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Evanescent waves in simulated ear canals: Experimental demonstration and method for compensation

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Evanescent waves emerge from a small sound source that radiates into a waveguide with a larger cross-sectional area, but unlike planar waves, do not propagate far from the source. Evanescent waves thus contaminate in-ear calibration of acoustic stimuli. Measurements with an otoacoustic-emission (OAE) probe inserted at the entrance of long tubes of various diameters show a decline in the evanescent wave with distance from the source when advancing a probe tube through the OAE probe and into the long tube. The amplitude of the evanescent pressure increases with frequency and depends strongly on the diameter of the long tube. Modifying the shape of the aperture of the probe’s sound source, thus effectively enlarging its diameter and redirecting acoustic flow, greatly reduced evanescent waves. The reduction in evanescent-wave pressure was observed in calibration cavities used to determine the Thévenin-equivalent source pressure and impedance of the probe. Errors in source calibrations were considerably larger in the unmodified configuration. An alternative method is proposed for calculation of acoustic source parameters that models the evanescent-wave pressure and reduces its influence on the calculation. This reduction greatly improves the quality of source calibrations, which should improve the accuracy of ear-canal impedance measurements and related quantities. © 2018 Acoustical Society of America.

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I. INTRODUCTION

A sound source at the entrance of a long, cylindrical waveguide with a diameter smaller than that of the waveguide generates a sound field composed of (1) plane waves that propagate far into the waveguide and (2) higher-order modes. If the frequency is low enough, these higher-order modes become evanescent and are attenuated exponentially along the waveguide (Miles, 1946; Ingård, 1948; Karal, 1953; Morse and Ingård, 1968; Keefe and Benade, 1981; Brass and Locke, 1997; Zebian et al., 2012; Nørgaard et al., 2017b). A microphone located near the sound source will record both the plane-wave pressure and the evanescent-wave pressure. Configuring the sound source and microphone to minimize the effect of evanescent waves has been explored previously in measurements of the ear’s acoustic impedance, e.g., by protruding the microphone beyond the sound source (Rabinowitz, 1981; Voss and Allen, 1994; Keefe et al., 1992; Huang et al., 2000). This study describes a physical modification to the sound source of a commercial otoacoustic-emission (OAE) probe (ER-10X, Etymotic Research) that has been found to physically decrease the amplitude of the evanescent wave. Following this demonstration, a method is described that decreases the undesirable effect of existing evanescent waves on Thévenin-equivalent source calibrations. These two approaches to account for the influence of evanescent waves lead to similar Thévenin source parameters.

To measure directly the planar and evanescent components emitted by an OAE probe, it is convenient to place the probe at the entrance of a long tube with diameter approximating that of the average ear canal. The sound field contains no reflections, so the decay of the evanescent pressure that is predicted by theory (Ingård, 1948; Morse and Ingård, 1968; Keefe and Benade, 1981), and can be measured directly by using a separate probe-tube microphone advanced through the OAE probe and into the long tube to sample the pressure at various locations along the tube in front of the OAE probe. The decay of the evanescent wave with distance is measured by advancing the probe tube into the long tube until there is little change in the measured pressure, indicating that the evanescent wave has decayed, and
that this plane-wave pressure thus establishes a reference for quantifying the evanescent wave. The plane-wave pressure amplitude declines slowly with distance due to frictional losses, but this factor is negligible for short distances that are typical for the spatial decay of the evanescent wave in a tube of radius similar to an adult ear canal.

Portions of this work were originally presented by Siegel and Neely (2017), who showed that not accounting for evanescent modes during the Thévenin calibration procedure causes the obtained source parameters to be non-causal. They also showed that a physical modification to the probe front tube effectively decreases the evanescent wave in front of the probe. Subsequently, Nørgaard et al. (2017b) showed that this error due to evanescent modes can be approximated as an acoustic compliance in parallel with the source parameters. They also proposed a method for incorporating the effect of the evanescent wave into the calibration method by estimating the evanescent-wave inertance rather than the waveguide lengths. Reported here are measurements of the decline in evanescent-wave pressure with increasing distance from the probe in long tubes of three different diameters. A simple modification of the replaceable sound-delivery front tube of the ER-10X1 showed greatly reduced evanescent waves, which led to improvements in Thévenin source calibrations, allowing better measures of the acoustic impedance of test cavities (and ear canals) by the probe and more accurate prediction of the termination pressure. Last, a signal-processing method is described to improve the determination of the probe’s Thévenin source pressure and impedance in the presence of evanescent waves that should yield similarly improved acoustic measurements in real ears. This approach is different from that of Nørgaard et al. (2017b) in that it does not require knowledge of the physical lengths of the calibration waveguides.

II. METHODS

A. Measurement procedure

A 1 mm outer-diameter stainless-steel probe tube, coupled to an electret microphone, was advanced through the access port in the body of an ER-10X OAE probe (Etymotic Research, Elk Grove Village, IL) and through the central lumen of the replaceable front tube under micrometer control [see Fig. 2(A)]. Inserting the probe tube disables the internal microphone of the ER-10X, but only minimally disturbs the sound field in front of the probe generated by the probe’s internal sound sources. Starting with the end of the probe tube flush with the end of the disposable multi-lumen front tube, the micrometer was advanced in small steps between measurements of the pressure using chirp excitation. The synchronous-averaging software used for these measurements was SysRes (Neely and Stevenson, 2002). The long tube was 50-ft in length of clear polyethylene tubing with inner diameter (id) 4.8, 7.9, or 12.7 mm.

B. Front-tube configurations

The long screw was removed from the back of the ER-10X to allow insertion of a calibrated probe tube through the body of the probe and the central lumen of the front tube to measure the pressure distal to the end of the probe [Fig. 1(A)]. The two configurations of the multi-lumen front tube used in the tests are shown in Fig. 1(B). The standard (unmodified) tube is as supplied by Etymotic Research. The beveled tube was modified to remove part of the outer lumen, leaving the central microphone lumen unaltered. The beveled tube allows the sound to be radiated, starting at the bottom of the small depression of the rubber ear tip [Fig. 1(C)]. The modification effectively increases the aperture of the sound source by a factor of about two, allowing the sound to begin to disperse proximal to the microphone inlet. Increasing the aperture of the source should reduce the amplitude of evanescent waves relative to the plane waves emitted by the probe (Ingård, 1948). A secondary effect of the beveled aperture was to direct acoustic flow away from the central microphone lumen, thus reducing the particle-velocity gradient near the edge of the microphone tube and minimizing potential flow losses (Nørgaard et al., 2017b).

The objective was to measure the influence of the diameter of the long tube on evanescent waves with the unmodified front tube and to test the prediction that modifying the source configuration as described would reduce the evanescent component of the pressure.

III. RESULTS

A. Dependence of the relative magnitude of the evanescent-wave pressure on the diameter of the long tube

To measure the relative amplitudes of evanescent- and plane-wave pressures along the long tubes, a series of measurements was made at increasing distance from the end of
the replaceable front tube of the probe. For each diameter of the long tube, a point was reached where the measured pressure became independent of position, and it was apparent that the pressure had become dominated by plane waves. One of these latter measurements was taken as a reference and the ratio of pressures along the tube to that of the reference was taken as the relative amplitudes of the evanescent and plane waves. When the plane wave travels along the guide, it undergoes a linear phase change with distance. Considering the plane-wave pressure response \( P_{pw} \) to a volume flow \( U \) at the position of the probe along a cylindrical waveguide with characteristic (or surge) acoustic impedance \( Z_0 \), the plane-wave pressure at the probe is purely real

\[
P_{pw}(x = 0) = U Z_0.
\]

(1)

Along the waveguide axis \( x \), however, the plane-wave pressure varies as

\[
P_{pw}(x) = U Z_0 e^{-j\omega(x/c)},
\]

(2)

where \( c \) is the speed of sound, and \( \omega = 2\pi f \). Conversely, the evanescent wave is purely imaginary along the waveguide and does not undergo a spatial phase change with the waveguide axis \( x \). Thus, the rotating phase of the plane wave pressure will combine constructively and destructively with the constant-phase evanescent wave pressure in a periodic manner over distance. The delay in the plane-wave reference pressure \( P_{pw,ref} \) measured at distance \( d_{ref} \) away from the probe was used to estimate the plane-wave pressure along the waveguide axis

\[
P_{pw}(x = 0) = U Z_0.
\]

This delay-corrected plane-wave pressure was then subtracted from the measured pressure \( P_{meas}(x) \) at a given position to obtain the evanescent-wave pressure

\[
P_{ew}(x) = P_{meas}(x) - P_{pw}(x).
\]

(4)

The delay compensation is needed to obtain the correct magnitude and phase of the evanescent wave. The magnitude and phase of the ratios \( P_{ew}(x)/P_{pw}(x = 0) \) are displayed in Fig. 2. The relative magnitude of the evanescent pressure increases with the diameter of the long tube and decreases with increasing distance of the probe tube from the front of the probe. The relative phase angle of the evanescent pressure hovers near \( +90^\circ \) because the evanescent wave can be approximated as an acoustic inertance in series with the plane-wave acoustic input impedance of a cylindrical waveguide. The phase is noisy at low frequencies because the evanescent pressure becomes relatively small and the relative pressure is subject to system noise.

The relative amplitude of the evanescent-wave pressure increased with the diameter of the long tube (Fig. 2). In the waveguide with id 12.7 mm, the evanescent-wave pressure appears to increase systematically until \( \approx 15.5 \) kHz (Fig. 2A). Each higher-order mode is only evanescent if the frequency is lower than the frequency at which the mode starts to propagate

\[
a_{mn} = \frac{\omega_{mn} c}{a},
\]

(5)

where \( \omega_{mn} \) is the \( n \)th zero-crossing of the Bessel-function derivative of order \( m \), \( c \) is the speed of sound, and \( a \) is the

![Figure 2](image_url)

FIG. 2. (Color online) Pressure in front of the probe normalized to the pressure measured at a location distant from the probe where plane waves were dominant and compensated for the propagation delay of the plane wave at the measurement position using Eqs. (3)–(4). Each column shows the relative magnitude and phase of the evanescent pressure for each tubing size. (A) The relative magnitude of the evanescent pressure was greatest for the 12.7 mm id tube, reaching \( +5 \) dB at 15 kHz. (B) Pressure variations became larger at high frequencies in the long tube with id of 7.9 mm and were strongly position dependent. (C) Only small variations in the normalized pressure response were noted for the long tube with id of 4.8 mm.
radius of the cylindrical waveguide. For $\lambda_1 = 1.841$ and the id 12.7 mm tube, this results in $f_1 = 15.8$ kHz. This means that above this frequency, the lowest-order azimuthal mode starts to propagate, and a proper reference consisting only of plane waves $p_{pw}$ cannot be established. As a result, we could only correctly estimate the magnitude of the plane wave, measured at a sufficient distance from the probe, for frequencies below 15.8 kHz for the largest tube. The relative magnitude of the evanescent pressure is successively smaller as the tubing diameter is reduced to 7.9 mm [Fig. 2(B)] and 4.8 mm [Fig. 2(C)] [note the different scaling of Fig. 2(A) vs Figs. 2(B) and 2(C)]. Note that the relative phase of the evanescent pressure appears reactive at all frequencies independent of tubing diameter. This consistent pattern in the phase suggests that our separation of the evanescent pressure from that of the plane wave was successful.

B. Beveling the front tube to change the source aperture

The effect of beveling the front tube is illustrated for the 7.9 mm id tube [Figs. 3(A) and 3(B)] and the 12.7 mm id tube [Figs. 3(C) and 3(D)]. In both cases, the relative magnitude of the evanescent pressure is reduced considerably for the beveled front tube [Figs. 3(B) and 3(D)], compared to the unmodified front tube [Figs. 3(A) and 3(C)]. The relative phase is consistently reactive, regardless of the front tube configuration. We did not measure the effect of beveling the

![Fig. 3. (Color online) Beveling the front tube reduced the relative amplitude of evanescent wave. (A), (B) Delay-compensated normalized pressure for the unmodified front tube [Fig. 3(A)] and for the beveled tube [Fig. 3(B)] for the 7.9 mm id long tube. (C), (D) Similar results were measured for the 12.7 mm id long tube. The relative phase of the evanescent pressure is reactive in all cases.](image_url)
front tube for the 4.8 mm tube because the relative evan
cent pressure was small even for the unmodified front tube
[Fig. 2(C)]. This is expected because the relatively small
difference between the aperture of the sound source of
the unmodified front tube (3 mm) and the 4.8 mm tube
(Ingård, 1948).

The decline in the evanescent-wave pressure with distance
from the probe is predicted from theory (Ingård, 1948). The
relative level of the evanescent pressure was evaluated at
20 kHz for the 4.8 mm and 7.9 mm long tubes and at 14 kHz
for the 12.7 mm tube. As previously mentioned, for the largest
tube, this was slightly below the 15.8 kHz frequency where the
lowest-order azimuthal mode starts to propagate, and a proper
reference consisting only of plane waves cannot be established.
The rate of decline of the evanescent pressure depended both
on the diameter of the long tube and on whether the front tube
was beveled (Fig. 4). The decline is approximately exponen
tial, with a generally shallower slope for the beveled front
tube. The different decay rates might be due to a difference in
which higher-order modes are elicited by each front tube. The
results suggest that the evanescent modes elicited by the bev
eled front tube are dominated by higher-modes modes of lower
order than the unmodified front tube.

It should be noted that the range of human ear-canal
diameters spans the range of long tube diameters shown here. Evanescent waves are thus likely to be a serious prob
lem in larger ear canals when the pressure measured by the
probe is relied on for calibration. Conversely, for relatively
small ear canals and for sound source apertures similar to
that of the ER-10X, evanescent waves are expected to be
small and can usually be ignored.

C. Beveled front tube allows more accurate prediction
of termination pressure

A good test of the accuracy of stimulus calibration using
an OAE probe is to compare the pressure measured by the
microphone that terminates a test cavity or an IEC 60318–4
occluded-ear simulator and the pressure at the termination
predicted from the pressure measured by the probe. This pred
iction adds the amplitudes of the forward and reflected
pressures, referred to as integrated pressure (Lewis et al.,
2009; Scheperle et al., 2011; Souza et al., 2014). In such
comparisons, the predicted and measured pressures com
monly agree within ~1 dB at frequencies up to ~10 kHz, but
steadily diverge at higher frequencies. An example of pre
dicting the pressure at the microphone of a 60318-4
occluded-ear simulator using the pressure measured by the
ER-10X probe is shown in Fig. 5. The distance between the
end of the OAE probe and the simulator's microphone is
approximately 21.2 mm, judged by the frequency of the first
half-wave resonance (8.1 kHz). At 20 kHz, the predicted
pressure became about 6 dB larger than the pressure mea
sured by a microphone at the termination. The discrepancy
between the predicted and actual termination pressure in the
ear simulator at high frequencies is much smaller for the
beveled front tube [Fig. 5(B)] than for the unmodified tube
[Fig. 5(A)]. The error in matching the target pressure of
nearly 6 dB implies that the amplitude of the evanescent-wave
pressure is similar to that of the plane wave at the inlet of
the emission probe. Note that the 3/4 wave null in the
measured probe pressure (arrows) is much more symmetric
ally related to the frequencies of half-wave resonance
(asterisks) for the beveled front tube [Fig. 5(B)] than for the
unmodified tube [Fig. 5(A)]. The null is shifted by ~2.5 kHz
but the frequencies of the half-wave resonances are essen
tially unchanged. The difference between the predicted and
measured termination pressures for the two front tube config
urations is shown in Fig. 5(C). This error becomes ~6 dB for
the unmodified front tube vs ~2 dB for the beveled tube.

D. Evanescent waves contaminate Thévenin source
calibrations

When using impedance or reflectance-based methods to
calibrate in-ear pressure stimuli in situ, it is important to
obtain an accurate Thévenin source calibration (Scheperle
et al., 2008; Souza et al., 2014). As expected, the pressure
responses in the cavities used to calibrate the ER-10X probe
depended on the configuration of the front tube. In the fre
quency domain, the differences between the unmodified and
beveled tube are mainly in the nulls above 5 kHz for each of
the five calibration cavities [Fig. 6(A)]. This asymmetry of
the nulls is similar to those seen in the EP pressure response
of the ear simulator in Fig. 5(A).

The pressure peaks for the beveled tube are to a higher
degree symmetrically positioned between the pressure nulls in
Fig. 6(B), similar to the ear simulator response in Fig. 5(B).
This symmetry is expected from uniform cavities with domi
nant plane waves, but its importance is uncertain. The calibra
tion algorithm is based on an optimization routine that
estimates the lengths of the waveguides. Table I lists the mea
sured and estimated calibration-waveguide lengths using
each set of probe pressures with the unmodified and beveled
front tubes. The actual lengths were measured within about
+/−0.2 mm using a mm ruler. The estimated lengths are

FIG. 4. (Color online) The relative magnitude of the evanescent pressure
decreases approximately exponentially with distance from the probe. The
decline appears steeper for the unmodified front tube.

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consistently larger for the unmodified front tube. The numbers in parentheses are the errors between the measured and estimated lengths. Notice that the estimated lengths with the beveled front tube in place correspond much more closely to the measured lengths. This suggests that the position of the probe-pressure nulls, which are translated due to the evanescent wave, is the primary factor determining the estimated lengths (Nørgaard et al., 2017b).

Further evidence of the evanescent wave was obtained in a time-domain representation of the cavity pressures. Figure 7 compares the cavity pressures in the time domain, shown as the inverse Fourier transforms of the pressures in Fig. 6. Here, the differences appear mainly in the first peak (between 1.9 and 2.1 ms). Because the cavities have a rigid perpendicular termination, secondary peaks, which are essentially delayed reflections of the initial peak, should have a similar shape. The pressure response for the beveled front tube comes closer to this expectation because the initial peak lacks the high-frequency oscillations that appear on the initial peak for the unmodified front tube.

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Thevenin-source impedances calculated by conventional methods from the cavity pressures are shown in Fig. 8. The magnitude [Fig. 8(A)] and phase [Fig. 8(B)] of the impedance exhibit less frequency-dependence for the beveled tube than for the unmodified tube. Although differences are apparent, the real part of the impedance remains positive at essentially all frequencies for both tube configurations, so there is no obvious reason to prefer one tube over the other. However, the set of pressure responses in the five cavities used to calculate the ER-10X source characteristics using an optimization that assumes plane wave propagation in the cavities reports consistently lower errors when using the beveled tube compared to the unmodified tube. This would be expected if the sound pressure in the calibration cavities is closer to ideal plane waves. Nørgaard et al. (2017b) showed that not accounting for evanescent modes when estimating the waveguide lengths introduces a parallel compliance into the source parameters. The behavior observed in the source impedance between the unmodified and beveled probe tubes are indeed similar to their observations.

An intriguing way to distinguish the two tube configurations was to compare the source impedances in the time domain (Fig. 9). Time-domain impedance is essentially the pressure response to an impulsive flow stimulus. In the context of discrete-time signals, it is prudent to select a flow stimulus that has a band-limited first derivative. For this reason, we use the inverse Fourier transform of a frequency-domain Blackman window that is centered at zero frequency and extends to 20 kHz. Additional smoothing was achieved by zero-padding in the frequency domain (to four times the Nyquist frequency) prior to computing the inverse Fourier transform. Figure 9 shows source impedances in the time domain for the unmodified and beveled front tubes.

In the time domain, much of the impedance for the unmodified front tube occurs prior to \( t = 0 \). This is obviously different from the impedance of the beveled tube, which is close to zero for \( t < 0 \). So, one can argue that the beveled front tube is preferable because the impedance for the unmodified tube is non-causal. The non-causality for the unmodified front tube can be explained as a negative parallel compliance (Nørgaard et al., 2018), thus discharging in reverse time.
IV. METHOD FOR DETERMINING THEVENIN-EQUIVALENT ACOUSTIC SOURCE PARAMETERS IN THE PRESENCE OF EVANESCENT WAVES

An essential prerequisite for determining the Thévenin-equivalent acoustic source parameters, specifically the source impedance $Z_s$ and the source pressure $P_s$, is a set of pressure measurements $P_n$ obtained from $N$, typically four or five, calibration waveguides. In addition, theoretical reference impedance $Z_n$ is required for each of these waveguides. The procedure is complicated by the need to estimate from the pressure measurements some of the parameters that are used in the calculation of the reference impedances, e.g., the waveguide lengths.

A. Reference impedance

The reference impedance $Z_n$ for the $n$th waveguide is modeled as the sum of three impedances

$$Z_n = Z_{fl} + Z_{ew} + Z_{lk},$$

where $\parallel$ represents the parallel combination (of $Z_{lk}$ and $Z_{pw,n}$). The primary impedance is the plane-wave impedance $Z_{pw,n}$ for the $n$th waveguide. The other three impedances, which are the flow loss $Z_{fl}$ (Nørgaard et al., 2017b), the evanescent-wave impedance $Z_{ew}$, and the leak impedance $Z_{lk}$, are modeled as being the same for all waveguides because the probe remains at a fixed location in the ER-10X calibrator. The leak impedance is resistive and is due to an intentional static-pressure relief near the entrance of the ER-10X calibrator. The reference impedance provides the acoustic loads that are used to determine the Thévenin-equivalent acoustic source parameters based on the circuit shown in Fig. 10.

The plane-wave impedance $Z_{pw,n}$ is modeled as a transmission line with an open-circuit termination

$$Z_{pw,n} = Z_0 \coth(\gamma L_n),$$

where $Z_0$ is the characteristic impedance and $\gamma$ is a propagation multiplier. In a lossless, uniform waveguide, $Z_0 = \rho c / A$ and $\gamma = i \omega / c$, where $\rho$ is the density of air, and $A$ is the cross-sectional area of the waveguide. Analytic solutions exist for propagation in circular uniform waveguides that have thermoviscous losses; however, we use approximations for $\gamma$ and $\omega$ suggested by Keefe (1984). The waveguide lengths $L_n$ and flow loss $Z_{fl}$ are estimated during the calibration procedure.

The evanescent-wave impedance $Z_{ew}$ is mainly inerance due to spreading of the pressure as it exits the sound output port of the ER-10X probe. However, because the effective inerance is observed to increase in the frequency range between 10 and 20 kHz, the evanescent-wave impedance is modeled as a polynomial function of frequency

$$Z_{ew} = i Z_0 \cdot \sum_{m=1}^{M} (\omega \tau_m)^m.$$

The parameters of this equation $\tau_1 \ldots \tau_M$ are estimated during the calibration procedure. Note that $Z_{ew}$ is purely imaginary, and so does not contribute any loss to the reference impedance.

Flow losses are caused by the large gradients due to the spreading flow close to the speaker aperture (Nørgaard et al., 2017b). In our reference impedance, the flow loss $Z_{fl}$ is modeled as a negative real quantity that is proportional to the square-root of frequency, since the particle-velocity gradient going away from the edge of the speaker aperture is inherently negative.

<table>
<thead>
<tr>
<th>Calibration waveguide</th>
<th>Measured waveguide lengths (cm)</th>
<th>Unmodified front tube estimated lengths (cm)</th>
<th>Beveled front tube estimated lengths (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.93</td>
<td>3.36 (0.43)</td>
<td>3.07 (0.14)</td>
</tr>
<tr>
<td>2</td>
<td>3.65</td>
<td>4.04 (0.39)</td>
<td>3.76 (0.11)</td>
</tr>
<tr>
<td>3</td>
<td>4.21</td>
<td>4.57 (0.36)</td>
<td>4.29 (0.08)</td>
</tr>
<tr>
<td>4</td>
<td>5.22</td>
<td>5.59 (0.37)</td>
<td>5.31 (0.09)</td>
</tr>
<tr>
<td>5</td>
<td>6.94</td>
<td>7.24 (0.30)</td>
<td>6.95 (0.01)</td>
</tr>
</tbody>
</table>

FIG. 6. (Color online) Set of pressure responses in the probe Thévenin-source calibration cavities for (A) the unmodified front tube and (B) the beveled front tube. The pressure nulls are consistently positioned midway between the frequencies of the resonant peaks in (B) and asymmetrically in (A).

TABLE I. Measured and estimated waveguide lengths. The numbers in parentheses are the errors in the estimated lengths relative to the measured lengths of the waveguides.
The flow-loss coefficient $R_{fl}$ is estimated during the calibration procedure.

### B. Waveguide length estimation

Waveguide lengths are estimated acoustically by minimizing the overall error in estimated waveguide pressures that are calculated by using the source parameters. When $N$ measured pressures $P_n$ and reference impedances $Z_n$ are known (for $n = 1 \ldots N$), then the source parameters can be calculated from a least-squares solution to the following equation (Allen, 1986):

$$
\begin{bmatrix}
Z_1 & -P_1 \\
\vdots & \vdots \\
Z_N & -P_N
\end{bmatrix}
\begin{bmatrix}
P_s \\
Z_s \\
Z_n
\end{bmatrix} =
\begin{bmatrix}
Z_1P_1 \\
\vdots \\
Z_NP_N
\end{bmatrix}. 
$$  

(10)

An estimate of each waveguide pressure is obtained from $P_s$, $Z_s$, and $Z_n$:

$$
\hat{P}_n = \frac{P_s Z_n}{Z_s + Z_n}. 
$$  

(11)

The waveguide pressure estimation error is quantified as (Schepelte et al., 2011)

$$
\epsilon_{pe|f_1<f<f_2} = \frac{\sum_{n=1}^{N} \sum_{f} |\hat{P}_n - P_n|^2}{\sum_{n=1}^{N} \sum_{f} |P_n|^2}. 
$$  

(12)

This estimation error is evaluated over the frequency range $f_1 < f < f_2$. A simplex search algorithm is used to find a set of waveguide lengths $L_n$ and flow-loss coefficient $R_{fl}$ that minimize this estimation error for fixed values of $Z_{ls}$ and $Z_{cw}$.

### C. Parallel-compliance delay

The presence of evanescent waves influences estimation of the Thévenin source parameters. When evanescent waves are physically present but their representation is omitted from the reference impedance, the source impedance is observed to have a negative compliance in parallel with its resistive component (Nørgaard et al., 2017b)

![FIG. 7. (Color online) Set of pressure responses in the time domain for (A) the unmodified front tube and (B) the beveled front tube. These waveforms were smoothed by zero-padding in the frequency domain (to four times the Nyquist frequency) prior to computing inverse Fourier transforms.](image)

![FIG. 8. (Color online) Evanescent waves contaminate Thévenin source calibrations. The magnitude (A) and phase (B) of the source impedances are both influenced over the entire frequency range with a much more dramatic change with frequency for the unmodified front tube than for the beveled tube. The impedance magnitude is about an order of magnitude higher at 20kHz for the beveled tube compared with the unmodified tube and the phase shows a corresponding divergence of about 1/4 period.](image)
parallel-compliance delay, is
gation time of the shorter transmission line, which we call
impedance \(Nørgaard\).

The reason that \(C_{par}\) is negative in this case is that when
placed across the input to a transmission line, a negative
compliance approximates a series inertance \(L_{ser}\) at the input
of a shorter transmission line with the same characteristic
impedance (Nørgaard et al., 2017b). The difference in propa-
gation time of the shorter transmission line, which we call
parallel-compliance delay, is

\[
\tau_{pc} = C_{par} Z_0. \tag{14}
\]

The series inerance in this approximation is

\[
L_{ser} = -\tau_{pc} Z_0. \tag{15}
\]

When an evanescent wave is physically present, but series
inerance has been omitted from the reference model, the
equations in this section provide a means to estimate the
series inerance that should have been added to the reference
model. This approach assumes that if \(C_{par} = 0\), the averaged
imaginary part of the source admittance across the frequency
bandwidth (\(\text{Im}\{1/Z_s\}\)) is 0.

D. Source-parameter calculation

Because the waveguide lengths \(L_n\), evanescent-wave
impedance \(Z_{ew}\), and flow loss \(Z_{fl}\) are not precisely known,
the source parameters are initially calculated using prelimi-
nary estimates of these quantities. Subsequently, the first
estimates of the source parameters are used to improve esti-
mates of all unknown quantities. The following steps outline
this procedure:

1. Let \(Z_{ew} = 0\) and find the set of \(Z_{fl}\) and \(L_n\) (for \(n = 1 \ldots N\))
   that minimize \(\epsilon_{pc}\) for \(2 < f < 8\) kHz.
2. Estimate \(\tau_{pc}\) from \(Z_s\). If \(\tau_{pc} < 0\), then let \(\tau = -\tau_{pc}\),
   otherwise let \(\tau = 0\).
3. Let \(Z_{ew} = iZ_0 \omega \tau_1\) and find the set of \(Z_{fl}\) and \(L_n\) (for
   \(n = 1 \ldots N\)) that minimize \(\epsilon_{pc}\) for \(2 < f < 8\) kHz.
4. Let \(Z_{ew} = iZ_0 \cdot \sum_{m=1}^{5} (\omega t_m)^m\) and find the set of \(t_m\) (for
   \(m = 1 \ldots 5\)) that minimize \(\epsilon_{pc}\) for \(2 < f < 18\) kHz.
5. Let \(Z_{ew} = iZ_0 \cdot \sum_{m=1}^{5} (\omega t_m)^m\) and find the set of \(Z_{fl}\) and
   \(L_n\) (for \(n = 1 \ldots N\)) that minimize \(\epsilon_{pc}\) for \(2 < f < 8\) kHz.

These five steps provide estimates for \(L_n, Z_{ew},\) and \(Z_{fl}\) in
addition to determining the source parameters \(Z_s\) and \(P_s\).

The characteristic impedance of a cylindrical waveguide
with a diameter of 8 mm is \(Z_0 = 81.55\) cgs acoustic ohms.
Based on comparisons between calibrations in the ER10X
calibrator and rigidly terminated waveguides that had no
static-pressure relief, the leak impedance of the ER10X cali-
ibrator may differ slightly across units and may differ
during the sampling rate to produce units in the
time-domain that are independent of
sampling rate.

E. Examples of source parameters for unmodified
and beveled front tubes

The unmodified and beveled front tubes differ in the
size of the evanescent wave present in waveguide pressure
measurements. Differences in the source parameters between
these two types of front tubes indicate sensitivity of the
source-parameter determination procedure to the presence of
evanescent waves. Because the geometry of the front of ER-
10X ear tip may differ slightly across units and may differ
sufficiently between the calibrator and ear canal, it is desirable

![FIG. 9. (Color online) The probe source impedance transformed into the
time domain reveals an initial peak for the unmodified tube that begins at neg-
ative time and is thus non-causal. The beveled configuration yields a time-
domain source impedance that occurs nearly completely in positive time and
is thus much closer to being causal. The impedance unit in the frequency
domain was cgs acoustic ohm prior to computing the inverse Fourier trans-
form, which has been multiplied by the sampling rate.](image-url)

![FIG. 10. Circuit representation of the waveguide reference impedance as the
acoustic load of a Thévenin-equivalent source. The characteristic impedance
and length of the ideal waveguide are \(Z_0\) and \(L_n\), respectively.](image-url)
that the procedure for determining source parameters is relatively insensitive to ear-tip geometry.

Two comparisons of source parameters for the unmodified and beveled front tubes are shown in Fig. 11. In the top row, the time-domain source parameters are compared after Step 1, which is prior to any evanescent-wave compensation. In the bottom row, the time-domain source parameters are compared after Step 5, which includes evanescent-wave compensation. Note that prior to evanescent-wave compensation, the discrepancy in peak impedance and peak pressure is 18% and 78%, respectively. The discrepancy after evanescent-wave compensation is reduced to 9% and 1%. The remaining discrepancy suggests that the current model for the reference impedance does not yet provide a complete characterization of evanescent waves; however, it is a significant improvement over previous parameter-determination methods that ignore the presence of evanescent waves. It should also be noted that the unmodified and beveled front tubes are physically different, and their true source parameters are thus not expected to be exactly identical.

It is instructive to consider how the estimated cavity lengths differ between Step 1 and Step 5. For the unmodified front tube, the five cavity lengths (cm) were estimated to be 3.37, 4.03, 4.57, 5.59, 7.23 in Step 1 and 3.06, 3.72, 4.25, 5.27, 6.91 in Step 5. The average difference between these two sets of measurements is 0.32 cm. This represents the length of an extension to the cavities of the same characteristic impedance that would have an effective series inductance equal to the inductance of the evanescent wave. The lack of any effective shunt capacitance in the evanescent wave, as would be required for a complete representation of this phantom cavity extension, is the reason that the source impedance acquires a parallel capacitance. In contrast, the five cavity lengths for the beveled tube were estimated to be 3.05, 3.73, 4.27, 5.29, 6.94 in Step 1 and 3.04, 3.72, 4.26, 5.28, and 6.93 in Step 5. The average difference between these two sets of measurements is only 0.01 cm. This indicates that the effective series inductance was much less for the beveled front tube compared to the unmodified front tube. Presumably, the reason for the smaller series inductance was that the evanescent wave was much smaller for the beveled front tube, as supported by the direct measurements described above.

V. DISCUSSION

The presence of evanescent waves near the microphone inlet of an acoustic probe can prevent accurate measurement of the plane-wave pressure that propagates away from the probe. As predicted by theory and demonstrated both here and in previous studies, the evanescent pressure becomes relatively larger with increasing frequency and as the diameter of a cylindrical tube into which the probe radiates becomes relatively large compared to the diameter of the sound source. The evanescent pressure is largest near the probe and declines rapidly with distance, unlike the propagating plane-wave pressure. The distance over which the evanescent pressure dissipates is a function of the same factors, also as predicted by theory (Ingard, 1948; Morse and Ingard, 1968; Keefe and Benade, 1981).

The inaccurate measurement of the plane-wave pressure due to the presence of evanescent waves degrades the accuracy of acoustic Thévenin calibrations and measurements of the ear’s acoustic impedance (Rabinowitz, 1981; Allen, 1986; Keefe et al., 1992; Huang et al., 2000) predicting the pressure at the termination of a cylindrical cavity or occluded-ear simulator from measurements made by an acoustic probe at the other end (Scheperle et al., 2011; Lewis et al., 2009). Changing the shape of the probe’s front tube/ear tip appeared to increase the effective aperture of the source reduced the relative amplitude of the evanescent wave pressure by allowing the sound to begin spreading before reaching the microphone inlet.

It is becoming routine to perform in-ear stimulus calibration by isolating the forward-going pressure from the reverse-going pressure traveling back toward the probe using Thévenin-equivalent source calibration (Scheperle et al., 2008; 2011;
Evanescent waves affect Thévenin calibrations, causing a drop in the magnitude of the source pressure and increasing the frequency-dependence in the source phase at high frequencies, similar to that of a parallel compliance (Nørgaard et al., 2017b; Nørgaard et al., 2018). Examining the source impedance in the time domain reveals substantial non-causality for the unmodified front tube that became causal after beveling the front tube. Contamination by evanescent waves also appears in in-ear stimulus calibrations.

The effect of evanescent waves can be reduced by protruding the microphone beyond the sound source (Rabinowitz, 1981; Voss and Allen, 1994; Keefe et al., 1992; Huang et al., 2000). However, this configuration complicates the definition of Thévenin-equivalent source parameters because the plane of the microphone inlet is spatially separate from the rigid plane of the probe where returning waves are reflected. Extending the microphone beyond the sound source leaves a residual volume between the microphone inlet and the end of the probe that complicates the acoustic circuit. However, it does not appear to be trivial to model the effect of the extension when estimating source parameters. Large errors in measuring admittance can result if the diameter of either test cavities or ear canals is different from the diameter of the calibration cavities, even below 10 kHz (Huang et al., 2000). Our method, with no extension, does not suffer from this problem and performs well to at least 20 kHz. Furthermore, microphone protrusion can be problematic for ear-canal measurements in cases where a deep insertion places the probe close to the eardrum. If a compelling reason to extend the microphone instead can be shown, then learning how to model it might be justified.

Beveling the front tube is inherently similar to protruding the microphone tube. However, this manipulation differs in that the shape of the flared recess in the rubber ear tip increases the area of the sound source rather than incorporating part of the ear canal itself. These features of the ear-tip are fixed across different ear-tip sizes and thus the source parameters remain independent of ear-canal diameter. This of course assumes that the shape of the depression at the end of the ear-tip is not deformed by placing the probe in an ear canal.

Refining the design of the ear tip and front tube of an emission probe is one way to reduce the magnitude of the evanescent wave relative to the plane wave because the plane of the microphone inlet remains at the front face of the probe. The evanescent wave is reduced both during calibration and during measurements in test loads and ears. It may be desirable to design modified front tubes and ear-tips to optimally couple to ear canals with the range of diameters seen in human ear canals. Since the beveling is currently a manual modification, there is an inherent variation between different front tubes. Thus, caution should be exercised in performing this modification unless it can be incorporated as part of the manufacturing process.

An alternative approach is to model the evanescent wave and then to compensate for it in the calculation of the source parameters. This method approximates the source parameters that would have been obtained in the absence of any evanescent pressure. The most effective approach to reducing the influence of evanescent waves may be to physically remove as much as possible and then compensate for the presence of any residual when calculating the source parameters. Although the evanescent wave is taken into account during the calibration procedure, subsequent measurements will still be affected by evanescent waves. Thus, additional compensation is required to obtain an estimate of the plane-wave impedance (e.g., Fletcher et al., 2005; Nørgaard et al., 2017a).

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