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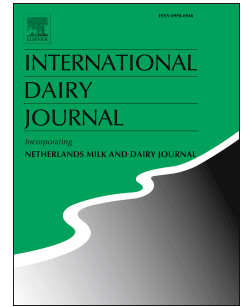
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1 **Modelling the influence of metabolite diffusion on non-starter lactic acid bacteria**
2 **growth in ripening Cheddar cheese**

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ABSTRACT

The influence of metabolite diffusion within the cheese matrix on growth of non-starter lactic acid bacteria (NSLAB) during Cheddar cheese ripening was mathematically modelled. The model was calibrated at a realistic range of diffusion of metabolites and the decay and growth parameters of immobilised starter LAB (SLAB) and NSLAB colonies, respectively.

Metabolite diffusion is the limiting factor for NSLAB growth only if essential metabolite molecules are extremely large or otherwise immobilised in the matrix. For relatively small molecules diffusion cannot be a limiting factor; the diffusive replenishment of small molecule nutrients around the NSLAB colonies consuming them is generally faster than the release rate from all possible sources within the curd. Assuming that the only nutrient source limiting NSLAB growth is the release of metabolites from lysed SLAB colonies, the decay rate of SLAB, rather than metabolite diffusion, most probably determines the rate of NSLAB growth during Cheddar cheese ripening.

42 1. Introduction

43

44 Cheese microbiota is pivotal to nearly all processes taking place during cheese
45 production. Starter lactic acid bacteria (SLAB) are responsible for the conversion of lactose to
46 lactate during the fermentation of milk and results in a pH decrease. The species of SLAB
47 used for the manufacture of cheese depends on the cheese type, with mesophilic species such
48 as *Lactococcus lactis* subsp. *lactis* and subsp. *cremoris* used for the production of Cheddar
49 and cottage cheese types. These mesophilic species can be supplemented with citrate
50 fermenting *Lactococcus lactis* and various *Leuconostoc* species for the production of Gouda
51 and Danbo cheese types. In addition to the mesophilic SLAB, thermophilic SLAB, such as
52 *Streptococcus thermophilus*, *Lactobacillus delbrueckii* subsp. *bulgaricus*, and *Lactobacillus*
53 *helveticus* are used for the production of pasta-filata and Swiss-type cheeses. Frequently, *S.*
54 *thermophilus* and/or *Lb. helveticus* cultures may be added to the core mesophilic cultures used
55 for Cheddar and Gouda cheese types to produce a meso-thermo blend. Such meso-thermo
56 blends give improved phage robustness and increased flavour properties. Besides the
57 fermentation of lactose, SLAB are also critical for degradation of casein into peptides and
58 free amino acids, and in the biotransformation of these free amino acids into a very diverse
59 range of aroma compounds (McSweeney, 2017; Yvon, Thirouin, Rijnen, Fromentier, &
60 Gripon, 1997). The SLAB used for the manufacture of cheese are carefully selected and
61 controlled by the cheese producer, and normally obtained from specialist suppliers in freeze-
62 dried or frozen format.

63 In contrast to the SLAB, the non-starter lactic acid bacteria (NSLAB) are not
64 controlled due to the non-aseptic nature of industrial cheese production. Pasteurisation of the
65 cheese milk only lowers NSLAB levels, but does not eliminate them from the cheese milk
66 (De Angelis et al., 2004). The NSLAB isolated from cheese belong to a very heterogeneous

67 group, frequently they are members of the *Lactobacillus* species and include *Lactobacillus*
68 *casei*, *Lactobacillus paracasei*, *Lactobacillus plantarum*, *Lactobacillus pentosus*,
69 *Lactobacillus curvatus*, *Lactobacillus rhamnosus*, *Lactobacillus fermentum*, *Lactobacillus*
70 *parabuchneri* and *Lactobacillus brevis*. The non-*Lactobacillus* species of NSLAB commonly
71 isolated from cheese comprise *Pediococcus acidilactici*, *Pediococcus pentosaceus*,
72 *Enterococcus durans*, *Enterococcus faecalis*, and *Enterococcus faecium* (Settanni &
73 Moschetti, 2010). NSLAB originate from the cheese milk and the cheese making
74 environment, and may vary significantly from one dairy plant to another (Banks & Williams,
75 2004; Settanni & Moschetti, 2010; Sgarbi et al., 2013).

76 The conversion of milk to cheese can be considered as a two-step process. In the first
77 step, milk is converted into a fresh curd, while in the second step the fresh curd is converted
78 into ripened cheese. In the first step, SLAB grow rapidly in the milk due to the abundance of
79 available substrate (lactose), and after approximately 24 hours reach levels of 10^9 cfu g⁻¹ in
80 the fresh curd. No further growth of SLAB occurs in the fresh curd due to the hostile
81 environment encountered (absence of a fermentable carbohydrate, high salt concentration, and
82 low pH). In the second step, which occurs over several months, the SLAB numbers begin to
83 decline, while the NSLAB numbers begin to increase. At the beginning of ripening the
84 NSLAB start at rather low levels of 10^1 – 10^3 cfu g⁻¹, and may in fact be undetectable using
85 conventional plating techniques. As ripening progresses their number increases to
86 approximately 10^7 – 10^8 cfu g⁻¹ (De Dea Lindner et al., 2008; Fitzsimons, Cogan, Condon, &
87 Beresford, 2001; Gatti et al., 2008; McMahon et al., 2014). The SLAB and NSLAB all grow
88 as immobilised colonies in the cheese during ripening, making them dependent on diffusion
89 of metabolites in the cheese matrix. The distribution of the immobilised bacteria cells in the
90 cheese matrix is random and therefore the mean distance between the colonies is strongly
91 affected by the initial inoculation levels (Jeanson et al., 2011).

92 The substrate source(s) and how they migrate to the regions in which the NSLAB cells
93 are sparsely distributed within the cheese matrix is not fully elucidated (Gobbetti, De Angelis,
94 Di Cagno, Mancini, & Fox, 2015). Considering the fact that sugars such as lactose, glucose
95 and galactose are rapidly depleted after a few days ripening (Budinich et al., 2011), then other
96 substrate sources must be present. In particular, SLAB derived substrate sources as a result of
97 cell death have been considered as potential carbon and nitrogen sources for NSLAB growth.
98 A typical SLAB cell composition, based on percentage of total dry weight, is approximately
99 45% protein, 12–15% polysaccharide, 10% teichoic acid, 6–8% RNA, 7% inorganic ions,
100 5.5% amino sugars, 4–4.3% lipid and 3–3.3% DNA (Novák & Loubiere, 2000). Other
101 possible substrate sources include either more complex milk-derived carbohydrates from κ -
102 casein or the milk fat globular membrane (Moe, Faye, Abrahamsen, Østlie, & Skeie, 2012)
103 and free amino acids and small peptides from the caseins. Strong evidence supports the theory
104 that it is the SLAB derived carbon that the NSLAB use as a growth source. In several studies
105 (Adamberg et al., 2005; Sgarbi et al., 2013; Thomas, 1987; Williams, Withers, & Banks,
106 2000) it has been demonstrated in vitro that NSLAB are able to grow on dead SLAB material
107 such as ribose and cell-wall originating sugars. Furthermore, it has been confirmed that
108 NSLAB grew faster in Cheddar cheese manufactured with a fast lysing SLAB than a slow
109 lysing SLAB (Lane, Fox, Walsh, Folkertsma, & McSweeney, 1997). More recently (Moe et
110 al., 2012), it has been demonstrated that NSLAB can utilise likely sources of nitrogen to
111 support the growth of NSLAB during cheese ripening are free amino acids and peptides
112 released from the casein due to the action of the rennet, as well as SLAB cell-wall associated
113 proteinase and intracellular peptidases (Cotter & Beresford, 2017). These sources of nitrogen
114 are abundantly available in the ripening cheese.

115 Very little is known about how and at what rate the SLAB cell components, in
116 particular the limited carbohydrate substrate sources (polysaccharide, teichoic acids, RNA,

117 DNA and amino sugars) migrate from the SLAB regions of the cheese matrix to the NSLAB
118 regions. Hydrolysis of the polymeric SLAB cell components (polysaccharide, teichoic acids,
119 RNA, DNA) into their constituent monomers (N-acetylglucosamine, N-acetylmuramic acid,
120 ribose and deoxyribose) would be a necessary step prior to diffusion in the cheese matrix.
121 Furthermore, high concentrations of casein-derived free amino acids and peptides are
122 expected to be localised in the SLAB regions, and it is unknown how and at what rate these
123 components migrate to the NSLAB regions. Recently, Floury et al. (2015) reported that milk
124 proteins (bovine serum albumin, lactoferrin and α_{S1} -casein) could not penetrate the inside of
125 bacterial colonies immobilised in a model cheese system. Interestingly, the effect of the
126 spatial distribution of *L. lactis* colonies (small colonies or large colonies) in the same model
127 cheese system was shown to influence the rate of degradation and production of various
128 cheese metabolites such as caseins, free amino acids and volatiles (Le Boucher et al., 2016).

129 This study, through the use of mathematical modelling, seeks to address the key
130 question regarding how the SLAB and NSLAB grow as immobilised colonies in the cheese
131 matrix during ripening. The micro-ecological approach we take here has not often been taken
132 in dairy research yet, even though ripening cheese provides an ideal setting for both
133 theoretical and experimental studies in microbial ecology (Wolfe & Dutton, 2014). The
134 essential features of the three component processes of cheese ripening – SLAB decay, nutrient
135 substrate diffusion and NSLAB growth – are concisely represented in a simple reaction-
136 diffusion system. Studying numerical realisations of the model with known values of the rates
137 of SLAB decay, nutrient diffusion and NSLAB growth, the potential effects of the component
138 processes on the course of ripening can be assessed separately, providing experimentally
139 testable predictions on the ripening process itself. Specifically, the model examines
140 metabolite diffusion rates and their influence on the space-time dynamics of SLAB death and
141 NSLAB growth. Furthermore, it seeks to examine the effect of a slow lysing SLAB versus a

142 fast lysing SLAB on the growth rate of the NSLAB population. The model proposed here is
143 applicable for dry-salted cheeses such as Cheddar, in which the SLAB are immediately
144 present in a high salt environment post manufacture.

145

146 **2. Material and methods**

147

148 *2.1. The model*

149

150 The presented mathematical model simulates the dynamics of SLAB lysis and NSLAB
151 growth during cheese ripening, based on a typical Cheddar cheese ripening scenario. In the
152 model the SLAB can be set to start decaying within the curd immediately after salting or at
153 any time after that. Upon SLAB lysis, nutrients are directly released from the cytosol or
154 produced by the hydrolytic enzymes of the decaying SLAB cells. These nutrients comprise
155 SLAB cell wall monomer components (N-acetylglucosamine, N-acetylmuramic acid), sugars
156 from hydrolysed nucleic acids (ribose and deoxyribose), free amino acids, and small peptides
157 (10–15 amino acid residues). All these are assumed to diffuse from the localised, lysed SLAB
158 cells within the cheese matrix. The model is set to predict the growth of NSLAB colonies
159 utilising the diffusing material as their nutrient source. We assume that the nutrients actually
160 limiting NSLAB growth in the curd (whether they are nucleotides, nucleobases, sugars or
161 essential co-factors) originate from lysed SLAB cells and need to diffuse to the localised
162 NSLAB colonies. For simplicity we assume that both the SLAB and the NSLAB are
163 homogeneous with respect to their dynamical properties, i.e., all SLAB strains have the same
164 probability of death/lysis, d_{SLAB} , and the expected maximum growth rates of NSLAB strains,
165 r_{NSLAB} , are also the same for all NSLAB bacteria in the cheese matrix. If diffusion is limiting

166 for NSLAB growth it is the diffusion parameter D that is the critical parameter for NSLAB
 167 growth. Fig. 1 explains the details of the corresponding dynamics.

168 With these simplifying assumptions, the model is implemented as a system of partial
 169 differential equations in two spatial dimensions, one equation for each of SLAB (S), nutrient
 170 (L) and NSLAB (N):

$$172 \quad \frac{\partial S(x,y,t)}{\partial t} = -d_{SLaB} S(x, y, t) \quad [1]$$

$$174 \quad \frac{\partial L(x,y,t)}{\partial t} = D \left(\frac{\partial^2 L(x,y,t)}{\partial x^2} + \frac{\partial^2 L(x,y,t)}{\partial y^2} \right) + d_{SLaB} S(x, y, t) - r_{NSLaB} \frac{L(x,y,t)}{L(x,y,t)+1} N(x, y, t) \quad [2]$$

$$176 \quad \frac{\partial N(x,y,t)}{\partial t} = r_{NSLaB} \frac{L(x,y,t)}{L(x,y,t)+1} N(x, y, t) \quad [3]$$

177
 178 where t is time from salting, x and y are coordinates in two spatial dimensions, representing
 179 spatial positions in the perpendicular projection of the 3D cheese volume onto a plane. The
 180 reduction of the number of spatial dimensions to two is necessary for computational reasons
 181 (diffusion in 3D is extremely slow to simulate), and it does not affect the conclusions in the
 182 qualitative sense. The decay coefficient d_{SLaB} of the SLAB population implies that SLAB
 183 decay is a random process, with each bacterium carrying the same risk of death and lysis
 184 within any small period of time, resulting in an exponential decay curve for the SLAB
 185 population. The constancy of the decay rate implies that the environment within the curd (salt
 186 concentration, pH, temperature) is essentially constant during the ripening process. Nutrients
 187 released from lysed SLAB cells move within the matrix following Fick's second law of
 188 diffusion, and they are locally consumed by NSLAB, which have a saturating consumption
 189 response to local nutrient concentration, i.e., the higher the nutrient concentration L around an

190 NSLAB colony the closer the actual growth rate of that colony to its maximum. We assume,
191 again without a loss of generality in the qualitative sense, that the conversion factor is 1,
192 meaning that the limiting component of the cell material from the lysis of a single SLAB cell
193 is sufficient for the production of one NSLAB cell. Note that nutrient sources other than
194 SLAB lysate (e.g., amino acids and small peptides from the proteolysis of the casein matrix)
195 may be present in excess, but in the model we assume that the nutrients supporting NSLAB
196 growth are released from lysed SLAB cells. Changing the conversion factor to any arbitrary
197 number less than 1 would just decrease the stationary density of NSLAB accordingly.

198 We follow the time course of the total masses of S , L and N within a small, square
199 shaped region (10 mm side length) of the cheese, using periodic boundary conditions.
200 Defining periodic boundaries amounts to assuming that the focal square region, and those of
201 the same size and shape adjacent to it, are identical, which means that the curd consists of an
202 infinite repetition (lattice) of the focal region in both spatial dimensions. Fig. 2 is an
203 illustration of the model dynamics within the focal region, with arbitrary parameters. Digital
204 Supplement 1 is a video showing the time course of the same process.

205 The initial patterns of both the SLAB and the NSLAB colonies are random, and the
206 number of NSLAB colonies per focal region is a parameter of the model. Since the cell count
207 of SLAB at salting is about 10^9 cfu g^{-1} and it is evenly distributed within the curd, it is safe to
208 assume that the actual initial SLAB distribution is continuous and uniform – so this is what
209 we assume in all the simulations. The initial NSLAB distribution is still random and discrete,
210 since NSLAB is present in the curd at very low density, represented by only a few individuals
211 within the focal region (about 10 cfu g^{-1}) initially. The founders of NSLAB colonies are
212 implemented as very narrow and very low Gaussian shaped $N(t,x,y)$ initial density curves
213 representing single colony forming units. The height, width and the number of these colonies

214 within the focal region are parameters of the model. Fig. 3 shows the time course of the log
215 total densities of S , L and N within the same sample.

216

217 2.2. Model calibration

218

219 We have determined a biologically/chemically feasible range for each of the three key
220 parameters of the model (SLAB decay rate: d_{SLAB} ; Nutrient diffusion rate from lysed SLAB:
221 D ; and NSLAB population growth rate under excess resource supply: r_{NSLAB}) based on
222 fundamental empirical data from the literature. SLAB decay rates and NSLAB growth rates
223 are calculated from known lysis times of different SLAB cultures and replication times of
224 NSLAB at different temperatures, respectively (see below). The metabolite diffusion rates
225 used in the simulations include the range from the diffusivity of small molecules in water to
226 that of large peptides in gels. These parameter ranges have been scanned for sections at which
227 SLAB decay, diffusion or NSLAB growth would be the limiting factor of NSLAB growth and
228 thus of the speed of cheese ripening. The calibrated parameter ranges are the following:

229

230 2.2.1. SLAB decay rate (d_{SLAB})

231 One of the features by which starter cultures are specified is their characteristic time to
232 complete lysis during the ripening process. For Cheddar this may extend from about 3 months
233 (Fox, Guinee, Cogan, & McSweeney, 2017) or 2160 h (using a “fast lysing” SLAB culture
234 like *L. lactis* AM1 or AM2) to approximately 9 months or 6480 h (“slowly lysing” SLAB
235 culture). Assuming exponential decay of the SLAB culture,

236

$$237 N(t) = N(0) \times e^{-d_{SLAB}t} \quad [4]$$

238

239 the relation between the time for 99% of SLAB to lyse, $t_{1\%}$, and the decay rate, d , is

$$240 \frac{N(t)}{N(0)} = e^{-d_{\text{SLAB}}t_{1\%}} = 0.01 \quad [5]$$

241 from which

$$242 -d_{\text{SLAB}}t_{1\%} = \ln 0.01 = -4.6 \quad [6]$$

243 that is,

$$244 d_{\text{SLAB}} = \frac{4.6}{t_{1\%}} \quad [7]$$

245 This yields $d_{\text{SLAB}} = 0.00213 \text{ (h}^{-1}\text{)}$ for “fast” SLAB cultures ($t_{1\%} = 2160 \text{ h}$), and

246 $d_{\text{SLAB}} = 0.00071 \text{ (h}^{-1}\text{)}$ for “slow” ones ($t_{1\%} = 6480 \text{ h}$). We have used $d_{\text{SLAB}} = 0.0025 \text{ (h}^{-1}\text{)}$

247 and $d_{\text{SLAB}} = 0.0007 \text{ (h}^{-1}\text{)}$ for the simulations.

248

249 2.2.2. Diffusion rate of nutrients from lysed SLAB (D_{Lys})

250 The sample of cheese that we simulate is $10 \times 10 \text{ mm}$ in size; therefore, the diffusion

251 rates of small-molecule nutrients (like monosaccharides or amino acids) from the literature

252 are rescaled to $\text{mm}^2 \text{ h}^{-1}$ dimensions for convenience. Literature data for the diffusion rates of

253 the smallest mono- and disaccharides in water at $25 \text{ }^\circ\text{C}$ range from 5×10^{-10} to $6 \times 10^{-10} \text{ m}^2 \text{ s}^{-1}$

254 (Ziegler, Benado, & Rizvi, 1987), which translates to 1.8 to $2.16 \text{ mm}^2 \text{ h}^{-1}$. Bovine serum

255 albumin (BSA), a peptide with 583 amino acid residues and of 66.5 kDa molecular mass has a

256 diffusion rate of $0.28 \text{ mm}^2 \text{ h}^{-1}$ at $25 \text{ }^\circ\text{C}$ and $\text{pH} = 7$ in water (Torres, Komiya, Okajima, &

257 Maruyama, 2012). Notice that the difference between the diffusivities of a small sugar

258 molecule and a rather large peptide is less than an order of magnitude, at least in water. We

259 consider $2.0 \text{ mm}^2 \text{ h}^{-1}$ an upper limit for the rate of nutrients diffusion in the curd, and scan the

260 range decreasing across 4 orders of magnitude to $0.0002 \text{ mm}^2 \text{ h}^{-1}$ during the simulations.

261 Note that we do not take the effect of the casein matrix as a physical barrier to the free

262 diffusion of molecules into account here. It is obvious that very large molecules, just as the

263 bacteria themselves, could be stuck within the matrix and thus immobilised. However,

264 Chapeau, Silva, Schuck, Thierry, and Flourey (2016) and Silva, Lortal, and Flourey (2015)
265 show that even very large dextrans (with molecular masses of 2000 kDa) diffuse freely within
266 the casein matrix of ripening cheese without being trapped in it. Therefore, we assume that all
267 molecules of the size readily ingested and metabolised by NSLAB also diffuse freely within
268 the aqueous phase of the curd. However, it should be noted here that the diffusion in cheese
269 depends on the actual water activity of the curd, which is in the range $a_w = 0.950\text{--}0.975$ in
270 ripening Cheddar (Schmidt & Fontana Jr., 2007), implying about an order of magnitude
271 decrease in diffusion rate compared with that in water. Until we have better data available on
272 the diffusion of different small-molecule metabolites (monosaccharides, amino acids) and
273 oligopeptides of different sizes (experimental work in progress), shapes and surface charges
274 in Cheddar we will use this approximation for model calibration.

275 Considering these facts, the estimated realistic range of diffusion for metabolites
276 available as nutrients for NSLAB growth is about $0.2\text{--}0.02\text{ mm}^2\text{ h}^{-1}$ in ripening Cheddar with
277 a water activity around 0.95.

278

279 2.2.3. NSLAB growth rate

280 Under ideal conditions (i.e., for unlimited food supply allowing for maximum
281 population growth rate, at optimal temperature, pH, etc.) the generation time of a typical lactic
282 acid bacteria strain is about half an hour ($t_{gen} = 0.5\text{ h}$) (Kunji, Slotboom, & Poolman, 2003).
283 Calculations similar to Eqs. 4–7 show that the maximum rate of population growth
284 corresponding to this generation time is $r_{NSLAB} = 1.386\text{ h}^{-1}$. Note that at ripening temperature
285 (9–10 °C) the growth rate is about an order of magnitude lower than at the temperature
286 optimal for population growth. Assuming nutrient limitation, we have defined the nutrient-
287 dependent growth rate to be of the form:

$$288 \quad r(L) = r_{NSLAB} \times \frac{L}{L+1} \quad [8]$$

289 We have set $r_{NSLAB} = 1.4 \text{ h}^{-1}$ to be the highest possible growth rate (at optimum
 290 conditions in all respects), and assumed that at ripening conditions (lower temperature and
 291 pH) the growth rate is at least an order of magnitude lower ($r_{NSLAB} = 0.14 \text{ h}^{-1}$). The
 292 simulations have been carried out using these two values of the NSLAB growth parameter.

293

294 2.3. *Parameter range of model simulations*

295

296 The part of the parameter space covered by the model simulations is shown in Table 1.

297 The model simulations are focussed on four combinations of two characteristic SLAB decay

298 rates and two characteristic NSLAB growth rates, corresponding to fast and slowly lysing

299 SLAB ($d_{SLAB} = 0.0025$ and 0.0007 h^{-1} , respectively) providing nutrients to fast and slowly

300 growing NSLAB ($r_{NSLAB} = 1.4$ and 0.14 h^{-1} , respectively). Each of the four possible (d_{SLAB} ,

301 r_{NSLAB}) combinations was simulated at nutrient diffusion rates varying across four orders of

302 magnitude (at $D = 2.0000, 0.2000, 0.0200, 0.0020$ and $0.0002 \text{ mm}^2 \text{ h}^{-1}$). The time courses of

303 changes in SLAB (S), lysed SLAB (nutrient; L) and NSLAB (N) density during the first 3

304 months of the ripening process are shown on Figs. 4–7. The curves on all these figures are

305 obtained by numerical integration with respect to the spatial dimensions x and y of the

306 corresponding $S(x, y, t)$, $L(x, y, t)$ and $N(x, y, t)$ functions across the 2D cheese sample:

307

$$\bar{S}(t) = \iint_{-5}^5 S(x, y, t) dx dy$$

$$\bar{L}(t) = \iint_{-5}^5 L(x, y, t) dx dy$$

$$\bar{N}(t) = \iint_{-5}^5 N(x, y, t) dx dy$$

308

309 where $\bar{S}(t)$, $\bar{L}(t)$ and $\bar{N}(t)$ are the total masses of SLAB, nutrients and NSLAB, respectively,
 310 within the $10 \times 10 \text{ mm}^2$ cheese samples, at time t .

311 The ripening process strongly depends on the activity of NSLAB that produces many
 312 of the aromatic compounds responsible for flavour development. Therefore, we can use the
 313 time integral of $\bar{N}(t)$, i.e.,

314

$$\bar{\bar{N}}(T) = \int_0^T \bar{N}(t) dt$$

315

316 as an approximate measure of the ripening accomplished within the cheese sample by time T
 317 from salting. In other words, $\bar{\bar{N}}(T)$ is the total microbial activity provided by the growing
 318 NSLAB population on ripening the curd. This is the target function of the model: the faster
 319 the NSLAB population grows, the shorter the time T needed to achieve a certain level of
 320 ripeness $\bar{\bar{N}}$.

321

322 3. Results

323

324 Simulation results for four different parameter scenarios are considered below: (i) fast
 325 decaying SLAB and fast growing NSLAB; (ii) slowly decaying SLAB and fast growing
 326 NSLAB; (iii) fast decaying SLAB and slowly growing NSLAB; (iv) slowly decaying SLAB
 327 and slowly growing NSLAB.

328

329 3.1. Fast decaying SLAB and fast growing NSLAB

330

331 This case corresponds to using a fast decaying SLAB culture and ripening Cheddar
332 cheese at room temperature (about 20 °C). Diffusion is almost completely irrelevant with
333 regard to NSLAB growth during ripening: the NSLAB growth curves corresponding to
334 different diffusion rates almost coincide, suggesting that there is no considerable diffusion
335 limitation on the ripening process at this parameter setting. Lysed SLAB density (i.e., the
336 concentration of the limiting metabolite) is close to zero almost all along the process,
337 indicating prompt consumption of the metabolites upon lysis (Fig. 4).

338

339 3.2. *Slowly decaying SLAB and fast growing NSLAB*

340

341 This case is essentially the same as the previous one, except that the SLAB population
342 lyses slower, and thus the ripening process also proceeds slower (Fig. 5). Diffusion does not
343 really make a substantial difference here either. Also, for these scenarios, ripening is limited
344 by SLAB decay rather than diffusion.

345

346 3.3. *Fast decaying SLAB and slowly growing NSLAB*

347

348 This is the most “realistic” scenario set to simulate the ripening of Cheddar inoculated
349 with a fast lysing starter culture such as *L. lactis* AM1 and AM2 (Fox et al., 2017) and
350 ripened at 9–10 °C (Fig. 6). The conspicuous difference in the dynamics relative to that of the
351 first case (which represents optimum conditions for NSLAB growth) is that diffusion is
352 obviously much more important in determining NSLAB growth in this case: the growth
353 curves are quite different at different nutrient diffusion rates. Slowly growing NSLAB seem
354 to be more sensitive to the rate of nutrient replenishment into the depletion zone around the
355 NSLAB colonies. However, the essential difference is confined to very low diffusion rates

356 (0.0200–0.0002 mm² h⁻¹ – which is most probably below the realistic range for the nutrients
357 that bacteria can readily utilise). At feasible diffusion rates (0.0200–0.2000 mm² h⁻¹) the
358 difference in the growth curves is small, meaning that moderate diffusion limitation is to be
359 expected only at the lowest realistic rate of metabolite diffusion, where the time integral of the
360 growth curve of NSLAB, $\bar{N}(T)$ is somewhat smaller than at faster diffusion rates. Note that
361 within the realistic range of diffusion rates the differences in total ripening accomplished
362 during the three months of simulated time are still very limited. In other words, the diffusion
363 of small molecules (like amino acids or monosaccharides, with diffusion rates in the range 0.2
364 to 0.3) is not limiting the speed of ripening, whereas the possibly limited accessibility of
365 larger molecules like oligopeptides may have some effect on NSLAB growth, and thereby
366 also on ripening time.

367

368 3.4. *Slowly decaying SLAB and slowly growing NSLAB*

369

370 Another “realistic” scenario set to simulate the ripening of Cheddar assuming that the
371 SLAB inoculum is a slowly lysing starter culture such as *L. lactis* Z8, ML1 and HP (Fox et
372 al., 2017), and the cheese is stored at 9–10 °C during the ripening phase (Fig. 7). The
373 dynamics are similar to that of case (iii): the NSLAB growth curves are quite similar within
374 the realistic range of nutrient diffusion rates (at $D = 0.0200$ and 0.2000 mm² h⁻¹), while at
375 lower rates the growth of NSLAB is significantly affected.

376

377 4. Discussion

378

379 Establishment of NSLAB flora during cheese ripening is considered essential for the
380 normal flavour development in long ripened cheeses such as Cheddar, Gouda and Grana type

381 cheeses (Crow, Curry, & Hayes, 2001; Santarelli, Bottari, Lazzi, Neviani, & Gatti, 2013).
382 Therefore, to control the ripening process it is important to understand how the NSLAB
383 population develops over time. In this study we have used mathematical modelling to
384 elucidate some of the possible limiting factors for NSLAB growth in a dry-salted cheese such
385 as Cheddar.

386 The only possible scenarios in which diffusion can limit the speed of cheese ripening
387 are either (i) some nutrient molecules essential for NSLAB growth are prevented from
388 moving freely within the Cheddar cheese matrix and, therefore, diffuse much slower than
389 measured in other systems, or (ii) NSLAB growth is dependent on the supply of some nutrient
390 consisting of rather large molecules like a large peptide, the diffusion rate of which could be
391 significantly lower than that of small metabolites.

392 Scenario (i) is not completely unrealistic, considering small metabolites possibly
393 immobilised on the casein matrix by covalent or strong secondary (e.g., ionic) bonds, but we
394 have no examples of such cases in mind. With respect to (ii), we do not know examples of
395 cultivated NSLAB strains requiring such large molecules for their growth either; therefore,
396 we see no reason why this should be the case when growing in the cheese matrix. Thus, we
397 see none of the above suggested scenarios realistic and, therefore, on the basis of the
398 simulation studies performed with the model, we conclude that diffusion is most probably not
399 limiting NSLAB growth. This applies to systems with slow or fast lysing SLAB cultures
400 alike.

401 It is important to stress that the model is built on the assumption that nutrients released
402 from autolysed SLAB cells diffuse into the surroundings immediately after lysis. During this
403 process the lysed cell carbohydrate components such as polysaccharides, teichoic acids, RNA
404 and DNA are expected to be rapidly hydrolysed into their constituent monomers (N-
405 acetylglucosamine, N-acetylmuramic acid, ribose and deoxyribose), which means that it is

406 most probably this pool of small-molecule nutrients that is released into the cheese matrix.
407 Similarly, released peptidases are expected to rapidly hydrolyse the casein and large casein-
408 derived peptides into small peptides and free amino acids, which will also rapidly diffuse into
409 the surrounding cheese matrix.

410 The simulation results indicated that for all four scenarios of the three component
411 processes (SLAB decay, limiting nutrient diffusion and NSLAB population growth) it is the
412 decay rate of the SLAB culture that is the main determining factor for the population growth
413 of NSLAB. Considering that the development of typical cheese flavour in Cheddar is
414 correlated with the development of NSLAB (Coolbear et al., 2008), then the model presented
415 here suggests that the supply of small-molecule nutrients (provided by lysed SLAB cells) to
416 the NSLAB, rather than their rate of diffusion in the cheese matrix that is the predominant
417 parameter for NSLAB growth. The model predicts that within the realistic small molecule
418 nutrient diffusivity ranges of 0.02 to 0.20 mm² h⁻¹ the rate of NSLAB growth is sufficiently
419 high to keep the limited SLAB carbohydrate sources concentration close to zero in the cheese
420 matrix even at the suboptimal conditions (9–10 °C, pH 4–5) of cheese ripening. The model
421 assumes that free amino acids and small peptides are in abundance. It is only at extremely
422 (and for the relatively small molecules of typical nutrients of NSLAB, unrealistically) low
423 diffusion rates that nutrient diffusion become limiting to such an extent that the limited
424 carbohydrate sources released due to SLAB lysis will not be immediately accessible to
425 NSLAB. In this situation, the limiting carbohydrate concentrations within the curd would
426 markedly exceed zero during the ripening period, which is not what we see: carbohydrates
427 disappear from the curd very early during ripening.

428 Thus, although we can conclude from these modelling studies that diffusion
429 limitations are most probably not limiting NSLAB growth, it should be emphasised that we
430 cannot exclude that other parameters rather than SLAB lysis could influence NSLAB growth.

431 For example, carbohydrates from κ -casein or the milk fat globular membrane present in the
432 cheese matrix, as previously suggested (Adamberg et al., 2005; Moe et al., 2012) could also
433 be important for NSLAB growth, and this needs to be still fully elucidated. Furthermore, the
434 situation may be more complex than that modelled here, as it has been shown (Hickey,
435 Fallico, Wilkinson, & Sheehan, 2018) that some starter cells may die but not lyse, and thus
436 prevent the release of cellular material into the cheese matrix. Another scenario not accounted
437 for in this model could be that the starter cells may be non-culturable but still alive.
438 Nevertheless, if these molecules are important for NSLAB growth it is most probably the
439 release rate of these molecules into the Cheddar cheese matrix that is determining NSLAB
440 growth rather than their ability to diffuse to the immobilised NSLAB colonies in the matrix.

441 Comparison of the simulation results at optimum growth temperature (25 °C) for
442 NSLAB (Figs. 4 and 5) with the corresponding results (Figs 6 and 7) for the temperature of
443 Cheddar cheese ripening (9–10 °C) indicates that the difference in the effect of very small
444 nutrient diffusions is conspicuous, and somewhat puzzling at first glance. One would expect
445 the effect of nutrient diffusion to be even weaker on a population of lower growth rate than on
446 a fast growing one, but in fact it is just the opposite: very slow nutrient diffusion affects
447 slowly growing NSLAB colonies considerably more than fast ones. This effect may be
448 explained by spatial constraints: slowly growing colonies cannot decrease the nutrient
449 concentration at their boundary as efficiently as the fast growing cells. Therefore, at any given
450 (very low) diffusion rate the concentration gradient at the boundary of slow growing NSLAB
451 colonies is less steep than that of fast growing NSLABs, resulting in slower nutrient
452 replenishment and, consequently, slower population growth. Note that this difference is
453 evident only at unrealistically low diffusion rates: changing nutrient diffusivity within the
454 realistic range does not have a significant dynamical effect either on a fast or a slowly
455 growing NSLAB population.

456

457 **5. Conclusions**

458

459 Mathematical modelling based on realistic assumptions and modelling scenarios of the
460 component processes of ripening of a dry-salted cheese such as Cheddar and applying
461 parameters for SLAB decay, nutrient diffusion and NSLAB growth taken from the literature
462 has shown that nutrient diffusion most probably cannot be the bottleneck for NSLAB growth
463 during ripening. Neither could the growth potential of the NSLAB colonies be the limiting
464 factor, not even at the suboptimal conditions at which they persist during ripening. The
465 component process determining the rate of NSLAB growth (and thus also the rate of the
466 ripening process) seems to be the supply of the nutrient that is present at limiting density
467 within the cheese, and that in its turn depends on the decay rate of SLAB cells. Which type of
468 nutrient is the one limiting NSLAB growth remains an open question that calls for further
469 experimental work.

470

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472

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476

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478

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Figure legends

Fig. 1. Schematic representation (a) and detailed explanation (b) of the component processes of the cheese ripening model. SLAB death and autolysis provides diffusible nutrients for NSLAB population growth. The nutrient supply of the NSLAB colonies, which are fixed within the curd, depends on the speed of SLAB decay and the diffusivity in the curd of the nutrients released from autolysed SLAB cells. The key parameters of the three component processes are: SLAB decay: d_{SLAB} ; nutrient diffusion: D_{Lys} ; NSLAB growth: r_{NSLAB} .

Fig. 2. Graphical demonstration of the space-time dynamics of the Cheddar ripening model with random initial patterns of 10 SLAB and 10 NSLAB colonies and arbitrary parameters: SLAB decay rate $d_{SLAB} = 0.01 \text{ h}^{-1}$, diffusion rate of nutrients from lysed SLAB $D_{Lys} = 0.18 \text{ mm}^2 \text{ h}^{-1}$, NSLAB population growth rate $r_{NSLAB} = 0.1 \text{ h}^{-1}$. Each row of panels shows the spatial density distribution of SLAB, lysed SLAB (nutrient source) and NSLAB colonies, respectively, at the corresponding time ($t = 0, 60, 160$ and 500).

Fig. 3. The dynamics of the total densities of SLAB (blue), lysed SLAB (orange) and NSLAB (green) within the $10 \times 10 \text{ mm}$ sample of curd, obtained by integration of local densities across the sample at every 4th hour. Data points are means and standard errors for 10 replicate simulations (produced with the same parameter set but different random number sequences). Parameters are the same as in Fig. 2; the graph represents the demo dynamics shown on Fig. 2.

Fig. 4. Simulated total densities of fast decaying SLAB (blue), lysed SLAB (orange) and fast growing NSLAB (green) cells at different nutrient diffusivities within the 10×10 mm cheese sample. Calibrated parameters of the model: SLAB decay rate $d_{\text{SLAB}} = 0.0025 \text{ h}^{-1}$; NSLAB growth rate $r_{\text{NSLAB}} = 1.4 \text{ h}^{-1}$; the diffusion rate of the nutrients released from lysed SLAB cells within the cheese matrix are $D = 2.0, 0.2, 0.02, 0.002$ and $0.0002 \text{ mm}^2 \text{ h}^{-1}$.

Fig. 5. Simulated total densities of slowly decaying SLAB (blue), lysed SLAB (orange) and fast growing NSLAB (green) cells at different nutrient diffusivities within the 10×10 mm cheese sample. Parameters are the same as in Fig. 4, except for SLAB decay rate: $d_{\text{SLAB}} = 0.0007 \text{ h}^{-1}$.

Fig. 6. Simulated total densities of fast decaying SLAB (blue), lysed SLAB (orange) and slowly growing NSLAB (green) cells at different nutrient diffusivities within the 10×10 mm cheese sample. Parameters are the same as in Fig. 4, except for NSLAB growth rate: $r_{\text{NSLAB}} = 0.14 \text{ h}^{-1}$.

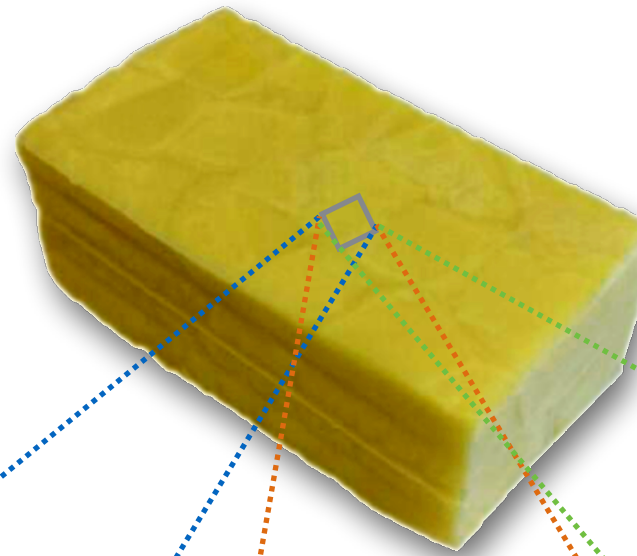
Fig. 7. Simulated total densities of slowly decaying SLAB (blue), lysed SLAB (orange) and slowly growing NSLAB (green) cells at different nutrient diffusivities within the 10×10 mm cheese sample. Parameters are the same as in Fig. 6, except for SLAB decay rate: $d_{\text{SLAB}} = 0.0007 \text{ h}^{-1}$.

Table 1

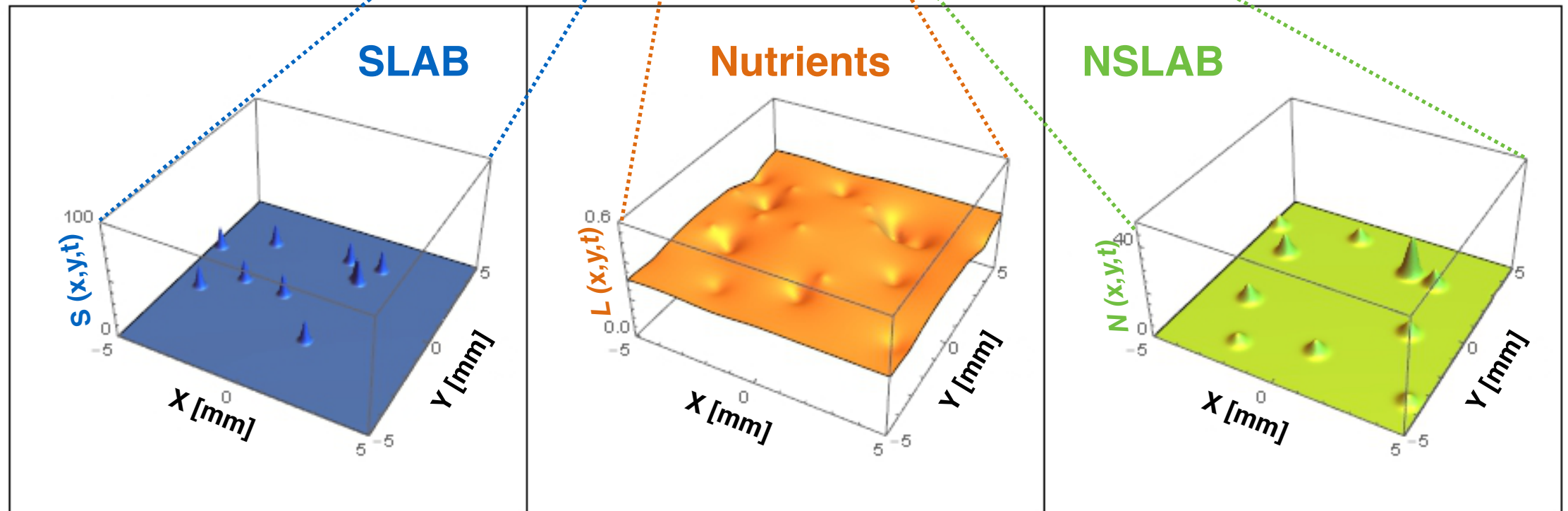
The parameter range of the simulations.

Parameter	Lower limit	Upper limit	Dimension
SLAB decay rate, d_{SLAB}	0.0007	0.0025	h^{-1}
Lysed SLAB (nutrient) diffusion rate, D_{Lys}	0.0002	2.0000	$\text{mm}^2 \text{h}^{-1}$
NSLAB maximum growth rate, r_{NSLAB}	0.0140	1.4000	h^{-1}

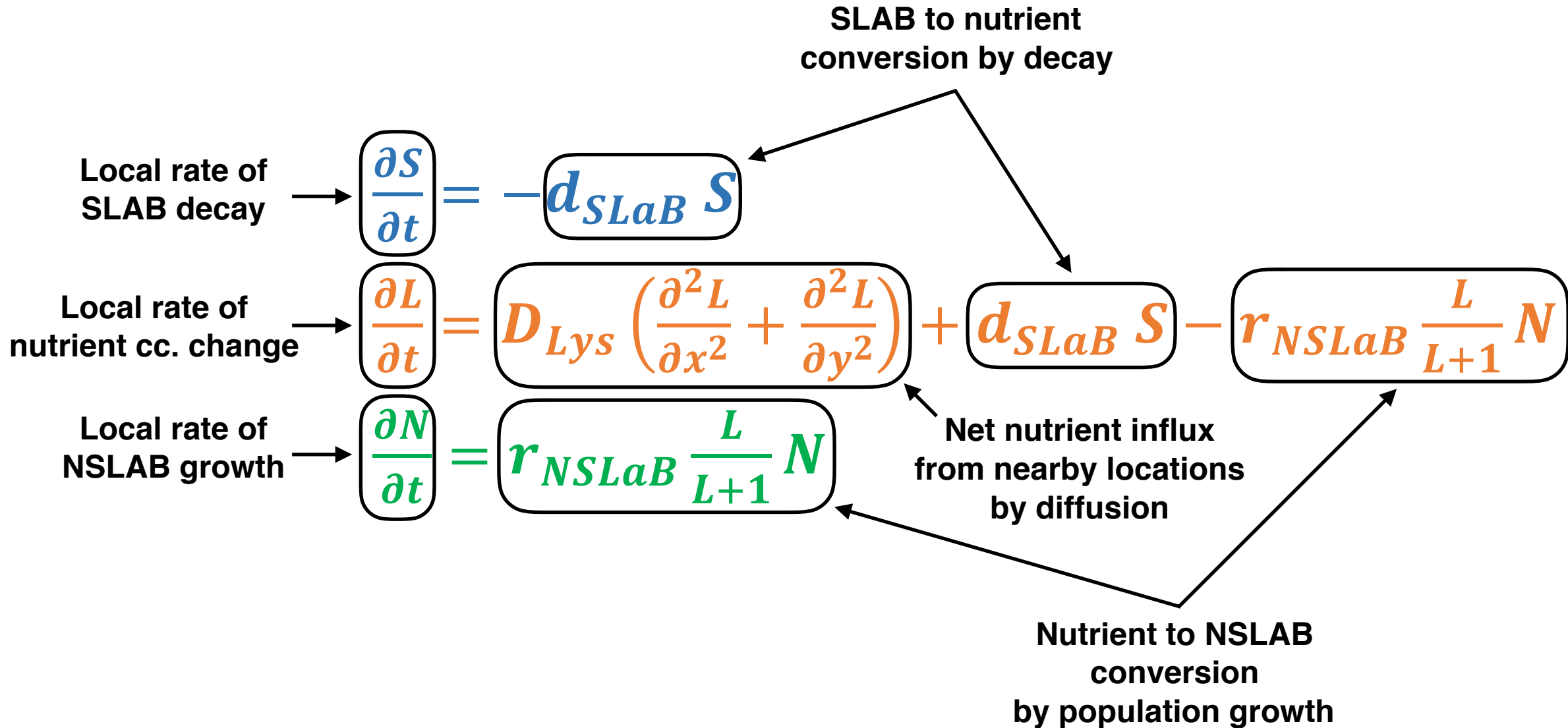
(a)



$t = 160$
[hours]

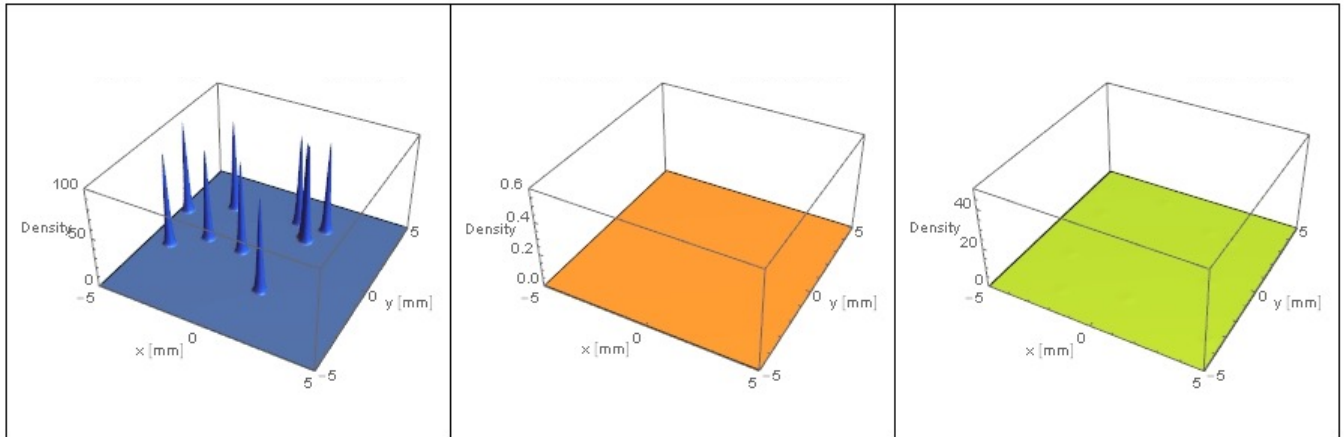


(b)

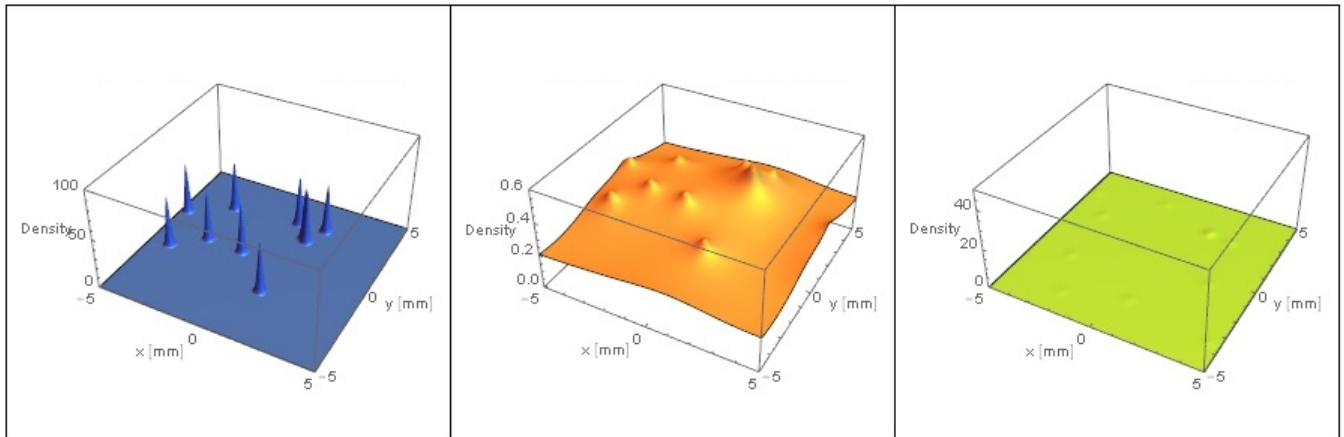


SLAB $\xrightarrow{\text{Lysis}}$ L-SLAB $\xrightarrow{\text{Diffusion}}$ NSLAB

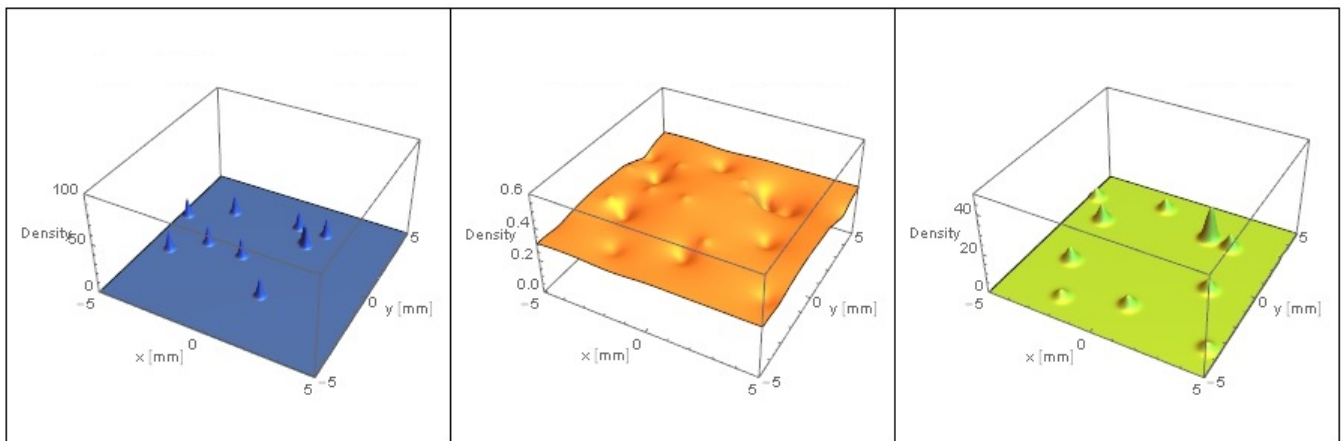
$t = 0$



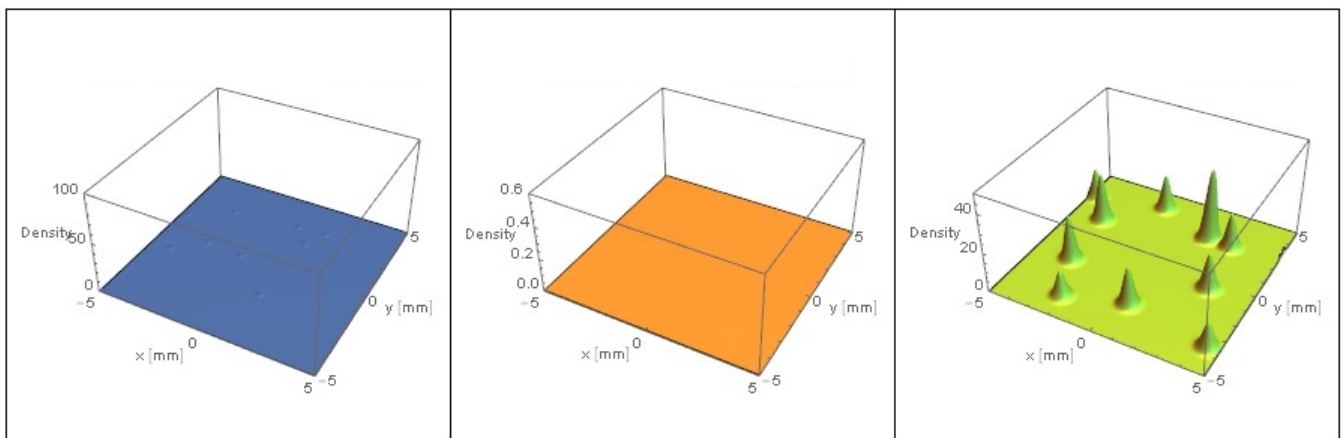
$t = 60$



$t = 160$



$t = 500$



“Demo” with 10 SLAB and 10 NSLAB colonies, Diff = 0.18 mm²/hour

