



Plastic Flow Localization and Ductile Fracture

Tvergaard, Viggo

Published in:
Journal of Physics: Conference Series

Link to article, DOI:
[10.1088/1742-6596/1063/1/012005](https://doi.org/10.1088/1742-6596/1063/1/012005)

Publication date:
2018

Document Version
Publisher's PDF, also known as Version of record

[Link back to DTU Orbit](#)

Citation (APA):
Tvergaard, V. (2018). Plastic Flow Localization and Ductile Fracture. *Journal of Physics: Conference Series*, 1063(1), Article 012005. <https://doi.org/10.1088/1742-6596/1063/1/012005>

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

PAPER • OPEN ACCESS

Plastic Flow Localization and Ductile Fracture

To cite this article: Viggo Tvergaard 2018 *J. Phys.: Conf. Ser.* **1063** 012005

View the [article online](#) for updates and enhancements.

Related content

- [Localization of plastic deformation and mechanical twinning in dynamical channel angular pressing](#)
E N Borodin and A E Mayer
- [Low-angle tilt boundaries in nanocrystalline materials](#)
S V Bobylev, M Yu Gutkin and I A Ovid'ko
- [Hydrogen-particle-matrix interactions in nickel](#)
Y Liang and P Sofronis



IOP | ebooks™

Bringing you innovative digital publishing with leading voices to create your essential collection of books in STEM research.

Start exploring the collection - download the first chapter of every title for free.

Plastic Flow Localization and Ductile Fracture

Viggo Tvergaard

Department of Mechanical Engineering, Solid Mechanics
Technical University of Denmark, DK-2800 Kgs. Lyngby, Denmark

viggo@mek.dtu.dk

Abstract. To treat tensile instabilities the main points of the theory of uniqueness and bifurcation in elastic-plastic solids are outlined. Also simple theories for localization of plastic flow are discussed. Instability predictions are highly sensitive to the material model applied. Ductile fracture in metals involves the nucleation and growth of micro-voids to coalesce. Such porous ductile materials also show localization of plastic flow. Analyses for low stress triaxiality show failure mechanisms involving collapse of voids into micro-cracks that subsequently interact.

1. Introduction

Tensile instabilities in ductile solids are often induced by imperfections and often related to a bifurcation that would occur under perfect conditions. Central to the understanding of these phenomena is the theory of uniqueness and bifurcation in elastic-plastic solids developed by Hill [1,2]. The main points of this theory are outlined for a smooth yield surface and normality of plastic flow. The instabilities are much affected by the constitutive model for the material, in particular whether or not a vertex forms on the yield surface. Besides a vertex, also the effect of non-normality of the plastic flow rule on bifurcation and the effect of an elastic-viscoplastic material model are discussed. Also the special, simplified formulations, assuming uniform straining inside and outside the neck or shear band, are presented.

The nucleation, growth and coalescence of small voids is the central mechanism in ductile fracture [3,4,5]. The presence of these damage mechanisms also has a strong effect on localization of plastic flow, as bifurcations are predicted at realistic strains for porous ductile materials. After localization the voids grow large inside the band, leading to the characteristic void-sheet fracture. In materials containing two size scales of voids, flow localization may develop between two larger voids, leading to local void-sheet fracture. Analyses for porous materials under low stress triaxiality, such as simple shear, show that even though the voids flatten out to micro-cracks, these cracks may interact in such a way that they give final fracture.

2. Basic equations

In the governing equations to be presented here, finite strains are accounted for and a convected coordinate Lagrangian formulation of the field equations is used, with a Cartesian x^i coordinate system as reference. Here, g_{ij} and G_{ij} are metric tensors in the reference configuration and the current configuration, respectively, with determinants g and G , and $\eta_{ij} = 1/2(G_{ij} - g_{ij})$ is the



Lagrangian strain tensor. The contravariant components τ^{ij} of the Kirchhoff stress tensor on the current base vectors are related to the components of the Cauchy stress tensor σ^{ij} by $\tau^{ij} = \sqrt{G/g} \sigma^{ij}$.

In terms of the displacement components u^i on the reference base vectors the Lagrangian strain tensor is

$$\eta_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i} + u_{,i}^k u_{k,j}) \tag{1}$$

The requirement of equilibrium can be specified in terms of the principle of virtual work

$$\int_V \tau^{ij} \delta \eta_{ij} dV = \int_S T^i \delta u_i dS \tag{2}$$

where V and S are the volume and surface, respectively, of the body in the reference configuration, T^i are the surface traction components on the reference base vectors, and there are no body forces.

Boundary value problems for elastic-plastic solids are solved incrementally, based on the incremental version of (2), to determine the displacement increments \dot{u}^i and the corresponding increments $\dot{\tau}^{ij}$ and $\dot{\eta}_{ij}$ of stress and strain. For the solution is used the incremental stress-strain relationship in the form $\dot{\tau}^{ij} = L^{ijkl} \dot{\eta}_{kl}$. Here, L^{ijkl} are the instantaneous moduli, which reduce to the elastic moduli R^{ijkl} when elastic unloading takes place in the material point.

3. Bifurcation and imperfection-sensitivity

A theory for the uniqueness and bifurcation in elastic-plastic solids has been developed by Hill [1,2]. At a current point of the loading history it is assumed that there are at least two distinct solution increments \dot{u}_i^a and \dot{u}_i^b corresponding to a given increment of the prescribed load or displacement.

The difference between these two incremental solutions is denoted by $\tilde{u}_i = \dot{u}_i^a - \dot{u}_i^b$, so that subtraction of the incremental principle of virtual work for the two solutions gives

$$\int_V \{ \tilde{\tau}^{ij} \delta \eta_{ij} + \tau^{ij} \tilde{u}_{,i}^k \delta u_{k,j} \} dV - \int_S \tilde{T}^i \delta u_i dS = 0 \tag{3}$$

where τ^{ij} are the current stresses. Thus, if the solution of the incremental boundary value problem is non-unique, (3) has a non-zero solution \tilde{u}_i .

For elastic-plastic solids that obey normality Hill [1,2] has made use of the expression

$$I = \int_V \{ \tilde{\tau}^{ij} \tilde{\eta}_{ij} + \tau^{ij} \tilde{u}_{,i}^k \tilde{u}_{k,j} \} dV - \int_S \tilde{T}^i \tilde{u}_i dS \tag{4}$$

to prove uniqueness. A comparison solid is defined by choosing fixed instantaneous moduli L_c^{ijkl} , which are equal to the current plastic moduli for every material point currently on the yield surface, and the elastic moduli elsewhere. For this comparison solid with fixed moduli (4) reduces to the quadratic functional

$$F = \int_V \{ L_c^{ijkl} \tilde{\eta}_{kl} \tilde{\eta}_{ij} + \tau^{ij} \tilde{u}_{,i}^k \tilde{u}_{k,j} \} dV - \int_S \tilde{T}^i \tilde{u}_i dS \tag{5}$$

A smooth yield surface and normality of the plastic flow rule are often used to model metal plasticity. Then the instantaneous moduli are of the form

$$L^{ijkl} = R^{ijkl} - \mu M^{ij} M^{kl} \tag{6}$$

where μ is zero or positive for elastic unloading or plastic loading, respectively, while M^{ij} is normal to the yield surface. For this type of material model it can be proved that the relation

$$\tilde{\tau}^{ij} \tilde{\eta}_{ij} \geq L_c^{ijkl} \tilde{\eta}_{kl} \tilde{\eta}_{ij} \tag{7}$$

is satisfied at every material point, and thus $F \leq I$. Therefore, since any non-trivial solution of (3) gives $I = 0$ the requirement $F > 0$ is a sufficient condition for uniqueness.

Equality in (7) and thus $I = 0$ for $F = 0$ requires that both solution increments, \dot{u}_i^a and \dot{u}_i^b , give plastic loading at all material points currently on the yield surface. Then, if \dot{u}_i^a is identified with the pre-bifurcation solution and \tilde{u}_i with the bifurcation mode, the variation of the prescribed load (or deformation) parameter λ with the bifurcation mode amplitude ξ initially after bifurcation can be written on the form

$$\lambda / \lambda_c = 1 + \lambda_1 \xi + \dots, \quad \xi \geq 0 \quad (8)$$

with λ_1 chosen sufficiently large. Generally, the minimum value of λ_1 is positive. In an extension of (8) to an actual asymptotic post-bifurcation expansion, accounting for elastic unloading zones that spread in the material, Hutchinson [6] has shown that the minimum value of λ_1 needs to be used.

When there is a relevant bifurcation point, determination of this point and the corresponding bifurcation mode, using the theory described by (3)-(8), is essential to the understanding of the problem. However, the occurrence of bifurcation is typical for perfect solids, and in practise the solid has imperfections, such as deviations from the perfect geometry, non-uniform material properties, or residual stresses. The imperfect solid will typically not reach a bifurcation point, and here the solution is obtained by a full incremental numerical solution for the elastic-plastic solid. Solutions for very small imperfections can be used to obtain a good understanding of the post-bifurcation behaviour. Plastic buckling of structures, such as thin-walled shells under compressive loading [7,8] provides many detailed studies of bifurcation and corresponding imperfection-sensitivity. In buckling problems small strain theory is usually sufficient.

In metal forming situations the relevant bifurcations, like necking or shear localization, occur under tensile loads and at large strains. Here the critical bifurcation mode is often not visible in the post-bifurcation range, e.g. in the round bar tensile test the first critical bifurcation mode has a long-wave sinusoidal shape, whereas the visible post-bifurcation shape is the well-known short neck, that develops rapidly due to massive elastic unloading. By contrast, in plastic buckling the visible post-bifurcation shapes are typically very similar to the bifurcation modes. A very detailed study of tensile instabilities in the plane strain tensile test has been carried out in [9], considering both the elliptic, hyperbolic or parabolic ranges of the governing equations. For the elliptic range the results include the classical long-wavelength result for necking, and the delay in the case of small aspect ratio specimens. A number of other authors have continued this study to consider compressive loads, axisymmetric conditions, or non-local plasticity.

If a vertex forms on the yield surface, so that the instantaneous moduli are functions of the stress-rate direction, the bifurcation theory (3)-(8) still applies in most cases. Such vertex formation is implied by physical models of polycrystalline plasticity, based on the concept of single crystal slip. If the plastic flow rule has non-normality, as occurs for the frictional dilatant behaviour of rocks and soils or for some metals with damage, upper bounds and lower bounds to the critical bifurcation can be found [10]. Finally, plastic instability problems are often described by using elastic-viscoplastic material models with a small value of the rate-hardening exponent m . Here, use of Hill's theory (3)-(8) shows that only elastic bifurcations are found. Still, knowledge of the bifurcation behaviour for the corresponding time-independent plasticity (i.e. for $m = 0$) gives important insight in the problem, and the effect of imperfections is analysed numerically as described above.

Equations (3)-(8) are concerned with bifurcation into a diffuse mode, which will often occur while the governing differential equations are elliptic. Localization of plastic flow in a narrow shear band is a different type of instability, which is observed as a rather sudden change from a smooth deformation pattern. Localization can be studied by a relatively simple model problem [11,12,13] for solids subject to uniform straining, assuming that localized shearing occurs in a thin slice of material with reference normal n_j , while the strain fields outside this band remain uniform. The quantities inside and outside this band are denoted by $()^b$ and $()^o$, respectively, and a Cartesian reference coordinate

system is used. Since uniform deformation fields are assumed both inside and outside the band, equilibrium and compatibility are automatically satisfied, apart from the necessary conditions at the band interface. Compatibility requires zero jump in tangential derivatives of \dot{u}_i over the band interface and equality of the nominal tractions on each side of the shear band interface

$$u_{i,j}^b = u_{i,j}^o + c_i n_j, \quad (T^i)^b = (T^i)^o \quad (9)$$

Analogous to (9), a simple plane stress model for localized necking in thin sheets is

$$u_{\alpha,\beta}^b = u_{\alpha,\beta}^o + c_\alpha n_\beta, \quad h^b (T^\alpha)^b = h^o (T^\alpha)^o \quad (10)$$

where Greek indices range from 1 to 2, and the initial sheet thicknesses inside and outside the band are h^b and h^o , respectively. The incremental forms of (9) or (10) account fully for nonlinear continuum mechanics, to determine the three or two unknown parameters c_i or c_α . Examples of sheet necking analyses based on (10) are given in [14,15] as forming limit diagrams for various anisotropic yield criteria, illustrating the effect of strain path change on limits to ductility.

In analyses based on the simple plane stress model (10) the neck geometry tends to be unprecise, as the length of the neck becomes very short relative to the thickness. A full 3D analysis represents the geometry well, but requires much more computer force. As an alternative, a non-local 2D analysis has been proposed, in which a characteristic length is used to incorporate the thickness effect [16].

4. Ductile fracture

Ductile fracture of structural metals occurs mainly by the nucleation, growth and coalescence of voids. Early micromechanical studies of void growth have been carried out for a single void in an infinite elastic-plastic solid, or as numerical studies for a material containing a periodic array of voids, which allows for including the effect of the interaction with neighboring voids, both in the early growth stages and in the final stages approaching coalescence [3,4,5]. The numerical studies consider a representative unit volume, typically containing a single void, with appropriate boundary conditions to represent the full material. Such unit cell analyses have become an important tool in the study of several different aspects of ductile fracture. It is also appreciated that unit cells containing many voids have advantages over those with only one void, as they can account for differences in void size or spacing and also for localized plastic flow due to void clustering or due to instabilities.

The most widely known porous ductile material model is that developed by Gurson [17] based on micromechanical studies, using averaging techniques. In this porous ductile material loss of ellipticity of the governing equations and thus localization of plastic flow in shear bands is predicted at realistic strains, even though the yield surface is smooth. In fact, studies of bifurcation into a shear band based on a cell model [18], were used early on to check the accuracy of predictions obtained by the Gurson model, and this resulted in some improvements of the model. After localization the voids grow large inside the band, while they do not grow outside, and failure in the band develops by void coalescence, so-called void-sheet fracture. Ductile fracture usually leaves a characteristic dimpled fracture surface, showing that failure occurred by void coalescence, but at void-sheet failure the fracture surface shows that the voids have been smeared out during coalescence. When studying imperfection-sensitivity it is often relevant to consider a larger volume fraction of voids or of void-nucleating particles inside the band than outside, thus leading to earlier failure in a shear band or in the neck of a biaxially stretched sheet.

A kinematic hardening version of the Gurson model predicts earlier localization of plastic flow [19], due to the higher yield surface curvature that acts like a rounded vertex on the yield surface. Here α^j denotes the center of the yield surface, and σ_F is the radius of the yield surface for the matrix material, given by

$$\sigma_F = (1-b)\sigma_y + b\sigma_M \quad (11)$$

where σ_y and σ_M are the initial yield stress and the matrix flow stress, respectively. Here, b is a constant between 0 and 1, so that $b=1$ gives the isotropic hardening Gurson model, while a pure kinematic hardening model appears for $b=0$. The approximate yield condition is of the form

$$\Phi = \frac{\tilde{\sigma}_e^2}{\sigma_F^2} + 2q_1 f^* \cosh \left\{ \frac{q_2 \tilde{\sigma}_k}{2\sigma_F} \right\} - 1 - (q_1 f^*)^2 = 0 \quad (12)$$

where $\tilde{\sigma}^{ij} = \sigma^{ij} - \alpha^{ij}$, $\tilde{\sigma}_e = (3\tilde{s}^{ij}\tilde{s}_{ij}/2)^{1/2}$ and $\tilde{s}^{ij} = \tilde{\sigma}^{ij} - G^{ij}\tilde{\sigma}_k^k/3$. Here, $f^* = f$ for small values of the void volume fraction f , and $f^*(f)$ can be stepped down at larger values of f , to approximately represent coalescence.

The Gurson model is limited by a number of assumptions, e.g. the voids are embedded in a standard Mises solid, and the voids are taken to remain spherical independent of the stress state. At low stress triaxiality, i.e. low mean tensile stress relative to the effective Mises stress, voids tend to elongate, and this has strong influence on predictions of ductile failure. Early studies that extended the Gurson model to account for void shape effects, and also to account for the effect of anisotropy have been discussed in [4,5]. The parameters q_1 and q_2 have been introduced in [18].

At void coalescence the ligament between neighbor voids necks down to zero thickness and leaves the characteristic fibrous fracture surface, as has been modeled in [20]. In materials containing two size scales of voids or inclusions from which voids nucleate, it is sometimes observed that plastic flow localization develops between two larger voids and that final failure involves void-sheet fracture by the small scale voids between the larger voids. In a model of this phenomenon the small scale voids have been represented in terms of the Gurson model and localization leading to void-sheet fracture between larger voids has been predicted [21].

Recently there has been increasing interest in the behavior of porous materials under low stress triaxiality, such as simple shear, where the standard material models do not predict void growth to coalescence. Full 3D analyses for shear specimens containing spherical voids have been carried out [22] in order to model experiments on ductile fracture in double notched tube specimens loaded in combined tension and torsion. In a number of plane strain cell model analyses for a material containing a periodic array of circular cylindrical voids it has been found [23,24,25] that in stress states similar to simple shear the voids are flattened out to micro-cracks, which rotate and elongate until interaction with neighbouring micro-cracks gives coalescence, where stresses pass through a maximum so that failure is predicted. This mechanism has also been found in 3D for initially spherical voids [26]. Thus, under high stress triaxiality the void volume fraction increases until ductile fracture occurs, whereas the void volume fraction disappears under low stress triaxiality, as the voids become micro-cracks. The significant void shape changes at low stress triaxiality are accounted in the models mentioned above, but to deal with failure in simple shear the models must be extended to describe void closure into micro-cracks and the interaction between these micro-cracks [27].

The early constitutive models for porous ductile solids did not incorporate an effect of the third stress invariant J_3 , but recently there has been more focus on this through the effect of the Lode parameter. It has been found in fracture tests under loads including shear [22] that the effective plastic strain does not decrease monotonically with increasing stress triaxiality. This has been further investigated in [28], where tension-torsion fracture experiments are modelled using an extension of the Gurson model [29], which has been made J_3 dependent by adding an extra damage term that allows for failure prediction even at zero hydrostatic tension. This extension of the Gurson model [29] has been compared with cell model studies for voids in shear fields [30] and it has been found that the model can capture quantitative aspects of softening and localization in shear. In [28] the tension-torsion fractures are modelled by finding the localization strain in a shear stress state with more or less tension superposed, and it is shown that the failure strain does not vary monotonically with the stress

triaxiality. Among the examples used to illustrate the models are various welded specimens, shear tests on butterfly specimens, and analyses of crack growth.

References

- [1] Hill R 1958 *J. Mech. Phys. Solids* **6** 236-49.
- [2] Hill R 1962 *Problems of Continuum Mechanics* (SIAM, Philadelphia) pp 155-64.
- [3] Tvergaard V 1990 *Adv. Appl. Mech.* **27** 83-151.
- [4] Benzerga A A and Leblond J-B 2010 *Adv. Appl. Mech.* **44** 169-305.
- [5] Benzerga A A, Leblond J-B, Needleman A and Tvergaard V 2016 *Int. J. Fracture* **201** 29-80.
- [6] Hutchinson J W 1973 *J. Mech. Phys. Solids* **21** 163-90.
- [7] Hutchinson J W 1974 *Adv. Appl. Mech.* **14** 67-144.
- [8] Needleman A and Tvergaard V 1982 *Mechanics of Solids, Rodney Hill 60th Anniversary Volume* (Pergamon Press, Oxford) pp 453-98.
- [9] Hill R and Hutchinson J W 1975 *J. Mech. Phys. Solids* **23** 239.
- [10] Raniecki B and Bruhns O T 1981 *J. Mech. Phys. Solids* **29** 153.
- [11] Marciniak K and Kuczynski K 1967 *Int. J. Mech. Sci.* **9** 609.
- [12] Stören S and Rice J R 1975 *J. Mech. Phys. Solids* **23** 239.
- [13] Tvergaard V 1989 *Theoretical and Appl. Mech., IUTAM Congress* (Elsevier Science) 349-68.
- [14] Kuroda M and Tvergaard V 2000 *Int. J. Solids Structures* **37** 5037-59.
- [15] Kuroda M and Tvergaard V 2000 *Int. J. Mechanical Sciences* **42** 867-87.
- [16] Mikkelsen L P and Tvergaard V 1999 *J. Mech. Phys. Solids* **47** 953-69.
- [17] Gurson A L 1977 *J. Engng. Materials Technol.* **99** 2-15.
- [18] Tvergaard V 1981 *Int. J. Fracture* **17** 389-407.
- [19] Tvergaard V 1987 *J. Mech. Phys. Solids* **35** 43-60.
- [20] Koplik J and Needleman A 1988 *Int. J. Solids Structures* **24** 835-53.
- [21] Tvergaard V 1982 *J. Mech. Phys. Solids* **30** 265-86.
- [22] Barsoum I and Faleskog J 2011 *Int. J. Solids Structures* **48** 925-38.
- [23] Tvergaard V 2009 *Int. J. Fracture* **158** 41-9.
- [24] Dahl J, Nielsen K L and Tvergaard V 2012 *ASME J. Appl. Mech.* **79** 021003-1-7.
- [25] Tvergaard V 2012 *Int. J. Solids Structures* **49** 3047-54.
- [26] Nielsen K L, Dahl J and Tvergaard V 2012 *Int. J. Fracture* **177** 97-108.
- [27] Morin L, Leblond J-B and Tvergaard V 2016 *J. Mech. Phys. Solids* **94** 148-66.
- [28] Xue Z, Faleskog J and Hutchinson J W 2013 *Int. J. Solids Structures* **50** 4258-69.
- [29] Nahshon K and Hutchinson J W 2008 *Eur. J. Mech. A/Solids* **27** 1-17.
- [30] Tvergaard V and Nielsen K L 2010 *J. Mech. Phys. Solids* **58** 1243-1252.