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Interferometric detection of OAM-carrying Helico-conical beams

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Abstract

Helico-conical beams (HCBs) are a class of orbital angular momentum (OAM)-carrying beams with spiral phase and intensity profiles. In this communication, we demonstrate the use of a common path interferometer (CPI) to phase-image incoming HCBs and directly detect their OAM-associated properties. The output intensity of the CPI is a direct mapping of the HCB's phase, hence both the topological charge value and sign manifest in the output intensity distribution. The topological charge value is calculated by fringe counting, while the helicity is determined by observing the intensity along the $\varphi = 0$ region. With our current CPI setup, we can firmly detect up to $l = \pm 20$. Higher values of l lead to a decrease in the fringe visibility, but this can in principle be improved by tailoring the parameters of the phase contrast filter (PCF) employed in the CPI. We present analytical expressions to optimize the CPI for HCB charge detection.

Keywords: *Diffractive optics, Physical optics, Interferometric imaging*

Beams with orbital angular momentum (OAM) have attracted much attention in recent years due to their potential applications in optical communications [1], high-resolution imaging [2] and optical trapping [3]. OAM arises from helical phased light beams with azimuthal phase dependence $\exp(il\varphi)$ [4], where the sign and value of l define the helicity and topological charge (TC) of the beam and φ is the azimuthal angle. Such beams include Laguerre-Gaussian (LG) and higher-order Bessel beams that carry an OAM equal to $l\hbar$ per photon. A striking feature of these OAM-carrying beams is the doughnut-shaped far-field intensity profile, which in a classical perspective results from the interference of the azimuthal component of the Poynting vectors.

Numerous methods have been proposed to introduce the helical phase to an incident fundamental Gaussian beam. These include employing tilted phase masks, spatial light modulators (SLMs) and meta-surfaces, amongst others. Of equal importance for most applications is the detection of both the topological charge magnitude and sign of incoming unknown beams. The OAM manifests in the intensity distribution when the beam diffracts through slits and apertures [5, 6], or when it is made to interfere with another beam [7]. Hence, from the diffraction or interference pattern, the OAM-content of the beam can be ascertained. Current methods for l -determination include employing amplitude diffractive apertures [8–11], interferometers [12–14] and spatial mode projections [15]. There has been a recent interest in employing phase-modulating elements because of their efficient detection of the topological charge. Pure-phase gratings have been reported to improve charge detection [16,17].

Helico-conical beams are a class of OAM-carrying beams whose phase function has a non-separable azimuthal and radial dependence [18,19],

$$\psi(r, \varphi) = l\varphi \left(K - \frac{r}{r_0} \right) \quad (1)$$

where K is between 0 and 1, and r_0 normalizes the radial coordinate r . HCBs have both twisted phase and intensity profiles in the far-field and maintain high photon concentration while still allowing higher topological charge values. By maintaining high photon densities even at higher values of l , HCBs may be suitable for applications that require large l 's [20]. Particularly, optical communication applications that could directly utilize the detected l values are candidates where using a HCB can be beneficial.

The generation of HCBs has already been demonstrated using SLMs [18,21], printed fork-type holograms [22] and plasmonic meta-surfaces [23]. Far-field intensity observations have been conducted in the past using a $2f$ lens-configuration where the input HCB phase is encoded on a SLM placed behind a focusing lens. The spiral profile of the HCB-intensity can be detected by a camera positioned at the focal plane of the Fourier lens. Fig. 1 shows examples of HCB intensity profiles for $K = 0$ and for different topological charge values.

Fig. 1. Spiral far-field intensity profiles of simulated Helico-conical beams (HCBs) for $K = 0$, $l = +1$, $+3$, $+5$, $+7$ and $+9$. The lateral spiral profile scales linearly with the topological charge.

The propagation dynamics of HCBs have also been investigated using far-field intensity measurements [24,25]. Although it has been shown that the intensity profiles laterally scale with l/r_0 [18], the reverse process of determining an unknown topological charge of the HCB from the spiral intensities can be a challenging task. This is commonly encountered in optical communications, where from one end a transmitter sends out optical beams for data transmission to a receiver on the other end for detection and subsequent decoding with no prior information on the incoming beam. HCB topological charge detection from far-field intensity measurements would require all other initial parameters to be known, which is not the case in most OAM-experiments.

In this study, we employ a well-established common path interferometer (CPI) to image the phase of incoming HCBs and extract their OAM-associated properties that are otherwise difficult to obtain when observing only their far-field intensity distributions via a $2f$ lens-setup. We take advantage of the HCB's localized spatial far field intensity distribution when $K = 0$. A filtering phase element can therefore be reliably aimed at the center of the beam. The higher photon density also ensures that the effective CPI-reference wave has a comparable amplitude compared to the spatial frequency distribution of the HCB being imaged. Had we instead chosen to encode LG beams, the filtering element would coincide with the null region of their ring-shaped profile, resulting in poor contrast, especially at higher l values. As the CPI provides direct mapping of the HCB phase, the intensity output embodies the topological charge value and the helicity of the beam. Another advantage of the CPI is its relatively small footprint compared with other interferometry configurations such as the Mach-Zehnder or Michelson interferometers. To optimize HCB charge detection performance, we also analytically examine several parameters that affect the CPI's output contrast.

Fig. 2. (a) Schematic diagram of a $4f$ common path interferometer (CPI) with a phase contrast filter (PCF) in the optical Fourier plane. (b) PCF parameters overlaid on an Airy function, the Fourier transform of a circular input aperture. A and B are transmittance factors, θ is the phase shift introduced by the PCF, R_1 and R_2 are the radii of the PCF and Airy main lobe.

The CPI employed in this study is constructed around a $4f$ optical setup as shown in Fig. 2 (a) [26,27]. A first Fourier transforming lens is used to separate the incoming beam into a focused light (zero-order) and an off-axis scattered light. A phase contrast filter (PCF) placed at the center of the optical Fourier plane is used to impart a θ -phase shift to the focused light, which serves as a so-called synthetic reference wave (SRW) for the interferometer. The output of the CPI is then the coherent superposition of the SRW and the scattered input field that results in the direct mapping of the HCB-phase into the CPI intensity output. The PCF's transmittance function is given by

$$H(f_x, f_y) = A \left(1 + (BA^{-1} \exp i\theta - 1) \text{circ} \left(\frac{f_r}{\Delta f_r} \right) \right) \quad (2)$$

where B and $A \in [0, 1]$ are transmittance factors associated with the focused and scattered light, respectively, $\theta \in [0, 2\pi]$ is the applied phase shift to the focused light within a circular region of radius Δf_r , (f_x, f_y) are the spatial frequency coordinates at the Fourier plane and $f_r = \sqrt{f_x^2 + f_y^2}$ [16, 17]. The filter factors A, B and θ , as depicted in Fig. 2 (b) can be adjusted accordingly to improve the visibility of the CPI and for maximum irradiance of the output. With an input HCB-phase described by the iris-truncated field $U(x, y) = U(r) = \text{circ}(r/r_0) e^{-il\varphi/r_0}$, the intensity at the CPI output becomes

$$I(r') = A^2 |U(r')|^2 + |\bar{\alpha}| (BA^{-1} (\exp i\theta) - 1) g(0) \quad (3)$$

where $|\bar{\alpha}|$ is the normalized Fourier zero-order of the input beam, defined as

$$\bar{\alpha} = |\bar{\alpha}| \exp i\phi_{\bar{\alpha}} = \frac{\int_0^{2\pi} \int_0^{r_0} U(r, \varphi) r dr d\varphi}{\int_0^{2\pi} \int_0^{r_0} \text{circ} \left(\frac{r}{r_0} \right) r dr d\varphi} \quad (4)$$

for integer values of l . The magnitude of $\bar{\alpha}$ determines the component of the beam that is filtered by the PCF. This parameter vanishes for an LG beam because of its dark center, and thus the CPI would fail to phase-image such a beam. The term describing the $4f$ -imaged object wave, $U(r')$, in Eq. (3) for $K = 0$ is given by

$$U(r') = \text{circ}\left(\frac{r'}{r_0}\right) \exp\left(-i\left(l\varphi\frac{r}{r_0} + \phi_{\bar{\alpha}}\right)\right) \quad (5)$$

while the central value of the SRW-profile $g(0)$ is [26]

$$g(0) = g(r' = 0) \approx 1 - J_0(1.22\pi\eta) \quad (6)$$

The CPI can also be optimized by tailoring the SRW using the η parameter which is the ratio of the radius of the filtering region R_1 to the radius of the mainlobe of the Airy disc R_2 ,

$$\eta = \frac{R_1}{R_2} = \frac{r_0 R_1}{0.61\lambda f} \quad (7)$$

where R_2 is written in terms of the input aperture radius r_0 , illumination wavelength λ and focal length f . Normally, for a circular input aperture, the PCF is fabricated to match the radius of the corresponding Airy disc of the Fourier transform distribution. However, it is much more convenient and practical to use a high quality static filter and, if the application imposes, a specific size on the beam to be detected.

Fig. 3 shows the experimental setup for the CPI. The HCB is generated using the collimated Gaussian output of a 532-nm diode laser impinging on an SLM (Hamamatsu, 12.5 μm pixel pitch) which contains the HCB's helical phase distribution. In actuality, the topological charge of the incoming HCB is completely unknown from the receiver. In our experiments, however, we generated HCBs with initially known topological charge to test our proposed detection scheme. The CPI is constructed using lenses with $f = 100$ mm, and its output is detected with a CMOS camera (Basler, 5.86 μm pixel size). Fig. 4 shows the CPI output for different values of η and input aperture. In the absence of an encoded phase, the axial alignment of the PCF can be calibrated so that the expected CPI-output is a dark area as shown in Fig. 4 (a) [28]. With a π -phase shifting filter, this is achieved when $K = 0.5$ in Eq. (3) that corresponds to $\eta = 0.4$ and $r_0 = 1.3$ mm for a PCF radius $R_1 = 9.8$ μm . After the PCF is aligned, the input aperture radius that corresponds to optimally contrasted HCBs is $r_0 = 2.1$ mm, corresponding to $\eta = 0.627$. With this value, the Bessel function in Eq. (6) is zero, the SRW and input amplitudes are matched to consequently lead to higher maximum irradiance I_{max} and visibility $\nu = (I_{\text{max}} - I_{\text{min}})/(I_{\text{max}} + I_{\text{min}})$ within the relevant region of the output intensity distribution. With the $\eta = 0.627$ condition, Fig. 4 (c) shows the CPI output intensity. In theory, the central intensity value of the output when $\eta = 0.627$ is four times the input's central intensity for a 25% π -phase fill factor. Fig. 4 (b) and (d) are the CPI-outputs for a checkerboard input phase pattern for $\eta = 0.4$ and $\eta = 0.627$, respectively, showing that higher maximum intensity and visibility is obtained for the latter.

Fig. 3. Experimental setup for HCB detection. An HCB is generated using a 532-nm Gaussian beam impinging on a LCOS-SLM. The detector or receiver is composed of a CPI interferometer constructed using a $4f$ optical setup. The PCF radius is 9.8 μm .

Fig 4. Simulated CPI output for (a), (b) $\eta = 0.4$ and (c), (d) $\eta = 0.627$. The dark output in (a) is obtained when the PCF is axially aligned for a circular input aperture. $\eta = 0.627$ is chosen to achieve matched amplitude for maximum visibility. (c) is the CPI output for an input circular aperture with $\eta = 0.627$.

Fig. 5 shows the phase-imaged HCBs for $K = 0$ and for different l . Visual inspection of the boxed region in the images reveals that the number of fringes directly matches the topological charge of the HCB. Given the good output visibility, it would be possible to count these fringes using image processing techniques. Scans of each interference pattern such as that in Fig. 6 (a) show that the number of fringes is consistent with the charge for up to $l = 20$. It becomes a challenge, however, to determine higher l 's due to the decreased fringe visibility. This highlights the importance of the $\bar{\alpha}$ parameter in Eq. (4)

Fig. 5. Experimental HCB ($K = 0$) images from the CPI with increasing topological charge.

Fig. 6. (a) Intensity line scans for $l = +15$ (top), $+20$ (bottom) and (b) normalized zero-order of the HCB with increasing topological charges.

We numerically calculated the normalized zero-order of simulated HCBs and plotted it in Fig. 6 (b) as a function of l . The inverse relation is verified analytically by evaluating Eq. (4) for $K=0$, $l \neq 0$, leading to

$$\bar{\alpha}_{K=0} = -\frac{i}{\pi l} \quad (8)$$

Thus, from the $|\bar{\alpha}|$ in Eq. (8), the available amount of zero-order for generating a phase-shifted SRW decreases as l increases. In our current setup, the PCF is based on glass, and hence the filter parameters A and B were both ~ 1 for a near lossless phase imaging system, allowing us to simplify the expression for the intensity distribution at the output from Eq. (3) into the short form

$$I_{K=0} = 1 + \frac{4}{\pi^2 l^2} + \frac{4}{\pi l} \cos l\varphi \frac{r}{r_0} \quad (9)$$

which then leads to the following fringe visibility dependence on l :

$$V = \frac{4\pi l}{4 + \pi^2 l^2} \quad (10)$$

The expression in Eq. (9) shows the direct conversion of the HCB's phase $l\varphi/r_0$ into the CPI output intensity apart from factors that depend on l . However, as described in Eq. (10), the off-axis scattered light overpowers the SRW for higher l 's, resulting in a low fringe visibility. The simple analytic expression in Eq. (8) can lead to possible further optimization of the CPI, although this is beyond the scope of this Communication. With the use of a dynamic PCF, by using a SLM for example [29], A can be adjusted so that matched amplitude is still achieved. However, this presents a drawback in possible losses in the system and a more complex implementation. The presence of moiré fringes due to the limited resolution of the SLM and camera in this demonstration also limits the detection of HCBs with large l 's that are not addressable with the CPI parameters.

We also examined the output's dependence on the beam's helicity. Fig. 7 shows the images obtained from the CPI with $l = \pm 1, \pm 2, \pm 4, \pm 10$ and ± 15 . As described by Eq. (9), the intensity distribution of negatively charged HCB's should exhibit fringe shifting with respect to those of positively charged ones. This is indeed observed in the images acquired in our experiments. The topological charge information is also preserved in the number of fringes for negative l . Moreover, the intensity along the $\varphi = 0$ region is maximum for positive l (Fig. 7, top row) and minimum for negative l (Fig. 7, bottom row). This is expected, as described by the intensity distribution in Eq. (9), where we achieve the maximum and minimum conditions when $\varphi = 0$ for positive and negative l , respectively, independent of l and r . Hence, by observing the intensity along the $\varphi = 0$ region, we can determine the helicity of the incoming HCB.

Fig. 7. (Top row) Positive and (bottom row) negatively charged HCBs ($K = 0$) acquired experimentally. A change in helicity manifests itself as a fringe shift at the output of the CPI, with maximum and minimum intensities at $\varphi = 0$ for positive and negative l , respectively.

Finally, using the same CPI-setup, we imaged HCBs with $K = 1$ and $K = \frac{1}{2}$. Fig. 8 shows the intensity distributions for select topological charges. As with the case of $K = 0$, the fringe visibility also decreases as l increases that in turn leads to the same difficulty of properly identifying the topological charge of the beam for higher orders.

Fig. 8. Experimental CPI output for (top row) $K = 1$ and (bottom row) $K = \frac{1}{2}$.

In conclusion, we have demonstrated the phase imaging of OAM-carrying HCBs using a common-path interferometer that enables determination of the beam's topological charge value and its sign. The sufficient photon density around the center in the far-field intensity of HCBs facilitates the formation of a SRW that is phase-shifted by a PCF and can subsequently interfere with the scattered light of the input beam. This enables the direct mapping of the HCB-phase at the interferometer's output intensity embodying both the charge magnitude and sign in the number and behavior of the generated interferometric fringes. Using this technique, up to $l = |20|$ can be firmly detected with our current CPI-embodiment. The limitation for higher topological charges is mainly due to the weaker fringe visibility associated with a decreasing SRW-strength for larger topological charges. This could be offset by changing the phase contrast filter parameters to achieve matched interferometric amplitudes and hence optimal interference. The phase imaging of HCBs with the use of a CPI provides a direct and robust determination of its properties. HCB properties have previously been assessed only indirectly, through the analysis of their far field intensities.

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References

1. A. E. Willner, H. Huang, Y. Yin, Y. Ren, N. Ahmed, G. Xie, C. Bao, L. Li, Y. Cao, Z. Zhao, J. Wang, M. P. J. Lavery, M. Tur, S. Ramachandran, A. F. Molisch, N. Ashrafi, and S. Ashrafi, "Optical communications using orbital angular momentum beams," *Adv. Opt. Photonics* **7**, 66–106 (2015).
2. C. Maurer, A. Jesacher, S. Fernet, and M. Ritsch-Marte, "What spatial light modulators can do for optical microscopy," *Laser Photonics Rev.* **5**, 81–101 (2011).
3. K. Ladavac and L. G. Grier, "Microoptomechanical pumps assembled and driven by holographic optical vortex arrays," *Opt. Express* **12**, 1144–1149 (2004).
4. L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman, "Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes," *Phys. Rev. A* **45**, 8185–8189 (1992).
5. Q. S. Ferreira, A. J. Jesus-Silva, E. J. S. Fonseca, and J. M. Hickmann, "Fraunhofer diffraction of light with orbital angular momentum by a slit," *Opt. Lett.* **36**, 3106–3108 (2011).
6. D. P. Ghai, P. Senthilkumaran, and R. S. Sirohi, "Single-slit diffraction of an optical beam with phase singularity," *Opt. Lasers Eng.* **47**, 123–126 (2009).
7. M. P. J. Lavery, A. Dudley, A. Forbes, J. Courtial, and M. J. Padgett, "Robust interferometer for the routing

of light beams carrying orbital angular momentum," *New J. Phys.* **13**, (2011).

8. J. M. Hickmann, E. J. S. Fonseca, W. C. Soares, and S. Chávez-Cerda, "Unveiling a truncated optical lattice associated with a triangular aperture using light's orbital angular momentum," *Phys. Rev. Lett.* **105**, 1–4 (2010).
9. C.-S. Guo, L.-L. Lu, and H.-T. Wang, "Characterizing topological charge of optical vortices by using an annular aperture," *Opt. Lett.* **34**, 3686–3688 (2009).
10. L. A. Melo, A. J. Jesus-Silva, S. Chávez-Cerda, P. H. S. Ribeiro, and W. C. Soares, "Direct Measurement of the Topological Charge in Elliptical Beams Using Diffraction by a Triangular Aperture," *Sci. Rep.* **8**, 1–8 (2018).
11. D. Hebri, S. Rasouli, and M. Yeganeh, "Intensity-based measuring of the topological charge alteration by the diffraction of vortex beams from amplitude sinusoidal radial gratings," *J. Opt. Soc. Am. B* **35**, 724–730 (2018).
12. H. I. Sztul and R. R. Alfano, "Double-slit interference with Laguerre-Gaussian beams," *Opt. Lett.* **31**, 1–3 (2006).
13. D. Jianji, Z. Hailong, and Z. Xinliang, "Dynamic interferometry measurement of orbital angular momentum of light," *Opt. Lett.* **39**, 7 (2014).
14. S. Slussarenko, V. D'Ambrosio, B. Piccirillo, L. Marrucci, and E. Santamato, "The Polarizing Sagnac Interferometer: a tool for light orbital angular momentum sorting and spin-orbit photon processing," *Opt. Express* **18**, 27205–27216 (2010).
15. A. D'Errico, R. D'Amelio, B. Piccirillo, F. C. C. de Faria, and L. Marrucci, "Measuring the complex orbital angular momentum spectrum and spatial mode decomposition of structured light beams," *Optica* **4**, 1350–1357 (2017).
16. S. Zheng and J. Wang, "Measuring Orbital Angular Momentum (OAM) States of Vortex Beams with Annular Gratings," *Sci. Rep.* **7**, 40781 (2017).
17. Z. Liu, S. Gao, W. Xiao, J. Yang, Y. Huang, Y. Feng, J. Li, W. Liu, and Z. Li, "Measuring high-order optical orbital angular momentum with a hyperbolic gradually changing period pure-phase grating," *Opt. Lett.* **43**, 3076–3079 (2018).
18. C. A. Alonzo, P. J. Rodrigo, and J. Glückstad, "Helico-conical optical beams: a product of helical and conical phase fronts," *Opt. Express* **13**, 1749–1760 (2005).
19. G. Overton and J. Glückstad, "Phase functions are inseparable in helico-conical optical beams," *Laser Focus World* (2005).
20. V. R. Daria, D. Z. Palima, and J. Glückstad, "Optical twists in phase and amplitude," *Opt. Express* **19**, 476–481 (2011).
21. T. Latychevskaya and H. W. Fink, "Inverted Gabor holography principle for tailoring arbitrary shaped three-dimensional beams," *Sci. Rep.* **6**, 26312 (2016).
22. N. P. Hermosa and C. O. Manaois, "Phase structure of helico-conical optical beams," *Opt. Commun.* **271**, 178–183 (2007).
23. Y. Zhang, X. Yang, and J. Gao, "Twisting phase and intensity of light with plasmonic metasurfaces," *Sci. Rep.* **8**, 4884 (2018).
24. N. Bareza and N. Hermosa, "Propagation dynamics of vortices in helico-conical optical beams," *Opt. Commun.* **356**, 236–242 (2015).

25. N. Hermosa, C. Rosales-Guzmán, and J. P. Torres, "Helico-conical optical beams self-heal," *Opt. Lett.* **38**, 383 (2013).
26. J. Glückstad and P. C. Mogensén, "Optimal phase contrast in common-path interferometry," *Appl. Opt.* **40**, 268–282 (2001).
27. P. J. Rodrigo, D. Palima, and J. Glückstad, "Accurate quantitative phase imaging using generalized phase contrast," *Opt. Lett.* **33**, 2498–2509 (2008).
28. M. J. Villangca, A. R. Bañas, D. Palima, and J. Glückstad, "Dark Grating extended nodal beam areas from binary-only phase," *Opt. Eng.* **55**, 125102 (2016).
29. F. Kenny, F. S. Choi, J. Glückstad, and M. J. Booth, "Adaptive optimisation of a generalised phase contrast beam shaping system," *Opt. Commun.* **342**, 109–114 (2015).