^2] & E[X(t)X(0)] & E[X(t)X(t_1)] \\ E[X(0)X(t)] & E[X(0)^2] & E[X(0)X(t_1)] \\ E[X(t_1)X(t)] & E[X(t_1)X(0)] & E[X(t_1)^2] \end{bmatrix} \quad (7)$$

167 and, after insertion of the normalised time-dependent autocorrelation function

168 $r(t)$,

$$\Sigma_2 = m_0 \begin{bmatrix} 1 & r(t_1) \\ r(t_1) & 1 \end{bmatrix} \quad (8)$$

169

$$\Sigma_3 = m_0 \begin{bmatrix} 1 & r(t) & r(t-t_1) \\ r(t) & 1 & r(t_1) \\ r(t_1-t) & r(t_1) & 1 \end{bmatrix} \quad (9)$$

170 where $r(t)$ is given by,

$$r(t) = \frac{1}{m_0} E[X(0)X(t)] \quad (10)$$

171 introducing the variance in terms of the 0-th order spectral moment m_0 , and

172 noting $r(t_1 - t) = r(t - t_1)$ for a stationary process. The i-th order spectral

173 moment m_i follows from

$$m_i = \int_0^\infty \omega^i S(\omega) d\omega \quad (11)$$

174 with the spectral density $S(\omega)$, at frequency ω , being the Fourier transform of

175 the (stationary) time domain process $X(t)$. It is noteworthy that the definition

176 of the time-dependent autocorrelation function (Eq. 10) is formulated in the

177 time domain, but for a stationary process the autocorrelation function may

178 alternatively be obtained by a frequency domain calculation,

$$r(t) = \frac{1}{m_0} \int_0^\infty S(\omega) \cos(\omega t) d\omega \quad (12)$$

179 The further steps in the development of the prediction procedure are to

180 insert Eq. (8) and Eq. (9), respectively, into Eq. (5), yielding analytical

181 expressions for the two marginal probability density functions $p(x_0, x_1)$ and

182 $p(x(t), x_0, x_1)$. Subsequently, substitution of these two expressions into Eq. (4)
 183 leads - through (algebraic) matrix multiplication - to an analytic expression for
 184 the conditional probability density function $p(x(t)|x_0, x_1)$. On the other hand,
 185 the assumption is that $p(x(t)|x_0, x_1)$ is given by a normal probability density
 186 function, $\varphi(x(t); \mu_1(t), \sigma_1(t))$, with given *processes* for the mean value $\mu_1(t)$ and
 187 the standard deviation $\sigma_1(t)$, cf. Eqs. (2) and (3). Hence, from the (explicit)
 188 analytic expression of the conditional probability density function it is possible
 189 to define analytic expressions for $\mu_1(t)$ and $\sigma_1(t)$; keeping in mind that the
 190 former yields the actual prediction in search, $\widehat{X(t)} = \mu_1(t)$. Thus, the expected
 191 mean variation, equivalently said the prediction ahead of current time t_0 , can
 192 be calculated from

$$\begin{aligned}\widehat{X(t)} &= \frac{(r(t) - r(t_1)r(t - t_1))x_0 + (r(t - t_1) - r(t)r(t_1))x_1}{1 - r^2(t_1)} \\ &= \frac{1}{1 - r^2(t_1)} [r(t), r(t - t_1)] \begin{bmatrix} 1 & -r(t_1) \\ -r(t_1) & 1 \end{bmatrix} [x_0, x_1]^T \quad (13)\end{aligned}$$

193 In the formula above, only the two most recent measurements, x_0 and x_1 ,
 194 are taken into account. In the general case with a set of n prior values, that is
 195 $n > 1$, the formula for predictions ahead of time t_0 changes accordingly:

$$\widehat{X(t)} = \mathbf{r}^T(t) \mathbf{R}^{-1} \mathbf{x} \quad (14)$$

196 using matrix notation with the 'measurement vector' $\mathbf{x} = [x_0, x_1, x_2, \dots, x_n]^T$.
 197 For at discrete set of (lagged) times, $t_k = k\Delta t, k = 0, 1, 2, \dots, n$ (i.e. $t_0 = 0$), the
 198 autocorrelation vector $\mathbf{r}(t)$ and autocorrelation matrix $\mathbf{R}$ are,

$$\mathbf{r}(t) = [r(t - 0), r(t - \Delta t), r(t - 2\Delta t), \dots, r(t - n\Delta t)]^T \quad (15)$$

199

$$\mathbf{R} = \begin{bmatrix} 1 & r(\Delta t) & r(2\Delta t) & \cdots & r(n\Delta t) \\ r(\Delta t) & 1 & r(\Delta t) & \cdots & r((n-1)\Delta t) \\ r(2\Delta t) & r(\Delta t) & 1 & \cdots & r((n-2)\Delta t) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r(n\Delta t) & r((n-1)\Delta t) & \cdots & r(\Delta t) & 1 \end{bmatrix} \quad (16)$$

200 where it is noted that the autocorrelation matrix is symmetric, with constant
 201 elements on any diagonal, and with ones on the centre diagonal. The autocorre-
 202 lation (row) vector has length $n + 1$, while the autocorrelation (square) matrix
 203 has dimension $(n + 1) \times (n + 1)$. For $n = 1$ it is evident that Eq. (14) becomes
 204 identical to Eq. (13).

205 2.2. Summary

206 The prediction procedure is complete with expression Eq. (14). As a kind
 207 of summary, and with attention to calculations in practice, where stationary
 208 conditions will be assumed, a few important points are worth mentioning:

- 209 1. The autocorrelation matrix (Eq. 16) does not change, and it needs there-
 210 fore to be calculated and inverted only once for the considered range of
 211 stationary data.
- 212 2. The autocorrelation vector (Eq. 15) does not depend on the (instanta-
 213 neous) measured values of $X(t)$ and can be precalculated and re-used for
 214 the set of prior time steps considered at the particular time step(s) of the
 215 (discretised) time t .
- 216 3. Combining (1) and (2) leads to the 'predictive vector' $\mathbf{y}(t)$ which is pre-
 217 calculated, or adapted, to the particular setup of prediction horizon and
 218 prior measurements considered,

$$\mathbf{y}(t) = \mathbf{r}^T(t) \mathbf{R}^{-1}, \text{ size}(\mathbf{y}) = 1 \times (n + 1). \quad (17)$$

- 219 4. In practice, one specific vector, $\mathbf{y}_m$, $m = 1, 2, \dots, M$, is computed/assigned
 220 corresponding to one particular time t_m ahead of current time t_0 . Thus,
 221 on a discrete time interval, t_m , $m = 1, 2, \dots, M$, predictions of the process
 222 $X(t)$ are calculated according to

$$\hat{x}_m = \mathbf{y}_m \mathbf{x} \quad (18)$$

223 noting that $\mathbf{y}_m = [y_{1,m}, y_{2,m}, \dots, y_{n+1,m}]$ and $\mathbf{x} = [x_0, x_1, x_2, \dots, x_n]^T$.

224 The outlined prediction procedure has some resemblance to predictions by the
 225 autoregressive (AR) predictor method, e.g. Zhao et al. [16], but with a main
 226 difference that for the AR procedure the future time step is assumed to be only
 227 the next time step and not a continuous variable as in Eq. (14) (and Eq. 18).
 228 Consequently, any 'standard' AR procedure needs some sort of offline training
 229 to facilitate predictions at *several* time steps ahead of current time. In contrast,
 230 the present prediction procedure (Eq. 14) makes directly use of the system's
 231 correlation structure in terms of the autocorrelation function and, thus, offline
 232 training is *not* needed to make predictions at a number of time steps ahead of
 233 time.

234 Some additional discussions, including comparisons, of theoretical concepts
 235 of various prediction procedures are given in, for instance, [20] and [9]. The
 236 present section is closed by a small theoretical example/illustration which serves
 237 to explain some general aspects of calculations, i.e. predictions, made with Eq.
 238 (14).

239 2.3. Theoretical example

240 A certain response has been monitored and recorded during a stationary pe-
 241 riod. Specifically, a time history recording of the past 30 minutes (see Figure 1)
 242 has been used to estimate the response spectrum and the associated autocorre-
 243 lation function. Some three minutes later is considered as the *current time*, i.e.
 244 "now", where a prediction ahead of time is made. Figure 1 shows the situation;
 245 the upper plot is the 30-minutes time recording while the lower plot is a zoom
 246 around the current time, which is taken to be three minutes later than the end
 247 of the 30-minutes time history recording providing the underlying correlation
 248 structure.

249 At time $t_0 = 33.0$ min., a value x_0 is measured and predictions ahead of
 250 t_0 are made using the past, say, $T_{past} = 20$ seconds of data. Thus, with
 251 the sampling rate to be, for instance, $\Delta t = 0.5$ s, it means that 41 prior
 252 points are considered for predictions, and the measurement vector $\mathbf{x}_{33min} =$
 253 $[x_0, x_1, \dots, x_{40}]^T$ is assigned accordingly. The autocorrelation matrix $\mathbf{R}_{20s}$ has

Figure 1: *Response measurements recorded during 30 minutes of stationary conditions (upper plot) and a zoom around current time $t_0 = 33.0$ min. at which a prediction of the future behaviour is made.*

254 been (pre)constructed with its 41×41 elements, cf. Eq. (16), using the cal-
 255 culated autocorrelation function derived from the 30-minutes (stationary) time
 256 recording. At any one time step ($m\Delta t$), $m = 1, 2, \dots$ ahead of t_0 the predictive
 257 vector $\mathbf{y}_{20s,m} = [y_1, y_2, \dots, y_{41}]_m$ has 41 elements and it is calculated according
 258 to Eq. (17), noting that the vector depends only on the autocorrelation func-
 259 tion, depending itself on just the initial 30-minutes time history recording. In
 260 this scenario, however, prediction is made $T_{predict} = 60$ seconds ahead of t_0 , so a
 261 set of 120 ($= \frac{T_{predict}}{\Delta t}$) predictive vectors is needed; a set that can be stored as a
 262 matrix $\mathbf{Y}_{20s,60s}$, which will be specific to the combination of T_{past} and $T_{predict}$.

263 As a consequence of the above "deduction", any new predictions, made also
 264 60 seconds ahead of a 'new current time' being different from t_0 , can be made
 265 by just changing $\mathbf{x}$, since $\mathbf{y}$ has not changed; assuming no change in the corre-
 266 lation structure of the process at the new *current time*. More generally, from
 267 the illustration-example, it is important to note (and to repeat) that in the
 268 prediction procedure;

- 269 • the measurement vector ($\mathbf{x}_{t_0}$) will be specific to the instant in time when
 270 predictions are made,
- 271 • the autocorrelation function is specific only to the considered stationary

time history recording; and thus the (specific) autocorrelation matrix $\mathbf{R}$, independent of the value of t_0 , can be used for predictions as long as conditions remain in the same 'stationary settings',

- consequently, or similarly, any predictive vector $\mathbf{y}$ does not change with the current time t_0 , whatever the value of t_0 , requiring just that t_0 is not (very) far away, measured on a time scale, from the initial stationary time history recording.

As a closing remark on the theoretical example, but focusing instead on the practical application of a predicted response sequence, one means to exploit such deterministic predictions (e.g., Fig. 1) is to provide the maximum and minimum values of the predicted time sequence. That is, it may not necessarily be important to know that, say, the heave motion will be +0.98 m, 28 seconds ahead of current time. Rather it will be beneficial to know that it is *likely* that the heave motion, *during* the next, say, 30 seconds, reaches a specific level (plus/minus) that makes a particular operation unsafe to carry out. Obviously, for a perfect prediction procedure the term 'likely' will be replaced by 'certain'. Consequently, the evaluation of the prediction procedure could be a matter of comparing *just* predicted max/min values to the corresponding measured max/min values for given prediction sequences. However, as will be addressed in the remaining sections, the evaluation is conducted significantly more thorough.

2.4. Kriging vs. non-Gaussian processes

It turns out¹ that Eq. (14) can be derived also from *Kriging* which is a statistical regression and/or prediction method, where the basic idea is *to predict the value of a function at a given point by computing a weighted average of the known values of the function in the neighborhood of the point* [25, 26, 27, 28]. Thus, the resemblance to the presented prediction procedure is clear.

As such, the derivations of the 'Kriging equations' do not need a specific assumption about (multivariate) Gaussian processes, although some authors will

¹Thanks to an anonymous reviewer.

claim that, in practice, this is necessary and why the Kriging models often are referred to a *Gaussian process models* [29]. Briefly said, the assumptions are 1) Stationarity and 2) Isotropy. However, one limitation is that, in general, the accuracy of interpolation by Kriging will be limited if the number of sampled observations is small, e.g. [26]. Consequently, the Gaussian assumption is implicitly imposed, because of the Central Limit Theorem.

Kriging will not be explored any further in the present article but, obviously, it should be interesting, as a future work, to look closer into Kriging to examine the method for potential use in the context of short-time, deterministic prediction of wave-induced processes. Some useful references can be found on the general topic of Kriging in relation to marine and offshore applications by consulting [30, 31]. To close the discussion about Kriging, and with given knowledge at hand, it appears that the Gaussian assumption can be relaxed, as Eq. (14) can be found by 'Simple Kriging'. Nonetheless, in the work by Lindgren [23], which is the original reference for the present work including Eq. (14), the assumption is a Gaussian process, and therefore the Gaussian assumption is kept herein.

3. Experimental data

3.1. Testing facility

The prediction procedure has been applied to experimental model-scale data obtained in a testing facility at the Marine Cybernetics Laboratory (MCLab) at the Norwegian University of Science and Technology (NTNU), Trondheim. The facility includes a basin with dimensions $40 \text{ m} \times 6.45 \text{ m} \times 1.41 \text{ m}$ ($L \times B \times D$), a vision-based positioning system that provides position and orientation measurements of a dynamic positioned (DP) vessel, and a wave flap² for generating long-crested waves derived from a given wave spectrum. Figure 2 shows the specific model, Cybership 3, in action. The particular ship is a 1:30 scale model

²DHI Wave Synthesizer, www.dhigroup.com.

327 of a platform supply vessel with dimensions $L_{pp} = 1.97$ m and $B = 0.44$ m. It
 328 is equipped with three azimuth thrusters; two at the stern with fixed angles of
 329 $\pm 30^\circ$ and one in the bow at 90° (Fig. 2). The vessel has eight 12 V batteries
 330 supplying power to the thrusters and a National InstrumentsTM CompactRIO
 331 (cRIO) that runs the DP control system. The operator supplies setpoints and
 332 specifies controller-gains from a laptop, and communication between the camera
 system, operator laptop and cRIO is via Ethernet.

Figure 2: *Cybership 3* deployed in the model-basin at NTNU (top), and thruster configuration
 of the vessel (bottom) with measures in meters. [32]

333

334 3.2. Experiments and motion measurements

335 The experimental tests have been run with *Cybership 3* exposed to different
 336 wave scenarios; in each case with the (irregular) sea state specified in terms of a
 337 parameterised wave spectrum that has as input significant wave height H_s and
 338 peak period T_p . The tests are made with different (relative) wave headings β ,
 339 and a summary of the experimental conditions are given in Table 1. All tests
 340 are made at zero-forward speed, and with the experimental conditions fixed for
 341 the single test case; since the prediction procedure requires/assumes stationary
 342 conditions (Section 2). The specific wave heading, used in subcases 'a', 'b' or
 343 'c', is given in the parenthesis {...}, where $\beta = 0^\circ$ is head sea (and $\beta = 180^\circ$

Table 1: *Experimental conditions of the test cases. Note, conditions apply to model scale.*

Case no.	Spectrum	H_s [m]	T_p [s]	β [°]
1a,b,c	JONSWAP	0.04	0.8	{0, 10, 20}
2a,b,c	JONSWAP	0.05	0.9	{0, 10, 20}
3a,b,c	JONSWAP	0.05	1.5	{0, 10, 20}
4a,b,c	Ochi-Hubble	(0.04+0.04)	(0.8+1.5)	{0, 10, 20}

is following sea). It is decided to keep data in model scale throughout; this includes all analyses and associated results.

The use of long-crested wave presumably does not influence the outcome of predictions, neither positively nor negatively, as the resulting stochastic properties of the wave-induced process, i.e. the motion of the vessel response, are unaffected. It should, however, be interesting to examine this hypothesis closer by conducting model-scale (or full-scale) experiment in short-crested seas. In the same line, it will be of no (theoretical) importance whether the ship advances with a *constant* forward speed or is at zero-forward speed, as the wave-induced process is stationary in either case; obviously, taking all other experimental conditions/parameters as constant too.

From Table 1 it is seen that totally 12 (sub)cases are investigated. For each subcase, the components of the six degrees-of-freedom motion of Cybership 3 have been measured and corresponding time history records thus exist. Onwards, it is chosen to focus almost entirely on the heave recordings, although analyses have been also made with roll and pitch; but leaving just a few comments, here and there, on these motion components. It is important to note that, in all of the considered cases, approximately ten minutes of stationary motion recordings are available. For the given sea states, noting the values of associated wave periods T_p (Table 1), 10-minutes recording lengths imply that the vessel encounters about 400-700 single waves, depending on the case (T_p) in study. The motion recordings were initially sampled at 100 Hz but, as a post-process, data has been resampled to 20 Hz. The reason to down-sample is merely a matter of saving memory/storage on the authors' personal computers, and increase computational efficiency as reduced sampling frequency leads to

369 smaller dimension of the autocorrelation matrix. Down-sampling to 20 Hz will
 370 *not* affect any of the global wave-induced responses; not even in model-scale
 371 (1:30).

372 3.3. Pre-analysis of measurements data

373 One example of a heave recording is shown in Figure 3, which shows both
 374 the time history recording and the corresponding periodogram, i.e. the re-
 375 sponse spectrum, of Case 1a. The response spectrum is shown without and
 376 with smoothing (legends 'No smoothing', respectively 'L = 2,400' and 'L =
 377 200') where smoothing is applied using a Parzen window on the estimated au-
 378 tocovariance function. In practice, the spectral calculation has been made with
 379 WAFO [33], and in this case (Fig. 3) the smoothing window functions have
 380 2,400 and 240 elements/weights.³ The one value, L = 2,400, is equivalent to
 381 one fifth of the total number of samples in the particular time history record-
 382 ing. It is noteworthy that this amount of smoothing is used throughout the

³In WAFO, the size of the smoothing window is controlled by parameter L.

Figure 3: *Time history recording (top) and corresponding response spectrum (bottom) with three versions of the spectrum; without and with smoothing controlled by the parameter 'L'.*

Figure 4: *Cumulative distribution functions and corresponding histograms ("internal" plots) of the heave level of Cases 1a and 2a.*

383 forthcoming analyses; however, with no special attempt to "justify" the value
 384 $L = 2,400$, although some further remarks on smoothing and its consequence
 385 are given in Subsection 4.1.

386 Mathematically, the prediction procedure assumes, or requires, data to be
 387 normal distributed; which for most wave-induced (global) vessel responses typ-
 388 ically is a reasonable assumption. Two visual evaluations of the (model-scale)
 389 data studied herein are shown in Figure 4, where the two plots apply to the time
 390 history recordings of cases 1a and 2a. The plots show the empirical cumulative
 391 distribution function (CDF) together with a cumulative normal distribution
 392 function having mean value and standard deviation as calculated from the em-
 393 pirical data of the single case. Additionally, each plot presents, as the smaller
 394 plot inside, a histogram of the empirical distribution including a fitted normal
 395 probability density function. It is seen that the considered cases, 1a and 2a,
 396 *seemingly* represent a normal distributed process, and albeit not shown herein
 397 similar findings/visualisations apply to all the other cases listed in Table 1. The
 398 visual evaluation can be supplemented with a quantitative/objective test using
 399 hypothesis testings, e.g. Anderson-Darling, Kolmogorov-Smirnov, and Lilliefors
 400 [34, 35], where data is tested against the null hypothesis [36, 37] that it follows
 401 some pre-specified distribution; or the alternative that data does not follow the
 402 specific distribution. The outcome of a hypothesis test is usually a logical, 1
 403 or 0, where '1' indicates that the hypothesis is rejected, and '0' means that

404 the test fails to reject the hypothesis. The result, 1 or 0, is based on the *test*
 405 *statistics*, considered as a metric/distance A^2 , relative to a certain significance
 406 level [38, 39]. In the present context 'rejection' thus implies that data is *not*
 407 normally distributed, and the alternative means it *is*. For the specific cases in
 408 Figure 4, the data sets have been tested using the Anderson-Darling test⁴ with
 409 a 5% significance level, and it is interesting to note that the data sets, i.e. the
 410 time history recordings, of Cases 1a and 2a are *not* normally distributed. There-
 411 fore, it will be interesting to see if predictions, on average, behave differently
 412 in terms of agreement relative to measurements, depending on the underlying
 413 probability distribution of the data.

414 The Anderson-Darling test has been applied to all time history recordings,
 415 and the result can be seen in Table 2 which specifies whether data follows a
 416 normal distribution or, the alternative, that it does not; with values 'Yes' or
 417 'No', respectively, in the specific column. The decision is, as mentioned, based
 418 on the test statistics A^2 relative to a 5% significance level where the latter, for
 419 the given time history recordings, directly translates into an associated critical
 420 value c_V . Thus, data is stationary if $A^2 < c_V$, equivalently $c_V - A^2 > 0$, and
 421 otherwise data is not, and Table 2 yields also the relative deviation $\frac{c_V - A^2}{c_V}$ to
 422 indicate the "degree of normality", or the opposite.

423 Additionally, Table 2 presents a summary of a pre-analysis made on the
 424 measurements data. Thus, the table provides some of the (spectral) parameters
 425 characterising the time history recordings; this includes the standard deviation
 426 σ , the mean zero-upcrossing period T_z and the spectral bandwidth parameter
 427 ε . These parameters can all be calculated using the spectral moments (cf. Eq.

⁴The actual computation is performed using the built-in function `adtest` of MATLAB®

428 11),

$$\sigma = \sqrt{m_0} \quad (19)$$

$$T_z = \sqrt{\frac{m_0}{m_2}} \quad (20)$$

$$\varepsilon = \sqrt{1 - \frac{m_2^2}{m_0 m_4}} \quad (21)$$

429 Finally, as the rightmost column in Table 2, the result of another hypothesis
 430 test is shown. This test looks into whether data can be considered as stationary
 431 or not, and is based on the outcome of an 'Augmented Dickey-Fuller test' which
 432 is used to indicate rejection of the presence of a unit root or failure to reject one
 433 in the given time history recording.⁵ Herein, it is understood that the presence
 434 of a unit root implies that data is non-stationary (and may have a trend). The
 435 principle of the test is much the same as the test for a normal distribution,
 436 which means that the actual outcome relies on some test statistics. Contrary
 437 to the test for a normal distribution it is, however, found that all time history
 438 recordings can be considered as stationary and no further remarks are given.

439 4. Results and discussions

440 The experimental data, including the pre-analysis, described in the previous
 441 section will be used to evaluate the prediction procedure outlined in Section
 442 2. The evaluation will be focused on merely the outcome of the prediction
 443 procedure when specific settings are applied. This leaves out any sensitivity
 444 and parameter studies in the following analysis. On the other hand, the work
 445 by Nielsen and Jensen [22] has some detailed studies in this respect for what
 446 reason 'guidance' from [22] is indeed valuable.

447 4.1. Prediction settings

448 In the particular work [22], studies were made on the *prediction settings*
 449 and their influence on any prediction. Specifically, efforts looked at the conse-

⁵The actual test is performed using `adftest` of MATLAB® without augmented difference terms.

Table 2: *Spectral parameters of the underlying time history recordings and results of hypothesis testings with regards to a normal distribution and stationarity, respectively.*

Case	σ [cm]	T_z [s]	ε [-]	Normally distr.	Stationary
1a	0.29	0.62	0.90	'No' (-0.28)	'Yes'
1b	0.25	0.62	0.90	'Yes' (0.21)	'Yes'
1c	0.26	0.64	0.90	'No' (-3.87)	'Yes'
2a	0.50	0.74	0.91	'No' (-5.33)	'Yes'
2b	0.42	0.65	0.90	'No' (-2.25)	'Yes'
2c	0.42	0.67	0.91	'Yes' (0.53)	'Yes'
3a	0.80	0.94	0.96	'No' (-0.97)	'Yes'
3b	0.75	0.92	0.96	'Yes' (0.35)	'Yes'
3c	0.81	0.93	0.95	'Yes' (0.16)	'Yes'
4a	0.64	0.83	0.94	'Yes' (0.43)	'Yes'
4b	0.62	0.79	0.94	'Yes' (0.20)	'Yes'
4c	0.69	0.82	0.94	'Yes' (0.57)	'Yes'

450 quence in applying different "amounts" of prior data, e.g. to consider the past
 451 10 seconds versus 20 seconds of data, relative to current time t_0 , for making
 452 predictions, say, 50 seconds ahead of time t_0 ; with all times in full-scale. More-
 453 over, the importance of settings related to the spectral calculation of the sample
 454 autocorrelation function (ACF) was addressed, since smoothing, as discussed in
 455 Section 3, affects significantly the shape of the periodogram from which the
 456 sample ACF is derived. The most "complete" sample ACF is obtained when no
 457 smoothing is applied to the periodogram, and, in this case, the spectral-version
 458 of the sample ACF will be identical to the sample ACF as if computed directly
 459 according to its definition in the time domain (Eq. 10). However, it is also
 460 known that for zero-smoothing, the sample autocorrelation function may fail
 461 to damp out according to expectation [21, 40]. Consequently, correlation may
 462 appear to last (be present) for longer duration than is actually true, and some
 463 smoothing is therefore necessary. On the other hand, if too much smoothing is
 464 applied to the periodogram, correlation will appear to vanish after only a short
 465 time, or equivalently said the sample ACF damps out too quickly.

466 Previously, it was explained that, in the present study, smoothing is applied
 467 to data using a Parzen window on the estimated autocovariance function and,

468 hence, contributions from covariance at large lags, which are generally not re-
 469 liable, will be small or zero. Three versions of sample ACFs, all obtained from
 470 exactly the same data, are visualised in Figure 5; with the underlying time
 471 history recording and the amounts of smoothing identical to what was studied
 472 previously (Fig. 3). Indeed, it is seen how varying degrees of smoothing may
 473 affect the sample ACF very much. Consequently, a prediction procedure relying
 474 fundamentally on the sample ACF will be influenced by the degree of smoothing
 475 being applied in the spectral calculations. Nonetheless, the conclusions drawn
 476 from the earlier study by Nielsen and Jensen [22], made on a very large set of
 477 numerical time history simulations, suggest, or "prescribe", that predictions,
 478 relative to corresponding measurements, are improved by taking into account
 479 correlation/autocovariance at large lags despite they are not necessarily always
 480 ("mathematically") reliable.

481 In summary, predictions will be made with the following settings, cf. Nielsen
 482 and Jensen [22], which apply to model scale:

- 483 • Predictions ahead of current time t_0 take into account N past measure-
 484 ment points (relative to t_0), where the value of N is equivalent to a time
 485 period $T_{past} = 25T_p$ with T_p given in Table 1.
- 486 • The periodogram is smoothed using a Parzen window on the estimated
 487 autocovariance function. The window function is of size L , where L is
 488 taken as one fifth of the total number of samples in the particular time

Figure 5: *Sample autocorrelation function with a zoom included as the "internal" plot. The underlying time history recording is as seen in Figure 3.*

489 history recording in study.

- 490 • Finally, predictions will be made 7.5 seconds ahead of any current time
491 t_0 , and a new prediction is made every 2.0 second on the 10-minutes time
492 strips. It is noted that 7.5 seconds correspond to about 5-8 wave periods,
493 depending on the case (T_p) in study.

494 These settings are applied to all time history recordings, cf. Tables 1 and 2. On
495 each 10-minutes time history strip, totally 200 prediction sequences have been
496 computed; taking note that the initial 50 seconds and the last approximately
497 90 seconds of any recording are not considered, and remembering also that
498 prediction sequences overlap each other.

499 4.2. Visual and statistical comparisons

500 Figure 6 shows four prediction sequences; all made for data strips taken from
501 Case 2a. The plots include the measured (i.e., the true) response sequences, and,
502 furthermore, two (statistical) numbers, R^2 and ρ , are printed in the upper right
503 corner of each plot. Leaving the two numbers to be defined and explained later,
504 the particular plots reveal good agreement between the predicted and measured
505 response sequences on almost the entire part of the individual data strips. How-
506 ever, generally it is not every prediction sequence of Case 2a which agrees as

Figure 6: Selected heave response sequences of Case 2a; blue full line is measurement and dashed red line is prediction. The determination and correlation coefficients, R^2 respectively ρ (defined later), are seen in the upper right-hand corner.

Figure 7: Entire set of prediction and measurement sequences of Case 2a; noting that there are 2×200 curves on top of each other. Additionally, the pairs of bold lines in cyan (predictions) and green (measurements) indicate the 'point-wise' standard deviation calculated from the 2×200 data points at given time instants.

507 accurate to the measured corresponding one, as seen from the plots/comparisons
 508 in Figure 6. Therefore, to get a better, or more "average", picture of the overall
 509 performance of the prediction procedure, a random excerpt of visual compar-
 510 isons can be seen in Figures A.13-A.15 in Appendix A, which presents sets of
 511 plots similar to those in Figure 6. The sets seen in Figures A.13, A.14, A.15
 512 apply to Cases 2a, 2b, and 2c, respectively, and the excerpts (of prediction vs.
 513 measurement sequences) are simply based on 16 data strips, for each case, cut
 514 out every 20 second starting at $t_0 = 50$ s. Thus, the average performance of
 515 the prediction procedure is better evaluated since no special focus is on "good"
 516 predictions, nor "bad" ones.

517 On the other hand, it is not practically possible, nor feasible, to visually
 518 compare - for all data sets (Cases 1-4), cf. Section 3 - every prediction sequence
 519 with the corresponding measurement sequence in single *and* detailed plots like
 520 studied in, e.g., Figures 6 and A.13-A.15. Therefore, to derive some sort of
 521 'statistical measure' of the average performance of the prediction procedure,
 522 another presentation of the outcome/comparison is studied. Collectively, Fig-
 523 ure 7 shows the results of *all* heave prediction sequences and the corresponding
 524 measurement sequences of Case 2a. The abscissa represent the normalised pre-
 525 diction time ahead of t_0 , where normalisation is made with respect to the wave
 526 peak period T_p (cf. Table 2). In addition to all pairs (prediction vs. mea-

Figure 8: Agreement between single pairs of measurements and predictions for Case 2a with separate results depending on the prediction horizon considered; e.g., $[0 - 3T_p]$ where T_p is the wave peak period.

surement) of data sequences, the curves of the *point-wise* standard deviation are included as pairs of plus/minus versions of it; shown as the green and cyan pairs of bold lines for measurements and predictions, respectively. Thus, keeping in mind, there are 2×200 prediction and measurement points at any instant, where the individual set of points has a mean value of zero and a standard deviation as visualised in Figure 7. From the plot in Figure 7, it is noted that until about $1T_p$ ahead of current time t_0 there is a very good agreement between predictions and measurements. Subsequently, the agreement reduces and at prediction instants from about $3T_p$ and further ahead until the end (taken as $8T_p$), the agreement remains, on average, at the same level. These findings are confirmed by the plot in Figure 8 which presents the agreement between any single pair of data values (prediction vs. measurement) obtained for Case 2a; with specified results dependent on the prediction horizon: $[0 - 1T_p]$, $[0 - 3T_p]$ and $[0 - 8T_p]$, respectively. Moreover, the theoretical line of perfect agreement is included in the plot as the black dashed line.

Appendix B contains pairs of plots equivalent to those in Figures 7 and 8 but considering subcases 2a, 2b, and 2c together. Generally, the observations from the (other) subcases, see Figures B.17 and B.18 in the appendix, are similar to what was addressed above, although the agreements for Cases

2b and 2c reduce to slightly lower levels than found for Case 2a. One particular "characteristic", evident from all cases (2a-c), is the decreasing amplitude levels, equivalently decreasing point-wise standard deviations, for prolonged prediction horizon/interval of the prediction sequences. This observation can be explained mathematically, cf. Lindgren [23], since the predicting process (Eq. 14) is non-stationary with properties resembling the autocorrelation function of a (stochastic) wave-induced process.

As a last visual comparison, see Figure 9, the relative error between corresponding set of heave sequences (prediction versus measurement), as seen in Figure 7, has been calculated for Case 2a; where normalisation is made with respect to the square root of the 0th-order spectral moment, cf. Table 2. The error curves for all corresponding sequences are shown in Figure 9 as the blue (thin) lines. Notably, the plot sheds light on four specific error curves (coloured in green); namely, those four obtained by considering the prediction and measurement sequences shown in Figure 6. Furthermore, the plot in Figure 9 includes the point-wise mean value curve and the ditto curves of plus/minus the point-wise standard deviation (StD) of the errors, where the former curve fluctuates around zero as expected. It is interesting to observe that even for sequences like those studied previously (Figure 6), where the agreement, based on a visual judgement, is apparently very acceptable, still the relative, normalised error is not insignificant; taking note that the four green curves in the plot in Figure 9 exceed the standard deviation of the point-wise error at several instants. Consequently, a large number of the prediction sequences seen in Figure 7 reveals just as good, or better, "visual agreement" as what can be seen from the four individual plots in Figure 6.

Obviously, the various plots like those discussed above are visual indicators of the general performance of the prediction procedure. Nonetheless, the discussion(s) can be supplemented with some quantitative error measures and/or statistical evaluations. In this context, it should be clear that *some* metric(s) must be computed to comprehensively evaluate the overall performance of the prediction procedure, since any visual comparison that can be made from the

Figure 9: *Normalised error between prediction and measurement for all sequences of Case 2a.*

plots in, for instance, Figures 6 and 8 always will be, to some degree, rather subjective. On the other hand, it is not straight forward to define unique and physically meaningful metrics to make comparisons from. This topic has been discussed in several of other similar works, e.g. [41, 18, 13, 22], and the matter is, to some extent, an entire topic in its own right. In the present study, attention is given to two metrics; the one taken as the Pearson Correlation Coefficient, ρ , and the other taken as the Determination Coefficient, R^2 , defined by, respectively,

$$\rho = \frac{\sum_{i=1}^N (\hat{x}_i - \mu_{\hat{x}})(x_i - \mu_x)}{\sqrt{\sum_{i=1}^N (\hat{x}_i - \mu_{\hat{x}})^2} \sqrt{\sum_{i=1}^N (x_i - \mu_x)^2}} \quad (22)$$

$$R^2 = 1 - \frac{\sum_{i=1}^N (\hat{x}_i - x_i)^2}{\sum_{i=1}^N (x_i - \mu_x)^2} \quad (23)$$

Here, the former is a direct measure of the linear dependence (correlation) between the two sequences; the prediction sequence, $\hat{\mathbf{x}}$, and the corresponding measurement sequence, $\mathbf{x}$, on a data strip with totally N pairs of observations $\{\hat{x}_i, x\}$ with mean values $\{\mu_{\hat{x}}, \mu_x\}$ on the specific data strip. The correlation coefficient ρ is 1 for perfect correlation, -1 for anti-correlation, and $-1 < \rho < 1$ for anything in between. The determination coefficient *r-squared* indicates to some degree the 'goodness of fit', on average, for the pairs of observations. It has a value of 1 if the fit is perfect, and otherwise $R^2 < 1$.

Although other metrics could be considered, see Nielsen and Jensen [22],

594 these two metrics have, in their *combined* use, the potential to quantitatively
 595 assess the agreement between predictions and measurements. It is noteworthy
 596 that some studies in the literature focus on only correlation as the measure for
 597 comparison. However, in principle, a correlation coefficient by itself has little
 598 meaning, if not the actual values of measurement and prediction are close to each
 599 other at any given observation point i . Therefore, it is necessary to introduce
 600 also a measure of the agreement between individual observations (prediction vs.
 601 measurement) for what reason the determination coefficient R^2 is used together
 602 with ρ . It is noteworthy that in this particular context, the r-squared value
 603 can lie outside $[0;1]$ with negative values; which is usually not the case for an
 604 r-squared value when the coefficient is calculated/applied in regression analysis
 605 [42, 43]. The issue here is that the determination coefficient, in the present
 606 application, is used in a different way than what is the typical way in (linear)
 607 regression analysis, where a regression model is fitted to data, so that the value
 608 of the coefficient is a measure of how well observed outcomes are replicated
 609 by the *regression* model itself, based on the proportion of total variation of
 610 outcomes explained by the model, cf. [44].

611 The correlation coefficient and the determination coefficient have been com-
 612 puted for every sequence (200 in total) within each of the test cases, cf. Table
 613 1, and specific outcomes of the coefficients are included in Figure 6, where the
 614 values of ρ and R^2 are seen in the upper right-hand corner of each plot. Like-
 615 wise, the values of the metrics appear in the plots of the sequences visualised
 616 in Appendix A. If focus is turned on all the sequences of Case 2a, the result
 617 is presented in Figure 10, and it is clear that the two coefficients, ρ and R^2 ,
 618 show some variation with both higher and lower values, indicating sequences
 619 with good agreement and the opposite, respectively, between predictions and
 620 measurements. The similar plots of Cases 2b and 2c have been included in
 621 Appendix C.

622 Table 3 presents the statistics of all cases, *including* results of roll and
 623 pitch, with the mean value and the coefficient of variation ($\text{CoV} = \text{"standard}$
 624 $\text{dev./mean}"$) noted for the correlation coefficient and the determination coeffi-

Table 3: *Statistics, i.e. mean values, of the correlation coefficient ρ and the determination coefficient R^2 , respectively, with results for heave, roll, and pitch. Note, the coefficient of variation (CoV) is included in parenthesis.*

Case	Heave		Roll		Pitch	
	ρ [-]	R^2 [-]	ρ [-]	R^2 [-]	ρ [-]	R^2 [-]
1a	0.54(0.50)	0.26(1.22)	0.67(0.33)	0.43(0.74)	0.42(0.65)	0.17(1.32)
1b	0.48(0.56)	0.21(1.34)	0.53(0.52)	0.27(1.10)	0.50(0.50)	0.24(1.12)
1c	0.42(0.56)	0.15(1.52)	0.50(0.55)	0.24(1.29)	0.46(0.53)	0.19(1.37)
2a	0.67(0.29)	0.44(0.59)	0.62(0.40)	0.35(0.98)	0.42(0.51)	0.17(1.18)
2b	0.56(0.45)	0.31(0.91)	0.62(0.46)	0.36(0.93)	0.47(0.48)	0.21(1.10)
2c	0.40(0.67)	0.16(1.55)	0.55(0.62)	0.30(1.24)	0.47(0.70)	0.22(1.42)
3a	0.54(0.42)	0.28(0.94)	0.65(0.36)	0.40(0.82)	0.49(0.46)	0.24(1.01)
3b	0.53(0.51)	0.26(1.25)	0.61(0.47)	0.35(1.00)	0.55(0.45)	0.28(0.96)
3c	0.53(0.45)	0.28(0.88)	0.61(0.40)	0.35(0.92)	0.54(0.42)	0.29(0.76)
4a	0.48(0.40)	0.23(0.83)	0.49(0.39)	0.32(0.96)	0.45(0.45)	0.20(0.93)
4b	0.48(0.39)	0.21(0.95)	0.58(0.45)	0.30(1.18)	0.44(0.45)	0.17(1.18)
4c	0.48(0.43)	0.23(0.85)	0.54(0.58)	0.27(1.36)	0.47(0.41)	0.21(0.96)
Average	0.51	0.25	0.58	0.33	0.47	0.22

cient, respectively.

It can be argued that the results presented in Table 3, i.e. the determination and the correlation coefficients, have the most meaning when they are discussed in relative terms and not considered as absolute statistical measures

Figure 10: *Determination coefficients R^2 (red) and correlation coefficients ρ (blue) of Case 2a. The results corresponding to the sequences from Figure 6 are indicated by the larger marker sizes in colours black and green in contrast to red and blue, respectively.*

629 and/or performance indicators. Thus, the coefficients should be rather used as
 630 *relative* indicators of the performance of the prediction procedure, when this is
 631 applied under different, but specific, settings and to various, but similar, exper-
 632 imental conditions (including vessel type, sea state, motions/responses, etc.).
 633 For instance, Table 3 suggests that heave, on average, may be predicted most
 634 accurately when the vessel faces the waves head sea (subcases 'a'), since the
 635 correlation coefficient and the determination coefficient consistently attain the
 636 highest average values in these cases; compared to headings off head sea (sub-
 637 cases 'b' and 'c'). An almost similar finding is observed for roll but not for
 638 pitch. The table also reveals that roll of the three responses, for the considered
 639 ship and sea states, can be predicted with the best accuracy. Albeit not shown
 640 (directly), it is in itself interesting that significant roll is actually induced even
 641 when the heading is straight head sea (and also slightly off). The physical expla-
 642 nation may be that some wave reflection occurs from the tank wall sides, and/or
 643 the explanation may be because of the DP system. This issue is, however, not
 644 considered any further in the present study but another should try to resolve
 645 the "problem".

646 Previously, all the time history recordings were tested for their probability
 647 distribution to be of a normal distribution type, cf. Table 2. It is interesting to
 648 note from Table 3 that it does not seem to be of any role whether the data is
 649 normally distributed or not, when the (average) agreement between prediction
 650 and measurement sequences are studied. Thus, the best results for heave are
 651 found for Cases 2a and 2b, where data - according to the Anderson-Darling
 652 test - should not be considered to be normally distributed. It is therefore of no
 653 fundamental importance that data, in practice, follows a normal distribution,
 654 despite the theoretical formulation of the prediction procedure assumes that
 655 data originates from a normal distributed process, cf. Lindgren [23].

656 Another way to make use of the correlation coefficient and the determination
 657 coefficient is to study their behaviour with the prediction horizon (ahead of
 658 current time). This sort of analysis can be used to evaluate, in relative terms,
 659 when predictions statistically will be less reliable. Table 4 presents the result of

Table 4: *Heave statistics: Behaviour of the correlation coefficient ρ and the determination coefficient R^2 with prediction horizon ahead of t_0 . Note, the coefficient of variation (CoV) is included in parenthesis.*

Case	[0-2] s		[0-4] s		[0-6] s	
	ρ [-]	R^2 [-]	ρ [-]	R^2 [-]	ρ [-]	R^2 [-]
1a	0.72(0.46)	0.42(1.55)	0.59(0.57)	0.31(1.68)	0.55(0.53)	0.28(1.38)
1b	0.66(0.50)	0.36(1.55)	0.53(0.59)	0.25(1.66)	0.49(0.58)	0.22(1.46)
1c	0.66(0.45)	0.32(1.69)	0.51(0.53)	0.22(1.56)	0.45(0.56)	0.17(1.46)
2a	0.80(0.31)	0.53(1.21)	0.72(0.31)	0.47(0.80)	0.69(0.29)	0.45(0.63)
2b	0.77(0.32)	0.51(0.92)	0.65(0.40)	0.38(0.96)	0.59(0.43)	0.34(0.89)
2c	0.70(0.39)	0.43(1.00)	0.53(0.56)	0.28(1.22)	0.44(0.64)	0.19(1.48)
3a	0.78(0.32)	0.53(0.88)	0.66(0.41)	0.38(1.15)	0.58(0.43)	0.32(1.02)
3b	0.80(0.27)	0.53(1.04)	0.66(0.40)	0.38(1.09)	0.57(0.49)	0.29(1.38)
3c	0.78(0.34)	0.58(0.67)	0.65(0.44)	0.41(0.88)	0.57(0.43)	0.32(0.86)
4a	0.69(0.39)	0.44(0.91)	0.57(0.40)	0.32(0.85)	0.51(0.39)	0.26(0.87)
4b	0.67(0.40)	0.41(1.15)	0.56(0.43)	0.29(1.12)	0.50(0.42)	0.24(1.06)
4c	0.69(0.38)	0.41(1.13)	0.57(0.43)	0.31(0.87)	0.51(0.43)	0.26(0.81)
Average	0.73	0.46	0.60	0.33	0.54	0.28

such an analysis made for the heave sequences alone; omitting results of roll and pitch. It is seen that the correlation coefficient and the determination coefficient have been calculated for prediction horizons: [0-2] s, [0-4] s, and [0-6] s. The content of Table 4 has been visualised in Figures 11 and 12, where the data from Table 3 is also included, since this data, of course, represent the full prediction horizon [0-7.5] s. Basically, Table 4 yields a (consistent) quantification of the graphical result presented previously in Figure 8, where the agreement at single time instants with varying prediction horizons was considered for one specific subcase.

Figures 11 and 12 confirm, not surprisingly, what was previously discussed about "reducing" agreement for prolonged prediction horizon. However, it is indeed interesting to see that the largest relative reduction occurs consistently, and for both ρ and R^2 , as the prediction horizon is increased from [0-2] s to [0-4] s, whereas the relative reduction is smaller for the larger horizons. This in-

674 dicates that successful predictions, with insignificant reduction in the accuracy,
675 may be obtained for even larger horizons than considered in the present study;
676 leaving the actual investigation for a future study.

677 5. Summary and conclusions

678 In the article, a procedure facilitating short-time, deterministic prediction
679 of wave-induced vessel responses has been presented. The predicted response
680 sequence applies to a given time horizon in the order 15-60 seconds ahead of cur-
681 rent time, and is deterministic in the sense that it is the actual (time-dependent)
682 response oscillation that is computed. The prediction procedure does not need
683 information about the exciting wave scenario; neither in terms of the sea surface
684 elevation nor in terms of a (statistical) wave spectrum. Merely, the procedure
685 requires discretely sampled measurements data of the vessel response to be
686 predicted, so that the only input is the measured time history recording. The
687 procedure is not dependent on off-line training and, thus, predictive calculations

Figure 11: *Heave correlation coefficients depending on the prediction horizon.*

Figure 12: *Heave determination coefficients depending on the prediction horizon.*

can run real-time. Mathematically, the procedure relies on the observed (measured) sample autocorrelation function of the particular wave-induced response in study. The response is considered to be of a normal distributed process and, in theory, stationary conditions should apply, since the sample autocorrelation function is not reliable otherwise.

The study herein was a direct continuation of earlier studies [20, 22] but, for the first time, the prediction procedure has been applied to model-scale data. The experimental data has been obtained from tests conducted at the 'MCLab' at the Norwegian University of Science and Technology, where a 1:30 scale model of a platform supply vessel was exposed to various long-crested, irregular wave scenarios. The main conclusions from the present study do not contradict any of the previous findings in [20, 22]. Especially, the following bullets are noteworthy; emphasising that the list not only draws conclusions from work explicitly shown herein but includes also findings from [22] that have been confirmed/elaborated on in the present work without including detailed discussions:

- 703 • Deterministic predictions ahead of time can be made successfully on a
704 given time horizon. In the present study, predictions were computed 7.5
705 seconds ahead of time. In full scale, this corresponds to a prediction hori-
706 zon of 41 seconds for the particular vessel. In the tested wave conditions,
707 this time horizon is equivalent to about 8-9 wave periods ahead of current
708 time.
- 709 • The accuracy of predictions reduces as the prediction horizon is increased.
710 Generally, for the shorter horizons ahead of time the deviations between
711 prediction and measurement sequences are explained primarily because of
712 a small delay/lag in the prediction. At times further ahead, deviations
713 are present also because the actual values of predictions, at particular
714 instants of time, are off compared to the measured values. This behaviour
715 is seen because the predicted response sequence is non-stationary with
716 properties resembling the autocorrelation function of a (stochastic) wave-
717 induced process.
- 718 • The accuracy of the prediction procedure is highly related to the correla-
719 tion structure of the actual process, as the autocorrelation function is a
720 direct measure of the hydrodynamic memory in the system. Thus, smooth-
721 ing of the autocorrelation function or, vice versa, the response spectrum
722 will be influencing the outcome of computed predictions.
- 723 • Albeit *some* smoothing must be applied to diminish the influence of co-
724 variance contributions at (very) large lags, which are generally not reli-
725 able, it is vital to keep *some* correlation, as the prediction horizon thus is
726 extended.

727 5.1. Further work

728 The presented work and the associated results show that the considered pre-
729 diction procedure has the potential to calculate accurately, in real-time, the
730 future wave-induced behaviour of a vessel. This ability will be indeed valuable,

731 as it can reduce significantly the probability of failure of many marine opera-
 732 tions. Nonetheless, and before the procedure is applied in real-case applications
 733 to assist in execution of practical operations, the prediction procedure should
 734 be examined further. Thus, it will be relevant to consider some, or all, of the
 735 points and/or questions below:

- 736 • Previously, the method, as is, has been applied to simulated data [22] and
 737 the present article considers model-scale data. This means that stationar-
 738 ity can be taken as a good assumption. It should be useful to examine the
 739 procedure with full-scale data; either obtained through dedicated sea tri-
 740 als, or from measurements recorded on an operating vessel, since strictly
 741 speaking stationarity does never occur in real-world conditions.
- 742 • The effect/influence of smoothing has not been fully explored, and sensi-
 743 tivity studies in this respect will provide useful knowledge. In the same
 744 line, it should be tested what is the maximum prediction horizon ahead of
 745 time, and what will it depend on; taking that any such 'maximum horizon'
 746 exists, i.e. can be calculated. In this context, statistical metrics/measures
 747 of the *goodness-of-fit* needs to be explored.
- 748 • Why are predictions good, when they are; or conversely, under which con-
 749 ditions are predictions typically not reliable/accurate. Obviously, keeping
 750 in mind here that in real-case scenarios it will be just as important to
 751 know when a marine operation should *not* be conducted, as it is to know
 752 when the operation (most likely) can be safely conducted.
- 753 • The prediction of the response, at any instant ahead of time, is the main
 754 objective. However, is it possible to associate some sort of 'limiting en-
 755 velope' which estimates upper and lower bounds on the actual prediction
 756 sequence. In practical applications/exploitations, this sort of knowledge
 757 is more useful than knowing that a response may take a given value at a
 758 specific time.
- 759 • Is it possible to use knowledge of the probability distribution, or other sta-

tistical properties, of the errors between previously-made sets of prediction and measurement sequences; both "good" ones and "bad" ones.

- Finally, in a more distant future, it could be interesting to set up a comparative study which will evaluate the performance of different (deterministic) prediction procedures, including the ones requiring offline training (e.g., neural networks, autoregressive-procedures).

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884 **Appendix A. Prediction and measurement sequences**

Figure A.13: Heave response sequences of Case 2a; blue full line is measurement and dashed red line is prediction. The determination and correlation coefficients, R^2 respectively ρ , are seen in the upper right-hand corner.

Figure A.14: Heave response sequences of Case 2b; blue full line is measurement and dashed red line is prediction. The determination and correlation coefficients, R^2 respectively ρ , are seen in the upper right-hand corner.

Figure A.15: Heave response sequences of Case 2c; blue full line is measurement and dashed red line is prediction. The determination and correlation coefficients, R^2 respectively ρ , are seen in the upper right-hand corner.

Figure B.16: *Case 2a: Heave data sequences (left) and pair-wise comparison (right) of predictions and measurement. The plots are identical to the plots in Figures 7 and 8.*

Figure B.17: *Case 2b: Heave data sequences (left) and pair-wise comparison (right) of predictions and measurement.*

Figure B.18: *Case 2c: Heave data sequences (left) and pair-wise comparison (right) of predictions and measurement.*

Figure C.19: *Determination coefficients R^2 (red) and correlation coefficients ρ (blue) of Case 2b.*

Figure C.20: *Determination coefficients R^2 (red) and correlation coefficients ρ (blue) of Case 2c.*