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COMPRESSIVE ONLINE ROBUST PRINCIPAL COMPONENT ANALYSIS WITH MULTIPLE PRIOR INFORMATION

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1. Motivation

- Applications: Computer vision, web data analysis, anomaly detection, and data visualization, etc.
- Robust Principal Component Analysis (RPCA): Batch-based, decomposes all data samples (matrix M) into low-rank (L) and sparse (S), e.g., all frames in a video, high computational and memory requirements

$$\min_{L, S} \|L\|_* + \lambda \|S\|_1 \text{ subject to } M = L + S$$

Challenges

- Online method processing a sequence of signals per time instance from a small set of measurements $y_t = \Phi(x_t + v_t)$

$$M_t = L_t + S_t \text{ into } S_t = [x_1 \ x_1 \dots x_t] \text{ and } L_t = [v_1 \ v_2 \dots v_t]$$

➤ Minimization at time instance t

$$\min_{x_t, v_t} \|[L_{t-1} \ v_t]\|_* + \lambda \|x_t\|_1 \text{ subject to } y_t = \Phi(x_t + v_t)$$

where $L_{t-1} = [v_1 \ v_2 \dots v_{t-1}]$, $S_{t-1} = [x_1 \ x_1 \dots x_{t-1}]$, Φ are given

2. Compressive Online RPCA (CORPCA) With Multiple Prior Information

Problem formulation

- Incorporating multiple prior information: at time instance t we observe $y_t = \Phi(x_t + v_t)$ with $y_t \in \mathbb{R}^m$ given priors Z_{t-1} and B_{t-1} from $\{\hat{x}_1, \dots, \hat{x}_{t-1}\}$ and $\{\hat{v}_1, \dots, \hat{v}_{t-1}\}$

- Solving the n - ℓ_1 minimization problem

$$\min_{x_t, v_t} \left\{ H(x_t, v_t | y_t, Z_{t-1}, B_{t-1}) = \frac{1}{2} \|\Phi(x_t + v_t) - y_t\|_2^2 + \lambda \mu \sum_{j=0}^J \beta_j \|\mathbf{W}_j(x_t - z_j)\|_1 + \mu \|[B_{t-1} \ v_t]\|_* \right\},$$

where $\lambda > 0$ and $\beta_j > 0$ are weights across the side information signals, and $\mathbf{W}_j$ is a diagonal matrix with weights for each element in the side information signal z_j ; namely, $\mathbf{W}_j = \text{diag}(w_{j1}, w_{j2}, \dots, w_{jn})$ with $w_{ji} > 0$ being the weight for the i -th element in the z_j vector.

The CORPCA algorithm

- Solving n - ℓ_1 minimization via the soft thresholding operator and the single value thresholding operator, at iteration $k + 1$

$$v_t^{(k+1)} = \arg \min_{v_t} \left\{ \mu h(v_t) + \left\| v_t - \left(v_t^{(k)} - \frac{1}{2} \nabla_{v_t} f(v_t^{(k)}, x_t^{(k)}) \right) \right\|_2^2 \right\}$$

$$x_t^{(k+1)} = \arg \min_{x_t} \left\{ \mu g(x_t) + \left\| x_t - \left(x_t^{(k)} - \frac{1}{2} \nabla_{x_t} f(v_t^{(k)}, x_t^{(k)}) \right) \right\|_2^2 \right\}$$

where $f(v_t, x_t) = (1/2) \|\Phi(x_t + v_t) - y_t\|_2^2$

$$g(x_t) = \lambda \sum_{j=0}^J \beta_j \|\mathbf{W}_j(x_t - z_j)\|_1, \text{ and } h(v_t) = \|[B_{t-1} \ v_t]\|_*$$

- Updating weights β_j and $\mathbf{W}_j$

- After solving for time instance t : Prior updates

$$Z_t := \{z_j = x_{t-J+j}\}_{j=1}^J$$

$$B_t = U_t(:, 1:d) \Gamma_{\frac{\mu_k}{2} g_1}(\Sigma_t)(1:d, 1:d) V_t(:, 1:d)^T$$

Algorithm 1: The proposed CORPCA algorithm.

```

Input:  $y_t, \Phi, Z_{t-1}, B_{t-1}$ ;
Output:  $\hat{x}_t, \hat{v}_t, Z_t, B_t$ ;
// Initialize variables and parameters.
1  $x_t^{(-1)} = x_t^{(0)} = 0$ ;  $v_t^{(-1)} = v_t^{(0)} = 0$ ;  $\xi_{-1} = \xi_0 = 1$ ;  $\mu_0 = 0$ ;
 $\bar{\mu} > 0$ ;  $\lambda > 0$ ;  $0 < \epsilon < 1$ ;  $k = 0$ ;  $g_1(\cdot) = \|\cdot\|_1$ ;
2 while not converged do
// Solve Problem (4).
3  $\tilde{v}_t^{(k)} = v_t^{(k)} + \frac{\xi_{k-1}-1}{\xi_k} (v_t^{(k)} - v_t^{(k-1)})$ ;
4  $\tilde{x}_t^{(k)} = x_t^{(k)} + \frac{\xi_{k-1}-1}{\xi_k} (x_t^{(k)} - x_t^{(k-1)})$ ;
5  $\nabla_{v_t} f(\tilde{v}_t^{(k)}, \tilde{x}_t^{(k)}) = \nabla_{v_t} f(v_t^{(k)}, x_t^{(k)}) = \Phi^T (\Phi(\tilde{v}_t^{(k)} + \tilde{x}_t^{(k)}) - y_t)$ ;
6  $(U_t, \Sigma_t, V_t) = \text{incSVD}([B_{t-1} \ (\tilde{v}_t^{(k)} - \frac{1}{2} \nabla_{v_t} f(\tilde{v}_t^{(k)}, \tilde{x}_t^{(k)}))])$ ;
7  $\Theta_t = U_t \Gamma_{\frac{\mu_k}{2} g_1}(\Sigma_t) V_t^T$ ;
8  $v_t^{(k+1)} = \Theta_t(:, \text{end})$ ;
9  $x_t^{(k+1)} = \Gamma_{\frac{\mu_k}{2} g}(\tilde{x}_t^{(k)} - \frac{1}{2} \nabla_{x_t} f(\tilde{v}_t^{(k)}, \tilde{x}_t^{(k)}))$ ; where
 $\Gamma_{\frac{\mu_k}{2} g}(\cdot)$  is given as in RAMSIA [18];
// Compute the updated weights [18].
10  $w_{ji} = \frac{n(|x_{ti}^{(k+1)} - z_{ji}| + \epsilon)^{-1}}{\sum_{i=1}^n (|x_{ti}^{(k+1)} - z_{ji}| + \epsilon)^{-1}}$ ;
11  $\beta_j = \frac{(\|\mathbf{W}_j(x_t^{(k+1)} - z_j)\|_1 + \epsilon)^{-1}}{\sum_{i=1}^J (\|\mathbf{W}_i(x_t^{(k+1)} - z_i)\|_1 + \epsilon)^{-1}}$ ;
12  $\xi_{k+1} = (1 + \sqrt{1 + 4\xi_k^2})/2$ ;  $\mu_{k+1} = \max(\epsilon \mu_k, \bar{\mu})$ ;
13  $k = k + 1$ ;
14 end
// Update prior information.
15  $Z_t := \{z_j = x_{t-J+j}\}_{j=1}^J$ ;
16  $B_t = U_t(:, 1:d) \Gamma_{\frac{\mu_k}{2} g_1}(\Sigma_t)(1:d, 1:d) V_t(:, 1:d)^T$ ;
17 return  $\hat{x}_t = x_t^{(k+1)}$ ,  $\hat{v}_t = v_t^{(k+1)}$ ,  $Z_t, B_t$ ;

```

3. Experimental Results

Synthetic data

- Generating low-rank components: $n = 500$, $d = 100$ (training), $q = 100$ (testing), $r = 5$ (rank)
 $L = UV^T$, where $U \in \mathbb{R}^{n \times r}$ and $V \in \mathbb{R}^{(d+q) \times r}$ yields $L = [v_1 \dots v_{d+q}]$
- Generating sparse components with $\|x_t\|_0 = s_0$ and $\|x_t - x_{t-1}\|_0 = s_0/2$ obtaining
 $S = [x_1 \dots x_{d+q}]$
- Testing on $M = [x_{d+1} + v_{d+1} \dots x_{d+q} + v_{d+q}]$
- Measuring probabilities of successful decomposition, $\text{Pr}(\text{success})$, success if $\|\hat{x}_t - x_t\|_2 / \|x_t\|_2 \leq 10^{-2}$

Fig. 1. Average success probabilities for CORPCA (for x_t, v_t), ReProCS (for x_t), and GRASTA (for v_t). The scale (f) is proportional to $\text{Pr}(\text{success})[\%]$ from black to white.

Compressive video foreground-background separation

- Considering two videos, Bootstrap (60x80 pixels) and Curtain (64x80 pixels) having a static and a dynamic background, respectively
- Background-foreground video separation with full access to the video data
- Compressive separating by varying rates on the number of measurements m over the dimension of the data n

Fig. 2. Background and foreground separation for the different separation methods with full data access Bootstrap#2213 and Curtain#2866.

Fig. 3. Compressive background and foreground separation of CORPCA with different measurement rates m/n .

Fig. 4. Compressive foreground separation of ReProCS with different measurement rates m/n .

4. Summary

Solution for an n - ℓ_1 minimization

- Incorporating efficiently multiple prior information
- Updating iteratively weights

The proposed COPRCA algorithm

- Processing a data vector per time instance using compressive measurements
- Solving the n - ℓ_1 minimization and updating priors for the next instance

Evaluation of COPRCA on synthetic data and actual video data

- Outperforming classical compressive sensing (CS) (ℓ_1 minimization) and CS with single prior information (ℓ_1 - ℓ_1 minimization)
- The superior performance improvement compared to the existing methods

CORPCA source code, test sequences, and the corresponding outcomes. [Online]. Available: <https://github.com/huynhvlvd/corpcap>