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Busch, N.E.; Kristensen, Leif

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N. E. Busch and L. Kristensen

Danish Atomic Energy Commission
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Abstract

Statistical considerations are applied to a general equation of motion for cup anemometers in a turbulent wind. It is shown that the relative overspeeding $\Delta S/S$ can be expressed as: $\Delta S/S = I_s^2 J_s(\ell_o/\Lambda_s) + c\, I_w^2$, where I_s and I_w are the horizontal and the vertical turbulence intensities, respectively. The function J_s depends on the shape of the spectrum of horizontal turbulent energy, ℓ_o is the distance constant for the anemometer and Λ_s is a characteristic length scale of the horizontal turbulence. The constant c is of order unity.

If Λ_s is suitably chosen as the scale of the energy-containing eddies, then J_s is satisfactorily approximated by $J_s = (1 + \Lambda_s/\ell_o)^{-1}$ in most atmospheric applications.

CONTENTS

1. INTRODUCTION

The cup anemometer is widely used because it is a simple, sturdy and reliable instrument that generally only requires a minimum of maintenance. A further major advantage from an operational point of view is that there is no need for alignment into the wind direction, so the cup anemometer is ideal for continuous measurements. A disadvantage is the co-called over-speeding caused by the nonlinear response to fluctuating winds. Cup anemometers respond more quickly to an increase in the wind speed than to a decrease of the same magnitude. Consequently, in a turbulent flow the mean wind speed will be overestimated, if the instrument has been calibrated in a laminar flow. It has been shown by Busch (1965) that the over-estimation is proportional to the variance of the horizontal wind speed divided by the square of the mean wind speed, i.e., proportional to the square of the horizontal turbulence intensity. However, it seems intuitively clear that a fast-responding cup anemometer will show less overspeeding than a slow-responding cup anemometer, if the two are exposed to the same wind field. In the following we establish a relation between overspeeding, turbulence intensity, and the ratios between the length scale of the instrument and the scales of the turbulence.

2. CUP ANEMOMETER DYNAMICS

We shall take our starting point in the work of Wyngaard et al. (1971), in the following referred to as WBL. They use the rather general equation of motion

$$IR\dot{\Omega} \equiv IR\frac{d\Omega}{dt} = T(R\Omega, S, W). \qquad (1)$$

Here I denotes the moment of inertia, R the cup arm, Ω the angular velocity, and T the torque exerted on the instrument by the wind field with the total horizontal component S and the vertical component W. The angular position of the cup wheel does not appear in the equation, because Ω and T represent quantities that are smoothed over the period of time that it takes the cup wheel to turn the angle $2\pi/N$, where N is the number of cups.

In a steady, horizontal wind, the torque is zero, so we have

$$\dot{\Omega} = 0 \qquad (2)$$

and

$$T(R\Omega = R\Omega_o,\ S = S_o,\ W = 0) = 0. \tag{3}$$

We consider the case in which S(t) and W(t) are second-order, stationary, stochastic processes satisfying

$$\langle S \rangle = S_o \tag{4}$$

and

$$\langle W \rangle = 0, \tag{5}$$

where the brackets here and in the following denote ensemble averaging.

The fluctuating quantities s(t), θ(t) and ω(t) are defined by

$$s(t) = S(t) - S_o, \tag{6}$$

$$\theta(t) = W(t)/S_o \tag{7}$$

and

$$\omega(t) = \Omega(t) - \Omega_o, \tag{8}$$

where Ω_o can be found by solving (3). The ensemble means of s(t) and θ(t) are zero.

From various symmetry considerations, WBL argued that the second-order dynamic equation for the fluctuating quantities can be written

$$\tau_o \frac{\dot{\omega}}{\Omega_o} + \frac{\omega}{\Omega_o} = \frac{s}{S_o} + a\frac{s^2}{S_o^2} - b\frac{\omega}{S_o}\frac{\omega}{\Omega_o} - (a-b)\frac{\omega^2}{\Omega_o^2} + c\,\theta^2, \tag{9}$$

where a, b and c are constants in the sense that they do not depend on the fluctuating quantities, and τ_o is a characteristic time. For later convenience, we introduce the so-called distance constant ℓ_o by

$$\ell_o = S_o \tau_o . \tag{10}$$

In order to find an expression for the overspeeding we take the ensemble average of (9):

$$\frac{\langle \omega \rangle}{\Omega_o} = a \frac{\langle s^2 \rangle}{S_o^2} - b \frac{\langle s\omega \rangle}{S_o \Omega_o} - (a-b) \frac{\langle \omega^2 \rangle}{\Omega_o^2} + c \langle \theta^2 \rangle . \tag{11}$$

The left-hand side of (11) is the relative overspeeding, since it is the mean of the excess response divided by the response to a constant wind speed of magnitude S_o. Unfortunately, the right-hand side contains terms proportional to $\langle s\omega \rangle$ and $\langle \omega^2 \rangle$. These terms cannot be evaluated without additional information on $\omega(t)$.

To overcome this problem, we assume that $|s|/S_o$, $|\omega|/\Omega_o$ and $|\theta|$ are so small that the solution to the linear perturbation equation

$$\frac{\ell_o}{S_o} \frac{\dot{\omega}}{\Omega_o} + \frac{\omega}{\Omega_o} = \frac{s}{S_o} \tag{12}$$

can be used to relate $\langle s\omega \rangle$ and $\langle \omega^2 \rangle$ to $\langle s^2 \rangle$ with sufficient accuracy.

The solution to (12) with the initial condition $\omega(-\infty) = 0$ is

$$\frac{\omega}{\Omega_o} = \frac{1}{\ell_o} \int_0^\infty s(t-t') \exp(- \frac{S_o}{\ell_o} t') \, dt' . \tag{13}$$

It is easily shown that

$$\frac{\langle \omega^2 \rangle}{\Omega_o^2} = \frac{\langle s\omega \rangle}{S_o \Omega_o} = \frac{\sigma_s^2}{S_o^2} \frac{S_o}{\ell_o} \int_0^\infty \rho_s(\tau) \exp(- \frac{S_o}{\ell_o} \tau) \, d\tau, \tag{14}$$

where $\sigma_s^2 = \langle s^2 \rangle$ is the variance and $\rho_s(\tau)$ the autocorrelation function for the time series $s(t)$.

Using Taylor's hypothesis to convert the temporal autocorrelation function to a spatial autocorrelation function, we can write the expression (11) for overspeeding

$$\frac{\langle \omega \rangle}{\Omega_o} = a \frac{\sigma_s^2}{S_o^2} (1 - \frac{1}{\ell_o} \int_0^\infty \mu_s(x) \exp(-x/\ell_o) dx) + c \frac{\sigma_w^2}{S_o^2}, \tag{15}$$

where

$$\mu_s(x) = \rho_s(x/S_o), \tag{16}$$

and σ_w^2 is the variance of the vertical wind velocity component.

If we introduce the speed energy spectrum $\Phi_s(\varkappa)$ by

$$\Phi_s(\varkappa) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \rho_s(x) \exp(- i\varkappa x)\, dx, \qquad (17)$$

then (15) may also be written

$$\frac{\langle \omega \rangle}{U_o} = a \frac{\sigma_s^2}{S_o^2} \int_{-\infty}^{\infty} \frac{(\ell_o \varkappa)^2}{1 + (\ell_o \varkappa)^2} \Phi_s(\varkappa)\, d\varkappa + c \frac{\sigma_w^2}{S_o^2}. \qquad (18)$$

From wind tunnel measurements, WBL found that a = 0.96 and c = 0.67 for their cup anemometer. Hence, both horizontal and vertical wind fluctuations should give rise to an overestimation of the mean wind speed. Indeed, more detailed theoretical considerations seem to show that a should be close to unity.

3. SCALE RELATIONS

Eq. (18) forms the basis for our evaluation of the overspeeding of a cup anemometer. As demonstrated by WBL, the coefficients a and c for a given cup anemometer can be determined once and for all in a wind tunnel. In each application we still have to estimate the variances σ_s^2 and σ_w^2 in addition to the quantity

$$J_s = \int_{-\infty}^{\infty} \frac{(\ell_o \varkappa)^2}{1 + (\ell_o \varkappa)^2} \Phi_s(\varkappa)\, d\varkappa. \qquad (19)$$

The distance constant ℓ_o can also be determined in a laminar wind tunnel, whereas the speed spectrum $\Phi_s(\varkappa)$ depends on the actual wind field.

As an example, let us consider a situation with small wind direction fluctuations. In this case the mean wind speed is approximately equal to the mean of the wind velocity component U_o in the mean wind direction. The reason is that up to the third order in the horizontal components u and v of the fluctuating part of the wind vector, the following relation holds

$$S_o = U_o (1 + \tfrac{1}{2} \langle v^2 \rangle / U_o^2). \qquad (20)$$

To the same approximation, we have a relation between the fluctuating parts of S and u and v:

$$s = u + \frac{1}{2}(v^2 - \langle v^2 \rangle)/U_o . \qquad (21)$$

We neglect the second-order terms in (20) and (21) and set the spectrum $\Phi_s(\kappa)$ equal to the one-dimensional horizontal spectrum $F_{11}(\kappa)$ for u:

$$\Phi_s(\kappa) = F_{11}(\kappa). \qquad (22)$$

If the fluctuating quantity u has an integral scale Λ_u, and if the spectrum $F_{11}(\kappa)$ fulfils certain rather weak conditions which are actually fulfilled in most atmospheric applications, then for $\Lambda_u \ll \ell_o$ we have

$$J_s = \int_{-\infty}^{\infty} \frac{(\ell_o \kappa)^2}{1 + (\ell_o \kappa)^2} F_{11}(\kappa) d\kappa = 1 - \int_{-\infty}^{\infty} \frac{F_{11}(\kappa) d\kappa}{1 + (\ell_o \kappa)^2}$$

$$\approx 1 - \frac{\Lambda_u}{\pi} \int_{-\infty}^{\infty} \frac{d\kappa}{1 + (\ell_o \kappa)^2} = 1 - \Lambda_u / \ell_o . \qquad (23)$$

If there is no integral scale, or if it is not much smaller than the distance constant, then this approximation is invalid. Instead, we may assume, for instance, that an inertial subrange exists, and that the corresponding expression for the one-dimensional spectrum,

$$F_{11}(\kappa) = \frac{9}{55} \alpha \frac{\epsilon^{2/3}}{\sigma_u^2} |\kappa|^{-5/3} , \qquad (24)$$

can be used over a sufficiently wide κ range. The meanings of the quantities in (24) are: ϵ is the rate of viscous dissipation of turbulent energy, σ_u the root mean square of u, and α the so-called Kolmogorov constant, which is approximately equal to 1.5. The expression for J_s becomes

$$J_s = \frac{18}{55} \frac{\pi}{\sqrt{3}} Re^{-2} \left(\frac{\ell_o}{\eta}\right)^{2/3} , \qquad (25)$$

where

$$\eta = (\nu^3/\epsilon)^{1/4} \qquad (26)$$

is the Kolmogorov dissipation scale and Re the Reynolds number based on η, σ_u, and the kinematic viscosity ν.

Eqs. (23) and (25) show examples of how overspeeding may be related to the ratios between the distance constant and the length scales of the horizontal turbulence.

Through (25) we may also relate the overspeeding to the friction velocity u_* and the height of measurement z, provided that the Monin-Obukhov similarity hypothesis holds and that there is local balance between the production of turbulent energy and viscous dissipation. Then we have

$$\frac{\langle\omega\rangle}{\omega_o} = a\,\frac{18}{55}\,\frac{\pi}{\sqrt{3}}\,\frac{\alpha}{k^{2/3}}\,\frac{u_*^2}{S_o^2}\left(\varphi_m\left(\frac{z}{L}\right)-\frac{z}{L}\right)^{2/3}\left(\frac{\ell_o}{z}\right)^{2/3} + c\,\frac{\sigma_w^2}{S_o^2}\,. \qquad (27)$$

The quantities k, L and φ_m are the von Karman constant, the Monin-Obukhov length scale and the non-dimensional wind-shear, respectively.

We may also introduce the function $\phi_m(\frac{z}{L})$ by

$$\phi_m\left(\frac{z}{L}\right) = \int_0^{z/L}\left(1-\varphi_m\left(\frac{z'}{L}\right)\right)\frac{dz'}{z'}\,, \qquad (28)$$

in which case (27) can be written

$$\frac{\langle\omega\rangle}{\omega_o} = a\,\frac{18}{55}\,\frac{\pi}{\sqrt{3}}\,k^{4/3}\,\frac{\left(\varphi_m\left(\frac{z}{L}\right)-\frac{z}{L}\right)^{2/3}}{\left(\ln\frac{z}{z_o}-\phi_m\left(\frac{z}{L}\right)\right)^2}\left(\frac{\ell_o}{z}\right)^{2/3} + c\,\frac{\sigma_w^2}{S_o^2}\,, \qquad (29)$$

where z_o denotes the roughness length.

In order to illustrate the meaning of (29) let us assume that the constant a is equal to one, and that σ_w^2 is equal to zero. We may rewrite (29) so that the relative overspeeding is expressed by

$$\frac{\langle\omega\rangle}{\Omega_o} = f(\ell_o/z_o,\, z/z_o,\, z/L)\,. \qquad (30)$$

For the function $\varphi_m(\frac{z}{L})$, we adopt the expression

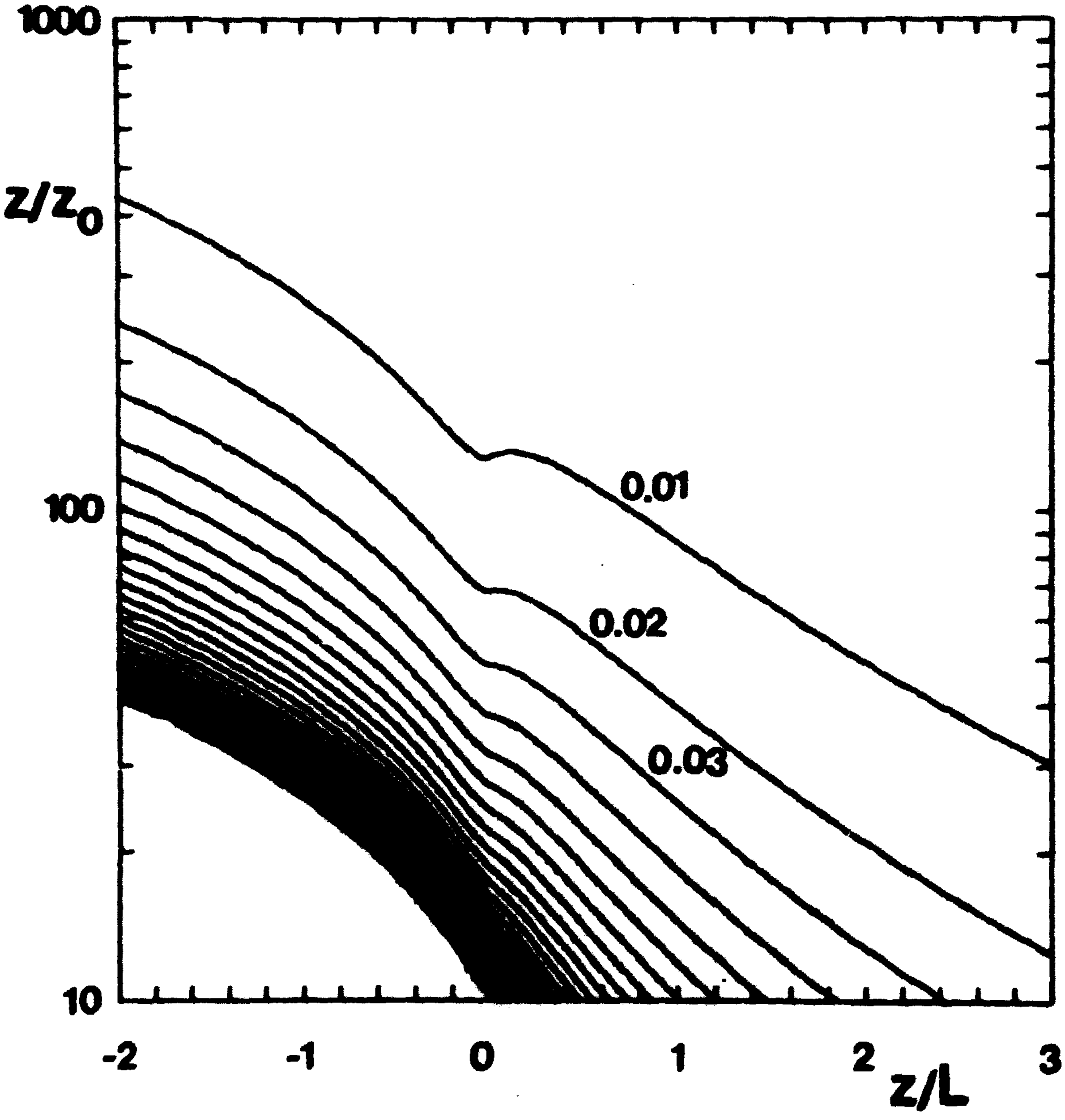

Fig. 1. Contour plot for $l_o/z_o = 200$ showing the relative overspeeding $\langle \omega \rangle / \Omega_o$.

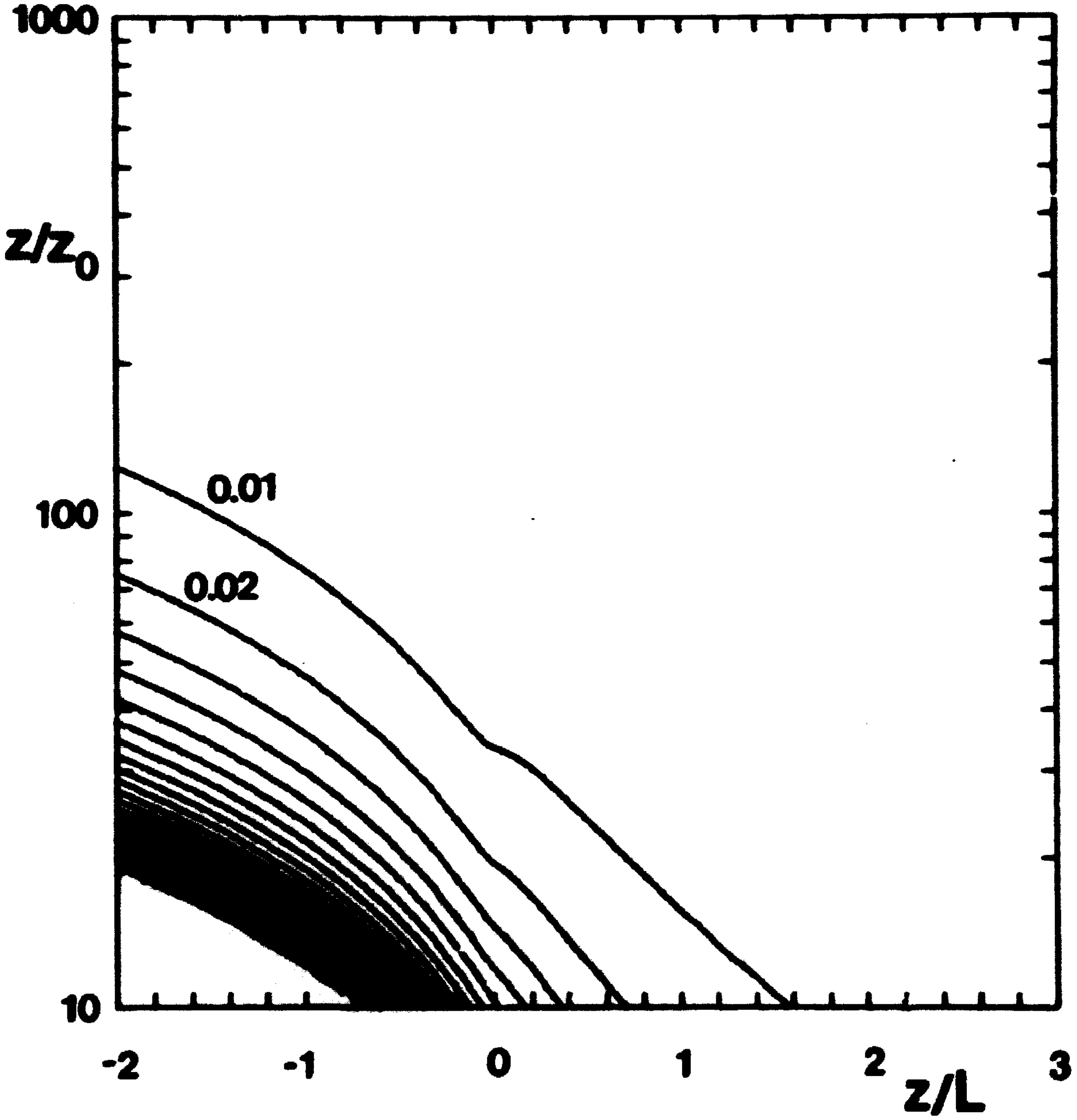

Fig. 2. Same as figure 1 for l_o/z_o = 20.

$$\phi_m(\frac{z}{L}) = \begin{cases} (1 - 15\frac{z}{L})^{-1/3} & \text{for} \quad \frac{z}{L} < 0 \\ 1 + 5\frac{z}{L} & \text{for} \quad \frac{z}{L} \geq 0 \end{cases} \tag{31}$$

suggested by Carl et al. (1973).

The function f is shown in figs. 1 and 2 as contour plots for two different values of ℓ_o/z_o.

As an example, suppose that a rather heavy, standard cup anemometer, with distance constant equal to 20 m, is located at a height of 2 m over a surface with roughness length equal to 10 cm. In this case the relative overspeeding will amount to about 10% under neutral conditions and to about 20% for $z/L = -0.2$. For a light cup anemometer with distance constant 2 m, the corresponding numbers are 2.5% and 4.5%, respectively.

The function f in (30) was evaluated under the assumption that the spectrum is given by (24) for all values of $\varkappa$, which implies that the integral scale is infinite. Eq. (30) will become inaccurate when ℓ_o is not small compared to the length scale of the horizontal turbulence. To estimate the error introduced by use of (24), we select five different expressions for the spectrum $F_{11}(\varkappa)$ and compute the ratio between the overspeeding pertaining to each of them and the overspeeding as computed with (24). The five different spectral expression are chosen so that they coincide with (24) in the limit $|\varkappa| \to \infty$. The five expressions are

$$F_{11}^{(1)}(\varkappa) = F_{11}^{(o)}(\varkappa_o) \frac{(\frac{3}{2})^{5/3}}{(1 + \frac{3}{2}|\frac{\varkappa}{\varkappa_o}|)^{5/3}} \quad \text{(Kaimal et al., 1972)} \tag{32}$$

$$F_{11}^{(2)}(\varkappa) = F_{11}^{(o)}(\varkappa_o) \frac{(\frac{3}{2})^{5/6}}{(1 + \frac{3}{2}(\frac{\varkappa}{\varkappa_o})^2)^{5/6}} \quad \text{(von Kármán, 1948)} \tag{33}$$

$$F_{11}^{(3)}(\varkappa) = F_{11}^{(o)}(\varkappa_o) \frac{3^{4/3}|\frac{\varkappa}{\varkappa_o}|}{(1 + 3(\frac{\varkappa}{\varkappa_o})^2)^{4/3}} \quad \text{(Davenport, 1961)} \tag{34}$$

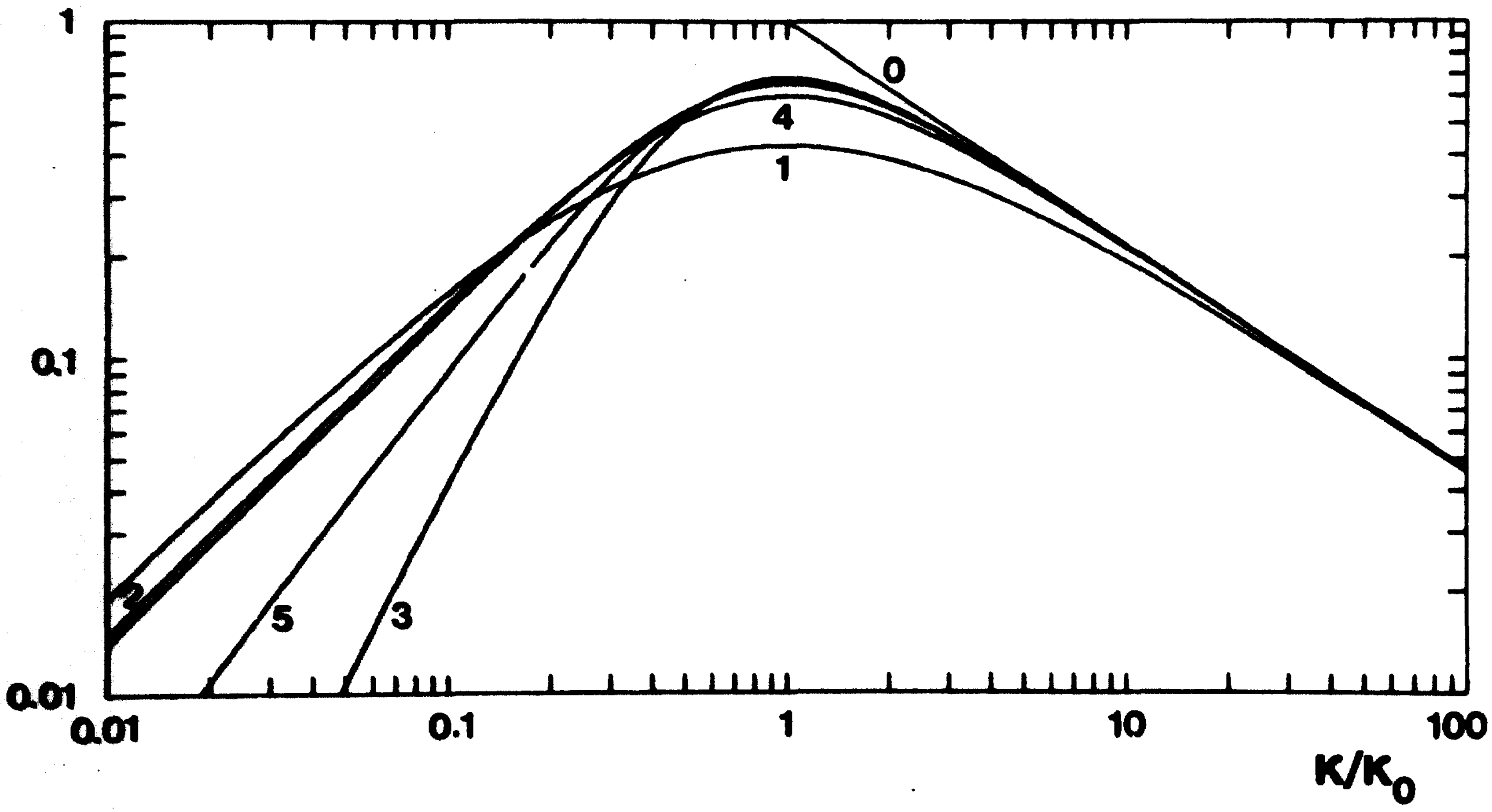

Fig. 3. Double logarithmic plot of $(\kappa/\kappa_o)\,(F_{11}^{(n)}(\kappa)/F_{11}^{(o)}(\kappa_o))$ versus κ/κ_o. The numbers on the curves correspond to superscripts n = 0, 1, ..., 5.

$$F_{11}^{(4)}(\varkappa) = F_{11}^{(o)}(\varkappa_o) \frac{\frac{3}{2}}{(1 + \frac{3}{2}|\frac{\varkappa}{\varkappa_o}|^{5/3})} \quad \text{(Kaimal, 1973)} \tag{35}$$

$$F_{11}^{(5)}(\varkappa) = F_{11}^{(o)}(\varkappa_o) \frac{2|\frac{\varkappa}{\varkappa_o}|^{1/3}}{(1 + 2(\frac{\varkappa}{\varkappa_o})^2)} , \tag{36}$$

where $F_{11}^{(o)}$ refers to the spectrum (24).

For each spectrum, the peak wave number $\varkappa_o$ must be adjusted so that

$$\int_{-\infty}^{\infty} F_{11}^{(n)}(\varkappa)\, d\varkappa = 1, \quad n = 1, 2, \ldots, 5 . \tag{37}$$

The reciprocal of $\varkappa_o$ is a horizontal length scale of the turbulence.

The five spectra are displayed in fig. 3, in which the peak positions have been made to coincide for the sake of comparison.

The result of the investigation is shown in fig. 4, where $J_s^{(n)}/J_s^{(o)}$ is plotted as a function of $\ell_o\varkappa_o$. Here $J_s^{(n)}$ is given by

$$J_s^{(n)} = \int_{-\infty}^{\infty} \frac{(\ell_o\varkappa)^2}{1 + (\ell_o\varkappa)^2} F_{11}^{(n)}(\varkappa)d\varkappa \tag{38}$$

which may be interpreted as the relative overspeeding divided by the square of the turbulence intensity. Fig. 4 shows that use of (24) generally leads to an overestimation of the overspeeding of between 0 and 50%.

Finally, (38) is evaluated for the five spectral shapes (32) through (36). In addition, the spectrum

$$F_{11}^{(6)}(\varkappa) = \frac{1}{\pi} \frac{1}{\varkappa_o} \frac{1}{1 + (\varkappa/\varkappa_o)^2} \tag{39}$$

is included in the investigation. For this spectrum the integration in (38) can be done analytically to yield (cf. (23))

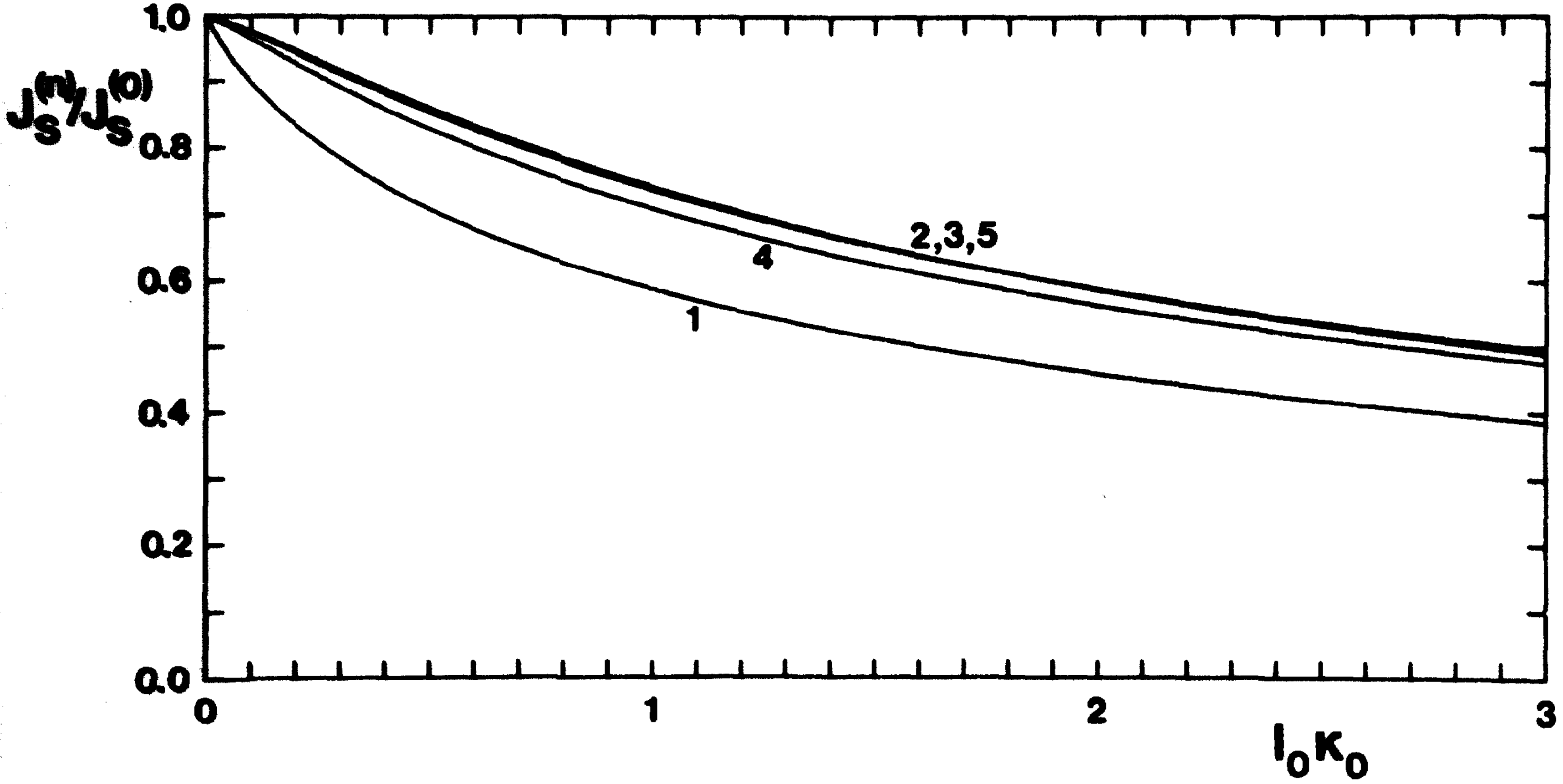

Fig. 4. Ratios of overspeeding (n = 1, 2, ..., 5), determined through (32-36). The anemometer distance constant ℓ_0 is made non-dimensional by the turbulent peak wavenumber κ_0.

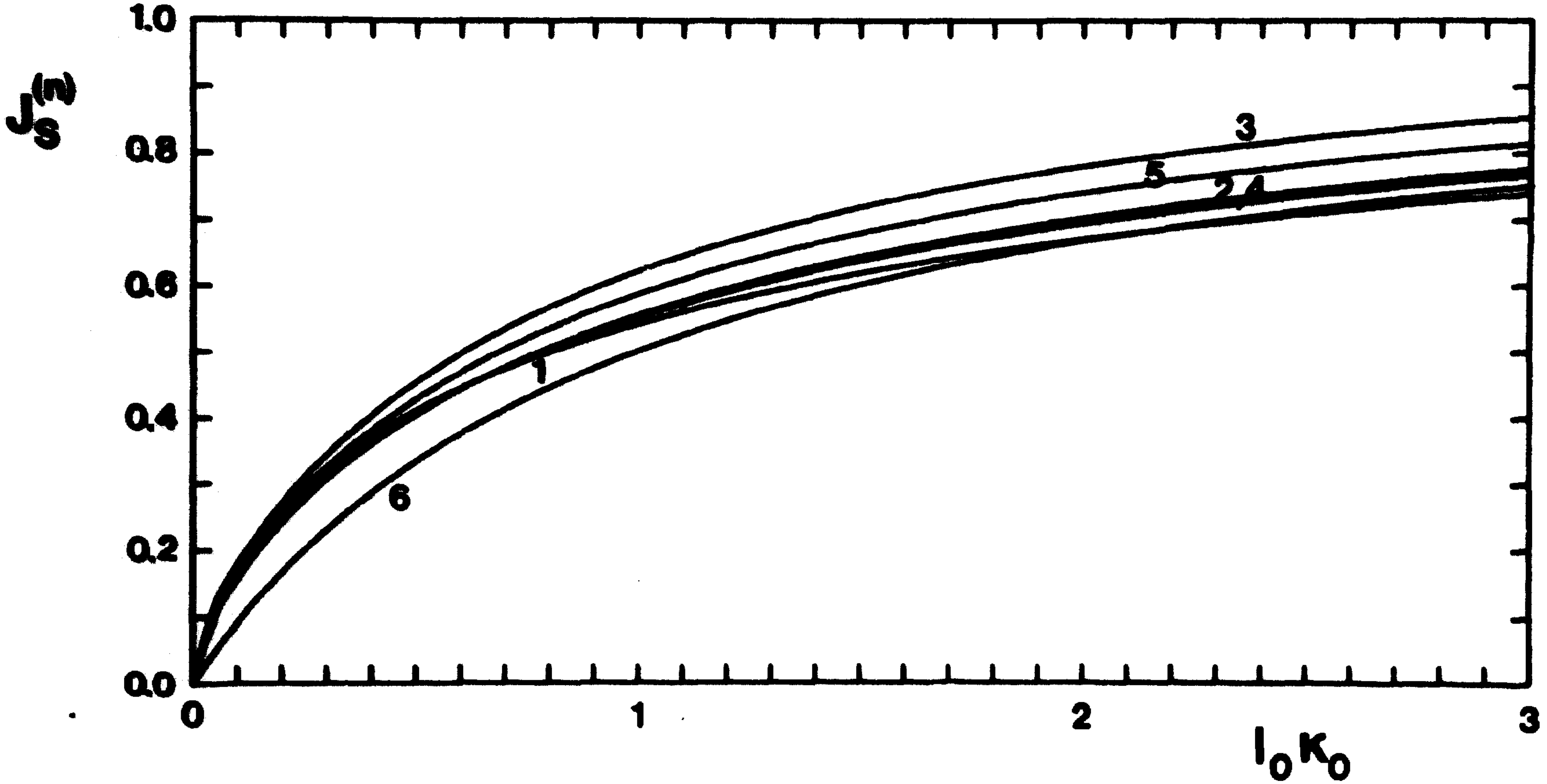

Fig. 5. The relative overspeeding divided by the square of the horizontal turbulence intensity as function of the ratio between the distance constant ℓ_o and the horizontal length scale of the turbulence $(1/\kappa_o)$. Curve no. 6 corresponds to (40) and (42).

$$J_s^{(6)} = \ell_o \kappa_o / (1 + \ell_o \kappa_o) . \tag{40}$$

Fig. 5 shows $J_s^{(n)}$, = 1, 2, ... 6, as a function of $\ell_o \kappa_o$.

4. CONCLUDING REMARKS

A statistical method was applied to a general cup-anemometer equation of motion in order to determine the overestimation of the mean wind speed in a turbulent wind. It was shown that the overspeeding depends not only on the turbulence intensity, but also on the shape of the speed energy-spectrum, $\varphi_s(\kappa)$. If the energy-containing part of the spectrum is well described in terms of the variance and a length scale Λ_s, then the relative overspeeding $\Delta S/S$ can be written

$$\frac{\Delta S}{S} = I_s^2 \, J_s(\ell_o/\Lambda_s) + c \, I_w^2 \tag{41}$$

where I_s and I_w are the horizontal and the vertical turbulence intensity, respectively, ℓ_o is the cup-anemometer distance constant, J_s a function of ℓ_o/Λ_s, and c a constant. Both J_s and c are believed to be less than unity, and consequently the relative overspeeding can never exceed the sum of I_s^2 and I_w^2.

If the formula is applied to a situation where ℓ_o is much less than the integral scale of the horizontal turbulence, then, from quite general assumptions about the existence of an inertial subrange and Monin-Obukhov similarity with local balance between production and dissipation of turbulent energy, we conclude that only in extreme cases does the overspeeding amount to more than 10%. Such cases may occur when the ratio between ℓ_o and the height of observation is of the order of 10 or more. Even then $\Delta S/S$ is overestimated if a correction for the finite magnitude of the integral scale is not taken into consideration (cf. fig. 4).

For practical purposes it is shown (cf. fig. 5) that if the characteristic scale Λ_s for the dominating turbulent eddies is chosen such that $\Lambda_s = \Lambda = \kappa_o^{-1}$, where κ_o is the peak wave number for the logarithmic energy spectrum (cf. fig. 3), then

$$J_s = (1 + \Lambda / \ell_o)^{-1} \tag{42}$$

will roughly describe the behaviour of the scale-dependent part of the overspeeding.

While the final manuscript was being prepared, Dr. J.C. Wyngaard of the Cooperative Institute for Research in Environmental Sciences, Boulder, brought to our attention an unpublished paper by E.I. Kaganov and A.M. Yaglom of the Institute of Atmospheric Physics, Academy of Sciences, Moscow, USSR, in which the overspeeding problem is treated in much the same way as in the present paper. A.M. Yaglom has informed us (personal communication) that he obtained the basic equation (18) in the early fifties and published the result in a paper (Yaglom, 1954) that has remained virtually unknown outside the USSR.

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