Compressive Online Robust Principal Component Analysis with Multiple Prior Information

Van Luong, Huynh; Deligiannis, Nikos; Seiler, Jürgen; Kaup, André; Forchhammer, Søren

Publication date:
2017

Document Version
Publisher's PDF, also known as Version of record

Link back to DTU Orbit

Citation (APA):
1. Motivation

- Applications: Computer vision, web data analysis, anomaly detection, and data visualization, etc.
- Robust Principal Component Analysis (RPCA): Batch-based, decomposes all data samples (matrix M) into low-rank (L) and sparse (S), e.g., all frames in a video, high computational and memory requirements

\[ \min_{L, S} \| L \|_*, \lambda \| S \|_1 \text{ subject to } M = L + S \]

Challenges

- Online method processing a sequence of signals per time instance from a small set of measurements: \( y_t = \Phi(x_t + v_t) \)

\[ M_t = L_t + S_t \]

- Minimization at time instance \( t \)

\[ \min_{L_t, S_t} \| L_t \|_*, \lambda \| S_t \|_1 \text{ subject to } y_t = \Phi(x_t + v_t) \]

where \( \lambda > 0 \) and \( \beta > 0 \) are weightings across the side information signals, and \( \Phi \) is a diagonal matrix with weights for each element in the side information signal \( x_t \), namely, \( \Phi = \text{diag}[w_1, w_2, \ldots, w_n] \)

2. Compressive Online RPCA (CORPCA) With Multiple Prior Information

Problem formulation

- Incorporating multiple prior information: at time instance \( t \) we observe \( y_t = \Phi(x_t + v_t) \) with \( y_t \in \mathbb{R}^n \) given priors \( Z_{t-1} = [z_{t-1,1}, \ldots, z_{t-1,m}] \) and \( B_{t-1} = [b_{t-1,1}, \ldots, b_{t-1,m}] \)

- Solving the \( n - 1 \) minimization problem

\[ \min_{x_t, v_t} \| \Phi(x_t + v_t) - y_t \|_2^2 + \lambda \| x_t \|_1 \]

- Solving the \( n - 1 \) minimization problem

\[ \min_{x_t, v_t} \| \Phi(x_t + v_t) - y_t \|_2^2 + \lambda \| x_t \|_1 \]

- Minimization at time instance \( t \)

\[ \min_{L_t, S_t} \| L_t \|_*, \lambda \| S_t \|_1 \text{ subject to } y_t = \Phi(x_t + v_t) \]

The CORPCA algorithm

- Solving \( n - 1 \) minimization via the soft thresholding operator and the single value thresholding operator, at iteration \( k + 1 \)

\[ w_k^{(1)} = \arg \min_{w_t} \| w_t \|_1 \text{ subject to } \| w_t \|_2 \leq 1 \]

\[ w_k^{(2)} = \arg \min_{w_t} \| w_t \|_2 \text{ subject to } \| w_t \|_1 \leq 1 \]

where \( f(w_t) = \| w_t \|_1 \text{ subject to } \| w_t \|_2 \leq 1 \)

- Updating weights \( j \) and \( W_j \)

- After solving for time instance \( t \): Prior updates

3. Experimental Results

- Synthetic data

- Generating low-rank components: \( n = 500, d = 100 \) (training), \( n = 100 \) (testing), \( r = 5 \) (rank)

\[ L = UV^T, \text{ where } U \in \mathbb{R}^{n \times r} \text{ and } V \in \mathbb{R}^{d \times r} \]

yields \( L = [v_1, \ldots, v_r] \)

- Generating sparse components with \( \| x_0 \|_0 = 0 \)

\[ \text{obtaining } S = [x_1, \ldots, x_r] \]

- Testing on \( M = [x_0, x_1, \ldots, x_r, v_0, v_1, \ldots, v_r] \)

- Measuring probabilities of successful decomposition, \( \text{Pr(success)} \), success if \( \| x_0 - x_1 \|_2 \| x_1 \|_2 \leq 10^{-2} \)

- Compressive video foreground-background separation

- Considering two videos, Bootstrap (60x80 pixels) and Curtain (60x80 pixels) having a static and a dynamic background, respectively

- Background-foreground video separation with full access to the video data

- Compressive separating by varying rates on the number of measurements \( m \) over the dimension of the data \( n \)

4. Summary

Solution for an \( n - 1 \) minimization

- Incorporating efficiently multiple prior information

- Updating iteratively weights

The proposed CORPCA algorithm

- Processing a data vector per time instance using compressive measurements

- Solving the \( n - 1 \) minimization and updating priors for the next instance

Evaluation of CORPCA on synthetic data and actual video data

- Outperforming classical compressive sensing (CS) (\( l_1 \) minimization) and CS with single prior information (\( l_1 + l_1 \) minimization)

- The superior performance improvement compared to the existing methods


Fig. 1. Average success probabilities for CORPCA (for \( m \), \( n \), \( d \), ReProCS for \( m \), and GRASTA for \( m \)). The scale \( n \) is proportional to \( \text{Pr(success)} \) from black to white.

Fig. 2. Background and foreground separation for the different separation methods with full data access: ReProCS, GRASTA, and Curse[246].

Fig. 3. Compressive background and foreground separation of CORPCA with different measurement rates \( m/n \).

Fig. 4. Compressive foreground separation of ReProCS with different measurement rates \( m/n \).