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An experimental and numerical study of floating breakwaters

Erik Damgaard Christensen¹, Harry B. Bingham², Andreas Peter Skou Friis³, Alexander Kruse Larsen⁴, and Karsten Lindegaard Jensen⁵

Abstract

Breakwaters are used to provide sheltered areas for loading and unloading of ships, and coastal protection. Often the breakwaters are bottom mounted such as rubble mound breakwaters. However, there can be several advantages to use a Floating Breakwater (FB). Therefore, the objective of this paper is to study the effect of two different damping mechanisms of a floating breakwater. Three basic cross-sections of FBs were tested and analysed in 2D; a regular pontoon (RG), a regular pontoon with wing plates attached (WP), and a regular pontoon with wing plates and porous media attached to the sides (WP P100). The damping of the FB motions was due to wave radiation and viscous damping. The viscous damping originated mainly from vortex generation around the edges of the structure and due to energy loss inside the porous material attached to the vertical sides of the floating breakwater. Attaching wing plates to the floating breakwater significantly reduced the motion, which was also anticipated. When the porous sides were attached the motion of the FB increased compared to the (WP) cross-section, but the wave transmission was reduced. The possibility for incorporating the effect of the damping in the radiation/diffraction code WAMIT was assessed. The study showed that the cross section with wing plates reduced the motions of the breakwater to the largest extent, while the cross section with wing plates and porous media attached to the sides reduced the reflection and transmission most effectively.

Keywords: Floating breakwater, Transmission, reflection and dissipation of waves, Porous breakwater.

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28 1 Introduction

29 Breakwaters are used to provide sheltered areas for loading and unloading of ships, and for
30 coastal protection. Often the breakwaters are bottom mounted such as rubble mound breakwaters.
31 However, there can be several advantages using a Floating Breakwater (FB). For instance, they can
32 be moved to another location with relatively little effort. When the water depth increases, the costs
33 of a bottom-mounted breakwater increase substantially, which makes the floating breakwater
34 concept economically attractive. Further, if the soil conditions are not suited for high loads, a FB
35 might be the only solution to attenuate the incoming wave field.

36 The use of FBs can get an enhanced attention in the coming years due to an anticipated
37 development of the ocean space. European oceans will be subject to massive development of
38 marine infrastructure in the near future, see (Christensen et al., 2015). The development includes
39 energy facilities, e.g. offshore wind farms, exploitation of wave energy, and development and
40 implementation of marine aquaculture. This change of infrastructure makes the concept of multi-use
41 offshore platforms (several functionalities in the same area/ or same platform) particularly
42 interesting, where FBs can play an important role in protecting service platforms and offshore
43 terminals.

44 The single pontoon FB has gained much attention. Most of these studies have been made with
45 the assumption of a very long structure, which allows for analysing the problem in 2D. For instance
46 (Drimer et al., 1992) developed an analytical model for a single pontoon. (Sannasiraj et al., 1998)
47 studied a single pontoon breakwater experimentally and theoretically, (Abul-Azm and Gesraha,
48 2000) and (Gesraha, 2006) studied the hydrodynamics under oblique waves. (Koutandos et al.,
49 2004) developed a Boussinesq model coupled with a 2DV elliptical model to study the
50 hydrodynamic behaviour of fixed and heave motion FBs. (Rahman et al., 2006) studied the single
51 pontoon breakwater with a VOF-type Navier-Stokes solver (see for instance (Hirt and Nichols,
52 1981) for the original introduction to VOF-method).

53 Other types of FBs have also been studied. For instance, (Dong et al., 2008) studied different
54 configurations of partly open breakwaters, i.e. single-box FB, double-box FB, and board-net FB.
55 (Wang and Sun, 2010) examined a porous breakwater where the structure was fabricated with large
56 numbers of diamond-shaped blocks arranged to reduce transmitted wave height and the mooring
57 force. Their results showed that the porous FB reduced transmission of a large part of the incident
58 wave energy through dissipation rather than reflection of the wave energy. (Ji et al., 2015) and (Ji et
59 al., 2016) used experiments to optimize the configuration of FBs. They found that a FB consisting

60 of two pontoons with a mesh between them gave the best performance in wave attenuation. Further,
61 they suggested that this could be combined with porous structures in order to improve the
62 functionality of the structure. (Tang et al., 2011) presented another dual pontoon floating structure,
63 where the pontoons supported a fish net for aquaculture. In this case the fish net acts as a very open
64 porous structure, which in (Wang and Sun, 2010) was found to increase the wave attenuation
65 caused by energy dissipation. Examples of full three-dimensional studies of FB can for instance be
66 found in (Loukogeorgaki and Angelides, 2005) and (Loukogeorgaki et al., 2014).

67 Traditional breakwaters can be divided into reflective, such as vertical wall breakwaters, and
68 dissipative such as rubble mound breakwaters. These two types of breakwaters have been
69 intensively studied and it is out of the scope of this paper to give a further introduction to them.
70 However, (CIRIA et al., 2007; Goda, 2010) give a good introduction to their function and design.
71 The transmission through a vertical breakwater will typical be very small and originates from
72 diffraction and overtopping processes. Diffraction and overtopping processes are also important for
73 rubble mound breakwaters, but rubble mound breakwaters are also subjected to transmission of
74 wave energy depending on the width and height of the structure and, furthermore, on the porous
75 material of the interior. The effect of the porous media on the incoming waves has in recent years
76 gained attention with the use of advanced numerical models as discussed for example by (Garcia et
77 al., 2004; Jacobsen et al., 2015; Jensen et al., 2014; Losada et al., 2005). The single pontoon FB
78 reflects rather than dissipates the wave energy. Compared to the vertical breakwater the
79 transmission of energy is of course much higher as the wave energy is translated under the pontoon
80 and through wave radiation caused by the motion of the breakwater. The wave radiation is often
81 related to the roll motion of the FB, which can be reduced by adding wings or in ship terminology,
82 bilge keels, to the pontoon, which increases the viscous damping. Another way could be active roll
83 control devices, see for instance (Perez and Blanke, 2012). The active roll control devices might be
84 less attractive for a FB as this will lead to a more complex and thus, a more expensive structure.
85 Wings increase wave attenuation by dissipating energy, and a smaller part of the incoming wave
86 energy is transmitted due to reduced wave radiation caused by rolling of the FB.

87 This paper presents experimental and numerical analyses of the motion of a FB, and its
88 reflection, dissipation and transmission of wave energy. The basic geometry of the cross section of
89 the FB was based on a single pontoon, which was modified in several steps in order to examine the
90 effect of roll damping wings and porous media on the side of the breakwater. Therefore, the
91 objective of this paper is to study the effect of two different damping mechanisms, and how they

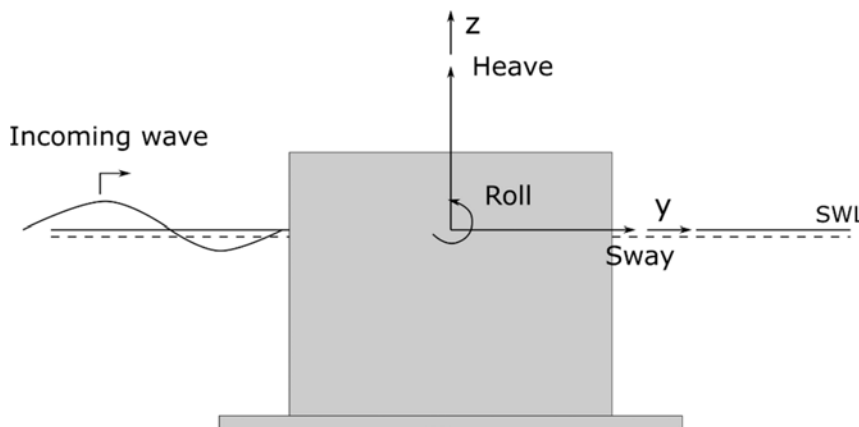
92 influence the reflection and transmission of waves. Section 2 describes how we measured the
 93 motion of the FB with particle tracking techniques as well as with accelerometers in a laboratory
 94 wave flume. The surface elevation was measured with wave gauges on the front and lee side of the
 95 FB. Section 3 presents the analyses of the measured data, where the response amplitude operator
 96 (RAO) and the derived wave characteristics from surface elevations are presented. The set-up of the
 97 numerical model is described in section 4 that included an attempt to account for external viscous
 98 damping, and to model the mooring system as an external stiffness matrix. The numerical analyses
 99 were compared to the experimental results in section 5.

100 2 Experiments with a floating breakwater (FB)

101 The objective of this study was to evaluate the effect of two different approaches to reduce the
 102 transmission of wave energy to the lee side of a single pontoon FB. Therefore, we evaluated three
 103 main cases; a regular pontoon (RG), a pontoon with wings (bilge keels) attached (WP), and a
 104 pontoon with wings and porous sidewalls (WP P100).

105 We use the common notation for the six DOF (degrees of freedom) of which only the sway,
 106 heave and roll were studied as sketched in Figure 1. The figure also indicates that a cross-section of
 107 the FB was examined in the study.

108



109

110 *Figure 1 Definition sketch of the DOF (Degrees Of Freedom) that was examined in this study.*
 111

112 The normalized Response Amplitude Operator (RAO) is the ratio of the amplitude of the FB
 113 motion to the amplitude of the incoming wave:

114
$$RAO_i(\omega) = \frac{\xi_i}{A}, i = 1 \sim 6 \quad (2.1)$$

115 Where RAO_i is the response amplitude operator for motion i , that is a function of frequency ω . ξ_i
 116 is the amplitude of the i^{th} DOF, and A is the amplitude of the incoming periodic waves.

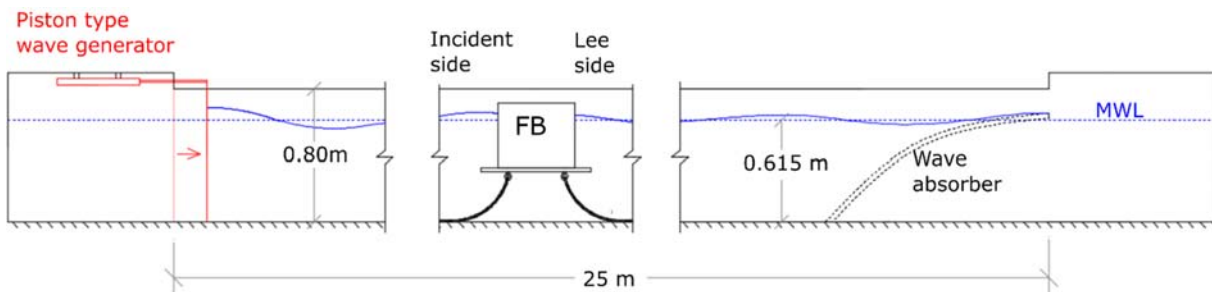
117 In the experimental and numerical analyses we only analysed the sway, heave and roll,
 118 corresponding to $i = 2,3,4$. To estimate the response amplitude and the reflection and transmission
 119 of waves, the wave elevation at several positions and the motion of the FB were evaluated. The
 120 following sub-section describes the wave flume and experimental set-up.

121

122 2.1 The wave flume and experimental set-up

123 The tests were carried out in a wave flume in the hydraulic laboratory at the Technical
 124 University of Denmark. The flume is 28 m long, 0.6 m wide, and the sidewalls are 0.8 m high. The
 125 pontoon took up almost the entire width of the wave flume to reduce side effects in the two-
 126 dimensional experiments.. The distance from the wave piston paddle to the end of the wave
 127 absorber was 25 m. The flume was used with waves alone even though it also had the ability to
 128 include currents. The sidewalls of the flume consisted of a long range of glass that made it possible
 129 to follow the motions of the FB. The flume was equipped with a piston-type wave maker at one end
 130 and a wave absorber at the other end. The water depth in the flume was 0.615 m.

131



132

133 *Figure 2 Sketch of the wave flume. The sketch is divided into sections to keep proportions.*

134

135 2.1.1 Wave measurements and analyses

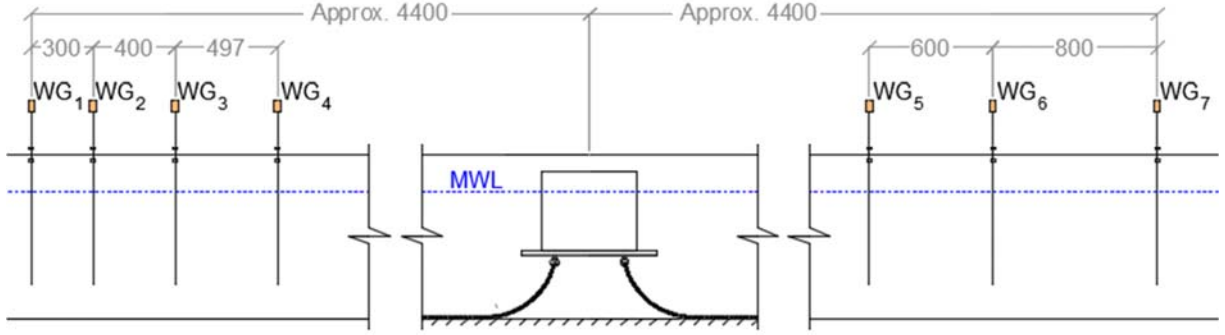
136 One of the major objectives of the experimental set-up was to distinguish between the
 137 incident, H_I , reflected, H_R , and transmitted, H_T , wave heights. From the measured wave heights, the
 138 reflection and transmission coefficients were defined as:

$$139 \quad C_R = H_R / H_I \quad (2.2)$$

$$140 \quad C_T = H_T / H_I \quad (2.3)$$

141 Seven wave gauges measured the surface elevation in the wave flume during the tests. We
 142 used conventional resistance-type wave gauges in the measurements with a sampling frequency of
 143 120 Hz. The surface elevation on the incident side of the FB was measured with 4 wave gauges
 144 while 3 wave gauges covered the lee side surface elevation, see Figure 3.

145
 146



147
 148

149 *Figure 3 Positions of wave gauges in unit [mm].*

150

151 The method used in this study to determined incoming and reflected waves was the one
 152 presented in (Jacobsen et al., 2012). To separate the incoming and reflected waves an over-
 153 determined set of equations was formed. Equations (2.4), (2.5), and (2.6) refer to a system with two
 154 wave gauges:

$$155 \quad \eta_{i,1} = a_0 + \sum_{j=1}^N a_j^I \cos(j(\omega t_i - kx_1)) + b_j^I \sin(j(\omega t_i - kx_1)) + \quad (2.4)$$

$$\sum_{j=1}^N a_j^R \cos(j(\omega t_i - kx_1)) + b_j^R \sin(j(\omega t_i - kx_1))$$

$$156 \quad \eta_{i,2} = a_0 + \sum_{j=1}^N a_j^I \cos(j(\omega t_i - kx_2)) + b_j^I \sin(j(\omega t_i - kx_2)) + \quad (2.5)$$

$$\sum_{j=1}^N a_j^R \cos(j(\omega t_i - kx_2)) + b_j^R \sin(j(\omega t_i - kx_2))$$

157 for

$$158 \quad t_i = t_0, t_0 + \Delta t, \dots, t_0 + M \Delta t \quad (2.6)$$

159 $\eta_{i,1}$ is the surface elevation at wave gauge 1, at time step i, a_j^I and b_j^I are amplitudes in the
 160 incoming wave and a_j^R and b_j^R in the reflected wave. x_1 and x_2 are the positions of wave gauge 1
 161 and wave gauge 2. The index j is the number harmonic out of N harmonics. k is the wave number

162 and reflects the assumption that all harmonics are bound. The overdetermined system of equations
163 (2.4) and (2.5) is solved numerically giving the incoming and reflected amplitudes from which
164 reflection coefficients etc. can be determined.

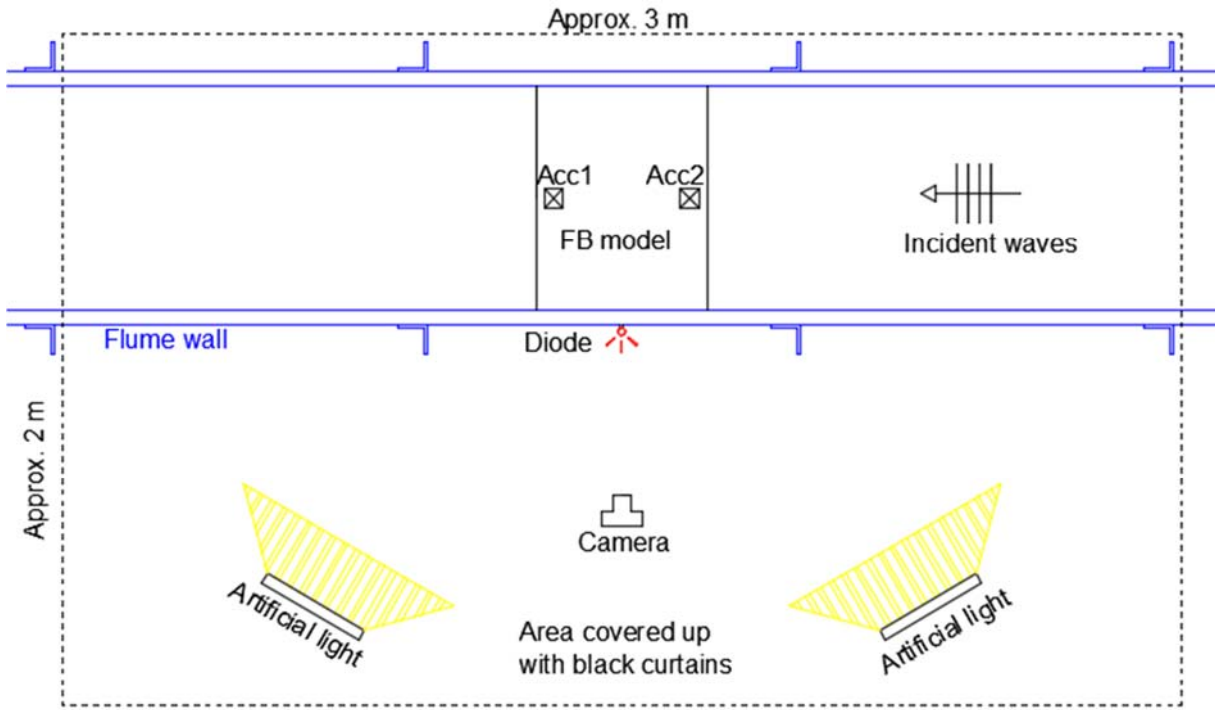
165 The number of harmonics, N , was three in our analyses. However, the difference between
166 using one harmonic instead of three was typically less than a percent on the result on for instance
167 the reflection coefficient, and the difference between using two and three harmonics even smaller.
168 Furthermore, to improve the accuracy the horizontal spacing between each wave gauge was not the
169 same. This prevents e.g. the nodes of a standing wave to coincide with the positions of more than
170 one of the wave gauges, which could lead to an ill-conditioned system of equations resulting in poor
171 accuracy.

172 The wave gauges were calibrated before conducting the experiments by a traditional
173 procedure where a relationship between the measured voltage and the water surface position was
174 established, see (Friis and Larsen, 2015). The data acquisition of the wave gauges began in calm
175 water before the wave maker was started and continued to measure up to approx. 60 s after the
176 wave maker was stopped. The early beginning and late closure were included in order to measure
177 the increasing/decreasing wave activity for the benefit of the Fast Fourier Transformation of the
178 accelerometer signals, which were stored in the same data file as the wave gauge measurements, cf.
179 section 2.1.3 for a description of the accelerometers.

180

181 2.1.2 Particle tracking

182 The motions of the FB model were determined by combining video recordings with the
183 software Particle Tracking, (Pastor, 2007). The camera used was a 50 Hz High Definition video
184 camera, and it delivered pictures with dimensions of 1920 x 1080 pixels. A sketch of the set-up with
185 camera and light sources is given in Figure 4.

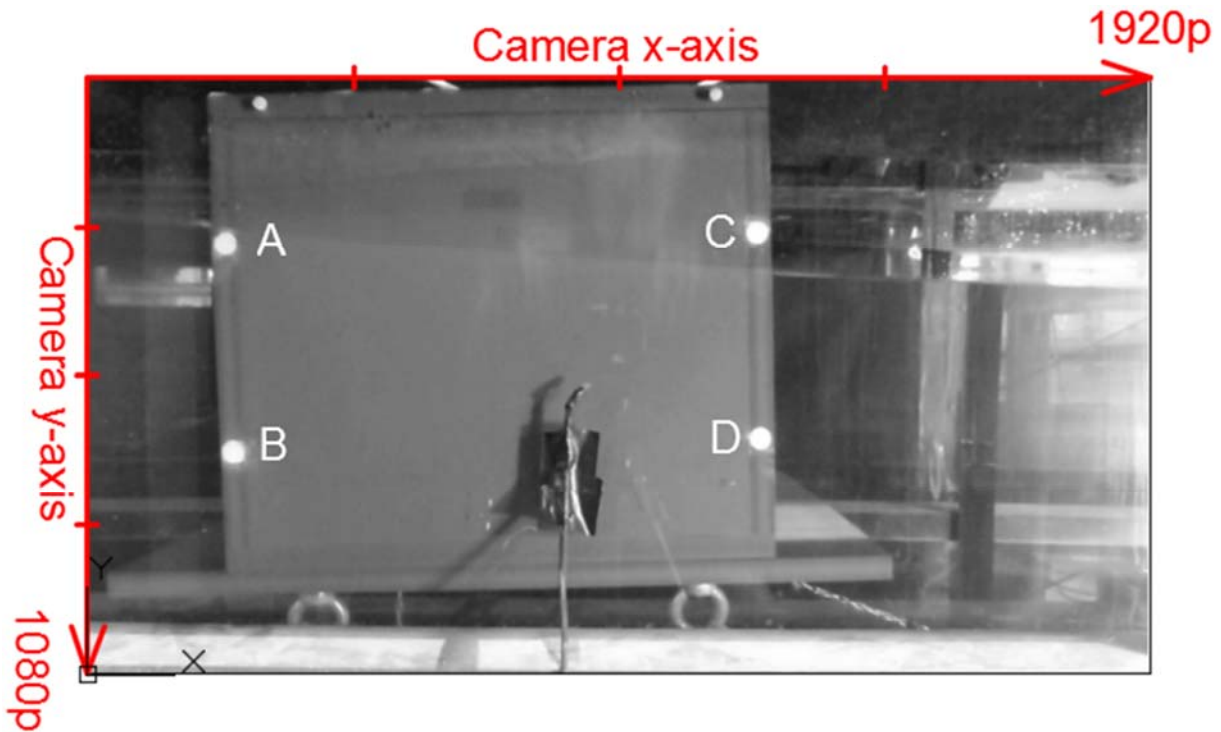


186
187

188 *Figure 4 Schematic top-view of the flume with black curtain set-up.*

189

190 The FB models were provided with 4 circular white spots, which were carefully positioned
191 with known individual distances making it possible to measure the distances on the pictures and
192 transform the tracked motions from pixels into meters, see Figure 5.



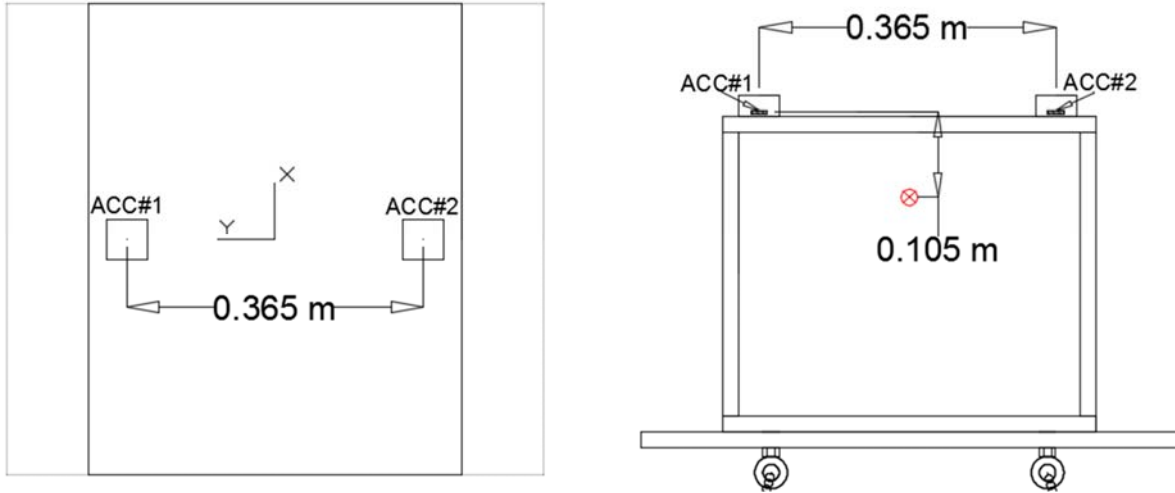
193
194 *Figure 5 Positions of 4 white points on the side of the floating breakwater (FB) model: A, B, C, and*
195 *D.*
196

197 In order to track the motions of the 4 white spots, they had to be easy to identify to avoid
198 confusion with other white objects in the picture like for instance reflections from blank metal
199 surfaces or sunlight. This was solved by covering up the flume with black curtains and using
200 artificial lights, cf. Figure 4.

202 2.1.3 Measuring motions with accelerometers

203 As a supplement to the Particle Tracking method described in section 2.1.2, the movements of
204 the FB models were evaluated by accelerometers as well. The reason for this additional
205 measurement was to verify the accuracy of the Particle Tracking method and to examine a suitable
206 alternative, which does not require as much processing of the output. Two 3-axis accelerometers
207 were used to measure the two translational motions and the roll rotation, see Figure 6. The
208 accelerometers were connected to a power supply and the output was a signal in conditioned
209 voltage stored in the software program DAQ (Data Acquisition). The accelerometers measured the
210 accelerations with full scale range of ± 3 g.

211 Both accelerometers were rigidly mounted on the lid of the FB model, with a horizontal
212 mutual distance of 0.365 m. The vertical distance from the MWL to the lid of the FB model was
213 0.085 m and 0.105 m without and with wing plate, respectively.

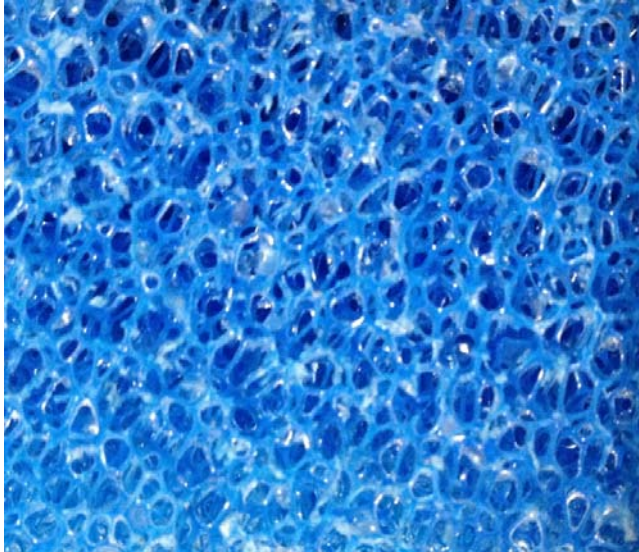


214
215 *Figure 6 Sketch of the set-up of the accelerometers for the FB model with wings. The vertical*
216 *distance from the MWL to the lid is indicated on the right panel.*
217

218 2.2 Cross-section of the FB

219 The physical model tests targeted a two-dimensional situation where cross-sections were
220 tested and analysed in the wave flume. The FB model had to fit into the wave flume and therefore
221 the regular pontoon (RG) cross-section had the dimensions 0.46 m wide, 0.39 m high and 0.58 m
222 long. The height of the other two cross-sections with a wing attached was 0.41 m, and the total
223 width was 0.66 m. The FB was ballasted to give a draft of 0.31 m that was kept constant for all
224 three cross-sections. With a total water depth of 0.615 m in the wave flume, the clearance was 0.305
225 m between the breakwater and “seabed”. Figure 7 illustrates the cross-section of the regular
226 pontoon (RG) with catenary anchor chains attached to the bottom of the pontoon.

227
228
229



244
245 *Figure 9 The porous material*
246

247 2.2.1 Mooring system

248 The design of the mooring system has not been one of the main objectives for this study, as
249 the focus was on the behaviour of the FB and the effect on the reflected and transmitted waves for
250 various damping mechanisms.

251 Four mooring lines kept the FB in place in the wave flume. The mooring system was designed
252 to be quite soft. The mooring system will therefore only have a small influence on the dynamics in
253 heave and roll, but still be able to fix the average position of the FB in the sway direction. In this
254 way, it mainly counteracted the mean drift forces while it appeared almost freely floating towards
255 wave forces from periodic waves. The mean drift forces were not measured, but the numerical
256 model gave an indication of their size, see for instance Figure 20. The mooring system consisted of
257 four mooring lines, two on each side of the cross-section. The submerged weight of the mooring
258 line was $w = 0.589 N / m$. The pretension in the mooring lines for calm water was $T_{pre} = 0.05 N$. The
259 total line length was, $l = 2.43 m$. The total horizontal extent was $X = 2.29 m$ and the active part was
260 $x = 0.17 m$. Figure 7 illustrates the cross-section of the mooring system for the (RG) cross-section.
261 The mooring system was set-up in the same way for all three cross-sections.

262 263 2.2.2 Test scenarios

264 It is desirable to keep the nonlinearity of the waves generated in the flume at a uniform level over
265 the entire range of frequencies. In deep water, the appropriate measure of nonlinearity is the wave

266 steepness, $S = H / L$, which has a maximum value of $S_{\max} = 14.2\%$. However, most of the test
 267 scenarios were in the intermediate range from deep to shallow water. This is also a realistic range
 268 for the application of FBs. To define a convenient uniformly valid measure for the nonlinearity, we
 269 related the wave height to the maximum wave height of progressive stable waves given by the
 270 following expression by (Fenton, 1990), which is a fit to the results of (Williams, 1981):

$$271 \quad \frac{H_{\max}}{h} = \frac{0.141063 \frac{L}{h} + 0.0095721 \left(\frac{L}{h}\right)^2 + 0.0077829 \left(\frac{L}{h}\right)^3}{1 + 0.0788340 \frac{L}{h} + 0.0317567 \left(\frac{L}{h}\right)^2 + 0.0093407 \left(\frac{L}{h}\right)^3} \quad (2.7)$$

272 This limit was then converted to an equivalent deep water steepness, S_{equiv} , defined by:

$$273 \quad S_{\text{equiv}} = 0.142 \frac{H}{H_{\max}} \quad (2.8)$$

274 where H is the actual wave height used in the tests. The equivalent deep-water steepness S_{equiv} , is
 275 then fixed at 2 %, 3 % and 4 %. As could be expected, the results showed a nearly linear increase in
 276 the response with increasing equivalent wave steepness over this range, and we therefore focus on
 277 the steepness of 2 % in the following.

278 The test scenarios were based on a first estimate of the design parameters relevant for a depth
 279 of 40 m, a draft of 20 m, a peak period of 7 s, and a significant wave height of 2.5 m. This can be
 280 considered as moderate waves, but most cargo handling situations need protection from conditions
 281 like this. The data were scaled by 1:65 assuming Froude scaling to be valid. Froude scaling was
 282 considered valid as long as viscous effects are negligible. This assumption is expected to hold for
 283 diffraction forces. However, viscous effects will have an effect in the analyses. The viscous effects
 284 from the wings will be scaled correctly as the eddies will detach at approximately the same points in
 285 both model- and full-scale. Other viscous effects, such as skin friction and friction in the porous
 286 media cannot be assumed to be modelled correctly using Froude scaling. As the focus was on the
 287 interaction with waves and transmission of waves we found the Froude scaling as the preferred
 288 scaling law. Table 1 shows the conditions tested in the experiments.

289
 290

291 *Table 1 Wave parameters for physical model tests in model scale.*

No.	Wave Length	Wave Period	Frequency	Depth ratio	Wave height
	$L[m]$	$T [s]$	$f [Hz]$	$h/L []$	$H_{(2\%)}$
1	1.174	0.868	1.152	0.524	0.023
2	1.519	0.992	1.008	0.405	0.030
3	1.883	1.116	0.896	0.327	0.036
4	2.252	1.240	0.806	0.273	0.041
5	2.618	1.364	0.733	0.235	0.045
6	2.979	1.488	0.672	0.207	0.049
7	3.333	1.613	0.620	0.185	0.052
8	3.681	1.737	0.576	0.167	0.054
9	4.024	1.861	0.538	0.153	0.055

292

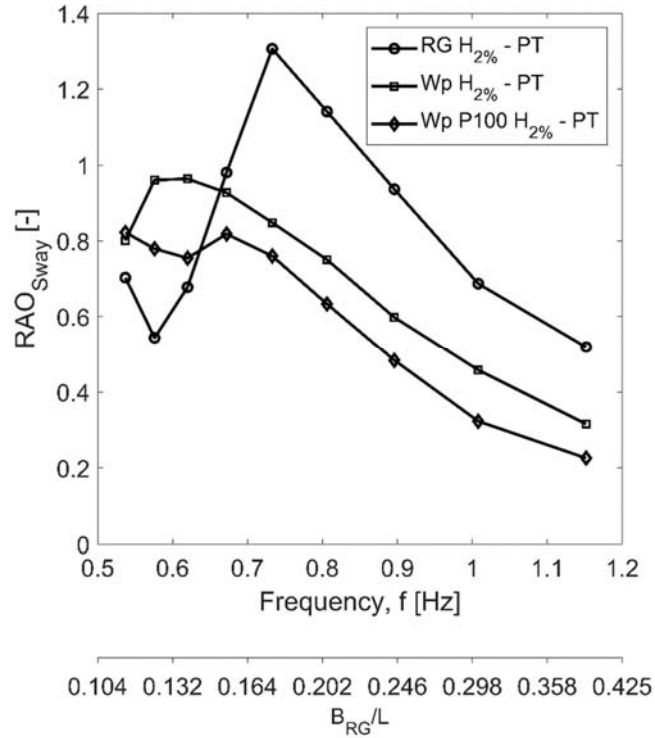
293 3 Experimental results

294 3.1.1 Response amplitude operators (RAO) from experiments

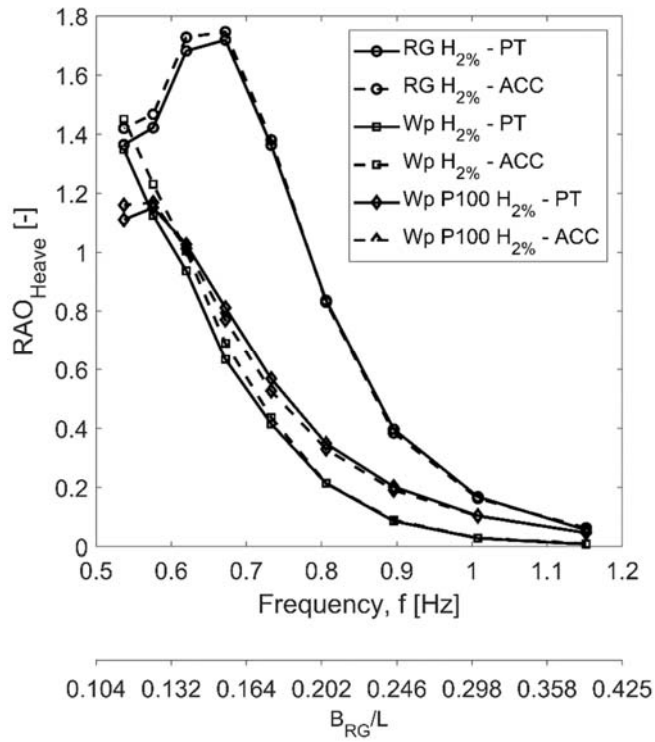
295 This section presents response amplitude operators derived from the experimental tests for the
 296 conditions outlined in Table 1. Two methods, particle tracking (PT) and accelerometers (ACC),
 297 tracked the motions of the FB. The sections 2.1.2 and 2.1.3 describe the methodology used. For the
 298 sway motion, only (PT) was used. The following figures have a primary axis that shows the
 299 frequency, and a secondary axis where B_{RG} / L (bream/wave length) is shown. Note that the
 300 secondary axis does not have an equidistant division.

301 Figure 10, Figure 11, and Figure 12 show that the motion was effectively reduced for the
 302 cross section with wings (WP) compared to the regular pontoon (RG). This was expected as the
 303 attached wings enhance both wave damping and frictional damping. The cross section with
 304 attached porous material (WP P100) showed less reduction of the motion in heave and roll
 305 compared to (WP). The smaller reduction of motion from the regular pontoon was in particular
 306 clear for the roll motion. The attached porous media reduced the effect of the wings on the roll
 307 motion as it apparently reduced the eddy generating mechanism because the surface of the wing
 308 plates was covered by the porous media. The reduction of the heave motion was comparable for the
 309 (WP) and (WP P100) cross sections, with slightly larger motion reduction for the cross section with

310 wings alone. The two techniques to measure the motion, particle tracking (PT) and accelerometers
 311 (ACC), showed a good agreement. The difference between the two methods was up to around 2-5%
 312 for the heave and roll motions.



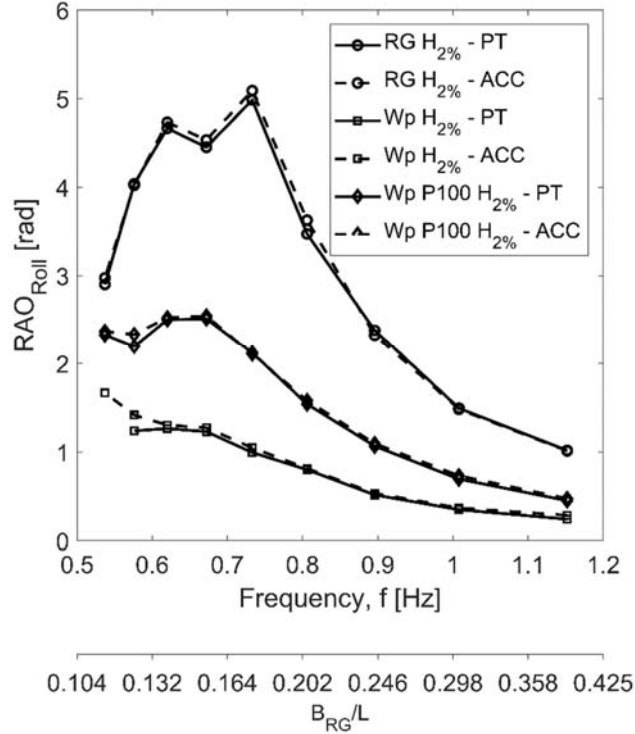
313
 314 *Figure 10 Response amplitude operator (RAO) for Sway derived from experiments for three*
 315 *different cross sections of the FB.*
 316



317

318 *Figure 11 Response Amplitude Operators (RAO) Heave derived from experiments for three*
 319 *different cross sections of the FB.*

320



321

322 *Figure 12 Response Amplitude Operators (RAO) Roll derived from experiments for three different*
 323 *cross sections of the FB.*

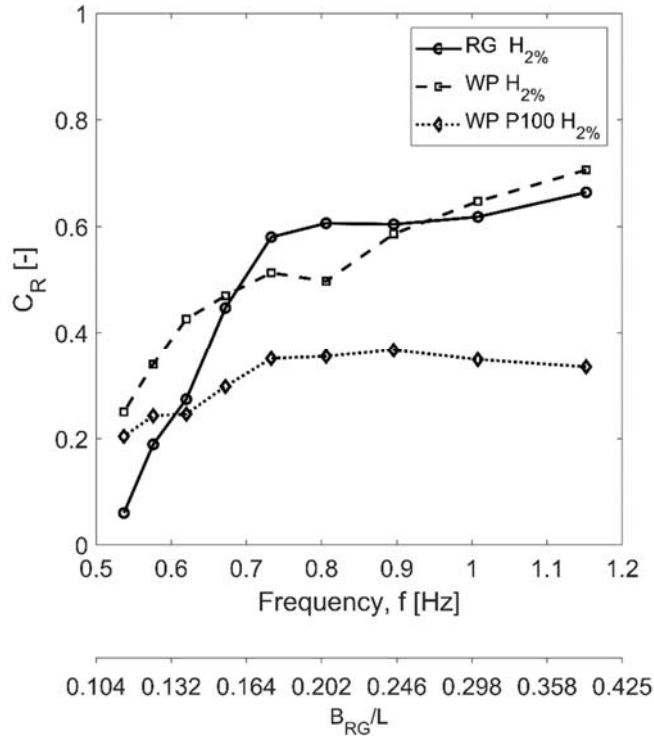
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325

326 3.1.2 Reflection and transmission of wave energy

327 The reflection and transmission of wave energy were analysed with standard wave gauges as
 328 outlined in section 2.1.1. Figure 13 shows the reflection coefficients, C_R , as a function of the wave
 329 frequency. The maximum reflection coefficient increases towards 0.7 in the area of the analysed
 330 frequencies for the regular pontoon (RG) and the pontoon with wings (WP), with an increasing
 331 tendency with increasing frequency. It could be argued that for very large frequencies the reflection
 332 would go towards 100% if there are no viscous effects, as the FB would appear as a vertical
 333 breakwater with a draft extending to deep water. Even though, including the wings indicated a
 334 substantial reduction of motion, this had only minor influence on the size of the reflection
 335 coefficient. Actually, the wings increased the reflection coefficient for smaller wave frequencies.
 336 When adding the porous media to the sides of the FB, the reflection decreased significantly as the
 337 porous media reduced the reflection coefficient from around 0.7 to around 0.3 for the largest wave

338 frequencies. The picture was not that clear for smaller frequencies as the regular pontoon (RG) gave
 339 the lowest reflection.



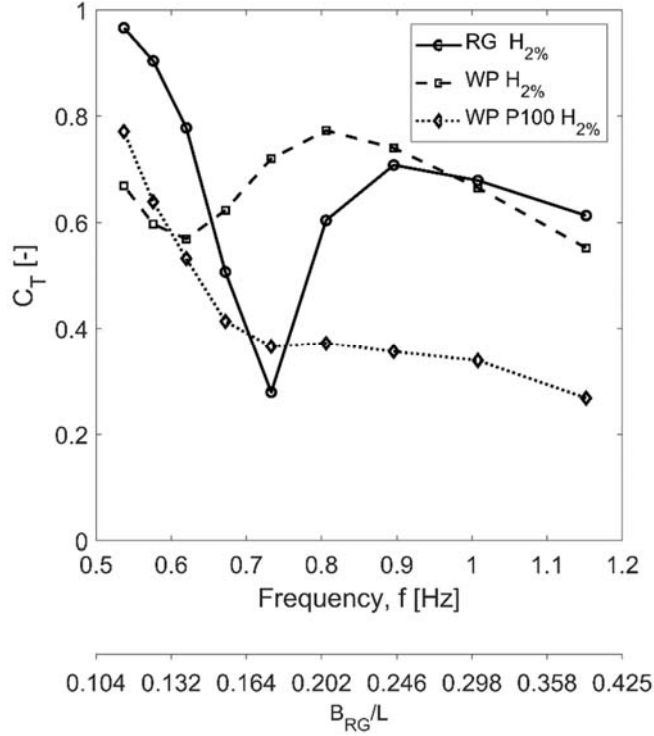
340

341 *Figure 13 Reflection coefficient, C_R , for three different cross sections of the FB.*

342

343 The transmission coefficient, C_T , was smaller for the cross section with porous media
 344 compared to the other two cross-sections for larger frequencies, cf. Figure 14. The transmission was
 345 similar for large frequencies for the two cross sections (RG) and (WP). The transmission of wave
 346 energy was largest for the regular pontoon (RG) for low frequencies. Attaching the porous media to
 347 the sides of the FB reduced the transmission coefficient in the order of 50 % for the largest
 348 frequencies. For frequencies larger than 0.9 Hz, a decreasing tendency in the transmission with
 349 increasing wave frequency was observed for all three cross sections. As wave frequencies increase,
 350 the draft of the FB appears deeper preventing transmission of wave energy. At the frequency 0.733
 351 Hz the (RG) cross section gave a relatively low transmission, which was at the same frequency that
 352 gave the maximum RAO for sway and roll.

353



354

355 *Figure 14* Transmission coefficient, C_T , for three different cross sections of the FB.

356

357 3.1.3 Wave energy balance

358 The total energy balance can be set-up based on 1st order wave theory and gives an indication
 359 of the amount of dissipated wave energy. After a transitional period, where a part of the wave
 360 energy is used to initiate the movement of the FB, the incoming wave energy will either be
 361 reflected, transmitted or dissipated into turbulence and eventually into heat. The reflected and
 362 transmitted wave energy were estimated based on analyses of the wave gauge measurements. The
 363 dissipated wave energy was not measured directly. Instead, the following relationship was
 364 anticipated to be valid:

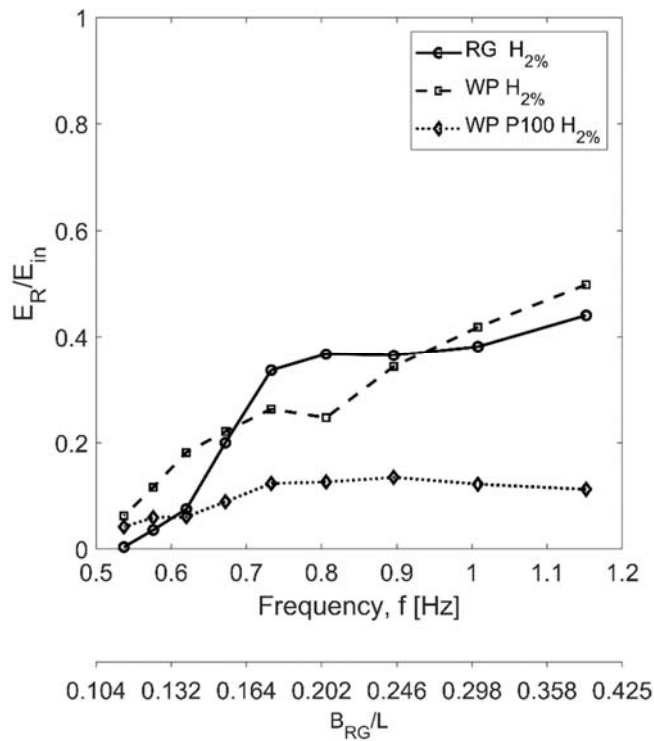
$$365 \quad E_{in} = E_R + E_T + E_{diss} \quad (3.1)$$

366 where E_{in} is the incoming wave energy, E_R and E_T are the reflected wave energy and transmitted
 367 energy, and finally E_{Diss} is the amount of energy that eventually is dissipated heat.

368 Analyses of reflected, transmitted, and dissipated energy are shown in Figure 15, Figure 16,
 369 and Figure 17. The transmission of wave energy had a minimum at a frequency of 0.733 Hz for the
 370 regular cross section (RG), cf. Figure 16, which was the frequency where the largest responses were

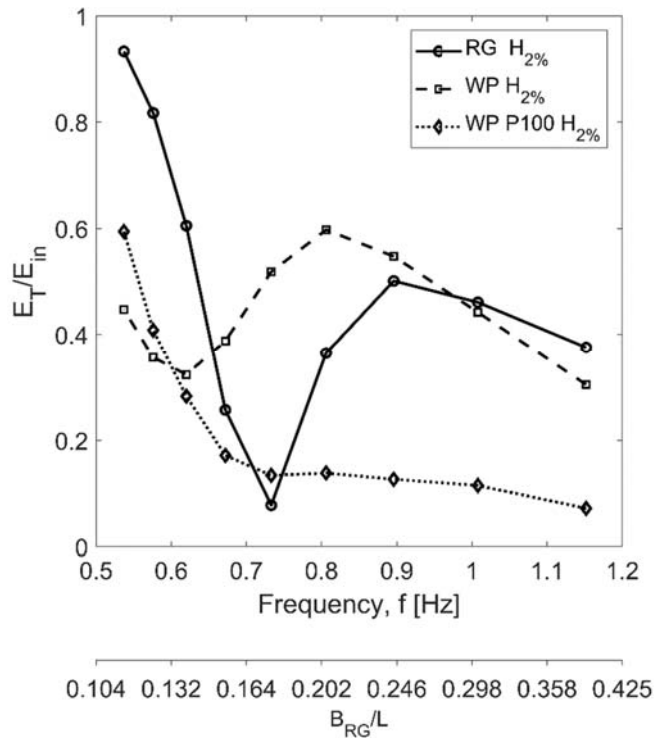
371 observed in the RAO for sway and roll. For larger frequencies, the amount of reflected energy was
 372 in the same order of magnitude for the regular cross section (RG) and the cross section with wings
 373 (WP). The FB will act as a relatively deep vertical solid wall for larger frequencies for the (RG) and
 374 (WP) cross sections, and thus, it was expected to reflect a major part of the wave energy for larger
 375 frequencies. In our results the width was typically smaller than 1/3 of the wave length, and the draft
 376 lower than 1/4 of the wave length, and therefore the structure was too small to reflect the major part
 377 of the energy, but the trend with increasing reflection as the frequencies increase (shorter wave
 378 lengths) can be seen in the figures. The cross section with the porous media reduced the amount of
 379 reflected wave energy. This trend became more pronounced with increasing wave frequency.

380 Similar tendencies were observed for the transmitted wave energy in Figure 16 where the RG
 381 and the WP cross sections reduced the transmitted energy. A minimum was observed for the RG
 382 cross section, which coincided with the maximum of the RAO for sway and roll. However, in
 383 general the cross section with porous media attached to the sides of the breakwater (WP P100)
 384 showed the smallest amount of wave transmission. The transmission of wave energy reduced to less
 385 than 20 % of the incoming wave energy for frequencies larger than 0.6 Hz.



386

387 *Figure 15 Relative reflection of energy, E_R / E_{in} , for three different cross sections of the FB.*



388

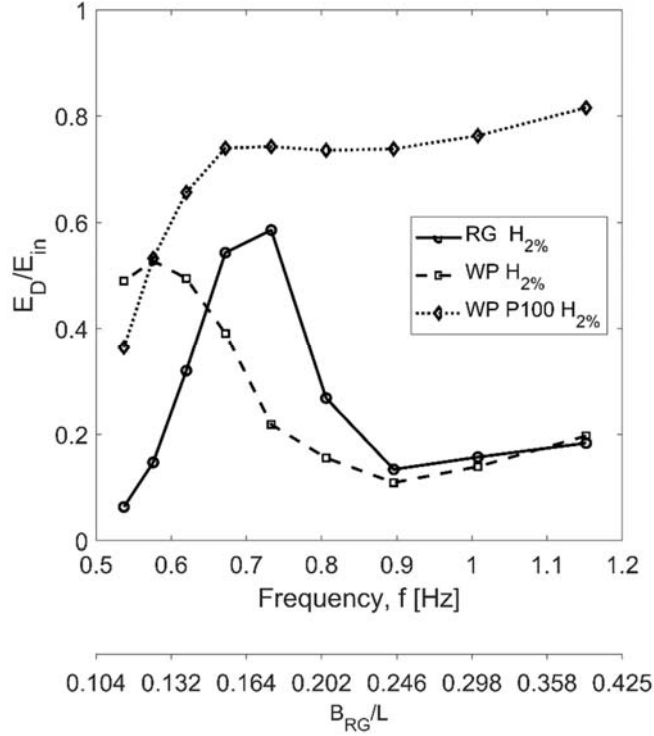
389 *Figure 16 Relative transmission of energy, E_T / E_{in} , for three different cross sections of the FB.*

390

391 Figure 17 shows the derived relative dissipation of wave energy. The regular cross section,
 392 RG, had its maximum dissipation at the same frequencies for maximum sway and roll motions. The
 393 transmitted wave energy was also low at these frequencies. The amount of dissipated energy
 394 reached an almost constant level when the porous media was attached to the sides of the FB. From a
 395 frequency of around 0.65 Hz, the level of energy dissipation was close to 75 %. For the larger
 396 frequencies, the increase in dissipated energy was a factor 4-5 times larger than the dissipation rates
 397 found for the two other cross sections, (RG) and (WP).

398

399



400

401 *Figure 17* Relative dissipation of energy, E_{Diss} / E_{in} , for three different cross sections of the FB.

402

403 3.2 Decay tests

404 Traditionally, the damping force is divided into three types; structural, material and fluid
 405 damping, see for instance (Sumer and Fredsøe, 2006). All three types of damping are present during
 406 the FB motions, however, the most prominent is the fluid damping, which is subdivided into linear
 407 wave damping and viscous damping. Structural damping accounts for external friction in e.g.
 408 structure joints while material damping covers the internal friction of a material at the molecular
 409 level.

410 The viscous damping effect is usually determined experimentally based on a decay test. In
 411 reality, the viscous damping is not linear, however, an approximate linear damping coefficient will
 412 be used in the numerical analyses in the next section.

413 The ratio δ between any two amplitudes following each other is constant. Therefore, we can
 414 use the logarithmic decrement to find the damping ratio, ζ_j . The logarithmic decrement is defined
 415 as:

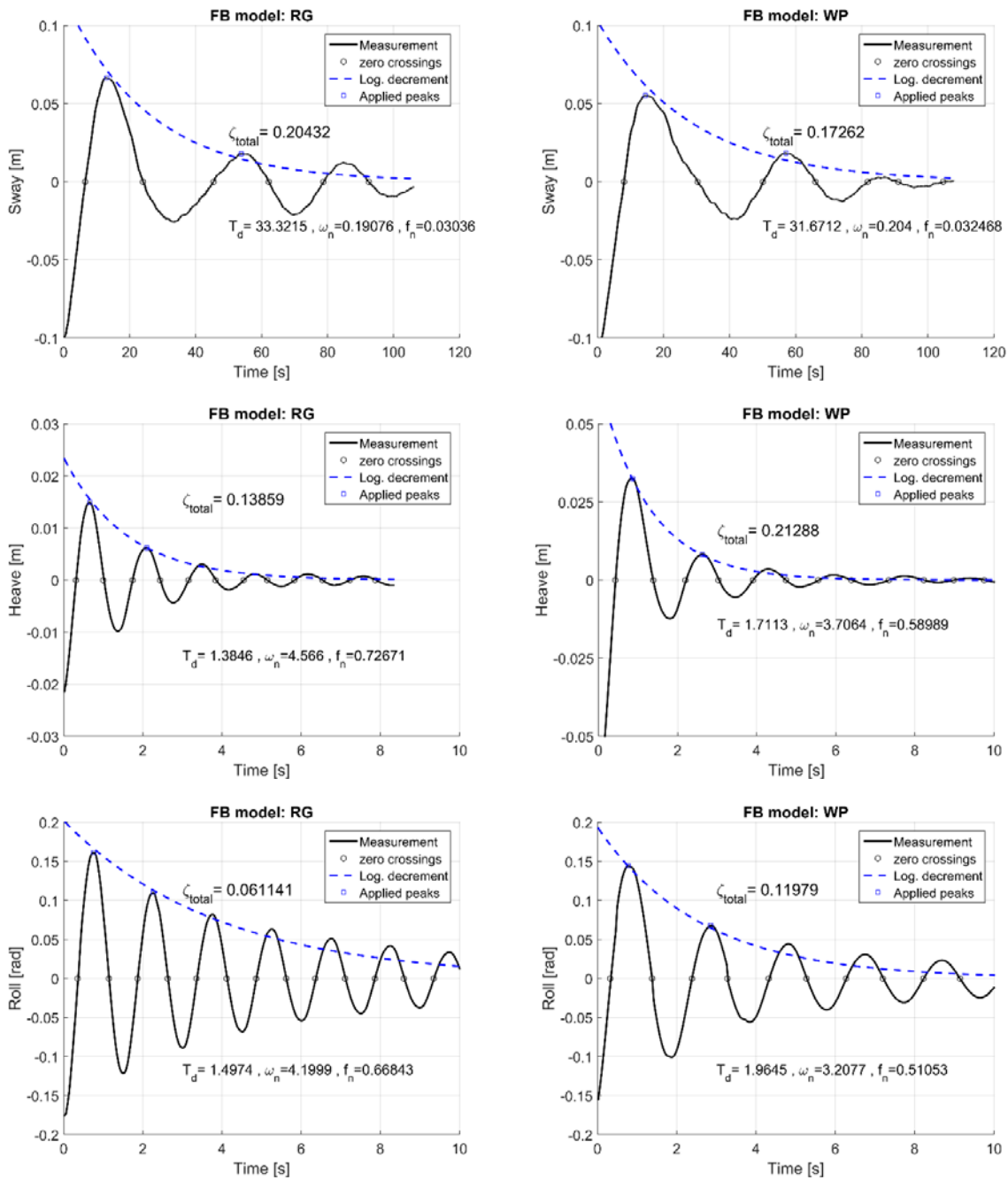
$$416 \quad \delta = \ln\left(\frac{x(t)}{x(t+T_d)}\right) = \ln(e^{-\zeta_j \omega_n T_d}) \quad (3.2)$$

417 where $x(t)$ and $x(t+T_d)$ are measured responses at time t and $t+T_d$. T_d is the period of damped
418 oscillations and ω_n is the natural angular frequency. While ζ_j is the ratio, which can be computed
419 from:

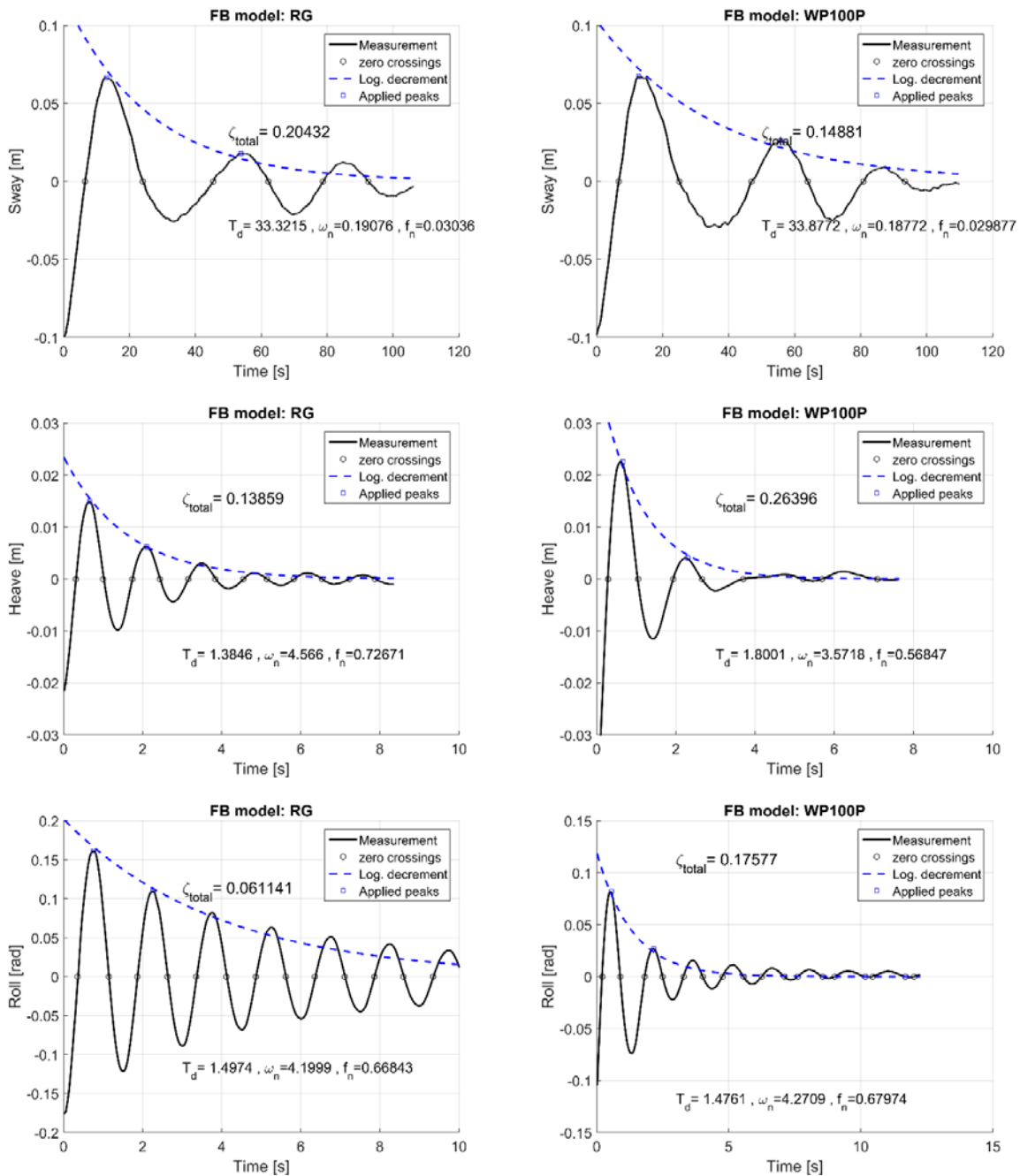
420
$$\zeta_j = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} \quad (3.3)$$

421 where ζ_j is proportional to the energy dissipated in one cycle of vibration.

422



423 Figure 18 Decay tests for regular and pontoon with wings for sway, roll and heave.
 424



425

426 *Figure 19 Decay tests for regular and pontoon with wings and porous sides for sway, roll and*
 427 *heave.*

428

429 Decay tests were carried out to support the numerical analyses. Damping is most important
 430 for frequencies close to the Eigenfrequencies of the system. Figure 18 and Figure 19 show how the
 431 damping ratio, ζ , has been estimated from decay tests. Each figure has the regular case (RG) to the

432 left, while the damping for wings (WP) and porous media (WP P100) is to the right. The sway
 433 damping appeared largest for the regular (RG) cross-section. For heave, the (RG) cross section had
 434 the smallest damping ratio while the other two cases had higher damping ratios. The case that
 435 included porous sides (WP P100) had the largest damping ratio in heave, which might be related to
 436 water filling and emptying the porous media. For the roll motion the (RG) cross section had the
 437 smallest damping ratio, while the other two cases appeared to be of same order.

438

439 4 Numerical modelling

440 The exciting force results in dynamical motion of the FB, which for a rigid body can be
 441 described in six degrees of freedom (DOF). However, in our case we analysed only the motion in
 442 three DOF in order to be able to make comparison with the experimental results. Under the
 443 assumption of linearity, and that the exciting force is sinusoidal, the frequency-domain equations of
 444 motion can be written, (see for instance (Newman, 1977)):

$$445 \sum_{j=1}^6 \left[-\omega^2 (M_{ij} + M_{ij}^E + A_{ij}) + i\omega (B_{ij} + B_{ij}^E) + (C_{ij} + C_{ij}^E) \right] \tilde{\xi}_j = \tilde{X}_i \quad (4.1)$$

446 where M_{ij} and M_{ij}^E are the inertia matrices, A_{ij} is the added mass matrix, B_{ij} and B_{ij}^E are damping
 447 matrices, C_{ij} and C_{ij}^E are the stiffness matrices, $\tilde{\xi}_j$ is the complex amplitude of the motion in 6
 448 DOF, and \tilde{X}_i is the complex exciting force. External contributions are shown with superscript E.

449 For instance, the anchor chain contributes to the stiffness through the term C_{ij}^E .

450 A complex exponential function describes the time-domain motion as follows:

$$451 \xi_j(t) = \text{Re}(\tilde{\xi}_j \cdot e^{i\omega t}) \quad (4.2)$$

452 where $\xi_j(t)$ is the periodic motion in 6 DOF, and ω is the angular frequency of the motion. In the
 453 same way, the excitation force can be described as:

$$454 F_i(t) = \text{Re}(\tilde{X}_i e^{i\omega t}) \quad (4.3)$$

455 where \tilde{X}_i is the complex amplitude of the exciting force.

456 Eqs. (4.1) does not account for non-linear behaviour of for instance damping or non-linear
 457 stiffness. However, to include the effect of the non-linear viscous damping, we will estimate an
 458 equivalent linear damping as described in section 4.1 based on decay tests in section 3.2. An
 459 approximation to the external stiffness matrix, C_{ij}^E , is given in section 4.2. The Response

460 Amplitude Operator (RAO) is the ratio of the amplitude of the FB motion to the amplitude of the
461 incoming wave:

$$462 \quad \bar{\xi}_i = \left| \frac{\tilde{\xi}_i}{A} \right|, i = 1 \sim 6 \quad (4.4)$$

463 An estimate of the single degree of freedom resonant or natural frequency of the FB motion
464 can be found from the following expression that has been used successfully in earlier studies:

$$465 \quad \omega_{ni} = \left(\frac{C_{ii} + C_{ii}^E}{M_{ii} + A_{ii}(\omega = \omega_{ni})} \right)^{\frac{1}{2}} \quad (4.5)$$

466 The index, i , refers to the i^{th} DOF. This study examined the $i = 2, 3$ and 4 modes.

467 The solution to the diffraction problem and the equation of motions up to 1st order can be
468 found with the panel method (WAMIT Inc., 2015). WAMIT is a radiation/diffraction panel program
469 developed for linear analyses of the interaction of surface waves with offshore structures. The
470 method is based on potential flow wave theory. The panel method, also referred to as the boundary
471 integral equation method (BIEM), is based on Green's theorem where the velocity potential at any
472 point in the fluid is represented by surface distributions of singularities over the fluid boundary
473 surfaces (Lee and Newman, 2003). The method is used to solve the diffraction and radiation
474 problem describing the interaction between the waves and the floating body.

475 The numerical modelling was carried out with WAMIT version 7. In this study we used the
476 higher-order method, where Green's velocity potential integrals over the body surface and
477 computational domains are carried out using Gauss quadrature in parametric space, (Lee and
478 Newman, 2003).

479 The intension of the study was to find the effect of a cross-section of the FB. This was
480 achieved with the sidewalls in the experiments, while this was not possible in the numerical
481 analyses. Instead, the length of the breakwater was set sufficiently large to minimise the 3D effects
482 around the ends of the breakwater. In the x-direction (surge, the length was set to 70 m, while the
483 dimensions in y- (sway) and z-direction (heave) were 0.31 m and 0.46 m, respectively. The
484 dimensions in the y and z directions varied slightly for the three different cross sections as sketched
485 in Figure 8. As the porous media cannot be modelled, a rigid geometry was used that had a total
486 width of 0.66 m as an approximation for cross-section (WP P100).

487 The laboratory data was scaled according to the difference between the length in the
488 experiments (0.58 m) and the length in the numerical analyses (70 m). For instance the mass was
489 scaled with a factor $70/0.58=120.7$, external stiffness coefficients and external damping coefficients

490 was also scaled with this factor. No attempt was made to account for three-dimensional effects
 491 induced by the ends of the structure.

492

493 4.1 Assessment of external damping

494 In the numerical set-up it was possible to include a linear external damping coefficient as a
 495 supplement to the wave damping. In general, the viscous damping is a nonlinear process, and
 496 cannot be fully represented in the linear numerical analysis. However, an attempt was made to find
 497 a realistic representation of the viscous damping from the decay tests that were presented in section
 498 3.

499 The actual damping coefficient is written as:

$$500 \quad B_{ij} = \zeta_j B_{cr,ij} = \zeta_j 2\sqrt{(M_{ij} + A_{ij})C_{ij}} \quad (4.6)$$

501 where $B_{cr,ij}$ is the critical damping coefficient, known as the smallest value of damping rate that
 502 yields non-oscillatory motion:

$$503 \quad B_{cr,ij} = 2\sqrt{(M_{ij} + A_{ij})C_{ij}} \quad (4.7)$$

504 The natural frequency, f_n , is calculated once the period T_d is identified from a decay test. T_d
 505 being the period of a damped free oscillation related to the angular damped frequency as $\omega_d = \frac{2\pi}{T_d}$.

506 While the connection to the angular frequency is defined as:

$$507 \quad \omega_d = \omega_n \sqrt{1 - \zeta^2} \Leftrightarrow \omega_n = \frac{\omega_d}{\sqrt{1 - \zeta^2}} \quad (4.8)$$

508 and the frequency in [Hz] is found as:

$$509 \quad f_n = \frac{\omega_n}{2\pi} \quad (4.9)$$

510 The natural frequency for each DOF is an important parameter since the FB is affected by
 511 periodic external wave forces. Hence, it is important to examine possible coincidence of wave
 512 frequencies and FB natural frequencies due to the risk of resonance and thus amplification of the
 513 oscillating amplitudes.

514 The damping ratio originating from viscous damping can be difficult to compare directly
 515 between cross sections, as the restoring coefficients also changed between the cross sections, as a
 516 consequence of the volume of porous media, and due to different non-linear processes. The
 517 numerical model finds the wave damping as a part of the solution. Therefore, we only had to add

518 the extra damping originating from viscous processes. As the experiments found the total damping
519 coefficient we subtracted the wave damping coefficient found by the calculations from the one
520 found though the experiments. Finally, the additional damping coefficients were adjusted to the
521 length of the FB in the numerical analyses that was 70 m compared to the 0.58 m in the
522 experiments. The equations used to find the damping coefficients are presented in eqs. (3.3), (4.6),
523 and (4.7). The tables, Table 2, Table 3 and Table 4, present the relevant damping parameters from
524 the decay tests and numerical modelling. The damping ratio, ζ_{total} , was found from decay tests,
525 ζ_{WAMIT} was the damping ratio estimated from the numerical modelling without introducing external
526 damping. The difference between ζ_{total} and ζ_{WAMIT} was assumed to be the viscous damping;
527 $\zeta_{viscous} = \zeta_{total} - \zeta_{WAMIT}$, and was added as external damping in the numerical analyses. The external
528 damping coefficient, $B_{viscous}$ was found for each degree of freedom, i.e. sway, heave and roll. Note
529 that mass, added mass, and restoring coefficients varied from cross section to cross section, which
530 means that the damping ratio scales differently depending on the cross section, (RG), (WP), or (WP
531 P100).
532

533 *Table 2 Sway decay test results.*

FB model	T_d [s]	ω_n [rad/s]	f_n [Hz]	ζ_{total} [-]	ζ_{WAMIT} [-]	$\zeta_{viscous}$ [-]	$B_{viscous}$ [kg/s]
RG	33.3	0.191	0.0300	0.204	0.0119	0.192	2393
WP	31.7	0.204	0.0325	0.173	0.0170	0.156	2725
WP P100	33.9	0.188	0.0299	0.149	0.0170	0.132	2234

534

535 *Table 3 Heave decay test results.*

FB model	T_d [s]	ω_n [rad/s]	f_n [Hz]	ζ_{total} [-]	ζ_{WAMIT} [-]	$\zeta_{viscous}$ [-]	$B_{viscous}$ [kg/s]
RG	1.38	4.57	0.727	0.138	0.113	0.025	$3.7 \cdot 10^3$
WP	1.71	3.71	0.590	0.213	0.0872	0.126	22850
WP P100	1.71	3.77	0.600	0.264	0.0872	0.177	32443

536

537 *Table 4 Roll decay test results.*

FB model	T_d [s]	ω_n [rad/s]	f_n [Hz]	ζ_{total} [-]	ζ_{WAMIT} [-]	$\zeta_{viscous}$ [-]	$B_{viscous}$ [kg/s · m ²]
RG	1.50	4.20	0.668	0.061	0.0234	0.037	193
WP	1.96	3.21	0.511	0.120	0.0074	0.113	855
WP P100	1.48	4.27	0.680	0.176	0.0074	0.169	1340

538

539 4.2 A mooring system modelled as an external stiffness matrix

540 The external restoring matrix, C_{ij}^E , was formulated with a general full three dimensional
 541 description. For each mooring component the mooring force at an attachment point and especially
 542 the gradient matrix were described as:

543

$$\vec{T} = (T_{x_1}, T_{x_2}, T_{x_3}) \quad (4.10)$$

544

$$T_{kl} = \begin{bmatrix} \frac{\partial T_{x_1}}{\partial x_1} & \frac{\partial T_{x_1}}{\partial x_2} & \frac{\partial T_{x_1}}{\partial x_3} \\ \frac{\partial T_{x_2}}{\partial x_1} & \frac{\partial T_{x_2}}{\partial x_2} & \frac{\partial T_{x_2}}{\partial x_3} \\ \frac{\partial T_{x_3}}{\partial x_1} & \frac{\partial T_{x_3}}{\partial x_2} & \frac{\partial T_{x_3}}{\partial x_3} \end{bmatrix} \quad (4.11)$$

545 Where T_{x1} is the force in the x -direction (surge), T_{x2} is the force in the y -direction (sway), and T_{x3} is
546 the force in the z -direction (heave). T_{kl} is the Jacobian matrix of the vector \vec{T} , k and l are 1,2,3
547 corresponding to the three spatial coordinates. In our study the forces in the x -direction could be
548 assumed to be zero, however, for the sake of completeness the derivation of the external stiffness
549 matrix included all three dimensions. The derivatives in the Jacobian matrix, T_{kl} , eqs. (4.11), were
550 estimated by eqs. (4.21)-(4.24) that are presented later.

551 The restoring forces and moments, ΔF_k^n and ΔM_k^n , from attachment point number n can under
552 the assumption of small motions be found as (all higher order terms are neglected):

$$553 \quad \begin{bmatrix} \Delta F_k^n \\ \Delta M_k^n \end{bmatrix} = \begin{bmatrix} T_{kl}^n \cdot u_l^n \\ R_{km}^n T_{ml}^n \cdot u_l^n \end{bmatrix} \quad (4.12)$$

554 Here u_l^n is the displacement of attachment point n from a mean position. With reference to a
555 coordinate system following the FB, the distance vector from origo to the attachment point was r_j .
556 Note that only 1st order terms were included in the derivation. The restoring moment, ΔM_k^n , can be
557 found as the cross-product between the distance vector, r_j , and the restoring force, ΔF_k^n , at the
558 attachment point. However, it was found convenient to formulate the cross-product as a matrix
559 multiplication with R_{kl} . Based on the distance vector, r_j , the formulation of R_{kl} is:

$$560 \quad R_{kl} = \begin{bmatrix} 0 & -r_3 & r_2 \\ r_3 & 0 & -r_1 \\ -r_2 & r_1 & 0 \end{bmatrix} \quad (4.13)$$

561 The translation u_l of the attachment point in three dimensions can be related to the motion of
562 the FB, ξ_j , as, $u_l = A_{lj} \xi_j$. Based on geometrical considerations the matrix A_{lj} were formulated:

$$563 \quad A_{lj} = [E_{kl}, -R_{kl}] = \begin{bmatrix} 1 & 0 & 0 & 0 & r_3 & -r_2 \\ 0 & 1 & 0 & -r_3 & 0 & r_1 \\ 0 & 0 & 1 & r_2 & -r_1 & 0 \end{bmatrix} \quad (4.14)$$

564 Eqs. (4.12) was rewritten as:

$$565 \quad \begin{bmatrix} \Delta F_k^n \\ \Delta M_k^n \end{bmatrix} = \begin{bmatrix} T_{kl}^n A_{lj}^n \\ R_{km}^n T_{ml}^n A_{lj}^n \end{bmatrix} \xi_j \quad (4.15)$$

566 Based on eqs. (4.15) we identified the contribution to the external stiffness matrix from mooring
567 line number, n , as:

568
$$C_{ij}^{E,n} = \begin{bmatrix} T_{kl}^n A_{ij}^n \\ R_{kn}^n T_{ml}^n A_{ij}^n \end{bmatrix} \quad (4.16)$$

569 Where the subscript, n , refers to the attachment point number. This means that the matrices
 570 R_{kl} , T_{kl} , and A_{ij} have to be found for each mooring line. Note that $T_{kl}^n A_{ij}^n$ is a 3x6 matrix in upper
 571 part of the stiffness matrix, $C_{ij}^{E,n}$, just as $R_{kn}^n T_{ml}^n A_{ij}^n$ is a 3x6 matrix in the lower part. Finally, the total
 572 external contribution to the restoring matrix was found as the sum of all the contributions:

573
$$C_{ij}^E = \sum_{n=1}^{n_{\max}} C_{ij}^{E,n} \quad (4.17)$$

574 To find the forces and derivatives of the forces at an attachment point from the mooring lines,
 575 eqs. (4.10) and (4.11), we used the following expressions for the horizontal and vertical forces. The
 576 expressions can be found in the literature, for instance in (Faltinsen, 1990). The horizontal force, T_H
 577 , in the line is:

578
$$T_H = \frac{w \cdot h}{\cosh\left(\frac{X - l + \sqrt{h^2 + 2ah}}{a}\right) - 1} \quad (4.18)$$

579 The vertical force, T_V , is:

580
$$T_V = T_H \sinh\left(\frac{x}{a}\right) \quad (4.19)$$

581 In order to describe the derivatives of the horizontal and vertical forces we described the total
 582 horizontal length, X , (i.e. the projection of l on a horizontal seabed) as:

583
$$X = l - h \left(1 + 2 \frac{a}{h}\right)^{\frac{1}{2}} + a \cosh^{-1}\left(1 + \frac{h}{a}\right) \quad (4.20)$$

584 Where w is the submerged weight of the mooring line per unit length, h is the vertical
 585 distance from seabed to attachment point, l the total length of the mooring line, X the total
 586 horizontal length of mooring line (i.e. the projection of l on a horizontal seabed), and a is,
 587 $a = T_H / w$.

588 The derivatives can be found numerically or analytically. Analytical formulations were used
 589 in this study and the expressions were tested numerically for their validity.

590 The derivative of T_H with respect to X can also be found in (Faltinsen, 1990):

591
$$\frac{dT_H}{dX} = \frac{w}{\cosh^{-1}\left(\frac{h}{a}+1\right) - \frac{2}{\sqrt{\frac{2a}{h}+1}}} \quad (4.21)$$

592 In the design of mooring systems, the main concern is to keep the position of a floating body
 593 (here floating breakwater) near an average position. However, the structure is allowed to move from
 594 wave to wave, which means that the mooring system is targeting the mean drift forces that are
 595 typically significantly smaller than the wave forces induced by the individual waves. In this context,
 596 eq. (4.21) plays a vital role in analysing mooring systems. In our case this was not expected to be
 597 sufficient, and a complete set-up of the external stiffness matrix was needed as expressed in eqs.

598 (4.16) and (4.17). Therefore, the derivatives, $\frac{dT_H}{dz}$, $\frac{dT_V}{dX}$, and $\frac{dT_V}{dz}$ were also found. The derivative
 599 of T_H with respect to the vertical coordinate z can be found to:

600
$$\frac{dT_H}{dz} = \left(\frac{aw \left(al_s \left(\cosh\left(\frac{x}{a}\right) - 1 \right) - h \sinh\left(\frac{x}{a}\right) (a+h) \right)}{a^2 l_s \left(\cosh\left(\frac{x}{a}\right) - 1 \right)^2 - h \sinh\left(\frac{x}{a}\right) (h^2 + ah + l_s (X-l))} \right) \quad (4.22)$$

601 The derivative of T_V with respect to X can be found to:

602
$$\frac{dT_V}{dX} = \frac{\left[\sinh\left(\frac{x}{a}\right) - \frac{x}{a} \cosh\left(\frac{x}{a}\right) \right] \frac{dT_H}{dX} + w \cosh\left(\frac{x}{a}\right)}{1 - \cosh\left(\frac{x}{a}\right)} \quad (4.23)$$

603 The derivative of T_V with respect to z (or h) have been derived in a similar way as eqs. (4.21):

604
$$\frac{dT_V}{dz} = w \sqrt{\frac{2a}{h}+1} - \frac{T_H}{h \sqrt{\frac{2a}{h}+1}} + \frac{1}{\sqrt{\frac{2a}{h}+1}} \frac{dT_H}{dz} \quad (4.24)$$

605 The Jacobian matrix, T_{kl} , as defined in eqs. (4.11) was found from eqs. (4.21), (4.22), (4.23),
 606 and (4.24). The horizontal direction was aligned with the sway motion while the vertical was
 607 aligned with the heave motion.

608 The first step was to find the horizontal force in the mooring lines. The sum of the four
 609 horizontal forces has to balance the mean drift force. When waves approach the FB from left to
 610 right we have:

611
$$F_{meandrift} = 2 \cdot T_{H,Left} - 2 \cdot T_{H,Right} \quad (4.25)$$

612 An iterative procedure solved eqs. (4.25) applying eqs. (4.18). With known mean forces at the
613 attachment points the derivatives in eqs (4.11) can be found, and finally the external stiffness matrix
614 given by eqs. (4.17).

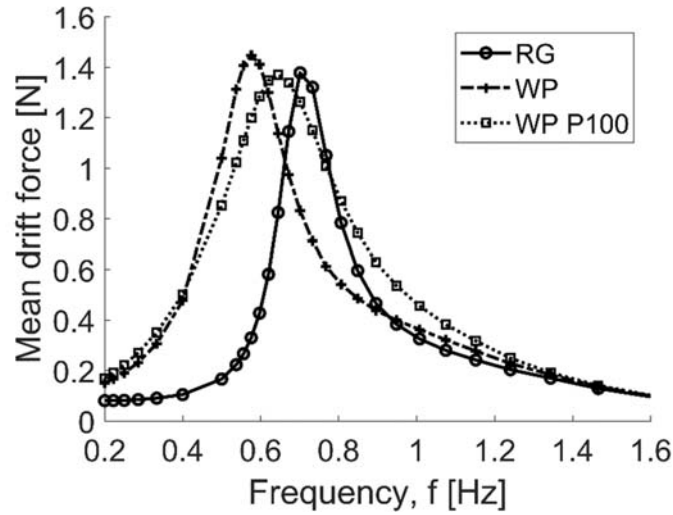
615

616 4.2.1 External Stiffness matrix for the three cross-sections

617 The estimation of the stiffness matrices followed the procedure presented above. The
618 submerged weight of the mooring line $w = 0.589 N / m$. The pretension in the mooring lines for
619 calm water was $T_{pre} = 0.05 N$, the total line length $l = 2.43 m$, and the active line length $l_s = 0.3 m$.
620 The total horizontal extent was $X = 2.29 m$ and the active part was $x = 0.17 m$. When a cross-section
621 interacted with periodic waves, it was exposed to a mean drift force. This changed the position of
622 the cross-section and, therefore, also the force in the mooring lines. Eqs. (4.25) estimated the
623 horizontal forces in the mooring lines for a range of frequencies to account for the mean drift force.
624 Figure 20 shows the mean drift forces that were estimated with the numerical model with a length
625 of 70 m. The length in the x-direction (surge) was significantly larger in the numerical analyses than
626 in the experiments in order to reduce the influence of 3D effects at the ends. Then the mean drift
627 force was multiplied with a factor $0.58/70 = 8.29 \cdot 10^{-3}$ for the three cross-sections to resemble the
628 laboratory length of cross-section of the FB. The wave height was set to the equivalent deep water
629 steepness of 2 %, as described in subsection 2.2.2.

630

631



632

633 *Figure 20 Calculated mean drift forces for cross-sections with a length of the FB corresponding to*
634 *the physical experiments.*

635

636 The only remaining quantities that were needed to find the external restoring coefficients were
637 the distance vectors from the local body coordinate system to the attachment points of the mooring
638 cables. The first vector that is located in one corner in the x-y-plane is:

639
$$r_j^1 = (0.23m, 0.17m, -0.385m) \quad (4.26)$$

640 The remaining vectors were found in a similar way, located at the other three corners in the x-y-
641 plane. The z-coordinate was constant in all four positions.

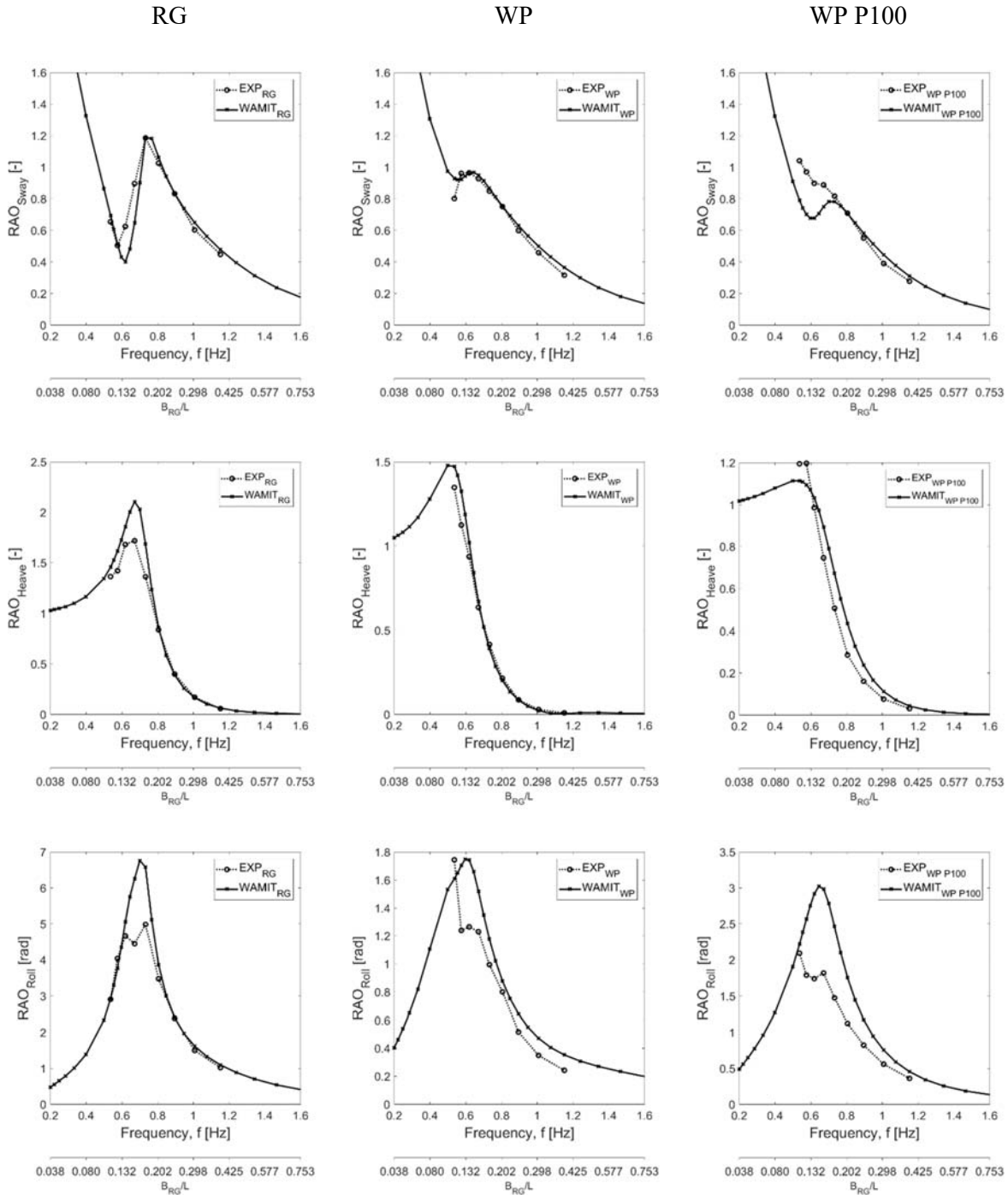
642

643 5 Comparison of experimental and numerical results

644 To examine the performance of the FB, numerical analyses were carried out and compared to
645 the experimental results. This will support the conclusions from the experiments and further show
646 how much of the non-linear processes can be captured with a linear radiation/diffraction analysis.
647 To improve the numerical analyses, rough estimates of the viscous damping in the porous media
648 and vortex generation around the edges of the FB, were included.

649 Based on the estimated damping ratios the final numerical setup was achieved. The numerical
650 model calculated the response amplitude operators for the three cross sections of the FB for the
651 three degrees of freedom sway, heave and roll. Figure 21 shows all 9 response amplitude operators
652 (RAO). For the regular FB (RG) we found a good consistency between the experiments and the

653 numerical results for sway and heave. The roll motion was smaller in the experiments compared to
654 the numerical results. Roll damping is often a non-linear process that cannot be fully accounted for
655 with linear damping. Therefore, this discrepancy could be expected. For the cross section with wing
656 plates (WP) a good consistency was also found for sway and heave, while the roll was less well
657 reproduced in the numerical analyses. This was again seen as a result of the non-linear damping
658 process that was considered even more important in this case due to the attached wing plates. In the
659 third cross section with porous sides (WP P100) a reasonable agreement was achieved for sway and
660 heave, although the differences between the numerical results and the experimental results were
661 larger compared to the other two cross sections; (RG) and (WP). Again, it was for the roll motion
662 that the differences were largest.
663



665 *Figure 21 Comparison between Response Amplitude operators from experimental results and*
 666 *WAMIT calculations.*
 667

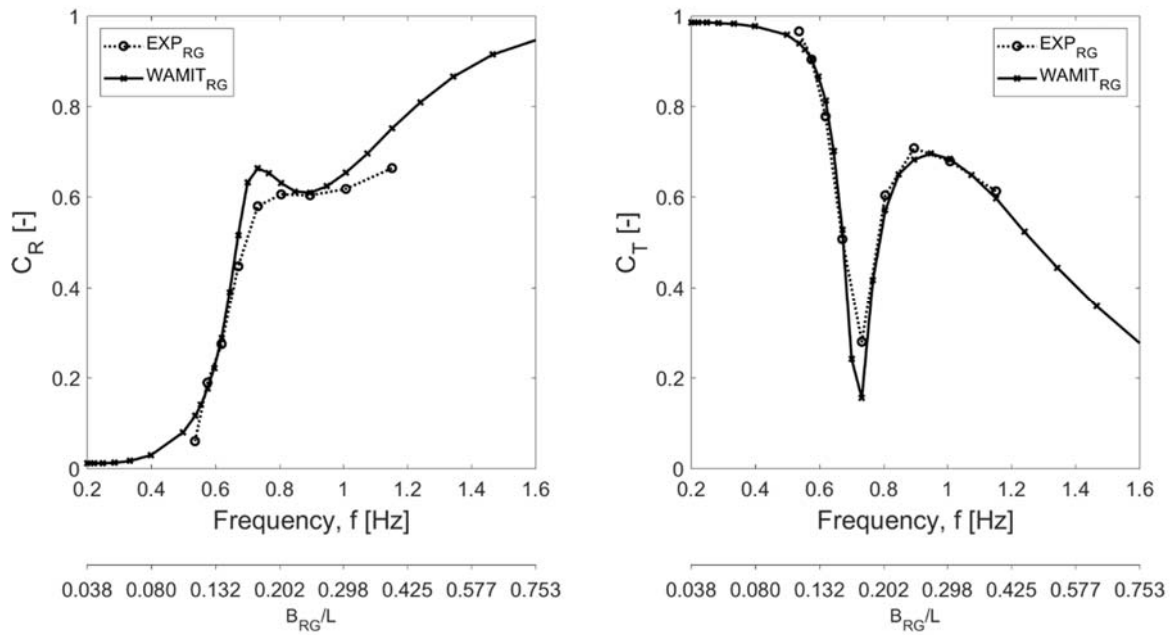
668 The process of filling and emptying of the porous media was clearly not well modelled by a
 669 simple equivalent linear damping. Besides the nonlinearity of the damping, this process can be

670 argued to also change the effective mass and inertia of the structure. However, given the simplicity
671 of the modelling, the results shown in Figure 21 are encouraging.

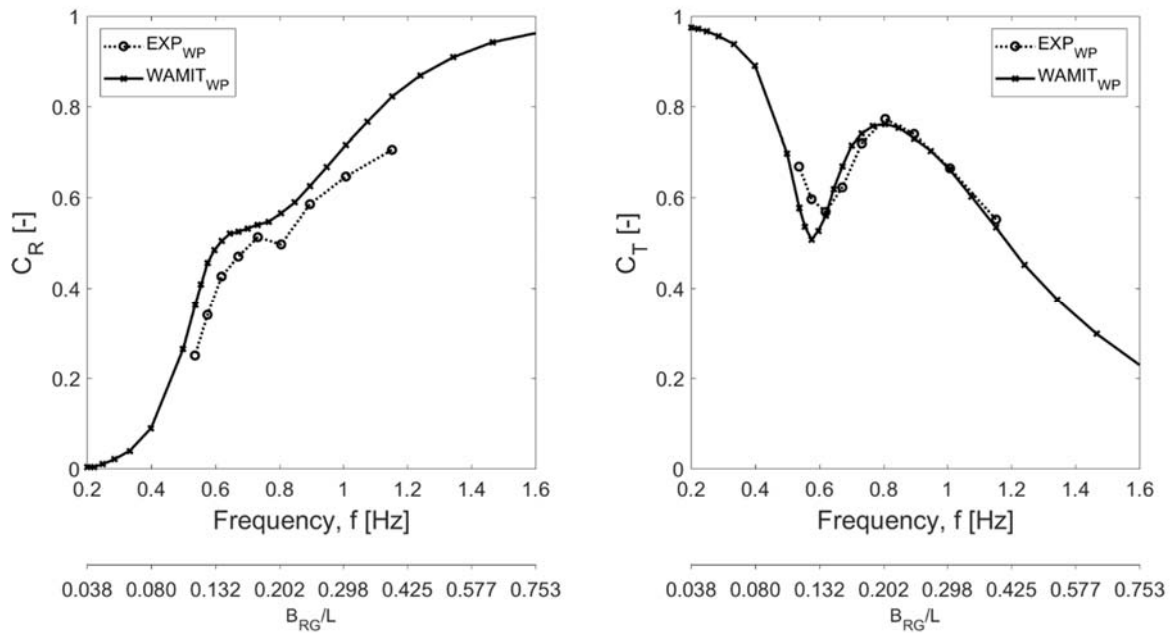
672 Wave characteristics were extracted in front of the FB and on the lee-side of the FB in the
673 middle of the FB (along the y-axis), over a distance of 10 m on each side. This gave consistent
674 estimates of the wave amplitudes. As the incoming wave amplitude was known the reflected
675 amplitude was derived from the amplitude on the front, while the amplitude on the lee side was
676 assumed to be equal to the transmitted wave. Based on these wave amplitudes transmission and
677 reflection coefficients were determined with eqs. (2.2) and (2.3). Note that the computational model
678 size in the surge-direction (x-axis) was taken to be 70 m in order to minimise the 3D end effects.
679 This length was chosen after sensitivity tests that showed the results to independent of the length of
680 the structure. Even for a smaller length of the FB, as for instance 10 m, the influence from the ends
681 was small. Figure 22, Figure 23, and Figure 24 compare the reflection and transmission coefficients
682 estimated from the experiments with the computed results. For the regular cross section (RG) we
683 found only a small difference between the results, but the reflection coefficient was slightly smaller
684 in the experiment compared to the numerical analyses. In the case with wing plates (WP) the
685 reflection coefficient, C_R , from the experiments was slightly smaller than the one found from
686 numerical analyses. The transmission coefficient, C_T , found from experiments and numerical
687 analyses was approximately the same.

688 For the two first cross sections, (RG) and (WP), we found a good agreement between the
689 experimental and numerical results, as outlined above. In the third cross section with attached
690 porous sides, (WP P100), this changed as illustrated in Figure 24. Here the reflection coefficient,
691 C_R , was lower than 0.4 for all frequencies in the experiments while it continued to increase with
692 increasing wave frequency in the numerical analyses. The transmission coefficient was also highly
693 modified by the porous sides in the experiments, while the numerical results behave in much the
694 same way as for the other two cross sections. The external damping that was included in the
695 calculations resulted in a satisfactory agreement for the body response, but it clearly failed to
696 capture the reflection and transmission coefficients correctly. While the added viscous damping
697 removes energy from the motion response and, thus the waves radiated by the body motion, it has
698 no effect on the diffraction problem, i.e. the waves scattered by the fixed body. This presumably
699 explains the large errors at higher frequencies where diffraction effects become more and more
700 important.

701



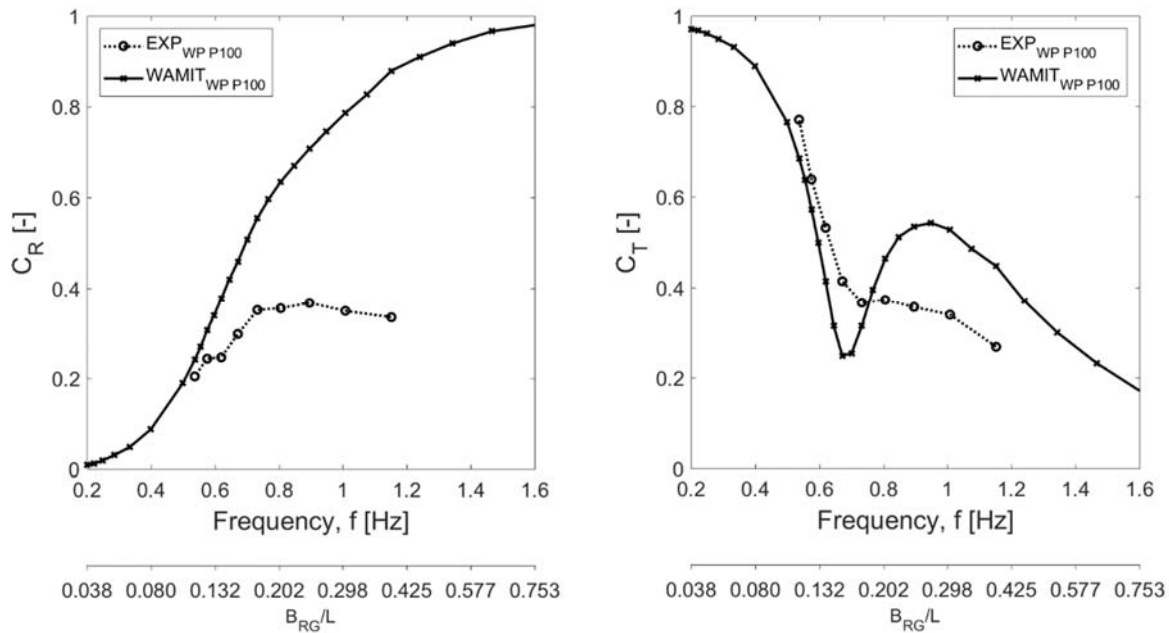
702 *Figure 22 Regular cross section (RG). Comparison of reflection and transmission coefficients*
 703 *between experimental results and numerical results based on WAMIT analyses.*



704 *Figure 23 Wings cross section (WP). Comparison of reflection and transmission coefficients*
 705 *between experimental results and numerical results based on WAMIT analyses.*
 706

707

708



709 *Figure 24 Wings with porous sides (WP P100). Comparison of reflection and transmission*
 710 *coefficients between experimental results and numerical results based on WAMIT*
 711 *analyses.*

712 6 Conclusions

713 Two different damping mechanisms for a FB were studied experimentally and numerically.
 714 The basis for the analysis was a regular box shaped breakwater (RG). The first damping mechanism
 715 was attached wing plates (WP), and the second, wing plates with a porous media (WP P100)
 716 attached to the vertical sides of the FB. The experiments were carried out in one of DTU's wave
 717 flumes. The motion of the FB was measured with two different systems; Particle Tracking (PT) and
 718 Accelerometers (ACC). The particle tracking system worked well for all the three investigated
 719 degrees of freedom, sway, heave and roll. The accelerometers worked well for the heave and roll
 720 motions. The difference between the two methods was up to around 2-5 %.

721 Comparison of the response amplitude operators (RAO) between the three cross sections
 722 showed the (RG)-cross section had the largest response to the waves. For the sway motion the (WP
 723 P100)-cross section damped the motion most significantly, while for the other two degrees of
 724 freedom, heave and roll, it was the (WP)-cross section that damped the motion to the largest extent.
 725 Both cross sections with damping mechanisms reduced the motion by factors of 0.5 to 0.8.

726 As the objective of breakwaters is to reduce the wave agitation on the lee side of the structure,
 727 the reflection and transmission coefficients were estimated from the experiments. The reflection

728 coefficient had approximately the same dependency on the wave frequency for the (RG)-cross
729 section and the (WP) cross section. Further, the reflection coefficient increased with increasing
730 wave frequency. The (WP P100) cross section reduced the reflection coefficient with an increasing
731 effect with increasing wave frequency. The largest reduction in the reflection coefficient was found
732 for the largest examined wave frequency. For wave frequencies larger than 0.9 Hz, the two cross
733 sections (RG) and (WP) had a transmission coefficient in the order of 0.6 to 0.8 that decreased with
734 increasing wave frequency. The cross section with wing plates and porous media (WP P100) had a
735 transmission coefficient in the order of 0.3 to 0.4 for the larger wave frequencies, and with a
736 decreasing tendency with increasing wave frequency. These analyses showed the cross section with
737 porous media attached to the sides could significantly reduce the reflection and transmission
738 coefficients. The effect was also studied from an energy point of view, where the effect obviously
739 was enhanced. More than 80 % of the incoming wave energy was dissipated with the cross section
740 with porous sides (WP P100) for wave frequencies larger than 0.6, while less than 20 % for the
741 other two cross sections, (RG) and (WP).

742 The motion, reflection and transmission coefficients were also examined with numerical
743 analyses using WAMIT. As the frictional damping is not modelled directly, an additional analysis
744 of the damping was carried out in the wave flume using decay tests. Based on these tests, an
745 external damping matrix accounting for the viscous damping was included in the numerical
746 analyses. Comparison between response amplitude operators found from experiments and
747 numerical calculations showed in general a good agreement for sway and heave. In roll, the
748 agreement was less good close to resonance, but better outside the area of resonance. For the cross
749 section with porous sides (WP P100) the agreement was still fair for frequencies larger than 0.8 Hz.
750 For smaller frequencies, the comparison between measured response amplitude operators was in
751 less good agreement. It was suggested that this might be related to filling and emptying the porous
752 media, which cannot be modelled by a simple external damping coefficient. The modelled
753 reflection and transmission coefficients for the regular cross section (RG) and the cross section with
754 wing plates (WP) agreed well with the coefficients estimated from the measurements. The cross
755 section with porous sides (WP P100) showed significantly smaller reflection and transmission
756 coefficients for wave frequencies larger than 0.8 Hz than those found by the numerical analysis.
757 This difference was attributed to dissipation of the diffraction wave in the experiments, a process
758 that cannot be modelled by a simple linear external damping coefficient.

759

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838

839	<i>Figure 1</i>	<i>Definition sketch of the DOF (Degrees Of Freedom) that was examined in this study. ...</i>	<i>4</i>
840	<i>Figure 2</i>	<i>Sketch of the wave flume. The sketch is divided into sections to keep proportions.</i>	<i>5</i>
841	<i>Figure 3</i>	<i>Positions of wave gauges in unit [mm].</i>	<i>6</i>
842	<i>Figure 4</i>	<i>Schematic top-view of the flume with black curtain set-up.</i>	<i>8</i>
843	<i>Figure 5</i>	<i>Positions of 4 white points on the side of the floating breakwater (FB) model: A, B, C, and</i>	
844		<i>D.</i>	<i>9</i>
845	<i>Figure 6</i>	<i>Sketch of the set-up of the accelerometers for the FB model with wings. The vertical</i>	
846		<i>distance from the MWL to the lid is indicated on the right panel.</i>	<i>10</i>
847	<i>Figure 7</i>	<i>Basic cross section of the regular pontoon (RG) floating breakwater.</i>	<i>11</i>
848	<i>Figure 8</i>	<i>The basic three different types of breakwaters, (RG), (WP), and (WP P100).</i>	<i>11</i>
849	<i>Figure 9</i>	<i>The porous material.</i>	<i>12</i>
850	<i>Figure 10</i>	<i>Response amplitude operator (RAO) for Sway derived from experiments for three</i>	
851		<i>different cross sections of the FB.</i>	<i>15</i>
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853		<i>different cross sections of the FB.</i>	<i>16</i>
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855		<i>cross sections of the FB.</i>	<i>17</i>
856	<i>Figure 13</i>	<i>Reflection coefficient, C_R, for three different cross sections of the FB.</i>	<i>18</i>
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858	<i>Figure 15</i>	<i>Relative reflection of energy, E_R / E_{in}, for three different cross sections of the FB.</i>	<i>20</i>
859	<i>Figure 16</i>	<i>Relative transmission of energy, E_T / E_{in}, for three different cross sections of the FB.</i>	
860		<i>21</i>	
861	<i>Figure 17</i>	<i>Relative dissipation of energy, E_{Diss} / E_{in}, for three different cross sections of the FB.</i>	
862		<i>22</i>	
863	<i>Figure 18</i>	<i>Decay tests for regular and pontoon with wings for sway, roll and heave.</i>	<i>24</i>
864	<i>Figure 19</i>	<i>Decay tests for regular and pontoon with wings and porous sides for sway, roll and</i>	
865		<i>heave. 25</i>	
866	<i>Figure 20</i>	<i>Calculated mean drift forces for cross-sections with a length of the FB corresponding to</i>	
867		<i>the physical experiments.</i>	<i>35</i>
868	<i>Figure 21</i>	<i>Comparison between Response Amplitude operators from experimental results and</i>	
869		<i>WAMIT calculations.</i>	<i>37</i>

870 *Figure 22 Regular cross section (RG). Comparison of reflection and transmission coefficients*
871 *between experimental results and numerical results based on WAMIT analyses.39*
872 *Figure 23 Wings cross section (WP). Comparison of reflection and transmission coefficients*
873 *between experimental results and numerical results based on WAMIT analyses.39*
874 *Figure 24 Wings with porous sides (WP P100). Comparison of reflection and transmission*
875 *coefficients between experimental results and numerical results based on WAMIT analyses. .40*
876