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Operational Limitations of Arctic Waste Stabilization Ponds - Insights from Modelling Oxygen Dynamics and Carbon Removal

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Abstract

Presented here is a mechanistic model of the biological dynamics of the photic zone of a single-cell arctic Waste

Stabilization Pond (WSP) for the prediction of oxygen concentration and the removal of oxygen demanding

substances. The model is an exploratory model to assess the limiting environmental factors affecting treatment

performance in arctic WSPs. A sensitivity analysis was utilized to provide a quantification of the relative

uncertainties of parameters that exist within the described modelling framework. The model was able to

qualitatively reproduce mesocosm experiment trends in phytoplankton growth, dissolved oxygen concentration, and

the reduction of CBOD₅ (Carbonaceous Biochemical Oxygen Demand – Day 5). These results demonstrated that

CBOD₅ reduction and oxygen state are very sensitive to organic loading regimes at cool temperatures (5-15 °C). The

sensitivity analysis identified that it was the difference in phytoplankton growth rates, and the associated change in

32 photosynthetic oxygen production, that mainly contribute to creating differences in CBOD₅ removal rates and the
33 development of aerobic conditions. The model was also sensitive to atmospheric aeration rates at low temperature
34 providing further evidence that low oxygen availability limits the treatment of CBOD₅ in cold climate WSPs. During
35 the development process, it was discovered that common formulations of depth-integrated phytoplankton growth
36 performed poorly for our modeled system, which was a quiescent eutrophic environment. This paper presents a new
37 phytoplankton growth formula within the paradigm of a poorly-mixed eutrophic system that may find utilization in
38 other eutrophic, colored or turbid systems. The novel aspect of the approach is that the depth integrated
39 phytoplankton growth function was formulated upon the premise that the phytoplankton population would be
40 capable to orient themselves to optimize their growth under poorly mixed conditions, and the average growth rate of
41 the phytoplankton population must decrease as crowding puts pressure on shared resources. The general agreement
42 of the model with the experiments, combined with the simplicity of the depth integrated box model, suggests there is
43 potential for further development of the model as a tool for assessing proposed arctic WSP designs. The sensitivity
44 analysis highlighted the uncertainty and importance of the parameterization of bacterial and phytoplankton
45 physiology and metabolism in WSP models.

46 1 Introduction

47 Waste Stabilization Ponds (WSPs) are, in essence, shallow highly eutrophic water bodies used for municipal
48 wastewater treatment, and operate by allowing biological (microbial degradation) and physical treatment processes
49 (settling) to reduce the CBOD₅ (Carbonaceous Biochemical Oxygen Demand – Day 5) concentration prior to
50 discharge from the treatment system. However, the design and operation of arctic WSPs is typically different than
51 those used in warmer climates due to the prevailing cold temperatures, and short ice-free time periods. Arctic WSPs
52 are operated as controlled discharge storage ponds; raw wastewater is continuously received into the WSP year
53 round, but effluent is only discharged once per year, typically during late summer/early fall for a period of 2-3
54 weeks. The surfaces of the arctic WSPs stay frozen for 9-10 months and influent wastewater temperatures quickly
55 approach 0 °C limiting the biological treatment capabilities of the system during this period. As a result, WSPs at the
56 start of the summer treatment season, or ice free period, contain high concentrations of oxygen demanding
57 substances (CBOD₅ >200 mg/l). The level of CBOD₅ treatment during the summer season is highly variable

58 (Ragush et al. 2017), and the limitations and best operational practices of single-cell WSPs operating in arctic
59 environments have not been deeply investigated.

60 The current design guidelines and "best practices" that are presently in use in the Arctic were developed from the
61 performance of systems operating in northern climates and expert experience (Dawson & Grainge, 1969; Heinke et
62 al., 1991). However, these design guidelines were meant to meet less stringent effluent regulations (Nunavut Water
63 Board, 2015) than are currently being implemented across Canada (Government of Canada, 2012). Also, the systems
64 used to develop the guidelines were generally i) located in cold temperate (such as northern interior United States or
65 Canada) or sub-arctic climates and/or were ii) continuous flow systems (US EPA, 1983). Since most northern
66 communities (e.g. 19 of 25 in Nunavut, Canada) depend on WSPs as a component of their municipal wastewater
67 treatment, the applicability of such guidelines for the design of arctic WSPs warrants further scrutiny.

68 To better understand the climatic and operational factors influencing the performance of single cell WSPs in cold
69 climates, Ragush et al. (2017) used a bench-scale factorial design experiment to examine the influences of
70 temperature, irradiance, organic loading and initial carbon concentration conditions at the onset of summer. The
71 focus was to observe how the aforementioned parameters impact the development of an aerobic environment and
72 CBOD₅ treatment performance. In this experiment, mesocosms were constructed to represent Arctic WSPs operating
73 for 40 days, which is roughly the length of the summer treatment season in many Nunavut, Canada communities.
74 Statistical analysis by Ragush et al. (2017) found that all four factors significantly impacted the oxygen state and
75 CBOD₅ removal rates.

76 Here, a mechanistic model is presented with the intent to represent the carbon and oxygen dynamics in arctic WSPs.
77 This model is to be used to explore existing knowledge gaps and uncertainties with respect to the dynamics
78 occurring in these systems and to determining limiting factors of system performance. Ultimately, the model can be
79 developed into a tool to assess arctic WSP design and optimization. With the focus of this study being on the
80 mechanisms of CBOD₅ removal and oxygen concentration dynamics, the model needed to adequately represent the
81 length of time required for algae populations to reach levels necessary to produce an aerobic (> 2 mg/L dissolved
82 oxygen) environment under arctic temperature and light conditions. One of the ultimate objectives of this work was
83 to identify organic loading regimes for arctic WSPs that facilitate the formation of aerobic environments within the
84 relatively short (approximately 40 - 60 days) summer treatment season. During the development of this model, it

85 was found that formulations from the literature poorly represented phytoplankton growth in our stagnant eutrophic
86 environment with high light attenuation. Thus, a mathematical representation for phytoplankton growth under these
87 particular conditions was developed. Here, we present this new phytoplankton growth model, for environments that
88 are eutrophic and have high light attenuation, which is likely to be applicable and have merit for simulations of other
89 ecosystems where phytoplankton are space limited due to a small vertical window in optimal photic depth.
90 Incorporating the new phytoplankton growth representation, we present a process-based model to predict dissolved
91 oxygen and CBOD₅ concentrations in WSPs and provide an assessment of the local sensitivity of associated
92 parameters of such a model through a one-factor-at-a-time (OFAT) sensitivity analysis. A brief discussion of
93 simulation results of the sensitivity analysis is provided in the context of WSP design. The formulation of the model,
94 specifically the depth-integrated phytoplankton growth function, and the results of the sensitivity analysis are likely
95 to be adaptable to other eutrophic systems.

96 2 Model Development

97 The use of process-based models to design and evaluate wastewater treatment processes, specifically activated
98 sludge systems, is well established (Orhon & Artan, 1994; Henze et al. 2000), and principles from these systems
99 have also been coupled with ecosystem models and applied to WSPs (e.g. Gehring et al., 2010; Fritz et al., 1979;
100 Buhr & Miller, 1983; Moreno-Grau et al., 1996; Banks et al., 2003; Beran & Kargi, 2005). These models display a
101 large range in complexity and formulations depending upon the studies' objectives and design characteristics of the
102 system. We reviewed the literature, assessing models for their applicability to our system and our focus on the
103 prediction of dissolved oxygen concentration and CBOD₅ removal in WSPs operating in arctic environments.
104 Banks et al. (2003), an adaptation of Buhr & Miller (1983), presented a box model of the photic zone (i.e. vertical
105 surface region where there is sufficient light for photosynthesis) that forms the cornerstone of the model presented in
106 this paper. However, when Banks et al.'s formulation was applied to the bench-scale system presented in Ragush et
107 al. (2017) we found that it was unable to adequately predict the oxygen state and CBOD₅ concentration. The
108 concentrations of oxygen, timing of when oxygen rose, and CBOD₅ removal could not be calibrated/validated
109 between the entire set of experiments. The poor agreement is believed to be due to the fact that the Buhr & Miller
110 (1983) system was a high-rate algal pond, which is shallow and has a paddle system engineered to create continually

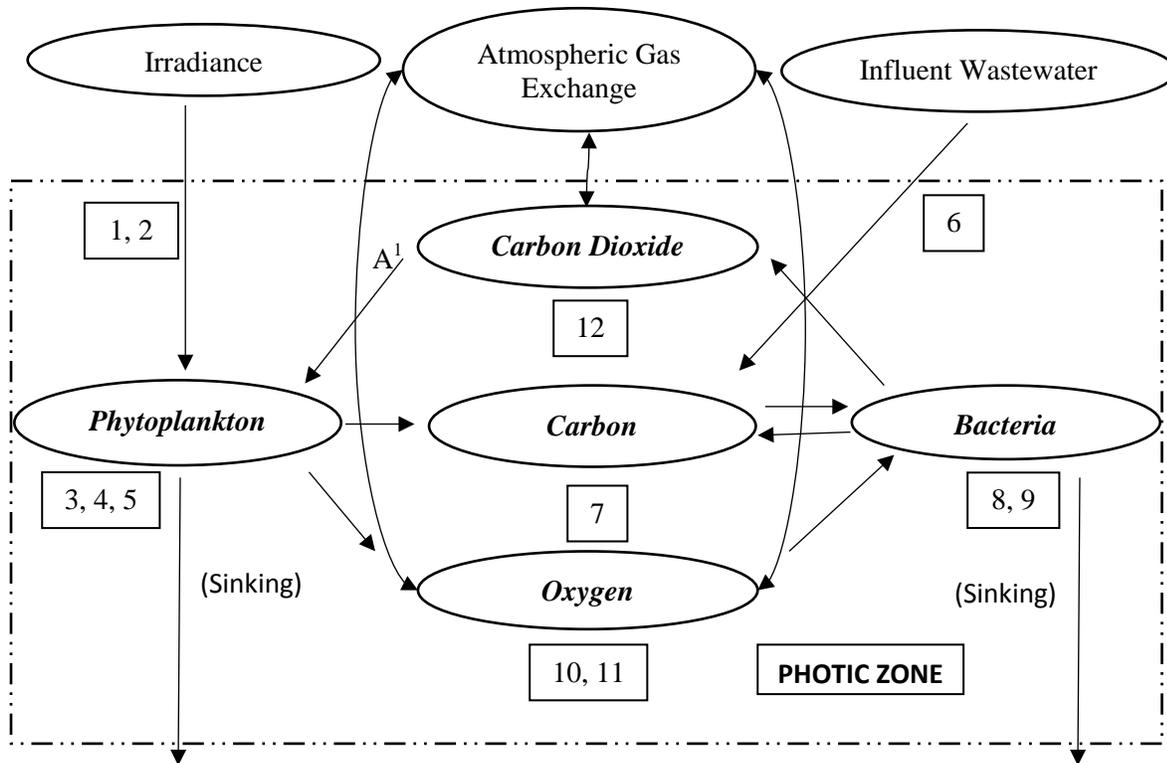
111 well-mixed conditions. This is inconsistent with single cell WSPs operating in the Arctic that have greater depth and
112 limited mixing. Thus, we made several modifications to the Buhr & Miller (1983) model.

113 2.1 Model overview

114 Figure 1 provides a schematic of the model along with references to the equations in Table 1 that were used to
115 represent the major processes. It is stressed to the reader that the model is a heuristic representation of arctic WSPs,
116 and accordingly is an abstraction of reality. This investigation uses the model to assess: i) the parameters that have
117 the greatest impact on treatment performance and ii) the environmental conditions that are limiting the treatment
118 performance in arctic WSPs, and the investigation does not aim to represent the model as an engineering design tool.
119 Omissions of phytoplankton respiration and anaerobic processes were based on heuristics. Extended daylight during
120 the summer in the North, allowing for continual photosynthesis, was the justification for the removal of
121 phytoplankton respiration from the model, and the relative low activity of anaerobic processes when temperatures
122 are less than 20 °C, as observed (Ragush et al. 2015; Ragush et al. 2017) in arctic WSPs, justified omission of
123 anaerobic processes. The model is a box model of the photic zone, and state variables and parameters were
124 vertically-integrated over the depth of the photic zone. External forcing into the photic zone were additional
125 wastewater, and surface irradiance. Exports from the photic zone were bacteria and phytoplankton through sinking.
126 Gas exchange of oxygen and carbon dioxide between the atmosphere and photic zone was included as a
127 transboundary interaction. Within the photic zone, the dynamics of bacteria and phytoplankton populations and their
128 metabolites of oxygen, carbon dioxide, and carbon (in the form of CBOD₅) were modeled. Nutrients other than
129 carbon, such as nitrogen and phosphorous, were excluded because their concentrations in both field scale and
130 experimental WSPs are high, and it was assumed that they would not impact biological processes by being limiting
131 (Ragush et al. 2015; Ragush et al. 2017).

132 The model formulation discussed in the following section will refer to equations by their number denoted in Tables
133 1 and 2 (for example Table 1 Equation 1 will be represented as equation 1.1 in the text). Table 1 contains the system
134 of differential equations, while Table 2 contains the supporting equations. Table 3 provides a list of model
135 parameters and their description. MATLAB, version R2015b, by MathWorks (Massachusetts, USA) was used to
136 implement a numerical solution to the system of ordinary differential equations presented in Table 1. The system of
137 ordinary differential equations is briefly discussed in section 2.1.1 and selected equations in Table 2 are discussed

138 where deemed appropriate following in section 2. Simulations were initialized using phytoplankton and bacteria
 139 concentrations that were reported by Ragush et al. (2017) at the beginning of their experiment.



140
 141 *Figure 1* Diagram of modeled processes with listed applicable equations next to process arrows. A^1 Respiration of phytoplankton
 142 omitted because of net uptake of CO_2 and continual solar irradiance leads to the potential of uninterrupted photosynthesis. Bold
 143 and Italicized text denotes state variables.
 144

145

146 *Table 1* List of Ordinary Differential Equations (ODEs) with brief descriptions.

#	Equation	Description & Comments
1	$\frac{dA}{dt} = (U_a - K_{ad} - K_{as}) * A$	Rate of change in phytoplankton = (specific rates of Growth – death – settling) * phytoplankton density
2	$\frac{dS}{dt} = -(OUR) * B + L * \frac{CBOD5inf}{V} * \frac{ColumnZ}{z} + K_{ad} * A * 0.5 + K_{db} * 0.7 * B$ Note $S \geq 0$	Rate of change in $CBOD_5$ = consumption by bacteria + daily loading + inputs from phytoplankton death + inputs from bacteria death
3	$\frac{dB}{dt} = (U_B - K_{bd} - K_{bs})B$	Rate of Change in bacteria = (rates of growth – death – settling) * bacteria density
4	$\frac{dO_2}{dt} = Y_{oa} * U_a * A - (OUR) * B + Kl_{O_2} * \frac{Area}{V} * (CS_{O_2} - O_2)$	Rate of Change of oxygen = Oxygenation by phytoplankton – Consumption by bacteria + aeration

5	$\frac{dCO_2}{dt} = \frac{Y_{cb}}{Y_{ob}} * (OUR) * B - Y_{ca} * U_a * A + Kl_{CO_2} * \frac{Area}{V} * (C_{S_{CO_2}} - CO_2)$	Rate of Change in carbon dioxide = production by bacteria – consumption by phytoplankton + aeration
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147

148

Table 2 List of supporting model equations and brief descriptions.

#	Equation	Description & Comments
1	$I_{av} = I_o \frac{1 - e^{-(K_w + K_p * A) * z_{1\%}}}{(K_w + K_p * A) * z_{1\%}}$	Average Irradiance across depth (considering shading by phytoplankton)
2	$z_{1\%} = \frac{\log(0.01)}{-K_w}$	Depth of 1% light transmittance (negating phytoplankton)
3	$F_{dis} = (1 - ED) * (1 - e^{-4e^{-(AGS)A}}) + ED$	Growth inhibition of phytoplankton as caused by crowding (Gompertz logistic growth model)
4	$U_a = U_{max} * F_{dis} * \frac{CO_2}{K_{CO_2} + CO_2} * \frac{I_{av}}{I_{av} + I_{halfsat}}$	Growth rate of phytoplankton = (Maximum phytoplankton growth rate * Crowding limitation * CO ₂ limitation * Light Limitation)
5	$CBOD_{5inf} = RAW * SOL_{CBOD_5}$	Addition of CBOD ₅ into photic zone = CBOD ₅ concentration * solubility
6	$U_b = U_{max_b} * \frac{S}{K_s + S} * \frac{O_2}{K_{O_2} + O_2} * (1 - BGS * B)$	Growth rate of bacteria rate Maximum bacteria growth rate * carbon substrate limitation * Oxygen limitation * self-limitation (logistic growth)
7	$\text{If } dO_2 > OUR_b * \frac{O_2}{O_2 * K_{O_2}} + (OUR)_b$ else $(OUR) = \frac{O_2}{B}$	Oxygen utilization rate: depends on the available oxygen and the bacterial population density
8	$CBOD_5(t) = S(t) + 0.5 * (A(t))$	CBOD ₅ = Carbon pool CBOD ₅ + CBOD ₅ of phytoplankton ((t) denoting at time t for clarity)

149

150

Table 3 List of model state variables and constants.

Symbol	Definition	Value & Units
<i>State Variables</i>		
A	Average phytoplankton concentration (algae)	mg/l (wet mass)
B	Bacteria concentration	mg/l (wet mass)
S	Substrate concentration (carbon)	mg/l (CBOD ₅)
O ₂	Oxygen concentration	mg/l
CO ₂	Carbon dioxide concentration	mg/l
<i>Variables</i>		
F _{dis}	Reduction in phytoplankton growth due to preferred distribution reducing irradiance	Unitless
I _{av}	Average irradiance across photic depth (Z) with phytoplankton	$\frac{\mu E}{m^2 s}$

<i>Constants</i>		
$CBOD_{5inf}$	Influent CBOD5 concentration	550 mg/l
Z	Depth of water column (total depth)	1.25 m
C_{SO_2}	Saturation concentration oxygen	11.3 mg/l (5 °C) 8.9 mg/l (15 °C) NIST (2015)
C_{SCO_2}	Saturation concentration carbon dioxide	1.01 mg/l (5 °C) 0.75 mg/l (15 °C) Benson & Krause (1984)
L	Daily volumetric loading	0.0125 or 0.05 l/d
I_{av}	Average irradiance across photic depth (Z) with no phytoplankton	(Eqn 1) $\frac{\mu E}{m^2} s^{-1}$
I_o	Surface incident light	225 & 1050 $\frac{\mu E}{m^2} s^{-1}$
K_w	Attenuation coefficient of the wastewater	14 m^{-1}
SOL_{CBOD_5}	Solubility Ratio of $CBOD_5$	0.5
V	Volume	0.0228 m^3
$Z_{1\%}$	Photic zone depth (1% measured irradiance)	(Eqn 2) m

151 *Manually calibrated constants provided in Table 3

152 2.1.1 State Variables and Ordinary Differential Equations

153 The model has five state variables: phytoplankton, bacteria, carbon, oxygen, and carbon dioxide, and the first four
154 were measured in the mesocosm study by Ragush et al. (2017), which were used to create a system of Ordinary
155 Differential Equations represented in Table 1 and briefly discussed below:

$$156 \quad \frac{dA}{dt} = (U_a - K_{ad} - K_{as}) * A \quad (1.1)$$

157 “A” represents phytoplankton (algae) as is used in many ecological models. The growth of phytoplankton
158 population is the balance of its growth rate (U_a) with some loss rates separated into death (K_{ad}) and settling (K_{as}).
159 The impact of death and settling has no mathematic functional difference and can be lumped with the same effect.
160 They were separated here, as it is a common practice in ecological models.

$$161 \quad \frac{dS}{dt} = -(OUR) * B + L * \frac{CBOD_{5inf}}{V} * \frac{Z}{Z_{1\%}} + K_{ad} * A * 0.5 + K_{bd} * 0.7 * B \quad (1.2)$$

162 “S” commonly represents substrate in ecological models; here it represents $CBOD_5$. The substrate is consumed by
163 the bacteria in a stoichiometric balance of the bacteria’s oxygen utilization rate (OUR). Additional $CBOD_5$ is added
164 daily, as wastewater is added to the system, and $CBOD_5$ is recycled in the death of phytoplankton (A) and bacteria
165 (B) according to stoichiometric carbon compositions.

$$166 \quad \frac{dB}{dt} = (U_B - K_{bd} - K_{bs})B \quad (1.3)$$

167 “B”, Bacteria is controlled analogously to phytoplankton with growth rate (U_b), death rate (K_{bd}), and settling rate
168 (K_{bs}).

169
$$\frac{dO_2}{dt} = Y_{oa} * U_a * A - (OUR) * B + Kl_{O_2} * \frac{Area}{V} * (Cs_{O_2} - O_2) \quad (1.4)$$

170 The differential equation for oxygen is governed by photosynthesis of phytoplankton, the utilization by bacteria and
171 finally oxygen transfer rate across the quiescent surface.

172
$$\frac{dCO_2}{dt} = \frac{Y_{bc}}{Y_{bo}} * (OUR) * B - Y_{ac} * U_a * A + Kl_{CO_2} * \frac{Area}{V} * (Cs_{CO_2} - CO_2) \quad (1.5)$$

173 Analogous to the equation for oxygen, the equation for carbon dioxide includes production from bacteria, uptake
174 from phytoplankton and carbon dioxide transfer across the surface.

175 Graphs are provided to compare the experimental and modelling result in figures 3 and 4 for carbon (measured
176 through $CBOD_5$) and dissolved oxygen, respectively. A graph of the phytoplankton and bacteria results are provided
177 for the most interesting case scenario of 80 mg/l initial $CBOD_5$ and 15 °C environmental temperature in Figure S1 in
178 the supplemental material. The model includes the state variable of carbon dioxide, however, no data was available
179 to create a comparison for this state variable, as this parameter was not measured by Ragush et al. (2017). Carbon
180 dioxide was included as a state variable because a state limitation was required to explain the decrease of
181 phytoplankton and bacteria populations in the later stage of some trials (Supplemental 1). Typically, a light
182 limitation would be expected to have caused the limit on population, however in this model with no mixing it would
183 result in a steady state phytoplankton population, and this was not observed. Since the decrease in phytoplankton
184 and bacteria populations coincided with the decrease in available organic carbon, it was hypothesized to be an
185 organic carbon/carbon dioxide limitation. It is noteworthy that carbon dioxide limitation has been identified as a
186 cause of phytoplankton population crashes in waste stabilization ponds (Shilton 2005). The authors recognize the
187 possibility that an alternative reason could be a micronutrient as the limiting agent, and the hypothesis that it is
188 carbon dioxide warrants further investigation. However, the existence of a different limiting agent has negligible
189 impact on the goals of this investigation and only a minor reformulation of the model would be required to
190 accommodate this realization.

191 The authors recognize that this model simplifies the complex inorganic carbon dynamics and does not explicitly
192 consider the potential uptake of other inorganic carbon species such as bicarbonate by phytoplankton. The data does

193 not exist to justify the incorporation of these complicating processes, and from a heuristic perspective their inclusion
194 is outside the scope of the model. The authors see their inclusion as an avenue of investigation for future model
195 improvements.

196 2.2 Temperature

197 The temperature dependences of chemical, physical and biological processes were modelled based on the van't
198 Hoff-Arrhenius relationship (equation 2.1) from Metcalf & Eddy (2003) who used a range of 1.024 -1.08 for θ
199 (equivalent to an approximate Q_{10} of 1.3-2.2) for biological processes. For the physical process of aeration Elmore
200 and West (1961) suggests a value of 1.024 (equivalent). Due to lack of data, and to maintain simplicity and focus of
201 the study, all temperature dependent processes were modelled with a θ of 1.024 except the phytoplankton maximum
202 growth rate. The van't Hoff-Arrhenius relationship was not applied to phytoplankton maximum growth rate because
203 literature supported a larger temperature dependence, and Dauta et al. (1990) estimated maximum phytoplankton
204 growth rates of 0.3 day⁻¹ at 5 °C and 0.72 day⁻¹ at 15 °C: equivalent to a Q_{10} of 2.4. During our study, the
205 phytoplankton growth rates were calibrated to 0.32 day⁻¹ at and 0.75 day⁻¹ at 5 °C and 15 °C; values remarkably
206 close to those recorded by Dauta et al. (1990). The model was found to be insensitive to any change in the growth
207 rate of bacteria in the range of literature values (Tables 5 & 6).

$$208 \quad \text{Rate}_{\text{Temperature}} = \text{Rate}_{20\text{C}} * \theta^{(T-20)} \quad (2.1)$$

209 2.3 Phytoplankton

210 Modeling of phytoplankton populations and their growth must account for the vertical distribution of the population
211 and the vertical gradients in irradiance, nutrients, and metabolites. As our system represents a special case of high
212 light attenuation, limited vertical mixing forces, and high nutrients, phytoplankton growth was formulated on the
213 premise that the phytoplankton population has the ability to optimize its growth rate and will distribute itself
214 accordingly. The formulation is significantly different than common formulations used for well-mixed environments
215 such as in Huisman and Weissing (1994). The deviation was out of a necessity as it was discovered that the unique
216 environmental conditions required approaching the problem from a different paradigm. Sections 2.3 focuses on the
217 process by which the novel formulation for phytoplankton growth was developed to describe the arctic WSP. The
218 development of the mathematical characterization of the depth integrated phytoplankton response for a WSP

219 requires careful consideration of three factors: i) phytoplankton-light response ii) population density limited growth,
220 and iii) photoinhibition.

221 2.3.1 Phytoplankton light response

222 Solar radiation provides the energy for photosynthesis, and the total (vertically integrated) phytoplankton production
223 will be proportional to the amount of energy absorbed by the phytoplankton. Not all of the irradiance that reaches
224 the surface of the water column can be utilized by the phytoplankton because light energy is also absorbed or
225 reflected by particles. Additionally, light photons are absorbed by the phytoplankton cells themselves, reducing the
226 available irradiance to other cells (specifically at greater depth) and as the vertically integrated population density
227 increases the available irradiance per individual must decrease, and is known as self-shading. Finally, the response
228 of the depth-integrated phytoplankton population in the photic zone is assumed to be related to the average
229 irradiance in the photic zone by a hyperbolic function.

230 The transmittance of light has been demonstrated to be successfully approximated to follow exponential decay with
231 distance through a media, and is commonly described by Beer-Lambert's law:

$$I_z = I_0 e^{-kz}$$

232 Where: I_0 = irradiance at surface (depth 0 m), I_z = irradiance at depth z ($\mu\text{E}/\text{m}^2/\text{s}$), k = attenuation coefficient (m^{-1}),
233 and z = depth (m)

234 The attenuation coefficient is a water quality property i.e. an expression of color and suspended solids (Lorenzen
235 1972). When modeling vertically varying phytoplankton growth, it is common to define the euphotic zone depth, as
236 the depth where 1% of the surface light may be measured in a water column with attenuation properties of k :

$$237 \quad -\frac{\text{Ln}(0.01)}{k} = z_{1\%} \quad (2.2) \quad (\text{Lorenzen 1972})$$

238 Phytoplankton concentrations change with depth and time, and therefore k was split into two contributors; k_w
239 (considered a property of the water and its constituents), and k_p (accounts for the absorption of light by
240 phytoplankton).

$$241 \quad k = k_w + k_p(A(t)) \quad (\text{Lorenzen 1972})$$

242 Where: k_w = Light attenuation coefficient of water and constituents (m^{-1}), k_p = Light attenuation coefficient of
243 phytoplankton ($m^{-1}/[mg/l]$), and A = phytoplankton concentration (mg/l)

244

245 k_w and k_p were considered homogenous and constant over the duration of the simulation.

246 The average light in the photic zone (between the surface and $Z_{1\%}$) can be approximated by incorporating
247 attenuation into Beer-Lambert's law and integrating over the photic zone and averaging over the depth:

248
$$I_{av} = \frac{1}{z_{1\%}} \int_0^{z_{1\%}} I_0 e^{-kz} dz = \frac{1 - e^{-(k_w + k_p * A)(z_{1\%})}}{(k_w + k_p * A)z_{1\%}} \quad (2.1)$$

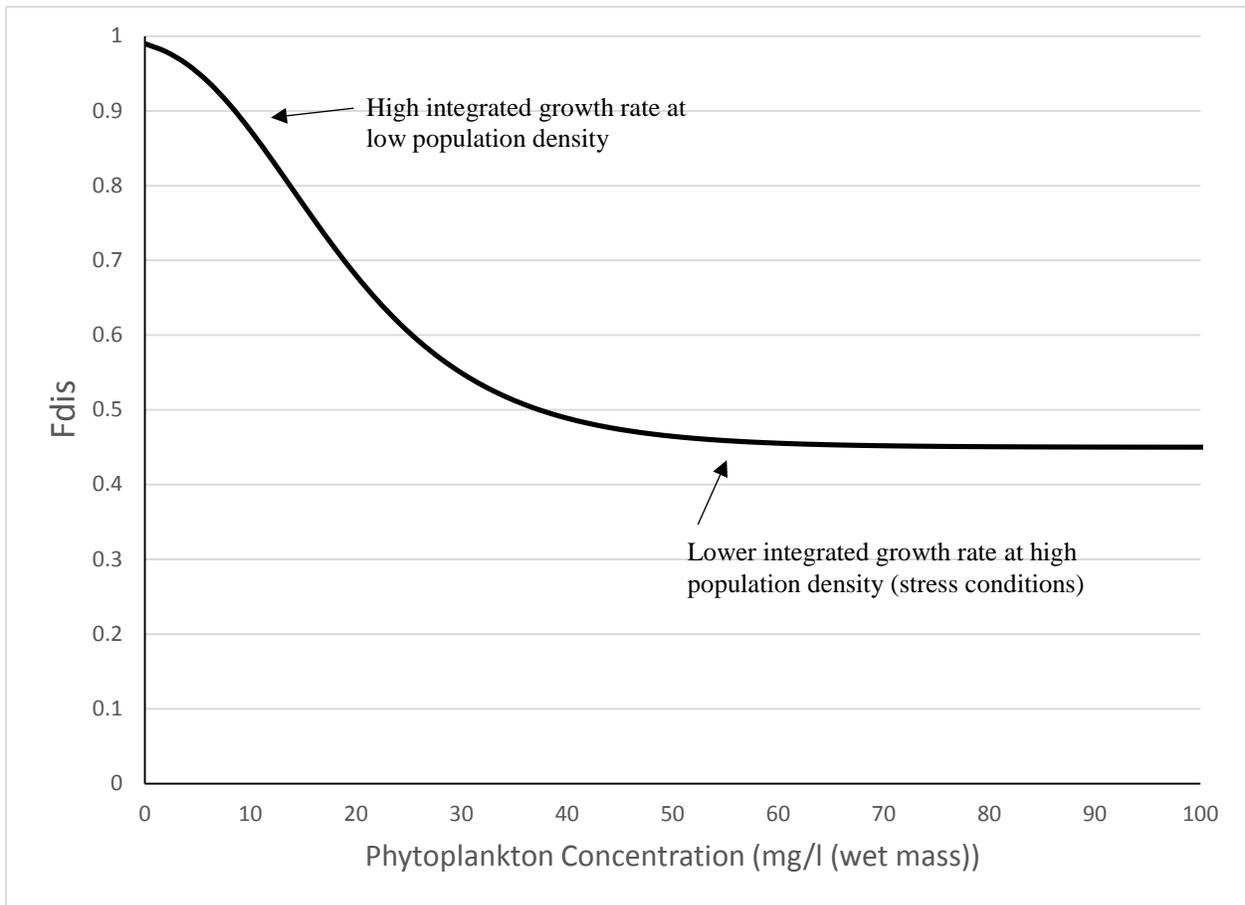
249 Note that in this formulation of the production-irradiance relationship average irradiance and phytoplankton values
250 are used, and results in an average growth rate in the photic zone. Although neither the phytoplankton concentration
251 nor the irradiance is constant with depth, an appropriately parameterized box model is not compromised by using the
252 average values; however, the parameterization is likely to be impacted (Behrenfeld & Falkowski 1997). The average
253 irradiance in the photic zone was used in the Michaelis-Menten equation for the production-irradiance response of
254 phytoplankton (Equation 4) with the half saturation constant of *Chlorella vulgaris* as reported by Dauta et al. (1990).
255 Alternatively, it was found that the exponential formulation for the phytoplankton-irradiance curve ($1 - \exp(-$
256 $\alpha I_{av} / U_{max})$) can be substituted with a value of 0.016 for α / U_{max} (assumed constant with temperature) with no impact
257 on model results.

258 2.3.2 Phytoplankton Distribution

259 If light was the only controlling factor of phytoplankton growth, it would be optimal for phytoplankton to grow in
260 large concentrations over a narrow depth where irradiance was optimal. Although phytoplankton populations
261 predictably reside in greatest concentrations near the depth of optimal irradiance (assuming no nutrient limitations)
262 (Mellard et al. 2011), as the population grows the vertical range inhabited expands out from the area of optimal
263 irradiance (Klausmeier and Litchman 2001). The physiological causes for this broadening of the vertical population
264 structure were not identified in this study, however the authors postulate that it is biological stressors related to
265 limitations in extracellular mass transfer rates (diffusion) of metabolites, such as carbon dioxide, oxygen, and
266 nutrients that create this vertical distribution. We assume phytoplankton can only obtain their maximum growth rate
267 at low population densities, when nutrients and light are abundant and the stressors associated with high population

268 densities are not present. We propose that the impact of self-limitation of phytoplankton growth is a critical element
269 in modeling phytoplankton dynamics in nutrient rich systems with minimal vertical mixing. Populations that
270 experience self-limitation with density are commonly described with logistic growth models, and in our model we
271 utilized a generalized logistic growth model, the Gompertz model (F_{dis}). The Gompertz model has been utilized
272 extensively in the modelling of bacteria populations (Contois, 1959, Zwietering et al. 1990), however the function is
273 difficult to visualize, and so a plot of F_{dis} (Figure 2) has been provided to clearly depict the response of this function
274 and demonstrate how it represents the aforementioned goals. The parameters of F_{Dis} , AGS (Phytoplankton Growth
275 Self-suppression) and ED (Equal Distribution), can be estimated through review of literature, however AGS and ED
276 were largely used in calibration of the model because more experimentation of the relationship between
277 phytoplankton density and phytoplankton growth is needed. The resulting formulation is:

278
$$F_{dis} = (1 - ED) * \left(1 - e^{-4e^{-(AGS)A}}\right) + ED \quad (2.3)$$



280

281 *Figure 2. Representation of the changes in integrated phytoplankton growth rate (Fdis) with increasing phytoplankton*
 282 *concentration.*

283

284 2.3.2.1 *Photoinhibition*

285 The growth rate of phytoplankton increases with increasing irradiance until an optimal irradiance results in a
 286 maximum growth rate, after which a decline in growth rate is typically observed (Dauta et al., 1990). The
 287 observation of such as photoinhibition has been documented in small batch reactors where phytoplankton are
 288 confined and subjected to high irradiance. However, photoinhibition is the result of a phytoplankton's inability to
 289 remove the stressor of excessive irradiance and UV radiation forming harmful reactive oxygen species, and it can be
 290 rationalized that provided the phytoplankton has adequate (i) space and (ii) mobility, they will avoid photoinhibition
 291 by migrating towards lower-light levels where their growth is optimized. This would result in a threshold irradiance
 292 where, the vertically integrated specific growth rate has reached a satiated maximum and is an implicit

293 representation of photoinhibition. The authors note that the exclusion of explicit photoinhibition is specific to a case
294 where currents or mixing do not overwhelm the phytoplankton's mobility and the growth of phytoplankton is
295 integrated over a control depth.

296 2.4 Bacteria

297 A logistic growth model (Equation 8), with a death term was used to describe heterotrophic bacteria growth.
298 Bacterial growth suppression (BGS) is approximately the inverse of the carrying capacity, as BGS multiplied by the
299 maximum bacteria population will provide a value of 1, resulting in bacterial growth rate of zero. The aerobic
300 metabolism was based on an oxygen utilization rate (OUR) (units of $\text{mg O}_2/\text{mg bacteria day}^{-1}$) that consisted of the
301 basal oxygen utilization rate (our_b) required to sustain the existing population, and an additional oxygen utilization
302 rate (our_m) required for the population to grow (Equation 10). It was reasoned that in the case there was less oxygen
303 available than desired by the bacteria, the bacteria would use all the oxygen (resulting in an OUR equal to the
304 available oxygen concentration divided amongst the bacteria concentration). The model does not consider the
305 potential for anaerobic growth of bacteria. The removal of CBOD_5 was equivalent to the amount of oxygen used,
306 and the production of CO_2 was computed based on the stoichiometry of carbon dioxide produced for every unit of
307 oxygen used (Y_{bc}/Y_{bo}).

308

309 2.5 Carbon Cycling

310 Phytoplankton and bacteria return carbon back into the organic pool upon death (Equation 7). From literature, it was
311 estimated that 1 mg of dry phytoplankton mass has a chemical oxygen demand (COD) of 1 mg (Boyd 1973) and
312 The general relationship of 1 mg/l CBOD_5 : 2 mg/l COD was used resulting in 0.5 mg CBOD_5/mg phytoplankton and
313 0.7 mg CBOD_5/mg bacteria. Additionally, the CBOD_5 of the phytoplankton was also accounted for in equation 13
314 by adding the CBOD_5 of the phytoplankton to the CBOD_5 of the organic pool. Bacteria was omitted from being
315 considered in the CBOD_5 pool because it forms the community that is responsible for the utilization of oxygen
316 within the CBOD_5 test and therefore cannot be enumerated.

317 3 Results and Discussion

318 The model was used to investigate the experimental results from Ragush et al. (2017). In the experiment,
319 mesocosms representative of arctic WSPs were constructed to assess the impact of temperature, irradiance, organic
320 loading rate, and initial organic concentration. The experiment was run either until steady state of oxygen and
321 CBOD₅ were achieved or for 40 days. The populations of phytoplankton and bacteria, and CBOD₅ concentration
322 were measured every 5-7 days while dissolved oxygen concentration was measured daily. The system was operated
323 in a manner that is analogous to systems in the North, with daily loading of carbon and nutrients being imitated with
324 a complex synthetic wastewater. Temperature and irradiance were maintained as constants for the duration of trials.
325 The water level was maintained through the addition of distilled water to replace evaporated volume to remove the
326 impact of any concentrating effect.

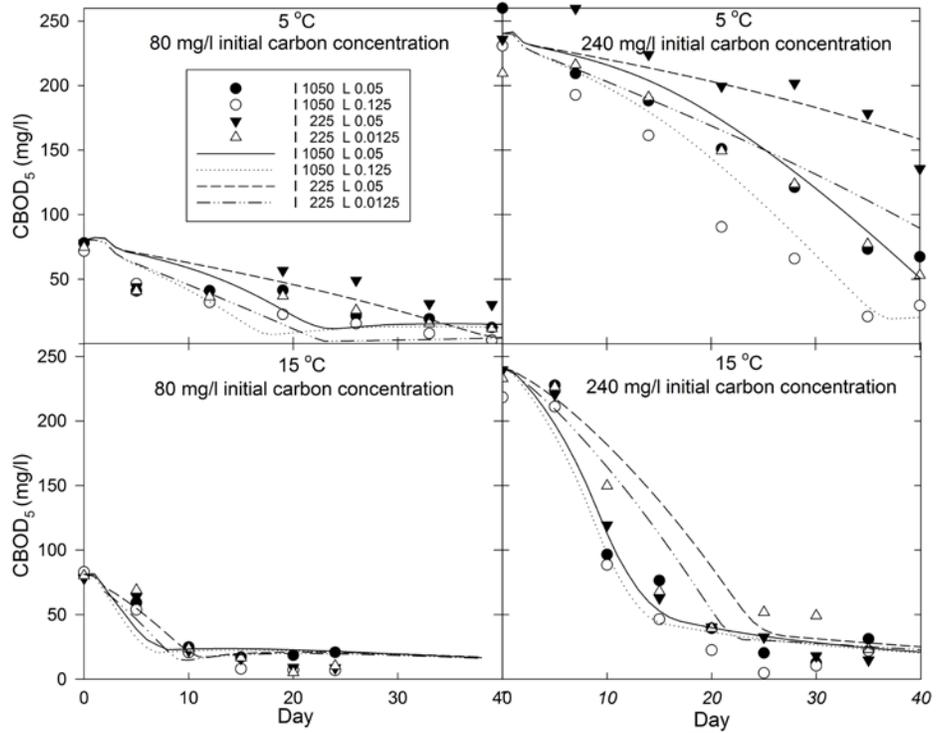
327 3.1 Model Calibration and Performance

328 Experimental results from Ragush et al. (2017) were used to calibrate the model. The calibration was performed by
329 fitting the model to the experimental results of CBOD₅ and dissolved oxygen concentrations obtained at 5 °C, and
330 then validating against experimental results generated at 15 °C. Maximum phytoplankton growth rates (U_{max_a}) were
331 set to the values provided by Dauta et al. (1990). U_{max_a} was then calibrated at both temperatures, however, the
332 calibration values (0.32 and 0.75 days⁻¹ at 5 and 15 °C, respectively) represented only a minor adjustment from
333 growth rates (0.3 and 0.7 days⁻¹) provided by Dauta et al. (2010). The model was calibrated at both temperatures
334 with a 240 mg/l initial carbon concentration, and validated at 80 mg/l initial carbon concentration. The values of the
335 calibrated parameters are provided in Table 4. Figures 3 and 4 depict the model predicted (lines) and experimental
336 observed (symbols) CBOD₅ and dissolved oxygen concentrations, respectively, under the different temperature and
337 initial loading conditions and show that the model is able to capture the general trends and effectively distinguishes
338 system dynamics for the various conditions. Such qualitative model-data comparison is sufficient for the purposes of
339 this paper, which focuses on exploration of the parameterization and impact of different environmental conditions.

340 *Table 4. Manually calibrated model parameters.*

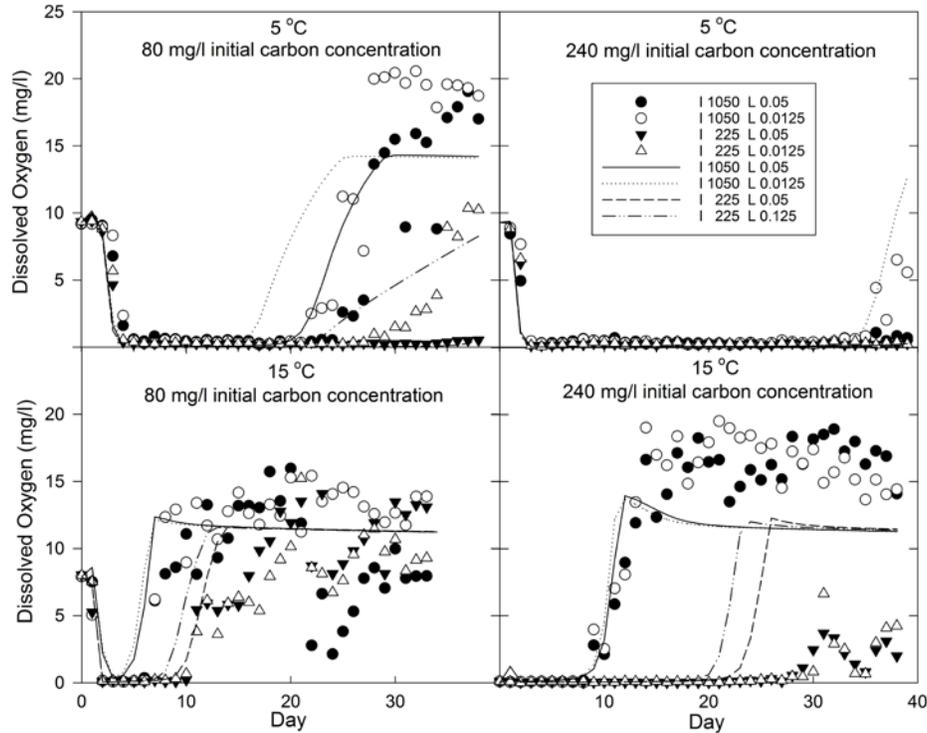
Parameter	Definition	Units	Value
Ihalfsat	Irradiance half saturation of phytoplankton	$\frac{\mu E}{m^2} s^{-1}$	30

K_{ad}	Phytoplankton death rate	day^{-1}	0.05
K_{as}	Phytoplankton settling	day^{-1}	0.05
K_{bd}	Bacteria death rate	day^{-1}	0.025
K_{bs}	Bacteria settling	day^{-1}	0
K_{CO_2}	Half saturation of phytoplankton on carbon dioxide	$\frac{\text{mg CO}_2}{\text{l}}$	0.044
K_{O_2}	Half saturation of bacteria on oxygen	$\frac{\text{mg O}_2}{\text{l}}$	0.256
K_p	Light abstraction by phytoplankton	$\frac{\text{m}^{-1}}{\text{mg/l}}$	0.13
Kl_{CO_2}	Carbon Dioxide transfer rate (piston velocity)	$\frac{\text{m}}{\text{day}}$	0.17 (@ 20 °C)
Kl_{O_2}	Oxygen transfer rate (piston velocity)	$\frac{\text{m}}{\text{day}}$	0.17 (@ 20 °C)
K_s	Half saturation of bacteria on substrate	$\frac{\text{mg CBOD}_5}{\text{l}}$	80
our_b	Basal oxygen utilization rate of bacteria	$\frac{\text{mg O}_2}{\text{mg bac}} \text{day}^{-1}$	0.10 (@ 20 °C)
our_m	Metabolic oxygen utilization rate of bacteria	$\frac{\text{mg O}_2}{\text{mg bac}} \text{day}^{-1}$	0.55 (@ 20 °C)
U_{max_a}	Max growth rate phytoplankton	day^{-1}	0.75 (@ 15 °C) 0.32 (@ 5 °C)
U_{max_b}	Max growth rate bacteria	day^{-1}	5
Y_{ac}	Yield factor of CO ₂ consumed per a mg of phytoplankton	$\frac{\text{mg CO}_2}{\text{mg Phytoplankton}}$	2.18
Y_{bc}/Y_{bo}	Carbon dioxide/oxygen produced	$\text{mg CO}_2/\text{mg O}_2$	1.38
BGS	Bacterial Growth Self Suppression	l/mg	0.01
AGS	Phytoplankton growth Self suppression	Unitless	0.1
ED	Equal Distribution Factor	Unitless	0.45



342

343 *Figure 3. Concentrations of CBOD5 in model waste stabilization ponds operating at 5 or 15°C with different initial carbon*
 344 *concentrations (80 or 240 mg/l). The model performance for CBOD5 concentration predictions under the different conditions is*
 345 *shown as lines, while the experimental results from Ragush et al. (2017) are shown as symbols. I denotes the modelled and*
 346 *experimental irradiance ($\mu\text{E}/\text{m}^2/\text{s}$) and L is the volumetric loading rate (m^3/day).*



347

348 *Figure 4. Dissolved oxygen concentrations in model waste stabilization ponds operating at 5 or 15°C with different initial carbon*
 349 *concentrations (80 or 240 mg/l). The model performance for dissolved oxygen concentration predictions under the different*
 350 *experimental conditions are shown as line, while the experimental results from Ragush et al. (2017) are shown as symbols. I is*
 351 *the modelled and experimental irradiance ($\mu\text{E}/\text{m}^2/\text{s}$) and L is the volumetric loading rate (m^3/day),*

352 While in general, good qualitative agreement between model and experimental results was observed, several
 353 inconsistencies provide insight into areas that are not well represented by the model and require further research. For
 354 example, the model underestimated the maximum dissolved oxygen concentrations (as measured at the surface), and
 355 predicted the development of measurable oxygen concentrations ($> 0.5 \text{ mg/l}$) earlier than was found experimentally
 356 (Figure 4). The model's prediction of lower maximum oxygen may be due to differences between what is modeled
 357 versus measured. Specifically, the model represents average concentrations over the photic zone, whereas measured
 358 values were taken at a depth where oxygen was likely at its maximum. To determine if it is a discrepancy between
 359 what is being measured vs modeled, increased measurement resolution by placing sensors throughout the photic
 360 zone would be necessary. The model's tendency to predict measureable oxygen concentration earlier than
 361 experimentally observed, especially under low light conditions, may be due to neglecting O_2 diffusion from the
 362 oxygen productive photic zone deeper into aphotic (anoxic) zone. Furthering this thought concerning the model's
 363 late prediction of measurable oxygen concentrations appearing as seen in Figure 4, the flux of oxygen from the
 364 photic to aphotic zone early in the experiment is of a similar magnitude to that of oxygen production of

365 phytoplankton in the early stages of phytoplankton growth. As phytoplankton populations increase the impact of
366 molecular diffusion on the oxygen concentration decreases relative to other factors such as oxygen production by
367 phytoplankton.

368 The model only considers aerobic metabolism of bacteria for the removal of CBOD₅, and due to the good agreement
369 with experimental results, this appears to be a reasonable simplification. However, when hypoxic conditions prevail,
370 especially under low light conditions with minimal oxygen production by photosynthesis, the model under-predicts
371 the treatment performance (Figure 3). The incorporation of anaerobic processes is likely to improve the robustness
372 and prediction under low light and cold conditions.

373 Finally, from a practical application, the model was able to capture the influence of organic loading rates and initial
374 carbon concentrations on dissolved oxygen and CBOD₅ concentrations (Figures 3 and 4). These are two key
375 parameters that WSP designers are able to control. Such findings suggest arctic WSPs can obtain an effluent
376 concentration for CBOD₅ that meet secondary wastewater treatment standards (25 mg/l) with lowered areal loading
377 rates, and more importantly lowered carbon concentrations at the onset of the summer treatment season.

378 3.2 Sensitivity Analysis

379 A one-factor-at-a-time (OFAT) method local sensitivity analysis (or nominal range analysis) was performed post
380 calibration of the model. An OFAT does not assess the parameter interactions and results of the OFAT may be
381 impacted by the values of other parameters set during the calibration. The sensitivity analysis was carried out on the
382 20 parameters in Table 1. The parameter range tested was chosen based upon values reported in the literature, listed
383 in Table 4. OFAT is an effective way of determining the model parameters that carry the most influence on output
384 results (Cullen and Frey 1999), and is useful for identifying where to focus data collection related to improving the
385 model (Salehi et al. 2000). These two strengths are directly in-line with the exploratory goals of this paper. In the
386 OFAT, parameters were set to the calibrated value (Table 4) and one parameter at a time was varied over 5 equally-
387 spaced levels that ranged between the high and low values reported in the literature when available (Table 5) or else
388 a range of (+/- 25%).

389 *Table 5 Parameter values from literature*

Parameter	Definition	Units	Reported Values	Sources
------------------	-------------------	--------------	------------------------	----------------

I _{halfsat}	Irradiance half saturation of phytoplankton	$\frac{\mu E}{m^2 s^{-1}}$	30	Dauta et al. (1990)
			60	Moreno-Grau et al. (1996)
			220	Beran & Kargi (2005)
K _{ad}	Phytoplankton death rate	day ⁻¹	0.05	Lawrence & McCarty (1970)
			0.001	Moreno-Grau et al. (1996)
			0.05-0.25	Schnoor (1996)
K _{as}	Phytoplankton settling/respiration	day ⁻¹	0.2 m/d	Schnoor (1996)
			0.05	Moreno-Grau et al. (1996)
			0.035	Moreno-Grau et al. (1996)
K _{bd}	Bacteria death rate	day ⁻¹	0.1	Buhr & Miller (1983)
			0.06 death	Beran (2005)
			0.06-0.015	Metcalf & Eddy (2003)
K _{bs}	Bacteria respiration/settling rate	day ⁻¹	0.085 (+/- 25%)	Moreno-Grau (1996)
K _{CO2}	Half saturation of phytoplankton on carbon dioxide	$\frac{mg CO_2}{l}$	0.044 (+/- 25%)	Buhr & Miller (1983)
K _{O2}	Half saturation of bacteria on oxygen	$\frac{mg O_2}{l}$	0.256	Buhr & Miller 1983
			0.128	Banks et al. (2003)
			1	Tchobanoglous et al. (2003)
K _p	Light abstraction by phytoplankton	$\frac{m^{-1}}{mg/l}$	0.138 – 0.0249	Lorenzen (1972) Li (2009)
K _s	Half saturation of bacteria on substrate	$\frac{m}{day}$	25-100 (60)	Metcalf & Eddy (2003)
			150	Lawrence & McCarty (1970)
K _{lCO2}	Carbon Dioxide transfer rate (piston velocity)	$\frac{m}{day}$	0.893	Boogerd et al. (1989)
			1	Schnoor (1996)
K _{lO2}	Oxygen transfer rate (piston velocity)	$\frac{mg CBOD_5}{l}$	0.15	Schnoor (1996)
			0.189	Chu & Jirka (2003)
			0.24	Deacon (1977)
our _b	Basal oxygen utilization rate of bacteria	$\frac{mg O_2}{mg bac} day^{-1}$	0.15 (+/- 25%)	Jenkins (1978)
our _m	Metabolic oxygen utilization rate of bacteria	$\frac{mg O_2}{mg bac} day^{-1}$	0.85 (+/- 25%)	Jenkins (1978)
U _{max_a}	Max growth rate phytoplankton	day ⁻¹	0.3 (5 °C) 0.7 (15 °C)	Dauta et al. (1990)
			0.5	Moreno-Grau et al. (1996)
			0.48 (5 °C) 0.78 (15 °C)	Buhr & Miller (1983)
			1.13 (@20 °C)	Banks (2003)
			1.5 (@20 °C)	Schoor (1996)
U _{max_b}	Max growth rate bacteria	day ⁻¹	4.95	Banks (2003)
			5.0	Moreno-Grau et al. (1996)
			2-10	Metcalf & Eddy (2003)
Y _{ca}	Yield factor of phytoplankton produced for CO ₂ consumed	$\frac{mg CO_2}{mg Phytoplankton}$	2.18	Fogg (1953)
			1.83	Cramer & Myers (1948)
			1.82	McKinney (2004)

Y_{Ca}/Y_{oa}	Carbon dioxide/ oxygen produced	mg CO ₂ / mg O ₂	1.25 – 1.37	Fogg (1953) Cramer & Myers (1948) McKinney (2004)
BGS	Bacterial Growth Self Suppression	mg/l ⁻¹	0.002– 0.05	Estimated
AGS	Phytoplankton growth Self suppression	Unitless	0.02 – 0.5	Estimated
ED	Equal Distribution	Unitless	0.25 – 0.6	Estimated

390

391 Sensitivity coefficients (SC) were developed for two chemical responses, i.e., when dissolved oxygen first exceeds 2
392 mg/l, and when CBOD₅ concentrations are reduced to 30 mg/l and four biological response metrics, i.e., the timing
393 of and maximum predicted phytoplankton and bacteria populations. The sensitivity coefficient provides a non-
394 dimensional measure of relative influence of parameters to the relative change in the response (Downing et al.
395 1985). The sensitivity coefficient was calculated according to Equation 3.1, and five parameter values (the original
396 and two higher and two lower) were used to determine an average SC over the parameter range (equation 3.2). The
397 SC was taken to be the average to smooth out non-linearities within the relationship. Sensitivity analysis was
398 performed at both lighting and temperature conditions at an initial carbon concentration of 240 mg/l and 0.0125 l/d
399 loading rate to examine if the sensitivity of the parameters varied with environmental conditions. Tables 6 and 7 list
400 the sensitivity coefficients for timing of dissolved oxygen concentration exceeding 2 mg/l and timing of CBOD₅
401 concentration below 30 mg/l. Insights from Table 5 and 6 will be discussed further in this section.

402

$$SC(P)_i = \left| \frac{dR}{dP_i} \right|_{P_0} = \frac{\frac{R_i - R_0}{R_0}}{\frac{P_i - P_0}{P_0}} \quad (3.1)$$

403 Where:

404 R = response vector, P = parameter vector, SC(P) = Sensitivity Coefficient of parameter p, and O =
405 origin of parameter value (middle value of range tested)

406

407

$$SC(P) = \frac{\sum_i^n SC(P)_i}{n} \quad (3.2)$$

408 *Table 6. Parameter sensitivity coefficient for timing of dissolved oxygen concentration exceeding 2 mg/l. Parameters with higher*
409 *values are more sensitive.*

Temperature (°C)	5	15	5	15
Light (μE/m²/s)	250	250	1000	1000
Ihalfsat	0.41	0.47	0.49	0.67

K_{ad}	0.54	0.38	0.77	0.45
K_{as}	0.11	0.14	0.16	0.13
K_{bs}	0.03	0.01	0.02	0.01
K_{bd}	0.70	0.17	0.31	0.03
K_s	0.02	0.10	0.06	0.17
K_{O_2}	0.00	0.04	0.02	0.09
K_C	0.00	0.01	0.00	0.01
K_p	0.03	0.10	0.02	0.04
Kl_{CO_2}	0.00	0.00	0.00	0.00
Kl_{O_2}	0.73	0.26	0.47	0.14
our_b	0.01	0.08	0.04	0.15
our_m	0.03	0.05	0.01	0.19
U_{max_a}	0.98	1.46	1.32	1.74
U_{max_b}	0.03	0.01	0.02	0.01
Y_{ca}	0.55	0.66	0.65	0.61
$Y_{caOY_{ca}}$	0.56	0.68	0.66	0.68
BGS	0.82	1.50	1.45	2.60
AGS	0.34	0.33	0.40	0.21
ED	0.15	0.57	0.37	0.74

410

411

412

Table 7. Parameter sensitivity coefficients for timing of CBOD5 concentration below 30 mg/l

	Temperature (°C)			
	5	15	5	15
Light ($\mu E/m^2/s$)	250	250	1000	1000
$I_{halfsat}$	0.29	0.86	0.46	0.47
K_{ad}	0.48	0.31	0.54	0.25
K_{as}	0.10	0.09	0.11	0.04
K_{bs}	0.02	0.00	0.01	0.01
K_{bd}	0.73	0.26	0.36	0.20
K_s	0.00	0.03	0.01	0.15
K_{O_2}	0.01	0.02	0.01	0.03
K_{CO_2}	0.00	0.01	0.00	0.01
K_p	0.03	0.06	0.02	0.01
Kl_{CO_2}	0.00	0.00	0.00	0.00
Kl_{O_2}	0.71	0.26	0.47	0.13
our_b	0.13	0.03	0.05	0.10
our_m	0.04	0.02	0.03	0.11
U_{max_a}	0.88	1.10	1.18	0.85
U_{max_b}	0.03	0.00	0.01	0.02

Yca	0.50	0.57	0.60	0.41
YcaOYca	0.51	0.58	0.59	0.43
BGS	0.23	0.68	0.03	0.91
AGS	0.29	0.28	0.41	0.34
ED	0.12	0.39	0.27	0.32

413

414 A cumulative sensitivity report was constructed to provide a qualitative assessment of parameter sensitivity across
415 the range of temperature and irradiance conditions, and a measure of relative parameter sensitivity in the model.

416 Table 7 provides a sensitivity index by tallying the number of sensitivity coefficients of the 6 tested responses that
417 exceeded 0.1 (a value that was arbitrarily assigned as being an indicator of a sensitive parameter) for a parameter
418 under the noted temperature and irradiance conditions. To provide a comparison of parameter sensitivity, the right
419 column total is a summation of exceedances for a parameter under all temperature/light conditions, and sensitivity
420 ranking of the parameters developed by blending the response sensitivity coefficients. Finally, to compare
421 sensitivity of the model under the four light/temperature pairings, a summation of the sensitivity index for each
422 pairing is provided in the bottom row of Table 7.

423 *Table 8 Cumulative sensitivity index by parameter or temperature/irradiance. Value denotes number of SI indices greater than*
424 *0.1 for 6 tested categories (see Tables 6 and 7).*

	Temperature (°C)	5	15	5	15		Sensitivity
	Irradiance (ue/m²/s)	225	225	1025	1025	Total	Ranking
Ihalfsat	4	6	5	5	20	2	
K _{ad}	4	5	6	5	20	5	
K _{as}	5	3	5	2	15	12	
K _{bs}	2	0	0	0	2	15	
K _{bd}	5	5	5	3	2	9	
K _s	0	1	0	3	4	14	
K _{O2}	0	0	0	0	0	18	
K _{CO2}	0	0	0	0	0	20	
K _p	0	1	0	0	1	16	
K _{lCO2}	0	0	0	1	1	19	
K _{lO2}	4	5	5	4	18	10	
our _b	3	3	4	3	13	11	
our _m	1	0	0	4	5	13	
Umaxa	6	5	5	5	21	1	
Umaxb	0	0	0	0	0	17	
Yca	6	5	5	5	21	4	
YcaOYco	5	5	5	5	20	6	

BGS	5	5	5	6	21	3
AGS	6	5	5	4	20	8
EDFactor	4	5	5	5	19	7
Total	60	59	60	60		

425

426 Parameter sensitivity was consistent for all the tested irradiance and temperature conditions (Table 7). However, the
 427 sensitivity of certain parameters, such as oxygen aeration rate (Kl_{O_2}) and bacterial growth self-suppression (BGS)
 428 can vary greatly with changing environmental conditions (Tables 5 and 6). Finally, the analysis highlighted the
 429 model's sensitivity to phytoplankton growth parameters as six of the seven most sensitive parameters are related to
 430 phytoplankton growth rate or metabolism (Table 7).

431 Critical assessment of the sensitivity analysis provides insight into model dynamics and limiting processes under
 432 different conditions. In general, the tested model was more impacted by changing parameter values at lower
 433 temperature, and this result reinforces findings that the $CBOD_5$ removal and oxygen dynamics in the WSP are less
 434 stable at lower temperature, as also noted by Ragush et al. (2017). The large increase in the sensitivity coefficient of
 435 Kl_{O_2} at low temperature identifies $CBOD_5$ removal at these low temperatures as being rate limited by the lack of
 436 oxygen. The observation of the importance of Kl_{O_2} at low temperature highlights the lack of impact of
 437 phytoplankton at a temperature of 5 °C, as well as illustrates the importance of phytoplankton in a system intended
 438 to remove $CBOD_5$. The BGS parameter, the bacterial carrying capacity, was more sensitive at an increased
 439 temperature of 15 °C compared to 5 °C. This would suggest that once the limitation of oxygen has been removed in
 440 WSPs, it is the activity (and size) of the bacterial community that will be the limiter of the $CBOD_5$ treatment rate.

441 4 Conclusion

442 Our model successfully linked aspects of ecosystem models (phytoplankton growth, irradiance) with wastewater
 443 treatment models (bacterial growth, $CBOD_5$) though the stoichiometry of reactions utilizing carbon dioxide and
 444 oxygen to create a model of arctic WSPs. Our efforts to model WSPs in the arctic shed light on the unique aspect of
 445 modeling phytoplankton under poorly mixed conditions, and we demonstrated, that in a poorly mixed system,
 446 approaching phytoplankton growth functions through a paradigm of growth optimization is a viable path to
 447 developing functions that are representative. A local sensitivity analysis was performed and illustrated the

448 importance of phytoplankton for the removal of CBOD₅ and the development of facultative conditions (> 2 mg/l
449 DO).

450 Our box model of the photic zone of WSPs operating under arctic conditions had the ability to predict the trends in
451 CBOD₅ and DO concentrations presented in Ragush et al. (2017) for different light and irradiance conditions.
452 Highlighted in the study, is that the difference in the phytoplankton growth rate was largely responsible for WSP
453 treatment performance in the temperature range of 5 – 15°C. The CBOD₅ removal rate was oxygen limited in
454 instances when phytoplankton concentrations were small, and point to the requirement of either supporting the
455 phytoplankton population's growth under these cold conditions or supplementing oxygen in WSPs with aeration to
456 achieve effective CBOD₅ treatment. In terms of supporting the phytoplankton population's growth, the most logical
457 method is increasing the temperature in these systems, and the most intuitive way of potentially doing so is
458 providing shallow summer treatment cells (less than 1.5 m deep).

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