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# Planning and Scheduling Operating Rooms for Elective and Emergency Surgeries with Uncertain Duration

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## Abstract

In this paper we investigate the planning of operating rooms at Rigshospitalet, a large Danish hospital. Each operation must be assigned to a specific operating room and also be scheduled for a specific time while taking into account clinical guidelines. Both elective and emergency operations are included, such that the elective operations are planned while still taking potential emergency operations into consideration. Furthermore, the duration of each operation is uncertain. The aim is to construct robust operating room schedules that minimise overtime work and release unused capacity.

Due to the uncertainty associated with arrival of emergency patients and also the duration of each operation, a deterministic model is not suitable for this problem. Therefore, we develop a stochastic model where operation duration can vary and where the arrivals of emergency patients are unknown. The stochastic model is computationally heavy, so two mixed integer programming based heuristics denoted 2-Step Relax-and-Fix and All Open Relax-and-Fix are developed to solve the problem.

The computational study is based on an extensive dataset comprising 304 days. The heuristics give good results with half of the operating rooms having less than 8 minutes of overtime work. To test the robustness of the solutions we carry out a simulated implementation of the operation plans. The simulation shows that the heuristic solutions are fairly robust. In general, results show a clear potential for implementing the method for planning and scheduling of operating rooms at Rigshospitalet.

*Keywords:* Operating room, Planning, Scheduling, Stochastic model, Heuristic  
*2000 MSC:* 90B36, 90B90, 90C15, 90C59

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## 1. Introduction

Hospitals are often very large and complex with many different functions, and Rigshospitalet, which is situated in central Copenhagen, is one of the largest hospitals in Denmark. In 2016 Rigshospitalet had 1,377 beds and around 10,200 employees covering more than 50 professional groups. One of the most expensive and resource demanding functions is to perform surgeries. Rigshospitalet performed 96,788 operations in 2014 including both elective and emergency patients. Elective patients are often planned well in advance after one or more consultations, while emergency patients show up unexpected on the day and have to be operated within a certain time limit. This can be anywhere from 30 minutes to 24 hours depending on how serious the emergency is. A total of 188,488 unique patients were treated in 2013. With so many activities going on it is of great importance that the resources, whether it is operating rooms (ORs), nurses, anaesthetists or surgeons, are used in the most efficient way. In Green (2004) they even write that “the efficient use of ORs, which are often bottlenecks, can be central to the smooth functioning of the hospital as a whole”.

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It requires complex planning to ensure efficient use of ORs. Planning is currently being done manually at Rigshospitalet with help from the planning tool “Orbit”. Orbit is a software tool, where nurses or doctors simply drag and drop operations either in order to add them to the schedule or in order to move an operation in time and/or space. Orbit indicates if some rules are violated, and the doctor then evaluates the violation.

As planning is currently done at Rigshospitalet, the date and time of each operation is decided very early for elective patients who are not staying a night at the hospital. Often it is decided during one of the preceding consultations, which can take place several weeks before the operation. This leaves little flexibility in the planning of a day as a whole, as an operation fixed at a certain time can cause either unnecessary wasted time, unnecessary overtime work or even that another operation has to be moved to another day. This might be avoided with better planning. For elective patients who are staying overnight at the hospital, typically only the day is given in advance and not the specific time, which significantly helps with regard to the flexibility in the planning process. In this paper we will assume that for all elective patients only the day is given well in advance, whereas the time of the day will be decided upon the day before the operation. As we at this point know all the elective operations for a given day, greater flexibility is achieved, which makes it possible to make a better plan. Apart from capacity utilisation a hospital must also adhere to strict clinical guidelines concerning quality and patient safety. In Denmark, as in most other countries, studies have found that too many patients experience adverse events, leading to a number of preventable deaths (Schiøler et al. (2001)).

The purpose of this paper is to explore how to efficiently plan which operations to assign to which ORs while simultaneously deciding what time each operation should be scheduled for and taking into account clinical guidelines. The schedules have to be robust with respect to changes during the operation day, such that unforeseen events do not ruin the entire plan. This can be very difficult since we have both elective and emergency patients. In addition, we can never be completely sure about the length of an operation.

We have received data from Rigshospitalet, which has been made anonymous. We have received data for two specialities here denoted S1 and S2. The data covers a period of nearly a year and includes more than 8,000 operations. Both of the selected specialities mostly deal with elective operations and only have a small amount of emergency operations.

The rest of the paper is organised in the following way. Section 2 contains a review of related literature while Section 3 gives a detailed description of the problem of allocating and scheduling operations to ORs at Rigshospitalet. A stochastic model is presented in Section 4. It has remained a challenge to get good solutions within an acceptable running time, and therefore we have developed a heuristic for solving the stochastic problem. Two variants of the heuristic are presented in Section 5. The results for the developed model and heuristics are presented in Section 6. Finally, the main conclusions of the paper can be found in Section 7.

## 2. Literature review

The survey by Cardoen et al. (2010) gives a comprehensive overview of more than 100 papers on OR planning and scheduling published in the period 2000-2009 by classifying the papers according to different aspects of the examined problems. Planning is described as dealing with capacity decisions, while scheduling is the sequence and time allocation of the activities of an operation. In this paper both planning and scheduling are considered, as we both decide which ORs to open and the time allocation and OR for each operation. The review is extended by Samudra et al. (2016) who also present a basic analysis of the research trends in the main areas of OR planning.

One of the aspects used for categorising papers in Cardoen et al. (2010) is performance criteria. A number of different performance criteria are used in the papers, where two of the main criteria are waiting time and utilisation. We use both general utilisation (whether to open or not) and overutilisation (overtime work) of the ORs. The same kind of objective function is used in Denton et al. (2010). Another important aspect of the problem is type and level of decisions. In Monteiro et al. (2015) the authors also include the skill level of the nurses and quality of teamwork as part of the objective function. We decide on the capacity on a discipline level, whereas we decide on the time and room on a patient level.

It is also important to specify what type of patients are considered: Elective and/or non-elective. Both types of patients can be further divided into subtypes. In total, 60 of the papers examined in Cardoen et al. (2010) deal with only elective patients, one paper deals with only non-elective patients, and 19 papers deal with both elective and non-elective patients. We deal with both types of patients in this paper. Interestingly, if we look at Samudra et al. (2016) their analysis on the research publications show that the level of elective vs. emergency patients have remained almost constant over the last 15 years; basically all research contributions deal with elective patients whereas much fewer papers look at emergency patients. As a review, Van Riet and Demeulemeester (2015) more specifically looks at the trade offs in OR planning, the sources of variability and approaches to tackle the trade offs. The allocation of time for emergency patients in the ORs can be done in different ways: One possibility is to allocate a number of ORs to only take care of emergency patients, such that the planning of the elective patients is not disturbed by emergency patients. This method is presented in Smith et al. (2013), where it is concluded that variability management improves the operational and financial performance of a hospital's surgical unit. However, this conclusion is not shared by all papers, as for instance Wullink et al. (2007) reaches the opposite conclusion by using a discrete event simulation model. In Rachuba and Werners (2014) the optimisation approach deals with elective and emergency patient using stochastic optimisation where rooms can be used by both elective and emergency patients. They ensure that the stochastic amount required for emergency patients is divided over a maximum number of ORs while deferring patients to the next planning period in case of insufficient capacity. Part of the focus is to obtain parameters for the stochastic optimisation that are fitted to avoid re-scheduling. Our approach will leave all rooms to be planned by the optimisation methods so we do not reserve specific rooms for emergency patients.

On a more strategic level we also have the case-mix problem for a surgery department. Here uncertainty into surgery duration, length of stay, surgery demand and availability of nurses has to result in a case-mix set up for the surgery department. A recent paper using stochastic optimisation models for this problem is Yahia et al. (2016).

Different solution techniques are used in the literature to solve the OR planning and scheduling problems. In Cardoen et al. (2010) the main solution techniques listed are: mathematical programming, simulation and heuristics. In Cardoen (2010) it is mentioned that especially linear programming gives satisfying results. This is also the basis of our approach. In addition, approaches combining mathematical programming and (meta-)heuristics have been used with success. In Landa et al. (2016) a neighborhood-based approach is used in combination with integer programming to solve the stochastic version of the problem. The solution approach in Tanfani and Testi (2010) is to use integrated simulation and optimisation for solving a problem consisting of waiting list management, OR planning and scheduling, and stay area sizing and organisation. Another way of using both optimisation and simulation is done in Zhang et al. (2009) where a deterministic MIP model is solved to allocate OR capacity to medical specialities. The allocation plan is then evaluated and fine-tuned by simulation. Wang et al. (2016) and Baesler et al. (2015) also use discrete event simulation and optimisation to schedule ORs. Lamiri et al. (2009) instead propose several heuristics to solve the stochastic surgery planning problem. Lately, Marchesi and Pacheco (2016) have used a genetic algorithm to solve the master surgical schedule problem while Hancerliogullari et al. (2016) use simulated annealing to generate the schedules for a set of ORs. Two quite different approaches can be found in Roshanaei et al. (2017) and Castro and Marques (2015). In Roshanaei et al. (2017) they use logic-based Benders' decomposition to schedule a distributed set of ORs across a network of hospitals while Castro and Marques (2015) use generalised disjunctive programming for the room scheduling problem.

An important aspect closely related to patient types is the incorporation of uncertainty. The two main kinds of uncertainty discussed in the literature is arrival of non-elective patients and duration for all patients. All uncertainty is ignored in deterministic models, whereas stochastic models take the uncertainty into account explicitly. 40 papers examined in Cardoen et al. (2010) consider only a deterministic model, whereas 42 papers consider only a stochastic model and six papers consider both a deterministic and a stochastic model. Addis et al. (2014) use a stochastic model to incorporate the uncertainty of operation durations, and Lamiri et al. (2008) use a stochastic model to incorporate uncertainty in capacity needed for emergency patients. Addis et al. (2014) generate all possible scenarios which satisfy that 1) a certain number of operations will need their

maximum operation time and 2) the rest of the operations will need their average operation time. The solution then has to be feasible for the worst case scenario. This is tested on instances with 20 or 40 operations. In Lamiri et al. (2008) a number of different scenarios are generated, each having a random capacity needed for emergency patients. The model calculates the overtime work for each scenario and finds the solution that gives the minimum average overtime work. This is tested on instances with 44 operations and between 2 and 1,000 samples. The study is continued in Lamiri et al. (2009), where some heuristics are proposed to solve the stochastic problem. In a more recent paper Gauthier and Legrain (2016) solve the problem in two phases: first a deterministic version using constraint programming and then a stochastic version in a sample average approximation scheme. In Addis et al. (2016) a rolling horizon approach is used to implement an integer programming based approach that can be used not only for scheduling but also for rescheduling of operating rooms. Aringhieri et al. (2015) puts a special focus on avoiding weekend stays when building the weekly plans.

More recent papers try to integrate different planning problems. The most “classical” planning problems to be integrated must be the planning of the rooms and the nurses. This is accomplished in Wang et al. (2015) using a metaheuristic. In Yin et al. (2016) they consider elective patient surgery in multiple operating theatres where the different stages of surgery is optimised as a complex connected patients flow problem. Nemati et al. (2016) integrate the transport of patients to the surgery unit with the optimisation of the operating schedule.

The OR planning problem has been widely studied. The novelty of this paper is the combination of using a large real-life dataset and including both elective and non-elective operations in the model, where the uncertainty of both operation duration and patient arrival are taken into account. At the same time the decision level is very detailed, as the exact time and location of each operation is determined.

### 3. Problem formulation

The problem considered in this paper is to assign and schedule operations to ORs in a both efficient and robust way while taking into account clinical guidelines. By efficient and robust we mean that we get as little wasted time in the ORs as possible, but at the same time the plan should also be able to handle sudden changes such as operation times being longer than planned or emergency operations.

When a patient has had a consultation and is referred to surgery, a rough estimate of the required operating time is automatically generated as the average operating time of the last operations of the same type performed by the consulting doctor. This information is then used to find an available date that suits the patient and has sufficient available time left. This estimate of “sufficient available time” is based on rough time estimates for all considered operations allocated to that day, a very rough estimate (even guess) on the required time to perform emergency surgeries, and the number of available ORs. The number of available ORs is a strategical long term decision though it is not required to use all available ORs each day. Since ORs can be closed and overtime is also allowed, there is therefore quite a broad definition of what constitutes “sufficient available time”.

As time passes the patients assigned to a given day changes dynamically as new patients are added and some are removed (rescheduled to other days or simply cancelled). The day before the operating day the assigned elective patients are fixed. The problem considered here is therefore a daily planning problem, meaning that for each day both the set of operations to perform and the set of available ORs are given.

For other specialities operations are sometimes performed in ORs belonging to S1 or S2, and we accommodate these operations in the plan. The reason for this is to not disturb the schedule of other specialities. It makes sense to plan each day separately since elective operations are, as far as possible, scheduled within the normal opening hours of the ORs, meaning that each day is a separate problem. We have chosen that all operations have to be performed on the given day, meaning that we never cancel any operations. It is assumed that surgical staff with the right qualifications is available for an open OR.

The aim is to minimise a weighted sum of overtime work and open ORs. We want to minimise the overtime work, because it is expensive to have people working overtime and it also causes

dissatisfied staff. We want to minimise the number of open ORs, because closing down an OR means that resources are released to perform other tasks. We have the following constraints given by Rigshospitalet, that need to be respected:

1. Operations can only be performed in compatible ORs.
2. Patients with diabetes have to start their operation before 11:00.
3. Patients using anticoagulant drugs have to start their operation after 10:00.
4. For operations performed on patients with a special infection there has to be at least a one hour break afterwards for that OR (for special cleaning).

The first constraint is important because different operations demand different equipment, which has to be present to perform the specific operation. The second constraint is important because patients have to fast before going under anaesthetic. Typically, they fast from the morning until the operation. For people with diabetes it can be dangerous to fast for too long since their blood sugar can become very low. Therefore patients with diabetes need to start their operation early. The third constraint is needed because patients who take anticoagulant drugs have to have taken a blood test before the operation. This blood test is taken in the morning and the test answer needs to be back before the beginning of the operation. The last constraint is needed because operations of people with a special infection demand more time consuming cleaning afterwards to make sure that the OR is clean enough to perform a new operation. We have decided not to include this cleaning time in the overtime work if the cleaning time is after the last operation in the OR. Because the cleaning is done by an external cleaning company, the cleaning in the end of the day does not affect the overtime work of surgeons and nurses.

We have chosen to model the overtime work so that we only have overtime work after the closing time of an OR and not before the opening of an OR. This means that in our model it is not possible to plan elective operations before the opening time but it is allowed, with a penalty, to plan operations after the closing time. Operations of emergency patients that arrive after the compatible ORs have closed is not included in the overtime work, as the night shift deals with these operations.

#### 4. The stochastic model

As seen in the literature the majority of the papers only consider elective patients, and just around half of the papers include uncertainty. However, uncertainty is important regarding the execution of the operating plans. In the literature, one strategy for incorporating uncertainty is to make a deterministic plan with only a certain utilisation of the ORs; often 85% (see for example Jebali et al. (2006)). This reserves some capacity for unforeseen events, but without considering whether the size and location of this extra capacity is appropriate. Another way to take the emergency patients into consideration is to use a stochastic model instead of a deterministic model. It is possible to take advantage of the fact that we have historic data from Rigshospitalet about the operations and use this in a stochastic model. It is also possible to consider not only emergency operations, but also other stochastic parameters in the model. We have decided to focus on stochastic arrivals and stochastic lengths of the operations, as these according to Rigshospitalet are the most important unknown quantities.

Even though the lengths of elective operations and the presence of emergency operations are not known, these can still be accounted for in the planning of the elective operations, because we have information about the behaviour of the operations. By taking the unknown into consideration when planning the elective operations, it is possible to make the operation plans more robust, so the arrival of an emergency patient or the delay of a planned operation does not ruin the flow of the ORs and make the plans infeasible.

##### 4.1. Monte Carlo simulation

Inspired by Lamiri et al. (2008) we have used Monte Carlo simulation and mixed integer programming to solve the stochastic problem. The stochastic model is based on samples, where a sample can be seen as a possible scenario for the actual running of an operation schedule. A sample consists of a set of emergency operations and lengths of all operations. Given a set of samples,

the goal for the stochastic model is to produce the solution that gives the best possible operation plan, when the scenario is unknown. “Best” in this context is defined later.

Lamiri et al. (2008) are also planning ORs, but they have a much simpler problem. They assume that the elective operations are deterministic and they do not look at the individual ORs. The only unknown is an emergency capacity for each time period. However, we can still use the same approach:

1. Generate a set  $\mathcal{K}$  of random samples. The distributions used for this are described in Section 4.3.
2. Solve a mixed integer program that uses the random samples, and where the objective function optimises the average best solution across the samples. The mathematical model for this problem is seen in Section 4.2.

Solving the stochastic model can be seen as solving a deterministic model  $|\mathcal{K}|$  times. The assignment of operations to ORs is the same across the samples and so is the order of elective operations in each OR. However, the start time of the operations vary depending on the sample. This way we will get a solution for the elective operations that leads to the average best solution for all samples, where the actual lengths and emergency operations are taken into account.

The overtime work will differ for each sample, but in the objective function the average overtime work through all samples is used. The objective function also uses variables to decide whether to open a given OR or not, and these variables do not change with the samples.

We would like to use the start times of the elective operations as part of the OR plans, but since the start times will differ depending on the samples, these cannot be used directly in order to get the final schedule. However, the assignment to ORs and the order of the elective operations are constant across the samples, so these can be used to generate the schedule. The schedules are made such that the first operation in an OR starts when the OR opens, and all following operations are scheduled to start right after the planned length of the previous operation. The only exceptions are if the previous patient has a special infection, in which case the start time of the operation will allow for extra cleaning, or the patient takes anticoagulant drugs, in which case it cannot start before 10:00.

#### 4.2. Mathematical model

All the sets, variables and parameters used in the stochastic model are listed below:

##### *Sets*

$\mathcal{K}$  The set of random samples.

$\mathcal{E}$  The set of elective operations.

$\mathcal{A}_k$  The set of emergency operations in sample  $k \in \mathcal{K}$ .

$\mathcal{O}_k$  The set of all operations in sample  $k \in \mathcal{K}$ , so  $\mathcal{O}_k = \mathcal{E} \cup \mathcal{A}_k$ .

$\mathcal{R}$  The set of OR types available. The ORs are grouped into types in order to improve the running time. Types are defined rooms with the same compatibility and opening hours.

$\mathcal{N}_r$  The set of ORs for OR type  $r \in \mathcal{R}$ .

##### *Variables*

$\bar{z}_{rn} \in \mathbb{R}_0^+$  The average overtime work in minutes for OR  $n \in \mathcal{N}_r$  of type  $r \in \mathcal{R}$ .

$z_{rnk} \in \mathbb{R}_0^+$  The overtime work in minutes for OR  $n \in \mathcal{N}_r$  of type  $r \in \mathcal{R}$  in sample  $k \in \mathcal{K}$ . This variable is naturally integer.

$t_{ik} \in \mathbb{R}_0^+$  The start time in minutes after midnight for operation  $i \in \mathcal{O}_k$  in sample  $k \in \mathcal{K}$ . This variable is naturally integer.

$q_{rn} \in \{0, 1\}$  Binary variable which is 1 if OR  $n \in \mathcal{N}_r$  of type  $r \in \mathcal{R}$  is used, and 0 otherwise.



$y_{irn} \in \{0, 1\}$  Binary variable which is 1 if operation  $i \in \mathcal{E}$  is performed in OR  $n \in \mathcal{N}_r$  of type  $r \in \mathcal{R}$ , and 0 otherwise.

$\dot{y}_{irnk} \in \{0, 1\}$  Binary variable which is 1 if operation  $i \in \mathcal{O}_k$  is performed in OR  $n \in \mathcal{N}_r$  of type  $r \in \mathcal{R}$  in sample  $k \in \mathcal{K}$ , and 0 otherwise.

$x_{ijrn} \in \{0, 1\}$  Binary variable which is 1 if operation  $i \in \mathcal{E}$  is performed before operation  $j \in \mathcal{E}$  in OR  $n \in \mathcal{N}_r$  of type  $r \in \mathcal{R}$ , and 0 otherwise.

$\dot{x}_{ijrnk} \in \{0, 1\}$  Binary variable which is 1 if operation  $i \in \mathcal{O}_k$  is performed before operation  $j \in \mathcal{O}_k$  in OR  $n \in \mathcal{N}_r$  of type  $r \in \mathcal{R}$  in sample  $k \in \mathcal{K}$ , and 0 otherwise.

#### Parameters

$w \in \mathbb{R}_0^+$  The weight of overtime work.

$o_r \in \mathbb{R}_0^+$  The opening time in minutes after midnight for OR type  $r \in \mathcal{R}$ .

$c_r \in \mathbb{R}_0^+$  The closing time in minutes after midnight for OR type  $r \in \mathcal{R}$ .

$a_{ik} \in \mathbb{R}_0^+$  The arrival time in minutes after midnight for emergency operation  $i \in \mathcal{A}_k$  in sample  $k \in \mathcal{K}$ .

$d_{ik} \in \mathbb{R}_0^+$  The maximum delay in minutes for the start time after arrival for emergency operation  $i \in \mathcal{A}_k$  in sample  $k \in \mathcal{K}$ .

$l_{ik} \in \mathbb{R}_0^+$  The random length in minutes of operation  $i \in \mathcal{O}_k$  in sample  $k \in \mathcal{K}$ . This includes pre-operation time, operation time, post-operation time and normal cleaning time.

$\kappa_{irk} \in \{0, 1\}$  Binary parameter which is 1 if operation  $i \in \mathcal{O}_k$ ,  $k \in \mathcal{K}$ , is compatible with OR type  $r \in \mathcal{R}$ , and 0 otherwise.

$h$  Conversion between minutes and hours, i.e. 60 min/hour.

$p_{ik}^s \in \{0, 1\}$  Binary parameter which is 1 if the patient having operation  $i \in \mathcal{O}_k$ ,  $k \in \mathcal{K}$ , has a special infection, and 0 otherwise.

$p_i^a \in \{0, 1\}$  Binary parameter which is 1 if the patient having operation  $i \in \mathcal{E}$  is taking anticoagulant drugs, and 0 otherwise.

$p_i^d \in \{0, 1\}$  Binary parameter which is 1 if the patient having operation  $i \in \mathcal{E}$  has diabetes, and 0 otherwise.

The mathematical formulation of the stochastic problem is as follows:

Model

$$\min \sum_{r \in \mathcal{R}} \sum_{n \in \mathcal{N}_r} (w\bar{z}_{rn} + (c_r - o_r)q_{rn}) \quad (1)$$

$$\text{s.t. } \bar{z}_{rn} = \frac{\sum_{k \in \mathcal{K}} z_{rnk}}{|\mathcal{K}|} \quad \forall r \in \mathcal{R}, n \in \mathcal{N}_r \quad (2)$$

$$y_{irn} = \dot{y}_{irn} \quad \forall k \in \mathcal{K}, i \in \mathcal{E}, r \in \mathcal{R}, n \in \mathcal{N}_r \quad (3)$$

$$x_{ijrn} = \dot{x}_{ijrn} \quad \forall k \in \mathcal{K}, i, j \in \mathcal{E}, r \in \mathcal{R}, n \in \mathcal{N}_r \quad (4)$$

$$t_{ik} \geq a_{ik} \quad \forall k \in \mathcal{K}, i \in \mathcal{A}_k \quad (5)$$

$$t_{ik} \leq (a_{ik} + d_{ik}) \quad \forall k \in \mathcal{K}, i \in \mathcal{A}_k \quad (6)$$

$$q_{r(n+1)} \leq q_{rn} \quad \forall r \in \mathcal{R}, n \in \mathcal{N}_r \quad (7)$$

$$M_1(1 - \dot{y}_{irn}) + z_{rnk} \geq t_{ik} + l_{ik} - c_r \quad \forall k \in \mathcal{K}, i \in \mathcal{O}_k, r \in \mathcal{R}, n \in \mathcal{N}_r \quad (8)$$

$$q_{rn} \geq \dot{y}_{irn} \quad \forall k \in \mathcal{K}, i \in \mathcal{O}_k, r \in \mathcal{R}, n \in \mathcal{N}_r \quad (9)$$

$$\sum_{r \in \mathcal{R}} \sum_{n \in \mathcal{N}_r} \dot{y}_{irn} = 1 \quad \forall k \in \mathcal{K}, i \in \mathcal{O}_k \quad (10)$$

$$\dot{y}_{irn} \leq \kappa_{irk} \quad \forall k \in \mathcal{K}, i \in \mathcal{O}_k, r \in \mathcal{R}, n \in \mathcal{N}_r \quad (11)$$

$$t_{ik} \geq o_r y_{irn} \quad \forall k \in \mathcal{K}, i \in \mathcal{E}, r \in \mathcal{R}, n \in \mathcal{N}_r \quad (12)$$

$$M_2(1 - \dot{x}_{ijrn}) + t_{jk} \geq t_{ik} + l_{ik} + hp_{ik}^s \quad \forall k \in \mathcal{K}, i, j \in \mathcal{O}_k, r \in \mathcal{R}, n \in \mathcal{N}_r \quad (13)$$

$$\dot{x}_{ijrn} + \dot{x}_{jirnk} \geq \dot{y}_{irn} + \dot{y}_{jrnk} - 1 \quad \forall k \in \mathcal{K}, i, j \in \mathcal{O}_k, i \neq j, r \in \mathcal{R}, n \in \mathcal{N}_r \quad (14)$$

$$t_{ik} \geq 10hp_i^a \quad \forall k \in \mathcal{K}, i \in \mathcal{E} \quad (15)$$

$$p_i^d t_{ik} \leq 11h \quad \forall k \in \mathcal{K}, i \in \mathcal{E} \quad (16)$$

$$\bar{z}_{rn} \in \mathbb{R}_0^+ \quad \forall r \in \mathcal{R}, n \in \mathcal{N}_r \quad (17)$$

$$z_{rnk} \in \mathbb{R}_0^+ \quad \forall k \in \mathcal{K}, r \in \mathcal{R}, n \in \mathcal{N}_r \quad (18)$$

$$t_{ik} \in \mathbb{R}_0^+ \quad \forall k \in \mathcal{K}, i \in \mathcal{O}_k \quad (19)$$

$$q_{rn} \in \{0, 1\} \quad \forall r \in \mathcal{R}, n \in \mathcal{N}_r \quad (20)$$

$$y_{irn} \in \{0, 1\} \quad \forall i \in \mathcal{E}, r \in \mathcal{R}, n \in \mathcal{N}_r \quad (21)$$

$$\dot{y}_{irn} \in \{0, 1\} \quad \forall k \in \mathcal{K}, i \in \mathcal{O}_k, r \in \mathcal{R}, n \in \mathcal{N}_r \quad (22)$$

$$x_{ijrn} \in \{0, 1\} \quad \forall i, j \in \mathcal{E}, r \in \mathcal{R}, n \in \mathcal{N}_r \quad (23)$$

$$\dot{x}_{ijrn} \in \{0, 1\} \quad \forall k \in \mathcal{K}, i, j \in \mathcal{O}_k, r \in \mathcal{R}, n \in \mathcal{N}_r \quad (24)$$

The objective function (1) minimises a weighted sum of the overtime work and the number of open ORs, where the overtime work is measured as the average overtime work across all samples. The cost of overtime work is a penalty weight saying how expensive it is to have one time unit of overtime work compared to one time unit of regular work. Cost can be interpreted as both money-wise cost and human cost because of dissatisfied staff. Whether to open an OR or not,  $q_{rn}$ ,  $r \in \mathcal{R}, n \in \mathcal{N}_r$ , does not depend on the sample. The cost of opening an OR corresponds to the open time for that room, so it is preferred to open a room with short opening hours compared to a room with longer opening hours, as long as it does not result in extra overtime work.

Constraints (2) set the average overtime work for each OR as the mean of the overtime work for that room over all the samples. Constraints (3) and (4) make sure that each elective operation is performed in the same OR for all samples and that the elective operations are performed in the same order, so the scheduling of the elective operations can be made from this. Constraints (5) and (6) give an upper and lower bound for the start time of emergency operations, as emergency operations cannot be conducted before the patient has arrived and they cannot be delayed more than a certain maximum amount of time. In the mathematical model this delay can be specified for a specific operation in a specific sample, so it is possible to differentiate between different levels of urgency. However, our information about the emergency operations is not that detailed, so in

our model all the emergency operations have to start before four hours after arrival.

To avoid symmetry, constraints (7) give a prioritised order for which OR to open for identical ORs, as an OR cannot be opened unless the previous OR for the same type is open. Constraints (8) set the overtime work for all operations. If operation  $i$  is performed in another OR, so  $y_{irnk} = 0$ , the inequality will not constrain the problem because  $M_1$  is a large constant. We have set  $M_1$  such that it corresponds to 36 hours, as it is reasonable to assume that all operations will end earlier than 36 hours after closure of the OR.

An OR has to be open for all samples if an operation in any of the samples uses that OR, which is ensured by constraints (9). Constraints (10) make sure that all operations in all samples are performed exactly once, and constraints (11) make sure that operations are only performed in compatible ORs.

All elective operations have to start after their OR has opened, which is ensured by constraints (12), so overtime work can only be planned in the afternoon and evening, and not in the morning. These constraints do not hold for the emergency patients, as the arrival of these patients and their maximum delay might make it impossible to perform these operations within the opening hours of the ORs. If emergency operations are to take place before the ORs open, this is not included in the overtime work, as the night shift takes care of it. Overlapping operations are handled by constraints (13), which make sure that the next operation in an OR does not start until the previous operation has ended. If the previous patient has a special infection there also needs to be time for extra cleaning of the OR. These constraints hold for all operations, both elective and emergency operations, and for all samples. Since the randomness in the lengths can cause some extreme cases we have set  $M_2 = 48 \cdot 60$  corresponding to 48 hours converted to minutes.

The connection between the  $x$ - and  $y$ -variables is ensured by constraints (14). Patients using anticoagulant drugs and patients with diabetes are taken into account in constraints (15) and (16). These constraints are made only for the elective operations, as it might not always be possible to consider these special cases in an emergency situation. Finally, constraints (17)-(24) set the domain of all the decision variables in the stochastic model.

#### 4.3. Sample generation

In order to generate samples for use in the stochastic model, we need random distributions for the elective as well as emergency operations. For elective operations, we need the lengths of the operations, whereas for emergency patients we need the number of emergency operations for a given day together with their arrival times and lengths of their operations.

For the emergency operations, we first need to generate the number of emergency operations for each day and their arrival time. From the data from Rigshospitalet 245 of the 8,273 operations are emergency operations, but they are not distributed equally across the weekdays. We have made some Analysis of Variance (ANOVA, see eg. Johnson (2011)) tests in order to see whether this difference is significant or not. This showed that there is no significant difference between the weekdays, but Saturday and Sunday are both significantly different from the weekdays and from each other. Therefore, we have made three different distributions for generating the emergency operations: One for the weekdays, one for Saturdays, and one for Sundays.

For all three subsets we assume that the number of emergency operations per day is Poisson distributed, as the Poisson distribution is often useful as a model to describe counts without a natural upper bound such as independent arrivals (Johnson (2011)). For Poisson distributed events, the time until the first arrival and the waiting time between successive arrivals are exponentially distributed with parameter  $\lambda$ . This parameter  $\lambda$  is the average number of emergency operations per day and is used to write up the inter-arrival times as exponentially distributed with mean inter-arrival time  $\frac{1}{\lambda}$  days. For weekdays there is on average 1.0 day between emergency operations (1.0 operation/day), for Saturdays there are 1.7 days on average (0.6 operation/Saturday), and for Sundays there are 4 days on average (0.3 operation/Sunday). The exponential distributions are used to generate arrival times for the emergency operations, and the number of emergency operations for each day is given directly from this.

In the provided data we have 8,242 operations that can be used to generate distributions for the actual lengths. For more information about how the distributions were derived we refer to Foverskov and Ravnskjær (2015).

When fitting distributions for the data of the relative lengths, we use the function `allfitdist` in Matlab. This function fits a number of different distributions to the data and ranks the distributions according to the Bayesian Information Criterion (BIC). For each operation type we choose the distribution with the highest rank, but only among the distributions that cannot be negative, as the relative length can never be negative. The used distributions have no upper limit on the range, so the maximum relative length is infinite, but in practice there is a limit on the maximum length of an operation. Therefore, if the relative length generated from a distribution is more than 3, we will instead fix it to 3. This means that the lengths of the operations in the samples cannot be more than three times the planned lengths. In practice, if an operation is too long, it will be stopped. The reason for this is either that something else has to be done, or because it is dangerous to keep the patient under anaesthesia for too long.

## 5. Heuristic solution approaches

Solving the stochastic model directly does not give useful results within a reasonable time limit. Therefore, we need another way to solve the problem. For this, heuristics are a prudent choice. Since we have a mixed integer programming (MIP) model that exhibits long running times, we will focus on MIP-based heuristics.

We have developed a new heuristic called *2-Step Relax-and-Fix*. It is inspired by the Relax-and-Fix heuristic (Ball (2011) and Wolsey (1998)). The main idea of the Relax-and-Fix heuristic is to solve easier problems, where we first decide on the most important variables, fix them and then decide on the remaining variables. The procedure for the heuristic can be seen in Algorithm 1.

---

### Algorithm 1 Relax-and-Fix

---

1. **Relax:** Relax the integrality requirements for the less important variables and solve the corresponding relaxation
  2. **Fix:** Fix the most important variables to their integer solution from the relaxation
  3. **Solution:** Solve the problem with fixed values to obtain the heuristic solution
- 

We present our 2-Step Relax-and-Fix heuristic via a simplified version of the stochastic model, where we do not distinguish between  $y$  and  $\hat{y}$ , nor  $x$  and  $\hat{x}$ . In the simplified version of the problem none of the non-integer variables are included. This gives a binary MIP problem which looks like the problem presented in (25)-(27). It seems natural that the most important decision is to decide which ORs to open, the second most important decision is to assign operations to ORs, and finally the least important decision is to decide on the order of the operations. This means that  $q$  is more important than  $y$ , which again is more important than  $x$ .

$$z = \min c^1 q + c^2 y + c^3 x \quad (25)$$

$$\text{s.t } A^1 q + A^2 y + A^3 x = b \quad (26)$$

$$q \in \{0, 1\}, y \in \{0, 1\}, x \in \{0, 1\} \quad (27)$$

An overview of the 2-Step Relax-and-Fix heuristic is seen in Algorithm 2, and each step is explained in details below.

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### Algorithm 2 2-Step Relax-and-Fix

---

1. **Relax:**  $(\underline{q}, \underline{y}, \underline{x}) \leftarrow$  Initial solution to linear programming (LP) relaxation
  2. **Fix:** Fix  $q$ -variables:  $q \leftarrow \lceil \underline{q} \rceil$
  3. **Relax:**  $(\tilde{y}, \tilde{x}) \leftarrow$  MIP solution with fixed  $q$ , integer  $y$  and relaxed  $x$
  4. **Fix:** Fix  $y$ -variables:  $y \leftarrow \tilde{y}$
  5. **Solution:** Order and start times of operations ( $x$ -variables)  $\leftarrow$  Algorithm 3
-

The first step of the heuristic is to relax the model. Instead of just relaxing the less important variables, as is the case in the standard Relax-and-Fix heuristic, we relax all the variables. So (27) is replaced by:

$$0 \leq q \leq 1, 0 \leq y \leq 1, 0 \leq x \leq 1 \quad (28)$$

Preliminary tests showed that a standard Relax-and-Fix heuristic takes too long to solve the problem, even though just one set of variables has to be binary.

The solution for this relaxed problem is denoted  $(q, y, x)$ . We can use the fractional solution  $(q, y, x)$  to fix the most important variables  $q$ . If the fractional  $q$ -variable is 0, it is fixed to 0 and in all other cases it is fixed to 1. As soon as a room has been open, even though it is just very little, i.e. the fraction is very small, the room has to be open in the binary solution.

When the  $q$ -variables are fixed the problem is solved again. This time only the least important variables,  $x$ , are relaxed. The solution for this problem is given by  $(\tilde{y}, \tilde{x})$ , where  $\tilde{y}$  is binary and  $\tilde{x}$  is still fractional. From the solution  $(\tilde{y}, \tilde{x})$ , we can now fix the  $y$ -variables to the values given by  $\tilde{y}$ .

At this point we know which ORs to open (corresponding to  $q$ ) and which operations to perform in which ORs (corresponding to  $y$ ). This means that we do not need to solve the final MIP problem to decide on a feasible order of the operations (corresponding to  $x$ ). Instead we have developed an algorithm to schedule the operations in a given OR.

---

**Algorithm 3** Order and start times of operations
 

---

**for** All open rooms in a given day **do**

Find order of operations:

1. Place all diabetes patients in ascending order according to planned operation length
2. Place all “normal” patients in descending order after the diabetes patients
3. Place all patients taking anticoagulant drugs in descending order after the “normal” patients
4. Place all patients with a special infection in descending order after the patients taking anticoagulant drugs

**end for**

Let the first operation start when the OR opens.

Assign start times to all following operations on the basis of the given order and planned operation lengths. Account for patients taking anticoagulant drugs cannot start until 10:00.

---

We always place diabetes patients first, because they have to be operated before 11:00. We place them in ascending order according to planned operation length, such that the shortest operation will be first, which ensures that the latest start time of a diabetes patient will be as early as possible. Next, we place all the “normal” patients in descending order, such that the longest operation will be first. We place the normal patients before the patients taking anticoagulant drugs, because patients taking anticoagulant drugs are not allowed to start before 10:00. Using this order avoid potential empty space. However, when assigning a start time to the patients taking anticoagulant drugs, it has to be checked that this time is not earlier than 10:00. Finally, we place the patients with a special infection at the end of the day to avoid that the time used to clean the room after such a patient is taking up time from another operation.

## 6. Experimental setup and testing

A running time of 10 minutes has been chosen as a fair compromise between the time used in the manual planning process as it is at Rigshospitalet now and the challenge of solving the entire data set with almost a year of instances. The heuristics are implemented using ILOG CPLEX version 12.6 with default settings and all tests are run on a PC with 3.0 GHz Intel(R) Core(TM)2 Duo processor and 4.0 GB memory.

### 6.1. Data description

We have received data from Rigshospitalet covering a period of 304 days for the two specialities. After editing data, where operations performed in special ORs were excluded, the data consists of 8,273 operations. Out of the total period 100 days are holidays or weekends even though all rooms are officially closed during weekends and holidays. In these 100 days only 203 operations are performed, i.e. around 2 operations per day. Hence, weekends and holidays are not really that interesting as they represent very simple and non standard planning problems. This leaves 8,070 operations to be performed in the remaining 204 weekdays. This gives an average of 39.6 operations per weekday with a range of 17-60 operations. The available ORs per weekday is on average 13.2 with a range of 7-16 ORs.

The average planned length of operations is 2.2 hours. The difference between actual lengths of operations and planned lengths are on average -8 minutes, which means that the operations on average are planned to take longer than their actual duration. Figure 1 shows a histogram of the delay in operation lengths, and it is clear that the majority of the operations are planned to take longer than their actual lengths.

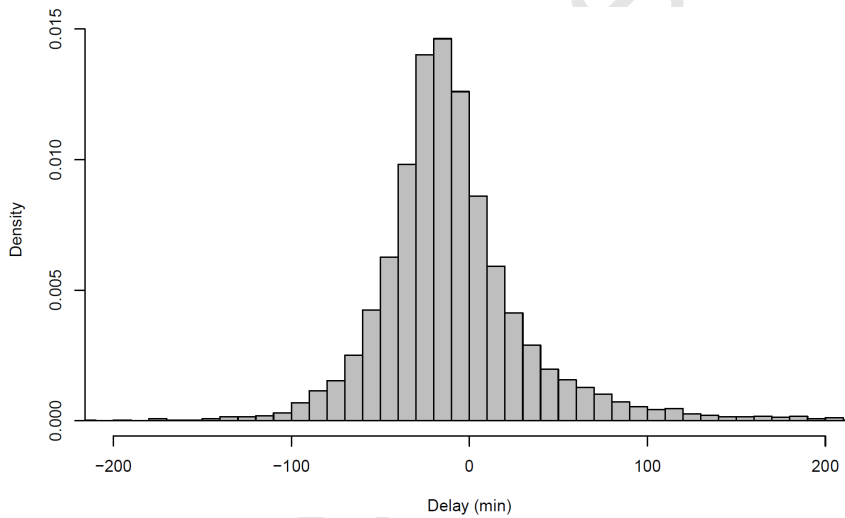


Figure 1: Histogram of difference between actual lengths of operations and planned lengths.

We have divided the dataset into smaller subsets consisting of chosen days from the entire period. These subsets are more or less “clean” subsets of the entire dataset with regard to the planned operations. We have decided to make such a division of data, because we want to be able to compare our solutions with the original plan, which contains some inconsistencies and infeasibilities such as overlaps of operations and operations outside opening hours. Since we cannot have this in our plans, it makes sense to compare our solution with a relatively clean original plan. We have defined clean on the basis of four criteria: Big overlaps of operations (more than 5 minutes), operations planned outside the opening hours, operations performed by other specialities, and operations performed in ORs belonging to other specialities.

In the data we have no weekdays, where all of the criteria are zero, so we have no “perfect” days. Instead we have decided to divide the data into five data sets A to E as described in Table 1.

It can be seen that  $A \subseteq B \subseteq C \subseteq D \subseteq E$ , with E corresponding to the original dataset. Characteristics of the five sets can be seen in Table 2, where the average values of the four criteria for clean datasets are shown together with the range in parenthesis. For each set the first column lists the number of instances. The second column lists the number of big overlaps (more than 5 minutes), and the third column lists the number of operations performed outside normal opening hours. The fourth column lists the number of operations performed by other specialities in rooms belonging to S1 or S2, and the fifth column lists the number of rooms used which belongs to other specialities.

Not even A is perfect, as we on average have 1.4 operations placed outside the opening hours and 0.8 of the used rooms belong to other specialities. Going from A to D the instances get less

Name	Description
A	Weekdays with no big overlaps, no more than 5 operations outside the opening hours, no more than 2 operations performed by other specialities and no more than 2 operations performed in rooms belonging to other specialities. This set consists of 5 days.
B	Weekdays with no more than 2 big overlaps, no more than 5 operations outside the opening hours, no more than 2 operations performed by other specialities and no more than 2 operations performed in rooms belonging to other specialities. This set consists of 13 days.
C	Weekdays with no more than 5 big overlaps. This set consists of 48 days.
D	All weekdays. This set consists of 204 days.
E	All days. This set consists of 304 days.

Table 1: The data sets developed for the analysis.

Set no.	No. of days	Big overlaps	Outside hours	Other operations	Other rooms
A	5	0.0 (0-0)	1.4 (0-3)	0.0 (0-0)	0.8 (0-2)
B	13	1.2 (0-2)	2.0 (0-5)	0.2 (0-1)	0.8 (0-2)
C	48	3.1 (0-5)	1.8 (0-10)	0.2 (0-1)	1.1 (0-8)
D	204	8.3 (0-19)	2.3 (0-10)	0.2 (0-3)	1.1 (0-9)
E	304	5.6 (0-19)	1.5 (0-10)	0.1 (0-3)	1.4 (0-9)

Table 2: Characteristics of subsets

and less clean, as the average number of the four criteria all increases. The reason why most of the numbers decrease in E compared to D is that weekends and holidays are included in set E. The weekends and holidays include so few operations that these days are almost always perfect. In Table 3 the instance sizes for the different sets are seen together with the range in parenthesis. For each set the first column lists the number of instances. The second column is the average number of operations per day, and the third column is the average number of ORs per day.

Set no.	No. of days	Operations	Rooms
A	5	33.4 (25-46)	12.2 (10-14)
B	13	37.5 (25-50)	12.9 (10-15)
C	48	35.9 (17-50)	12.5 (7-16)
D	204	39.6 (17-60)	13.2 (7-16)
E	304	27.2 (0-60)	8.9 (0-16)

Table 3: Size of subsets

## 6.2. Results

The weight  $w$  in the objective function is not only an expression of the cost of extra salary for overtime but also represents the personal cost of working overtime. Therefore, the weight cannot simply be chosen based on a monetary tradeoff between overtime work and closing an OR. The best balance will in the end be a matter of opinion and different departments at Rigshospitalet have different approaches and valuation regarding overtime work. The management at the considered department did not at the time of implementation have strong opinions on what constituted the best balance. We have therefore simplified the selection procedure for  $w$ . In case more specific input can be retrieved, more advanced selection procedures can of course be utilised. We have run the model with different values of  $w$  and afterwards inspected the plots of the solutions visually. We have looked at the days for which the solutions differ with regard to number of open ORs and considered whether closed ORs were obtained at the cost of too much overtime, and vice versa whether reduction in overtime was obtained at the cost of underutilised ORs. This visual approach

indicates that  $w = 2$  gives a good balance between overtime work and number of closed operating rooms. Therefore we will use the weight  $w = 2$  from now on.

For the 2-Step Relax-and-Fix heuristic the LP relaxation, corresponding to (25), (26), and (28), is run without any time limit and the MIP model with relaxed  $x$ - and  $\hat{x}$ -variables is run with a time limit of 10 minutes per day. Finally, Algorithm 3 is run. This time is not included in the total time, but for all instances it only took a few seconds. To keep the computational running time relatively low, the number of samples used for all tests except for those in Section 6.4 is  $|\mathcal{K}| = 10$ . In Section 6.4 we double the number of samples to 20 to investigate the effect.

The results for the 2-Step Relax-and-Fix heuristic is presented in Table 4. Notice that the gap shown is the average relative difference between the best integer solution and the lower bound from the LP relaxation with fixed  $q$ -variables, and not the original LP relaxation. 'Time' is the average total time for solving both the LP relaxation and the MIP model. 'Overtime work' is the average total overtime work per day in minutes. 'Closed rooms' is the average number of rooms per day that are closed.

Set no.	No. of days	Gap (%)	Time (s)	Overtime work (min)	Closed rooms
A	5	1.2	515	102	1.0
B	13	2.9	574	215	0.8
C	48	3.4	566	259	0.8
D	204	5.5	612	399	0.7
E	304	3.7	412	273	0.6

Table 4: Results for 2-Step Relax-and-Fix heuristic

For set D the average gap is only 5.5%. We can also see that the average time used is 612 seconds, i.e. just over 10 minutes. Except for set E the average gap decreases as the sets get cleaner, and for set A the gap is down to merely 1.2%. The solution time is correlated with the average problem size as given in Table 3. The amount of overtime work decreases from set D with 399 minutes to set A with only 102 minutes. At the same time the number of closed rooms increases from set D with 0.7 closed rooms to set A with 1.0 closed rooms. It may seem like much overtime work in set D; however, it should be remembered that this overtime work is the average overtime work taken over 10 samples, where we do have samples with very long operation lengths being up to three times the planned length. The overtime work includes emergency operations, which contribute with a lot of uncertainty and thereby increase the expected overtime work.

### 6.2.1. Utilisation

We want to see how well the ORs are utilised when the heuristic method is compared to the original planning. This is done by calculating the percentage of time that the rooms are open and unused. In order to get a more fair comparison we also include the operations that could be performed within the opening hours for the original plan since operations in the original plan might be placed outside the opening hours without being categorised as emergency operations. This does not happen in the heuristic plan due to the way the operations are scheduled (see Algorithm 3). In the heuristic we distinguish between elective and emergency operations and therefore we exclude emergency operations in the utilisation calculations. This is done such that the utilisation is an expression for how well the planning is done beforehand, where the number and lengths of emergency operations are unknown. Table 5 contains the utilisation calculated as the percentage of unused capacity for the original and the 2-Step Relax-and-Fix heuristic planning without emergency operations.

Comparing the results in Table 5 we get less unused capacity for the heuristic in sets A to D. Note that it does not make sense to calculate the utilisation for set E, as this set includes all weekends and holidays where all rooms are officially closed. The unused capacity for the heuristic increases as the sets go from A to D, whereas the trend is more unclear for the original planning. However, the difference in unused capacity between the original and the heuristic planning have a clear trend and decreases from 4.4% in set A to 1.6% in set D. Not surprisingly, this indicates that



Set no.	No. of days	Original planning (%)	Heuristic planning (%)
A	5	11.9	7.5
B	13	10.9	7.9
C	48	11.0	9.0
D	204	10.8	9.2
E	304	-	-

Table 5: Comparison of utilisation in original and 2-Step Relax-and-Fix heuristic planning

the instances included in e.g. set A may have had too many open rooms in the original planning leaving a lot of unused capacity, which is avoided in the heuristic planning by closing some rooms.

### 6.2.2. Extreme cases

It is important for the staff to avoid extreme overtime work, as this might negatively impact their private lives. What is considered extreme varies between departments, but one hour of overtime work in an OR is extreme for most of the departments at Rigshospitalet. Figure 2 shows the distribution of overtime work per OR for all weekdays, i.e. set D, for the solutions from the 2-Step Relax-and-Fix heuristic. The boxplot in Figure 2(a) has all outliers included, while Figure 2(b) shows the boxplot without outliers. Notice the different scales. As can be seen there are some very extreme cases with up to 1855 minutes (30.9 hours) of overtime work. This very extreme overtime work can be explained by a couple of reasons. First, the model is solved with a heuristic, so we have no guarantee of the quality of the solution and second, there is high uncertainty connected to the lengths of operations. Despite of the outliers with very extreme overtime work, most of the ORs have a reasonable amount of overtime work. Half of the ORs have less than 8 minutes of overtime work and 75% of the ORs have less than 28 minutes of overtime work.

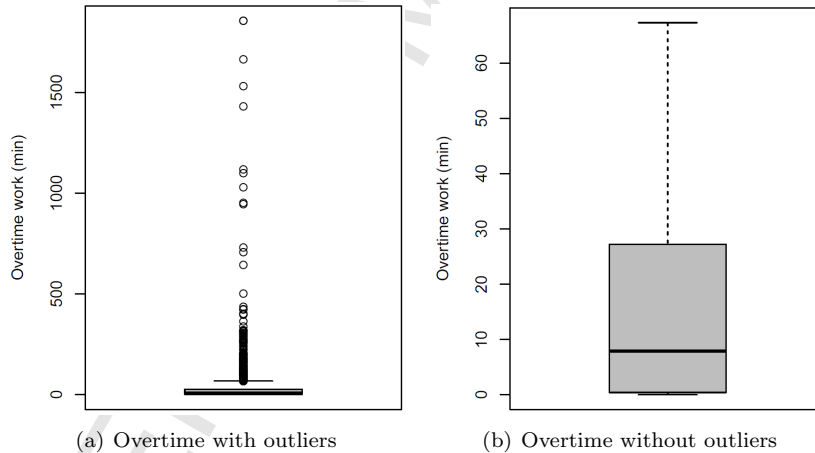


Figure 2: Overtime work per OR for each day in set D for the 2-Step Relax-and-Fix heuristic

Out of all the ORs, 69 (2.6%) have more than three hours of overtime work. Five of these are solved to optimality after the  $q$ -variables are fixed and 6 of them only have a gap of up to 2%. However, the gap can be as high as 52%.

Instances solved to optimality or near-optimality which still have much overtime work is because not enough resources are available. This could potentially be caused by an unfortunate fixing of the  $q$ -variables such that too many or the wrong ORs are closed. However, as we shall see in Section 6.2.3, where we leave all ORs open, most of the overtime is actually caused by a shortage in OR capacity. On the other hand, a large gap does not on its own explain a high amount of overtime work. Instead, we can use the amount of unused capacity. Instances without enough available resources are characterised by a high amount of overtime work and little unused capacity,

while the instances where the heuristic fails are characterised by having a high amount of overtime work, while at the same time having much unused capacity.

Table 6 shows how many days that have at least one OR with more than one hour of overtime work in the 2-Step Relax-and-Fix solutions. For each set, the first column gives the number of instances in the set and the second column shows the number of days with at least one OR with more than one hour of overtime work. Finally, the third column gives the average number of ORs per day with more than one hour of overtime work and the corresponding range in parenthesis. For set D there are 135 days with extreme overtime work in an OR. This corresponds to 66.2% of the days, so this happens very often. On average 1.5 of the available ORs during a day will have more than one hour of overtime work, and this ranges from zero to six ORs.

Set no.	No. of days	Days with >1h overtime work	Operating rooms per day
A	5	1 (20.0%)	0.2 (0-1)
B	13	4 (30.8%)	0.8 (0-5)
C	48	24 (50.0%)	1.1 (0-6)
D	204	135 (66.2%)	1.5 (0-6)
E	304	141 (46.4%)	1.0 (0-6)

Table 6: Days that include ORs with more than one hour of overtime work in the 2-Step Relax-and-Fix solutions

Of all the ORs on weekdays, 11.2% have more than one hour of overtime work. Table 7 shows how these ORs are spread across the weekdays. In the heuristic planning Wednesday, closely followed by Tuesday and Monday, are the days with most rooms with extreme overtime work. This could indicate that too many operations are planned on the first three days of the week.

Monday	Tuesday	Wednesday	Thursday	Friday
75	79	82	33	34

Table 7: ORs with more than one hour of overtime work in the 2-Step Relax-and-Fix solutions

It is also important to look into days with extreme underutilisation. Here it could be advantageous to close one or more ORs. On average 0.7 ORs are closed in the solutions for set D, so even though we have solutions with significant amounts of overtime work, we also find that many rooms can actually be closed.

Three or more ORs are closed on 6 of the 204 weekdays, all of which are Mondays. In addition, two or more ORs are closed on 33 days (16.2%). There is a very clear tendency that these days are Mondays as can be seen in Table 8.

Monday	Tuesday	Wednesday	Thursday	Friday
23	1	8	0	1

Table 8: Days with two or more ORs closed in the 2-Step Relax-and-Fix solutions

The boxplots in Figure 3 show the number of closed ORs for each weekday. Also here it is seen that ORs are often closed on Mondays, with at least one OR closed on 75% of Mondays. From these results we see that we have quite a lot of overtime work for Mondays, but at the same time this is the day of the week where most ORs are closed. This could indicate that the heuristic closes too many rooms for Mondays. We see the same tendency for Wednesday. This could be caused by the heuristic decision of which rooms to close. Some of the overtime work might be avoided with another value of the weight  $w$ , or it could be looked into whether it was actually better to just keep all rooms open. This will be done in the next section.

### 6.2.3. All Open Relax-and-Fix heuristic

It could be argued that the 2-Step Relax-and-Fix heuristic may close too many rooms, since we for set D have an average overtime work of 399 minutes and at the same time close 0.7 rooms.

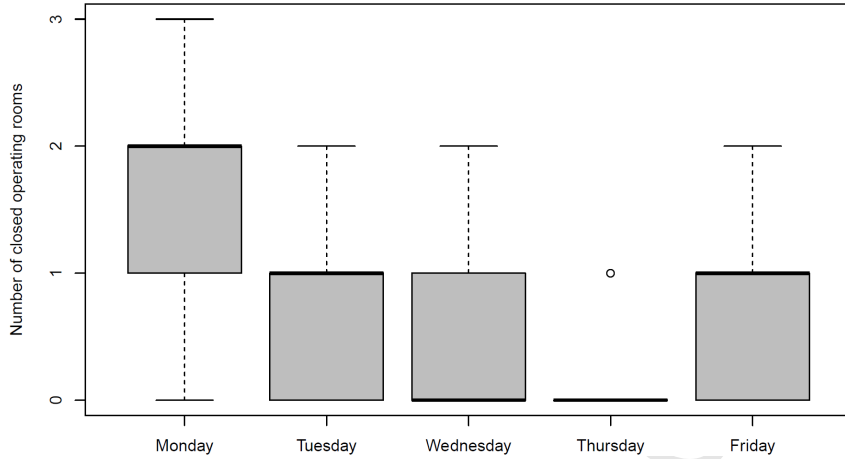


Figure 3: Number of closed ORs in the 2-Step Relax-and-Fix solutions

This is in spite of the fact that we only close those rooms where the  $q$ -variable is exactly zero in the LP relaxation. To test if we close too many rooms, we have derived a new heuristic called *All Open Relax-and-Fix*. It is almost identical to the 2-Step Relax-and-Fix heuristic except that we skip the first step where the  $q$ -variables are relaxed. Instead we start out in step 2 by fixing all the  $q$ -variables to 1, which means that all rooms are open. From here, the All Open Relax-and-Fix heuristic is exactly as step 3 to step 5 in the 2-Step Relax-and-Fix heuristic in Algorithm 2.

The All Open Relax-and-Fix heuristic is run using the same 10 samples as for the 2-Step Relax-and-Fix heuristic. The results are shown in Table 9 together with the results for the 2-Step Relax-and-Fix heuristic. 'Gap' is the average relative difference between the integer solution and the lower bound from the LP relaxation with fixed  $q$ . 'Time' is the average total solution time while 'Overtime work' is the average total overtime work per day in minutes.

Set no.	No. of days	All Open Relax-and-Fix			2-Step Relax-and-Fix		
		Gap (%)	Time (s)	Overtime work (min)	Gap (%)	Time (s)	Overtime work (min)
A	5	0.9	382	94	1.2	515	102
B	13	1.5	511	157	2.9	574	215
C	48	1.9	514	196	3.4	566	259
D	204	4.2	588	354	5.5	612	399
E	304	2.8	396	243	3.7	412	273

Table 9: Comparing results for the All Open Relax-and-Fix heuristic and the 2-Step Relax-and-Fix heuristic

Like with the 2-Step Relax-and-Fix heuristic, there is a trend in the amount of overtime work as it increases from 94 minutes in set A to 354 minutes in set D. Compared to the 2-Step Relax-and-Fix heuristic the gap for each set is smaller for the All Open Relax-and-Fix heuristic and, as expected, for each set the amount of overtime work is also less. However, looking at set D, the difference is not that big, as it is only 45 minutes shared between all the open rooms in the 2-Step Relax-and-Fix heuristic. Therefore, a very large part of the overtime work in the 2-Step Relax-and-Fix solutions must be caused by a shortage in OR capacity rather than closure of too many of the wrong ORs.

The amount of overtime work is reduced, which is natural as we increase the number of open rooms. However, it is also important to look at the utilisation of the rooms. In Table 10 the percentage of unused capacity for the original planning, the All Open Relax-and-Fix heuristic and the 2-Step Relax-and-Fix heuristic is shown, where all emergency operations have been excluded in the calculations as described earlier.

The percentage of unused capacity for the All Open Relax-and-Fix heuristic is higher for all sets compared to the original planning. The opposite is true for the 2-Step Relax-and-Fix heuris-

Set no.	No. of days	Original (%)	All Open Relax-and-Fix (%)	2-Step Relax-and-Fix (%)
A	5	11.9	15.3	7.5
B	13	10.9	12.4	7.9
C	48	11.0	13.0	9.0
D	204	10.8	12.7	9.2
E	304	-	-	-

Table 10: Percentage unused capacity for the original, the All Open Relax-and-Fix heuristic, and the 2-Step Relax-and-Fix heuristic planning.

tic, where the original planning has a higher unused percentage of capacity. For the All Open Relax-and-Fix heuristic the unused capacity is 12.7% for set D, whereas the unused capacity for the 2-Step Relax-and-Fix heuristic is 9.2%, which is a difference of 3.5%. It is natural that the unused proportion is higher for the All Open Relax-and-Fix heuristic than the 2-Step Relax-and-Fix heuristic, as we have more capacity available, but the same amount of operations. The question is if the 45 minutes less overtime work per day makes up for the 3.5% extra unused capacity. This can be difficult to answer, as it depends on how the prioritisation is between the two factors. To get closer to an answer, we have also looked at extreme cases for the All Open Relax-and-Fix heuristic, to see if the extra capacity can reduce the amount of extreme cases. The results are not presented, but it turned out not to be the case.

It can be seen from the results that it makes a difference to the average overtime work to force all rooms to be open, but it does not make a big difference for the extreme cases. At the same time, the percentage of unused capacity increases. Therefore it could be argued, that the 2-Step Relax-and-Fix heuristic actually performs excellently.

### 6.3. Simulated implementation

We would like to test the quality of the generated heuristic solutions and evaluate how the operation plans would perform in real life; however, since it has not been possible for us to implement the solutions in real life, we have made a simulated implementation instead. The plan evaluation is based on Monte Carlo simulation like that of Lamiri et al. (2008).

The idea is to generate a set  $\mathcal{L}$  of random samples. These are generated the same way as described in Section 4.3. While the set of samples  $\mathcal{K}$  was used to produce operation plans, the set of samples  $\mathcal{L}$  is used to evaluate the robustness of these plans.

In the simulated implementation it is assumed that the room and order of the elective operations cannot be changed, as these are given by the operation plans. The first operation will start on time when the OR opens, and all following operations will start depending on the duration of the previous operations in the same OR. For the emergency patients we do not know their OR and start time, so we need to find this in order to simulate the performance of the day. It is assumed that operations in progress cannot be disturbed, so if an emergency operation arrives while an operation is taking place in the assigned OR, the emergency operation is delayed until the current operation has ended.

In order to allocate the emergency operations, the ORs are prioritised according to the amount of underutilisation. Each emergency operation is allocated to the feasible room with the highest priority. It is feasible to allocate an emergency operation to an OR, if two conditions are satisfied: First, the operation and OR are compatible, secondly, when the emergency operation arrives at the hospital, a possible current operation in the room needs to end such that the emergency operation can start before a given maximum delay. When an emergency operation is scheduled, all the following planned operations are delayed. If no feasible room is found, the algorithm has failed, and no feasible solution is found for this sample on the given day.

Table 11 shows the results for the simulated implementation of the 2-Step Relax-and-Fix heuristic with  $\mathcal{L} = 100$  samples. 'Overtime work' is the average total overtime work per day in minutes. '>60 min overtime work' is the average percentage of rooms, where the overtime work is more than 60 minutes. 'Unused cap.' is the average percentage of unused capacity for the ORs. Finally, 'No sol.' is the average percentage of the samples that did not find a feasible solution with the simulation. The average overtime work per weekday is 418 minutes, which is only 4.8% more than the

optimised overtime work for the heuristic (see Table 4). The small increase in the overtime work means that the heuristic solution is quite robust, even though it is based on only 10 samples. On average 15.1% of the ORs on weekdays have more than one hour of overtime work. The solution for the 2-Step Relax-and-Fix heuristic had 11.2% ORs with more than one hour of overtime work, so again there is a little increase in the simulation. The unused capacity in set D for the heuristic was 9.2%, while the unused capacity in the simulation is 20.7%, i.e. more than twice as large. This might be caused by the few samples in the model, if they in general have longer operation times than the 100 samples in the simulation.

Set no.	No. of days	Overtime work (min)	>60 min overtime work (%)	Unused cap. (%)	No sol. (%)
A	5	183	9.1	21.6	0.00
B	13	263	12.5	20.5	0.00
C	48	285	13.0	20.7	0.00
D	204	418	15.1	20.7	0.02
E	304	280	-	-	0.01

Table 11: Results for the simulated implementation of the 2-Step Relax-and-Fix heuristic

On average the operations in the simulated implementation start 17 minutes earlier than planned, with a range from 12.6 minutes earlier than planned to 17.4 minutes later than planned. It is worth noting that 18.7% of the operations in the simulated implementation are set to start more than one hour before planned start, while 6.6% are set to start more than one hour after planned start.

For set D we get that the majority of the rooms have a reasonable overtime work. The mean overtime work for an OR is 32 minutes; however, 75% of the ORs have less than 31 minutes of overtime work. This is only a bit worse than the optimised results, where 75% of the ORs had less than 28 minutes of overtime work.

The results for the simulated implementations are generally a bit worse than the results found with the heuristic; however, this is to be expected since the heuristic solution is optimised to the samples, while that same solution is used in this simulated implementation with new samples. On the other hand, the overtime work is not much worse for the simulated implementation, so this could be an indication that the heuristic method is robust.

#### 6.4. Extra samples

In this section we want to see how it affects the results if we increase the number of samples and at the same time allow longer running times. The number of samples will be doubled to a total of 20 samples. We have chosen three days to test the longer running times and extra samples on: An “easy” day, an “average” day, and a “hard” day. The easy day has 27 operations and 10 operating rooms. When solving this instance with the stochastic model it had a gap of 4.8%, which is well below the average gap of 53.2% for the solved instances. The average day has 36 operations and 13 operating rooms. It had a gap of 47.3%, which is close to the average gap. Finally, the hard day has 56 operations and 15 operating rooms. This instance found a solution for the stochastic model with a gap of 100%, so this solution is not useful. If we include more than 20 samples, CPLEX runs out of memory for the hard day.

The test is run for 10 minutes, 2 hours and 5 hours, respectively, for the MIP problem. There is no time limit for solving the LP solution, even though these vary from around 30 seconds to 10 minutes. The results for the heuristic with 20 samples are presented in Table 12. It can be seen that the easy day has been solved to optimality within the 10 minutes time limit, when the  $q$ -variables are fixed. In fact, this instance was solved within 4 minutes. For the easy and average days we get satisfying results after just 10 minutes, whereas the hard day has a very large gap and an extreme amount of overtime work. For the hard day with only 10 samples we got a gap of 5.5% and 244 minutes of overtime work, so for this day the extra samples make a big difference. Looking at the results found with extra samples, it seems that the heuristic provides good results for long running times. However, 20 samples are too much for some of the instances to give good results, if the running time is kept at 10 minutes.

Level	<u>10 min. time limit</u>		<u>2 hour time limit</u>		<u>5 hour time limit</u>	
	Gap (%)	Overtime work (min)	Gap (%)	Overtime work (min)	Gap (%)	Overtime work (min)
Easy	0.0	115	0.0	115	0.0	115
Average	2.1	240	1.2	223	1.1	222
Hard	56.8	4,416	3.7	211	2.1	155

Table 12: Results for the 2-Step Relax-and-Fix heuristic with 20 samples and different running times. Gap is the average relative difference between the integer solution and the lower bound from the LP relaxation with fixed  $q$ . Overtime work is the total overtime work for the day in minutes.

## 7. Conclusion

We have developed and presented a stochastic integer programming model to solve the OR planning and scheduling problem with both elective and emergency patients as well as stochastic lengths of all operations. We have run this model with 10 samples, where each sample consists of stochastic arrivals of emergency operations and stochastic lengths of all operations. Because of large gaps and poor solution quality within the time limit, we have developed the 2-Step Relax-and-Fix heuristic to solve the problem. This heuristic takes advantage of relatively fast running times for LP relaxations and a prioritisation of the binary variables. We have also tested another version of the heuristic, the All Open Relax-and-Fix heuristic, in which all ORs are forced to be open.

The average overtime work for weekdays is 6.7 hours for the 2-Step Relax-and-Fix heuristic and 5.9 hours for the All Open Relax-and-Fix heuristic. Half of the ORs have less than 8 minutes of overtime work in the 2-Step Relax-and-Fix heuristic, and the heuristic closes 0.7 ORs on average. The heuristics give solutions where the first days of the week have significantly more overtime work than the last days, and Mondays are the days where it is most likely to close ORs. It is a bit better with regard to overtime work and utilisation in the average case to open all ORs; however, there is no difference between the two heuristics when it comes to extreme cases. Most of the overtime work is not caused by closed ORs but is caused by a shortage in OR capacity, which could be compensated for by distributing the available capacity better. An interesting extension of this work would therefore be to investigate the effect of extending the model to plan several days at a time. This would make it possible for the model to distribute the operations intelligently between the days included in the planning to avoid too much overtime work on any of the days.

The 2-Step Relax-and-Fix heuristic has been implemented with a simulation in order to see how it performs with new data and thereby determine the robustness of the plans produced. The overtime work for the simulated implementation is a bit larger than for the original heuristic solutions. However, there is not a significant difference, indicating that the operation plans are robust.

The problem dealt with in this paper is part of a greater flow taking place in the hospital. This means that to improve the final solution, it could be very interesting to look at what takes place before we start scheduling the operations and what takes place afterwards, when changes are necessary because of e.g. emergency patients.

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