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COMPRESSIVE ONLINE ROBUST PRINCIPAL COMPONENT ANALYSIS WITH MULTIPLE PRIOR INFORMATION

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1. Motivation

- Applications: Computer vision, web data analysis, anomaly detection, and data visualization, etc.
- Robust Principal Component Analysis (RPCA): Batch-based, decomposes all data samples (matrix $\boldsymbol{M}$) into low-rank ($\boldsymbol{L}$) and sparse ($\boldsymbol{S}$), e.g., all frames in a video, high computational and memory requirements

$$\min_{\boldsymbol{L},\boldsymbol{S}} \|\boldsymbol{L}\|_* + \lambda\|\boldsymbol{S}\|_1 \text{ subject to } \boldsymbol{M} = \boldsymbol{L} + \boldsymbol{S}$$

Challenges

- Online method processing a sequence of signals per time instance from a small set of measurements $\boldsymbol{y}_t = \boldsymbol{\Phi}(\boldsymbol{x}_t + \boldsymbol{v}_t)$

$$\boldsymbol{M}_t = \boldsymbol{L}_t + \boldsymbol{S}_t \text{ into } \boldsymbol{S}_t = [\boldsymbol{x}_1\ \boldsymbol{x}_1\ \dots\ \boldsymbol{x}_t] \text{ and } \boldsymbol{L}_t = [\boldsymbol{v}_1\ \boldsymbol{v}_2\ \dots\ \boldsymbol{v}_t]$$

➢ Minimization at time instance t

$$\min_{\boldsymbol{x}_t,\boldsymbol{v}_t} \|[\boldsymbol{L}_{t-1}\ \boldsymbol{v}_t]\|_* + \lambda\|\boldsymbol{x}_t\|_1 \text{ subject to } \boldsymbol{y}_t = \boldsymbol{\Phi}(\boldsymbol{x}_t + \boldsymbol{v}_t)$$

where $\boldsymbol{L}_{t-1} = [\boldsymbol{v}_1\ \boldsymbol{v}_2\ \dots\ \boldsymbol{v}_{t-1}]$, $\boldsymbol{S}_{t-1} = [\boldsymbol{x}_1\ \boldsymbol{x}_1\ \dots\ \boldsymbol{x}_{t-1}]$, $\boldsymbol{\Phi}$ are given

2. Compressive Online RPCA (CORPCA) With Multiple Prior Information

Problem formulation

- Incorporating multiple prior information: at time instance t we observe $\boldsymbol{y}_t = \boldsymbol{\Phi}(\boldsymbol{x}_t + \boldsymbol{v}_t)$ with $\boldsymbol{y}_t \in \mathbb{R}^m$ given priors $\boldsymbol{Z}_{t-1}$ and $\boldsymbol{B}_{t-1}$ from $\{\hat{\boldsymbol{x}}_1, \dots, \hat{\boldsymbol{x}}_{t-1}\}$ and $\{\hat{\boldsymbol{v}}_1, \dots, \hat{\boldsymbol{v}}_{t-1}\}$
- Solving the n-ℓ_1 minimization problem

$$\min_{\boldsymbol{x}_t,\boldsymbol{v}_t}\Big\{H(\boldsymbol{x}_t,\boldsymbol{v}_t|\boldsymbol{y}_t,\boldsymbol{Z}_{t-1},\boldsymbol{B}_{t-1}) = \frac{1}{2}\|\boldsymbol{\Phi}(\boldsymbol{x}_t+\boldsymbol{v}_t)-\boldsymbol{y}_t\|_2^2 + \lambda\mu\sum_{j=0}^{J}\beta_j\|\mathbf{W}_j(\boldsymbol{x}_t-\boldsymbol{z}_j)\|_1 + \mu\Big\|[\boldsymbol{B}_{t-1}\ \boldsymbol{v}_t]\Big\|_*\Big\},$$

where $\lambda > 0$ and $\beta_j > 0$ are weights across the side information signals, and $\mathbf{W}_j$ is a diagonal matrix with weights for each element in the side information signal $\boldsymbol{z}_j$; namely, $\mathbf{W}_j = \mathrm{diag}(w_{j1}, w_{j2}, \dots, w_{jn})$ with $w_{ji} > 0$ being the weight for the i-th element in the $\boldsymbol{z}_j$ vector.

The CORPCA algorithm

- Solving n-ℓ_1 minimization via the soft thresholding operator and the single value thresholding operator, at iteration $k + 1$

$$\boldsymbol{v}_t^{(k+1)} = \arg\min_{\boldsymbol{v}_t}\Big\{\mu h(\boldsymbol{v}_t) + \Big\|\boldsymbol{v}_t - \Big(\boldsymbol{v}_t^{(k)} - \frac{1}{2}\nabla_{\boldsymbol{v}_t} f(\boldsymbol{v}_t^{(k)}, \boldsymbol{x}_t^{(k)})\Big)\Big\|_2^2\Big\}$$

$$\boldsymbol{x}_t^{(k+1)} = \arg\min_{\boldsymbol{x}_t}\Big\{\mu g(\boldsymbol{x}_t) + \Big\|\boldsymbol{x}_t - \Big(\boldsymbol{x}_t^{(k)} - \frac{1}{2}\nabla_{\boldsymbol{x}_t} f(\boldsymbol{v}_t^{(k)}, \boldsymbol{x}_t^{(k)})\Big)\Big\|_2^2\Big\}$$

where $f(\boldsymbol{v}_t, \boldsymbol{x}_t) = (1/2)\|\boldsymbol{\Phi}(\boldsymbol{x}_t + \boldsymbol{v}_t) - \boldsymbol{y}_t\|_2^2$
$g(\boldsymbol{x}_t) = \lambda\sum_{j=0}^{J}\beta_j\|\mathbf{W}_j(\boldsymbol{x}_t - \boldsymbol{z}_j)\|_1$, and $h(\boldsymbol{v}_t) = \|[\boldsymbol{B}_{t-1}\ \boldsymbol{v}_t]\|_*$

- Updating weights β_j and $\mathbf{W}_j$
- After solving for time instance t: Prior updates

$$\boldsymbol{Z}_t := \{\boldsymbol{z}_j = \boldsymbol{x}_{t-J+j}\}_{j=1}^{J}$$

$$\boldsymbol{B}_t = \boldsymbol{U}_t(:, 1:d)\boldsymbol{\Gamma}_{\frac{\mu_k}{2}\,g_1}(\boldsymbol{\Sigma}_t)(1:d, 1:d)\boldsymbol{V}_t(:, 1:d)^{\mathrm{T}}$$

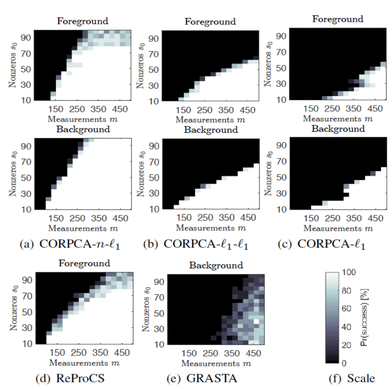
Algorithm 1: The proposed CORPCA algorithm.

Input: $\boldsymbol{y}_t$, $\boldsymbol{\Phi}$, $\boldsymbol{Z}_{t-1}$, $\boldsymbol{B}_{t-1}$;
Output: $\widehat{\boldsymbol{x}}_t$, $\widehat{\boldsymbol{v}}_t$, $\boldsymbol{Z}_t$, $\boldsymbol{B}_t$;
// Initialize variables and parameters.
$\boldsymbol{x}_t^{(-1)} = \boldsymbol{x}_t^{(0)} = 0$; $\boldsymbol{v}_t^{(-1)} = \boldsymbol{v}_t^{(0)} = 0$; $\xi_{-1} = \xi_0 = 1$; $\mu_0 = 0$; $\bar{\mu} > 0$; $\lambda > 0$; $0 < \epsilon < 1$; $k = 0$; $g_1(\cdot) = \|\cdot\|_1$;
while *not converged* **do**
  // Solve Problem (4).
  $\widetilde{\boldsymbol{v}}_t^{(k)} = \boldsymbol{v}_t^{(k)} + \frac{\xi_{k-1}-1}{\xi_k}(\boldsymbol{v}_t^{(k)} - \boldsymbol{v}_t^{(k-1)})$;
  $\widetilde{\boldsymbol{x}}_t^{(k)} = \boldsymbol{x}_t^{(k)} + \frac{\xi_{k-1}-1}{\xi_k}(\boldsymbol{x}_t^{(k)} - \boldsymbol{x}_t^{(k-1)})$;
  $\nabla_{\boldsymbol{v}_t} f(\widetilde{\boldsymbol{v}}_t^{(k)}, \widetilde{\boldsymbol{x}}_t^{(k)}) = \nabla_{\boldsymbol{x}_t} f(\widetilde{\boldsymbol{v}}_t^{(k)}, \widetilde{\boldsymbol{x}}_t^{(k)}) = \boldsymbol{\Phi}^{\mathrm{T}}\big(\boldsymbol{\Phi}(\widetilde{\boldsymbol{v}}_t^{(k)} + \widetilde{\boldsymbol{x}}_t^{(k)}) - \boldsymbol{y}_t\big)$;
  $(\boldsymbol{U}_t, \boldsymbol{\Sigma}_t, \boldsymbol{V}_t) = \mathrm{incSVD}\Big(\Big[\boldsymbol{B}_{t-1}\ \Big(\widetilde{\boldsymbol{v}}_t^{(k)} - \frac{1}{2}\nabla_{\boldsymbol{v}_t} f(\widetilde{\boldsymbol{v}}_t^{(k)}, \widetilde{\boldsymbol{x}}_t^{(k)})\Big)\Big]\Big)$;
  $\boldsymbol{\Theta}_t = \boldsymbol{U}_t\boldsymbol{\Gamma}_{\frac{\mu_k}{2}\,g_1}(\boldsymbol{\Sigma}_t)\boldsymbol{V}_t^{\mathrm{T}}$;
  $\boldsymbol{v}_t^{(k+1)} = \boldsymbol{\Theta}_t(:, \mathrm{end})$;
  $\boldsymbol{x}_t^{(k+1)} = \Gamma_{\frac{\mu_k}{2}\,g}\Big(\widetilde{\boldsymbol{x}}_t^{(k)} - \frac{1}{2}\nabla_{\boldsymbol{x}_t} f(\widetilde{\boldsymbol{v}}_t^{(k)}, \widetilde{\boldsymbol{x}}_t^{(k)})\Big)$; where $\Gamma_{\frac{\mu_k}{2}\,g}(\cdot)$ is given as in RAMSIA [18];
  // Compute the updated weights [18].
  $w_{ji} = \frac{n(|x_{ti}^{(k+1)} - z_{ji}| + \epsilon)^{-1}}{\sum_{l=1}^{n}(|x_{tl}^{(k+1)} - z_{jl}| + \epsilon)^{-1}}$;
  $\beta_j = \frac{\Big(\|\mathbf{W}_j(\boldsymbol{x}_t^{(k+1)} - \boldsymbol{z}_j)\|_1 + \epsilon\Big)^{-1}}{\sum_{l=0}^{J}\Big(\|\mathbf{W}_l(\boldsymbol{x}_t^{(k+1)} - \boldsymbol{z}_l)\|_1 + \epsilon\Big)^{-1}}$;
  $\xi_{k+1} = (1 + \sqrt{1 + 4\xi_k^2})/2$; $\mu_{k+1} = \max(\epsilon\mu_k, \bar{\mu})$;
  $k = k + 1$;
end
// Update prior information.
$\boldsymbol{Z}_t := \{\boldsymbol{z}_j = \boldsymbol{x}_{t-J+j}^{(k+1)}\}_{j=1}^{J}$;
$\boldsymbol{B}_t = \boldsymbol{U}_t(:, 1:d)\boldsymbol{\Gamma}_{\frac{\mu_k}{2}\,g_1}(\boldsymbol{\Sigma}_t)(1:d, 1:d)\boldsymbol{V}_t(:, 1:d)^{\mathrm{T}}$;
return $\widehat{\boldsymbol{x}}_t = \boldsymbol{x}_t^{(k+1)}$, $\widehat{\boldsymbol{v}}_t = \boldsymbol{v}_t^{(k+1)}$, $\boldsymbol{Z}_t$, $\boldsymbol{B}_t$;

3. Experimental Results

Synthetic data

- Generating low-rank components: n = 500, d = 100 (training), q = 100 (testing), r = 5 (rank)
  $\boldsymbol{L} = \boldsymbol{U}\boldsymbol{V}^{\mathrm{T}}$, where $\boldsymbol{U} \in \mathbb{R}^{n\times r}$ and $\boldsymbol{V} \in \mathbb{R}^{(d+q)\times r}$ yields $\boldsymbol{L} = [\boldsymbol{v}_1\ \dots\ \boldsymbol{v}_{d+q}]$
- Generating sparse components with $\|\boldsymbol{x}_t\|_0 = s_0$ and $\|\boldsymbol{x}_t - \boldsymbol{x}_{t-1}\|_0 = s_0/2$ obtaining $\boldsymbol{S} = [\boldsymbol{x}_1\ \dots\ \boldsymbol{x}_{d+q}]$
- Testing on $\boldsymbol{M} = [\boldsymbol{x}_{d+1} + \boldsymbol{v}_{d+1}\ \dots\ \boldsymbol{x}_{d+q} + \boldsymbol{v}_{d+q}]$
- Measuring probabilities of successful decomposition, Pr(success), success if $\|\widehat{\boldsymbol{x}}_t - \boldsymbol{x}_t\|_2/\|\boldsymbol{x}_t\|_2 \le 10^{-2}$

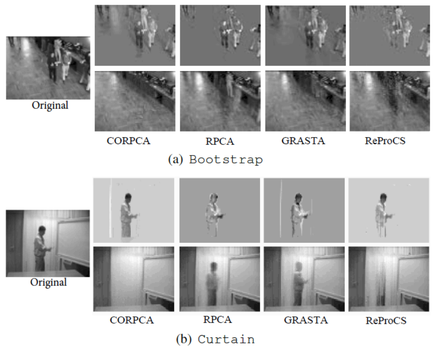


(a) CORPCA-n-ℓ_1 (b) CORPCA-ℓ_1-ℓ_1 (c) CORPCA-ℓ_1 (d) ReProCS (e) GRASTA (f) Scale

Fig. 1. Average success probabilities for CORPCA (for $\boldsymbol{x}_t, \boldsymbol{v}_t$), ReProCS (for $\boldsymbol{x}_t$), and GRASTA (for $\boldsymbol{v}_t$). The scale (f) is proportional to Pr(success)[%] from black to white.

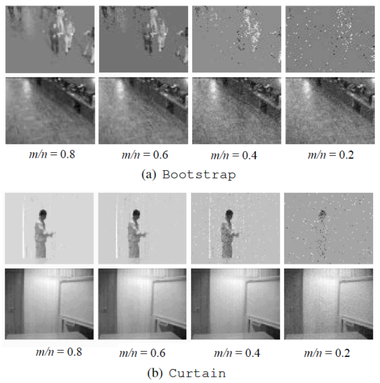
Compressive video foreground-background separation

- Considering two videos, Bootstrap (60x80 pixels) and Curtain (64x80 pixels) having a static and a dynamic background, respectively
- Background-foreground video separation with full access to the video data
- Compressive separating by varying rates on the number of measurements m over the dimension of the data n



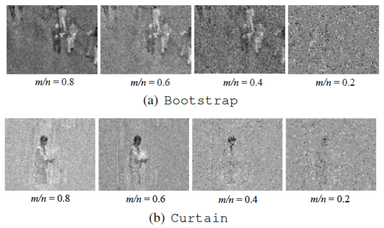
(a) Bootstrap (b) Curtain

Fig. 2. Background and foreground separation for the different separation methods with full data access `Bootstrap#2213` and `Curtain#2866`.



(a) Bootstrap (b) Curtain

Fig. 3. Compressive background and foreground separation of CORPCA with different measurement rates m/n.



(a) Bootstrap (b) Curtain

Fig. 4. Compressive foreground separation of ReProCS with different measurement rates m/n.

4. Summary

Solution for an n-ℓ_1 minimization

- Incorporating efficiently multiple prior information
- Updating iteratively weights

The proposed COPRCA algorithm

- Processing a data vector per time instance using compressive measurements
- Solving the n-ℓ_1 minimization and updating priors for the next instance

Evaluation of COPRCA on synthetic data and actual video data

- Outperforming classical compressive sensing (CS) (ℓ_1 minimization) and CS with single prior information (ℓ_1-ℓ_1 minimization)
- The superior performance improvement compared to the existing methods

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