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The profit maximizing liner shipping problem with flexible frequencies: logistical and environmental considerations[1]

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Abstract

The literature on liner shipping includes many models on containership speed optimization, fleet deployment, fleet size and mix, network design and other problem variants and combinations. Many of these models, and in fact most models at the tactical planning level, assume a fixed revenue for the ship operator and as a result they typically minimize costs. This treatment does not capture a fundamental characteristic of shipping market behavior, that ships tend to speed up in periods of high freight rates and slown down in depressed market conditions. This paper develops a simple model for a fixed route scenario which, among other things, incorporates the influence of freight rates, along with that of fuel prices and cargo inventory costs into the overall decision process. The objective to be maximized is the line's average daily profit. Departing from convention, the model is also able to consider flexible service frequencies, to be selected from a broader set than the standard assumption of one call per week. It is shown that this may lead to better solutions and that the cost of forcing a fixed frequency can be significant. Such cost is attributed either to additional fuel cost if the fleet is forced to sail faster to accommodate a frequency that is higher than the optimal one, or to lost income if the opposite is the case. The impact of the line's decisions on CO_2 emissions is also examined and illustrative runs of the model are made on three existing services.

Key words: *ship speed optimization, liner shipping, container shipping, CO_2 emissions.*

Introduction

The literature on liner (and specifically container) shipping problems is rich and growing, and encompasses routing, network design, fleet deployment, fleet size and mix, speed optimization, and other related problems. For instance, the survey of Meng et al (2014) lists more than 90 related references.

A typical modeling assumption that is reflected in many papers in this area, is that the models *do not* include the state of the market (that is, the freight rate) as part of their formulation. This is particularly true for most models at the *tactical* planning level (see Christiansen et al. (2007) for the distinction among strategic, tactical and operational planning in shipping). In these models, the income component of the problem is typically assumed fixed, and as a result the typical objective is to minimize cost. Income is assumed fixed because the quantity of cargo to be transported along a given route or a given service is assumed known and constant. However, it is well known in shipping that an important tradeoff in pursuit of higher profits for a ship operator is the balance between more trips and hence more income at a higher speed and less trips because the cost of fuel gets higher. In that sense, the ship operator would like to take advantage of high freight rates by hauling as much cargo as possible within a given period of time. Conversely, if the market is low, ships tend to reduce speed, as the additional revenue from hauling more cargo is less than the additional cost of the fuel. A main reason for slow steaming in recent years is the depressed state of the market (although the fact that fuel prices have also dropped has masked the extent of this phenomenon). Whereas this is

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true in all shipping markets, nowhere this is more prevalent than in liner shipping and specifically in container shipping.

The fact that slow steaming is being practised in periods of depressed market conditions can be confirmed by the fact that whatever fleet overcapacity existed has been virtually absorbed. In the years following the 2008 economic crisis, and according to Alphaliner (2011), of the approximately 15 million TEU global containership capacity in 2011, less than 0.3 million TEU were idle. A similar situation pertains in the tanker and drybulk markets (Devanney, 2011). Moreover, and according to the third Greenhouse Gas (GHG) study of the International Maritime Organization (IMO), the reduction of global maritime CO_2 emissions from 885 million tonnes in 2007 to 796 million tonnes in 2012 is mainly attributed to slow steaming due to the serious slump in the shipping markets after 2008 (Smith et al., 2014).

A number of papers, see for instance the early work of Ronen (1982) as applied to tramp vessels, the speed model taxonomy of Psaraftis and Kontovas (2013) and more recently the paper by Magirou et al. (2015) on the economic speed of a ship in a dynamic setting, have considered the impact of freight rates (among other parameters) on the speed decision. However, none of these papers developed optimization models for the liner sector. In fact, most liner shipping optimization models that do include ship speed as a decision variable typically do not consider the possible impact of the state of the market on ship speed as being within the scope of their analysis. These include (among others) Alvarez (2009), Brouer et al. (2017), Cariou (2011), Doudnikoff and Lacoste (2014), Eefsen and Cerup-Simonsen (2010), Guericke and Tierney (2015), Karsten et. al. (2017), Lang and Veenstra (2010), Meng and Wang (2011), Notteboom and Vernimmen (2010), Qi and Song (2010), Reinhardt et al. (2016), Song et al. (2017), Yao et al (2012), and Zis et al. (2015). An exception is the recent paper by Xia et al. (2015), where a model which incorporates income considerations is presented, even though there is no analysis on the impact of higher or lower freight rates on how fast or slow containerships may go. We also note the survey by Christiansen et al. (2007), which provides a formulation for route design in liner shipping that maximizes profits, and as such incorporates the state of the market into the model. However, the problem examined only concerns decisions at the strategic planning level and speeds are considered known and fixed, therefore capturing the possible impact of freight rates on ship speed would (at a minimum) require some modifications in the model formulation and the solution method. This is actually true for any optimization approach that does *not* consider ship speed as a decision variable, even though it may include the state of the market in its formulation.

The scenario of our paper deals with a *fixed route* served by a fleet of identical container ships. In that sense, decisions at the strategic level are assumed to have been already made, and the only decisions at play are (a) the speeds of the vessels along the route, (b) the number of vessels deployed, and (c) the service frequency. In that sense, our problem falls within the *tactical* planning level. The objective of the problem is to *maximize the operator's average daily profit*. This is equivalent to maximizing the operator's *annual profit* for the fleet, as the latter is equal to the average daily profit for the fleet multiplied by the average annual operating days of a ship in the fleet, which can be safely assumed to be a constant (equal to 365 minus time for annual surveys, maintenance, etc). Speeds along the legs of the route are allowed to be different. The model takes as inputs, among other things, the fuel price (expressed in USD/tonne) and the market freight rate for each specific origin/destination port pair (expressed in USD/TEU). Outputs include, among other variables, ship speeds, fleet size, and fleet CO_2 emissions.

A critical parameter that links containership speed and fleet size is *service frequency*. In most liner services worldwide the standard practice (at least for mainline services) is that such frequency is *once per week*, and this is also reflected in the models developed in the literature. However from an optimization perspective, fixing the frequency to one call per week is tantamount to a constraint in the problem to be solved, and one may wonder what might be the potential benefits of relaxing that constraint. To do so, and departing from convention, this paper will also investigate *flexible* service frequencies, namely ones that belong to a broader set than the standard one call per week. This would also allow one to quantify the cost (or lost income) of a fixed frequency.

Last but not least, the model will also take into account *in transit cargo inventory costs*. In shipping such costs express the loss of revenue to the cargo owner due to the cargo being in transit. Even though these costs are borne by the cargo owner, they can be important especially for high valued products and long hauls and may influence a the cargo owner's choice of carrier, if for instance a specific carrier can haul the cargo faster. These costs are also important for the ship owner, as a shipper will prefer a ship that delivers his cargo earlier than another ship that sails slower, everything else being equal. Thus, if the owner of the slower ship would like to attract that cargo, he may have to rebate to the charterer the loss due to delayed delivery of cargo. In that sense, the in-transit inventory cost is very much relevant in the ship owner's profit equation, as much as it is relevant in the shipper's cost equation. This is a typical situation in liner trades. In fact, a defining difference between liner and tramp trades is the higher value of liner (unitized) cargoes as opposed to tramp (bulk) cargoes, and this is reflected in higher operating speeds in liner shipping versus those in tramp shipping.

Even though the fixed route scenario examined in the paper is much simpler as compared to other scenarios involving route selection, network design, fleet deployment, fleet size and mix, or others, the main contribution of this paper is twofold: (a) incorporating income (freight rate) considerations into the problem, which allows one to capture a basic facet of liner shipping behavior, and (b) examining flexible frequencies, and this allows one to investigate their potential benefits.

The rest of the paper is organized as follows. Section 2 presents the model formulation. Section 3 performs a linearization that allows the use of simple tools to solve the problem. Section 4 performs some runs of the model on three existing services including sensitivity analysis. Finally Section 5 makes some concluding remarks.

1 Model formulation

1.1 Route topology

Our model considers a general and *fixed* route such as the example shown in Figure 1:

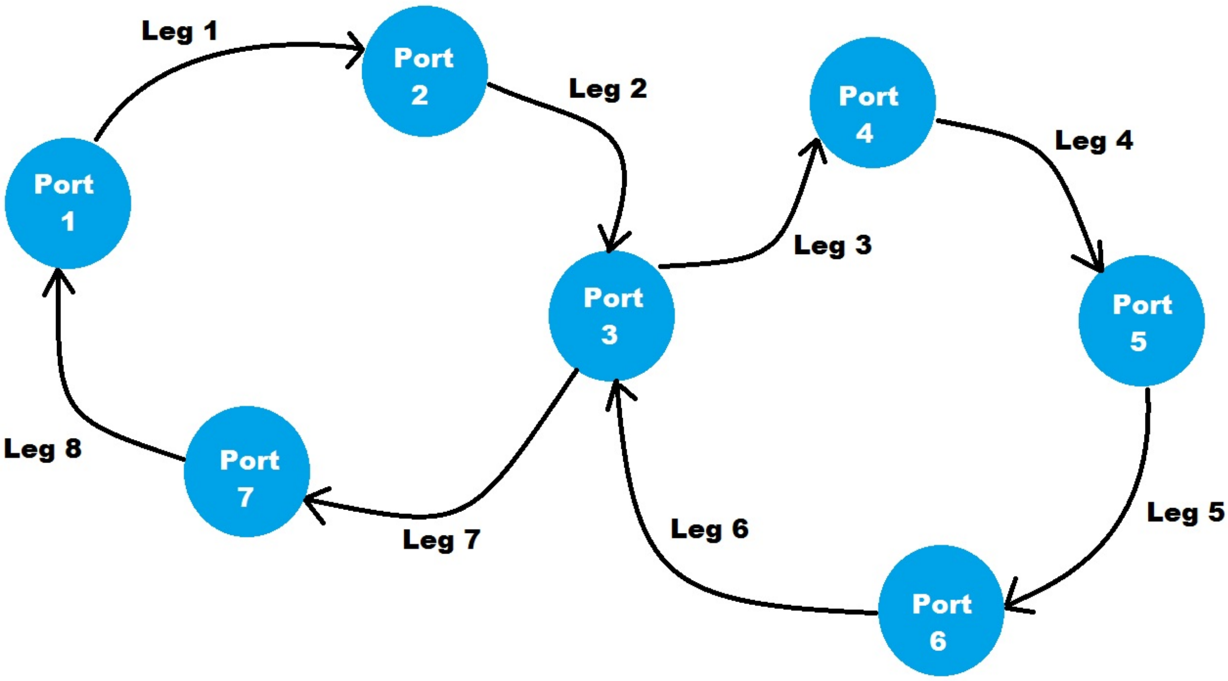

Fig. 1: Example of a route

Any route topology can be considered, so long as the ports and legs among them are defined. In Figure 1, seven ports and eight route legs are shown. In all cases the route is assumed to start from port 1, sail the given route following a prescribed sequence of port visits, and start a new round trip at port 1 after the entire route is covered.

1.2 Model assumptions, inputs and decision variables

The model assumes without loss of generality a fleet of *identical* containerships deployed on a given route. An important assumption that is critical in our ability to model the line's decision making process is that *the line has no monopoly or oligopoly power over the market of the specific route*. This assumption may not necessarily be true in segments of the market in which a shipping line has a dominant position (for instance, holds an important share of the traffic) which it can use to its advantage in setting rates or controlling capacity on the route. However, in view of anti-trust policies being recently pursued in many parts of the world (see for instance the repeal of EU Regulation 4056/86 on block exemption of liner shipping conferences in 2008), and in view that most ports are served by many lines, we think that such a scenario is rare and the above assumption is reasonable.

A main implication of this assumption is that the freight rates used in the model (in USD/TEU) are *exogenous inputs*, meaning that they are dictated by overall supply and demand, both of which are outside the operator's control. In our model the freight rates F_{zx} are supposed to be known from every port pair (z, x) of the route, assuming of course that cargoes are carried from z to x. The fact that such freight rates may depend on the direction of travel has been historically documented in various markets. See for instance Figure 2 between Asia and the US (a similar situation pertains to the trade between Asia and Europe).

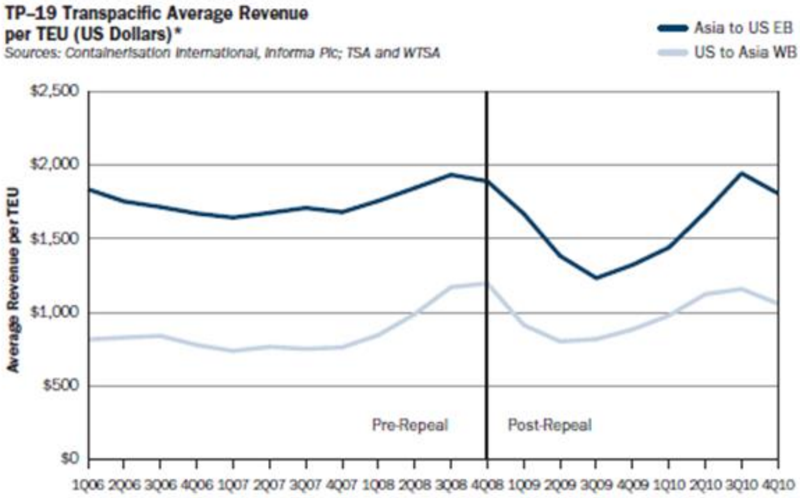

Fig. 2: Freight rate imbalances between Asia and the US. The vertical line in 4Q08 is the repeal of EU Regulation 4056/86 in 2008. Source: FMC (2012).

A second assumption in our model (which may be partially implied by the first assumption) is that the cargo demand c_{zx} between any two ports z and x is known and fixed on a per call basis and is independent of both the service period (or frequency) and of the number of ships deployed by the line on the route. The (partial) implication may be because if the line had a dominant position in the market, per port call cargo demand would generally depend on factors such as service frequency, among others (for instance if the line carries all of the traffic in a route, it is conceivable that increasing the frequency would reduce demand per port call). Irrespective of this, the assumption is consistent with what is typically assumed in the literature. In our case it means that the line can increase revenue by hauling more cargo in a given period of time, provided of course this is profitable. This can be done by speeding up or modifying the frequency of service so as to increase the number of times the line can visit a specific port within that period. Even though in the real world the per port call demand may depend on frequency and many other factors (for instance the overall demand, the freight rate, the company's intermodal services, who the line's competitors are, what capacity they put on the line, and others), here we will assume that quantity to be a known and exogenous input. As with the freight rates, demand is not necessarily symmetric and generally depends on the direction of travel. See for instance Figure 3 between the Far East and Europe (a similar situation pertains between the Far East and North America).

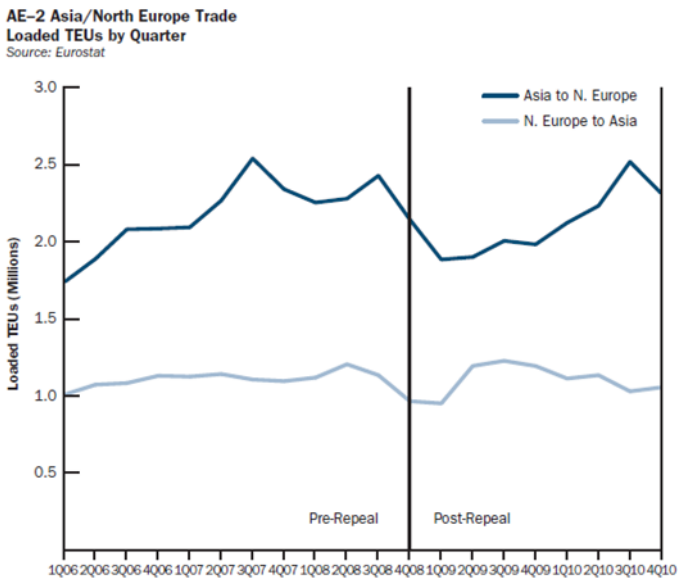

Fig. 3: Trade imbalances between Far East and Europe. The vertical line in 4Q08 is the repeal of EU Regulation 4056/86 in 2008. Source: FMC (2012).

A third assumption in our model (which may be partially due to lack of market dominance and partially due to the chronic overcapacity that is pervading the liner trades) is that there is no unsatisfied demand for cargo to be carried by the line and thus, in an optimization sense, our problem is an uncapacitated one. This means that there is always capacity on the ship to carry all cargo. This assumption is also consistent with the scenarios that were used to test our model. Overcapacity in container shipping has been recorded historically, and especially after the 2008 economic crisis. For the US to Europe trade lane and for year 2010, FMC (2012) reports an average capacity utilization of 88% eastbound and 87% westbound. For the US to Asia trade lane these numbers are 85% eastbound and 57% westbound, whereas for the Asia to Europe trade lane these numbers are 92% westbound and 59% eastbound. Moreover, UNCTAD (2016) documents a continuing *sluggish demand* challenged by an accelerated massive global *expansion* in container supply capacity, estimated at 8% in 2015 – its highest level since 2010. Based on the above, we think that the above assumption is reasonable and that, given the current state of the industry, the cases in which a carrier that has no dominant market position may encounter capacity problems are rare.

With the above in mind, inputs and decision variables of the model are shown in Table 1 below.

Table 1: Problem inputs and decision variables

A. Problem inputs			
Symbol	Definition	Units	Comments
J, I	Route geometry, represented by a set of ports J and a set of legs I representing the route	--	See Figure 1.
L_i	Length of leg $i \in I$ of the route	NM	

F_{zx}	Freight rate for transporting a TEU from port z to port x, $x \in I, z \in I$	USD/TEU	Freight rate value depends on the direction of travel, hence in general $F_{xz} \neq F_{zx}$
c_{zx}	Per port call transport demand from port z to port x, $x \in I, z \in I$	TEU	Quantity in TEU from port z to port x that will be loaded on the ship at port z. As with the freight rate, the transport demand depends on the direction of travel, hence in general $c_{xz} \neq c_{zx}$
P	Bunker price	USD/tonne	We assume without loss of generality that there is one (known) price for the fuel, even though the ship may be burning several different kinds of fuel for the main engine and the auxiliary engines. Generalization for many fuel prices is straightforward.
E	Daily operating fixed costs per vessel	USD/day	These are all, other than fuel, operating costs incurred by the ship operator. In case the ship is time chartered, they are the time charter rate.
v_{min}	Minimum allowable ship speed	Knots	Dictated by the technology of the main engine (more modern engines can run at lower speeds)
v_{max}	Maximum allowable ship speed	Knots	Dictated by the maximum allowable main engine horse power
$f(v)$	Daily fuel consumption function of ship if speed is v	Tonnes/day	Even though ship speed is a decision variable (see below), function $f(v)$ is assumed a known function and includes both main engine and auxiliaries. No dependency of f on ship payload is assumed[2].
A	Daily auxiliary engine fuel consumption at port	Tonnes/day	
W_i	Average monetary value of ship cargo on leg $i \in I$	USD/TEU	This value depends on the leg involved and on cargo composition on the ship on that leg and it may be different in different directions
R	Operator's annual cost of capital	%	
α_i	Daily unit cargo inventory costs on leg i of the route $i \in I$	USD/TEU/ day	α_i is connected to W_i and R. See note (i).

[2] This is of course an approximation. A payload-dependent function could also be used, but data to model it was not available.

D_j	Total cargo loaded to and unloaded from ship at port $j \in J$	TEU	Can be computed as a function of the demand matrix [c_{zx}]. See note (ii).
G_j	Time spent at port $j \in J$	Days	Is generally a known function of both D_j and the port's cargo handling rate.
H	Cargo handling cost per TEU	USD/TEU	We assume without loss of generality that this is the same in all ports.
C_i	Total cargo on the ship along leg i of the route $i \in I$	TEU	Can be computed as a function of the demand matrix [c_{zx}] and of the route topology. See note (ii).
Θ	Ship's capacity	TEU	See note (ii)
B. Decision variables			
Symbol	Definition	Units	Comments
t_0	Service period, defined as the period between two consecutive port visits by ships in the fleet	Days	The service period is the inverse of the service frequency. In the general case this is a decision variable, however typically the service period is fixed, eg is equal to 7 for a weekly service. See also Section 2.3.
N	Number of ships deployed on the route	Integer	
v_i	Ship speed along leg *i* of the route $i \in I$	Knots	Ship speed is bounded above and below as follows: $v_{min} \leq v_i \leq v_{max} \quad i \in I$
T_i	Time to sail leg *i* of the route $i \in I$	Days	See note (iii)
T_0	Time for one ship to complete the route	Days	See notes (iv) and (v)

The following notes clarify relationships among problem inputs and decision variables:

(i) $\alpha_i = \frac{R\,W_i}{365} \qquad i \in I \qquad (1)$

(ii) Cargo inputs D_j and C_i, the ship's capacity Θ and the cargo demand matrix [c_{zx}] should be consistent with each other and are expected to lead to feasible solutions. It is thus expected that these inputs satisfy the following relations:

$$D_j = \sum_x c_{jx} + \sum_z c_{zj} \qquad j \in J \qquad (2)$$

$$C_i = C_{i-1} + \sum_x c_{kx} - \sum_z c_{zk} \qquad i \in I \qquad (3)$$

where in (3) $k \in J$ is defined as the port between legs *i-1* and *i* of the route.

$$C_i \leq \Theta \quad i \in I \qquad (4)$$

(iii) $T_i = \frac{L_i}{24\,v_i} \qquad i \in I \qquad (5)$

(iv) $T_0 = \sum_i T_i + \sum_j G_j \qquad (6)$

(v) $T_0 = Nt_0 \qquad (7)$

Note that equation (2) defines D_j as the total cargo loaded and unloaded at port *j*. Equation (3) expresses the total cargo on the ship along leg *i* as a function of the cargo along the previous leg and the set of cargoes loaded and unloaded at port *k*, and inequality (4) states that cargo on the ship should not exceed ship capacity. Note that as these are relations *among problem inputs* and involve no decision variables, none of these are constraints on the optimization problem to be solved (of which more below). However, these relations should be satisfied so that input is consistent.

1.3 Problem formulation

In order to understand how the objective function of the problem is defined, one can first formulate the total carrier's profit π (in USD) if the considered route is sailed (once) by the fleet of *N* ships:

$$\pi = N \left(\sum_x \sum_z F_{zx}\, c_{zx} - P \sum_i f(v_i)\, T_i - PA \sum_j G_j - \sum_i \alpha_i\, C_i\, T_i - H \sum_j D_j - E\, T_0 \right) \tag{8}$$

Note that this is *not* the objective function of our problem (of which more below). Expression (8) is the product of the number of ships *N*, times the per ship profit for the route, which is the expression in parentheses that encompasses six terms. These are listed below in order of appearance in (8):

1. Per ship revenue for route
2. Per ship fuel at sea expenditure for route
3. Per ship fuel at port expenditure for route
4. Per ship in transit cargo inventory cost for route
5. Per ship cargo handling expenditure for route
6. Per ship other than fuel operating cost for route.

Once the total profit π is stated for the route and for the entire fleet, one can formulate an expression for the corresponding *maximum average daily profit* $\dot{\pi}$ (in USD/day). To do so, one can divide equation (8) by the total time for each ship to complete the route T_0. After some algebraic manipulations, and also taking into account equations (5), (6) and (7), one can eliminate T_0 and T_i and arrive at the following optimization problem:

$$\dot{\pi} = Max_{v_i, t_0, N} \left\{ \frac{1}{t_0} \left(\sum_x \sum_z F_{zx}\, c_{zx} - P \sum_i f(v_i)\, \frac{L_i}{24\, v_i} - PA \sum_j G_j - \sum_i \alpha_i\, C_i\, \frac{L_i}{24\, v_i} - H \sum_j D_j \right) - N\, E \right\} \tag{9}$$

subject to the following constraints:

$$v_{min} \le v_i \le v_{max} \qquad i \in I \tag{10}$$

$$N t_0 = \sum_i \frac{L_i}{24\, v_i} + \sum_j G_j \tag{11}$$

and $N \in \mathbb{N}^+$. (12)

Constraints (10) are the upper and lower bounds on ship speed for each leg of the route, constraint (11) links the three decision variables of the problem (number of ships, service period and ship speeds) together, and constraint (12) is the integrality constraint. Non-negativity constraints could also be added for t_0, but they are redundant because of (11).

The model, as formulated above, is rather simple. However, a departure from many other liner shipping optimization problem formulations is that our objective function deals with profit, not cost. Another, perhaps more important difference is that in contrast to other formulations in which the objective function is defined on a *per route* basis, in our formulation the objective function is defined on a *per unit time* basis. In that sense, our objective function is the *maximization of a ratio*, that of total route profit divided by the total duration of the route. Both numerator and denominator of the ratio are *nonlinear* functions of ship speed, and of course so is the ratio itself. Constraint (11) is also nonlinear. Last but not least, another difference from other models is that the service period (or frequency) is not fixed, but flexible.

1.4 CO_2 Emissions

Based on the above, and as an aside not directly connected to the optimization problem (being only a post-processing of its results), one can also compute the daily CO_2 emissions of the deployed fleet as follows.

The total CO_2 emissions M (in tonnes) produced by the fleet for one route are:

$$M = N\, m_{CO_2}\, (\Phi_{Sea} + \Phi_{Port}) \tag{13}$$

where m_{CO_2} is the CO_2 emissions factor (tonnes of CO_2/tonnes of fuel), Φ_{Sea} is the total per route fuel consumption at sea for one ship and Φ_{Port} is the total per route fuel consumption at port for one ship. For the main types of fossil fuels such as Heavy Fuel Oil (HFO), Marine Diesel Oil (MDO), and Marine Gas Oil (MGO), typical values of m_{CO_2} are in the range between 3.02 and 3.08 (Buhaug at al., 2009).

The per ship fuel consumption at sea, as per above, is equal to:

$$\Phi_{Sea} = \sum_i f(v_i)\, \frac{L_i}{24\, v_i} \tag{14}$$

Whereas the per ship fuel consumption at port, also as per above, is equal to:

$$\Phi_{Port} = A \sum_j G_j \tag{15}$$

Therefore the total CO_2 per route for the whole fleet can be expressed as:

$$M = N\, m_{CO_2}\, \left(\sum_i f(v_i)\, \frac{L_i}{24\, v_i} + A \sum_j G_j\right) \tag{16}$$

To evaluate the daily CO_2 emissions produced by the fleet M_d (tonnes of CO_2/day), one must divide M by the total route time T_0. Considering also (7), this value is equal to:

$$M_d = \frac{m_{CO_2}\, \left(\sum_i f(v_i)\, \frac{L_i}{24\, v_i} + A \sum_j G_j\right)}{t_0} \tag{17}$$

1.5 Fixed versus flexible service period (or frequency)

Note that the non-linear optimization problem of section 2.3 has *three* decision variables, N, t_0, and the ship speeds v_i $(i \in I)$. A *constrained* version of the above problem is if one of these decision variables, t_0, the service period, is fixed, that is, is considered an exogenous input and cannot vary freely. It is obvious that the optimal value of t_0 generally depends on the values of all problem inputs. Thus, the constrained version of the problem (t_0 fixed) will generally achieve inferior results vis-à-vis the case in which t_0 is allowed to a range or a set of values. In that sense, a fixed t_0 will generally come at a cost. We shall provide estimates of such such cost later in the paper.

In considering a flexible service period, we recognize that it would be absurd if t_0 takes on a value of (say) 3.8, 7.3, square root of 10, or any other similarly unconventional value. A value of t_0=7 (weekly service) is the standard assumed value in most liner services worldwide. t_0=14 corresponds to a biweekly service, and t_0=3.5 to a service twice a week. However, such practices are not very common. A fortiori, considering t_0 being equal to 4, 5, 6, 8 or 9 may be, at least for the time being, well outside the realm of what may be at play in liner shipping. Maybe this is so because the benefits of a regular weekly service are deemed paramount for all players involved, or because adopting a different regime may involve a nontrivial reconfiguration of a liner's feeder networks (and possibly of its rail and truck connections), or finally because the force of habit is just too important. However, as liner services schedules are published in each carrier's web site and other media well in advance, there is really nothing fundamental that prevents a carrier from organizing a service with t_0 equal to *any* prescribed value, and particularly if these 'unconventional' service frequencies happen to achieve better results for the carrier, in terms of the objective function examined. In that sense, current practice does not prevent us from investigating a thus far unexplored option, and the case of a flexible service period will be one of the alternatives considered in this paper.

Constraint (11) links all three decision variables of the problem together, or the two decision variables (number of ships and speeds) if t_0 is fixed. As such, it has a strong influence on the feasible solution space. If t_0 is fixed, the carrier has only *one* degree of freedom: either employ less vessels and speed up the fleet, or add more ships to the route and slow down the fleet. Instead, if a flexible service frequency is considered, a wider set of alternatives may be available to the carrier in order to optimize his per day profit. We shall come back to this point later.

1.6 Bounding the number of ships

Since the speeds v_i are bounded from below and above (constraints (10)), one can clearly observe that *for any specific service period* t_0, the possible values for N are restricted. In fact, assuming that ships sail at the maximum allowable speed or at the minimum allowable speed in all legs of the route, one can find upper and lower bounds on the required number of ships, as follows:

$$N_{min}(t_0) = \left\lceil \frac{\sum_i \frac{L_i}{24\, v_{max}} + \sum_j G_j}{t_0} \right\rceil \tag{18}$$

$$N_{max}(t_0) = \left\lfloor \frac{\sum_i \frac{L_i}{24\, v_{min}} + \sum_j G_j}{t_0} \right\rfloor \tag{19}$$

Therefore the number of ships *N* is bounded as follows:

$$N_{min}(t_0) \leq N \ \leq N_{max}(t_0) \qquad (20)$$

One can exploit this fact in order to limit the set of values of *N* and hence the computational effort of the solution procedure.

2 Linearization

The objective function of the model is clearly non-linear. In fact, variables v_i and t_0 are in the denominator and the daily at sea fuel consumption function is a nonlinear function of the speeds. Moreover, the constraint in (11) is also non-linear, as the speeds are in the denominator. A non-linear problem is not trivial to solve, not to mention that the computational time could be long. However, in order to simplify the solution of the problem, both the objective function and the constraints can be linearized. By doing that, any solution software such as CPLEX can find the optimal solution quickly. The way to linearize the problem is explained below.

First of all, the service period t_0 can reasonably be assumed to be drawn from a prescribed and finite set of values. Therefore, one can fix a set S of plausible possible values for t_0 (not only 7, but also possibly other values), optimize the problem for each value of t_0 in S, and then pick the best t_0. Section 4.3 provides more details on set S for the runs that were made.

Consequently, *and for a given* $t_0 \in$ S, the objective function can be written as:

$$\xi(t_0) = Max_{v_i,N}\left\{\frac{1}{t_0}\left(\sum_x\sum_z F_{zx}\, c_{zx} \,-\, P\sum_i f(v_i)\,\frac{L_i}{24\,v_i} - PA\sum_j G_j - \sum_i \alpha_i\, C_i\,\frac{L_i}{24\,v_i} - H\sum_j D_j\right) - N\,E\right\} \qquad (21)$$

Variable v_i can be replaced by variable u_i which is its reciprocal as follows:

$$u_i = \frac{1}{v_i} \qquad (22)$$

Substituting u_i into the objective function, equation (21) can be expressed as:

$$\xi(t_0) = Max_{u_i,N}\left\{\frac{1}{t_0}\left(\sum_x\sum_z F_{zx}\, c_{zx} \,-\, P\sum_i g(u_i)\,\frac{L_i\,u_i}{24} - PA\sum_j G_j - \sum_i \alpha_i\, C_i\,\frac{L_i\,u_i}{24} - H\sum_j D_j\right) - N\,E\right\} \qquad (23)$$

where $g(u) := f(v)\,|_{\,v=1/u}$ (24)

After this substitution, the objective function becomes linear in the new variable u_i , except for the daily at sea fuel consumption function g, which is still non-linear. Moreover, constraint (11) also becomes linear as follows:

$$N\,t_0 = \sum_i \frac{L_i\,u_i}{24} + \sum_j G_j \tag{25}$$

The bounds concerning the speed which come from (10) can be rewritten as follows:

$$u_{min} \le u_i \le u_{max} \qquad\qquad i \in I \tag{26}$$

where u_{max} and u_{min} are defined as:

$$u_{max} = \frac{1}{v_{min}} \tag{27}$$

$$u_{min} = \frac{1}{v_{max}} \tag{28}$$

We assume that the daily fuel consumption function at sea can be expressed as a power function of speed as follows:

$$f(v) = a\,v^b \tag{29}$$

with a and b being known positive constants (usually $b \geq 3$).

Given (24), this leads to

$$g(u) = a\,u^{-b} \tag{30}$$

Define now $Q(u)$ as the per nautical mile at sea fuel consumption. It is clear that

$$Q(u) = \frac{ug(u)}{24} = \frac{a\,u^{1-b}}{24} \tag{31}$$

And the objective function can be rewritten as follows:

$$\xi(t_0) = Max_{u_i,N}\left\{\frac{1}{t_0}\left(\sum_x\sum_z F_{zx}\,c_{zx} - P\sum_i Q(u_i)L_i - PA\sum_j G_j - \sum_i \alpha_i\,C_i\,\frac{L_i\,u_i}{24} - H\sum_j D_j\right) - N\,E\right\} \tag{32}$$

We would then maximize $\xi(t_0)$ over all values of $t_0 \in$ S.

$$\dot{\pi} = Max_{t_0}\,\xi(t_0) \tag{33}$$

Function $Q(u)$ is a convex function of u. As explained in Wang and Meng (2012), it is possible to use a piecewise linear approximation of such a function, and we have used their approach to solve our problem, by coding the respective procedure in MATLAB and solving the linearized problem using an Excel spreadsheet. Note that our use of this approach is limited only to how the problem is linearized. Our problem formulation, including objective function, set of decision variables and constraints differs from those of the above paper, which examines speed optimization in a container network including transhipment but without income or flexible frequency considerations.

3 Application

3.1 Mainlane East-West

The so-called Mainlane East-West represents a major set of liner routes. During 2015 this lane transported about 52.5 million TEU (UNCTAD, 2016). It connects the three major economic centers that are Europe, North America and the Far East (especially China). As depicted in Figure 4, these continents are connected by three trade routes: the Trans-Atlantic lane, the Europe-Asia lane, and the Trans-Pacific lane. The Trans-Pacific lane counts for 46% of the overall container trade on the East-West route, whereas the Europe-Asia lane counts for 41% of the trade and the Trans-Atlantic lane counts for 13% of the trade (UNCTAD, 2016). Figure 5 shows the quantities of TEU moved along these three lanes.

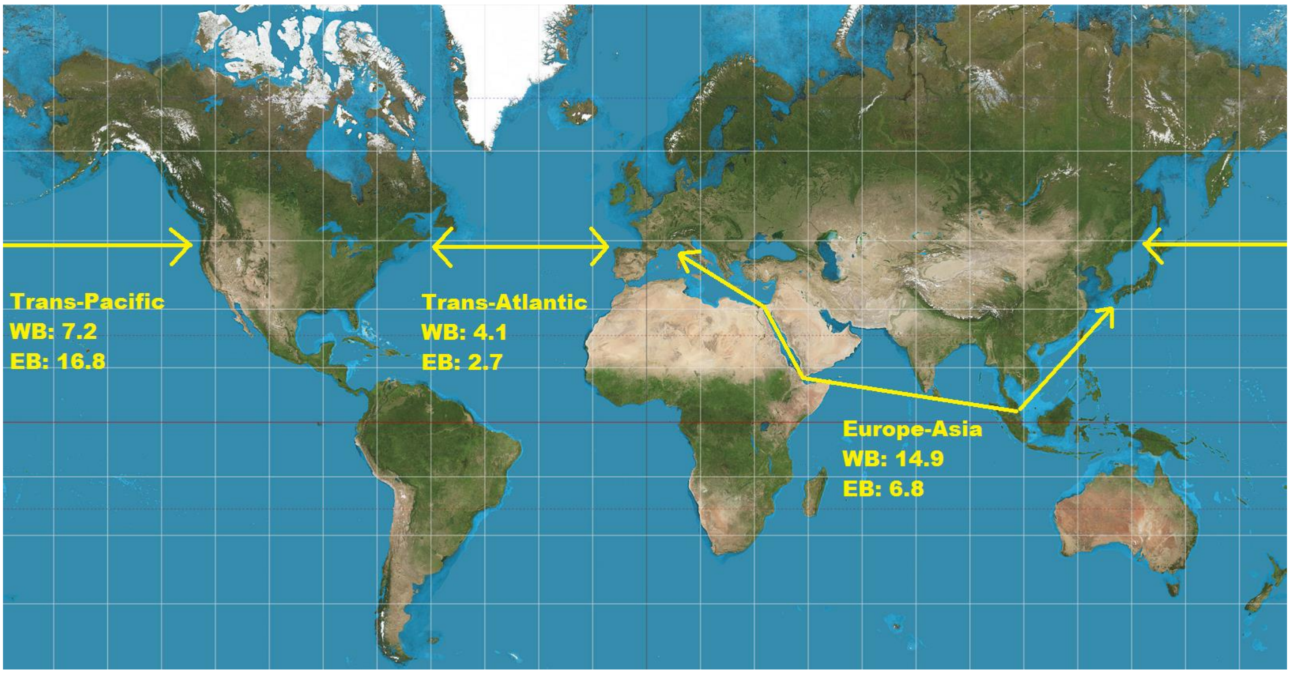

**Fig. 4: Container flows on Mainlane East-West route [million TEUs], 2015.
WB: westbound, EB: eastbound. Adapted from UNCTAD (2016), Table 1.7.**

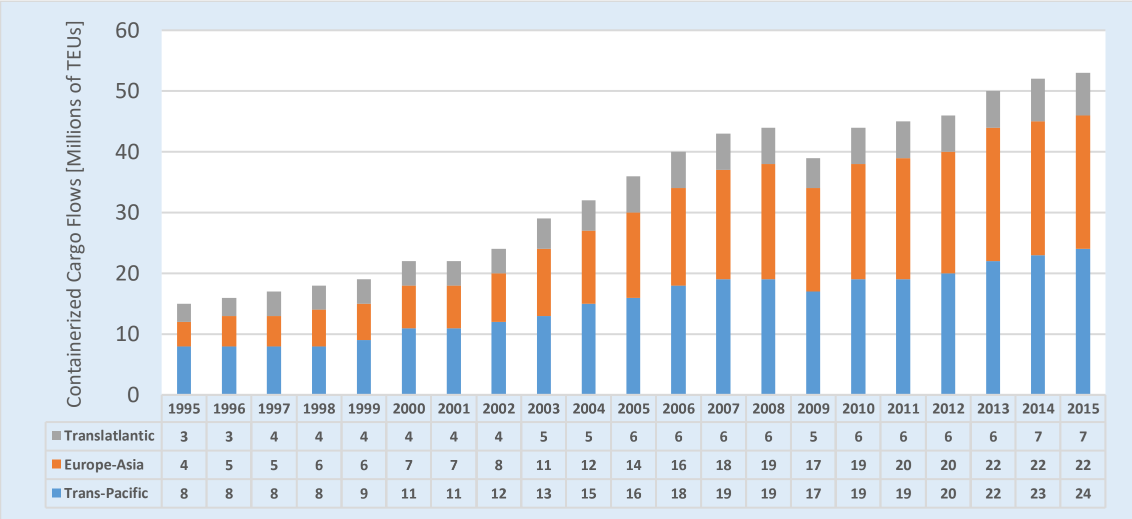

Fig. 5: Containerized trade on Mainlane East-West route, 1995-2015.
Adapted from UNCTAD (2016), Figure 1.7.

The following are general characteristics of these trade routes:

Freight rate imbalance: As mentioned earlier (see also Figure 2), there is a freight rate imbalance. As shown in Table 2, freight rates typically depend on the travel direction, especially in the Asian trades where freight rate imbalances are maximum. The reason for such imbalances is more complex to ascertain, and can be attributed to the differences in the mix of products carried in the two directions, as well as in the values of these products.

Table 2: Eastbound and Westbound freight rates in the fourth quarter of 2010.
Adapted from FMC (2012), Table TE-20, AE-19 and TP-19.

Route	Eastbound [USD/TEU]	Westbound [USD/TEU]	Ratio
North Europe-US	800	650	1.231
Far East-North Europe	1200	1900	1.583
Far East-Us	1800	1100	1.636

Flow imbalance: As mentioned earlier (see also Figures 3 and 4), and in addition to the freight rate imbalance, the quantity of cargoes hauled along a trade route is typically different in the two directions. In 2015 the ratio of TEUs transported westbound divided by TEUs transported westbound was equal to 2.33 in the Trans-Pacific lane, 2.2 in the Europe-Asia lane 1.52 in the Trans-Atlantic lane (UNCTAD, 2016). The reason for this difference is the trade imbalance among these regions. It is something that shipping companies can do little or nothing to change.

Capacity utilization: the capacity utilization is the percentage of payload carried by a ship in respect to its potential capacity. As already mentioned, it has been observed that in the Europe-Asia and in the Asia- North America lanes this value is significantly different between the eastbound direction and the westbound directions. This difference is a direct result of the flow imbalance.

Average value of cargo: as reported in Psaraftis and Kontovas (2013), the monetary value of containers is influenced by the specific trade. Indeed, the paper claims that in the Europe-Asia lane the average cargo values are about double in the westbound direction than in the eastbound direction.

As mentioned before, the cargo value influences the optimal speed of the vessel hence it is significant to consider such aspect.

3.2 Routes under study

In this paper we shall examine the following three actual liner routes:

AE2 - North Europe and Asia: such service links Asia to North Europe and is provided by Maersk. Nevertheless, the same service is also provided by MSC under the name SWAN. Indeed, both Maersk and MSC vessels are deployed along this route. The average vessel size deployed on this route has a capacity of 18,459 TEU.

TP1 - North America (West Coast) and Asia: the route connects Asia to the West Coast of North America. Maersk offers this service, however the same service is also provided by MSC and it is called EAGLE. As for the AE2 service, along the TP1 route both Maersk and MSC vessels are deployed. The average vessel size deployed on this route has a capacity of 7,073 TEU.

NEUATL1 - North Europe and North America (East Coast): the NEUATL1 lane links North Europe to the US East Coast. The service is furnished by MSC or similarly by Maersk under the name TA1. The average vessel size deployed on this route has a capacity of 4,739 TEUs.

Table 3 shows the ports involved in these routes.

Table 3: Ports in the routes under study.

Ports					
AE2		**TP1**		**NEUATL1**	
Felixstowe	1	**Vancouver**	1	**Antwerp**	1
Antwerp	2	**Seattle**	2	**Rotterdam**	2
Wilhelmshaven	3	**Yokohama**	3	**Bremerhaven**	3
Bremerhaven	4	**Busan**	4	**Norfolk**	4
Rotterdam	5	**Kaoshiung**	5	**Charleston**	5
Colombo	6	**Yantian**	6	**Miami**	6
Singapore	7	**Xiamen**	7	**Houston**	7
Hong Kong	8	**Shanghai**	8	**Norfolk**	8
Yantian	9	**Busan**	9		
Xingang	10				
Qingdao	11				
Busan	12				
Shanghai	13				
Ningbo	14				
Yantian	15				
Tanjung Pelepas	16				
Algeciras	17				

All three routes are characterized by many input parameters, such as ports distances, freight rates, transport capacity utilization along the legs and many others. Sources that have been used to draw data for this set of runs include:

- UNCTAD www.unctad.org for general information on liner shipping statistics

- EQUASIS (2015), database with information on the world merchant fleet in 2015
- FMC (2012) for transport demand tables, capacity utilization on various trade lanes
- Drewry (2015) for miscellaneous vessel operating cost information
- Maersk Line www.maersk.com for information on routes and schedules including port times
- www.shipowners.dk/en/services/beregningsvaerktoejer, for the SHIP DESMO spreadsheet that calculates fuel consumption and emissions as a function of speed- developed for Danish Shipowners Association (now Danish Shipping)
- https://shipandbunker.com/prices for bunker price information
- www.worldfreightrates.com for freight rate information
- www.searates.com for distances among ports
- www.marinetraffic.com for information on ship deadweight, length overall and breadth
- www.containership-info.com for information on ship power.

Finally, the model considers the maximum speed v_{max} equal to 24 knots while the minimum speed v_{min} equal to 15 knots.

3.3 Scenarios under study

Consistent with the considerations of section 2.5, this paper will analyze three different classes of scenarios:

Scenario 1: the service period (or frequency) is constant and the number of ships can vary. Therefore the main decision variables in such scenario are two, the speeds and the number of deployed vessels.

Scenario 2: the number of ships is constant and the frequency can vary. Hence the main decision variables are again two, the speeds and the service period.

Scenario 3: both the frequency and the number of ships can vary, in which case the main decision variables are three. However, in this case the number of ships will be bounded from above. This bound is imposed because otherwise the optimal number of ships may reach unrealistic values.

In addition to the above three scenarios, some combinations of these scenarios will be examined.

For each scenario, or combinations thereof, the effect of the following inputs on the problem solution can be examined: (a) bunker price, (b) freight rate, (c) daily operating fixed costs, and (d) cargo inventory costs. Due to space limitations we shall not report all of the above combinations but provide a representative sample. In order to assess the influence of these inputs, a sensitivity analysis is performed. To do so, we consider a variation of each of these inputs as a percentage from its *base value*. The base values are shown in Table 4 for the three routes involved. A scenario in which the input parameters take on their base values is called the *base scenario*. It is assumed that the model deals with two sets of ports: the East ports and the West ports, with transport demand supposed to be only from East ports to West ports and reversely. Each route is associated with two different values of the freight rate, which are used as benchmark values to compute the freight rates among ports F_{xz}: the freight rate *westbound* and the freight rate *eastbound*.

Table 4: Base values for the three routes involved

Base input values				
Route	**Bunker price, P [USD/tonne]**	**Freight rate westbound, F-WB [USD/TEU]**	**Freight rate eastbound, F-EB [USD/TEU]**	**Daily operating fixed costs, E [USD/day]**
AE2	365	690	675	34557
TP1	365	360	1070	22625
NEUATL1	365	1260	1150	18230

The sensitivity analysis also employs a parameter called average speed $v_{average}$, defined as follows:

$$v_{average} = \frac{\sum_i L_i}{24\, T_0'} \tag{34}$$

where the numerator is the overall length of the route (in NM) and T_0' is the travel time (in days) for the route excluding time spent in ports. Given that $T_o' = \sum_i \frac{L_i}{24\, v_i}$, it follows that

$$v_{average} = \frac{\sum_i L_i}{\sum_i \frac{L_i}{v_i}} \tag{35}$$

The average speed is the sailing speed if the vessel sailed at a constant speed. Such speed is useful for assessing easily how the speed changes in different scenarios.

The assumed alternative service periods t_0 used in these runs are drawn from the following set S (9 alternative values):

S={3.5, 4, 5, 6, 7, 8, 9, 10, 14}

In practical terms, a service period of 3.5 means two calls per week. This would not necessarily mean two calls each week separated by 3.5 days, but they could be performed (say) on Tuesdays and Fridays, or any two distinct days of the week. A service period of 14 days means a call every two weeks. But a service period of 6 days means that if a call is on Sunday this week, next week the call will be on Saturday, the following week on Friday, etc. A service period of 8 days would mean the opposite. Similar considerations can be made for the other values. Certainly adopting such practices is currently unconventional, however these might be worth being looked upon if they are better for the operator. The sections below provide some insights on this issue.

3.4 Results of runs

This section shows a set of results derived from the sensitivity analysis concerning the three routes studied in the paper. Table 5 provides an overview of the various runs.

Table 5: Overview of runs

Scenario	Route	Fixed (base value) inputs	Varying inputs	Effect on	Reference
Fixed service period	AE2	Bunker price Freight rates Inventory costs	Daily operating fixed costs	Daily CO_2 emissions	Section 4.4.1, Fig. 6
Fixed service period	AE2	Daily operating fixed costs Freight rates Inventory costs	Bunker price	Number of ships Average speed	Section 4.4.1, Fig. 7
Fixed number of ships	TP1	Daily operating fixed costs Bunker price Inventory costs	Freight rates	Service period Average speed	Section 4.4.2, Fig. 8 Table 7
Bounded above number of ships	TP1	Daily operating fixed costs Bunker price Inventory costs	Freight rates	Service period Average speed	Section 4.4.3, Fig. 9
Bounded above number of ships	AE2	Daily operating fixed costs Freight rates Inventory costs	Bunker price	Service period Average speed	Section 4.4.3, Fig. 10
Bounded above vs unlimited number of ships	AE2	Daily operating fixed costs Freight rates Inventory costs	Bunker price	Average speed	Section 4.4.3, Fig. 11
Bounded above vs unlimited number of ships	AE2	Daily operating fixed costs Freight rates Inventory costs	Bunker price	Number of ships	Section 4.4.3, Fig. 12
Bounded above vs unlimited number of ships	AE2	Daily operating fixed costs Freight rates Inventory costs	Bunker price	Daily CO_2 emissions	Section 4.4.3, Fig. 13
Fixed service period Fixed number of ships	NEUATL1	Daily operating fixed costs Freight rates Bunker price	Inventory costs	Leg speeds	Section 4.5, Fig. 14
Fixed service period Fixed number of ships	AE2	Daily operating fixed costs Freight rates	Inventory costs Bunker price	Leg speeds	Section 4.5, Fig. 15

Some details of these runs follow.

3.4.1 Fixed service period scenario

Here the service period employed in the simulations is a constant and taken equal to 7 days for each route, as this is the most common practice in the containership market.

The daily operating fixed costs E influence the average speed of the vessels, the number of ships employed and hence the CO_2 emissions. A higher value of E reduces the number of ships deployed. However, since the service frequency is constant, this effect also leads to an increase of the average sailing speed and therefore also of the fuel costs and daily CO_2 emissions produced by the fleet. Figure 6 shows the effect of E on daily CO_2 emissions for route AE2, when E ranges from 6911 USD/day to 62203 USD/day (plus or minus 80% from base value).

Fig. 6: Fixed service period scenario, effect of the daily operating fixed costs on daily CO_2 emissions (route AE2)

Increasing the bunker price has the opposite effect, that is, it decreases the average speed of the vessels and hence also their emissions, and it also increases the number of ships deployed to maintain the service frequency constant. Figure 7 depicts the effect of the bunker price on the average speed and the number of ships for the fixed service period scenario. In this chart, bunker price ranges from 146 USD/tonne to 583 USD/tonne (plus or minus 60% from base value).

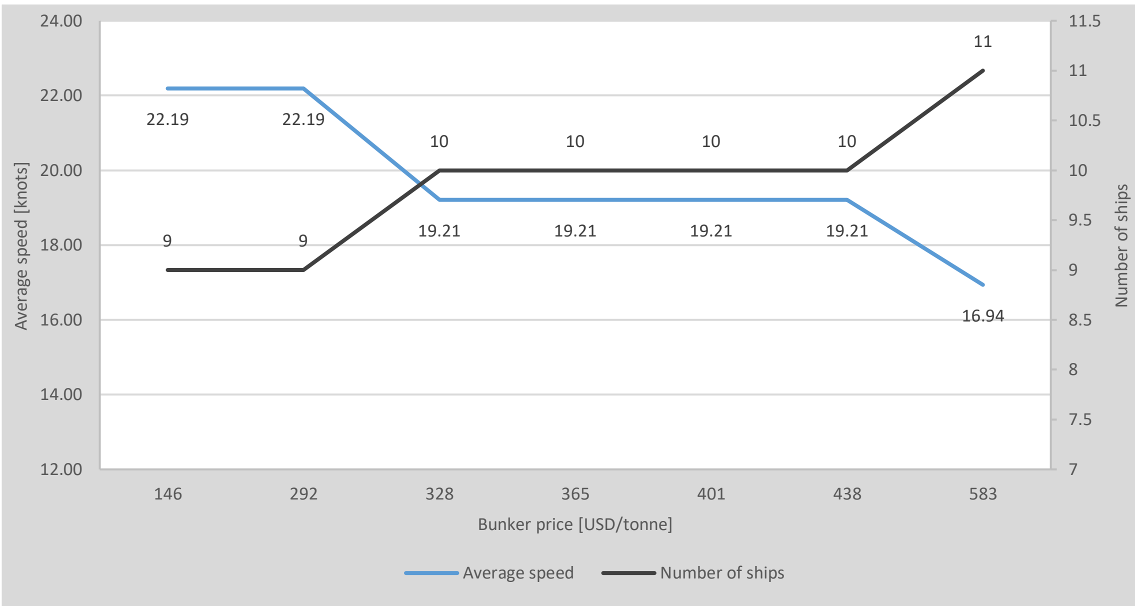

Fig. 7: Fixed service period scenario: optimal number of ships and optimal average speed at different bunker prices (route AE2)

Since the frequency of service is constant, in this scenario the revenue is also constant because the quantity of delivered goods is fixed. Therefore in this scenario variations in the freight rate do not influence any of the problem's decision variables.

3.4.2 Fixed number of ships scenario

The number of ships in the base scenarios is the actual number of ships deployed in the examined routes. These numbers are shown in Table 6.

Table 6: Number of ships employed in the fixed number of ships scenario for each route

Number of ships	
Scenario	**Number of ships**
AE2	10
TP1	5
NEUATL1	5

Since this scenario considers the number of ships constant, the daily operating fixed costs *E* do not influence the results. As in the fixed service period scenario, if the bunker price goes up, the average speed decreases, and so do CO_2 emissions. If the number of ships is constant, the only way to reduce speed is by decreasing service frequency (or increasing the service period).

Figure 8 depicts service frequency and average speed at different freight rates for route TP1. To make a pictorial representation easy to understand, the X axis shows what is labelled the *average freight rate*, that being the average between the eastbound and westbound freight rates. The freight rate (both eastbound and westbound, and obviously also the average) is allowed to vary plus or minus 40% from the base value. This translates into a range for the average freight rate from 393 USD/TEU to 1001 USD/TEU. It can be seen that if the freight rates are low enough, a service period of 8 days is better, whereas for higher rates a service period of 7 or even 6 days is better. Also average speed is seen to be a non-decreasing function of the average freight rate.

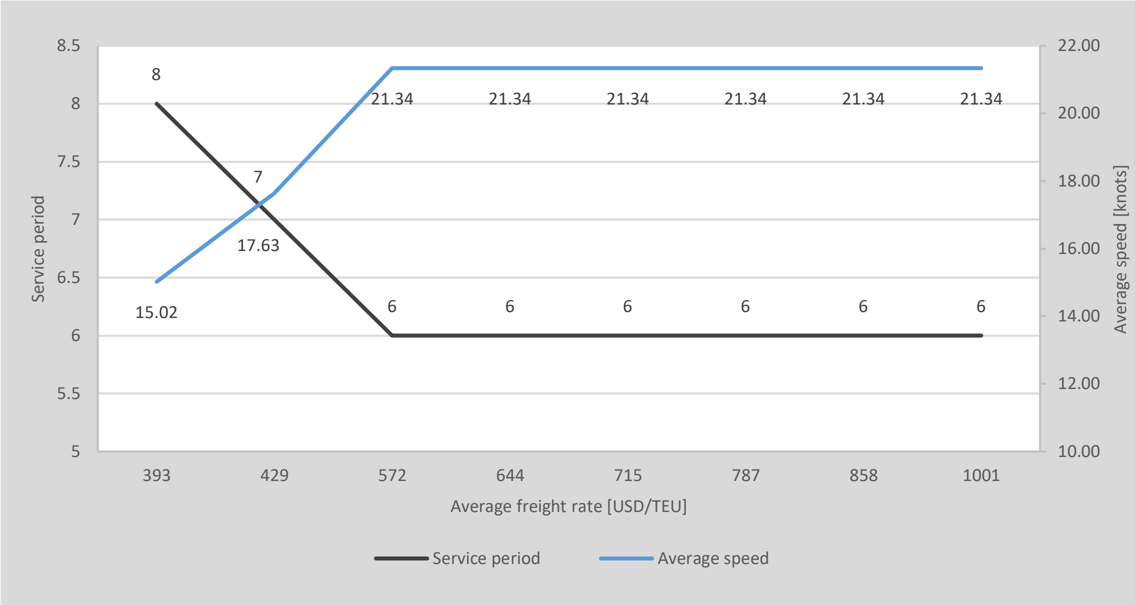

Fig. 8: Fixed number of ships scenario, optimal service period and optimal average speed at different average freight rates (route TP1)

The above means that if we force t_0=7, the solution will be suboptimal in 7 out of the 8 cases. We can compute the differential in the objective function between the optimal solution and the solution in which t_0 is forced to be equal to 7. This differential Δ (difference in per day profit between the optimal t_0 case and the t_0=7 case) is essentially the extra per day cost the company will have to incur for having a weekly service instead of a service in which the service frequency is allowed to be different. This extra cost has been computed and is shown in Table 7. Instances correspond to the average freight rates shown in Figure 8.

Table 7: Cost of forcing t_0=7 days in a fixed number of ships scenario.
Speed if t_0=7 days is 17.63 knots.

Instance	Average freight rate (USD/TEU)	Optimal t_0 (days)	Δ (USD/day)
1	393	8	4,132
2	429	7	0
3	572	6	15,717
4	644	6	35,029
5	715	6	54,341
6	787	6	73,653
7	858	6	92,965
8	1,001	6	131,590

At the low end of the freight rate spectrum (instance 1), the model chooses an 8-day service period as optimal and a (relatively) low corresponding average speed, 15.02 knots. If one forces a higher frequency (and specifically a call every 7 days) and the number of ships is constant, this would only be achievable if the average speed increases to 17.63 knots. The higher frequency would increase the amount of cargo transported and the associated revenue, but as the freight rate is low the additional revenue cannot match the increased cost due to the higher speed, hence daily profit is lower by 4,132 USD/day.

The situation at the high end of the freight rate spectrum is the opposite, but its effect is the same. At instance 8, the high average freight rate of 1001 USD/TEU suggests a 6-day service period as optimal and a (relatively) high corresponding average speed, 21.32 knots. If one forces a lower frequency (a call every 7 days) and the number of ships is constant, this would only be achievable by a lower average ship speed, again 17.63 knots. The lower frequency would decrease cargo transported, but given the freight rate is high the associated loss of revenue would be greater than the savings in fuel cost due to the lower speed, hence again a lower daily profit (in this instance lower by 131,590 USD/day for the entire fleet). The situation in instances 3 to 7 is similar.

The above rudimentary example shows that the model can capture the impact of different freight rates and that costs of having a fixed service frequency (in this case a weekly service) can potentially be significant.

4.4.3 Bounded above number of ships scenario

The scenario analysed in this section allows both service period and number of ships to vary, but imposes a limit on the maximum number of available ships. This limit is imposed in order to avoid the optimal number of ships reaching unrealistic values. As an example for the AE2 scenario, if one considers a service period of 3.5 days, the number of ships would be equal to 24 (versus 10 ships in the equivalent base scenario for that route). The upper bound on the number of ships for each route is arbitrarily chosen as a number greater than the number of ships in the base case scenario (as per Table 6). The values of the upper bound on the number of ships for each route are shown in Table 8.

Table 8: Upper bound on the number of ships for each route

(shown in parentheses is the number of ships in each base scenario, as per Table 6)

Route	Upper bound
AE2	18 (10)
TP1	8 (5)
NEUATL1	7 (5)

Before analysing the effect of freight rate, bunker price and daily operating fixed costs in this scenario, it is useful to be aware of the effect caused by an increase of the service frequency. Providing a higher service frequency implies a higher revenue. However, in order to increase the service frequency it is necessary to deploy more vessels. Moreover, with the number of ships bounded above, a higher service frequency also entails a higher average speed.

Figure 9 depicts the service frequency and the average speed at different average freight rates for route TP1. The average freight rate ranges from 429 USD/TEU to 1001 USD/TEU.

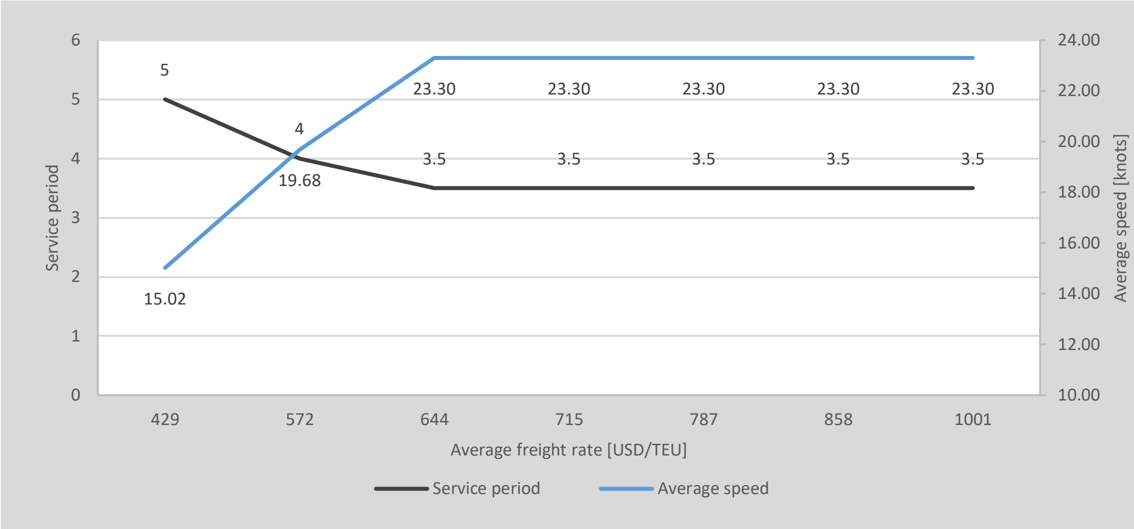

Fig. 9: Number of ships bounded above scenario, optimal service period and optimal average speed at different average freight rates (route TP1)

Again, one can see from Figure 9 that if the number of ships is not fixed but is bounded above, the optimal service period is quite different from t_0=7 and can go as low as 3.5 (two calls a week). Obviously the possibility of adding more ships makes more calls per week an easier option.

In this case forcing t_0=7 entails a cost (difference in objective function value) which has been calculated to range from 27,257 USD/day at the low end of the freight rate to 883,528 USD/day at the high end, for the whole fleet. The difference is mostly attributable to loss of revenue if frequency is forced to stay at one call per week even though the freight rates (even the low ones) justify more calls and ships are available to serve them.

Figure 10 shows the bunker price effect on the average speed and the service period for route AE2. Bunker prices range again from 146 USD/tonne to 583 USD/tonne.

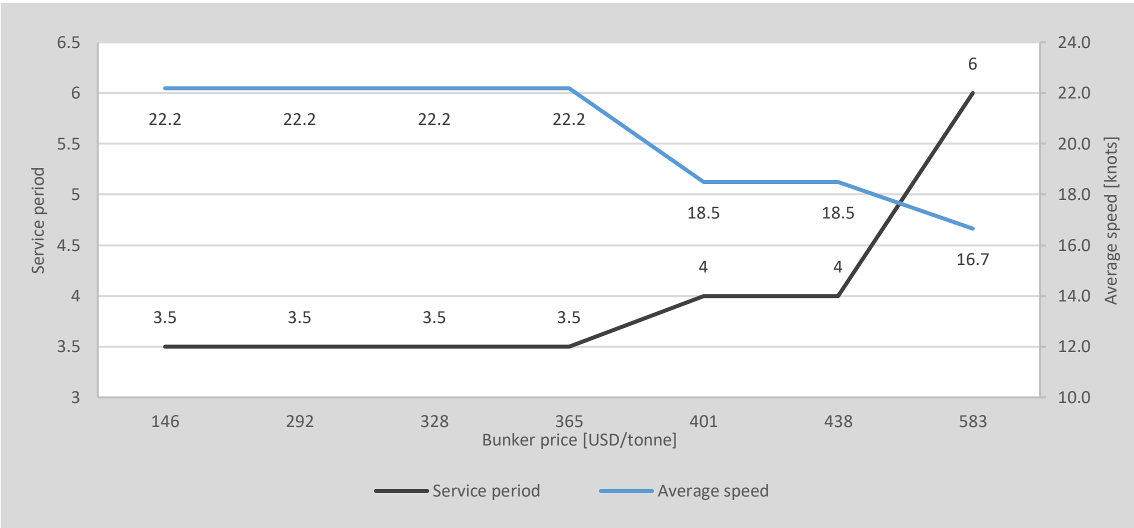

Fig. 10: Number of ships bounded above scenario, optimal service period and optimal average speed at different bunker prices (route AE2)

This example confirms that under certain circumstances service periods different from 7 days may achieve better results for the operator, and that speed generally is a non-decreasing function of the freight rate. Also, a higher bunker price makes the high service frequency disadvantageous since this would entail deploying more ships and increasing the average speed, hence a higher fuel expenditure.

Figures 11, 12 and 13 show a comparison between the results of the *N* limited scenario and the *N* unlimited scenario, in terms of the effect of bunker price on average speed, number of ships and daily CO_2 emissions (respectively).

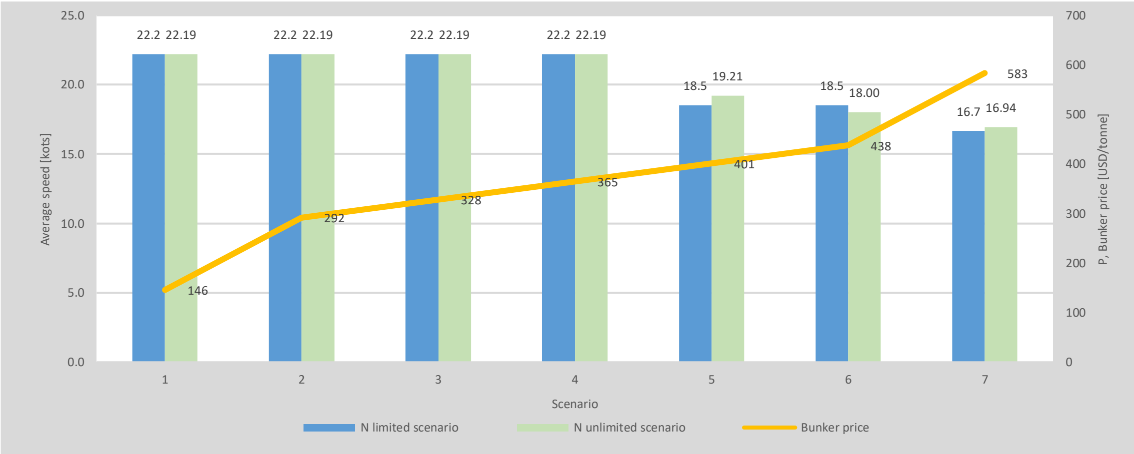

Fig. 11: Comparison between the *N* limited scenario and the *N* unlimited scenario, effect of the bunker price on the average speed (route AE2)

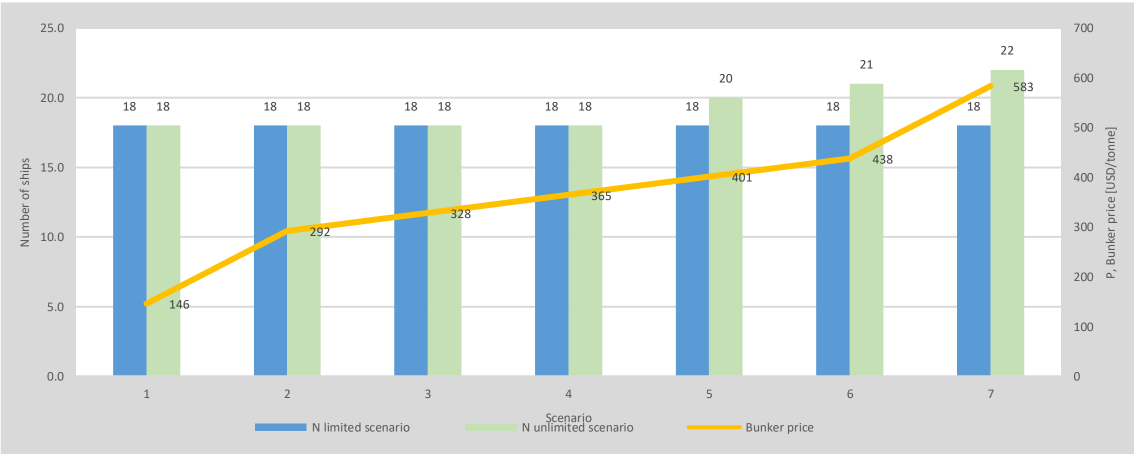

Fig. 12: Comparison between the *N* limited scenario and the *N* unlimited scenario, effect of the bunker price on the number of ships (route AE2)

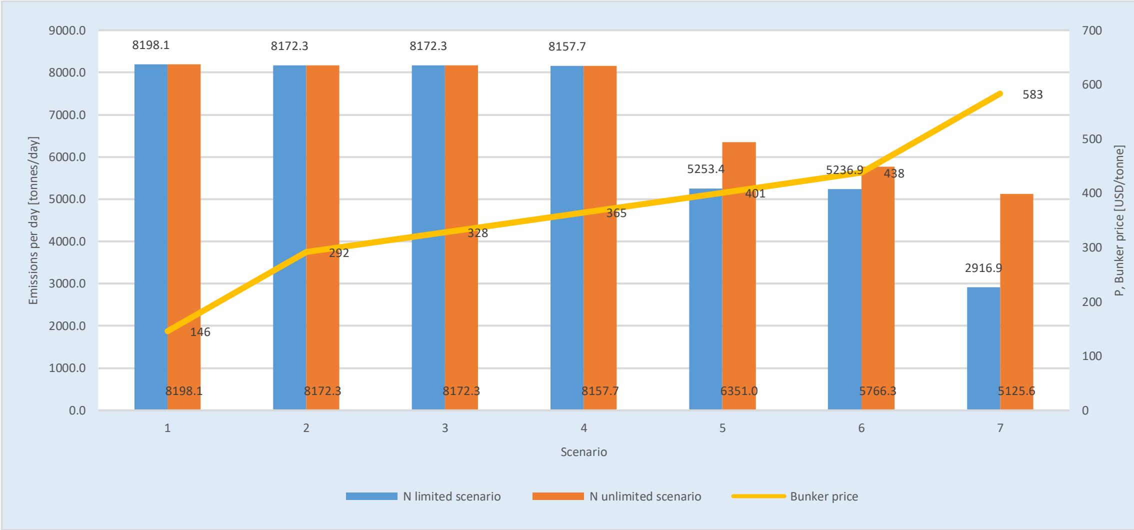

Fig. 13: Comparison between the *N* limited scenario and the *N* unlimited scenario, effect of the bunker price on the daily CO_2 emissions (route AE2)

That CO_2 emissions can be reduced by a bunker price increase (Figure 13) points to the importance of a bunker levy as a potential CO_2 emissions reduction measure. For a discussion of Market Based Measures for the reduction of GHG emissions from ships, see Psaraftis (2012).

3.5 Inventory Costs Effect

As one can see in the objective function, expression (9), given a specific service period and a specific number of ships, the optimal sailing speeds along the legs v_i depend essentially on two factors, the bunker price and the cargo inventory costs. The influence of these two factors is opposite: the fuel consumption factor leads to a reduction of the speeds v_i, so as to respect the service frequency, whereas the inventory costs factor leads to an increase of the speeds along the legs of the route in order to reduce the sailing time on each leg and therefore the in transit cargo inventory costs. In order to assess the inventory costs impact on the speeds v_i it is useful to introduce the daily inventory costs along leg i $K_{d,i}$ (in USD/day):

$$K_{d,i} = \alpha_i C_i \qquad (36)$$

where α_i and C_i are as defined in Table 1. The effect of the daily inventory costs on the speeds v_i is easy to understand: a higher inventory cost value implies a higher speed along the relevant leg. One can verify this via Figure 14, which depicts the speeds along the eight legs of the NEUATL1 route (legs are as defined in Table 3).

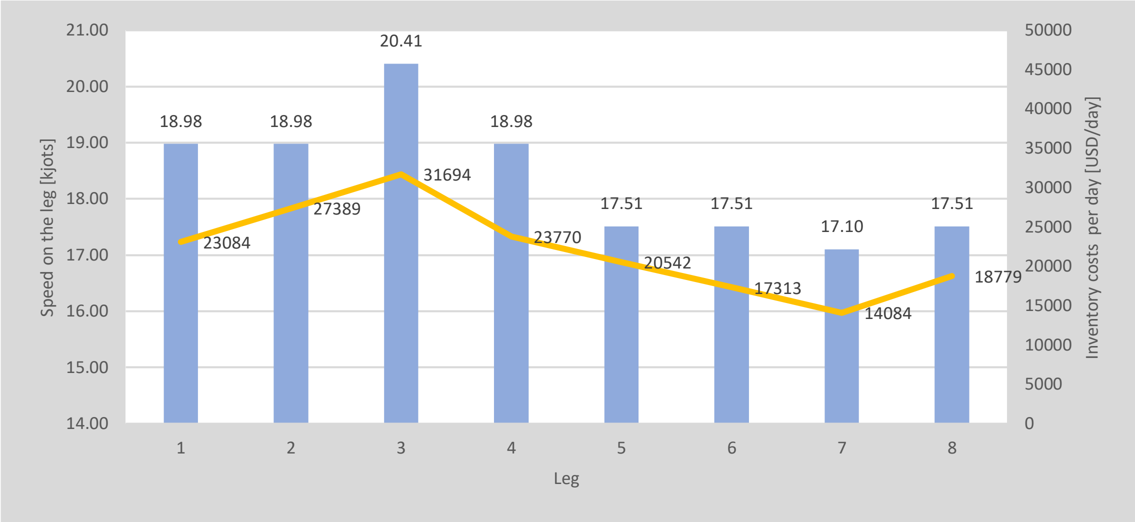

Fig. 14: Effect of inventory costs on the speeds along the legs (route NEUATL1). The figure refers to a base scenario in which *N*=5 and t_0=6.

At the same time, it is important to note that the influence of the inventory costs on the optimal speeds along the legs of the route is stronger for low values of bunker price, whereas their influence is weak when the bunker price is high. Indeed, if the bunker price is high, the carrier will slow down in order to curb fuel costs, which are higher than the inventory costs in such case.

Figure 15 confirms this statement. It refers to the route AE2. The scenarios are base scenarios except for the bunker price, which has three different values as shown in Table 8. Besides, the scenarios consider a fixed service period and a fixed number of ships (and specifically N=10 and t_0=7, which are the actual values for the route considered). In the first instance, in which the bunker price is low, the optimal speeds closely follow the trend of the inventory costs, namely the speeds are low along the legs on which the daily inventory costs are low, whereas the speeds are high along the legs in which the daily inventory costs are high. On the contrary, in the third instance in which the bunker price is high, the optimal speeds are nearly constant because in order to reduce fuel costs the speeds must be as low as possible on each leg. However, one can still see the effect of the inventory costs: along legs 5 to 11, in which the daily inventory costs are lower than what they are on the other legs, the optimal speeds are lower.

Table 9: Inventory costs effect scenario, bunker price (route AE2)

	Bunker price	
Scenario	**P [USD/tonne]**	**Variation**
1	146	-60%
2-base	365	/
3	583	+60%

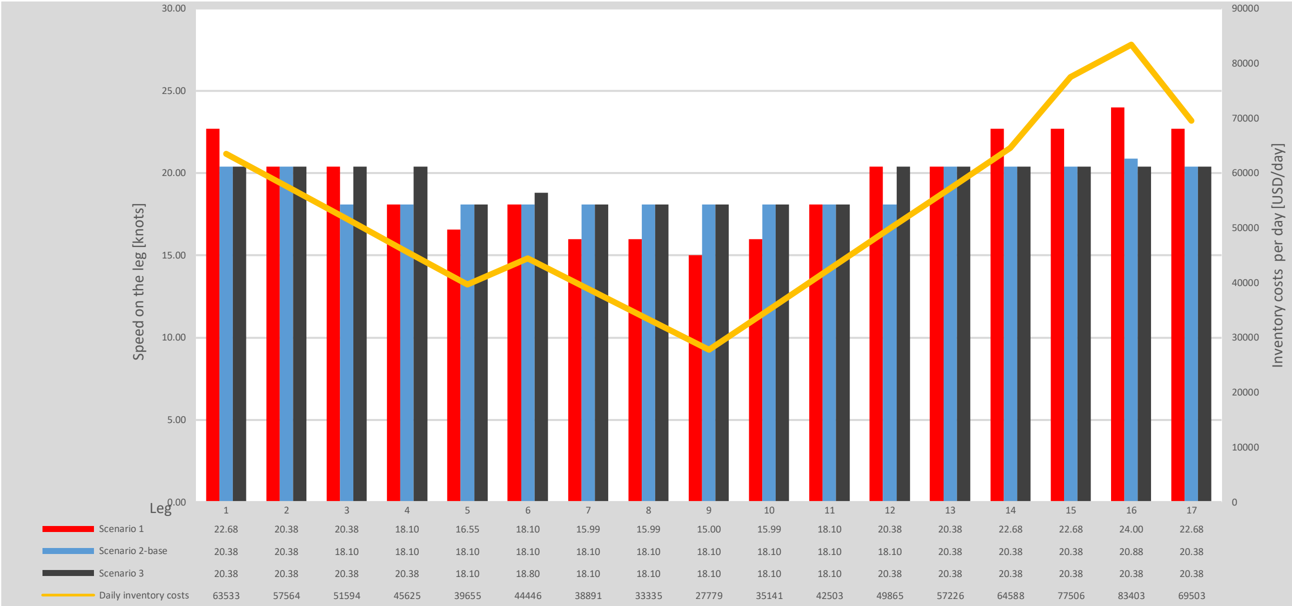

Fig. 15: Effect of inventory costs and bunker price on the optimal speeds (route AE2). The speeds are higher on the legs on which the daily inventory costs are higher.

4 Concluding remarks

The main contribution of this paper has been the development of a model that is simple on the one hand, but captures some basic facets of liner shipping behavior on the other. This is done by incorporating the impact of freight rates on the decision process of a container line regarding speeds, number of deployed ships and frequency. This allows one to consider the income part of the equation and the results reflect the actual practice of lines, speeding up when the market is high and slowing down otherwise. Fuel prices, cargo inventory costs and other ship costs are also taken into account.

An additional contribution of the paper is that service frequencies different from the standard assumption of one call per week were also considered. Even though this is well outside the current spectrum of practices in the liner sector, the potential benefits of an enlarged set of alternatives as regards frequency were investigated. In that sense, it was shown that the cost of forcing a fixed (weekly) frequency can sometimes be significant. This cost is attributed either to additional fuel cost if the fleet is forced to sail faster to accommodate a frequency that is higher than the optimal one, or to lost income if the fleet is forced to sail slower to meet a lower than optimal frequency. As regards the impact of inventory costs**,** the model can capture the fact that higher valued cargoes induce higher speeds.

In terms of possible further work, a straightforward extension of the model would be to assume a mix of different types of ships, which is the actual case that occurs in the shipping industry. This is currently being worked on and will be reported in a future publication. We do not believe that such an extension would change the major trends identified in the paper, however the issue would be to find an efficient way to solve the more complex prolem. In addition, embedding the approach of this paper into more complex optimization models such as fleet deployment, fleet size and mix and network design would constitute some extensions worthy of note. Last but not least, examining alternative forms of cargo demand functions and/or monopoly/oligopoly scenarios could be interesting.

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