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Friction-resilient position control for machine tools - adaptive and sliding-mode methods compared

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Abstract

Robust trajectory tracking and increasing demand for high-accuracy tool positioning have motivated research in advanced control design for machine tools. State-of-the-art industry solutions employ cascades of Proportional (P) and Proportional-Integral (PI) controllers for closed-loop servo control of position and velocity of the machine axes. Although these schemes provide the required positioning accuracy in nominal conditions, performance deteriorates with increased friction and wear of the machine. With conventional control, re-tuning is necessary during the lifetime if specified accuracy shall be maintained. This paper investigates whether nonlinear and adaptive controllers can cope with typical levels of friction increase without loss of performance. It evaluates the performance of a state-of-art industry solution with that obtainable with adaptive and sliding mode positioning controls. The main finding is that an adaptive backstepping control is resilient to unknown and increasing friction at realistic levels of wear, where the P-PI control fall short with respect to accuracy. A single-axis test rig with adjustable friction is used to assess the performance of different controllers.

Keywords: Machine tools, mechanical drive-train, robustness to friction, nonlinear adaptive control, sliding-mode control, controller comparison, high-accuracy positioning.

1. Introduction

Mechanical components are manufactured using automated machine tools for cutting, drilling, milling, shearing etc, on a wide range of materials. Sophisticated mechanical designs lead to narrow workpiece tolerances, which necessitate high-accuracy tool/axis positioning. Several mechanical phenomena could compromise workpiece tolerances if not properly handled by closed-loop control. This paper will focus on friction phenomena.

To meet modern manufacturing requirements, fully automated machining must be robust to equipment degradation due to wear and tear. State-of-the-art axis-positioning solutions use combinations of P and PI controllers with additional feedforward compensation. The reader is indicatively referred to Koren (1997, 1980); Gross et al. (2001); Chiu and Tomizuka (2001) and Settineri et al. (2013) for a general overview of single and multiple-axes machine tool positioning. Additional variable-gain P-PI solutions have been proposed for high-accuracy motion control systems van de Wouw et al. (2008); Hunnekens et al. (2012). Variable integral gain PI controllers for transient performance improvement were discussed in Hunnekens et al. (2015), who proposed a saturation element to limit the integral action in case the tracking error exceeds a predefined bound.

Friction compensation for machine tool axes has several times been addressed in the literature, where various static and dynamical descriptions of the frictional phenomena have been

proposed. A detailed survey was presented in Armstrong-Houviy et al. (1994) and in Bona and Indri (2005). Most of the compensating algorithms include additional feedforward terms in the control law to counteract the effect of friction. These feedforward terms provide a description of the friction phenomena at different levels of detail depending on the regime of the axis motion (see Liu et al. (2004); Marton and Lantos (2007) for modelling and compensation of friction during slow axis motion and Swevers et al. (2000) for a two-mode motion profile). Dedicated disturbance observers Huang et al. (2010); Lampaert et al. (2004) or model inversion are then utilized to estimate friction Jamaludin et al. (2008, 2009). In most of these compensation schemes, the friction model parameters are considered constant and they are obtained from offline identification experiments De Wit and Lischinsky (1997); Indri et al. (2013). Nonlinear presliding friction compensation was addressed in Ruderman (2014), where a feedforward observer, introduced in Ruderman and Bertram (2012), was used to estimate the frictional forces during the presliding regimes. The design of a more general nonlinear friction observer was provided in Ruderman and Iwasaki (2015), especially for motion control applications. The performance limitations of the cascaded P-PI motion controls with respect to nonlinear friction rejection at micro-positioning were discussed in Ruderman and Iwasaki (2016); Ruderman (2016). The authors demonstrated that zero steady-state positioning error is nearly impossible with linear controllers.

Despite their inherent design limitations Middleton (1991); Seron et al. (2012), linear controllers achieve adequate performance under nominal conditions. However, with gradually increasing friction over the lifetime of equipment, control loops need to be frequently re-tuned. The need for achieving nominal performance even in the presence of increased friction, motivates the investigation of nonlinear control strategies for machine tool axis positioning.

Reference tracking methods based on nonlinear control principles have powerful stability and robustness properties. Specifically, Sliding Mode Control (SMC) techniques can ensure model-free disturbance rejection and reference tracking in finite time by injecting high-frequency switching terms in the control signal Utkin (1978); Edwards and Spurgeon (1998); Yu and Efe (2015). The general principles of applying conventional first-order SMC algorithms to various electromechanical systems were presented in Utkin et al. (2009) and, specifically for feed-drive systems, in Utkin (1993). The design of a SMC for high-speed machine feed drives was presented in Altintas et al. (2000). The authors experimentally demonstrated that the proposed algorithm ensured accurate tracking of different machining profiles under friction and cutting-force disturbances. High-order sliding-mode controllers, detailed in Levant (1993), were shown to provide robust tracking with reduced actuator chattering Jin et al. (2009).

Nonlinear adaptive control methods have also been largely explored in connection to positioning of machine or robot axes, although emphasizing stability of the closed-loop error dynamics Khorasani (1992); Ray et al. (2001); Avila-Becerril et al. (2016). Compensation of friction in high-precision machine tool drives was achieved in Garagić and Srinivasan (2004) via a nonlinear model-reference adaptive control scheme. The Adaptive backstepping Controller (ABSC) was presented in detail in Krstic et al. (1995). Such controllers were designed for a Permanent Magnet Synchronous Motor (PMSM) Zhou and Wang (2002) and a double-motor driving servo system Zhang and Ren (2014). LuGre friction compensation was achieved in Tan and Kanellakopoulos (1999) via an ABSC. The unknown friction dynamics was estimated by a double adaptive observer system and the estimate was fed to the control law. The application of an adaptive backstepping sliding mode control design for flexible ball screw drives was investigated in Dong and Tang (2014), who demonstrated robustness against cutting-forces-induced vibrations.

Although the friction compensating properties and robust performance of sliding-mode and nonlinear adaptive algorithms have frequently been reported, there is little work on systematic comparison of these two control families in the context of high-accuracy machine tool positioning under incipient wear. This paper focuses on the applicability of advanced control schemes for machine tools under unknown and increasing friction. Specifically, three nonlinear controllers are proposed for high-accuracy positioning of machine tool axis. The designs are based on sliding-mode and nonlinear adaptive control principles and utilize a direct-position control architecture, i.e. they do not consist of position-velocity cascades, in contrast to the conventional P-PI solutions used in the industry. This architecture

Figure 1: (Top) Machine-tool axis, where ω_m, ω_I are the motor and load angular velocities, respectively. (Bottom) Equivalent double mass oscillator with J_m, J_I being the drive motor and generalized load inertias, respectively, K_s the spring constant corresponding to the shaft stiffness and D_s being the damping coefficient of the shaft. The torque generated by the drive motor is denoted with T_m , the interconnecting torque is T_I , $N = 1$ is the gearing ratio between motor and load and $T_{F,m}, T_{F,I}$ is the friction on the motor and the load side, respectively.

allows for frictional phenomena and model uncertainties to be addressed both for the drive motor and the axis itself. A machine tool specific design is made using each of the suggested methods and details on the design are provided. The objective of this study is to investigate the ability the proposed nonlinear control algorithms to maintain positioning tolerances under unknown and increasing friction.

The paper is organised as follows: Section 2 gives a description of the physical system and presents the problem formulation. Modelling details are presented in Section 3. Section 4 presents the different control schemes. The design of each controller is provided and the theoretical features of each method are highlighted. Experimental results of the controllers' performance are presented in Section 5. Section 6 makes an overall comparison of the control schemes and conclusions are discussed in Section 7.

2. System description and problem formulation

Typical single-axis machine tools consist of a drive motor connected to an angular-to-linear motion conversion system (e.g. a ball-screw), which linearly positions the tool Gross et al. (2001). All the mechanical components connected to the machine spindle can be viewed as the serial interconnection of inertias and torsional springs. The lumped masses and damping of these components motivate the description of the single-axis machine tool as mechanical drive-train comprising the drive motor, a flexible shaft with damping and a generalized load with friction as shown in Figure 1. This drive-train abstraction of the single-axis machine tool will be the basis for comparing the different control algorithms presented in this paper.

2.1. Physical system and positioning accuracy

The physical system used in this paper comprises two identical Siemens 1FT7 Permanent Magnet Synchronous Motors

(PMSMs) Siemens (2017a) connected via a steel shaft (see Figure 10). The system is equipped with a Siemens SINAMICS S120 drive converter Siemens (2017b). The first motor acts as the driving actuator, while the second motor emulates the load of the axis as pure inertia.

The objective in machine tool axis control is to ensure that the axis end-point (e.g. tool piece, load) tracks a given motion profile. In applications where high accuracy is required, machine tools drives are equipped with high resolution absolute or incremental encoders (in the setup used in this study each of the motors is equipped with an 22-bit incremental encoder for the angular position) that allow linear positioning accuracy to the order of $1 - 10 \mu\text{m}$. Typical linear axes have lead screws with pitch size ranging from 2 mm to 10 mm , which leads to an angular positioning tolerance between $5 \cdot 10^{-3}$ and $2 \cdot 10^{-2} \text{ rad}$ Gross et al. (2001). The accuracy requirement considered in this study is 10^{-2} rad .

2.2. Friction and positioning degradation

The friction that develops between the surfaces of the mechanical parts of the machine acts as a decelerating torque both on the drive motor and the load. Its positioning-degrading effect is usually addressed by adding appropriate feed-forward terms in the low-level control algorithms. Although calculation of the friction characteristics is typically done during the commissioning of the machine drive, these compensation schemes assume constant friction characteristics. As a result, they fail to preserve positioning accuracy when friction parameters change due to equipment wear Rebelein and Zaeh (2016) (for example deformation of the bearing surfaces of the motors and linear axes, lubrication film failure etc.) or environmental changes, such increased workspace temperature, which can cause increased Coulomb friction and viscosity Straffelini (2015).

Consequently, an axis-positioning control strategy that can ensure nominal performance irrespectively of friction changes is a highly desirable feature for machine tools. This is the reason that friction with variable characteristics is chosen to be the degrading phenomenon, against which the nonlinear controllers designed for the drive-train will be compared and evaluated.

The friction acting on the drive motor can be different than the one affecting the generalized load. The first is primarily due to the contact of the bearing surfaces. On the other hand, load friction is the combination of all the frictional torques. These extend from the gearing mechanism to the linear axis ball screw system and the various intermediate contact surfaces. Different friction models can be used depending on the motion profile of the machine. When the motor moves fast (above 30 rpm) Coulomb and viscous friction dominate. In the case of very low motor speed (below 5 rpm) more complex phenomena, such as hysteresis and Stribeck pre-sliding take place. In such cases more complicated models, such as the *LuGre* friction model De Wit et al. (1995) or the Maxwell-Slip model Boegli et al. (2012), are required to describe these phenomena. (For a more detailed review of the various different friction models and their evolution the reader may consider the survey done in Armstrong-Hlouvry et al. (1994) and more recently in Johansson and Canudas-De-Wit (2008), as well as Rebelein and

Zaeh (2016), where the focus is exclusively on machine tool feed drives). Since in the physical system used in this study the generalized load is identical to the drive motor (same inertias and no gearing in between), the nominal friction is considered to be the same for both the motor and the load. This assumption is valid on the basis of the friction sources being the same for the drive motor and the load.

2.3. Problem formulation

The collective objective with respect to which the different control architectures will be compared to each other can be summarized in the following formulation of the problem:

Problem 1 (Friction-resilient accuracy control for single-axis machine tool). *Consider a single-axis machine tool system consisting of a drive motor connected to a load with a flexible shaft. Let the tool positioning error be denoted by e_θ and let $T_{C,m}^{\max}$ be an upper bound for the Coulomb friction magnitude $T_{C,m}$ on the motor side. Design a closed-loop control strategy that ensures:*

$$|e_\theta(t)| \leq 10 \text{ mrad} \quad \forall t \geq t_0 > 0 \text{ and for } T_{C,m} \leq T_{C,m}^{\max}$$

where t_0 denotes a time after the starting up of the positioning task for the machine.

In the above problem formulation the bound $T_{C,m}^{\max}$ describes the maximum value of Coulomb friction, above which alleviation of the positioning degradation is not addressed by means of low-level axis control.

3. Mathematical model

3.1. Drive train mechanical dynamics

The drive train system consists of a drive motor connected to a load via a flexible shaft. Based on the abstraction shown in Figure 1, the mechanical dynamics of the drive axis can be described as the interaction between the motor and the load inertias. This description corresponds to a two-mass-spring-damper system, with dynamics given by:

$$\dot{\omega}_m = \frac{1}{J_m} T_m - \frac{1}{J_m} T_{F,m}(\omega_m) - \frac{1}{NJ_m} T_l \quad (3.1)$$

$$\dot{\theta}_m = \omega_m \quad (3.2)$$

$$\dot{\omega}_l = -\frac{1}{J_l} T_{F,l}(\omega_l) + \frac{1}{J_l} T_l \quad (3.3)$$

$$\dot{\theta}_l = \omega_l \quad (3.4)$$

where ω_m, θ_m and ω_l, θ_l are the velocities and positions of the motor and the load, respectively, T_m is the torque generated by the drive motor and the interconnection torque T_l is given from:

$$T_l = K_S \left(\frac{1}{N} \theta_m - \theta_l \right) + D_S \left(\frac{1}{N} \omega_m - \omega_l \right). \quad (3.5)$$

The friction torques $T_{F,m}, T_{F,l}$ acting on the motor and load, respectively, are described by the Coulomb-viscous model Egeland and Gravdahl (2002):

$$T_{F,i}(\omega_i) = T_{C,i} \text{sgn}(\omega_i) + \beta_i \omega_i \quad i \in \{m, l\}. \quad (3.6)$$

The signum function $\text{sgn}(\cdot)$ is strictly defined as

$$\text{sgn}(y) = \begin{cases} 1 & \text{if } y > 0 \\ v \in [-1, 1] & \text{if } y = 0 \\ -1 & \text{if } y < 0 \end{cases} . \quad (3.7)$$

However, the convention $\text{sgn}(0) = 0$ is used throughout the entire study, which implies a discontinuity of the friction model at zero velocity. To ensure the Lipschitz continuity of the vector field of the drive train system, which is needed for the theoretical analysis of the closed-loop systems later on, the friction model introduced above is modified by replacing $\text{sgn}(\cdot)$ with a smooth approximation $v(p, y)$ defined in the following:

$$v(p, y) \triangleq \frac{2}{\pi} \arctan(py) \approx \text{sgn}(y) . \quad (3.8)$$

The parameter p is a positive number denoting the slope of the function near 0 (see also Figure B.21). The modified friction expression reads

$$T_{F,i}(\omega_i) = T_{C,i}v(p, \omega_i) + \beta_i\omega_i \quad i \in \{m, l\} . \quad (3.9)$$

This static friction model captures the Coulomb and viscous components and although it dominates in the friction compensation schemes of commercial drive controllers due to its simplicity, it does not include stiction or the nonlinear presliding phenomenon (see Johansson and Canudas-De-Wit (2008)). During very slow axis motion profiles (almost zero speed) the stick-slip and hysteresis are not negligible. More advanced descriptions of frictional torques have been proposed for such regimes in Ferretti et al. (2004a,b); Indri et al. (2013); Ruderman and Bertram (2013). However, these dynamical models significantly increase the complexity of the control designs, especially in combination to time-varying friction parameters, such is the case in this study. The reason for this is that the proposed control algorithms, as it will become clearer in Section 4, do not compensate for friction via feedforward terms (which would allow for more complex friction estimation strategies to be used) but they integrate the rejection of friction in their design. Moreover, the changes in friction, on which this study focuses, relate to incipient equipment wear. Hence, they are better captured by parameter variations rather than internal states of dynamical friction models. In fact only one of the proposed methods requires accurate friction modelling, hence could be improved by using a more complex friction model.

3.2. Closed-loop electrical dynamics

The input torque T_m to the mechanical drive train is the output of the drive motor's electrical subsystem, which includes the dynamics of the direct and quadrature axis currents i_d and i_q , respectively Krause et al. (2012). The control of i_d, i_q such that a given torque command u is generated by the drive motor is done separately from the rest of the system dynamics and it is not considered in this study. Moreover, the main degrading disturbance addressed in this study, namely friction, appears as a decelerating torque, i.e. at the mechanical dynamics of the system. This reasons lead to the following modelling assumption, common in axis positioning problems:

Figure 2: (Top) Torque command u and actual motor torque T_m . (Bottom) Input torque disturbance.

Assumption 3.1. *The closed-loop electrical dynamics can be considered as a unit gain perturbed by a finite number of torque ripples, i.e. the produced torque by the motor is the torque command plus a bounded disturbance:*

$$T_m(t) = u'(t) + d_e(t) . \quad (3.10)$$

This torque perturbation is the combined effect of cogging torques, current control imperfections and sensor offsets of the current transducers which provide an indirect torque measurement. Analytical modelling and identification of such parasitic torque ripples have been addressed in the literature Jahns (1984); Bolton and Ashen (1984); Dai et al. (2004); Tripathi et al. (2005); Gebregergis et al. (2015). Compensation of the input torque perturbation is mainly achieved by correcting the quadrature current reference in the current control loop with an appropriate feed-forward term Carlson et al. (1989); Ferretti et al. (1998) and is not addressed in the current study. This is expressed in the following assumption:

Assumption 3.2. *There exists an appropriate feedforward term $\hat{d}_e$ that if inserted in the control law $u' = u - \hat{d}_e$, it compensates for the disturbance d_e leading to:*

$$T_m \cong u . \quad (3.11)$$

For the drive motor used in the study, the maximum amplitude of the torque ripples was measured to be 0.02 N m, which approximately corresponds to 0.7% of the rated torque and 0.15% of the maximum input (2.7 and 13 N m, respectively Siemens (2017a)). Figure 2 shows the motor torque T_m produced after a torque profile u , as well as the associated perturbation d_e . As it can be seen from the bottom plot, d_e is contained in a ± 0.02 N m envelope.

Table 1 provides a complete explanation of the notation used in the equations above.

Table 1: System model nomenclature

Symbol	Description	Units
States and Outputs		
ω_m	Motor angular velocity	rad s^{-1}
θ_m	Motor angular position	rad
ω_l	Load angular velocity	rad s^{-1}
θ_l	Load angular position	rad
Inputs		
θ_r	Load position reference	rad
ω_r	Motor velocity reference	rad s^{-1}
u	Torque command	N m
T_m	Generated torque	N m
Constant parameters		
N	Gearing ratio	–
J_m	Motor inertia	kg m^2
J_l	Load inertia	kg m^2
K_S	Shaft stiffness	N m rad^{-1}
D_S	Shaft damping coefficient	N m s rad^{-1}
$T_{C,m}$	Coulomb friction on the motor	N m
$T_{C,l}$	Coulomb friction on the load	N m
β_m	Motor viscous friction coefficient	N m s rad^{-1}
β_l	Load viscous friction coefficient	N m s rad^{-1}
Disturbances		
d_e	Input torque disturbance	N m
$T_{F,m}$	Motor friction	N m
$T_{F,l}$	Load friction	N m
T_l	Load torque	N m

4. Axis position control methods

State-of-the-art industrial solutions for load positioning include a cascade of P and PI controllers, that command the necessary accelerating torque for the drive motor. Two individual PI controllers (for the direct and quadrature currents) ensure that this torque is actually generated by the motor. The control scheme is shown in Figure 3. This section presents the design of three direct-position nonlinear controllers based on adaptive and sliding-mode methods. The control of the load position θ_l is achieved directly by each nonlinear controller without the use of any cascaded structure. Consequently, the design of all these three controllers take into account the dynamics of the entire drive-train system and not just of the drive motor, as is the case in conventional P-PI cascades. This does not imply that direct position controllers do not include velocity control (e.g. the ABSC presented later) but rather that the design of these control loops is done in an integrated way and not separately from the positioning control loops. This generic control scheme is illustrated in Figure 4.

Notation: In the following we will denote an estimate of a

Figure 3: State of the art in industrial position control using cascaded P and PI controllers.

parameter or state ξ by $\hat{\xi}$ and the associated estimation error by $\tilde{\xi}$, such that it holds:

$$\tilde{\xi} = \xi - \hat{\xi}. \quad (4.1)$$

4.1. Nonlinear adaptive backstepping control

Backstepping controllers for systems with relative degree larger than 1 are based on the concept of *virtual input*. At each level of the control design a state variable is regulated so that it serves as control input for the next step. In the adaptive version of backstepping controllers the unknown quantities can be expressed as uncertain or unknown parameters. The control design follows similar steps but now update laws for the unknown parameters are designed as well.

The adaptive backstepping control scheme for the drive-train system is shown in Figure 5. The system has relative degree 3, since it is desired to control the load position via applying torques at the drive motor inertia. This implies that three steps of backtracking will be needed. Instead of doing that, we define the manifold

$$\mathcal{M} = \{y \in \mathbb{R} \mid \dot{y} + cy = 0, c > 0\} \quad (4.2)$$

and design the control law such that $\mathcal{M}$ is rendered invariant with respect to the trajectories of the position tracking error. Once the system reaches $\mathcal{M}$, the positioning error converges exponentially (see (4.2)) to zero with a rate of convergence c . The friction parameters and the shaft characteristics are considered to be unknown or uncertain.

Figure 4: Direct-position control architecture.

Figure 5: ABCS direct position controller.

The drive-train dynamics is written in parametric form as

$$\dot{\omega}_m = \frac{1}{J_m} (u + \phi_1^T(x)\vartheta) \quad (4.3)$$

$$\dot{\theta}_m = \omega_m \quad (4.4)$$

$$\dot{\omega}_l = \frac{1}{J_l} (\phi_2^T(x)\vartheta + b\omega_m), \quad b = \frac{D_S}{N} \quad (4.5)$$

$$\dot{\theta}_l = \omega_l \quad (4.6)$$

$$x = [\omega_m \quad \theta_m \quad \omega_l \quad \theta_l]^T \quad (4.7)$$

$$\vartheta = [K_S \quad D_S \quad T_{C,m} \quad \beta_m \quad T_{C,l} \quad \beta_l]^T \quad (4.8)$$

$$\phi_1(x) = \begin{bmatrix} -\frac{1}{N}(\frac{1}{N}\theta_m - \theta_l) \\ -\frac{1}{N}(\frac{1}{N}\omega_m - \omega_l) \\ -v(p, \omega_m) \\ 0 \\ 0 \end{bmatrix}, \quad \phi_2(x) = \begin{bmatrix} \frac{1}{N}\theta_m - \theta_l \\ -\omega_l \\ 0 \\ -v(p, \omega_l) \\ -\omega_l \end{bmatrix}, \quad (4.9)$$

where the function $v(p, \omega_i)$ was defined in Equation (3.8). The following assumption, common in adaptive control literature is made:

Assumption 4.1. (Constant unknown parameter vector)

The unknown parameters ϑ are assumed to be constant or at least slowly varying, i.e. their time derivatives are considered to be zero

$$\dot{\vartheta} = 0.$$

The design of the controller, which is detailed in the next theorem, involves the derivation of few complicated expressions. For greater readability a number of auxiliary variables (e.g. z_1, z_2, ψ_1, ψ_2) are introduced in the control and adaptation laws and are later elaborated in the proof of the theorem.

Theorem 4.1. Consider the drive-train system described in (4.3)-(4.9) with unknown parameters ϑ defined in (4.8), the uncertain virtual input gain $b = \frac{D_S}{N}$ with known sign and a bounded reference signal $\theta_r(t) \in C^3$ for the load position. The collective control law

$$u = -\phi_1^T(x)\hat{\vartheta} + J_m \left(\psi_2 - k_2 z_2 - \frac{1}{J_l} z_1 \hat{b} \right) \quad (4.10)$$

where

$$z_1 = \dot{e}_\theta + c e_\theta$$

$$e_\theta = \theta_l - \theta_r$$

$$z_2 = \omega_m - \alpha(x, \hat{\rho}, \hat{\vartheta}, \dot{\theta}_r, \ddot{\theta}_r)$$

$$\alpha = \alpha(x, \hat{\rho}, \hat{\vartheta}, \dot{\theta}_r, \ddot{\theta}_r) = \hat{\rho} [J_l (\ddot{\theta}_r - c\omega_l + c\dot{\theta}_r - k_1 z_1) - \phi_2^T(x)\hat{\vartheta}] \quad (4.11)$$

$$\psi_1 = \dot{\rho} [-\phi_2^T(x)\hat{\vartheta} + J_l (\ddot{\theta}_r - c\omega_l + c\dot{\theta}_r - k_1 z_1)] \quad (4.12)$$

$$\psi_2 = \psi_1 + \hat{\rho} \left\{ -\phi_2^T(x)\hat{\vartheta} - \hat{K}_S \left(\frac{\omega_m}{N} - \omega_l \right) + J_l (\theta_r^{(3)} + c\ddot{\theta}_r) + \left(\frac{\hat{D}_s + \hat{\beta}_l}{J_l} - c - k_1 \right) (\phi_2^T(x)\hat{\vartheta} + \hat{b}\omega_m) - J_l k_1 (c\omega_l - \ddot{\theta}_r - c\dot{\theta}_r) \right\} \quad (4.13)$$

together with the adaptation laws

$$\dot{\hat{\vartheta}} = \Gamma \left\{ \phi_2(x) \left[\frac{z_1}{J_l} - \hat{\rho} z_2 \left(\frac{\hat{D}_s + \hat{\beta}_l}{J_l} - c - k_1 \right) \right] + \frac{z_2}{J_m} \phi_1(x) \right\} \quad (4.14)$$

$$\dot{\hat{\rho}} = -\gamma_1 \cdot \text{sgn}(b) z_1 \left(-\frac{1}{J_l} \phi_2^T(x)\hat{\vartheta} + \ddot{\theta}_r - c\omega_l + c\dot{\theta}_r - k_1 z_1 \right) \quad (4.15)$$

$$\dot{\hat{b}} = \gamma_2 \left[\frac{1}{J_l} z_1 z_2 - \hat{\rho} z_2 \left(\frac{\hat{D}_s + \hat{\beta}_l}{J_l} - c - k_1 \right) \omega_m \right] \quad (4.16)$$

where $k_1, k_2, \gamma_1, \gamma_2, c > 0$ and Γ is a 6×6 symmetric positive definite real matrix, ensures that the position tracking error e_θ converges to the origin $e_\theta^* = 0$ as $t \rightarrow \infty$, i.e.

$$\lim_{t \rightarrow \infty} e_\theta(t) = 0.$$

Moreover, the parameter estimation errors $\tilde{\vartheta}, \tilde{\rho}, \tilde{b}$ remain bounded for all future times.

Proof. The control design is done in two steps, starting from the position error dynamics and moving towards the angular velocity of the drive-motor, which is directly affected by the actuator.

Step 1

Define the off-the-manifold $\mathcal{M}$ variable $z_1 \triangleq \dot{e}_\theta + c e_\theta$ and a corresponding Lyapunov function candidate

$$V_1(z_1) = \frac{1}{2} z_1^2$$

The time derivative of V_1 along the dynamics of z_1 is written

$$\dot{V}_1 = z_1 \dot{z}_1 = z_1 \left[\frac{1}{J_l} (\phi_2^T(x)\vartheta + b\omega_m) - \ddot{\theta}_r + c\omega_l - c\dot{\theta}_r \right] \quad (4.17)$$

From the equation above it can be seen that if ω_m is seen as a virtual input to the subsystem, then a stabilizing function

$$\alpha(x, \rho, \vartheta, \dot{\theta}_r, \ddot{\theta}_r) = \rho [-\phi_2^T(x)\vartheta + J_l (\ddot{\theta}_r - c\omega_l + c\dot{\theta}_r - k_1 z_1)]$$

$$\rho \triangleq \frac{1}{b}$$

where $k_1 > 0$, will make $\dot{V}_1$ negative definite. However, since both $\boldsymbol{\theta}$ and ρ are unknown, this control law is not implementable. A certainty equivalence law, where $\boldsymbol{\theta}$ and ρ are substituted with their estimates $\hat{\boldsymbol{\theta}}$ and $\hat{\rho}$, respectively, is used instead. The goal then is to regulate ω_m such that it tracks $\alpha(\mathbf{x}, \hat{\rho}, \hat{\boldsymbol{\theta}}, \dot{\theta}_r, \ddot{\theta}_r)$ at all times. Subsequently, V_1 is augmented with the quadratic parameter estimation errors as

$$V_\alpha(z_1, \tilde{\boldsymbol{\theta}}, \tilde{\rho}) = V_1 + \frac{1}{2} \tilde{\boldsymbol{\theta}}^T \mathbf{\Gamma}^{-1} \tilde{\boldsymbol{\theta}} + \frac{1}{2\gamma_1} |b| \tilde{\rho}^2. \quad (4.18)$$

Taking the time derivative of V_α and substituting ω_m with $\alpha(\mathbf{x}, \hat{\rho}, \hat{\boldsymbol{\theta}}, \dot{\theta}_r, \ddot{\theta}_r)$ while noting that $\rho b = 1$ leads to

$$\begin{aligned} \dot{V}_\alpha = & -k_1 z_1^2 + \frac{1}{J_l} z_1 \boldsymbol{\phi}_2^T(\mathbf{x}) \tilde{\boldsymbol{\theta}} - \tilde{\boldsymbol{\theta}}^T \mathbf{\Gamma}^{-1} \dot{\tilde{\boldsymbol{\theta}}} - \frac{1}{\gamma_1} |b| \tilde{\rho} \dot{\tilde{\rho}} \\ & - \frac{1}{J_l} z_1 \tilde{\rho} b \left[-\boldsymbol{\phi}_2^T(\mathbf{x}) \hat{\boldsymbol{\theta}} + J_l (\ddot{\theta}_r - c\omega_l + c\dot{\theta}_r - k_1 z_1) \right] \end{aligned} \quad (4.19)$$

where assumption 4.1 was used. From (4.19) it can be seen that selecting an appropriate adaptation law for $\dot{\tilde{\boldsymbol{\theta}}}$

$$\dot{\tilde{\boldsymbol{\theta}}} = \frac{1}{J_l} \mathbf{\Gamma} \boldsymbol{\phi}_2(\mathbf{x}) z_1$$

eliminates the parameter estimation error. However the term $\boldsymbol{\theta}$ appears again in the next steps of the design. In order to avoid introducing a second estimate of the parameter vector (see (Krstic et al., 1995, Section 3.2.1)), elimination of $\tilde{\boldsymbol{\theta}}$ and $\tilde{\rho}$ will be treated in the next step.

Step 2

Define the motor velocity tracking error variable $z_2 \triangleq \omega_m - \alpha$. The dynamics of z_1 can now be written as

$$\dot{z}_1 = \frac{1}{J_l} (\boldsymbol{\phi}_2^T(\mathbf{x}) \boldsymbol{\theta} + b z_2 + b \alpha) - \ddot{\theta}_r + c\omega_l - c\dot{\theta}_r \quad (4.20)$$

and therefore

$$\begin{aligned} \dot{V}_\alpha = & -k_1 z_1^2 + \frac{1}{J_l} z_1 (\boldsymbol{\phi}_2^T(\mathbf{x}) \tilde{\boldsymbol{\theta}} + b z_2) - \tilde{\boldsymbol{\theta}}^T \mathbf{\Gamma}^{-1} \dot{\tilde{\boldsymbol{\theta}}} - \frac{1}{\gamma_1} |b| \tilde{\rho} \dot{\tilde{\rho}} \\ & - \frac{1}{J_l} z_1 \tilde{\rho} b \left[-\boldsymbol{\phi}_2^T(\mathbf{x}) \hat{\boldsymbol{\theta}} + J_l (\ddot{\theta}_r - c\omega_l + c\dot{\theta}_r - k_1 z_1) \right] \end{aligned} \quad (4.21)$$

The dynamics of z_2 reads

$$\begin{aligned} \dot{z}_2 = & \frac{1}{J_m} (u + \boldsymbol{\phi}_1^T(\mathbf{x}) \boldsymbol{\theta}) - \dot{\alpha} \\ = & \frac{1}{J_m} (u + \boldsymbol{\phi}_1^T(\mathbf{x}) \hat{\boldsymbol{\theta}}) \\ & - \frac{d}{dt} \left\{ \hat{\rho} \left[J_l (\ddot{\theta}_r - c\omega_l + c\dot{\theta}_r - k_1 z_1) - \boldsymbol{\phi}_2^T(\mathbf{x}) \hat{\boldsymbol{\theta}} \right] \right\} \\ = & \frac{1}{J_m} (u + \boldsymbol{\phi}_1^T(\mathbf{x}) \hat{\boldsymbol{\theta}}) - \psi_2 - \hat{\rho} \left(\frac{\hat{D}_s + \hat{\beta}_l}{J_l} - c - k_1 \right) \boldsymbol{\phi}_2^T(\mathbf{x}) \tilde{\boldsymbol{\theta}} \\ & - \hat{\rho} \left(\frac{\hat{D}_s + \hat{\beta}_l}{J_l} - c - k_1 \right) \tilde{\rho} \omega_m \end{aligned} \quad (4.22)$$

where ψ_2 is defined in (4.12)-(4.13). Detailed calculation of $\dot{\alpha}$ is presented in Appendix B.

Extending V_α to the new Lyapunov function candidate

$$V_2(z_1, z_2, \tilde{\boldsymbol{\theta}}, \tilde{\rho}, \tilde{b}) = V_\alpha + \frac{1}{2} z_2^2 + \frac{1}{2\gamma_2} \tilde{b}^2 \quad (4.23)$$

and taking its derivative along the trajectories of $z_1, z_2, \tilde{\boldsymbol{\theta}}, \tilde{\rho}, \tilde{b}$, while considering also (4.10), (4.20), (4.21) and (4.22) yields after straightforward calculations:

$$\begin{aligned} \dot{V}_2 = & -k_1 z_1^2 - k_2 z_2^2 \\ & + \tilde{\boldsymbol{\theta}}^T \left\{ \boldsymbol{\phi}_2(\mathbf{x}) \left[\frac{z_1}{J_l} - \hat{\rho} z_2 \left(\frac{\hat{D}_s + \hat{\beta}_l}{J_l} - c - k_1 \right) \right] + \frac{z_2}{J_m} \boldsymbol{\phi}_1(\mathbf{x}) - \mathbf{\Gamma}^{-1} \dot{\tilde{\boldsymbol{\theta}}} \right\} \\ & - \tilde{\rho} |b| \left[\text{sgn}(b) z_1 \left(-\frac{1}{J_l} \boldsymbol{\phi}_2^T(\mathbf{x}) \hat{\boldsymbol{\theta}} + \ddot{\theta}_r - c\omega_l + c\dot{\theta}_r - k_1 z_1 \right) + \frac{1}{\gamma_1} \dot{\tilde{\rho}} \right] \\ & + \tilde{b} \left[\frac{1}{J_l} z_1 z_2 - \hat{\rho} z_2 \left(\frac{\hat{D}_s + \hat{\beta}_l}{J_l} - c - k_1 \right) \omega_m - \frac{1}{\gamma_2} \dot{\tilde{b}} \right]. \end{aligned} \quad (4.24)$$

Substituting the adaptation laws (4.14), (4.15), (4.16) into (4.24) yields

$$\dot{V}_2 = -k_1 z_1^2 - k_2 z_2^2 \leq 0 \quad (4.25)$$

with $k_1, k_2 > 0$. From this it can be seen that V_2 is non-increasing and since $V_2(z_1(0), z_2(0), \tilde{\boldsymbol{\theta}}(0), \tilde{\rho}(0), \tilde{b}(0)) < \infty$, this implies that $z_1, z_2, \tilde{\boldsymbol{\theta}}, \tilde{\rho}, \tilde{b}$ are bounded for all future times, i.e. $z_1, z_2, \tilde{\boldsymbol{\theta}}, \tilde{\rho}, \tilde{b} \in \mathcal{L}_\infty$. From Equation (4.20), since all the signals on the right side are bounded, it follows that $\dot{z}_1 \in \mathcal{L}_\infty$. Moreover, integrating (4.25) with respect to time gives

$$\begin{aligned} 0 \leq & \int_0^\infty \begin{bmatrix} z_1 & z_2 \end{bmatrix} \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} dt \\ \leq & V_2(z_1(0), z_2(0), \tilde{\boldsymbol{\theta}}(0), \tilde{\rho}(0), \tilde{b}(0)), \end{aligned}$$

which implies that $z_1, z_2 \in \mathcal{L}_2$. Then, by Corollary A.7 in (Krstic et al., 1995, p. 491), it is shown that $\lim_{t \rightarrow \infty} \begin{bmatrix} z_1(t) & z_2(t) \end{bmatrix}^T = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$, which means that the off-the-manifold $\mathcal{M}$ variable z_1 decays to 0.

The dynamics of the load positioning error e_θ can be written according to (4.2) as

$$\dot{e}_\theta = -c e_\theta + z_1 \quad (4.26)$$

which can be viewed as the cascaded interconnection of a Uniformly Globally Asymptotically Stable (UGAS) and a convergent system (the dynamics of the unperturbed e_θ and those of z_1 , respectively). Moreover, the system in (4.26) is Input-to-State Stable (ISS) with respect to the input variable z_1 (see Appendix C for a proof). Then $\lim_{t \rightarrow \infty} z_1(t) = 0$ implies that $\lim_{t \rightarrow \infty} e_\theta(t) = 0$, i.e. the positioning error system is also convergent, which completes the proof. ■

Remark 1. Regarding the adaptive law in Equation (4.15), the approximation of the $\text{sgn}(b)$ with $\frac{2}{\pi} \arctan(pb)$, $p > 0$ is not necessary for the theoretical analysis. However, for implementation purposes, such an approximation is employed with not necessarily the same value for the slope factor p . Using this approximation alleviates the effect of noise on the adaptation of the inverse input gain ρ , which is important since this gain multiplies the entire motor velocity reference in the expression (4.11) for α .

Remark 2. It should be noted that bounded perturbations in the system dynamics coming from low-speed friction modelling errors or the input torque ripples will affect the tracking accuracy. Since these perturbations are not accounted for in the design of the ABSC, the adaptive part of the controller will try to adapt the parameter estimates to drive the positioning error to zero. This eventually leads to erroneous, yet bounded, estimates of the unknown parameters and some loss of accuracy depending on how large these unmodelled perturbations are. It is interesting to point out that this is the case with the typical PI controller, where the model mismatch comes from the fact that all lumped system disturbances are expressed as the product of a unit regressor function by a constant parameter.

4.2. Second-order sliding-mode control

High-order SMCs discussed in detail in Levant (1993) combine the finite-time convergence and robustness features of conventional SMC schemes with chattering attenuation, and more importantly, they can be applied to systems with relative degree higher than 1. The *Super-twisting Sliding Mode controller (STSMC)* presented in Levant (2002, 2007); Bartolini et al. (2000) is a second-order SMC, which for systems with relative degree 1 ensures finite-time convergence and reduced chattering. Given a scalar dynamical system perturbed by a bounded perturbation $\psi(t, \mathbf{x}(t))$

$$\dot{s}(\mathbf{x}(t)) = \psi(t, \mathbf{x}(t)) + bu, \quad b \neq 0, \quad (4.27)$$

where $\mathbf{x}(t)$ are bounded internal states, the STSMC

$$u \triangleq u_{SM} = -\frac{1}{b} \left(c_1 |s|^{\frac{1}{2}} \text{sgn}(s) + c_2 \int_0^t \text{sgn}(s(\tau)) d\tau \right) \quad (4.28)$$

with c_1, c_2 appropriate positive gains ensures that the system reaches the manifold

$$\mathcal{S} = \{s(t) \in \mathbb{R} | s = \dot{s} = 0\}, \quad (4.29)$$

referred to as the *sliding manifold*, in finite time. In Moreno and Osorio (2012) it is proven by using Strict Lyapunov functions that if $|\psi(t, \mathbf{x})| \leq \delta |s|^{\frac{1}{2}}$, where $\delta > 0$, finite-time convergence is always possible for appropriate selection of the positive gains c_1, c_2 . The variable s is often called the *sliding variable*, it is a function of the states of the system and their derivatives and it can explicitly depend on time.

Two variations of the STSMC for direct load position control will be presented in the following, namely the Output Super-twisting Sliding Mode controller (OSTSMC) and the Adaptive Super-twisting Sliding Mode controller (ASTSMC).

4.2.1. OSTSMC

The architecture of the OSTSMC is shown in Figure 6. The main idea relates to appropriately designing the sliding variable s , such that the relative degree of the tracking error system is reduced Shtessel et al. (2014); Edwards et al. (2000), i.e. the control input directly affects the sliding surface. In this way any sliding-mode control algorithm can be applied to systems with relative degree higher than 1. For the regulation of the

load position error at zero the following auxiliary variables are defined:

$$\begin{aligned} e_1 &\triangleq \theta_l - \theta_r \\ e_2 &\triangleq \dot{e}_1 = \omega_l - \dot{\theta}_r. \end{aligned}$$

Selecting the sliding variable s as a linear combination of $e_1, e_2, \dot{e}_2$ allows establishing a desired dynamical behavior for the positioning error (and its derivative) once the system reaches the sliding manifold $\mathcal{S}$ defined in (4.29). With this approach the STSMC algorithm can be applied circumventing the problem that the drive-train system has relative degree 3.

The sliding variable is defined as

$$s \triangleq \dot{e}_2 + (\lambda_1 + \lambda_2) e_2 + \lambda_1 \lambda_2 e_1. \quad (4.30)$$

where λ_1, λ_2 are real positive constants. Its dynamics reads:

$$\begin{aligned} \dot{s} &= \ddot{\omega}_l - \ddot{\theta}_r^{(3)} + (\lambda_1 + \lambda_2) (\dot{\omega}_l - \ddot{\theta}_r) + \lambda_1 \lambda_2 (\omega_l - \dot{\theta}_r) \\ &= \frac{1}{J_l} \left[K_S \left(\frac{1}{N} \omega_m - \omega_l \right) - D_S \dot{\omega}_l - \dot{T}_{F,l} - \frac{D_S}{J_m} \left(\frac{1}{N} T_l + T_{F,m} \right) \right] \\ &\quad - \ddot{\theta}_r^{(3)} + (\lambda_1 + \lambda_2) (\dot{\omega}_l - \ddot{\theta}_r) + \lambda_1 \lambda_2 (\omega_l - \dot{\theta}_r) + \frac{D_S}{J_m J_l} u. \end{aligned}$$

Defining the control signal

$$u = u_{SM} + \frac{J_m J_l}{D_S} \left[\ddot{\theta}_r^{(3)} + (\lambda_1 + \lambda_2) \ddot{\theta}_r - \lambda_1 \lambda_2 (\omega_l - \dot{\theta}_r) \right] \quad (4.31)$$

to cancel known terms the sliding surface dynamics finally reads:

$$\dot{s} = \psi(t, \mathbf{x}) + bu_{SM} \quad (4.32)$$

where

$$\begin{aligned} \psi(t, \mathbf{x}) &= \frac{1}{J_l} \left[K_S \left(\frac{1}{N} \omega_m - \omega_l \right) - D_S \dot{\omega}_l - \dot{T}_{F,l} - \frac{D_S}{J_m} \left(\frac{T_l}{N} + T_{F,m} \right) \right] \\ &\quad + (\lambda_1 + \lambda_2) \dot{\omega}_l \\ b &= \frac{D_S}{J_m J_l}. \end{aligned}$$

Application of the control law (4.28) will bring the system (4.32) on the sliding manifold $\mathcal{S}$ in finite-time, i.e. $\dot{s} = s = 0$.

Figure 6: OSTSMC direct position controller.

From the definition of s and e_1, e_2 , this leads to the following dynamical system

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\lambda_1 \lambda_2 & -(\lambda_1 + \lambda_2) \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \quad (4.33)$$

which is a Linear Time-Invariant (LTI) system with its eigenvalues $-\lambda_1, -\lambda_2$ being design parameters. Choosing λ_1 and λ_2 positive ensures that the position error and its first time-derivative decay to 0 exponentially, i.e. $e_1^* = 0$ is Exponentially Stable (ES).

Remark 3. Modelling friction errors, especially at low speeds, as well as input disturbance compensation residuals are always bounded and can be lumped into the bounded perturbation $\psi(t, \mathbf{x})$. As such, no changes in the sliding mode algorithm are necessary.

Remark 4. The design of the sliding surface s requires knowledge of $\dot{e}_2$, i.e. of $\dot{\omega}_l$, which is not measured. For the estimation of this signal any dynamic differentiator or observer can be used (e.g. sliding-mode exact differentiators Levant (2001)). A Luenberger linear observer is used in this design. Any bounded acceleration estimation error will affect the accuracy of the algorithm since it will affect the calculation of the sliding surface. Indeed, if s_{real} is the real sliding variable, i.e. assuming that $\dot{\omega}_l$ is available, then, based on Equation (4.30), the expression used for the implementation of the algorithm reads $s = s_{real} - \dot{\hat{\omega}}_l$, where the acceleration estimation error $\dot{\hat{\omega}}_l$ is bounded. This means that the algorithm will ideally lead to $s_{real} = \dot{\hat{\omega}}_l$ instead of $s_{real} = 0$, which shows that the acceleration error has to be kept as small as possible.

Remark 5. As it can be seen from the design of the algorithm, there are two possible strategies for tuning the controller. One is increasing the gains c_1, c_2 . By doing so, the system reaches the sliding manifold $\mathcal{S}$ in finite time and remains thereafter for all future times. Then, according to (4.33), the positioning error will decay exponentially to zero. Increasing the controller gains too much, however, is usually not desirable since it often leads into exerting large torques at high frequency switching, which is damaging for the actuator. The second tuning strategy pertains to increasing the magnitude of the dominant pole λ_1 (and consequently of λ_2), while keeping c_1, c_2 low. In this case, the system does not reach the sliding manifold in finite-time and if it does, it may move away from it temporarily (e.g. during change of motion direction). Then $s \neq 0$. Let $s = \Delta(t)$, where $\Delta(t)$ is a bounded scalar function. Then one can obtain from Equation (4.30):

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\lambda_1 \lambda_2 & -(\lambda_1 + \lambda_2) \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Delta(t) \quad (4.34)$$

From (4.34) and the general solution of LTI systems it is clear that faster poles λ_1, λ_2 will decrease the effect of the input $\Delta(t)$ on the states, i.e. the positioning error and its first derivative.

4.2.2. ASTSMC

The design of ASTSMC presented in Krimpmann et al. (2016) extends the OSTSMC by an adaptive law for the dominant pole λ_1 of the system (4.33). The core control algorithm is

also given by (4.28), (4.31). The architecture of the ASTSMC can be seen in Figure 7.

The adaptive law for λ_1 is based on the average absolute positioning error. Indeed, by using Lyapunov stability theory and the analysis of the STSMC done in Moreno and Osorio (2012), the authors in Krimpmann et al. (2016) proved that the adaptive law

$$\dot{\lambda}_1 = -\gamma_\lambda \text{sgn}(e_1^{av} - \varepsilon_\theta), \quad \gamma_\lambda > 0 \quad (4.35)$$

in combination with the OSTSMC described in (4.28), (4.30) ensures finite time convergence of the load position error e_1 to the compact set $\mathcal{E}_\theta = \{\zeta \in \mathbb{R} \mid |\zeta| \leq \varepsilon_\theta\}$, where $\varepsilon_\theta > 0$ expresses the positioning accuracy limit. Considering λ_1 as the dominant (slower) eigenvalue of system (4.33), λ_2 can be selected proportionally to λ_1 , i.e.

$$\lambda_2 = a\lambda_1, \quad a > 1. \quad (4.36)$$

The average absolute positioning error e_1^{av} used in (4.35) is given over a horizon of n samples by Krimpmann et al. (2016)

$$e_1^{av} = \frac{1}{n} \sum_{i=0}^{n-1} |e_1(t - iT_s)|$$

where T_s is the sampling period. Since it is the maximum error peaks that define the accuracy in the positioning, the threshold ε_θ in (4.35) can be compared to the maximum peak e_1^{peak} of the error signal within the same time horizon of n samples, instead of the error average e_1^{av} . Specifically, by defining

$$e_1^{peak} = \sup_{0 \leq i \leq n} |e_1(t - iT_s)| \quad (4.37)$$

the sliding surface poles change depending on how narrow the $\pm e_1^{peak}$ envelope that contains the position tracking error is. This, in principle, improves the overall efficiency of the controller since the poles, and by extension the control effort, increase only to the level where the positioning error is no larger than the pre-specified bound. Increasing the magnitude of λ_1 beyond a certain value may lead to instability of the closed-loop system. For this reason, a saturation functionality is used for λ_1 .

Remark 6. As in the case of the ABSC, the signum function is replaced by its approximation introduced in Equation (3.8). This substitution alleviates the chatter in the control signal and reduces the sensitivity of the algorithm to measurement

Figure 7: ASTSMC direct position controller.

Table 2: Methods' requirements and key properties.

Requirements	Theoretical properties
ABSC	
<ul style="list-style-type: none"> • Unknown parameters are constant (4.1) • Regressor functions are Lipschitz continuous 	<ul style="list-style-type: none"> • Position error converges to zero • Uniformly bounded parameter estimation error • Parameter estimation if the regressors are persistently exciting
OSTSMC	
<ul style="list-style-type: none"> • Perturbation is Lipschitz continuous 	<ul style="list-style-type: none"> • Position error is exponentially stable • Robustness against any type of Lipschitz disturbances and model uncertainties
ASTSMC	
<ul style="list-style-type: none"> • Perturbation is Lipschitz continuous 	<ul style="list-style-type: none"> • Position error is exponentially stable • Robustness against any type of Lipschitz disturbances and model uncertainties • Sliding surface poles are adjusted to the positioning error magnitude

noise, at the cost of some loss in positioning accuracy. Even if the gains c_1, c_2 are appropriately chosen to ensure finite-time tracking, the sliding variable will converge to a narrow band around zero, where the error will remain confined. The width of this error band depends on how close the approximation $\nu(s, p)$ is to the signum function. Hence, the scaling factor p in (3.8) defines the trade-off between chatter levels and loss of accuracy.

The theoretical requirements considered for each control method, as well as the main features of the four nonlinear controllers are summarised in Table 2.

5. Experiments and evaluation

5.1. Experimental setup

The experimental setup consists of two Siemens 1FT7042-5AF70 PMSMs connected through a stainless steel shaft. The system is equipped with a Siemens SINAMICS S120 drive converter with 22-bit IC22DQ incremental encoders for the position of both motors. Consequently, the theoretical (sensor-based) angular resolution of the drive is approximately $1.5 \mu\text{rad}$ as shown in Figure 9. The motors velocities readings are obtained by numerical differentiation (already integrated into the commissioning software) of the position values and are inflicted with zero-mean white Gaussian noise with standard deviation approximately $\sigma_{meas} = 9 \cdot 10^{-3} \text{rad s}^{-1}$ (see Figure 9). As already mentioned in Section 2, the second motor is inactive and serves as pure inertia (emulating the load). A cylindrical Vari-tork 279.25.22 adjustable-friction clutch Dyratok (2017) is mounted on a steel base and houses the interconnecting shaft (see Figure

Figure 8: Schematic of Vari-tork 273.25.22 adjustable-friction clutch: (1) inner bearing housing the interconnecting shaft, (2) outer bearing.

Figure 9: (Top left) Drive motor zero velocity measurement. (Top right) Probability distribution of velocity measurement noise. (Bottom): Drive motor zero position measurement.

8). The friction is developed between the inner cylinder, housing the shaft, and the outer bearing of the component. A ring adjusts the friction between these two surfaces, increasing it as it turns clockwise. The entire drive train with the friction component is shown in Figure 10.

The current control loop in the drive motor consists of two PI controllers, one for magnetizing and one for torque-generating. The design and tuning of these controllers was not included in this study. All the control algorithms (including the P-PI) were implemented as custom-made software upgrades and integrated into the Siemens commissioning software STARTER Siemens (2018). The sampling time of the S120 control unit, at which the algorithms run, was $125 \mu\text{s}$.

5.2. Test scenarios

Fifteen different scenarios were considered for the experimental assessment of the performance of the control algorithms. A sinusoidal reference signal

$$\theta_r(t) = \Theta_0 \sin(2\pi f_r t),$$

Table 3: Test scenarios. In connection to Problem 1, the motor Coulomb friction bound is taken equal to $T_{C,m}^{max} = 0.15$ N m.

No	Θ_0 (rad)	$T_{C,m}$ (N m)	f_r (Hz)	Increase in friction
1	1	0.035	0.1	0
2	1	0.035	0.5	0
3	1	0.035	2	0
4	1	0.11	0.1	215%
5	1	0.11	0.5	215%
6	1	0.11	2	215%
7	1	0.15	0.1	330%
8	1	0.15	0.5	330%
9	1	0.15	2	330%
10	1	0.25	0.1	615%
11	1	0.25	0.5	615%
12	1	0.25	2	615%
13	1	0.35	0.1	900%
14	1	0.35	0.5	900%
15	1	0.35	2	900%

where Θ_0 is the position amplitude in rad and f_r is the frequency in Hz, was used as a basis for the test scenarios. Such reference signal could describe the motion profile of one axis of a machine tool that follows a circular contour. The choice of this reference signal was also motivated by the fact that the largest tool-positioning errors in machine tools occur during the change of motion direction of the axes Gross et al. (2001). Five cases of friction (including the nominal case) were considered, with the last two of them corresponding to extreme friction values. These values were used to assess the performance limits of the designed controllers. Three speed profiles were considered (very slow, slow, fast) by varying the sinusoidal signal frequency. The fifteen different scenarios are shown in Table 3.

The controllers were tuned in the nominal friction case at 0.5

Figure 10: Experimental setup: (1) 1FT7 drive PMSM, (2) 1FT7 load PMSM, (3) shaft, (4) friction component base, (5) Siemens SINAMICS S120 converter, (6) shaft housing, (7) friction adjustment ring.

Hz, such that the maximum absolute positioning error is no larger than $5 \cdot 10^{-3}$ rad for all control schemes (whenever this was possible). In the P-PI scheme $\dot{\theta}_r$ was added as a feedforward term to the position P controller output. The conventional P-PI controller was tuned based on tests at 0.5Hz operation (gradually increasing the PI proportional gain, then decreasing the integrator reset time constant and finally increasing the position proportional gain Gross et al. (2001)).

The gains for the nonlinear controllers were selected as close as possible to the equivalent P-PI parameters. For instance, from Equations (4.10)-(4.13) it can be seen that the expression $\frac{J_m D_s}{J_r N} c$ is equivalent to the outer loop position proportional gain k_{pos} in the P-PI scheme. This suggests that a starting point for the selection of c can be the value $k_{pos} \frac{J_r N}{J_m D_s}$. A similar line of thinking was followed for the tuning of the velocity proportional gains k_1, k_2 . The adaptation gains in Equations (4.14)-(4.16) were selected to be small to avoid large overshoots in parameter estimation.

The adaptation gain for the motor Coulomb friction, i.e. the phenomenon associated to the emulated wear, was selected larger than the other gains, so that changes in this parameters (which could indicated wear increase) are captured fast. Lastly, the switching gains of the sliding-mode controllers were selected in a similar way to the PI tuning. First the integral switching gains c_2 were tuned up to provide a basic tracking accuracy and then the proportional gains c_1 were increased until before significant actuator chatter appears. The values of the controllers tuning parameters are presented in Table 4.

5.3. Controllers comparison criteria

A combination of quantitative and qualitative criteria is employed for the comparison of the control algorithms. The first group consists of three performance indices that describe the positioning accuracy and efficiency of each method. Specifically, these indices are Dorf and Bishop (2011):

- Maximum Absolute Error (MAE). This corresponds to the maximum peak of the positioning error $e_\theta = \theta_l - \theta_r$ and it is defined over a time interval $T > 0$ as

$$MAE = \sup_{t_0 \leq t \leq t_0 + T} |e_\theta(t)|. \quad (5.1)$$

- Control Power (CP). This is metric of the average power of the control signal. It is defined over a time interval $T > 0$ as

$$CP = \frac{1}{T} \int_{t_0}^{t_0 + T} u^2(t) dt. \quad (5.2)$$

- Error-Control Power product (ECP). This describes the efficiency of the control algorithm. Lower ECP corresponds to greater efficiency. It is defined over a time interval $T > 0$ as

$$ECP = MAE \cdot CP = \sup_{t_0 \leq t \leq t_0 + T} |e_\theta(t)| \cdot \frac{1}{T} \int_{t_0}^{t_0 + T} u^2(t) dt. \quad (5.3)$$

Table 4: Selected values for the parameters of the positioning methods.

Symbol	Description	Value
Outer loop P and $\dot{\omega}_l$ estimation		
k_{pos}	Proportional gain	9
k_{obs}	$\dot{\omega}_l$ observer gain	75.4
PI		
k_p	Proportional gain	0.9
T_n	Integral reset time	0.06
ABSC		
k_1	z_1 gain	200
k_2	z_2 gain	200
c	Proportional gain	54
$\text{diag}(\mathbf{T})$	Adaptation gains	$[10^{-5}, 0.012 \cdot (1, 10, 1, 1, 1)]$
$[\gamma_1, \gamma_2]$	Adaptation gains	$[0.2, 0.01]$
$\hat{\mathbf{d}}_0$	Initial conditions	$[17, 0.01, 0, 0.01, 0, 0.01]^T$
$[\hat{\rho}_0, \hat{b}_0]$	Initial conditions	$[19, 0.01]$
OSTSMC		
c_1	Switching gain	0.001
c_2	Integral sw. gain	5.4
λ_1	Sliding mode pole	17
λ_2	Sliding mode pole	35
ASTSMC		
c_1	Switching gain	0.001
c_2	Integral sw. gain	5.4
$\lambda_{1,0}$	Initial conditions	10
γ_λ	Adaptation gain	0.5
a	Pole scaling	2
ε_θ	Accuracy limit	10 (mrad)
n	Number of samples	500
T_s	e_1 sampling period	4 (ms)

- Complexity Index (CI). This reflects the design complexity of each control strategy. It is defined as the sum of the number of tunable parameters N_p plus the number of signals N_s required for the implementation of each controller:

$$CI = N_p + N_s. \quad (5.4)$$

An additional criterion that is indicative for the lag between reference and actual position of the tool is the Maximum Absolute Phase Error (MAPE). This error is important in machines with more than two axes, since delays in positioning, in combination with no synchronization between the axes, may result in a distorted contour Gross et al. (2001); Smith (2016). Since there is only one axis, the experiments are run twice with the reference being shifted by $\frac{\pi}{2}$ during the second time in order to emulate a circular trajectory. Specifically, the position signals

are scaled appropriately so that they both have amplitude 1. The scaled x,y positions are given by

$$\theta_{x,y}(t) = \frac{1}{\Theta_0} \theta_l(t)$$

and the corresponding reference signals

$$\begin{aligned} \theta_x^r(t) &= \frac{1}{\Theta_0} \theta_r(t) = \sin(2\pi f_r t) \\ \theta_y^r(t) &= -\frac{1}{\Theta_0} \theta_r \left(t - \frac{\pi}{2} \right) = \cos(2\pi f_r t). \end{aligned}$$

Assuming that the axes have identical characteristics and control system, the MAPE is defined over a time interval $T > 0$ as

$$MAPE = \sup_{t_0 \leq t \leq t_0 + T} |e_\phi(t)| \quad \text{with} \quad (5.5)$$

$$e_\phi(t) = \tan^{-1} \left(\frac{\theta_y^r(t)}{\theta_x^r(t)} \right) - \tan^{-1} \left(\frac{\theta_y(t)}{\theta_x(t)} \right). \quad (5.6)$$

It should be noted that, although phase errors are primarily relevant for machines with multiple-axes, the MAPE is indicative of the positioning accuracy of just one axis. In the case of machines with more degrees of freedom, additional lag can be induced due to lack of synchronization between the different axes Gross et al. (2001). Added to the total lag coming from response delays at each axis, this distorts the actual machining trajectory profiles.

Lastly, a qualitative criterion is also used for the evaluation of the controllers along with the metrics introduced above. It is a graphical representation of the MAPE in an $\theta_x - \theta_y$ plot, similar to the ones used in a Circular Interpolation Test (CIT) in machine tools Smith (2016). Larger deviations from the unit-radius circular path imply increased radial or lag errors.

5.4. Results

The section presents the results obtained by the experimental evaluation of the control methods. The controllers were evaluated over the last 20s of each experiment, which had duration of 540s. This ensured that starting-up errors did not influence the performance assessment and that there was sufficient time for the adaptation of any parameters.

The MAE for each control in all friction cases and reference profiles is presented in Table 5. It can be seen that the ABSC has consistently the best performance in all tests. Specifically, the MAE of the ABSC is kept well below the accuracy limit in the first four friction cases at 0.1 and 0.5 Hz. During operation at 2 Hz the controller marginally performs above the accuracy limit. The performance gradually deteriorates as friction increases but the MAE is 30-60% smaller than that of the P-PI. In the worst "realistic" scenario (Test 13) the ABSC MAE is 4.2 mrad above the accuracy limit and it is 35% smaller than in the P-PI case. The accuracy of ABSC can be improved by increasing the gains k_1, k_2 at the cost, however, of making the method more sensitive to measurement noise. Figure 11 illustrates the estimation of the motor Coulomb friction and the product $\hat{\rho}\hat{b}$ during Test

Table 5: MAE in mrad for all controllers in all scenarios. The indices 1-4 denote the four different friction cases.

Controller	nominal	1	2	3	4
MAE in mrad at 0.1 Hz					
-	Test 1	Test 4	Test 7	Test 10	Test 13
P-PI	12	17.9	20.3	24.4	30.3
ABSC	4.3	5.5	6.2	6.2	12.8
OSTSMC	6.6	5.6	7.4	9.8	15.6
ASTSMC	6.4	6.5	8.9	12.4	16.9
MAE in mrad at 0.5 Hz					
-	Test 2	Test 5	Test 8	Test 11	Test 14
P-PI	4.9	15.6	17	23.7	37.2
ABSC	3.4	4.7	5.7	7	13.8
OSTSMC	10.9	13.6	14.4	19.6	62.5
ASTSMC	10.5	14.2	19.1	28.7	57.5
MAE in mrad at 2 Hz					
-	Test 3	Test 6	Test 9	Test 12	Test 15
P-PI	12.3	20.1	21.8	31.6	45.7
ABSC	12.9	13.7	14.2	15.3	19.2
OSTSMC	49.1	118.9	177.7	256.5	448.6
ASTSMC	60.1	102.4	214.1	242.6	480.7

5. It can be seen that the friction estimate approaches a value close to the real Coulomb friction. Moreover, the product $\hat{\rho}\hat{b}$ converges to 1, which is its real value since by definition $\rho = \frac{1}{b}$. The rest of the parameter estimates do not necessarily converge to the real system parameter values but the estimation errors are bounded for all future times, as this can be seen in Figure 12. Regarding the implicit velocity control part of the ABSC, it can be seen from Figure 13, the drive motor velocity follows a smooth profile, similar to the ones observed in the case of position-velocity cascaded control.

Both of the sliding mode designs outperform the P-PI only in the first three tests (0.1 Hz with no extreme friction values) while they are consistently worse than the ABSC. At 0.5 Hz they have similar accuracy to the P-PI (except for the last friction case) and at 2 Hz their performance rapidly deteriorates, with MAE up to 10 times larger than the one in the P-PI case. Figures 14 and 15 show the positioning errors and corresponding torque commands for all the controllers during Tests 7 and 9, respectively.

From Table 5 it can be seen that the OSTSMC and ASTSMC have similar performance in terms of both accuracy and control input profile. This is expected since the only difference in their design is that in the case of ASTSMC the poles shaping the sliding surface are adapted to the positioning error magnitude rather than being fixed. The saturation and scaling of the poles is tuned such that whenever the accuracy requirement is not met, the ASTSMC design converges to the one of OSTSMC, i.e. the selected values for λ_1, λ_2 in OSTSMC are the pole saturation

Figure 11: (Top): Estimation of motor Coulomb friction $T_{C,m}$ from the ABSC algorithm during Test 5. (Bottom): Product of the uncertain input gain b and its inverse ρ during Test 5.

limits in ASTSMC. This can also be seen in Figure 16, where the adaptation of the sliding surface poles and the position error envelope are depicted.

Another reason for which the OSTSMC and ASTSMC show worse performance for fast reference profiles is the fact that the design of the sliding variable requires knowledge of $\dot{\omega}_l$, which is not measured. Since the load acceleration is estimated by a linear observer, the quality of the estimation depends on how large the observer proportional gain is in comparison to how fast the velocity changes. The effect of noise on the measured signals does not allow arbitrary increase of the observer gain and, as such, faster reference profiles lead to less accurate estimation of $\dot{\omega}_l$. Using high observer gains require appropriate lowpass filtering that will remove the effect of high-frequency noise. This, however, introduces additional lag to the estimation signal that further reduces the estimation accuracy, as can be seen in Figure 17. Improvement of the performance of these two sliding mode methods requires considerably increasing the gains c_1, c_2 . This however also induces more chattering in the control signal (see bottom plot in Figure 15) that can be damaging for the actuator. The average MAE and ECP of the controllers for all friction cases over the three operation frequencies are depicted in Figure 18.

A comparative visualisation of the MAPE for all the controllers is given in Figure 19, where the $\theta_x - \theta_y$ plots for the largest "realistic" friction value (Tests 7-9) are shown for each controller. It can clearly be seen from the distortion of the circular paths that the ABSC outperforms the P-PI and maintains the positioning tolerances, with the MAE being marginally above the performance limit only at 2 Hz. On the contrary, both the OSTSMC and the ASTSMC perform sufficiently well only at 0.1 Hz, where their degradation at 2 Hz is clearly worse than that of the P-PI. These plots also suggest that phase lag between reference signal and actual position response significantly contributes to the positioning error in the sliding mode designs at 2 Hz. On the contrary, the MAPE in the ABSC case is the smallest compared to all the other methods, which indicates that the ABSC positioning error is mainly due to amplitude deviations

Figure 12: Drive train parameters estimation during Test 5. Although the estimates do not necessarily converge to the real (offline identified) parameter values, the estimation errors remain bounded for all future times.

from the position setpoint at each time instant.

Lastly, the design and implementation complexity of each control strategy can be deduced from the number of tunable parameters and required signals, shown in Table 6. As it can be seen, the ABSC is the most demanding algorithm with respect to tuning effort, since it has 19 tunable parameters and its design requires 8 signals to be available. The ASTSMC and the OSTSMC follow in complexity with 9 and 5 tunable parameters, respectively and the same number of required signals, namely 6. The P-PI has the simplest design with only 3 tunable parameters and 4 required signals.

6. Discussion

The experimental results showed that the ABSC outperformed all the other methods in all the tests. Specifically, the positioning tolerances were kept for all the cases of "realistic" friction degradation. The two sliding-mode schemes, namely the OSTSMC and ASTSMC have no substantial advantages compared to the state-of-the-art P-PI solution, in terms of

Figure 13: (Top) Motor velocity ω_m during operation at 2 Hz with the ABSC. The produced velocity is a smooth signal similar to the ones obtained in the cascaded control architectures. (Bottom) Motor velocity tracking errors e_ω and z_2 for the P-PI and ABSC, respectively.

either positioning accuracy or resilience to unknown and increasing friction. The design of the sliding surface on the basis of the "direct-position control" architecture requires the use of high gains, so that the finite-time convergence and disturbance-rejection features of the sliding-mode algorithms are achieved. This comes from the well known condition in second-order sliding-mode algorithms that the integral switching gain (here it corresponds to c_2) has to be chosen larger than the second time-derivative of the perturbation that needs to be rejected Chalanga and Plestan (2017). In the case of the drive train system in this study, however, such a selection leads to high controller gains, which is impractical due to the significant actuator chattering. Consequently, the performance degradation of the OSTSMC and the ASTSMC was more severe than in the case of the P-PI.

An overall ranking of the methods based on the average MAE over all the experiments and the complexity index is illustrated in Figure 20. It can be seen that the ABSC provides the best accuracy and friction resilience at the cost, however, of higher design and application complexity. It is also interesting to note

Table 6: Number of tunable parameters and signals required for each controller.

	Tunable	Total	Signals	Total
P-PI	k_{pos}, k_p, T_n	3	$\theta_r, \dot{\theta}_r, \theta_l, \omega_m$	4
ABSC	$c, k_1, k_2, \gamma_1, \gamma_2$ $\Gamma, \hat{\theta}_0, \hat{\rho}_0, \hat{b}_0$	19	$\theta_r, \dot{\theta}_r, \ddot{\theta}_r, \theta_r^{(3)} \mathbf{x}$	8
OSTSMC	$c_1, c_2, \lambda_1, \lambda_2, k_{obs}$	5	$\theta_r, \dot{\theta}_r, \ddot{\theta}_r$ $\theta_r^{(3)}, \theta_l, \omega_l$	6
ASTSMC	$c_1, c_2, \lambda_{1,0}, \gamma_\lambda$ $a, \varepsilon_\theta, \nu, T_s, k_{obs}$	9	$\theta_r, \dot{\theta}_r, \ddot{\theta}_r$ $\theta_r^{(3)}, \theta_l, \omega_l$	6

Figure 14: Positioning errors and torque commands for Test 7.

Figure 15: Positioning errors and torque commands for Test 9.

that the modelling error for the friction especially at low speed operation, where all the nonlinear presliding phenomena dominate, mostly affects the ABSC, since it is the only algorithm that benefits from an accurate description of friction. This is also reflected in the parameter estimation signals, especially in the Coulomb constant estimate, where small spikes, consistent to the reversal of motion, can be observed (see Figure 11). This is due to the fact that the algorithm tries to compensate for the stick-slip phenomenon observed in the velocity tracking error by adjusting the estimated parameters.

7. Conclusions

This paper discussed the design and application of three friction-resilient nonlinear control methods, namely the adaptive backstepping controller, the output super-twisting sliding-

Figure 16: (Top): Evolution of the ASTSMC sliding surface poles λ_1, λ_2 during Test 8. (Bottom): Position error e_1 and its amplitude envelope $\pm e_1^{peak}$ during Test 8.

mode controller and the adaptive super-twisting sliding-mode controller, for high-accuracy positioning in machine tools. All three strategies shared a direct-position control architecture, i.e. no cascaded structure was employed. In this way, friction and model uncertainties on both the drive-motor and the load were addressed. The controllers were implemented and tested on a real single-axis drive train with state-of-the-art Siemens equipment. The performance of each technique was assessed in terms of both maximum deviation from the reference trajectory and overall efficiency at nominal and increased-friction operation. The design and implementation complexity of each method was also considered. A systematic comparison with conventional P-PI solutions showed that the adaptive backstepping controller outperformed the P-PI scheme in all tests, maintaining the positioning tolerances even under unknown increased friction.

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Appendix A. Basic definitions

Definition 1 (Persistence of Excitation (PE) Ioannou and Sun (1996)). A piecewise continuous signal vector $\phi : \mathbb{R}^+ \rightarrow \mathbb{R}^n$ is Persistently Exciting with a level of excitation $\alpha_0 > 0$ if

Figure 17: Lowpass filtered (*LPF*) and unfiltered estimation of $\dot{\omega}_l$ for OSTSMC at 2 Hz. (Top) Load acceleration estimates. (Bottom) Velocity estimation errors. Smaller observer gains (thick blue line) give similar estimates to those obtained from high-gain observers with filtering.

Figure 18: Average MAE (top) and ECP (bottom) for all friction cases over all three frequency profiles. The dashed line corresponds to the $\pm 10^{-2}$ rad accuracy bound and the stripped area denotes the extreme friction cases.

there exist constants $\alpha_1, T_0 > 0$ such that

$$\alpha_0 \mathbf{I} \leq \frac{1}{T_0} \int_t^{t+T_0} \boldsymbol{\phi}(\tau) \boldsymbol{\phi}^T(\tau) d\tau \leq \alpha_1 \mathbf{I} \quad (\text{A.1})$$

Appendix B. Calculation of $\dot{\alpha}$

$$\begin{aligned} \dot{\alpha} &= \frac{d}{dt} \left\{ \hat{\rho} \left[-\boldsymbol{\phi}_2^T(\mathbf{x}) \hat{\boldsymbol{\theta}} + J_l (\ddot{\theta}_r - c\omega_l + c\dot{\theta}_r - k_1 z_1) \right] \right\} \\ &= \underbrace{\dot{\hat{\rho}} \left[-\boldsymbol{\phi}_2^T(\mathbf{x}) \hat{\boldsymbol{\theta}} + J_l (\ddot{\theta}_r - c\omega_l + c\dot{\theta}_r - k_1 z_1) \right]}_{\psi_1} \\ &\quad + \hat{\rho} \frac{d}{dt} \left[-\boldsymbol{\phi}_2^T(\mathbf{x}) \hat{\boldsymbol{\theta}} + J_l (\ddot{\theta}_r - c\omega_l + c\dot{\theta}_r - k_1 z_1) \right] \\ &= \psi_1 + \hat{\rho} \left[-\frac{d}{dt} (\boldsymbol{\phi}_2^T(\mathbf{x}) \hat{\boldsymbol{\theta}}) + J_l (\theta_r^{(3)} + c\ddot{\theta}_r) \right] \\ &\quad + J_l \hat{\rho} \left[-\frac{c}{J_l} (\boldsymbol{\phi}_2^T(\mathbf{x}) \boldsymbol{\theta} + b\omega_m) - k_1 \dot{z}_1 \right] \\ &= \psi_1 + \hat{\rho} \left[-\frac{d}{dt} (\boldsymbol{\phi}_2^T(\mathbf{x}) \hat{\boldsymbol{\theta}}) + J_l (\theta_r^{(3)} + c\ddot{\theta}_r) \right] \\ &\quad - c\hat{\rho} (\boldsymbol{\phi}_2^T(\mathbf{x}) \boldsymbol{\theta} + b\omega_m) \\ &\quad - J_l k_1 \hat{\rho} \left(\frac{\boldsymbol{\phi}_2^T(\mathbf{x}) \boldsymbol{\theta} + b\omega_m}{J_l} - \ddot{\theta}_r + c\omega_l - c\dot{\theta}_r \right). \end{aligned} \quad (\text{B.1})$$

Moreover, we have

$$\begin{aligned} \frac{d}{dt} (\boldsymbol{\phi}_2^T(\mathbf{x}) \hat{\boldsymbol{\theta}}) &= \boldsymbol{\phi}_2^T(\mathbf{x}) \dot{\hat{\boldsymbol{\theta}}} + \dot{\mathbf{x}}^T \frac{\partial \boldsymbol{\phi}_2^T(\mathbf{x})}{\partial \mathbf{x}} \hat{\boldsymbol{\theta}} \\ &= \boldsymbol{\phi}_2^T(\mathbf{x}) \dot{\hat{\boldsymbol{\theta}}} + \left(\frac{\omega_m}{N} - \omega_l \right) \hat{K}_S - \left(\frac{\hat{D}_S + \hat{\beta}_m}{J_l} \right) (\boldsymbol{\phi}_2^T(\mathbf{x}) \boldsymbol{\theta} + b\omega_m). \end{aligned} \quad (\text{B.2})$$

Substituting (B.2) in (B.1) and using (4.1) yields after some manipulation of the terms:

$$\begin{aligned} \dot{\alpha} &= \underbrace{\psi_1 + \hat{\rho} \left[-\boldsymbol{\phi}_2^T(\mathbf{x}) \dot{\hat{\boldsymbol{\theta}}} - \left(\frac{\omega_m}{N} - \omega_l \right) \hat{K}_S + J_l (\theta_r^{(3)} + c\ddot{\theta}_r) \right]}_{\psi_2} \\ &\quad + \underbrace{\left(\frac{\hat{D}_S + \hat{\beta}_m}{J_l} - c - k_1 \right) (\boldsymbol{\phi}_2^T(\mathbf{x}) \hat{\boldsymbol{\theta}} + \hat{b}\omega_m) - J_l k_1 (c\omega_l - \ddot{\theta}_r - c\dot{\theta}_r)}_{\psi_2} \\ &\quad + \hat{\rho} \left(\frac{\hat{D}_S + \hat{\beta}_l}{J_l} - c - k_1 \right) \boldsymbol{\phi}_2^T(\mathbf{x}) \tilde{\boldsymbol{\theta}} + \hat{\rho} \left(\frac{\hat{D}_S + \hat{\beta}_l}{J_l} - c - k_1 \right) \tilde{b}\omega_m = \\ &= \psi_2 + \hat{\rho} \left(\frac{\hat{D}_S + \hat{\beta}_l}{J_l} - c - k_1 \right) \boldsymbol{\phi}_2^T(\mathbf{x}) \tilde{\boldsymbol{\theta}} + \hat{\rho} \left(\frac{\hat{D}_S + \hat{\beta}_l}{J_l} - c - k_1 \right) \tilde{b}\omega_m. \end{aligned} \quad (\text{B.3})$$

Remark 7. In the calculations above the derivative of the approximated signum function $v(p, \omega_l)$ with respect to the load angular velocity shows up when evaluating the term $\frac{\partial \boldsymbol{\phi}_2^T(\mathbf{x})}{\partial \mathbf{x}}$. This derivative is practically everywhere equal to 0 except in

Figure 19: $\theta_x - \theta_y$ plot for one cycle during Tests 7 (left column), 8 (middle column) and 9 (right column). From top to bottom: P-PI, ABSC, OSTSMC, ASTSMC. The dashed circles correspond to the $\pm 10^{-2}$ rad accuracy limit. For increased clarity, both the error and the accuracy limits have been enlarged by a factor of 5.

Figure 20: (Top): Average MAE over all tests for all controllers. The dashed line corresponds to the $\pm 10^{-2}$ rad accuracy bound. (Bottom): Complexity index as the sum of tunable parameters and required signals for each controller. The associated value is written on the top of each column.

a neighbourhood around zero velocity. Indeed, evaluating this derivative gives

$$\frac{\partial}{\partial \omega_l} v(p, \omega_l) = \frac{2}{\pi} \frac{p}{1 + (p\omega_l)^2} \quad (\text{B.4})$$

which is equal to $\frac{2p}{\pi}$ at $\omega_l = 0$ and vanishes when $|\omega_l| \rightarrow \infty$. For large values of the scaling factor p (e.g. $p \geq 100$), for which $v(p, \omega_l)$ approximates $\text{sgn}(\omega_l)$ with sufficient accuracy, the derivative with respect to ω_l has a spike around $\omega_l = 0$, proportionally large to p . The larger the p factor is, however, the smaller the 0-neighbourhood in which the non-zero derivative exists is. In real-life applications the effect of measurement noise may introduce unwanted compensation terms at low (close to zero) speeds if the term in (B.4) is included in the control design, even if the direction of motion has not changed (see Figure B.21). For this reason and without loss of generality, the derivative of $v(p, \omega_l)$ is considered zero everywhere.

Appendix C. Proof of ISS property of e_θ

Define the continuously differentiable function $V_e(e_\theta) = \frac{1}{2}e_\theta^2$ for which the following inequality holds:

$$\alpha_1(|e_\theta|) \leq V_e \leq \alpha_2(|e_\theta|)$$

with $\alpha_1(x) = \frac{1}{4}x^2$ and $\alpha_2(y) = x^2$ being class $\mathcal{K}_\infty$ functions. Taking the time derivative of V_e along the trajectories of the error system in (4.26) yields

$$\begin{aligned} \dot{V}_e &= -ce_\theta^2 - e_\theta z_1 \leq -c|e_\theta|^2 + |e_\theta| \cdot |z_1| = \\ &= -c(1-q)|e_\theta|^2 - |e_\theta|(cq|e_\theta| - |z_1|) \\ &\leq -c(1-q)|e_\theta|^2 \triangleq \alpha_4(|e_\theta|), \\ \forall |e_\theta| &\geq \frac{1}{cq}|z_1| \triangleq \alpha_3(|z_1|) \end{aligned}$$

Figure B.21: (Top): Comparison of the signum function and its approximation $v(\cdot)$ during a change of the drive motor motion. (Bottom): For small values of the scaling factor p (poor approximation) the derivative of the approximated signum function takes undesired non-zero values even before the direction of motion changes.

where $0 < q < 1$, $\alpha_4(\cdot)$ is a class $\mathcal{K}_\infty$ function and $\alpha_3(\cdot)$ is positive definite in $\mathbb{R}$. Then according to Theorem 4.19 in (Khalil, 2002, p. 176) the system in (4.26) is ISS with respect to the input z_1 .

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