Evanescent waves in simulated ear canals: Experimental demonstration and method for compensation

Siegel, Jonathan H. ; Nørgaard, Kren Rahbek; Neely, Stephen T.

Published in:
Journal of the Acoustical Society of America

Link to article, DOI:
10.1121/1.5058683

Publication date:
2018

Document Version
Publisher's PDF, also known as Version of record

Link back to DTU Orbit

Citation (APA):
Evanescent waves in simulated ear canals: Experimental demonstration and method for compensation

Jonathan H. Siegel
Department of Communication Sciences and Disorders and Knowles Hearing Center, Northwestern University, 2240 Campus Drive, Evanston, Illinois 60208, USA

Kren Rahbek Nørgaard
Acoustic Technology, Department of Electrical Engineering, Technical University of Denmark, Ørsteds Plads, Building 352, Kongens Lyngby, DK-2800, Denmark

Stephen T. Neely
Boys Town National Research Hospital, 555 North 30th Street, Omaha, Nebraska 68131, USA

(Received 16 July 2018; revised 14 September 2018; accepted 17 September 2018; published online 12 October 2018)

Evanescent waves emerge from a small sound source that radiates into a waveguide with a larger cross-sectional area, but unlike planar waves, do not propagate far from the source. Evanescent waves thus contaminate in-ear calibration of acoustic stimuli. Measurements with an otoacoustic-emission (OAE) probe inserted at the entrance of long tubes of various diameters show a decline in the evanescent wave with distance from the source when advancing a probe tube through the OAE probe and into the long tube. The amplitude of the evanescent pressure increases with frequency and depends strongly on the diameter of the long tube. Modifying the shape of the aperture of the probe’s sound source, thus effectively enlarging its diameter and redirecting acoustic flow, greatly reduced evanescent waves. The reduction in evanescent-wave pressure was observed in calibration cavities used to determine the Thévenin-equivalent source pressure and impedance of the probe. Errors in source calibrations were considerably larger in the unmodified configuration. An alternative method is proposed for calculation of acoustic source parameters that models the evanescent-wave pressure and reduces its influence on the calculation. This reduction greatly improves the quality of source calibrations, which should improve the accuracy of ear-canal impedance measurements and related quantities.

Copyright © 2018 Acoustical Society of America. https://doi.org/10.1121/1.5058683

I. INTRODUCTION

A sound source at the entrance of a long, cylindrical waveguide with a diameter smaller than that of the waveguide generates a sound field composed of (1) plane waves that propagate far into the waveguide and (2) higher-order modes. If the frequency is low enough, these higher-order modes become evanescent and are attenuated exponentially along the waveguide (Miles, 1946; Ingård, 1948; Karal, 1953; Morse and Ingård, 1968; Keefe and Benade, 1981; Brass and Locke, 1997; Zebian et al., 2012; Nørgaard et al., 2017b). A microphone located near the sound source will record both the plane-wave pressure and the evanescent-wave pressure. Configuring the sound source and microphone to minimize the effect of evanescent waves has been explored previously in measurements of the ear’s acoustic impedance, e.g., by protruding the microphone beyond the sound source (Rabinowitz, 1981; Voss and Allen, 1994; Keefe et al., 1992; Huang et al., 2000). This study describes a physical modification to the sound source of a commercial otoacoustic-emission (OAE) probe (ER-10X, Etymotic Research) that has been found to physically decrease the amplitude of the evanescent wave. Following this demonstration, a method is described that decreases the undesirable effect of existing evanescent waves on Thévenin-equivalent source calibrations. These two approaches to account for the influence of evanescent waves lead to similar Thévenin source parameters.

To measure directly the planar and evanescent components emitted by an OAE probe, it is convenient to place the probe at the entrance of a long tube with diameter approximating that of the average ear canal. The sound field contains no reflections, so the decay of the evanescent pressure that is predicted by theory (Ingård, 1948; Morse and Ingård, 1968; Keefe and Benade, 1981), and can be measured directly by using a separate probe-tube microphone advanced through the OAE probe and into the long tube to sample the pressure at various locations along the tube in front of the OAE probe. The decay of the evanescent wave with distance is measured by advancing the probe tube into the long tube until there is little change in the measured pressure, indicating that the evanescent wave has decayed, and
that this plane-wave pressure thus establishes a reference for quantifying the evanescent wave. The plane-wave pressure amplitude declines slowly with distance due to frictional losses, but this factor is negligible for short distances that are typical for the spatial decay of the evanescent wave in a tube of radius similar to an adult ear canal.

Portions of this work were originally presented by Siegel and Neely (2017), who showed that not accounting for evanescent modes during the Thévenin calibration procedure causes the obtained source parameters to be non-causal. They also showed that a physical modification to the probe front tube effectively decreases the evanescent wave in front of the probe. Subsequently, Nørgaard et al. (2017b) showed that this error due to evanescent modes can be approximated as an acoustic compliance in parallel with the source parameters. They also proposed a method for incorporating the effect of the evanescent wave into the calibration method by estimating the evanescent-wave inertance rather than the waveguide lengths. Reported here are measurements of the decline in evanescent-wave pressure with increasing distance from the probe in long tubes of three different diameters. A simple modification of the replaceable sound-delivery front tube of the ER-10X¹ showed greatly reduced evanescent waves, which led to improvements in Thévenin source calibrations, allowing better measures of the acoustic impedance of test cavities (and ear canals) by the probe and more accurate prediction of the termination pressure. Last, a signal-processing method is described to improve the determination of the probe’s Thévenin source pressure and impedance in the presence of evanescent waves that should yield similarly improved acoustic measurements in real ears. This approach is different from that of Nørgaard et al. (2017b) in that it does not require knowledge of the physical lengths of the calibration waveguides.

II. METHODS

A. Measurement procedure

A 1 mm outer-diameter stainless-steel probe tube, coupled to an electret microphone, was advanced through the access port in the body of an ER-10X OAE probe (Etymotic Research, Elk Grove Village, IL) and through the central lumen of the replaceable front tube under micrometer control [see Fig. 2(A)]. Inserting the probe tube disables the internal microphone of the ER-10X, but only minimally disturbs the sound field in front of the probe generated by the probe’s internal sound sources. Starting with the end of the probe tube flush with the end of the disposable multi-lumen front tube, the micrometer was advanced in small steps between measurements of the pressure using chirp excitation. The synchronous-averaging software used for these measurements was SysRes (Neely and Stevenson, 2002). The long tube was 50-ft in length of clear polyethylene tubing with inner diameter (id) 4.8, 7.9, or 12.7 mm.

B. Front-tube configurations

The long screw was removed from the back of the ER-10X to allow insertion of a calibrated probe tube through the body of the probe and the central lumen of the front tube to measure the pressure distal to the end of the probe [Fig. 1(A)]. The two configurations of the multi-lumen front tube used in the tests are shown in Fig. 1(B). The standard (unmodified) tube is as supplied by Etymotic Research. The beveled tube was modified to remove part of the outer lumen, leaving the central microphone lumen unaltered. The beveled tube allows the sound to be radiated, starting at the bottom of the small depression of the rubber ear tip [Fig. 1(C)]. The modification effectively increases the aperture of the sound source by a factor of about two, allowing the sound to begin to disperse proximal to the microphone inlet. Increasing the aperture of the source should reduce the amplitude of evanescent waves relative to the plane waves emitted by the probe (Ingård, 1948). A secondary effect of the beveled aperture was to direct acoustic flow away from the central microphone lumen, thus reducing the particle-velocity gradient near the edge of the microphone tube and minimizing potential flow losses (Nørgaard et al., 2017b).

The objective was to measure the influence of the diameter of the long tube on evanescent waves with the unmodified front tube and to test the prediction that modifying the source configuration as described would reduce the evanescent component of the pressure.

III. RESULTS

A. Dependence of the relative magnitude of the evanescent-wave pressure on the diameter of the long tube

To measure the relative amplitudes of evanescent- and plane-wave pressures along the long tubes, a series of measurements was made at increasing distance from the end of

FIG. 1. (Color online) Measurement configuration. (A) With the long screw removed from the access port on the back of the ER-10X probe, a probe-tube microphone was advanced through the body of the probe and beyond the end of the replaceable front tube into the long reflectionless tube. (B) The unmodified and beveled front tubes shown from the side and (C) looking at the front of the ear-tip with the beveled front tube in place.
the replaceable front tube of the probe. For each diameter of
the long tube, a point was reached where the measured
pressure became independent of position, and it was appar-
ent that the pressure had become dominated by plane
waves. One of these latter measurements was taken as a
reference and the ratio of pressures along the tube to that
of the reference was taken as the relative amplitudes of the
evanescent and plane waves. When the plane wave travels
along the waveguide, it undergoes a linear phase change
with distance. Considering the plane-wave pressure
response \( P_{pw} \) to a volume flow \( U \) at the position of the
probe along a cylindrical waveguide with characteristic (or
surge) acoustic impedance \( Z_0 \), the plane-wave pressure at
the probe is purely real

\[
P_{pw}(x = 0) = U Z_0. \tag{1}
\]

Along the waveguide axis \( x \), however, the plane-wave pres-
sure varies as

\[
P_{pw}(x) = U Z_0 e^{-j\omega x/c}, \tag{2}
\]

where \( c \) is the speed of sound, and \( \omega = 2\pi f \). Conversely, the
evanescent wave is purely imaginary along the waveguide
and does not undergo a spatial phase change with the wave-
guide axis \( x \). Thus, the rotating phase of the plane wave pres-
sure will combine constructively and destructively with the
constant-phase evanescent wave pressure in a periodic man-
ner over distance. The delay in the plane-wave reference
pressure \( P_{pw,ref} \) measured at distance \( d_{ref} \) away from the
probe was used to estimate the plane-wave pressure along
the waveguide axis

\[
P_{pw}(x) = P_{pw,ref} e^{-j\omega(x-d_{ref})/c}. \tag{3}
\]

This delay-corrected plane-wave pressure was then sub-
tracted from the measured pressure \( P_{meas}(x) \) at a given posi-
tion to obtain the evanescent-wave pressure

\[
P_{ew}(x) = P_{meas}(x) - P_{pw}(x). \tag{4}
\]

The delay compensation is needed to obtain the correct magni-
tude and phase of the evanescent wave. The magnitude and
phase of the ratios \( P_{ew}(x)/P_{pw}(x = 0) \) are displayed in Fig. 2.
The relative magnitude of the evanescent pressure increases
with the diameter of the long tube and decreases with increasing
distance of the probe tube from the front of the probe. The rela-
tive phase angle of the evanescent pressure hovers near \(+90^\circ\)
as expected as the evanescent wave can be approximated as an
acoustic inertance in series with the plane-wave acoustic input
impedance of a cylindrical waveguide. The phase is noisy at low
frequencies because the evanescent pressure becomes relatively
small and the relative pressure is subject to system noise.

The relative amplitude of the evanescent-wave pressure
increased with the diameter of the long tube (Fig. 2). In the
waveguide with id 12.7 mm, the evanescent-wave pressure
appears to increase systematically until \( \sim 15.5 \text{ kHz} \) (Fig. 2A).
Each higher-order mode is only evanescent if the frequency is
lower than the frequency at which the mode starts to propagate

\[
\omega_{mn} = \frac{x''_m c}{a}, \tag{5}
\]

where \( x''_m \) is the \( n \)th zero-crossing of the Bessel-function
derivative of order \( m \), \( c \) is the speed of sound, and \( a \) is the

![FIG. 2. (Color online) Pressure in front of the probe normalized to the pressure measured at a location distant from the probe where plane waves were domi-
nant and compensated for the propagation delay of the plane wave at the measurement position using Eqs. (3)–(4). Each column shows the relative magnitude
and phase of the evanescent pressure for each tubing size. (A) The relative magnitude of the evanescent pressure was greatest for the 12.7 mm id tube, reaching
\(+5 \text{ dB}\) at 15 kHz. (B) Pressure variations became larger at high frequencies in the long tube with id of 7.9 mm and were strongly position dependent. (C) Only
small variations in the normalized pressure response were noted for the long tube with id of 4.8 mm.](image)
radius of the cylindrical waveguide. For $a'_{10} = 1.841$ and the id 12.7 mm tube, this results in $f'_{10} = 15.8$ kHz. This means that above this frequency, the lowest-order azimuthal mode starts to propagate, and a proper reference consisting only of plane waves $p_{pw}$ cannot be established. As a result, we could only correctly estimate the magnitude of the plane wave, measured at a sufficient distance from the probe, for frequencies below 15.8 kHz for the largest tube. The relative magnitude of the evanescent pressure is successively smaller as the tubing diameter is reduced to 7.9 mm [Fig. 2(B)] and 4.8 mm [Fig. 2(C)] (note the different scaling of Fig. 2(A) vs Figs. 2(B) and 2(C)]. Note that the relative phase of the evanescent pressure appears reactive at all frequencies independent of tubing diameter. This consistent pattern in the phase suggests that our separation of the evanescent pressure from that of the plane wave was successful.

B. Beveling the front tube to change the source aperture

The effect of beveling the front tube is illustrated for the 7.9 mm id tube [Figs. 3(A) and 3(B)] and the 12.7 mm id tube [Figs. 3(C) and 3(D)]. In both cases, the relative magnitude of the evanescent pressure is reduced considerably for the beveled front tube [Figs. 3(B) and 3(D)], compared to the unmodified front tube [Figs. 3(A) and 3(C)]. The relative phase is consistently reactive, regardless of the front tube configuration. We did not measure the effect of beveling the
front tube for the 4.8 mm tube because the relative evanescent pressure was small even for the unmodified front tube [Fig. 2(C)]. This is expected because the relatively small difference between the aperture of the sound source of the unmodified front tube (3 mm) and the 4.8 mm tube (Ingård, 1948).

The decline in the evanescent-wave pressure with distance from the probe is predicted from theory (Ingård, 1948). The relative level of the evanescent pressure was evaluated at 20 kHz for the 4.8 mm and 7.9 mm long tubes and at 14 kHz for the 12.7 mm tube. As previously mentioned, for the largest tube, this was slightly below the 15.8 kHz frequency where the lowest-order azimuthal mode starts to propagate, and a proper reference consisting only of plane waves cannot be established. The rate of decline of the evanescent pressure depended both on the diameter of the long tube and on whether the front tube was beveled (Fig. 4). The decline is approximately exponential, with a generally shallower slope for the beveled front tube. The different decay rates might be due to a difference in which higher-order modes are elicited by each front tube. The results suggest that the evanescent modes elicited by the beveled front tube are dominated by higher-modes modes of lower order than the unmodified front tube.

It should be noted that the range of human ear-canal diameters spans the range of long tube diameters shown here. Evanescent waves are thus likely to be a serious problem in larger ear canals when the pressure measured by the probe is relied on for calibration. Conversely, for relatively small ear canals and for sound source apertures similar to that of the ER-10X, evanescent waves are expected to be small and can usually be ignored.

C. Beveled front tube allows more accurate prediction of termination pressure

A good test of the accuracy of stimulus calibration using an OAE probe is to compare the pressure measured by the microphone that terminates a test cavity or an IEC 60318–4 occluded-ear simulator and the pressure at the termination predicted from the pressure measured by the probe. This prediction adds the amplitudes of the forward and reflected pressures, referred to as integrated pressure (Lewis et al., 2009; Scherperle et al., 2011; Souza et al., 2014). In such comparisons, the predicted and measured pressures commonly agree within ~1 dB at frequencies up to ~10 kHz, but steadily diverge at higher frequencies. An example of predicting the pressure at the microphone of a 60318-4 occluded-ear simulator using the pressure measured by the ER-10X probe is shown in Fig. 5. The distance between the end of the OAE probe and the simulator’s microphone is approximately 21.2 mm, judged by the frequency of the first half-wave resonance (8.1 kHz). At 20 kHz, the predicted pressure became about 6 dB larger than the pressure measured by a microphone at the termination. The discrepancy between the predicted and actual termination pressure in the ear simulator at high frequencies is much smaller for the beveled front tube [Fig. 5(B)] than for the unmodified tube [Fig. 5(A)]. The error in matching the target pressure of nearly 6 dB implies that the amplitude of the evanescent-wave pressure is similar to that of the plane wave at the inlet of the emission probe. Note that the 3/4 wave null in the measured probe pressure (arrows) is much more symmetrically related to the frequencies of half-wave resonance (asterisks) for the beveled front tube [Fig. 5(B)] than for the unmodified tube [Fig. 5(A)]. The null is shifted by ~2.5 kHz but the frequencies of the half-wave resonances are essentially unchanged. The difference between the predicted and measured termination pressures for the two front tube configurations is shown in Fig. 5(C). This error becomes ~6 dB for the unmodified front tube vs ~2 dB for the beveled tube.

D. Evanescent waves contaminate Thévenin source calibrations

When using impedance or reflectance-based methods to calibrate in-ear pressure stimuli in situ, it is important to obtain an accurate Thévenin source calibration (Scherperle et al., 2008; Souza et al., 2014). As expected, the pressure responses in the cavities used to calibrate the ER-10X probe depended on the configuration of the front tube. In the frequency domain, the differences between the unmodified and beveled tube are mainly in the nulls above 5 kHz for each of the five calibration cavities [Fig. 6(A)]. This asymmetry of the nulls is similar to those seen in the EP pressure response of the ear simulator in Fig. 5(A).

The pressure peaks for the beveled tube are to a higher degree symmetrically positioned between the pressure nulls in Fig. 6(B), similar to the ear simulator response in Fig. 5(B). This asymmetry is expected from uniform cavities with dominant plane waves, but its importance is uncertain. The calibration algorithm is based on an optimization routine that estimates the lengths of the waveguides. Table 1 lists the measured and estimated calibration-waveguide lengths using each set of probe pressures with the unmodified and beveled front tubes. The actual lengths were measured within about +/-0.2 mm using a mm ruler. The estimated lengths are...
consistently larger for the unmodified front tube. The numbers in parentheses are the errors between the measured and estimated lengths. Notice that the estimated lengths with the beveled front tube in place correspond much more closely to the measured lengths. This suggests that the position of the probe-pressure nulls, which are translated due to the evanescent wave, is the primary factor determining the estimated lengths (Nørgaard et al., 2017b).

Further evidence of the evanescent wave was obtained in a time-domain representation of the cavity pressures. Figure 7 compares the cavity pressures in the time domain, shown as the inverse Fourier transforms of the pressures in Fig. 6. Here, the differences appear mainly in the first peak (between 1.9 and 2.1 ms). Because the cavities have a rigid perpendicular termination, secondary peaks, which are essentially delayed reflections of the initial peak, should have a similar shape. The pressure response for the beveled front tube comes closer to this expectation because the initial peak lacks the high-frequency oscillations that appear on the initial peak for the unmodified front tube.

Thevenin-source impedances calculated by conventional methods from the cavity pressures are shown in Fig. 8. The magnitude [Fig. 8(A)] and phase [Fig. 8(B)] of the impedance exhibit less frequency-dependence for the beveled tube than for the unmodified tube. Although differences are apparent, the real part of the impedance remains positive at essentially all frequencies for both tube configurations, so there is no obvious reason to prefer one tube over the other. However, the set of pressure responses in the five cavities used to calculate the ER-10X source characteristics using an optimization that assumes plane wave propagation in the cavities reports consistently lower errors when using the beveled tube compared to the unmodified tube. This would be expected if the sound pressure in the calibration cavities is closer to ideal plane waves. Nørgaard et al. (2017b) showed that not accounting for evanescent modes when estimating the waveguide lengths introduces a parallel compliance into the source parameters. The behavior observed in the source impedance between the unmodified and beveled probe tubes are indeed similar to their observations.

An intriguing way to distinguish the two tube configurations was to compare the source impedances in the time domain (Fig. 9). Time-domain impedance is essentially the pressure response to an impulsive flow stimulus. In the context of discrete-time signals, it is prudent to select a flow stimulus that has a band-limited first derivative. For this reason, we use the inverse Fourier transform of a frequency-domain Blackman window that is centered at zero frequency and extends to 20 kHz. Additional smoothing was achieved by zero-padding in the frequency domain (to four times the Nyquist frequency) prior to computing the inverse Fourier transform. Figure 9 shows source impedances in the time domain for the unmodified and beveled front tubes.

In the time domain, much of the impedance for the unmodified front tube occurs prior to $t = 0$. This is obviously different from the impedance of the beveled tube, which is close to zero for $t < 0$. So, one can argue that the beveled front tube is preferable because the impedance for the unmodified tube is non-causal. The non-causality for the unmodified front tube can be explained as a negative parallel compliance (Nørgaard et al., 2018), thus discharging in reverse time.
IV. METHOD FOR DETERMINING THEVENIN-EQUIVALENT ACOUSTIC SOURCE PARAMETERS IN THE PRESENCE OF EVANESCENT WAVES

An essential prerequisite for determining the Thévenin-equivalent acoustic source parameters, specifically the source impedance $Z_s$ and the source pressure $P_s$, is a set of pressure measurements $P_n$ obtained from $N$, typically four or five, calibration waveguides. In addition, theoretical reference impedance $Z_n$ is required for each of these waveguides. The procedure is complicated by the need to estimate from the pressure measurements some of the parameters that are used in the calculation of the reference impedances, e.g., the waveguide lengths.

A. Reference impedance

The reference impedance $Z_n$ for the $n$th waveguide is modeled as the sum of three impedances

$$Z_n = Z_{fl} + Z_{ew} + Z_{lk},$$

where $\parallel$ represents the parallel combination of $Z_{fl}$ and $Z_{ew}$ for the $n$th waveguide. The primary impedance is the plane-wave impedance $Z_{pw,n}$, and the leak impedance $Z_{lk}$, are modeled as being the same for all waveguides because the probe remains at a fixed location in the ER-10X calibrator. The leak impedance is resistive and is due to an intentional static-pressure relief near the entrance of the ER-10X calibrator. The reference impedance provides the acoustic loads that are used to determine the Thévenin-equivalent acoustic source parameters based on the circuit shown in Fig. 10.

The plane-wave impedance $Z_{pw,n}$ is modeled as a transmission line with an open-circuit termination

$$Z_{pw,n} = Z_0 \coth(\gamma L_n),$$

where $Z_0$ is the characteristic impedance and $\gamma$ is a propagation multiplier. In a lossless, uniform waveguide, $Z_0 = \rho c/A$ and $\gamma = i\omega/c$, where $\rho$ is the density of air, and $A$ is the cross-sectional area of the waveguide. Analytic solutions exist for propagation in circular uniform waveguides that have thermoviscous losses; however, we use approximations for $\gamma$ and $\omega$ suggested by Keefe (1984). The waveguide lengths $L_n$ and flow loss $Z_{fl}$ are estimated during the calibration procedure.

The evanescent-wave impedance $Z_{ew}$ is mainly inerter due to spreading of the pressure as it exits the sound output port of the ER-10X probe. However, because the effective inerterance is observed to increase in the frequency range between 10 and 20 kHz, the evanescent-wave impedance is modeled as a polynomial function of frequency

$$Z_{ew} = iZ_0 \cdot \sum_{m=1}^{M} (\omega \tau_m)^m.$$

The parameters of this equation $\tau_1\ldots\tau_M$ are estimated during the calibration procedure. Note that $Z_{ew}$ is purely imaginary, and so does not contribute any loss to the reference impedance.

Flow losses are caused by the large gradients due to the spreading flow close to the speaker aperture (Nørgaard et al., 2017b). In our reference impedance, the flow loss $Z_{fl}$ is modeled as a negative real quantity that is proportional to the square-root of frequency, since the particle-velocity gradient going away from the edge of the speaker aperture is inherently negative.
The flow-loss coefficient $R_{fl}$ is estimated during the calibration procedure.

### B. Waveguide length estimation

Waveguide lengths are estimated acoustically by minimizing the overall error in estimated waveguide pressures that are calculated by using the source parameters. When $N$ measured pressures $P_n$ and reference impedances $Z_n$ are known (for $n = 1 \ldots N$), then the source parameters can be calculated from a least-squares solution to the following equation (Allen, 1986):

\[
\begin{bmatrix}
Z_1 & -P_1 \\
\vdots & \vdots \\
Z_N & -P_N
\end{bmatrix}
\begin{bmatrix}
P_s \\
Z_s
\end{bmatrix}
= 
\begin{bmatrix}
Z_1P_1 \\
\vdots \\
Z_NP_N
\end{bmatrix}.
\]

(10)

An estimate of each waveguide pressure is obtained from $P_s$, $Z_s$, and $Z_n$

\[
P_n = P_s \frac{Z_n}{Z_s + Z_n}.
\]

(11)

The waveguide pressure estimation error is quantified as (Scheperle et al., 2011)

\[
\epsilon_{pe}|f_1 < f < f_2| = \frac{\sum_{n=1}^{N} \sum_{f} |P_n - P_n'|^2}{\sum_{n=1}^{N} \sum_{f} |P_n|^2}.
\]

(12)

This estimation error is evaluated over the frequency range $f_1 < f < f_2$. A simplex search algorithm is used to find a set of waveguide lengths $L_n$ and flow-loss coefficient $R_{fl}$ that minimize this estimation error for fixed values of $Z_{lk}$ and $Z_{ew}$.

### C. Parallel-compliance delay

The presence of evanescent waves influences estimation of the Thévenin source parameters. When evanescent waves are physically present but their representation is omitted from the reference impedance, the source impedance is observed to have a negative compliance in parallel with its resistive component (Nørgaard et al., 2017b)

**FIG. 7.** (Color online) Set of pressure responses in the time domain for (A) the unmodified front tube and (B) the beveled front tube. These waveforms were smoothed by zero-padding in the frequency domain (to four times the Nyquist frequency) prior to computing inverse Fourier transforms.

**FIG. 8.** (Color online) Evanescent waves contaminate Thévenin source calibrations. The magnitude (A) and phase (B) of the source impedances are both influenced over the entire frequency range with a much more dramatic change with frequency for the unmodified front tube than for the beveled tube. The impedance magnitude is about an order of magnitude higher at 20kHz for the beveled tube compared with the unmodified tube and the phase shows a corresponding divergence of about 1/4 period.
The reason that $C_{par}$ is negative in this case is that when placed across the input to a transmission line, a negative compliance approximates a series inertance $L_{ser}$ at the input of a shorter transmission line with the same characteristic bandwidth.

\[
C_{par} \approx \frac{1}{\omega^3} \left\{ \frac{1}{Z_s} \right\}.
\]

The series inertance in this approximation is

\[
L_{ser} = -\tau_{pc} Z_0.
\]

When an evanescent wave is physically present, but series inertance has been omitted from the reference model, the equations in this section provide a means to estimate the series inertance that should have been added to the reference model. This approach assumes that if $C_{par} = 0$, the averaged imaginary part of the source admittance across the frequency bandwidth $\langle \text{Im}\{1/Z_s\} \rangle = 0$.

**D. Source-parameter calculation**

Because the waveguide lengths $L_n$, evanescent-wave impedance $Z_{ew}$, and flow loss $Z_{fl}$ are not precisely known, the source parameters are initially calculated using preliminary estimates of these quantities. Subsequently, the first estimates of the source parameters are used to improve estimates of all unknown quantities. The following steps outline this procedure:

1. Let $Z_{ew} = 0$ and find the set of $Z_{fl}$ and $L_n$ (for $n = 1 \ldots N$) that minimize $\epsilon_{pc}$ for $2 < f < 8$ kHz.
2. Estimate $\tau_{pc}$ from $Z_s$. If $\tau_{pc} < 0$, then let $\tau_1 = -\tau_{pc}$, otherwise let $\tau_1 = 0$.
3. Let $Z_{ew} = iZ_0 \tau_1$ and find the set of $Z_{fl}$ and $L_n$ (for $n = 1 \ldots N$) that minimize $\epsilon_{pc}$ for $2 < f < 8$ kHz.
4. Let $Z_{ew} = iZ_0 \cdot \sum_{m=1}^{5} (or_m)^m$ and find the set of $\tau_m$ (for $m = 1 \ldots 5$) that minimize $\epsilon_{pc}$ for $2 < f < 18$ kHz.
5. Let $Z_{ew} = iZ_0 \cdot \sum_{m=1}^{5} (or_m)^m$ and find the set of $Z_{fl}$ and $L_n$ (for $n = 1 \ldots N$) that minimize $\epsilon_{pc}$ for $2 < f < 8$ kHz.

These five steps provide estimates for $L_n$, $Z_{ew}$, and $Z_{fl}$ in addition to determining the source parameters $Z_s$ and $P_s$.

The characteristic impedance of a cylindrical waveguide with a diameter of 8 mm is $Z_0 = 81.55$ cgs acoustic ohms. Based on comparisons between calibrations in the ER10X calibrator and rigidly terminated waveguides that had no static-pressure relief, the leak impedance of the ER10X calibrator is estimated to be about 100 times its characteristic impedance, so we set $Z_{lk} = 8000$ cgs acoustic ohms, a value that appears to be established by the small leak designed into the calibrator by Etymotic Research to limit static pressure changes. It should be noted that the leak impedance may differ across ER-10X units due to manufacturing tolerances.

**E. Examples of source parameters for unmodified and beveled front tubes**

The unmodified and beveled front tubes differ in the size of the evanescent wave present in waveguide pressure measurements. Differences in the source parameters between these two types of front tubes indicate sensitivity of the source-parameter determination procedure to the presence of evanescent waves. Because the geometry of the front of ER-10X ear tip may differ slightly across units and may differ slightly between the calibrator and ear canal, it is desirable.
that the procedure for determining source parameters is relatively insensitive to ear-tip geometry.

Two comparisons of source parameters for the unmodified and beveled front tubes are shown in Fig. 11. In the top row, the time-domain source parameters are compared after Step 1, which is prior to any evanescent-wave compensation. In the bottom row, the time-domain source parameters are compared after Step 5, which includes evanescent-wave compensation. Note that prior to evanescent-wave compensation, the discrepancy in peak impedance and peak pressure is 18% and 78%, respectively. The discrepancy after evanescent-wave compensation is reduced to 9% and 1%. The remaining discrepancy suggests that the current model for the reference impedance does not yet provide a complete characterization of evanescent waves; however, it is a significant improvement over previous parameter-determination methods that ignore the presence of evanescent waves. It should also be noted that the unmodified and beveled front tubes are physically different, and their true source parameters are thus not expected to be exactly identical.

It is instructive to consider how the estimated cavity lengths differ between Step 1 and Step 5. For the unmodified front tube, the five cavity lengths (cm) were estimated to be 3.37, 4.03, 4.57, 5.59, 7.23 in Step 1 and 3.06, 3.72, 4.25, 5.27, 6.91 in Step 5. The average difference between these two sets of measurements is 0.32 cm. This represents the length of an extension to the cavities of the same characteristic impedance that would have an effective series inertance equal to the inertance of the evanescent wave. The lack of any effective shunt capacitance in the evanescent wave, as would be required for a complete representation of this phantom cavity extension, is the reason that the source impedance acquires a parallel capacitance. In contrast, the five cavity lengths for the beveled tube were estimated to be 3.05, 3.73, 4.27, 5.29, 6.94 in Step 1 and 3.04, 3.72, 4.26, 5.28, and 6.93 in Step 5. The average difference between these two sets of measurements is only 0.01 cm. This indicates that the effective series inertance was much less for the beveled front tube compared to the unmodified front tube. Presumably, the reason for the smaller series inertance was that the evanescent wave was much smaller for the beveled front tube, as supported by the direct measurements described above.

V. DISCUSSION

The presence of evanescent waves near the microphone inlet of an acoustic probe can prevent accurate measurement of the plane-wave pressure that propagates away from the probe. As predicted by theory and demonstrated both here and in previous studies, the evanescent pressure becomes relatively larger with increasing frequency and as the diameter of a cylindrical tube into which the probe radiates becomes relatively large compared to the diameter of the sound source. The evanescent pressure is largest near the probe and declines rapidly with distance, unlike the propagating plane-wave pressure. The distance over which the evanescent pressure dissipates is a function of the same factors, also as predicted by theory (Ingard, 1948; Morse and Ingard, 1968; Keefe and Benade, 1981).

The inaccurate measurement of the plane-wave pressure due to the presence of evanescent waves degrades the accuracy of acoustic Thévenin calibrations and measurements of the ear’s acoustic impedance (Rabinowitz, 1981; Allen, 1986; Keefe et al., 1992; Huang et al., 2000) predicting the pressure at the termination of a cylindrical cavity or occluded-ear simulator from measurements made by an acoustic probe at the other end (Scheperle et al., 2011; Lewis et al., 2009). Changing the shape of the probe’s front tube/ear tip appeared to increase the effective aperture of the source reduced the relative amplitude of the evanescent wave pressure by allowing the sound to begin spreading before reaching the microphone inlet.

It is becoming routine to perform in-ear stimulus calibration by isolating the forward-going pressure from the reverse-going pressure traveling back toward the probe using Thévenin-equivalent source calibration (Scheperle et al., 2008; 2011;
Evanescent waves affect Thévenin calibrations, causing a drop in the magnitude of the source pressure and increasing the frequency-dependence in the source phase at high frequencies, similar to that of a parallel compliance (Nørgaard et al., 2017b; Nørgaard et al., 2018). Examining the source impedance in the time domain reveals substantial non-causality for the unmodified front tube that became causal after beveling the front tube. Contamination by evanescent waves also appears in in-ear stimulus calibrations.

The effect of evanescent waves can be reduced by protruding the microphone beyond the sound source (Rabinowitz, 1981; Voss and Allen, 1994; Keefe et al., 1992; Huang et al., 2000). However, this configuration complicates the definition of Thévenin-equivalent source parameters because the plane of the microphone inlet is spatially separate from the rigid plane of the probe where returning waves are reflected. Extending the microphone beyond the sound source leaves a residual volume between the microphone inlet and the end of the probe that complicates the acoustic circuit. However, it does not appear to be trivial to model the effect of the extension when estimating source parameters. Large errors in measuring admittance can result if the diameter of either test cavities or ear canals is different from the diameter of the calibration cavities, even below 10 kHz (Huang et al., 2000). Our method, with no extension, does not suffer from this problem and performs well to at least 20 kHz. Furthermore, microphone protrusion can be problematic for ear-canal measurements in cases where a deep insertion places the probe close to the eardrum. If a compelling reason to extend the microphone instead can be shown, then learning how to model it might be justified.

Beveling the front tube is inherently similar to protruding the microphone tube. However, this manipulation differs in that the shape of the flared recess in the rubber ear tip increases the area of the sound source rather than incorporating part of the ear canal itself. These features of the ear-tip are fixed across different ear-tip sizes and thus the source parameters remain independent of ear-canal diameter. This of course assumes that the shape of the depression at the end of the ear-tip is not deformed by placing the probe in an ear canal.

Refining the design of the ear tip and front tube of an emission probe is one way to reduce the magnitude of the evanescent wave relative to the plane wave because the plane of the microphone inlet remains at the front face of the probe. The evanescent wave is reduced both during calibration and during measurements in test loads and ears. It may be desirable to design modified front tubes and ear-tips to optimally couple to ear canals with the range of diameters seen in human ear canals. Since the beveling is currently a manual modification, there is an inherent variation between different front tubes. Thus, caution should be exercised in performing this modification unless it can be incorporated as part of the manufacturing process.

An alternative approach is to model the evanescent wave and then to compensate for it in the calculation of the source parameters. This method approximates the source parameters that would have been obtained in the absence of any evanescent pressure. The most effective approach to reducing the influence of evanescent waves may be to physically remove as much as possible and then compensate for the presence of any residual when calculating the source parameters. Although the evanescent wave is taken into account during the calibration procedure, subsequent measurements will still be affected by evanescent waves. Thus, additional compensation is required to obtain an estimate of the plane-wave impedance (e.g., Fletcher et al., 2005; Nørgaard et al., 2017a).

ACKNOWLEDGMENTS

The research reported in this publication was supported by Northwestern University and by the NIDCD of the National Institutes of Health under Award No. R01DC008318 to S.T.N., PI. The content is solely the responsibility of the authors and does not necessarily represent the official views of the National Institutes of Health. The Etymotic Research ER-10X otoacoustic emission probe was developed with support from STTR Phase II contract N00014-12-C-0108 from the U.S. Office of Naval Research, J.H.S., PI.

Our choice of the ER-10X probe was dictated by several features that were likely important in this study. First it has a wide bandwidth and excellent linearity that allow good measurements to beyond 20 kHz where the evanescent pressure was relatively large. It also has a fixed geometry with precisely formed replaceable front tubes, assuring good repeatability. Also, other probes do not allow for convenient use of a probe tube that can experimentally measure the decay of the evanescent waves with distance. This was important in our study because this data provided a benchmark for reducing the evanescent pressure that could be compared with our compensation method. However, all probes are affected by evanescent modes to various degrees. This report will hopefully encourage explorations of this issue in other probes and be helpful to those interested in designing new probes.


