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# Reliability analysis and updating of deteriorating systems with subset simulation

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## 15 Abstract

16 An efficient approach to reliability analysis of deteriorating structural systems is presented, which 17 considers stochastic dependence among element deterioration. Information on a deteriorating 18 structure obtained through inspection or monitoring is included in the reliability assessment 19 through Bayesian updating of the system deterioration model. The updated system reliability is 20 then obtained through coupling the updated deterioration model with a probabilistic structural 21 model. The underlying high-dimensional structural reliability problems are solved using subset 22 simulation, which is an efficient and robust sampling-based algorithm suitable for such analyses. 23 The approach is demonstrated in two case studies considering a steel frame structure and a Daniels 24 system subjected to high-cycle fatigue.

25

# 26 Keywords

27 Structural reliability, deterioration, Bayesian analysis, inspection, monitoring, subset simulation

# 28 **1** Introduction

Engineering structures are generally subjected to deterioration processes such as fatigue and corrosion, and their structural reliability may thus reduce over time. Predictions of the deterioration progress with quantitative models are uncertain due to the simplified representation of the actual deterioration phenomena, the inherent variability of the influencing parameters and limited information on those parameters. These uncertainties must be addressed when modeling

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34 deterioration of structures (Lin and Yang 1985; Madsen et al. 1986; Melchers 1999a). Inspections 35 and monitoring are effective means of obtaining information on the actual condition of 36 deteriorating structures. This information should be utilized to reduce uncertainties in probabilistic 37 models. A consistent framework for this task is provided by Bayesian analysis, in which prior 38 probabilistic models are updated with inspection and monitoring outcomes (e.g. Tang 1973; 39 Madsen 1987; Enright and Frangopol 1999). This approach facilitates the quantification of the 40 effect of inspection and monitoring results on the structural reliability, and forms the basis for 41 decisions on maintenance actions and future inspection efforts (e.g. Thoft-Christensen and 42 Sørensen 1987; Faber et al. 2000; Moan 2005; Straub and Faber 2005).

43 Deterioration processes are generally correlated among structural elements within a system due to 44 common influencing factors, such as environmental conditions and material characteristics (e.g. 45 Moan and Song 2000; Vrouwenvelder 2004; Straub and Faber 2005; Stewart and Mullard 2007). 46 This leads to a correlation among deterioration failures of different elements whose effect on the 47 system reliability has to be assessed as a function of structural redundancy (Straub and Der 48 Kiureghian 2011). Correlation among element deterioration is especially relevant when inspection 49 and monitoring outcomes are considered in the reliability assessment (Vrouwenvelder 2004). An 50 observation at one location within a structure contains more indirect information on the 51 deterioration progress at another location if the correlation among element deterioration is high. 52 For these reasons, the reliability of deteriorating structures should be analyzed and updated 53 considering the structure as a whole.

54 A number of publications propose methods for computing the time-variant reliability of 55 deteriorating structures, including works by Mori and Ellingwood (1993), Li (1995), Ciampoli 56 (1998), Estes and Frangopol (1999), Stewart and Val (1999) and Li et al. (2015). They consider 57 the time-dependent characteristics of both the load and resistance, but do not account for 58 correlation among element deterioration. More recently, a number of researchers have considered 59 modeling and updating the system deterioration state of structures, taking into account the aspect of spatial correlation among element deterioration (Moan and Song 2000; Li et al. 2004; Faber et 60 61 al. 2006; Straub 2011b; Qin and Faber 2012; Maljaars and Vrouwenvelder 2014). Therein, the 62 effect of inspections and monitoring results on the probability of either reinforcement corrosion in 63 concrete structures or fatigue failures in steel structures is quantified using Bayesian analysis. 64 However, the impact of deterioration on the structural reliability is not included in these works. 65 Such integrated system reliability analyses are proposed in (Lee and Song 2014; Schneider et al. 2015; Luque and Straub 2016). Lee and Song (2014) consider sequential fatigue failures taking 66 into account the effect of stress redistribution within a structural system. They identify critical 67 68 failure sequences through a branch-and-bound scheme and iteratively compute and update bounds 69 on the system failure probability. Luque and Straub (2016) and Schneider et al. (2015) propose the 70 use of hierarchical Dynamic Bayesian Network (DBN) models for probabilistically representing 71 deterioration in structural systems and for updating deterioration probabilities as well as the system

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reliability with inspection and monitoring results. While they can be powerful, DBN models arerather demanding in the implementation.

74 To enable an integrated system reliability analysis of inspected and monitored deteriorating structures, which is computationally efficient and simple to implement, we here develop a 75 framework using two coupled sub-models: a probabilistic system deterioration model, which 76 77 considers stochastic dependence among element deterioration, and a probabilistic structural model 78 for calculating the failure probability of the weakened system. Motivated by the work of Straub 79 and Der Kiureghian (2011), the system deterioration state is assessed at discrete time intervals and 80 is considered constant within each interval. Information on the deteriorating structure obtained 81 through inspection or monitoring is included in the reliability assessment through Bayesian 82 updating of the system deterioration model. The updated system reliability is then obtained through 83 coupling this updated model with a probabilistic structural model. The resulting structural reliability problems are high-dimensional since they include all (correlated) deteriorating 84 85 elements. To solve these problems, we apply subset simulation, which is a sampling-based algorithm that can robustly and efficiently handle problems involving a large number of random 86 87 variables. The method is demonstrated in two case studies considering welded steel structures 88 subjected to fatigue deterioration.

## 89 2 System reliability analysis of deteriorating structures

#### 90 2.1 Deterioration modeling

Deterioration is modeled at the element level at discrete time steps. An element may be a structural member, a welded connection or a segment of a continuous surface (Straub and Der Kiureghian 2011). The state of deterioration of an element *i* at time *t* is represented by a random variable or random vector  $D_{i,t}$ . For example, in the context of reinforcement corrosion in concrete structures,  $D_{i,t}$  may represent the loss of reinforcement cross section. Deterioration of all elements is influenced by a set random variables  $\mathbf{X} = (X_1, ..., X_n)$ . The relationship between  $\mathbf{X}$  and  $D_{i,t}$  is described by a parametric deterioration model  $h_i$ , which is written in generic form as:

$$D_{i,t} = h_i(\mathbf{X}, t) \tag{1}$$

98 The joint probability density function (PDF) of **X** is denoted by  $f_{\mathbf{X}}(\mathbf{x})$ . Model uncertainties arising 99 from a simplified representation of the actual deterioration phenomenon are included through 100 additional random variables in **X**.

101 All random variables describing the deterioration state of the individual elements at time *t* are 102 summarized in a vector  $\mathbf{D}_t = (D_{1,t}, ..., D_{n_E,t})$ , where  $n_E$  is the number of elements considered in 103 the system reliability analysis. This vector represents the overall deterioration state of the structural 104 system at time t. The relationship between the system deterioration state  $D_t$  and the deterioration 105 model parameter X is described by a function h as:

$$\mathbf{D}_{t} = \mathbf{h}(\mathbf{X}, t) = \left(h_{1}(\mathbf{X}, t), \dots, h_{n_{E}}(\mathbf{X}, t)\right)$$
(2)

#### **2.2 Modeling dependence among deterioration model parameters**

107 Deterioration of different elements of a structural system is generally interdependent due to the 108 spatial correlation among the uncertain parameters **X** influencing their condition. Such spatial 109 dependencies are often due to geometrical proximity, but they mainly exist due to common factors 110 influencing the element condition such as environmental conditions and material characteristics 111 (Luque and Straub 2016). The aspect of spatial correlation of deterioration is especially relevant 112 when inspection and monitoring outcomes are considered in the reliability assessment of 113 deteriorating structures. The effect of such observations on the reliability strongly depends on the 114 spatial correlation among the parameters X. An observation at one location contains more indirect information on the deterioration progress at another location if the correlation among the 115 116 parameters **X** is high.

There is only limited information available on modeling statistical dependence of deterioration in structural systems (e.g. Vrouwenvelder 2004; Malioka 2009; Luque et al. 2016). For example, Vrouwenvelder (2004) estimated the correlation among uncertain parameters influencing fatigue crack growth in welded connections by comparing the scatter of the parameters within one production series to the scatter in the overall population. In most applications, however, correlation among the uncertain parameters **X** has to be estimated based at least partially on engineering judgment.

- Hierarchical models and random field models are commonly applied to represent spatial 124 125 dependence among the uncertain parameters **X**. The latter are suitable for representing parameters 126 with inherent spatial variability (e.g. Hergenröder 1992; Stewart and Mullard 2007; Malioka 127 2009). The random field approach models a spatially varying parameter X as a random variable 128 X(z) at each location z, and describes the correlation structure of the different random variables 129 X(z) in terms of a suitable correlation function. Such random fields are typically discretized to 130 enable their numerical representation (see, for example, Betz et al. 2014a). As a result, a random 131 field of a spatially varying parameter is defined by a discrete set of correlated random variables, 132 which are part of **X**. The joint distribution of the variables in a random field is commonly 133 represented by the Nataf model, also known as the Gaussian copula (Liu and Der Kiureghian
- 134 1986).
- 135 Hierarchical models may be applied if common influencing factors can be modeled explicitly (e.g.
- 136 Maes and Dann 2007; Luque et al. 2016). Such models represent correlation among random
- 137 variables by defining different levels. The random variables within one level are linked through

138 common influencing factors, which are modeled as random variables at a higher level in the 139 hierarchy. The random variables at the highest level are often called hyper-parameters (see, for 140 example, Maes and Dann 2007). The additional random variables representing common influencing

- 140 Example, waes and Dami 2007). The additional random variables representing common influencing
- 141 factors in a hierarchical model are included in **X**. The probability distributions of the random
- 142 variables in each level are defined conditional on the random variables at the next higher level in
- the hierarchy. Such a hierarchical dependence structure among the variables in **X** can be implemented through the Rosenblatt transformation (Hohenbichler and Rackwitz 1981).
- In many instances common influencing factors can, however, not be modeled explicitly. Instead, statistical dependence among the variables in **X** is often represented by correlation coefficients. As an example, statistical dependence of fatigue deterioration among welded connections due to common fabrication quality may be modeled by defining a correlation coefficient between the initial crack sizes at different hotspots (Vrouwenvelder 2004). In this case, the Nataf model can be applied to model the joint distribution of the correlated deterioration model parameters.
- Parameters influencing deterioration can also be time variant. Such parameters are ideally modeled by stochastic processes (see, for example, Lin and Yang 1985; Straub and Faber 2007; Altamura and Straub 2014). Similar to a random field, a stochastic process represents a time-varying parameter X as a random variable X(t) at each time t, and describes the correlation among the random variables X(t) through a suitable correlation function. Continuous-time stochastic processes are discretized to facilitate their numerical representation. The resulting set of correlated random variables is included in **X**. The joint distribution of the variables in a stochastic process
- 158 may be represented by the Nataf model. In case a stochastic process has the Markov property, the
- 159 Rosenblatt transformation may be applied (see, for example, Altamura and Straub 2014).

## 160 **2.3 Prior system failure probability**

The system failure probability is assessed conditional on the system deterioration state  $D_t$ . In 161 162 agreement with Straub and Der Kiureghian (2011), the deterioration state of a structure is 163 considered constant over a period  $\Delta t$ . The value of  $\Delta t$  depends on how fast deterioration progresses 164 and on the lifetime of the structure. In most applications, a good choice is  $\Delta t = 1$  year, which is 165 short compared to the typical lifetime of structural systems. Conservatively, the system deterioration state in the period  $[t - \Delta t, t]$  is set equal to the state at time t, **D**<sub>t</sub>. The relationship 166 between the system deterioration state  $D_t$  and the deterioration model parameters **X** is described 167 by Equation (2). Let  $F_t$  denote the event of system failure in the period  $[t - \Delta t, t]$ . The probability 168 169 of this event conditional on a realization of the deterioration model parameters  $\mathbf{x}$  is written as:

$$p_F(\mathbf{x}, t) = \Pr(F_t | \mathbf{X} = \mathbf{x}) = \Pr(F_t | \mathbf{D}_t = \mathbf{h}(\mathbf{x}, t))$$
(3)

170 To include the uncertainty in the deterioration model parameters, the total probability theorem is 171 applied. The overall probability of system failure in the reference period  $[t - \Delta t, t]$  is:

$$\Pr(F_t) = \int_{D_X} \Pr(F_t | \mathbf{X} = \mathbf{x}) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} = \int_{D_X} p_F(\mathbf{x}, t) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}$$
(4)

where  $D_X$  denotes the domain of definition of **X**. The total-probability-form of the structural reliability problem is advantageous when random variables not contained in **X** also influence the event of system failure  $F_t$  (Straub and Der Kiureghian 2010). Such random variables are considered in the computation of  $p_F(\mathbf{x}, t)$ . By defining the problem in this form, the deterioration model is decoupled from the structural model.

- The conditional system failure probability  $p_F(\mathbf{x},t) = \Pr(F_t | \mathbf{D}_t = \mathbf{h}(\mathbf{x},t))$  is computed by 177 178 performing system reliability analyses of the damaged structure. To this end, the structural model 179 is defined with element properties according to the system deterioration state  $\mathbf{D}_t = \mathbf{h}(\mathbf{x}, t)$ . 180 Random variables influencing the system reliability which are not contained in X are typically load 181 and resistance parameters. While resistance parameters, such as material strengths and structural 182 dimensions, are usually modeled as time-invariant random variables, load parameters are mostly 183 stochastic processes. However, it is typically sufficient to represent the load process by its extreme 184 value distribution for the reference period  $[t - \Delta t, t]$  (Melchers 1999b; Straub and Der Kiureghian 2011). The computation of  $p_F(\mathbf{x}, t)$  then reduces to a time-invariant reliability analysis of the 185 weakened system. This approach leads to an accurate solution if the load process is ergodic and 186 187 the maximum loads in two different time periods are statistically independent of each other. This 188 holds at least approximately for most relevant applications.
- Equation (4) can be transformed into a component reliability problem following Wen and Chen (1987). To this end, we introduce an auxiliary standard uniform random variable *P* with PDF  $f_P(p) = 1$  and cumulative distribution function (CDF)  $F_P(p) = p$ . We now note that the following identity holds:

$$p_F(\mathbf{x},t) = F_P(p_F(\mathbf{x},t)) = \Pr(P \le p_F(\mathbf{x},t))$$
(5)

193 The right hand side of Equation (5) corresponds to a component reliability problem with limit-194 state function:

$$g_F(\mathbf{x}, p, t) = p - p_F(\mathbf{x}, t) \tag{6}$$

195 The limit-state function  $g_F(\mathbf{x}, p, t)$  describes a domain  $\Omega_F(t)$  in the augmented outcome space of

196 **X** and *P* as  $\Omega_F(t) = \{(\mathbf{x}, p) : g_F(\mathbf{x}, p, t) \le 0\}$ . The conditional probability  $p_F(\mathbf{x}, t)$  can now be

197 expressed as:

$$p_F(\mathbf{x},t) = \int_{p \in \Omega_F(t)} f_P(p) \, dp = \int_0^1 I(g_F(\mathbf{x},p,t) \le 0) \, f_P(p) \, dp \tag{7}$$

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where  $I(\cdot)$  is the indicator function:  $I(\cdot) = 1$  if the condition ( $\cdot$ ) is true and  $I(\cdot) = 0$  otherwise. Inserting Equation (7) into Equation (4) gives:

$$\Pr(F_t) = \int_{D_{\mathbf{X}}} \left[ \int_0^1 I(g_F(\mathbf{x}, p, t) \le 0) f_P(p) dp \right] f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}$$
$$= \int_{D_{\mathbf{X}}} \int_0^1 I(g_F(\mathbf{x}, p, t) \le 0) f_{\mathbf{X}}(\mathbf{x}) f_P(p) d\mathbf{x} dp$$
$$= \int_{(\mathbf{x}, p) \in \Omega_F(t)} f_{\mathbf{X}}(\mathbf{x}) f_P(p) d\mathbf{x} dp$$
(8)

200 Equation (8) can be solved using structural reliability methods (e.g. Ditlevsen and Madsen 1996).

201 As discussed earlier, such a calculation requires the computation of the conditional system failure 202 probability  $p_F(\mathbf{x},t) = \Pr(F_t | \mathbf{D}_t = \mathbf{h}(\mathbf{x},t))$ , which can be determined using system reliability analysis methods (see, for example, Hohenbichler and Rackwitz 1983; Ditlevsen and Bjerager 203 204 1986; Thoft-Christensen and Murotsu 1986; Naess et al. 2009; Song and Kang 2009). If the number of distinct system deterioration states  $\mathbf{D}_t = \mathbf{d}_t$  is limited,  $\Pr(F_t | \mathbf{D}_t = \mathbf{d}_t)$  may be pre-205 computed for all  $\mathbf{D}_t = \mathbf{d}_t$ . If this is not possible,  $\Pr(F_t | \mathbf{D}_t = \mathbf{d}_t)$  has to be computed during the 206 207 evaluation of Equation (8). The computation of  $Pr(F_t | \mathbf{D}_t = \mathbf{d}_t)$  is discussed in more detail in 208 Section 7.

For the purpose of applying structural reliability methods, it is convenient to transform the reliability problem defined in Equation (8) to standard normal space. To this end, the auxiliary random variable *P* and the deterioration model parameters **X** are transformed to independent standard normal random variables  $\mathbf{U} = (U_0, U_1, ..., U_n)$ . *P* and **X** are independent and can be transformed separately. The inverse transformation from **U** to *P* and **X** is as follows (see also Straub and Papaioannou 2015b):

$$P = \Phi(U_0) \tag{9}$$

215 where  $\Phi(\cdot)$  is the standard normal CDF, and

$$\mathbf{X} = \mathbf{T}^{-1}(U_1, \dots, U_n) \tag{10}$$

- 216  $\mathbf{T}(\cdot)$  is a probability preserving one-to-one mapping from the original outcome space of **X** to the
- standard normal space for which the Rosenblatt transformation (Hohenbichler and Rackwitz 1981)
- or the Nataf transformation (Liu and Der Kiureghian 1986) can be applied (see also Section 2.2).
- 219 The limit-state function  $g_F$  is now transformed to **U**-space as:

$$G_F(\mathbf{u}, t) = u_0 - \Phi^{-1}(p_F(\mathbf{T}^{-1}(u_1, \dots, u_n), t))$$
(11)

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220  $G_F$  describes the domain  $\Omega_F^U(t)$  in the transformed space as  $\Omega_F^U(t) = {\mathbf{u} : G_F(\mathbf{u}, t) \le 0}$ . 221 Therefore, the system failure probability  $\Pr(F_t)$  can be expressed in **U**-space as:

$$\Pr(F_t) = \Pr(G_F(\mathbf{U}, t) \le 0) = \int_{\mathbf{u} \in \Omega_F^U(t)} \varphi_{n+1}(\mathbf{u}) \, d\mathbf{u}$$
(12)

where  $\varphi_{n+1}(\mathbf{u}) = \prod_{i=0}^{n} \varphi(u_i)$  is the (n + 1)-variate standard normal PDF and  $\varphi(\cdot)$  is the standard normal PDF.

#### **3** System reliability updating of deteriorating structures

#### **3.1 Modeling observations with likelihood functions**

226 In the context of deteriorating structures, inspections and monitoring systems typically provide 227 direct or indirect information on the uncertain deterioration model parameters **X**. This information 228 is usually imperfect due to measurement uncertainties and random errors. Generally, parameters 229 influencing the deterioration process, the deterioration state of the structure itself, and quantities 230 indirectly related to the deterioration state of the structure can be observed. Examples of such 231 observations are results from half-cell potential measurements that provide indirect information 232 on corrosion initiation in reinforced concrete structures and measurements of fatigue cracks in 233 welded steel structures.

- Mathematically, an inspection or monitoring outcome *i* at time *t* is an event, which is denoted by  $Z_i(t)$  in the following. The relationship between  $Z_i(t)$  and the uncertain deterioration model parameters **X** is modeled through a likelihood function  $L_i(\mathbf{x}, t)$ , which is proportional to the
- 237 conditional probability of observing  $Z_i(t)$  when the uncertain parameters **X** take a value **x**:

$$L_i(\mathbf{x}, t) \propto \Pr(Z_i(t) | \mathbf{X} = \mathbf{x})$$
<sup>(13)</sup>

238 Generally, two types of observations can be distinguished: observations providing equality 239 information and observations providing inequality information (Madsen et al. 1986; Straub 2011a). 240 Observations providing equality information are observations that can be described by an 241 observation event such as  $Z_i(t) = \{y_i(t) = q_i(\mathbf{X}, t) + E_i\}$ , where  $y_i(t)$  is a measurement of a 242 continuous quantity predicted by the model  $q_i(\mathbf{X}, t)$  and  $E_i$  is an additive measurement error with PDF  $f_{E_i}(\epsilon_i)$ . The following equality holds  $Y_i(t) - q_i(\mathbf{X}, t) = E_i$ , where  $Y_i(t)$  is the uncertain 243 244 measurement outcome. In this special but common case, the likelihood of observing  $Y_i(t) = y_i(t)$ given  $\mathbf{X} = \mathbf{x}$  is equal to the probability density of the random measurement error  $E_i$  taking the 245 value  $y_i(t) - q_i(\mathbf{x}, t)$ . The corresponding likelihood function can be written as (Straub and 246 247 Papaioannou 2015b):

$$L_i(\mathbf{x}, t) = f_{E_i}(y_i(t) - q_i(\mathbf{x}, t))$$
(14)

In the general case, the likelihood function of an observation  $Z_i(t)$  of the equality-type is defined as (Straub and Papaioannou 2015b):

$$L_i(\mathbf{x},t) = f_{Y_i(t)|\mathbf{X}}(y_i(t)|\mathbf{x})$$
(15)

where  $f_{Y_i(t)|\mathbf{X}}(y_i(t)|\mathbf{x})$  is the conditional PDF of the uncertain measurement outcome  $Y_i(t)$  given X = x, which is typically defined in terms of the PDF of the associated measurement error  $E_i$ . The likelihood function defined in Equation (15) includes the evaluation of the model predicting the measured quantity as in Equation (14).

Observations providing inequality information are observations such as "corrosion progress is larger than a limit" or "no fatigue crack detected". An observation  $Z_i(t)$  of the inequality type is modeled through a function  $q_i(\mathbf{X}, t)$  as follows (Madsen et al. 1986):

$$Z_i(t) = \{q_i(\mathbf{X}, t) \le 0\}$$

$$\tag{16}$$

A function  $q_i(\mathbf{X}, t)$  of this type can be interpreted as a limit-state function. The corresponding likelihood function is written as (Straub and Papaioannou 2015b):

$$L_i(\mathbf{x}, t) = \Pr(Z_i(t) | \mathbf{X} = \mathbf{x}) = I(q_i(\mathbf{x}, t) \le 0)$$
(17)

- 259 The value of such a likelihood function is either 0 or 1.
- All observations obtained in the period [0, t] are expressed by a combined event  $Z_{0:t}$  as follows:

$$Z_{0:t} = \bigcap_{j=1}^{n_Z(t)} \left( \bigcap_{i \in S_j} Z_i(t_j) \right)$$
(18)

where  $n_Z(t)$  is the number of times at which inspections or measurements are performed in the period [0, t] and  $S_j$  is an index set containing the indices of all observations at time  $t_j$ . The likelihood function describing the relationship between  $Z_{0:t}$  and the uncertain deterioration model parameters **X** is defined as:

$$L(\mathbf{x}, t) \propto \Pr(Z_{0:t} | \mathbf{X} = \mathbf{x}) \tag{19}$$

265 Under the common assumption that all individual observations are statistically independent given 266 the deterioration model parameters  $\mathbf{X} = \mathbf{x}$ ,  $L(\mathbf{x}, t)$  is computed as:

$$L(\mathbf{x},t) = \prod_{j=1}^{n_Z(t)} \left( \prod_{i \in S_j} L_i(\mathbf{x},t_j) \right)$$
(20)

In the case of statistically dependent observations, the combined likelihood has to be formulated
as a function of the joint distribution of all measurement errors. Straub and Papaioannou (2015a)
provide further details on how to model observations with likelihood functions.

#### 270 **3.2** Posterior system failure probability

The goal here is to assess the effect of inspection and monitoring outcomes on the failure probability of deteriorating structural systems. In Bayesian analysis, this is achieved by computing the conditional probability of the failure event  $F_t$  given the observation event  $Z_{0:t}$ , which is defined as follows:

$$\Pr(F_t|Z_{0:t}) = \frac{\Pr(F_t \cap Z_{0:t})}{\Pr(Z_{0:t})}$$
(21)

We make the fundamental assumption that the system failure event  $F_t$  and the observation event Z<sub>0:t</sub> are conditionally independent given the deterioration model parameters  $\mathbf{X} = \mathbf{x}$ . The joint probability of the events  $F_t$  and  $Z_{0:t}$  can therefore be written as:

$$\Pr(F_t \cap Z_{0:t}) = \int_{\boldsymbol{D}_{\mathbf{X}}} \Pr(F_t | \mathbf{X} = \mathbf{x}) \Pr(Z_{0:t} | \mathbf{X} = \mathbf{x}) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}$$
(22)

The probability of the observation outcome event can also be expressed in the total-probabilityform:

$$\Pr(Z_{0:t}) = \int_{\boldsymbol{D}_{\mathbf{X}}} \Pr(Z_{0:t} | \mathbf{X} = \mathbf{x}) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}$$
(23)

Following Straub (2011a), the integrals in Equations (22) and (23) are transformed such that they can be solved using structural reliability methods. This method follows the same principles as presented in Section 2.3 for the computation of  $Pr(F_t)$ .

For the purpose of transforming Equations (23), an auxiliary standard normal random variable *P* is again introduced. In addition, let *c* be a positive constant that ensures  $0 \le cL(\mathbf{x}, t) \le 1$  for all **x**. In this case, the following relationship holds:

$$cL(\mathbf{x},t) = F_P(cL(\mathbf{x},t)) = \Pr(P \le cL(\mathbf{x},t))$$
(24)

The right hand side of Equation (24) corresponds to a component reliability problem with limitstate function:

$$g_{Z_e}(\mathbf{x}, p, t) = p - cL(\mathbf{x}, t)$$
<sup>(25)</sup>

288 The limit-state function  $g_{Z_e}(\mathbf{x}, p, t)$  defines a domain  $\Omega_{Z_e}(t)$  in the augmented outcome space of

289 **X** and *P* as 
$$\Omega_{Z_e}(t) = \{(\mathbf{x}, p) : g_{Z_e}(\mathbf{x}, p, t) \le 0\}$$
. The quantity  $cL(\mathbf{x}, t)$  can be interpreted as the

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290 conditional probability of *P* taking a value in  $\Omega_{Z_e}(t)$  given  $\mathbf{X} = \mathbf{x}$ . It can thus be computed by 291 integrating  $f_P(p)$  over the failure domain  $\Omega_{Z_e}(t)$  when **X** take a value **x**:

$$cL(\mathbf{x},t) = \int_{p \in \Omega_{Z_e}(t)} f_P(p) \, dp = \int_0^1 I(g_{Z_e}(\mathbf{x},p,t) \le 0) \, f_P(p) \, dp \tag{26}$$

292 Consequently, the likelihood function  $L(\mathbf{x}, t)$  can be written as:

$$L(\mathbf{x},t) = \frac{1}{c} \int_{0}^{1} I(g_{Z_{e}}(\mathbf{x},p,t) \le 0) f_{P}(p) dp$$
(27)

293 Let *a* denote the proportionality constant in the likelihood definition given in Equation (19). It 294 follows that:

$$\Pr(Z_{0:t}|\mathbf{X} = \mathbf{x}) = aL(\mathbf{x}, t) = \frac{a}{c} \int_{0}^{1} I(g_{Z_{e}}(\mathbf{x}, p, t) \le 0) f_{P}(p) dp$$
(28)

295 Inserting Equation (28) into Equation (23) gives:

$$\Pr(Z_{0:t}) = \frac{a}{c} \int_{D_{\mathbf{X}}} \left[ \int_{0}^{1} I(g_{Z_{e}}(\mathbf{x}, p, t) \le 0) f_{P}(p) dp \right] f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}$$
  
$$= \frac{a}{c} \int_{(\mathbf{x}, p) \in \Omega_{Z_{e}}(t)} f_{\mathbf{X}}(\mathbf{x}) f_{P}(p) d\mathbf{x} dp$$
(29)

Similarly, it can be shown that the probability of the joint event  $F_t \cap Z_{0:t}$  can be written as:

$$\Pr(F_t \cap Z_{0:t}) = \frac{a}{c} \int_{(\mathbf{x}, p) \in \Omega_{F \cap Z_e}(t)} f_{\mathbf{X}}(\mathbf{x}) f_P(p) \, d\mathbf{x} \, dp \tag{30}$$

where the domain  $\Omega_{F \cap Z_e}(t)$  is defined in the augmented outcome space of **X** and *P* in terms of the limit-state function:

$$g_{F \cap Z_e}(\mathbf{x}, p, t) = p - p_F(\mathbf{x}, t) \cdot cL(\mathbf{x}, t)$$
(31)

- 299 as  $\Omega_{F\cap Z_e}(t) = \{(\mathbf{x},p) : g_{F\cap Z_e}(\mathbf{x},p,t) \leq 0\}.$
- 300 Inserting Equation (29) and Equation (30) into Equation (21) gives:

$$\Pr(F_t|Z_{0:t}) = \frac{\int_{(\mathbf{x};p)\in\Omega_{F\cap Z_e}(t)} f_{\mathbf{X}}(\mathbf{x})f_P(p)\,d\mathbf{x}\,dp}{\int_{(\mathbf{x};p)\in\Omega_{Z_e}(t)} f_{\mathbf{X}}(\mathbf{x})f_P(p)\,d\mathbf{x}\,dp} = \frac{\Pr(g_{F\cap Z_e}(\mathbf{X},P,t)\leq 0)}{\Pr(g_{Z_e}(\mathbf{X},P,t)\leq 0)}$$
(32)

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301 Note that the proportionality constant *a* vanishes. The numerator and the denominator in Equation

302 (32) correspond to component reliability problems, which can be solved using structural reliability303 methods.

The solution given in Equation (32) can be interpreted as follows. The denominator in Equation (32) corresponds to the probability of an inequality observation event  $Z_{e,0:t} = \{g_{Z_e}(\mathbf{X}, P, t) \le 0\}$ and the numerator is equal to the probability of the joint event  $F_t \cap Z_{e,0:t} = \{g_{F \cap Z_e}(\mathbf{X}, P, t) \le 0\}$ . In the context of Bayesian updating, the event  $Z_{e,0:t}$  is equivalent to the original observation event

308  $Z_{0:t}$  in the sense that:

$$\Pr(F_t|Z_{0:t}) = \Pr(F_t|Z_{e,0:t}) = \frac{\Pr(F_t \cap Z_{e,0:t})}{\Pr(Z_{e,0:t})}$$
(33)

309 For applying structural reliability methods, the component reliability problems defined in Equation

310 (32) are also transformed to standard normal space following Section 2.3. The corresponding limit-

311 state functions  $G_{F \cap Z_e}$  and  $G_{Z_e}$  in **U**-space are:

$$G_{F \cap Z_e}(\mathbf{u}, t) = u_0 - \Phi^{-1} \left( p_F(\mathbf{T}^{-1}(u_1, \dots, u_n), t) \cdot cL(\mathbf{T}^{-1}(u_1, \dots, u_n), t) \right)$$
(34)

312 and

$$G_{Z_e}(\mathbf{u},t) = u_0 - \Phi^{-1}(cL(\mathbf{T}^{-1}(u_1,\dots,u_n),t))$$
(35)

These limit-state functions respectively describe the domains  $\Omega_{F \cap Z_e}^U(t) = \{\mathbf{u} : G_{F \cap Z_e}(\mathbf{u}, t) \le 0\}$ and  $\Omega_{Z_e}^U(t) = \{\mathbf{u} : G_{Z_e}(\mathbf{u}, t) \le 0\}$  in the transformed space. Consequently, the probabilities

315  $\Pr(F_t \cap Z_{e,0:t})$  and  $\Pr(Z_{e,0:t})$  can be computed as:

$$\Pr(F_t \cap Z_{e,0:t}) = \Pr(G_{F \cap Z_e}(\mathbf{U}, t) \le 0) = \int_{\mathbf{u} \in \Omega_{F \cap Z_e}^U(t)} \varphi_{n+1}(\mathbf{u}) \, d\mathbf{u}$$
(36)

316 and

$$\Pr(Z_{e,0:t}) = \Pr(G_{Z_e}(\mathbf{U}, t) \le 0) = \int_{\mathbf{u} \in \Omega_{Z_e}^U(t)} \varphi_{n+1}(\mathbf{u}) \, d\mathbf{u}$$
(37)

The computation of the integrals in Equations (36) and (37) requires the selection of the constant *c*. A discussion on how to select *c* is provided in (Betz et al. 2014b; Au et al. 2015; Straub and Papaioannou 2015b). In the general case, the optimal choice is  $c = 1/\sup(L(\mathbf{x}, t))$  where  $\sup(\cdot)$ is the supremum of the expression ( $\cdot$ ). In some cases,  $\sup(L(\mathbf{x}, t))$  can be readily selected. For instance, in the special case of a single measurement with measurement error *E*, the supremum of the likelihood function is  $\sup(L(\mathbf{x}, t)) = \max(f_E(\epsilon))$  where  $f_E(\epsilon)$  is the PDF of *E*.

#### 323 **4** Computing system failure probabilities with subset simulation

Subset simulation, originally proposed by Au and Beck (2001), is an adaptive Monte Carlo method particularly suitable for evaluating the high-dimensional reliability problems defined in Equations (12), (36) and (37). The method is robust and computationally efficient, and it can be implemented relatively easily. The algorithm is here implemented following Papaioannou et al. (2015).

328 First, consider the computation of the prior system failure probability  $Pr(F_t) = Pr(G_F(\mathbf{U}, t) \le 0)$ .

329 The basic idea of subset simulation is to express the event  $F_t$  as an intersection of M intermediate 330 events:

$$F_t = E_1 \cap E_2 \cap \dots \cap E_M \tag{38}$$

331 The intermediate events are nested, i.e.  $E_1 \supset E_2 \supset \cdots \supset E_M = F_t$ . Consequently, the probability 332 of the event  $F_t$  can be computed by a product of conditional probabilities:

$$\Pr(F_t) = \Pr(E_1 \cap E_2 \cap ... \cap E_M) = \prod_{i=1}^M \Pr(E_i | E_{i-1})$$
(39)

In this formulation, the event  $E_0$  is the certain event. The intermediate events are selected such that the conditional probabilities  $Pr(E_i|E_{i-1})$ , i = 1, ..., M are much larger than  $Pr(F_t)$ . In this way, the original problem of evaluating the small probability of the rare event  $F_t$  reduces to computing a sequence of *M* larger conditional probabilities.

The intermediate events  $E_i$ , i = 1, ..., M are defined as  $E_i = \{G_F(\mathbf{U}, t) \le b_i\}$  where  $b_1 > b_2 > b_1$ 337  $\cdots > b_M = 0$ . The values of  $b_i$  are selected adaptively such that the conditional probabilities are 338 339 equal to a chosen value  $p_0$ . For this purpose, N samples of **U** are simulated at each subset level *i*, 340 conditional on the previous intermediate event  $E_{i-1}$ . For each generated sample, the limit-state 341 function  $G_F(\mathbf{u}, t)$  is evaluated and  $b_i$  is set equal to the  $p_0$ -percentile of the N resulting values of 342  $G_F(\mathbf{u}, t)$ . This procedure is repeated until the  $p_0$ -percentile becomes negative. At this stage, the failure event  $E_M = F_t$  is reached, for which  $b_M = 0$ . The samples conditional on the event  $E_0$  are 343 344 obtained by crude Monte Carlo sampling. The samples conditional on the events  $E_i$ , i = 1, ..., M - M1 are generated by simulating states of Markov chains starting from the samples conditional on 345  $E_{i-1}$ , for which  $G_F(\mathbf{u}, t) \leq b_i$ . This is achieved by application of Markov Chain Monte Carlo 346 (MCMC) sampling. An estimator  $\hat{P}_{SuS}$  of the prior system failure probability  $Pr(F_t)$  can now be 347 348 written as:

$$\Pr(F_t) \approx \hat{P}_{SuS} = p_0^{M-1} \hat{P}_M \tag{40}$$

 $\hat{P}_{M} \text{ is the estimate of the conditional probability } \Pr(E_{M}|E_{M-1}), \text{ which is given by the ratio of the number of samples for which } G_{F}(\mathbf{u}, t) \leq 0 \text{ over the number of samples } N \text{ simulated conditional on } E_{M-1}.$ 

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- 352 Note that the MCMC samples are generally not statistically independent. Their correlation has an
- effect on the efficiency and accuracy of subset simulation (see, for example, Au and Beck 2001;
- 354 Schuëller and Pradlwarter 2007; Papaioannou et al. 2015). It is important to adopt an MCMC
- 355 sampling algorithm that produces samples with low correlation such that the conditional
- probabilities  $Pr(E_i|E_{i-1})$  can be estimated with a minimum number of samples. We adopt the
- adaptive MCMC sampling algorithm of Papaioannou et al. (2015).
- The value  $p_0$  of the conditional probabilities and the number of simulated samples *N* at each subset level can be chosen freely. A value of  $p_0 = 0.1$  is a suitable choice. *N* should be selected large enough to give accurate estimates of  $p_0$ . Note that the total number of required samples for estimating  $Pr(F_t)$  increases linearly with  $-\log_{10}(Pr(F_t))$  when using subset simulation instead of with  $1/Pr(F_t)$  when using crude Monte Carlo simulation (Au and Beck 2001).
- The probabilities  $\Pr(F_t \cap Z_{e,0:t}) = \Pr(G_{F \cap Z_e}(\mathbf{U}, t) \le 0)$  and  $\Pr(Z_{e,0:t}) = \Pr(G_{Z_e}(\mathbf{U}, t) \le 0)$  are calculated accordingly. The posterior system failure probability  $\Pr(F_t|Z_{0:t})$  is then computed using Equation (21). Alternatively, the conditional probability  $\Pr(F_t|Z_{0:t}) = \Pr(F_t|Z_{e,0:t})$  can be estimated directly with a new subset simulation run following the estimation of  $\Pr(Z_{e,0:t})$  (see also Schneider et al. 2013; Straub et al. 2016). For this purpose, a set of nested intermediate events  $E_0 \supset E_1 \supset \cdots \supset E_M$  is defined where  $E_0 = Z_{e,0:t}$ ,  $E_i = \{G_{F \cap Z_e}(\mathbf{U}, t) \le b_i\}$ , i = 1, ..., M and  $b_1 >$  $b_2 > \cdots > b_M = 0$ . The conditional probability  $\Pr(F_t|Z_{0:t})$  can now be expressed as:

$$\Pr(F_t|Z_{0:t}) = \Pr(E_1 \cap E_2 \cap ... \cap E_M|E_0) = \prod_{i=1}^M \Pr(E_i|E_{i-1})$$
(41)

The first threshold  $b_1$  defining the intermediate event  $E_1 = \{G_{F \cap Z_e}(\mathbf{U}, t) \le b_1\}$  is determined from the samples conditional on  $E_0 = Z_{e,0:t}$ , which are obtained as a by-product of estimating  $\Pr(Z_{e,0:t})$ with subset simulation. The remaining thresholds  $b_i$ , i = 2, ..., M - 1 are determined following the original subset simulation procedure. When applying this approach, the estimator  $\hat{P}_{SuS}$  defined in Equation (40) provides an estimate of the conditional probability  $\Pr(F_t | Z_{0:t})$ .

## **5** Application a: Zayas frame subjected to fatigue deterioration

We consider the two-dimensional welded steel frame shown in Figure 1, which is known as Zayas frame (Zayas et al. 1980). The critical load scenario is an environmental load *L*. In addition, the frame is subjected to fatigue loads throughout its service life of T = 50 years. The effect of inspections on the fatigue reliability of the elements and on the reliability of the complete structural system is studied.



Figure 1. Zayas frame (Zayas et al. 1980). OD is the outer diameter and WT is the wall thickness of thetubular steel members.

#### 383 5.1 System model

The Zayas frame consists of tubular steel elements with welded connections. The state of fatigue deterioration of any element *i* depends on the condition of the associated welded connections. Fatigue cracks usually develop at locations with local stress concentrations; welded connections are especially vulnerable due to material inhomogeneities, imperfections, high stress concentrations and residual stresses (Fricke 2003). Locations where fatigue cracks may develop are called hotspots. A welded connection may contain multiple hotspots.

Fatigue crack growth reduces the capacity of welded connections. In the current example, we assume that fatigue deterioration occurs at the welds connecting the braces with the legs and with the upper horizontal element as well as at the welds at the intersection of the X-braces. Furthermore, we assume that each deteriorating welded connection contains only one critical hotspot. Thus, there are  $n_E = 13$  deteriorating elements and  $n_H = 22$  hotspots as indicated in Figure 1.

- The approach of Straub and Der Kiureghian (2011) is adopted to determine the reliability of the
- 397 welded steel structure subjected to fatigue. At system level, no gradual degradation of weld
- 398 capacities is considered. At a given time t, a welded connection has either its full capacity or it has
- 399 completely lost its capacity because of fatigue crack growth. In the current example, we assume
- 400 that a welded connection loses its capacity if a fatigue crack at any of the associated hotspots grows
- 401 beyond a critical size (e.g. Madsen 1997). Thus, the deterioration state of any hotspot j at time t

402 is modeled by a binary random variable  $D_{H,j,t}$ , where  $\{D_{H,j,t} = 1\}$  is the hotspot fatigue damage 403 event and  $\{D_{H,j,t} = 0\}$  is the compliment. The event of fatigue damage of hotspot *j* at time *t* is 404 defined by a limit-state function  $g_{H,j}(\mathbf{x}, t)$  as  $\{D_{H,j,t} = 1\} = \{g_{H,j}(\mathbf{X}, t) \le 0\}$  where **X** denotes the 405 vector of all uncertain parameters that describe fatigue deterioration of all hotspots considered in 406 the system reliability analysis.  $g_{H,j}(\mathbf{x}, t)$  is written as:

$$g_{H,j}(\mathbf{x},t) = a_{c,j} - a_j(\mathbf{x},t) \tag{42}$$

407 where  $a_{c,j}$  is the critical crack size and  $a_j(\mathbf{x}, t)$  is the fatigue crack size at hotspot j at time t. 408  $a_j(\mathbf{x}, t)$  is computed by means of a probabilistic fatigue crack growth model presented in Section 409 5.2.  $a_{c,j}$  may be defined such that failure modes such as plastic collapse or unstable crack growth 410 are approximately accounted for.

411 A structural element loses its capacity if any of the associated welded connections loses its 412 capacity. It follows that an element fails as soon as any of the associated hotspots fails due to 413 fatigue deterioration; this corresponds to a series system. The deterioration state of any element *i* 414 at time *t* is, therefore, also modeled by a binary random variable  $D_{i,t}$  where  $\{D_{i,t} = 1\}$  is the event 415 of element fatigue failure and  $\{D_{i,t} = 0\}$  is the compliment. From system reliability theory it 416 follows that the event of fatigue failure of element *i* can be written as:

$$\{D_{i,t} = 1\} = \bigcup_{j \in C_i} \{D_{H,j,t} = 1\}$$
(43)

417 where  $C_i$  is an index set containing the indices of all hotspots associated with element *i*. The event 418 of fatigue failure of element *i* can also be expressed by a limit-state function  $g_i(\mathbf{x}, t)$  such that 419  $\{D_{i,t} = 1\} = \{g_i(\mathbf{X}, t) \le 0\}$ .  $g_i(\mathbf{x}, t)$  is defined as a combination of the individual hotspot limit-420 state functions  $g_{H,i}(\mathbf{x}, t), \forall j \in C_i$  as:

$$g_i(\mathbf{x}, t) = \min_{j \in C_i} g_{H,j}(\mathbf{x}, t)$$
(44)

421 The function  $h_i$  defining the relationship between the fatigue model parameters **X** and the element 422 deterioration state  $D_{i,t}$  can now be written as:

$$D_{i,t} = h_i(\mathbf{X}, t) = I(g_i(\mathbf{X}, t) \le 0)$$

$$\tag{45}$$

423 Using Equation (2), the system deterioration state  $\mathbf{D}_t = (D_{1,t}, \dots, D_{n_E,t})$  of the Zayas frame can

424 subsequently be calculated as a function of the uncertain fatigue model parameters **X**. In the current

425 example,  $\mathbf{D}_t$  is a binary random vector with  $2^{n_E}$  states.

426 The Zayas frame is subjected to a time-variant horizontal load whose annual maximum *L* has the 427 Gumbel distribution with a coefficient of variation (c.o.v.)  $\delta_L = 0.35$ . The CDF of *L* is denoted by 428  $F_L(l)$ . Material and geometry properties are modeled as deterministic parameters as listed in Figure 429 1. This simplification is reasonable since the uncertainties associated with these quantities are 430 small compared to the uncertainties associated with the system deterioration state and the load *L*. 431 It is thus possible to determine a deterministic ultimate horizontal capacity  $r(\mathbf{d}_t)$  of the damaged 432 Zayas frame for any realization of the system deterioration state  $\mathbf{D}_t = \mathbf{d}_t$ . Consequently, the 433 conditional probability of system failure  $p_F(\mathbf{x}, t)$  of the Zayas frame corresponds to the probability 434 that the annual maximum load *L* exceeds the ultimate capacity  $r(\mathbf{d}_t)$ :

$$\Pr(F_t | \mathbf{D}_t = \mathbf{d}_t) = \Pr(r(\mathbf{d}_t) \le L) = 1 - F_L(r(\mathbf{d}_t))$$
(46)

435 The mean of *L* is selected such that the undamaged Zayas frame has an annual probability of 436 system failure  $Pr(F_t | \mathbf{D}_t = \mathbf{0}) = 1.3 \times 10^{-6}$ , leading to  $\mu_L = 62$  kN.

In the current example,  $r(\mathbf{d}_t)$  is computed by performing pushover analysis of the structure with 437 438 all elements damaged according to  $\mathbf{D}_t = \mathbf{d}_t$ , i.e. all elements with fatigue failure are removed from 439 the model used in the pushover analysis. Through such analyses the ultimate capacity of framed 440 steel structures can be quantified. Non-linear effects associated with non-linear material behavior, 441 imperfections, large displacements and deformations (large strains) are modelled explicitly. The 442 analysis captures load redistribution within the structural system resulting from local stiffness 443 changes. It simulates the collapse process of the structural system including initial yielding, 444 formation of plastic hinges, member buckling as well as formation of a global system collapse 445 mechanism (see e.g. Ultiguide 1999; Skallerud and Amdahl 2002).

In the current study,  $2^{13} = 8192$  pushover analyses are carried out using USFOS (2014) to pre-446 447 calculate the maximum resistance  $r(\mathbf{d}_t)$  for all possible realizations of the system deterioration 448 state  $\mathbf{D}_t$ . The corresponding conditional system failure probabilities  $\Pr(F_t | \mathbf{D}_t = \mathbf{d}_t)$  are computed 449 according to Equation (46). These failure probabilities have a reference period  $\Delta t = 1$  year but are 450 independent of time. In the subsequent reliability analysis of the deteriorating Zayas frame, the 451 computation of  $p_F(\mathbf{x}, t)$  is thus reduced to a lookup operation in which a realization of the fatigue 452 model parameters  $\mathbf{x}$  is matched to a pre-calculated conditional system failure probability  $\Pr(F_t | \mathbf{D}_t = \mathbf{d}_t)$  at time t by means of Equation (3), i.e.  $\mathbf{d}_t = \mathbf{h}(\mathbf{x}, t)$ . 453

The influence of individual element failure on the reliability of the Zayas frame depends on the structural importance of the failed element. Following Straub and Der Kiureghian (2011), the structural importance of an element *i* is quantified in terms of the single-element importance measure  $SEI_i$ , which is defined as the difference in the failure probability of the undamaged system and the failure probability of the system in which only element *i* has failed due to fatigue deterioration.

$$SEI_{i} = Pr(F_{t}|D_{1,t} = 0, ..., D_{i-1,t} = 0, D_{i,t} = 1, D_{i+1,t} = 0, ..., D_{n_{E},t} = 0) - Pr(F_{t}|\mathbf{D}_{t} = \mathbf{0})$$
(47)

Table 1 summarizes the single-element importance measures for all deteriorating elementsconsidered in the system reliability analysis of the Zayas frame.

462 Table 1. Single-element importance (SEI) measure and structural importance category of all deteriorating463 elements of the Zayas frame.

Element <i>i</i>	SEI <sub>i</sub>	Structural importance category	
1, 3	$1.14 \times 10^{-5}$	Medium	
2, 4	$1.06 \times 10^{-5}$	Medium	
5,7	$1.99 \times 10^{-3}$	High	
6, 8	$2.00 \times 10^{-3}$	High	
9, 10	$7.25 \times 10^{-7}$	Low	
11	$8.26 \times 10^{-8}$	Low	
12	$6.31 \times 10^{-7}$	Low	
13	$2.27 \times 10^{-7}$	Low	

464

The lower X-braces (elements 5 to 8) are the most important elements followed by the X-braces at the level above (elements 1 to 4). The top braces (elements 9 and 10) and the horizontal braces (elements 11 to 13) are the least important elements.

#### 468 **5.2 Fatigue model**

In the current example, we adopt the widely used Paris' law (Paris and Erdogan 1963) to describe fatigue crack growth at a given hotspot. For illustration purposes, we consider a through-thickness fatigue crack in an infinite plate subjected to fluctuating stresses in the plane of the plate and orthogonal to the crack. In this case, the fatigue crack is fully characterized by its length 2a and Paris' law is written as:

$$\frac{da(n)}{dn} = C \left(\Delta S(n) \sqrt{\pi a(n)}\right)^m \tag{48}$$

474 da(n)/dn is the crack growth rate, *n* is the number of applied fatigue stress cycles, *C* and *m* are 475 empirical material parameters and  $\Delta S(n)$  is the varying far-field fatigue stress range. The quantity 476  $\Delta K = \Delta S(n)\sqrt{\pi a(n)}$  is the stress intensity factor (SIF) range. This model can be extended to 477 account for more complex fatigue crack and hotspot geometries as well as more complex fatigue 478 stress distributions (Straub 2004). If desired, the model can be replaced altogether with a more 479 advanced crack growth model (e.g. Altamura and Straub 2014). This will not affect the method as 480 described in the remainder of the paper.

481 Fatigue loads are generally random and the load sequence  $\Delta S(n)$  is ideally modeled by a stochastic 482 process (Altamura and Straub 2014). Under the condition that the fatigue stress process is 483 stationary, ergodic and sufficiently mixing, a simplified approach can be adopted where the crack 484 growth rate da(n)/dn given by Equations (48) is approximated by its expected value with respect 485 to  $\Delta S$ :

$$\frac{da(n)}{dn} \approx E_{\Delta S} \left[ C \left( \Delta S(n) \sqrt{\pi a(n)} \right)^m \right] = C \left( \sqrt{\pi a(n)} \right)^m E_{\Delta S} [\Delta S(n)^m]$$
(49)

The fatigue stress process is described by its stationary distribution  $f_{\Delta S}(\Delta S)$  and an annual stress cycle rate  $\nu$  (e.g. Madsen et al. 1986). The quantity  $\Delta S_e = (E_{\Delta S}[\Delta S(n)^m])^{1/m}$  is interpreted as an equivalent stress range. In the current example, we assume that the stationary distribution of the fatigue stress ranges  $f_{\Delta S}(\Delta S)$  can be modeled by a Weibull distribution. The equivalent stress range is hence given by:

$$\Delta S_e = (E_{\Delta S}[\Delta S(n)^m])^{1/m} = k\Gamma \left(1 + \frac{m}{\lambda}\right)^{1/m}$$
(50)

491  $\Gamma(\cdot)$  denotes the Gamma function and k and  $\lambda$  are the Weibull scale and shape parameters. k is 492 modeled as a lognormal random variable to model statistical uncertainties in the calculation of 493  $\Delta S_e$ ;  $\lambda$  is assumed to be deterministic.

The parameters C and m of Paris' law are modeled as time-invariant random variables to capture uncertainties due to the variability of the material properties and material inhomogeneities. Proper attention has to be paid to modeling the correlation among C and m. They are empirical parameters generally derived from the same experiments and are therefore strongly correlated. To model the dependence among the Paris' law parameters, the linear relationship between ln C and m given in (Gurney 1978) is adopted:

$$\ln C = -15.84 - 3.34m \tag{51}$$

500 Equation (51) is valid if stresses are given in N/mm<sup>2</sup> and the crack growth rate is given in m/cycle. 501 In the following, *C* is modeled as a lognormally distributed random variable. *m* is thus normal 502 distributed due to the linear relationship between  $\ln C$  and *m*.

503 To capture uncertainties in the fabrication quality, the initial crack size  $a_0$  is modeled as a random 504 variable with exponential distribution. Uncertainties in the calculation of the hotspot stress and in 505 the calculation of the SIF range are captured by introducing lognormal random bias factors  $B_{\Delta S}$ 506 and  $B_{SIF}$ , which are multiplied with the calculated equivalent stress range  $\Delta S_e$ . The one-507 dimensional crack growth model given in Equation (48) is rewritten as:

$$\frac{da(n)}{dn} = C \left( B_{SIF} B_{\Delta S} \Delta S_e \sqrt{\pi a(n)} \right)^m$$
(52)

508 With  $a_j(n = 0) = a_{0,j}$  as initial condition, the differential equation given by Equation (52) is 509 solved for the fatigue crack size  $a_j$  at hotspot *j* as a function of time *t* (Madsen et al. 1986):

$$a_{j}(\mathbf{X},t) = \begin{cases} \left[ \left(1 - \frac{m_{j}}{2}\right) C_{j} B_{SIF,j}^{m_{j}} B_{\Delta S,j}^{m_{j}} \Delta S_{e,j}^{m_{j}} \pi^{\frac{m_{j}}{2}} \nu_{j} t + a_{0,j}^{\left(1 - \frac{m_{j}}{2}\right)} \right]^{\left(1 - \frac{m_{j}}{2}\right)^{-1}}, & m_{j} \neq 2 \\ a_{0,j} \exp\left(C_{j} B_{SIF,j}^{2} B_{\Delta S,j}^{2} \Delta S_{e,j}^{2} \pi \nu_{j} t\right), & m_{j} = 2 \end{cases}$$
(53)

where *t* is the time in years,  $v_j$  is the annual stress cycle rate and  $v_j t$  is the total number of stress cycles in the period [0, t].  $\Delta S_{e,j}$  is computed as a function of  $k_j$ ,  $\lambda_j$  and  $m_j$  according to Equation (50). The vector of all uncertain parameters describing fatigue deterioration of all hotspots considered in the system reliability analysis is defined as:

$$\mathbf{X} = (C_1, m_1, a_{0,1}, B_{SIF,1}, B_{\Delta S,1}, k_1, \dots, C_{n_H}, m_{n_H}, a_{0,n_H}, B_{SIF,n_H}, B_{\Delta S,n_H}, k_{n_H})$$
(54)

514 The same probabilistic models are applied to describe the crack growth model parameters for all 515 hotspots  $j = 1, ..., n_H$ . They are summarized in Table 2.

Parameter	Dimension	Distribution	Mean	Standard deviation
ln k <sub>j</sub>	corresponding to N/mm <sup>2</sup>	normal	2.0	0.275
$\lambda_j$	-	deterministic	0.8	-
$\nu_j$	yr <sup>-1</sup>	deterministic	$5 \cdot 10^{6}$	-
a <sub>0,j</sub>	mm	exponential	0.11	0.11
$a_{c,j}$	mm	deterministic	20	-
ln C <sub>j</sub>	corresponding to N and mm	normal	-29.97	0.514
$m_j$	-	normal	calculated from $\ln C_j$	$= -15.84 - 3.34m_j$
$B_{\Delta S,j}$	-	lognormal	1.0	0.1
$B_{SIF,j}$	-	lognormal	1.0	0.1

516 Table 2. Probabilistic models of the fatigue crack growth parameters for all hotspots  $j = 1, ..., n_H$ .

517

518 The mean and standard deviation of the equivalent stress range  $\Delta S_{e,j}$  are a function of the 519 distributions of ln  $k_j$  and ln  $C_j$  through Equations (50) and (51). They are  $\mu_{\Delta S_{e,j}} = 20.1 \text{ N/mm}^2$ 520 and  $\sigma_{\Delta S_{e,j}} = 5.65 \text{ N/mm}^2$ .

521 Statistical dependence among hotspot fatigue behavior is modeled through correlation coefficients

among the fatigue model parameters. In the current example, the fatigue model parameters  $a_{0,i}$ ,

523  $C_j, k_j, B_{\Delta S,j}$  and  $B_{SIF,j}$  are equi-correlated among all hotspots  $j = 1, ..., n_H$  with correlation

524 coefficients  $\rho_{a_0}$ ,  $\rho_{\ln c}$ ,  $\rho_{\ln k}$ ,  $\rho_{B_{\Delta S}}$  and  $\rho_{B_{SIF}}$ . The correlation coefficient  $\rho_{a_0}$  represents the

statistical dependence due to common fabrication quality;  $\rho_{\ln C}$  reflects the statistical dependence

- 526 due to common material characteristics;  $\rho_{\ln k}$  models the statistical dependence due to common 527 loading characteristics; and  $\rho_{B_{\Delta S}}$  and  $\rho_{B_{SIF}}$  describe statistical dependence due to common 528 uncertainties in the calculation of hotspot fatigue stress ranges and SIF ranges. The joint 529 distribution of all fatigue model parameters in **X** is subsequently modeled through a Gaussian 530 copula (Nataf) model (Liu and Der Kiureghian 1986).
- 531 To study the influence of different levels of statistical dependence among hotspot fatigue behavior,
- three different dependence cases are considered (low, medium, high), which are defined in terms

of the correlation coefficients  $\rho_{a_0}$ ,  $\rho_{\ln C}$ ,  $\rho_{\ln k}$ ,  $\rho_{B_{\Lambda S}}$  and  $\rho_{B_{SIF}}$  as listed in Table 3.

	low dependence	medium dependence	high dependence	
$\rho_{a_0}$	0.2	0.5	0.8	
$ ho_{\ln C}$	0.2	0.5	0.8	
$\rho_{lnk}$	0.2	0.5	0.8	
$ ho_{B_{\Delta S}}$	0.2	0.5	0.8	
$ ho_{B_{SIF}}$	0.2	0.5	0.8	

Table 3. Correlation coefficients among the fatigue crack growth parameters.

#### 535 5.3 Inspection model

536 In the context of fatigue deterioration, relevant inspection outcomes are (a) no detection, (b) 537 detection but no measurement, and (c) detection and measurement of a fatigue crack. These 538 inspection outcomes are directly related to the crack size  $a_j(\mathbf{X}, t)$  predicted for a given hotspot *j* 539 at inspection time *t*. In the current study, we consider inspection outcomes of the type (a) and (b). 540 The ability of an inspection method to detect a fatigue crack with a certain size A = a is commonly 541 described by a probability of detection curve  $\pi(a)$ , which is defined as:

$$\pi(a) = \Pr(\text{detection of a fatigue crack}|A = a)$$
(55)

542 Such a probability of detection curve describes the performance of the applied inspection method; 543 it accounts for uncertain factors such as measurement errors, inspector performance and 544 environmental conditions (Straub 2004). In the current case study, an exponential probability of 545 detection curve is applied:

$$\pi(a) = 1 - \exp(-a/\lambda_D) \tag{56}$$

546 with  $\lambda_D = 1.95$  mm. This probability of detection model is representative of magnetic particle 547 inspection (Moan et al. 2000).

548 The likelihood function describing the inspection outcome  $Z_i(t) = \{\text{fatigue crack detected at} 549 \text{ hotpot } j \text{ at time } t\}$  is thus equal to the probability of detection:

$$L_i(\mathbf{x},t) = \pi \left( a_j(\mathbf{x},t) \right) \tag{57}$$

550 The likelihood function of the complementary inspection outcome  $Z_i(t) = \{\text{no fatigue crack} 551 \text{ detected at hotpot } j \text{ at time } t\}$  is:

$$L_i(\mathbf{x},t) = 1 - \pi \left( a_j(\mathbf{x},t) \right)$$
(58)

552 Under the common assumption that individual inspection outcomes are statistically independent 553 given the crack sizes  $a_j(\mathbf{x}, t), j = 1, ..., n_H$ , the combined likelihood function  $L(\mathbf{x}, t)$  of all inspection outcomes  $Z_{0:t}$  in the time period [0, t] is given by Equation (20). If individual 554 555 inspections are not statistically independent due to, for example, common influencing factors such 556 as environmental conditions and inspector characteristics, the combined likelihood has to be 557 formulated such that this aspect is captured. Approaches to modeling dependence among 558 inspection outcomes are, for example, presented in (Straub and Faber 2003)(Maljaars and Vrouwenvelder 2014). 559

560 Since detection/no-detection events provide inequality information, the constant *c* that ensures  $0 \le cL(\mathbf{x}, t) \le 1$  for all **x** can be chosen as c = 1 (see also Section 3.1).

#### 562 **5.4 Prior system reliability analysis**

The prior annual system failure probability  $Pr(F_t)$  of the Zayas frame is computed for each degree of dependence among hotspot fatigue behavior according to Equation (12). The results are shown in Figure 2. The problem is solved using subset simulation as described in Section 4 with conditional probabilities  $p_0 = 0.1$  and N = 1000 samples per subset level. The statistics of the system failure probability are determined from 500 independent simulation runs. This approach is applied in all subsequent analyses presented in this paper.



Figure 2. Median and 95% credible interval of the prior annual system failure probability  $Pr(F_t)$  of the Zayas frame as a function of different degrees of dependence among hotspot fatigue behavior. Computations are performed with subset simulation as summarized in Section 4 with conditional probabilities  $p_0 = 0.1$  and N = 1000 samples per subset level. (d) compares the respective medians of the prior annual system failure probability.

574 As expected, the annual system failure probability  $Pr(F_t)$  increases with time t, due to fatigue

deterioration. Furthermore, Figure 2(d) indicates that a higher dependence among hotspot fatigue
behavior leads to a larger system failure probability due to an increase in the probability of joint

577 occurrence of several element fatigue failures. This result is expected for a redundant structural

578 system (Straub and Der Kiureghian 2011).

579 The width of the 95% credible interval indicates the accuracy of the employed subset simulation. 580 The interval has 0.95 probability of containing the true value of the system failure probability 581 (within the confines of the model). From Figure 2(a) to (c) it can be seen that the accuracy of the 582 computation varies with time t since the number of samples per subset level used in the simulation 583 is the same for all years. Results are less accurate for low values of t, because of the associated 584 smaller system failure probability. Note, however, that the variability of the simulated failure 585 probabilities at the beginning of the structure's service life (t < 5yr) is small. In this period, the 586 probability of fatigue failures is very small, and they have little effect on the system failure probability (the failure probability of the undamaged Zayas frame is  $\Pr(F_t | \mathbf{D}_t = \mathbf{0}) = 1.3 \cdot 10^{-6}$ ). 587

#### 588 **5.5 Posterior system reliability analysis**

589 In this section, different inspection scenarios in terms of inspection times, coverage and outcomes 590 are considered to study their effect on the reliability of the Zayas frame. Firstly, hotspots 591  $\{5, 6, 13, 14, 21, 22\}$  are inspected at time t = 10 years. These hotspots are associated with the 592 least important braces of the Zayas frame (see Figure 1 and Table 1). No fatigue cracks are detected 593 during the inspection. The posterior annual system failure probability  $Pr(F_t|Z_{0:t}), t = 1, ..., 50$  are 594 computed for each degree of dependence among hotspot fatigue behavior with subset simulation 595 as described in Section 4 with conditional probabilities  $p_0 = 0.1$  and N = 1000 samples per subset 596 level. The results are shown in Figure 3.



Figure 3. Median and 95% credible interval of the posterior annual system failure probability  $Pr(F_t|Z_{0:t})$ of the Zayas frame as a function of different degrees of dependence among hotspot fatigue behavior. Hotspots {5, 6, 13, 14, 21, 22} are inspected at time t = 10 years. No fatigue cracks are detected. Computations are performed with subset simulation as summarized in Section 4 with conditional probabilities  $p_0 = 0.1$  and N = 1000 samples per subset level. (d) compares the respective medians of the posterior annual system failure probability.

When considering the posterior medians of the estimated posterior system failure probabilities shown in Figure 3 (a) to (c), it can be seen that the system failure probability reduces after the inspection due to the positive inspection result. The effect increases with increasing degree of dependence among hotspot deterioration behavior.

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Table 4 lists the probabilities  $Pr(Z_{e,0:t})$  and  $Pr(F_t|Z_{0:t})$  computed at time t = 10 years. The subset 607 simulation (SuS) results are presented together with those from additional Monte Carlo simulations 608 609 (MCS). The number of model evaluations is also provided for each simulation of  $Pr(F_t|Z_{0:t})$  to 610 indicate the computational efforts, since the accuracy can always be improved by increasing the 611 number of samples. The results in Table 4 show that the probability of the inspection outcome is 612 large. This is because the initial defects at each hotspot considered in the current case study are small, and hence the fatigue cracks are unlikely to grow to a detectable size within the first 10 613 614 years of the structure's service life.

Table 4 Probability of the inspection outcome  $Pr(Z_{e,0:t})$  and the posterior system failure probability Pr $(F_t|Z_{0:t})$  at time t = 10 years. Subset simulation (SuS) is performed as summarized in Section 4 with conditional probabilities  $p_0 = 0.1$  and N = 1000 samples per subset level. Results in square brackets represent the 95% credible interval. MCS is performed with  $10^7$  samples. Results are shown as 95% confidence interval. The total number of model runs are provided for the computation of  $Pr(Z_{e,0:t})$  and  $Pr(F_t|Z_{0:t})$ .

Case	Method	$\Pr(Z_{e,0:t})$	$\Pr(F_t Z_{0:t})$	# model runs
Low dependence	SuS	[0.642; 0.7]	$[0.0467; 2.3] \cdot 10^{-4}$	$5.9 \cdot 10^{3}$
	MCS	[0.671; 0.6716]	$[7.08; 8.49] \cdot 10^{-5}$	10 <sup>7</sup>
Medium dependence	SuS	[0.659; 0.717]	$[0.0267; 3.11] \cdot 10^{-4}$	5.9 · 10 <sup>3</sup>
	MCS	[0.6875; 0.688]	$[0.864; 1.01] \cdot 10^{-4}$	10 <sup>7</sup>
High dependence	SuS	[0.673; 0.734]	$[0.00758; 1.08] \cdot 10^{-4}$	$6.8 \cdot 10^{3}$
	MCS	[0.7042; 0.7048]	$[2.94; 3.79] \cdot 10^{-5}$	10 <sup>7</sup>

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In the second example, hotspots {15,16,17,18,19,20} are inspected at time t = 10 years. These hotspots are associated with the most important structural members of the Zayas frame (see Figure 1 and Table 1). We assume again that each inspection results in a no-detection event. The posterior annual system failure probability  $Pr(F_t|Z_{0:t})$  is shown for all three dependence cases in Figure 4. In contrast to the first scenario, an inspection of the most important structural elements has a significant effect on the system reliability regardless of the degree of dependence among element deterioration.

629



Figure 4. Median and 95% credible interval of the posterior annual system failure probability  $\Pr(F_t|Z_{0:t})$ of the Zayas frame as a function of different degrees of dependence among hotspot fatigue behavior. Hotspots {15,16,17,18,19,20} are inspected at time t = 10 years. No fatigue cracks are detected. Computations are performed with subset simulation as summarized in Section 4 with conditional probabilities  $p_0 = 0.1$  and N = 1000 samples per subset level. (d) compares the respective medians of the posterior annual system failure probability.

636 The probability of the inspection outcome  $Pr(Z_{e,0:t})$  and the posterior system failure probability

637  $Pr(F_t|Z_{0:t})$  at year 10 are summarized in Table 5 for each dependence case. The computed

638 probabilities  $Pr(Z_{e,0:t})$  are the same as in the first scenario (see Table 4) because the applied

639 probabilistic models of the crack growth parameters are identical for all hotspots (see Table 2).

640 Comparing the bounds of the SuS and MCS results for  $Pr(F_t|Z_{0:t})$ , it is seen that their accuracy is

similar even though the number of samples in the SuS is only N = 1000 samples per subset level.

Table 5. Probability of the inspection outcome  $Pr(Z_{e,0:t})$  and the posterior system failure probability Pr( $F_t|Z_{0:t}$ ) at time t = 10 years. Subset simulation is performed as summarized in Section 4 with conditional probabilities  $p_0 = 0.1$  and N = 1000 samples per subset level. Results represent the 95% credible interval. MCS is performed with  $10^7$  samples. Results represent the 95% confidence interval.

Case	Method	$\Pr(Z_{e,0:t})$	$\Pr(F_t Z_{0:t})$	# model runs
Low dependence	SuS	[0.642; 0.701]	$[0.635; 3.15] \cdot 10^{-6}$	$6.8 \cdot 10^{3}$
	MCS	[0.671; 0.6716]	$[1.22; 3.54] \cdot 10^{-6}$	10 <sup>7</sup>
Medium dependence	SuS	[0.656; 0.715]	$[0.543; 3.12] \cdot 10^{-6}$	$6.8 \cdot 10^{3}$
	MCS	[0.6876; 0.6882]	$[1.89; 4.53] \cdot 10^{-6}$	10 <sup>7</sup>
High dependence	SuS	[0.673; 0.73]	$[0.55; 2.77] \cdot 10^{-6}$	$6.8 \cdot 10^3$
	MCS	[0.7042; 0.7048]	$[1.06; 3.2] \cdot 10^{-6}$	10 <sup>7</sup>

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In the third scenario, hotspots {15,16,17,18,19,20} are again inspected in year 10. No fatigue cracks are detected at hotspots {15,16,17,18} whereas defects are detected at hotspots {19,20}. The corresponding posterior annual system failure probabilities are shown for all three dependence cases in Figure 5. The system failure probability increases after the inspection since fatigue cracks are detected in welds connecting two of the most important braces with the legs (see Figure 1 and Table 1). The effect is most pronounced in the low dependence case.

653



Figure 5. Median and 95% credible interval of the posterior annual system failure probability  $Pr(F_t|Z_{0:t})$ of the Zayas frame as a function of different degrees of dependence among hotspot fatigue behavior. Hotspots {15,16,17,18,19,20} are inspected in year 10. No fatigue cracks are detected at hotspots {15,16,17,18} whereas defects are detected at hotspots {19,20}. Computations are performed with subset simulation as summarized in Section 4 with conditional probabilities  $p_0 = 0.1$  and N = 1000 samples per subset level. (d) compares the respective medians of the posterior annual system failure probability.

Table 6 shows that the current inspection outcome is approximately two orders of magnitude less probable than the no-detection outcomes in the previous scenarios (see Table 4 and Table 5). When comparing the subset simulation results in Table 5 and Table 6, it can also be seen that the number of model evaluations are similar in both examples although the posterior system failure probabilities  $\Pr(F_t|Z_{0:t})$  are multiple orders of magnitude larger in the current example. The reason is that the simulations of the smaller probabilities of the observation event  $\Pr(Z_{e,0:t})$  require here more model evaluations.

Table 6. Probability of the inspection outcome  $Pr(Z_{e,0:t})$  and the posterior system failure probability Pr( $F_t|Z_{0:t}$ ) at time t = 10 years. Subset simulation is performed as summarized in Section 4 with conditional probabilities  $p_0 = 0.1$  and N = 1000 samples per subset level. Results represent the 95% credible interval. MCS is performed with  $10^7$  samples. Results represent the 95% confidence interval.

Case	Method	$\Pr(Z_{e,0:t})$	$\Pr(F_t Z_{0:t})$	# model runs
Low dependence	SuS	$[2.45; 5.18] \cdot 10^{-3}$	$[0.00429; 1.02] \cdot 10^{-2}$	$7.1 \cdot 10^{3}$
	MCS	$[3.66; 3.74] \cdot 10^{-3}$	$[2.64; 3.79] \cdot 10^{-3}$	107
Medium dependence	SuS	$[2.66; 5.71] \cdot 10^{-3}$	$[0.0121; 1.67] \cdot 10^{-2}$	$6.2 \cdot 10^{3}$
	MCS	$[3.82; 3.9] \cdot 10^{-3}$	$[6.14; 7.79] \cdot 10^{-3}$	107
High dependence	SuS	$[2.57; 5.78] \cdot 10^{-3}$	$[0.0319; 9.84] \cdot 10^{-3}$	$7.0 \cdot 10^{3}$
	MCS	$[3.86; 3.93] \cdot 10^{-3}$	$[2.96; 4.13] \cdot 10^{-3}$	107

671

672 In the last scenario, regular inspections are performed at 10 year intervals. Hotspots associated 673 with elements of each importance category are inspected at each inspection apart from the last 674 inspection where only hotspots associated with the upper braces (low importance category) are 675 inspected (see Figure 1 and Table 1). This inspection strategy ensures that each hotspot is inspected 676 at least once throughout the service life of the structure. The inspection strategy is as follows: 677 hotspots {15,16,7,8,5,6} are inspected at time t = 10 years, hotspots {17,18,9,10,13,14} are 678 inspected at time t = 20 years, hotspots {19,20,11,12,21,22} are inspected at time t = 30 years and hotspots  $\{1,2,3,4\}$  are inspected at time t = 40 years. Each inspection results in a no-detection 679 680 events. The results are shown in Figure 6. As expected, the positive inspection outcome causes a 681 reduction in the annual system failure probability after each inspection. This effect increases with 682 increasing degree of dependence among hotspot fatigue behavior.



Figure 6. Median and 95% credible interval of the posterior annual system failure probability  $Pr(F_t|Z_{0:t})$ of the Zayas frame as a function of different degrees of dependence among hotspot fatigue behavior. Hotspots {15,16,7,8,5,6} are inspected at time t = 10 years, hotspots {17,18,9,10,13,14} are inspected at time t = 20 years, hotspots {19,20,11,12,21,22} are inspected at time t = 30 years and hotspots {1,2,3,4} are inspected at time t = 40 years. No fatigue cracks are detected. Computations are performed with subset simulation as summarized in Section 4 with conditional probabilities  $p_0 = 0.1$  and N = 1000 samples per subset level. (d) compares the respective medians of the posterior annual system failure probability.

Table 7 summarizes the median probability of the inspection outcome  $Pr(Z_{e,0:t})$  after each inspection. Each additional inspection provides more information on the actual condition of the structure. With increasing amount of information, the probability of the inspection outcome  $Pr(Z_{e,0:t})$  decreases. It also decreases with decreasing degree of dependence among hotspot fatigue behavior.

Year t	low dependence	medium dependence	high dependence	
10	0.67	0.69	0.70	
20	0.42	0.47	0.52	
30	0.25	0.33	0.40	
40	0.17	0.26	0.34	

Table 7. Median of the probability of the inspection outcome  $Pr(Z_{e,0:t})$  as a function of the number of inspections and the degree of dependence among hotspot fatigue behavior.

## 697 6 Application b: Daniels system subjected to fatigue deterioration

We apply the proposed approach to the idealized structural system shown in Figure 7, known as Daniels system (Daniels 1945). We here assume that the Daniels system consists of welded steel members, which are subjected to fatigue deterioration throughout the structure's service life of T = 50 years. The properties of the Daniels system, in particular the exchangeability of the elements, facilitate numerical investigations.

703



Figure 7. Daniels system with  $n_E$  elements.

#### 705 6.1 System model

The considered Daniels system consists of  $n_E = 100$  elements with independent and identically distributed (i.i.d.) capacities  $R_i$ ,  $i = 1, ..., n_E$ . The applied load is shared equally among all elements; its annual maximum is denoted by *L*. In the current example, we assume that each element is associated with one welded connection. Furthermore, we assume that each welded connection contains only one critical hotspot, i.e.  $n_H = n_E = 100$ .

The same deterioration model presented in Section 5 is applied to model fatigue deterioration of

712 the Daniels system. At any time t there are  $N_{F,t}$  failed elements and  $n_E - N_{F,t}$  elements are

available to resist the applied loads. Because of the exchangeability of its elements,  $N_{F,t}$  represents

the deterioration state of the Daniels system at time t.  $N_{F,t}$  is computed as a function h of the

715 deterioration model parameters **X** as:

$$N_{F,t} = h(\mathbf{X}, t) = \sum_{i=1}^{n_E} I(g_i(\mathbf{X}, t) \le 0)$$
(59)

716 where  $g_i(\mathbf{X}, t)$  is the limit-state function defining the event of fatigue failure of element *i*; see

Figure 717 Equation (44). The system failure probability of the Daniels system in the reference period  $[t - t_{1}]$ 

718  $\Delta t, t$ ] conditional on a realization of the fatigue model parameters **X** = **x** can now be written as:

$$p_F(\mathbf{x},t) = \Pr\left(F_t | N_{F,t} = h(\mathbf{x},t)\right)$$
(60)

For given probability distributions of the component capacities  $R_i$  and the annual maximum load L, the conditional system failure probability  $Pr(F_t|N_{F,t} = k)$ ,  $k = 0, ..., n_E$  is readily determined

from Daniels system formulation. This failure probability has a reference period  $\Delta t = 1$  year but

it is independent of time t. For ductile element behavior (steel elements), the solution is given by

723 (Gollwitzer and Rackwitz 1990):

$$\Pr(F_t|N_{F,t} = k) = \Pr\left(\sum_{i=1}^{n_E - k} R_i \le L\right)$$
(61)

The right hand side of Equation (61) corresponds to a component reliability problem, which canbe solved using structural reliability methods.

726 In the current example, the component capacities  $R_i$ ,  $i = 1, ..., n_E$  are modeled as i.i.d. normal random variables with c.o.v.  $\delta_R = 0.15$ . The annual maximum of the applied load L is modeled as 727 a lognormal random variable with c.o.v.  $\delta_L = 0.25$ . The ratio of the mean values of  $n_E R_i$  and L is 728 selected such that the undamaged Daniels system has a probability of failure  $Pr(F_t|N_{F,t} = 0) =$ 729  $1.3 \times 10^{-6}$ . The resulting ratio is  $n_E \mu_{R_i} / \mu_L = 3.09$ .  $\Pr(F_t | N_{F,t} = k)$ ,  $k = 0, ..., n_E$  is pre-730 calculated for each realization of the system deterioration state by solving Equation (61) using the 731 732 first-order reliability method (FORM). The results are illustrated in Figure 8. In the subsequent 733 reliability analysis of the deteriorating Daniels system, the computation of  $p_F(\mathbf{x}, t)$  is again 734 reduced to a lookup operation.



Figure 8. Failure probability of the Daniels system as a function of the number of elements failed due tofatigue.

Each element of the Daniels system is equally important due to the perfect load sharing among the structural elements. The single element importance measure of an individual element *i* of the Daniels system is  $SEI_i = Pr(F_t|N_{F,t} = 1) - Pr(F_t|N_{F,t} = 0) = 2.9 \times 10^{-7}$ . The Daniels system is highly redundant with respect to single element failure when compared to the Zayas frame studied in Section 5 where failure of elements of the highest importance category lead to a significant reduction in system reliability, see Table 1.

#### 743 6.2 Prior system reliability analysis

The computed prior annual system failure probability  $Pr(F_t)$  of the Daniels system is shown in Figure 9 for each degree of dependence among hotspot fatigue. Computations are performed with subset simulation as summarized in Section 4 with conditional probabilities  $p_0 = 0.1$  and N =

747 1000 samples per subset level.



Figure 9. Median and 95% credible interval of the prior annual system failure probability  $Pr(F_t)$  of the Daniels system as a function of different degrees of dependence among hotspot fatigue behavior. Computations are performed with subset simulation as summarized in Section 4 with conditional probabilities  $p_0 = 0.1$  and N = 1000 samples per subset level. (d) compares the respective medians of the prior annual system failure probability.

In general, a large dependence among element deterioration behavior increases the probability of joint occurrence of more than one element deterioration failures. Figure 9 shows that this behavior has a significant influence on the reliability of the Daniels system. This outcome is expected for a structural system with a large redundancy. In contrast, the results computed for the Zayas frame show that the influence of correlation among element deterioration failures is less pronounced for structural systems with limited or no redundancy (see Figure 2).

#### 759 6.3 Posterior system reliability analysis

To study the effect of inspections on the reliability of the Daniels system, different inspection scenarios in terms of inspection times and coverage are considered. Each inspection is assumed to result in a no detection event. The same inspection model as presented in Section 5.3 is applied.

In the first scenario, hotspots {1 to 10} are inspected at time t = 10 years. The updated annual system failure probabilities  $Pr(F_t|Z_{0:t})$  of the Daniels system are shown in Figure 10 for each degree of dependence among hotspot fatigue behavior. In all three dependence cases, the system

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failure probability decreases after the inspection due to the positive inspection outcome. After the inspection, the system failure probability is reduced to its lower limit, which corresponds to the reliability of the undamaged structure at the beginning of its service live. The subsequent increase in the annual system failure probability is most pronounced in the high-dependence case.



Figure 10. Median and 95% credible interval of the posterior annual system failure probability  $Pr(F_t|Z_{0:t})$ of the Daniels system as a function of different degrees of dependence among hotspot fatigue behavior. hotspots {1 to 10} are inspected in year 10. No fatigue cracks are detected. Computations are performed with subset simulation as summarized in Section 4 with conditional probabilities  $p_0 = 0.1$  and N = 1000samples per subset level. (d) compares the respective medians of the posterior annual system failure probability.

In the second scenario, different sets of hotspots are inspected at 10 year intervals. The inspection

strategy is as follows: hotspots {1 to 10} are inspected at time t = 10 years, hotspots {11 to 20}

are inspected at time t = 20 years, hotspots {21 to 30} are inspected at time t = 30 years and

hotspots {31 to 40} are inspected at time t = 40 years. The results are shown in Figure 11. In all

- three dependence cases, the posterior annual system failure probability is close the annual failure
- 781 probability of the undamaged structures after all inspections are performed.



Figure 11. Median and 95% credible interval of the posterior annual system failure probability  $\Pr(F_t|Z_{0:t})$ of the Daniels system as a function of different degrees of dependence among hotspot fatigue behavior. Hotspots {1 to 10} are inspected at time t = 10 years, hotspots {11 to 20} are inspected at time t =20 years, hotspots {21 to 30} are inspected at time t = 30 years and hotspots {31 to 40} are inspected at time 40 years. Each inspection results in a no-detection event. Computations are performed with subset simulation as summarized in Section 4 with conditional probabilities  $p_0 = 0.1$  and N = 1000 samples per subset level. (d) compares the respective medians of the posterior annual system failure probability.

### 789 7 Discussion

790 We propose a modeling and computational framework for analyzing the reliability of deteriorating 791 structural systems and updating it with inspection and monitoring data. It enables an integral 792 assessment of deterioration at the element level together with the structural system performance 793 and structural condition information. The interdependences among the element deterioration states 794 are included. Only few previous works have addressed such an integral system analysis (e.g. Lee 795 and Song 2014; Schneider et al. 2015; Luque and Straub 2016). In contrast to these approaches, 796 the main advantage of the proposed framework is the fact that it can be implemented easily through 797 the use of subset simulation. It is computationally robust since it provides reasonably accurate 798 solutions without a need for tailoring the algorithm to specific applications. It is also 799 computationally efficient for many applications, as discussed further below.

800 The results in the paper demonstrate the importance of considering dependence among element 801 deterioration when evaluating the structural system reliability. For the considered redundant 802 systems, the dependence leads to a decrease in the prior (unconditional) system reliability. This 803 effect is more pronounced as the system redundancy increases (from the Zayas frame to the Daniels 804 system). When including inspection results, dependence among element deterioration means that 805 the state of non-inspected elements can be inferred from the inspection results. As long as 806 inspections do not indicate serious problems, this additional learning leads to a reduction of 807 uncertainty and hence to an increase in the overall system reliability. In the considered case studies, 808 the posterior (conditional) reliability after the inspections is fairly similar for the different degrees 809 of dependence. However, this is not expected to occur if inspections do indicate larger damage.

810 The framework can handle any type of information on the deterioration state of the structure, as 811 long as a corresponding likelihood function is formulated. In particular, the framework can also 812 include information from monitoring systems. For monitoring systems, which provide potentially 813 large amount of data, it might be beneficial to pre-process the data. In such a pre-processing step 814 (e.g. a system identification), the probability of the observed data given the deterioration states of 815 the structure is determined. This probability is the likelihood function that is inputted into 816 Equations (34) and (35). Such an approach is similar to a two-stage Bayesian analysis for system 817 identification (see Au and Zhang 2015).

The use of subset simulation is computationally rather efficient, as demonstrated in the case studies. Here, no attempt was made to optimize the efficiency of subset simulation. The number of samples per subset level was chosen such that the results have a reasonable accuracy. Their accuracies can always be improved or reduced by increasing or decreasing the number of samples per subset level. It should be noted, however, that the number of required subsets increases with increasing amount of information, i.e. with decreasing  $Pr(Z_{e,0:t})$ .

824 The proposed framework relies on the separation of the computation of the system deterioration 825 state  $\mathbf{D}_t$  and structural system reliability conditional on  $\mathbf{D}_t = \mathbf{d}_t$ . Here, two situations must be distinguished: Applications, in which the conditional probability  $Pr(F_t | \mathbf{D}_t = \mathbf{d}_t)$  can be pre-826 computed, and those in which it cannot. The former occurs if the numbers of distinct states in  $\mathbf{D}_t$ 827 828 is limited. If the structural system reliability analysis is demanding, it might take some computation 829 time for establishing a database with all values of  $Pr(F_t | \mathbf{D}_t = \mathbf{d}_t)$ , but this is typically not critical, 830 as this computation must be carried out only once and the database can be used for all subsequent 831 reliability updating calculations. If the number of states in  $D_t$  is too large to enable pre-832 computation, because there are too many elements or because continuous damage states are 833 considered,  $\Pr(F_t | \mathbf{D}_t = \mathbf{d}_t)$  must be computed on the fly. If such calculations are inexpensive (e.g. 834 through a FORM analysis), the separation of the system deterioration state and structural system reliability is still computationally beneficial. In cases where pre-computation of  $Pr(F_t | \mathbf{D}_t = \mathbf{d}_t)$ 835 is not an option, and in which it is expensive to compute it on the fly, there are two possible 836 837 strategies: (a) One can investigate the possibility of developing a response surface for

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- 838  $Pr(F_t | \mathbf{D}_t = \mathbf{d}_t)$ . Since  $\mathbf{D}_t$  is typically discrete, many of the classical response surface techniques 839 used in structural reliability will not be suitable. This is an area of future research. (b) Alternatively, 840 the proposed framework can be modified to solve the system deterioration updating and the system 841 reliability jointly. In this case, however, the advantages of the de-coupling are lost.
- Potentials for further developments are seen in integrating the presented method into the framework of pre-posterior decision analysis to identify optimal inspection, monitoring and maintenance strategies for engineering structures (e.g. Straub and Faber 2005; Thöns and Faber 2013; Straub 2014).

## 846 8 Conclusions

We propose a novel approach to modeling and analyzing the system reliability of deteriorating
structural systems in conjunction with structural condition information, which considers stochastic
interdependence among the deterioration states of the structural elements. The approach provides
the means to consistently utilize inspection and monitoring information on the deterioration state

851 of structures to update the system failure probability. Through the application of subset simulation,

the approach can be implemented relatively easily and is considerably more efficient than crude

- 853 Monte Carlo simulation.
- 854

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