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Improved Analysis and Modelling of Leakage Inductance for Planar Transformers

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Abstract— Planar transformers have often been mistaken to essentially have lower leakage inductances. The “radial effect” is a nature characteristic for planar windings due to a higher aspect ratio of conductor width to conductor thickness, which gives a reduction in leakage inductance. Traditional formulas for leakage inductance in traditional transformers where the winding width is much smaller than the winding height are not suitable for planar transformers. This paper specifically tailors the traditional 1-D solution of leakage inductance by decomposing the leakage flux into longitudinal and transversal flux. In this manner the “eddy current effect” and the “radial effect” in leakage inductance can be analyzed individually. The proposed new formula including both ac (high frequency eddy current effect) and dc effects (radial effect) offers an accurate prediction of leakage inductance in planar transformers. Finite Element Analysis (FEA) and measurements are carried out to validate the proposed formula.

Index Terms— leakage inductance, transformer, switched mode power supply, magnetic field.

I. INTRODUCTION

Planar magnetics have gained the popularity in recent years because they have several advantages such as low profile, modularity, good thermal characteristics, ease of manufacture, and predictable parasitics etc. [1]-[9]. In traditional wire-wound components, it is very difficult to control the winding layout, which means significant variations in leakage inductance and winding capacitance appear in devices manufactured at the same time. On the other hand, with planar magnetics, the windings manufactured by automated PCB machines are more precise and consistent, yielding magnetic designs with highly controllable and predictable parasitic parameters [5]. Developments in wide bandgap devices (such as GaN and SiC semiconductors) and in advanced magnetic materials (such as Hitachi ML91S and TDK PC200) push the switching frequency into the MHz range, to minimize the size of the inductive components, however, the switching loss increases with frequency. PCB windings with enclosed planar magnetic cores have been widely used in such high-frequency and high-power-density converters [6]-[9]. Therefore, an accurate prediction of leakage inductance for planar transformers is needed in order to implement an optimized design in terms of high efficiency. Planar transformers have also been studied in resonant converters, such as LLC

converters, where the leakage inductance is critical for a well-matched resonant frequency [10]-[14].

Characterization and modeling of the planar transformers have been widely published [15]-[28]. Most of the efforts are oriented to obtain winding loss based on modifications to Dowell’s formula [15]. Dowell’s formula may also be used to find the leakage inductance in conventional transformers [16]. A new set of formulas for calculating self and mutual impedances of planar coil on homogenous ferromagnetic substrates was proposed in [21]. These formulas are well suitable for planar air coils, thick film and thin film cases, but they are not directly applicable to planar PCB transformers where the core encloses the winding. But it is difficult to apply these results to planar PCB transformers. 1-D magnetic component model for planar structures based on transmission lines has been described in [24], [25], and an important characteristic of the “radial effect” for planar transformers has been reported as well. A general 2-D analytical method which is based on the partial-element-equivalent-circuit (PEEC) method was proposed in [30], [31] to compute the window field, and thus to calculate the leakage inductances of planar transformers. Publications [32] and [33] studied the leakage inductance influenced by the high frequency eddy current effect, and accurate expressions of frequency-dependent leakage inductance were proposed. However, these expressions can only be applied in traditional transformer structures where the “radial effect” of planar transformer was not taken into account.

It has often been misunderstanding assumed that planar transformers intrinsically have low leakage inductances compared to that of conventional counter parts. In fact, leakage inductance is proportional with the mean turn length (MTL) of the conductors. Planar transformers intrinsically have a longer mean turn length than that of the traditional vertical structures, resulting in a higher leakage inductance [1]. However, the important benefit of planar transformers in this regard is the relative ease with which primary and secondary windings can be heavily interleaved in a manufacturing or automation environment. Dowell and many of its modified forms for calculating the leakage inductance has been widely applied in traditional wire-wound transformers. However, significant errors have been observed when they are applied to planar transformers. The errors are mainly due to: 1) the traditional analysis only express the leakage inductance at low frequency (with frequency

Fig. 1 Leakage flux path across half-core window area of a planar transformer.

independency), and the high frequency eddy current effect was not taken into account; 2) the traditional analytical expressions ignores the insulators' thickness between layers because they are so small compared to the thickness of the conductor layers. However, PCB windings usually have much thicker dielectric layers ($>0.2\text{mm}$) than the copper thickness. For this reason, the error of the traditional analytical expression can reach to $\sim 30\%$ or even more [34], depending on the dimensions of the layout; 3) the “radial effect” did not arise in conventional transformers. Planar transformer windings situated in a horizontal plane typically have an extremely high aspect ratio of width to height of a section, causing the currents to concentrate the inner edge, and thus impact on the amplitude of magnetic field tangential to the surface of conductors. A new analytical expression of leakage inductance in planar transformers covering all above 3 considerations especially for the “radial effect” is proposed in this paper. The “radial effect” has been reported in planar transformer modelling before, but it is rare to link it with frequency-dependent leakage inductance calculation. This paper specifically tailors the traditional 1-D solution of leakage inductance calculation by decomposing the leakage flux into longitudinal and transversal flux fields, and thus the “eddy current effect” and the “radial effect” in leakage inductance can be separated. The proposed new formula including both ac (high frequency eddy current effect) and dc effects (radial effect) gives rise to an accurate prediction of leakage inductance in planar transformers.

II. IMPROVED CALCULATION OF LEAKAGE INDUCTANCE FOR PLANAR TRANSFORMER

Some flux will leak from the core and return to the air, winding layers and insulator layers, causing imperfect coupling to the secondary winding in a planar transformer as shown in Fig. 1. If the secondary is short-circuited, the main flux in the core which links both windings will be negligible because the primary and secondary ampere turns almost cancel. This means only the leakage energy is retained. Assuming that the permeability of core material is infinite ($\mu_r \rightarrow \infty, \sigma \rightarrow 0$), the magnetic field intensity goes to zero inside the core. The magnetic core window height h_w is much smaller than the core window width b_w , and the conductors fully fill the core window width, so that end effects can be assumed to be negligible. The leakage inductance is related to

Fig. 2. Inner and outer boundary magnetic field for the n^{th} layer.

the energy stored in the core window area. Therefore, the magnetic field strength accommodated in the conductors and dielectric insulators are very important to give an accurate account of leakage inductance.

A. Radial Effect

Planar transformer windings situated in a horizontal plane typically have a very high aspect ratio of width (b_w) to height (h_c) of the conductor cross-section, with the result that the current is concentrated in the inner path due to lower resistance in the shorter path. In any winding with spiral or circular coils conductors, the shorter path on the inside edge (at $r = r_1$ in Fig. 2) of the conducting section means that the resistance to current flow is lower and, therefore, the current density is higher than that on the outside. On the basis of this observation, it is reasonable to assume that there is an inverse relationship between the dc current density $J(r)$ and the radius r [35]:

$$h \int_{r_1}^{r_2} J(r) \cdot dr = h \int_{r_1}^{r_2} \frac{k}{r} \cdot dr = I \quad (1)$$

where h is the conductor thickness, r_1, r_2 are the distances from the center to the inner edge and the outer edge of the core window respectively. The constant k can be obtained from equation (1):

$$k = \frac{I}{h \cdot \ln\left(\frac{r_2}{r_1}\right)} \quad (2)$$

Therefore, the dc current density $J(r)$ can be expressed:

$$J(r) = \frac{I}{r \cdot h \cdot \ln\left(\frac{r_2}{r_1}\right)} \quad (3)$$

This relationship has been verified by Finite Element Analysis (FEA) simulation as shown in Fig. 6.

Fig. 3. MMF distribution of planar transformer windings at high frequency (FEA simulation).

B. High Frequency Eddy Current Effect

We often observe from the measurement that the leakage inductance is reduced when the frequency increases. This is mainly due to the fact that the high frequency effect concentrates the current on the surface of the conductors, and thus equivalently reduces the thickness of the conductors in which the part of leakage energy is stored. The magnetic field is no longer linearly distributed within the conductors at high frequency because of the eddy current effect, this is illustrated in Fig.3. The area of the MMF curve at high frequency is smaller than the one at low frequency, which means the stored leakage energy is actually lower at high frequency.

In a previous contribution [33], the authors gave a concrete analysis in connection to frequency-dependent leakage inductance particularly for traditional transformer structures. Maxwell's equations can be rewritten in Cartesian coordinates as a second-order ordinary differential equation, namely the Helmholtz differential equation:

$$\frac{d^2 H_r}{dy^2} = j\omega\mu_0\sigma H_r \quad (4)$$

The general solution of the Helmholtz equation is given by,

$$H_r(y) = H_1 e^{\gamma y} + H_2 e^{-\gamma y} \quad (5)$$

where H_1 and H_2 are determined by the boundary conditions and complex constants. The complex propagation constant is:

$$\gamma = \sqrt{j\omega\mu_0\sigma} = \frac{1+j}{\delta_w} \quad (6)$$

and δ_w is the skin depth and $\delta_w = \frac{1}{\sqrt{\pi f \mu_0 \sigma}}$.

However, this analysis only considers the magnetic field along the window height (y -component that is parallel to the surface of conductors). This part of flux is the main contribution to the

Fig. 4. The magnetic fields due to AC effect and DC effect are decoupled.

high frequency eddy current (AC) effect. The DC current distribution $J(r)$ in equation (3) due to the radial effect induces a magnetic field only having r -component, $H_r(r)$, (changing along with r axis). The two magnetic fields are orthogonal and decoupled as shown in Fig. 4, so that they can be treated individually. A piecewise decomposition approach is needed to ensure a uniform current distribution in each piece, in order to apply the r -component magnetic field $H_r(r)$ into the Helmholtz differential equation.

C. Piecewise Decomposition

The first conduction layer may be decomposed into many small segments with the width Δr ($\Delta r \rightarrow 0$), as shown in Fig. 2. Taking Ampere's law at radius r :

$$J(r) \cdot \Delta r \cdot h = [H_o(r, 0) + H_o(r, h)] \cdot \Delta r + [H_o(r - \Delta r) + H_o(r + \Delta r)] \cdot h \quad (7)$$

where the magnetic field intensity in the boundary between the first layer and the high permeability core material $H_o(r, 0) = 0$; and $H_o(r - \Delta r) = -H_o(r + \Delta r)$ due to the condition of $\Delta r \rightarrow 0$, this leads to a simplification of equation (7):

$$H_o(r, h) = J(r) \cdot h = \frac{I}{r \cdot \ln\left(\frac{r_2}{r_1}\right)} \quad (8)$$

Invoking Ampere's law for the closed loops L_n and L_{n-1} (see in Fig. 2) in a high permeability core, the upper and lower boundary conditions for the n^{th} layer of the winding are obtained,

$$H(r, y_{ni}) = (n-1) \cdot H_o(r, h) \quad (9)$$

$$H(r, y_{no}) = n \cdot H_o(r, h) \quad (10)$$

and y_{ni} and y_{no} are the distance from the upper surface of the first conductor to the upper surface and the lower surface of the n^{th} layer respectively. Applying the boundary conditions (9) and (10) to the Helmholtz equation (5), the magnetic field inside the n^{th} layer is obtained:

$$H(r, y) = H_o(r, h) \cdot \frac{n \sinh(\gamma y) + (n-1) \sinh(\gamma h - \gamma y)}{\sinh(\gamma h)} \quad (11)$$

D. Leakage Energy in Each Element

With the consideration of AC and DC effects described above, the double integrals are used to calculate the leakage energy stored in each element. In the corresponding [33] for conventional transformer, this was not necessary since the radial effect was not presented. The differential volume of

each element is $2\pi r \cdot dr \cdot dy$, therefore the energy stored in each layer is:

$$E_i = \frac{1}{2} \cdot \mu_0 \cdot \int_0^h \int_{r_1}^{r_2} H(r, y)^2 \cdot 2\pi r \cdot dr \cdot dy \quad (12)$$

Substituting (11) into (12), the energy stored in the primary winding is:

$$E_p = \sum_{i=1}^{n_p} E_i = \frac{\mu_0 \cdot \pi \cdot I_p^2 \cdot n_p [k_1(2n_p^2 + 1) + 4k_2(n_p^2 - 1)]}{\ln\left(\frac{r_2}{r_1}\right) \cdot 12 \cdot \gamma \sinh^2(\gamma h_p)} \quad (13)$$

where,

$$k_1 = \sinh(2\gamma h_p) - 2\gamma h_p$$

$$k_2 = \gamma h_p \cosh(\gamma \cdot h_p) - \sinh(\gamma \cdot h_p)$$

The same approach can be applied to secondary winding. The expression for the leakage energy stored in the secondary winding E_s is similar to equation (13). The dielectric layers are placed between each conducting layer, and the magnetic field inside the n^{th} dielectric layer remains constant and is equal to the magnetic field of the lower surface of the n^{th} conductor layer. The energy stored in the dielectric layer is, therefore,

$$\begin{aligned} E_d &= \frac{1}{2} \cdot \mu_0 \cdot h_i \cdot \int_{r_1}^{r_2} H(r, h)^2 \cdot 2\pi r \cdot dr \\ &= \frac{\mu_0 \cdot \pi \cdot h_i}{\ln\left(\frac{r_2}{r_1}\right)} \left[I_p^2 \sum_{i=1}^{n_p} i^2 + I_s^2 \sum_{i=1}^{n_s-1} i^2 \right] \\ &= \frac{\mu_0 \cdot \pi \cdot h_i}{6 \ln\left(\frac{r_2}{r_1}\right)} \left[I_p^2 \cdot n_p(n_p + 1)(2n_p + 1) \right. \\ &\quad \left. + I_s^2 \cdot n_s(n_s - 1)(2n_s - 1) \right] \quad (14) \end{aligned}$$

where h_p , h_s and h_i are the thickness of primary winding layers, secondary winding layers, and dielectric layer, respectively. The total leakage energy is sum of the energy stored in each elementary layer which can be expressed by:

$$E_{total} = E_p + E_s + E_d \quad (15)$$

For a configuration with one turn in each layer, the turns ratio $n = \frac{n_s}{n_p}$ is defined, and the thickness of all conductors is kept the same ($h_p = h_s$), then the total leakage inductance is:

$$\begin{aligned} L_{lk} &= \frac{\mu_0 \cdot \pi \cdot n_p}{3 \ln\left(\frac{r_2}{r_1}\right)} \left\{ \frac{n_p^2(k_1 + 2k_2)(n + 1)}{\gamma \sinh^2(\gamma h_p)} \right. \\ &\quad \left. + \frac{(k_1 - 4k_2)(n + 1)}{2n\gamma \sinh^2(\gamma h_p)} \right. \\ &\quad \left. + \left[2(1 + n) \cdot n_p^2 + \frac{1}{n} + 1 \right] h_i \right\} \quad (16) \end{aligned}$$

Equation (16) can be also modeled for the case with multiple turns in each layer. It should be noted that more turns in each layer leads to a smaller effect from the dc current distribution.

Fig.5. Photos of planar PCB transformer with 8 primary winding turns and 8 secondary winding turns.

Table I Transformer Parameters

Parameters	Values
Number of turns in primary (n_p)	8
Turns ratio (n)	1
Primary conductor thickness (h_p)	0.15 mm
Secondary conductor thickness (h_s)	0.15 mm
Insulator thickness (h_i)	0.25 mm
Core type	ER 51/10/38
Core material	Ferrite 3C96
Distance from core center to inner edge (r_1)	10 mm
Distance from core center to outer edge (r_2)	20.9 mm

Fig. 6. Current distribution J_r along with the r axis for the first conductor layer

III. FEA SIMULATIONS AND EXPERIMENTAL VERIFICATIONS

An example of the planar magnetic core ER 51/10/38 was selected to experimentally validate the proposed model for planar transformers. The parameters of the planar transformer are shown in Table I, the actual PCB windings are shown in Fig. 5. Both primary winding and secondary winding have 8 turns with one turn in each layer. The FEA simulation is carried out with ANSYS MAXWELL, and the transformer model is built in a 2D plane under the “eddy current” type.

Fig. 6 and Fig. 7 show the current density along with the r axis (horizontal) and the y axis (vertical), respectively. At low frequency, the current distribution J_r along the r axis well matches the equation (3) where the dc current concentrates on

Fig. 7. Current distribution J_y along with the y axis for the first five conductor layers.

Fig. 8. Magnetic field strength $H_o(r, h)$ along with the r axis for the first conductor layer

the inner edge of conductors. At high frequency, the current concentrate on the surface of conductors due to the eddy currents.

Fig. 8 shows the magnetic field strength $H_o(r, h)$ of the first conductor layer along with the r axis. The FEA simulation for the magnetic strength matches the equation (8) very well and validates its correctness. Fig. 9 shows the FEA simulation results of the current density J in the conductors. Fig. 9(a) is simulated at 1MHz with the cylindrical axis in which both the radial effect and the high frequency eddy current effect are included. Fig. 9(b) is simulated at low frequency (dc current) with cylindrical axis which means only the radial effect is included. Fig. 9(c) is simulated at 1MHz with the Cartesian XY plane where the radial effect is not included, and only the high frequency ac current effect is included. It may be observed from Fig. 9 that the currents are concentrated on the inner edge of the conductor. The current density J_{ry} can be decomposed into two orthogonal components J_r and J_y . The two components are decoupled in which J_r is caused by the radial effect (DC current distribution) due to the unequal impedance path, and J_y is caused by the skin and the proximity effects (AC current distribution) due to the transverse flux at high frequency. Fig. 10 shows the measurement results for leakage inductances and ac resistances of the transformer at different frequencies. The measurement was carried out by the

Fig. 10. Measurement results from impedance analyzer HP-4294A.

Fig. 11. Comparisons on leakage inductance of the planar transformer.

impedance analyzer HP-4294A. The leakage inductances were obtained by connecting the primary side to the impedance analyzer and shorting the secondary winding. The measured leakage inductances in Fig. 10 are referred to the primary side. 1.44- μ H leakage inductance is obtained at 100 kHz, and 1.22- μ H leakage inductance is measured at 1 MHz. The measurement shows the reduction of the leakage inductances with increased frequencies, this is due to the high frequency eddy current effect. Fig. 11 shows a comparison on leakage inductance between the Dowell's calculation [15], authors' previous work [33], the proposed calculation, the FEA simulation, and the experimental measurement. References [15] and [33] have not taken the "radial effect" into account, and therefore the leakage inductances are overestimated. The overall agreement between the proposed calculation, the FEA simulation, and the experimental measurement is very good.

IV. CONCLUSIONS

Planar transformer windings situated in a horizontal plane typically have an extremely high aspect ratio, thus concentrating the current in the inner edge of the conductors.

This phenomenon gives rise to a lower leakage inductance because the area of the stored leakage energy is effectively reduced. This so-called “radial effect” has been reported in planar transformer modelling before, but it is rare to link with frequency-dependent leakage inductance calculation. This paper gives an accurate prediction of leakage inductance for planar transformers with considerations of the high frequency eddy current (AC) effect and the radial (DC) effect. FEA simulations and experimental measurements show good agreement with the predictions. The proposed approach is an improved 1-D solution for leakage inductance, and thus only suitable for the cases that ac magnetic flux is substantially parallel to the surface of rectangular conductors, meaning that the external ac magnetic flux perpendicular to the surface of conductors (2-D consideration) is negligible. The proposed 1-D solution is applicable to most of planar transformers with PCB windings enclosed by high magnetically permeability cores, and it is also applicable for interleaved windings where each layer is interleaved, but not applicable for complex interleaved cases such as primary and secondary windings on the same layer where 2-D consideration may be needed.

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