



Decision Making under Uncertainty in Sustainable Energy Operations and Investments

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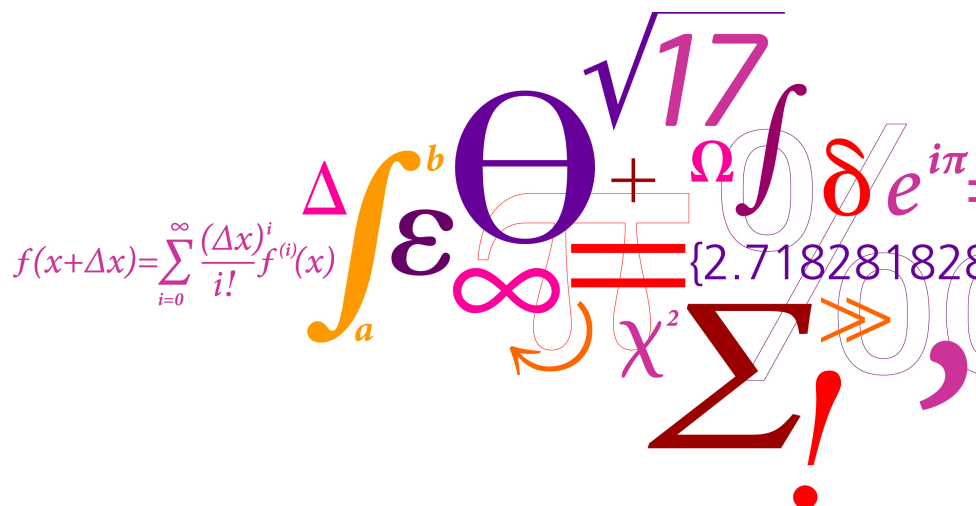


Decision Making under Uncertainty in Sustainable Energy Operations and Investments

PhD Thesis

Alessio Trivella

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Summary

The energy sector is undergoing a rapid transformation that gives rise to a variety of new optimization problems to solve. The goal of this thesis is to develop advanced analytical solutions for sustainable energy operations and investment problems that are subject to uncertainty. We consider case studies related to both industrial applications and consumer models, and cover problems related to energy efficiency, renewable energy, and corporate social and environmental responsibility.

The problems that we consider in this thesis are the following: (i) constructing a power contract portfolio for companies that commit to reach a renewable energy target, which means, to procure a specific percentage of electricity demand from renewable energy sources by a future date; (ii) managing shutdown decisions in commodity and energy production assets from a social commerce perspective, that is, taking into account the indirect consequences that a plant shutdown has on the society; (iii) modeling the optimal market bidding strategies of virtual power generators under a novel proposed electricity market structure that would favor the integration of renewable production units; and (iv) investigating the factors behind the consumer investments in energy efficient household appliances, and the optimal energy saving investments from a consumer and energy system perspectives.

To tackle these problems, we leverage tools from operations research to design novel methodology and perform energy analysis. Several problems encountered in this thesis can be formulated as intractable Markov decision processes (MDPs) with high-dimensional exogenous and/or endogenous component of the state. To overcome this intractability, we develop approximate dynamic programming (ADP) methods to compute near optimal operating and investment policies, and lower and upper bounds on the optimal MDP value. In particular, the ADP methods that we develop include: (i) an extension of the regress-later least squares Monte Carlo (LSML) to approximate risk-averse MDPs, (ii) a combination of LSML and classification to learn decision rules, (iii) a shortest path reformulation of the reoptimization heuristic, and (iv) a novel use of the information relaxations and duality framework to extract non-anticipative decision rules from sample action distributions. We also use more classical scenario-based stochastic programming.

The contributions of this thesis cover modeling, methodology, and applications. We contribute to the definition, understanding, and resolution of emerging optimization problems in sustainable energy operations and investments, and provide results and insights that can be useful for companies and policy makers. We contribute to the development of new operations research models, algorithms, and theory for solving large-scale stochastic optimization problems, with particular focus on ADP techniques. Some of the methodology developed in this thesis has potential broader relevance in other application contexts, such as inventory control and financial portfolio optimization.

Resumé (Danish Summary)

Energisektoren er under hastig forandring, hvilket skaber behov for at løse en lang række nye optimeringsproblemer. Formålet med denne afhandling er at udvikle avancerede analytiske løsninger til bæredygtig energiforsyning og investeringsproblemer underlagt usikkerhed. Vi betragter case studier relateret til både industrielle anvendelser og husholdninger, og studerer problemer relateret til energieffektivitet, vedvarende energi og virksomhedernes sociale og miljømæssige ansvar.

Problemer studeret i denne afhandling omfatter: (i) opbygning af en energikontrakt portefølje for virksomheder, der forpligter sig til at nå et mål for vedvarende energi, hvilket betyder at de skal have en bestemt procentdel af elforbruget dækket af vedvarende energikilder inden for en fremtidig horisont; (ii) optimering af nedluknings-beslutning i råvare- og energiproduktionsaktiver med focus på et socialt perspektiv, det vil sige under hensyntagen til de indirekte konsekvenser som en nedlukning af anlægget har for samfundet; (iii) modellering af optimale markedsbud-strategier for virtuelle kraftgeneratorer under en ny foreslået elektricitetsmarkedsstruktur, der ville fremme integrering af vedvarende produktionsenheder; og (iv) undersøge faktorerne bag forbrugerinvesteringer i energieffektive husholdningsapparater, og finde de optimale energibesparende investeringer ud fra et forbruger- og energisystemperspektiv.

For at løse disse problemer anvender vi værktøjer fra operationsanalyse til at designe ny metodik og udfører energianalyser. Flere problemer i denne afhandling kan formuleres som uhåndterlige Markov-beslutningsprocesser (MDP'er) med multidimensional eksogene og/eller endogene komponenter i tilstanden. For at overvinde denne uhåndterbarhed udvikler vi approximerede dynamisk programmeringsmetoder (ADP) til beregning af nær-optimale drifts- og investeringsbeslutninger, samt nedre og øvre grænser på den optimale MDP-værdi. I særdeleshed omfatter ADP-metoderne som vi udvikler: (i) en udvidelse af regress-senere mindste kvadrater Monte Carlo (LSML) metoden til at tilnærme risikoavers MDP'er, (ii) en kombination af LSML og klassifikation for at lære beslutningsregler (iii) en kortest-vej reformulering af reoptimiserings heuristikken, og (iv) en ny anvendelse af informationsrelaxeringerne og dualitetsrammen til at udtrække beslutningsregler uaf-

hængigt på fremtiden fra testfordelinger. Vi anvender også mere klassisk scenarie-baseret stokastisk programmering.

Bidragene fra denne afhandling omfatter modellering, metodologi og applikationer. Vi bidrager til definitionen, forståelsen og løsningen af nye optimeringsproblemer inden for bæredygtig energi og investering, og rapporterer resultater og indsigt som kan være nyttige for virksomheder og beslutningstagere. Vi bidrager til udviklingen af nye operationsanalyse, algoritmer og teori til løsning af stor-skala stokastiske optimeringsproblemer med særlig fokus på ADP teknikker. Nogle af de metoder, der er udviklet i denne afhandling, har potentiel bredere relevans i andre anvendelses sammenhæng, såsom lagerstyring og finansiell porteføljeoptimering.

Preface

This thesis was carried out at the Division of Management Science, DTU Management Engineering, Technical University of Denmark, in partial fulfillment of the thesis requirements for the degree of Doctor of Philosophy (Ph.D.) in Operations Research.

The project has been supervised by Professor David Pisinger and co-supervised by Professors Stein-Erik Fleten and Selvaprabu Nadarajah. This thesis consists of an introduction and six research chapters, five of which are based on academic papers that have been published or are currently under review in an international peer-reviewed journal and one is a supplement chapter. The five papers are co-authored, and they are each self contained in terms notation and with separate bibliographies.

This project has been supported by the Innovation Fund Denmark under the “SAVE-E” project and by DTU Management Engineering, and was carried out between April 2015 and June 2018. Part of this study has been conducted at the Norwegian University of Science and Technology (one month in spring 2016), and at the University of Illinois at Chicago (five months in spring 2017).

Kgs. Lyngby, Denmark, June 2018



Alessio Trivella

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I thank my co-supervisors Stein-Erik Fleten and Selvaprabu Nadarajah. Stein-Erik has introduced me to the real option analysis discipline with interesting case studies, and hosted me at the Norwegian University of Science and Technology which was a valuable experience. Selva has taught me most of what I know now about approximate dynamic programming, helped me to improve my writing and to lift the quality of my PhD, and given me career advice. My visit to the University of Illinois at Chicago has been very productive as well as my entire collaboration with Selva.

In short, this thesis would not have been possible without such a capable and enthusiastic supervision team.

I also want to thank my co-authors Mattia Baldini, Denis Mazieres, Nicolò Mazzi, Danial Mohseni-Taheri, Juan Miguel Morales, and Jordan William Wentz for the fruitful research collaborations and teamwork that I have truly enjoyed.

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Part I

Introduction

Introduction and thesis outline

Driven by the growing international effort towards the climate change mitigation and the falling costs of solar and wind power production, the energy sector has been undergoing a transformation during the last decade. Globally, it is estimated that 10 trillion US dollars will be invested by 2040 in new power generation, with more than 7 trillion of which invested in clean power assets (BNEF, 2017). In the European Union (EU), the share of renewable energy consumption has increased from 9% in 2005 to 17% in 2015 due to strong supporting policy measures and long term targets, and this figure is projected to double by 2030 reaching 34% (IRENA, 2018). Denmark is at forefront with a non-dispatchable renewable energy share of 44% in 2017 that is estimated to reach up to 70% in 2022 (IEA, 2017b). Besides renewable energy, energy efficiency is central to the global energy transition to improve energy security and to reduce energy costs and greenhouse gas emissions. Without energy efficiency improvements in the period 2000–2016, it is estimated that 12% more energy would have been consumed worldwide (IEA, 2017a). Moreover, global investments in energy efficiency are increasing and in 2016 they amounted to 230 billion US dollars, representing 13.6% of the total investments across the entire energy sector for that year (IEA, 2017a).

The development of a clean energy economy is a major opportunity for new investments, jobs, and environmental conservation around the world. However, it also introduces a variety of new challenges such as determining optimal investments in renewable energy and energy saving projects, implementing effective energy regulations, ensuring the reliability of energy systems with high penetration of renewable energy, and designing efficient electricity markets (Conejo et al., 2010; Morales et al., 2013). Addressing these challenges

usually requires making good decisions in the presence of uncertainty (in energy prices, demand, technology, etc.), which makes modeling and solving these problems generally harder, but also makes this an exciting time for doing energy research.

In this thesis, we consider problems related to sustainable energy operations and investments. With *sustainability*, we mean a broader concept which includes energy efficiency and savings, renewable energy, and corporate social and environmental responsibility. The thesis consists of a number of case studies aimed at covering industrial applications as well as consumer/household models. The focus of this thesis is at the intersection between application side and methodological side, and the main two objectives of this work are the following:

- New applications to SUSTAINABLE ENERGY OPERATIONS AND INVESTMENTS. We study emerging optimization problems related to renewable energy, energy efficiency, and corporate social and environmental responsibility. We contribute to the definition, understanding, and resolution of these problems and provide insights for companies and policy makers, with a focus on Denmark in some of the studies.
- Methodological advances in LARGE-SCALE STOCHASTIC OPTIMIZATION. Many of the problems encountered in this PhD are large-scale sequential decision making problems under uncertainty. We contribute to the development of new operations research methodologies, including models, algorithms, and theory, for solving such problems with particular focus on approximate dynamic programming techniques.

This thesis is divided into two parts. Part I introduces the problems considered in this thesis, summarizes the contributions and dissemination of the work, and provides an overview of the background material relevant for understanding models and methods in the rest of the thesis. Part II is the core part of the thesis and contains the research work that has been carried out during this PhD project.

The remainder of this chapter is organized as follows. In Section 1.1 we introduce the context of this PhD project. In Section 1.2, we present the scientific papers that are part of this thesis and describe the business problems and contributions of each paper. Conclusions are drawn in Section 1.3 and directions for further research are discussed in Section 1.4. Additional work done during the PhD study but outside the main scope of this thesis is briefly presented in Section 1.5.

1.1 Context: SAVE-E

This PhD thesis is part of a larger interdisciplinary research project called “SAVE-E Energy Savings: Closing the Energy Efficiency Gap” funded by the Danish Innovation Fund. The SAVE-E project is composed of seven work packages and the overall research examines what makes households and companies invest in energy saving solutions, with a focus on Denmark. The long term goal of this research is to reduce the CO₂ emissions and the dependency of fossil fuels as well as to contribute to meeting the ambitious Danish energy policy targets in relation to climate and energy efficiency. Each SAVE-E work package tackles specific aspects of the problem by developing qualitative and quantitative models, methods, and performing economic analyses.

Within the SAVE-E context, the objective of this PhD project is to develop advanced analytical tools for decision making problems in energy and energy saving investments that are subject to uncertainty. We approach this task by considering different case studies that are related to both industrial and consumer applications, and by covering with our research problems in energy production, operations, and investments. Some of the studies in the thesis pertain specifically to a Danish context while in some others the perspective is broader.

1.2 Papers overview and contributions

Part II of this thesis consists of six research chapters. Chapters 3 and 5–8 are based on academic journal papers while Chapter 4 is a supplement to Chapter 3. Two of these five papers are published in international peer-reviewed journals, two papers are currently under review, and one will be submitted to a journal in the next months. In this section we introduce the content of these papers, discuss the scientific contributions and give an overview of the dissemination activities.

Our first case study involves commodity and energy production assets embedding *real optionality*, that is managerial flexibility, to convert a set of inputs into a set of outputs and with the option to permanently shut down. Merchant commodity and energy producers maximize the value of the production asset by adapting this flexibility to the evolution of uncertain market factors such as prices of commodities and energy sources. However, given the gravity of a plant shutdown on the society in the form of e.g. loss of employment, adverse publicity, and political resistance from unions, companies can benefit from deviating from a pure asset value perspective. Motivated by an aluminum producer, in

Chapter 3, Managing shutdown decisions in merchant commodity and energy production: A social commerce perspective, we focus on the management of shutdown decisions in a social commerce perspective. To mitigate adverse societal impacts of a shutdown, we prefer plant’s operating policies that (i) delay shutdown decisions to later parts of the planning horizon and (ii) reduce the likelihood of shutdown within this horizon. The goal of this paper is thus finding operating policies that can delay and/or reduce the use of the plant’s shutdown option for small asset value losses.

We approach this problem by formulating a constrained Markov decision process (MDP) that maximizes the asset value with constraints on the shutdown decisions. Solving our constrained MDP is more challenging than an unconstrained MDP, which is by itself hard under realistic high-dimensional models of the market uncertainty. We therefore approximate this intractable constrained MDP using unconstrained MDPs. Our first strategy modifies the shutdown cost in a way that is consistent with anticipated regret theory in behavioural psychology. We compute operating policies from this model by extending the regress-later least squares Monte Carlo (LSML) method and also defined a deterministic version of this policy as a shortest path-based reoptimization heuristic. Our second strategy extends practice-based methods that use switching thresholds on production margins by determining better thresholds via a combination of LSML and machine learning classification. We test our methods using real data from an aluminum producer and an eight-factor stochastic model of the uncertainty calibrated on market data. We find that our methods can substantially delay and/or reduce the use of the shutdown option for small asset value losses. Thus, taking a social commerce perspective in managing a plant’s operating flexibility appears financially viable.

This work contributes to the literature on socially responsible operations and approximate dynamic programming by developing intuitive and efficient operating policies to manage the trade-off between asset value and shutdown decisions. These policies approximate in a tractable manner a complex stochastic optimization problem, that is, a high-dimensional constrained MDP. We establish properties of the anticipated regret policies and characterize the behaviour of these policies for limiting shutdown aversion preferences. Our extension of the LSML method to approximate high-dimensional shutdown-averse stochastic dynamic programs (or, in general, with an objective different than the expected asset value) is new, as it is combining LSML and machine learning techniques to learn policies. We also show that the reoptimization heuristic presents an efficient directed acyclic shortest path formulation in the class of stochastic dynamic programs with finite endogenous state and action sets. Our findings are relevant in commodity and energy production including metal smelters, refineries, power plants, and also in renewable energy production, e.g. in biogas plants, where managing shutdown decisions is important to uphold the clean asset as a source of power.

Chapter 4, Managing shutdown decisions in merchant commodity and energy production: The performance of popular strategies, provides additional material related to the analysis performed in Chapter 3. When we first approached the problem of managing shutdown decisions, we defined operating policies based on the dynamic conditional value-at-risk (CVaR). The dynamic CVaR is in fact a popular risk measure that has been successfully applied to model risk aversion in multi-stage stochastic dynamic optimization problems. In our context, however, we find that when risk aversion increases CVaR favors shutdown as oppose to discouraging such decisions. Therefore, we define an intuitive modification denoted reverse CVaR (RCVaR) and prove it to reduce the use of the shutdown option as risk aversion increases. We also formalize a financial strategy used in practice based on procuring input power using long term forward contracts. We carry out numerical experiments of CVaR-based and forward contracting strategies using LSML extensions and find that, unlike the policies in Chapter 3, both these strategies are unable to manage the shutdown option effectively. Our results suggest caution in using methods that are not tailored to manage shutdown decisions (e.g., risk measures used for cash flow risk) as they can lead to incorrect shutdown decisions.

The contributions of this chapter include the formalization and numerical study of CVaR-based policies and forward contracting policies in the context of shutdown aversion, and the establishment of some theoretical properties describing the behaviour of CVaR-based policies. Our assessment of popular strategies used in academia and industry to manage shutdown decisions can be useful for merchant energy and commodity producers and for operational researchers who apply dynamic risk measures. The work of Chapters 3–4 has been disseminated as follows:

- A journal paper co-authored with Selvaprabu Nadarajah, Stein-Erik Fleten, Denis Mazieres, and David Pisinger is under 2nd round of review in *Manufacturing & Service Operations Management* (Trivella et al., 2018);
- Presentation by Alessio Trivella at ECSO 2017, the *2nd European Conference on Stochastic Optimization*, Rome, Italy (peer-reviewed abstract, invited talk);
- Presentation by Alessio Trivella at IFORS 2017, the *21st Conference of the International Federation of OR Societies*, Quebec City, Canada (peer-reviewed abstract, invited talk);
- Presentation by Alessio Trivella at POMS 2017, the *28th Conference of the Production and Operations Management Society*, Seattle, USA (peer-reviewed

abstract, invited talk);

- Two seminar presentations by Alessio Trivella held in 2017 at the Norwegian University of Science and Technology, Trondheim, Norway, and at the Technical University of Denmark, Kgs. Lyngby, Denmark;
- Poster presentation by Alessio Trivella at the 2017 *PhD Winter School on Stochastic Programming*, Passo del Tonale, Italy;
- Presentation by Alessio Trivella at OR 2016, the *International Conference of the German OR Society*, Hamburg, Germany (initial version of this work with title: “Optimal operation and electricity sourcing in the aluminum industry”).

The rapid evolution of the energy sector and the increasing environmental concerns constantly introduce fresh optimization problems to solve. In **Chapter 5, Meeting corporate renewable power targets using a dual reoptimization scheme**, we study an emerging problem in the context of corporate energy procurement. In the last few years, many companies (for instance, half of the Fortune 500) have announced commitments to meet sustainability and climate targets, which include targets on greenhouse gas emissions reduction, energy efficiency, and renewable energy procurement. In particular, meeting a renewable energy target for a company refers to satisfying a percentage of its electricity demand by renewable energy sources by a future target date. The goal of Chapter 5 is to investigate how a company can set up a power sourcing policy to reach a renewable energy target and then sustain this target at minimum expected cost.

Constructing a multi-year procurement portfolio to meet a renewable target is challenging because of the long term planning horizon and also the number of buying options in the market. We approach this problem by considering two dominant procurement strategies used by corporations: (i) buying power from the utility (i.e. akin to a spot purchase) and supplementing it with renewable energy certificates and (ii) entering bilateral contracts known as power purchase agreements (PPAs) to buy power directly from a renewable generator for a predefined number of years. We formulate a multi-period stochastic dynamic program (SDP) to minimize the expected procurement cost, where the company has flexibility to enter at each stage into new contracts of varying size and length. Computing an optimal policy of this SDP is intractable because its state space has high-dimensional endogenous and exogenous components, and approximate dynamic programming techniques to approach this problem are very limited. To overcome this intractability, we consider the information relaxation approach typically used to obtain dual bounds, and develop a novel dual average reoptimization heuristic (DRH) that ex-

tracts a non-anticipative procurement policy by looking at sample action distributions obtained from solving information relaxation models over a set of Monte Carlo sample paths of the uncertainty. We find that our approach outperforms commonly used primal reoptimization methods and simple heuristics on realistic instances.

This work contributes to the corporate energy procurement literature by developing procurement policies for a relevant problem faced by an increasing number of companies worldwide. Our DRH method contributes to the approximate dynamic programming literature by providing tractable non-anticipative decision rules for high-dimensional SDPs. In fact, computing the DRH decision rules only requires (i) simulating the evolution of the uncertainty in Monte Carlo, and (ii) solving a deterministic dual mathematical program along individual sample paths. This method thus emerges as a promising approach for tackling high-dimensional stochastic dynamic programs. The work has been disseminated as follows:

- A journal paper co-authored with Danial Mohseni-Taheri and Selvaprabu Nadarajah is being written and will be submitted in the next few months to a journal among *Management Science*, *Operations Research*, and *Manufacturing & Service Operations Management* (Mohseni-Taheri et al., 2018);
- Presentation by Alessio Trivella at CMS 2018, the *15th Conference on Computational Management Science*, Trondheim, Norway (peer-reviewed abstract);
- Presentations planned by Alessio Trivella at EURO 2018, the *29th European Conference on Operational Research*, Valencia, Spain (accepted peer-reviewed abstract, invited talk);
- A peer-reviewed extended abstract is accepted for MSOM 2018, the *Manufacturing and Service Operations Management Conference*, Dallas, USA;
- Seminar presentation by Alessio Trivella held in 2018 at the Technical University of Denmark, Kgs. Lyngby, Denmark.

Next, we consider an optimization problem that is related to the participation of power producers to electricity markets. Electricity markets around the world have different structures but typically are composed by at least a day-ahead market and a balancing (real-time) market. The market participants, e.g., conventional and renewable producers, place a market bid at the day-ahead market for the delivery of electricity during the next day. In determining their day-ahead bid, market participants account for the uncertainty

in power prices and renewable energy production during the next-day delivery period, which are not known when the offer is made. Thus, bidding in electricity markets generally arises as a stochastic optimization problem. The increasing share of renewable (stochastic) energy sources poses severe challenges to the security and efficiency of energy systems and markets, consequently, electricity market offering strategies constitute an active area of research for stochastic optimization experts.

In **Chapter 6, Enabling active/passive electricity trading in dual-price balancing markets**, we propose a novel electricity market framework that differs from the current market structure in the way the balancing market participation is thought. Currently, controllable production units participate in the balancing market as “active” actors that offer regulating energy to the system, whereas renewable units are “passive” actors that create imbalances and hence receive less competitive prices. Motivated by the need of additional flexibility in the system, our proposed market framework enables instead participants in the balancing market to be active in some trading intervals and passive in some others. However, the two participation modes are enforced to be complementary and agents submitting regulating energy offers for a given trading interval are prevented from creating imbalances in that interval. To evaluate our “active/passive” balancing market, we consider the case of a virtual power plant (VPP) trading in a two-settlement electricity market composed of a day-ahead and a dual-price balancing market (e.g., the market setup in place in Denmark). A VPP is a cluster of generating units (both controllable and renewable) and storage systems which acts as a single actor in the market and is indeed a natural market agent who would benefit from our market framework. We formulate the optimal active/passive VPP bidding strategy as a three-stage stochastic program, and show with numerical experiments using 300 scenarios that the VPP expected revenue can increase significantly compared to an as-is market status.

This work contributes to the literature on energy markets and offering strategies by proposing a new balancing market participation model. This market model is relevant because (i) the system operator would benefit from more flexible regulating energy to schedule in real-time, and (ii) it can be seen as a lever to facilitate the integration of renewable power in the system through the aggregation into VPPs. Another contribution of this work is the definition of the VPP active/passive offering model as a multi-stage stochastic program. The work has been disseminated as follows:

- A journal paper co-authored with Nicolò Mazzi and Juan Miguel Morales has been submitted to *IEEE Transactions on Power Systems* (Mazzi et al., 2018);

- Two seminar presentations by Alessio Trivella held in 2017 at the Norwegian University of Science and Technology, Trondheim, Norway, and at the Technical University of Denmark, Kgs. Lyngby, Denmark.

In Chapter 7, Modelling of electricity savings in the Danish households sector: From the energy system to the end-user, the focus moves from companies to energy systems and consumers. In particular, in this chapter we are interested in examining the value of investing in energy efficient (i.e. high-labelled) household electric appliances. Purchasing energy efficient appliances implies higher upfront costs compared to conventional, less efficient ones, but also lower electricity consumption and consequently economic savings over the appliance lifetime. Would these savings be sufficient to recoup the higher investment cost? The answer to this question depends on many factors including the appliance investment cost, lifetime, and the amount of savings that is not known exactly due to uncertain future power prices. Our research question is to quantitatively investigate this trade-off with the goal to identify the most cost-effective investments. To perform such analysis, we consider a set of appliance categories constituting the majority of the electricity consumption in the Danish private household sector (e.g., refrigerator, freezer, dishwasher, washing machine, dryer), and collect data from multiple sources and vendors to determine average characteristics for representative energy label classes. Using this data, we take the perspective of two different and complementary entities: the society and the individual end-user.

We start considering the societal optimal investments by using an energy system model. Energy system models are large-scale mathematical models developed to examine the functioning of a whole energy system with a national or international perspective. Examples of such models include the well-known TIMES (Loulou et al., 2016), EnergyPLAN (Lund, 2015), and Balmorel (Ravn et al., 2001; see also Wiese et al., 2018 for a recent review of studies performed with Balmorel). In particular, in Chapter 7 we use Balmorel, which is a partial equilibrium model for the heat and power sectors implemented as a linear programming optimization model, and extend it to endogenously determine the best investments in energy efficient home appliances. Specifically, a decision to invest is undertaken if the improved energy efficiency can compete with the cost of electricity supply from existing or new power plants under different fuel price and availability scenarios. Next, given that optimal energy system choices do not match with the actual end-user choices due to different reference electricity prices, we develop a consumer investment model and link it with Balmorel to compare the two optimal investment solutions.

This work contributes to the energy efficiency investments literature in several directions.

First, we extend the energy system model Balmorel by including the option to invest in a variety of major energy efficient appliances. Our extension is coded in the form of an add-on so that the investment option can be easily activated in Balmorel by the user. Second, we develop a method to relate the optimal system and consumer investment decisions by soft-linking the energy system model with a consumer behaviour model designed for the study. We provide insights on the trade-off between investment cost and energy saving, quantify average economic and electricity savings on both energy system and consumer side, and describe the evolution of the energy system under optimal investment decisions. Our findings are related to the Danish case, however, the methodology that we have introduced could be employed to different countries or geographical regions if the data needed to model the energy system as well as the appliance investments (e.g., investment costs and electricity consumption for low and high energy labels) is available. Our results can be relevant for researchers on energy systems and for policy makers. The work has been disseminated as follows:

- A journal paper co-authored with Mattia Baldini has been published in *Energy Efficiency* (Baldini and Trivella, 2017);
- Presentation by Alessio Trivella at BEHAVE 2016, the *4th European Conference on Behaviour and Energy Efficiency*, Coimbra, Portugal (peer-reviewed extended abstract; Trivella and Baldini, 2016);
- Seminar presentation by Alessio Trivella held in 2016 at the Technical University of Denmark, Kgs. Lyngby, Denmark.

The consumer investment model developed in Chapter 7 is elementary due to the primary focus of the study on (i) the energy system, and (ii) the link between energy system and consumer's investments. Under this consumer model, investment choices are rational decisions based on an investment cost recoup criterion (that is, a positive investment net present value), and they are affected by a generic behavioural uncertainty that is related to the income level. In reality, though, many more socioeconomic and behavioural factors may be argued to be connected with the consumer choices.

We address this gap by deepening the consumer behavioural side in relation to energy efficiency investments in **Chapter 8, The impact of socioeconomic and behavioural factors for purchasing energy efficient household appliances: A case study for Denmark**. The purpose of Chapter 8 is to understand which characteristics lead Danish consumers to choose energy efficient household electric appliances at the moment

of purchase. To perform this analysis, we employ empirical data from a survey conducted by the Danish Energy Agency over a representative set of Danish households, and extract a number of socioeconomic, demographic, and behavioural variables. Based on this data, we develop and calibrate a statistically sound logistic regression model to calculate consumer purchase propensities of choosing an energy efficient investment. We compute propensity curves to decouple the effect of different variables, and provide a visual representation of the most relevant factors. Eventually, based on the outcome of the analysis, we draw suggestions for improved energy efficiency policies and information campaigns by targeting key demographics.

This paper presents some novel methodology and its findings have practical relevance. On the methodological side, the contributions of the paper consist on (i) the construction of an energy efficiency index (EE-index) that gathers and synthesizes a rich set of consumer behavioural characteristics and daily actions in relations to energy end-use and energy savings, and (ii) the integration of such index in a consumer choice model to study the joint effect of socioeconomic, demographic, and behavioural variables on consumer investment in EE appliances. Moreover, unlike previous studies, (iii) we perform an extensive investigation of a behavioural index by determining correlation matrices and examining interrelations between its constituent parts. On the practical side, we find from our statistical results that socioeconomic and behavioural characteristics are highly significant when explaining the choice of purchasing EE appliances. Specifically, income, housing type, quantity of inhabitants, age, and end-use behaviour are predictors for choosing energy efficient appliances, with EE-index and housing type being the strongest of these predictors while income is weaker. By providing empirical results on the influence of socioeconomic and behavioural characteristics on the consumer's choice, the paper narrows the knowledge gap on household energy consumption behaviour and the drivers of high-labelled household appliances purchasing. The outcome of work is relevant for policy makers and energy actors who are in fact increasingly interested in the determining factors behind the consumer choice of conventional versus high efficiency labeled appliances. This work has been disseminated as follows:

- A journal paper co-authored with Mattia Baldini and Jordan Wentz has been published in *Energy Policy* (Baldini et al., 2018);
- A conference paper co-authored with Mattia Baldini and Jordan Wentz describing an earlier version of this work was published in the proceeding of EEDAL 2017, the *9th International Conference on Energy Efficiency in Domestic Appliances and Lighting*, Irvine, USA (Baldini et al., 2017b);

- A peer-reviewed extended abstract was published in IAEE 2017, the *40th Conference of the International Association for Energy Economics*, Singapore. This abstract was selected as one of the conference top 5% contributions and appeared in the *IAEE Energy Forum* (Baldini et al., 2017a).

1.3 Conclusions

The constantly evolving energy sector is a rich playground for operational researcher since it gives rise to a variety of important and complex optimization problems under uncertainty to solve. As part of the SAVE-E project, in this thesis we have studied some emerging problems in the field of energy sustainability, in particular, related to energy efficiency investments, renewable energy operations and investments, and corporate social and environmental responsibility. To tackle these problems, we have leveraged tools from operations research to design novel methodology and perform energy analysis. A special focus has been given to extending or developing approximate dynamic programming techniques in order to solve large-scale stochastic dynamic optimization problems. These models include high-dimensional constrained MDPs or MDPs with endogenous and exogenous state components which are both high dimensional, for which methods available in the literature are limited.

Overall, the results and insights from this thesis can be relevant for a number of different actors including

1. Companies: we provide managerial insights and define well-performing operating and investment policies in contexts like managing a production asset shutdown decisions and meeting a corporate renewable energy target;
2. Policy makers: our results suggest energy policies to increase consumer awareness towards energy efficiency and savings, for instance, or give ideas on how to design more flexible electricity markets;
3. Operational researchers: the new approximate dynamic programming methods we have developed have potential broader relevance in other application contexts.

Through the SAVE-E project, this thesis develops decision support tools for energy saving and renewable energy investments from the perspective of both companies and consumers, with the long term goal of reducing greenhouse gas emissions.

1.4 Further work

Within the scope of the SAVE-E project, it would be worth examining other industrial case studies than the ones considered in this thesis. For example, many companies in different industrial sectors have the possibility to invest in various energy saving options but deciding which investment to undertake can be hard. Consider the case of a liner shipping company. Due to the increasingly stricter carbon emission regulations in the shipping industry (IMO, 2018), liner shipping companies often have to undertake large investments in energy saving technologies for vessels. However, many investment options are available such as waste heat recovery systems, better coating of vessels, more efficient engines, better propulsion, LNG and bio-fuels upgrades, or investing in new vessels. Assessing the profitability, risk, and correct timing of such investments is challenging due to long term uncertainty in bunker price, cost of emissions, and regulations. Therefore, a decision support tool for optimizing an energy saving investment portfolio would be useful in this context.

Regarding the chapters of this thesis, some of the models and methods could be extended to handle additional features and/or could be applicable to different contexts. Below we indicate some directions for further research in connection with our papers.

- Chapters 3–4. The models in these chapters could be applied to other contexts that involve a non-reversible decision under uncertainty, also beyond energy. One of such problems could be renting production facilities versus buying production facilities. Buying a production facilities is a non-reversible decision but it may be a good strategy to rent the facilities until a given maturity of a production has been reached. Another problem is modeling the life cycle of new products on the market. Every new product has a product life cycle where it is first introduced, then the market grows, it reaches a mature level, and declines. The model could be used to decide when a portfolio of different products should be terminated.
- Chapter 5. We are currently working on the following two extensions of this chapter that will be incorporated in a future version of the paper:
 - Establishing analytical results and managerial insights that can be useful for companies using stylized models. We have performed an initial analysis on simplified two- and three-period stochastic models to minimize the expected cost of power under a renewable target with one or two PPA options available. We obtained some preliminary analytical insights that shed light on the relation between the following three key elements of the problem: (i) the optimal procurement cost, (ii) the level of the renewable target, and (iii) the PPA

price. These insights can help companies in structuring an electricity portfolio, negotiating a PPA contract, and quantifying the implications of contract price and renewable target choice on the long-term procurement costs. More work has to be done to finalize this analytical study which would so complement the numerical study focus of current version of the paper.

- Improving the linear dual penalties used in the information relaxation dual optimization. In the paper, we make use of simple linear dual penalties where the weight coefficients are chosen manually based on experimentation. This penalty seems to work well in practice, however, determining such weights in a more principled manner could improve the performance of both the dual bound and our dual reoptimization method. To this end, we are investigating a two-step process where simple penalties are used first to obtain a policy, but then better weights are learned by using a linear regression over the values of this policy.
- Chapters 6. The VPP offering model presented in this chapter could be enriched to take into account more operating features of the conventional power generators. Moreover, the model currently handles well up to 300 scenarios. Further research could be done in developing efficient algorithms (e.g. scenario decomposition techniques) to solve the model for a larger number of scenarios.
- Chapters 7–8. In Chapter 7 we integrate an energy system model with a simple consumer model while in Chapter 8 we develop a more sophisticated consumer behaviour model. An obvious extension of these papers would be to integrate the energy system model in Chapter 7 with the more advanced consumer model of Chapter 8, and examine the effect on system and consumer investment choices.

1.5 Work not included in the thesis

Additional research has been carried out to model energy saving in transportation and logistics operations. In particular, we use combinatorial optimization techniques to repack the cargo of a fleet of vehicles in a more efficient and balanced manner so that fuel consumption can be reduced. The work is based on modelling and solving complex bin-packing problems and has not been included in this thesis to keep it more focused on energy applications. However, in the following we provide a minimal description of its content and contributions.

The bin-packing problem (BPP) is one of the most applicable models in combinatorial

optimization, and consists in packing a given set of items of different size into the minimum number of identical bins. The multi-dimensional version of the BPP involves two- or three-dimensional, typically rectangular-shaped items and is notoriously hard to solve in practice, with mixed-integer programming solvers frequently failing to solve even small instances of 30 items. In this work, we extend the classical multi-dimensional BPP by integrating constraints arising from real-world transport applications, such as load balancing and stability of the packing. In particular, our goal is to find the packing requiring the minimum number of bins while ensuring that the average center of mass of the loaded bins falls as close as possible to an idea location inside the bin. The idea is that packing and balancing the load of a set of items represent two conflicting objectives and it can be convenient to incorporate them in a single model. This joint problem is relevant in a number of transportation and logistics applications. For example, loading a truck or an aircraft cargo in a way that the center of mass remains low and central in the cargo area is important to make the travel safer and more fuel-efficient. However, this problem has been scarcely studied in the literature and only in simplified versions, e.g., that consider a single bin.

This work contributes to the combinatorial optimization and BPP literature. To solve the joint packing and balancing problem exactly, we developed new mixed-integer linear formulations addressing both the case where we want to optimally balance a set of items already assigned to a single bin, and the general load-balanced bin-packing problem. These formulations are considerably more complex to derive than the standard BPP as several new sets of support continuous and binary variables as well as conditional constraints are needed to model the load balancing. Using this model, we were able to solve to optimality instances up to 20 items. To deal with larger instances, we designed a novel heuristic based on a multi-level local search concept. A starting bin-packing solution is found using a greedy randomized constructive heuristic. Our algorithm then takes advantage of a particular interval graph-based representation of a feasible packing, and iteratively improves the load balancing of a bin-packing solution using three nested search levels. The first level explores the space of transitive orientations of the complement graphs associated with the packing. This is done by exploiting some theoretical properties of the interval graphs and a map between transitive orientations and feasible packings. The second level modifies the structure of the interval graphs. The third level exchanges items between bins by repacking proper n -tuples of weakly balanced bins, and is coded in the form of a variable depth neighborhood search framework. We also derived a combinatorial lower bound for the optimal objective value of the joint problem that enabled us to assess an optimality gap of the heuristic solutions. Computational experiments on instances up to 200 items showed that our new approach can provide near optimal solutions, that is, a very effective load-balancing, in seconds or minutes. The dissemination activities related

to these BPP topics include:

- A journal paper co-authored with David Pisinger has been published in *Computers & Operations Research* (Trivella and Pisinger, 2016);
- Presentation by Alessio Trivella at EURO 2016, the *28th European Conference on Operational Research*, Poznan, Poland (peer-reviewed abstract);
- Presentation by Alessio Trivella at INFORMS TSL 2017, the *1st Conference of the INFORMS Transportation and Logistics Society*, Chicago, USA (peer-reviewed extended abstract; Trivella and Pisinger, 2017);
- A master thesis at DTU has been co-advised with David Pisinger in 2016, with title: “A new column generation-based heuristic for the 3D bin-packing problem”.

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Methodological background

Making decisions in the presence of uncertainty is part of everyday life and spans practically any business area including healthcare, manufacturing, communications, transportation, finance, and energy. Optimization under uncertainty, or stochastic optimization, refers to a collection of quantitative methods that help us making better decisions in the presence of uncertainty about the future or about our data. While deterministic optimization is handled using a rather universal mathematical programming framework, stochastic optimization encompasses different modeling techniques and solution approaches including, for example, stochastic programming, robust optimization, optimal control, optimal stopping, Markov decision processes, and approximate dynamic programming. Each of these sub-fields has been developed by a different community and has its own theoretical background, style, and language. Powell (2018) describes this mix of styles and approaches as “the jungle of stochastic optimization” and proposes a unified framework for modeling stochastic optimization problems. Typically, stochastic optimization entails additional challenges compared to deterministic optimization such as calibrating a stochastic model for the uncertainty or searching over policies (i.e., collections of functions) instead of scalars or vectors.

Energy operations and investments usually require making decisions in the presence of uncertainty that stems from unpredictability, for instance, of energy prices and demand. Furthermore, these optimization problems are often large scale, which poses additional challenges to their resolution. In this thesis we indeed faced several large-scale stochastic optimization problems and some of which were hard to approach by existing techniques requiring the development of new methodology (e.g. the problems in Chapters 3–5). In

Figure 2.1, we show the stochastic optimization methods applied in some chapters of the thesis (3, 4, 5, 6). In this chapter, we present some of these methods keeping our focus

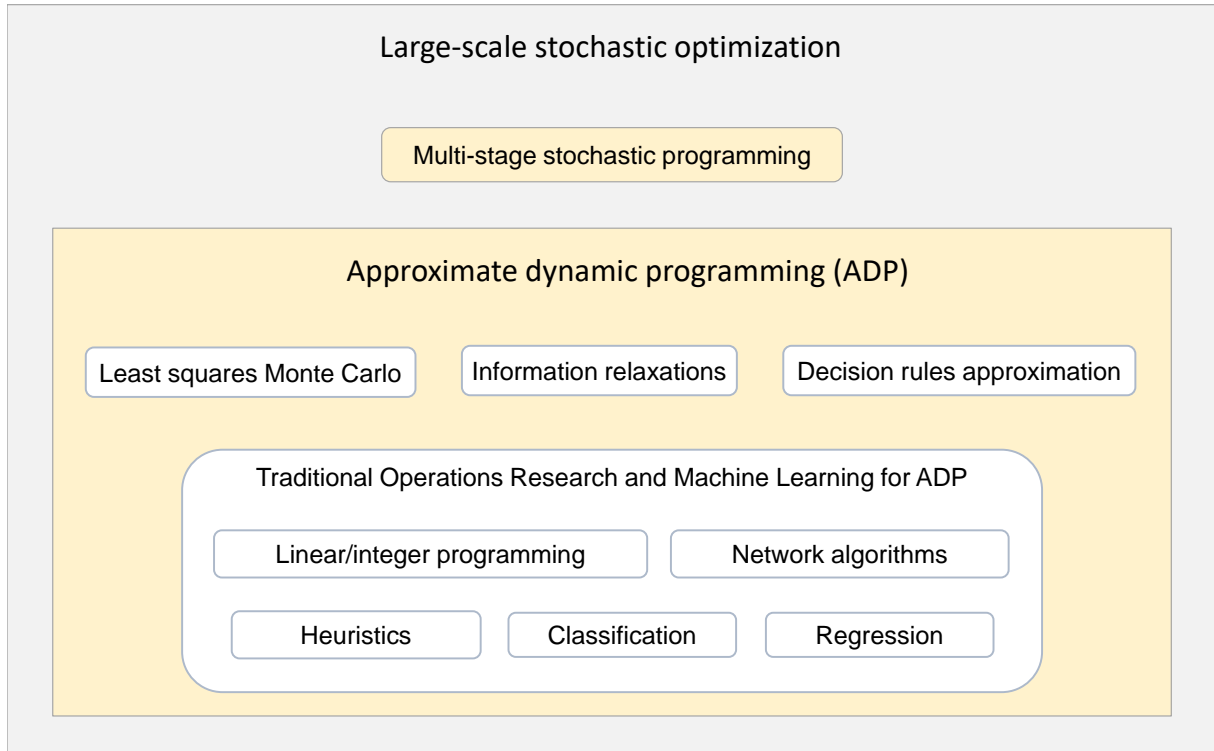


Figure 2.1: Stochastic optimization methods used in the thesis.

on energy applications. We start in Section 2.1 by discussing the types of uncertainty arising in these problems and how to model the uncertainty. We introduce stochastic programming in Section 2.2. We present the stochastic dynamic programming framework and some approximate dynamic programming techniques in Section 2.3. In the context of approximate dynamic programming, as shown in Figure 2.1, we also applied in this thesis several traditional operations research (OR) and machine learning (ML) techniques that we assume a reader with basic knowledge in OR/ML is familiar with. Thus, we do not describe such methods here but refer to the textbooks of Hillier and Lieberman (2010) and Bishop (2006).

2.1 Uncertainty modeling in energy

Modeling the uncertainty is a key element in any stochastic optimization problem as it affects the reliability of the model and its solution. In Section 2.1.1, we propose a categorization of the different types of uncertainty when dealing with energy operation and investment problems. In Section 2.1.2, we introduce some of the common techniques

used to model the uncertainty. In Section 2.1.3, we show that modeling the uncertainty might be challenging by presenting an example based on one of the papers in the thesis.

2.1.1 Where is the uncertainty?

Numerous sources of uncertainty affect the decision making processes arising in energy operations and investments, and identifying the relevant sources of uncertainty in a specific problem is the first step in developing a stochastic optimization model. In our research, we faced different types of uncertainty which we categorize, together with other types that we identified, into six groups displayed in Figure 2.2. This categorization does not attempt to be comprehensive but it does cover all sources of randomness that show up in our papers as well as most of the ones considered in the energy optimization literature. Note that many of these categories are linked and some uncertainty could easily be represented by more categories. For instance, renewable energy certificates (RECs) prices could fall under market uncertainty as RECs are tradable commodities but also under regulatory uncertainty because their procurement is often required from energy regulation. Below we summarize these categories and relate them to our research.

1. *Market uncertainty*: Electricity price uncertainty is important in all the chapters and includes both short-term price uncertainty, that is, day-ahead and balancing market prices [ch. 6], and uncertainty in long-term electricity prices or futures contracts [ch. 3, 4, 5, 7, 8]. Moreover, we consider uncertainty in the price of other energy-related commodities and fuels [ch. 3, 4, 7] and exchange rates [ch. 3, 4].
2. *Weather uncertainty*: This uncertainty is particularly relevant when dealing with electricity production from renewable energy sources. For example, forecasting and modeling the uncertainty in wind and solar power production is crucial for wind and solar power generators participating in an electricity market [ch. 6].
3. *Energy supply/demand uncertainty*: Uncertainty in power supply and demand is common when analyzing energy systems [ch. 7] and energy markets. We consider electricity consumption uncertainty also at a company level [ch. 5].
4. *Regulatory uncertainty*: By regulatory uncertainty we refer not only to uncertainty in the energy regulation itself, for example, in the type of support scheme granted to renewable generators and its level, but also to uncertainty that is induced by energy regulations such as the trading of RECs or CO₂ allowances. We consider uncertainty in the RECs prices [ch. 5], CO₂ prices [ch. 7] and discuss uncertainty in energy efficiency subsidies [ch. 8].

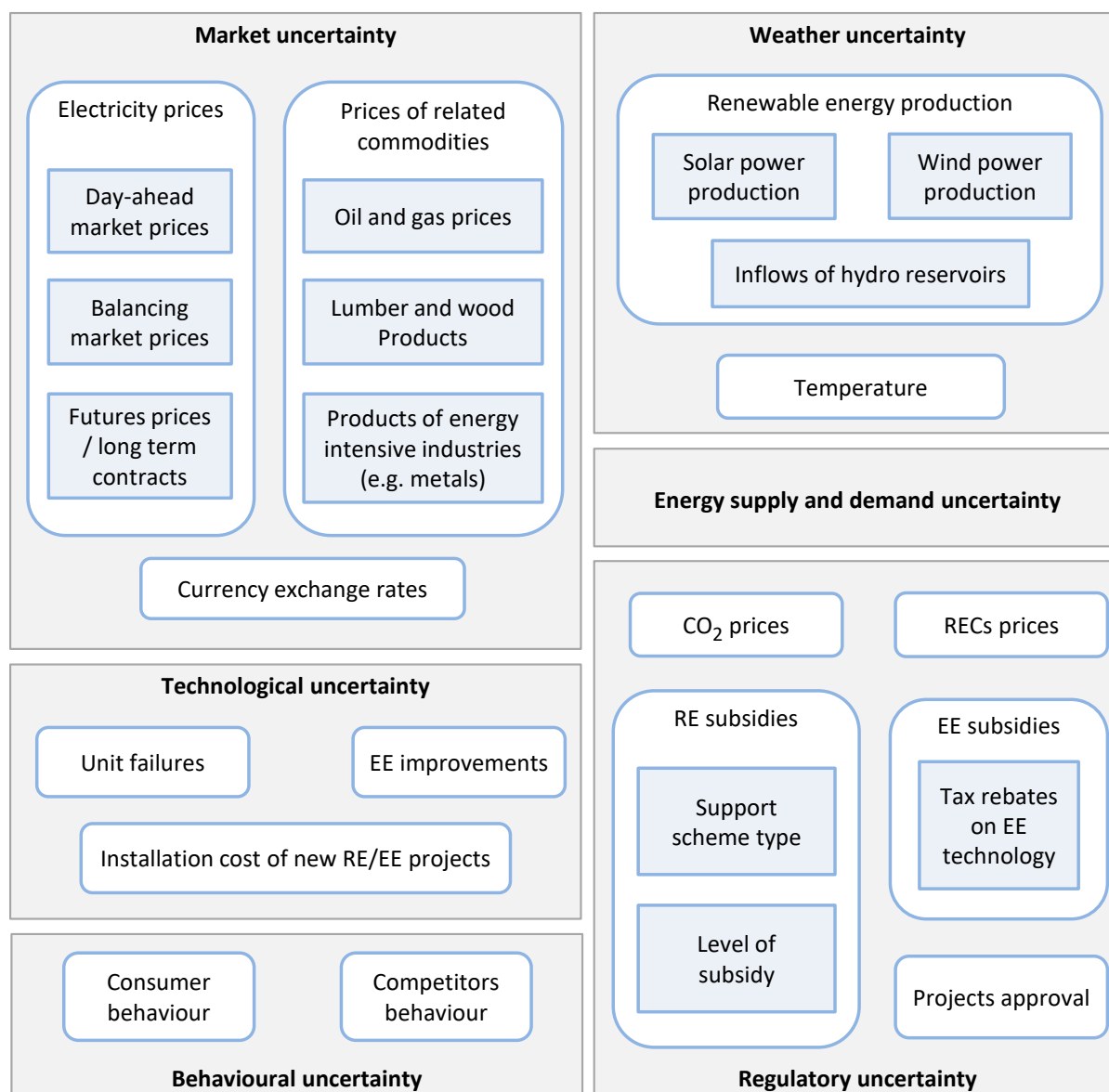


Figure 2.2: Uncertainty in energy operations and investments.

5. *Behavioural uncertainty*: This uncertainty is associated with the stochastic nature of human behaviour in relation, for instance, to the energy end-use or to consumer investments in energy efficient household electric appliances [ch. 7, 8]. We also include in this category uncertainty in the strategy adopted by competitors, which is common in problems such as the trading of power in electricity markets.
6. *Technological uncertainty*: Technological uncertainty is associated, for instance, to the development of new energy production technologies or to the extent of energy efficiency advances in industrial processes and household appliances. We model the future investment cost of a renewable energy project [ch. 5] but do not directly

account for uncertainty in this cost.

2.1.2 How to represent the uncertainty

As shown, randomness of different nature arises in energy applications, and this randomness can affect the parameters of the objective function and/or constraints of an optimization model. It is often assumed that the uncertain model parameters follow a probability distribution that is known or can be estimated. A variety of probability distributions are used in the energy stochastic optimization literature, from the familiar Gaussian noise to heavy-tail distributions (e.g., to model electricity spot prices) and rare events (e.g., in the case of production unit failures). In several of the problems that we consider, decisions are taken sequentially over time and the uncertainty thus evolves over time in the form of a time series process or stochastic process. Simple single-factor stochastic processes include the popular geometric Brownian motion that is commonly used to model the evolution of commodity market prices, and the Ornstein-Uhlenbeck process. More complex processes are multi-factor models used to describe uncertainty driven by multiple stochastic factors, which is common to represent the term structure of commodity prices, for instance. Depending on the problem and the type of uncertainty, in our research we make use of different stochastic processes including an eight-factor and multi-commodity stochastic process in Chapter 3. We do not discuss in detail probability distributions, time series processes and stochastic processes but refer to Madsen (2007) and Pinsky and Karlin (2010) for a more comprehensive introduction.

Modeling the uncertainty and representing it in a way that is suitable for an optimization model usually involves the following steps:

1. Selecting a model for the uncertainty, e.g., a stochastic process;
2. Calibrating the parameters of the model using real data, for example, historical or market data;
3. Generating scenarios of the uncertainty from the calibrated stochastic process.

To elaborate on the latter item, depending on the structure of the underlying decision making problem and our solution approach, the evolution of the stochastic process over time can be approximated by a discrete-state scenario process using different techniques. In Figure 2.3, we illustrate a four-period horizon ($i = 0, \dots, 3$) with uncertainty represented in the form of a scenario tree 2.3(a) and a scenario fan 2.3(b).

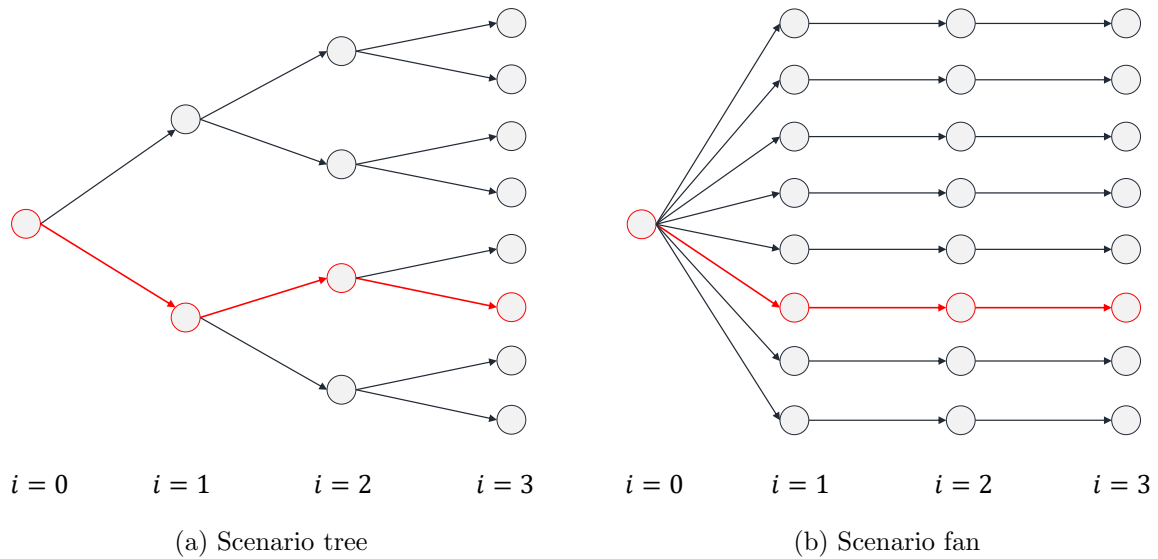


Figure 2.3: Uncertainty representation using a scenario tree (a) and a scenario fan (b)

In both the scenario tree and the scenario fan, a node w_i represents a stochastic process outcome at stage i . In the scenario tree, given w_i , the distribution of the next-period uncertainty w_{i+1} is characterized by finitely many possible outcomes defined by the branches exiting w_i and probabilities associated with each branch. A scenario is then a path from the root ($i=0$) to a leaf ($i=3$ in the figure) as the one marked in red in Figure 2.3(a). This representation is common in stochastic programming (see Section 2.2), and we use it in Chapter 6 to model a three-stage stochastic program. The number of nodes and scenarios in the tree, however, grows exponentially with the number of time periods, making scenario trees hard to handle in problems with many decision stages. In contrast, a scenario fan consists of a set of sample paths of the uncertainty typically generated in Monte Carlo simulation. This representation is rather common in high-dimensional multi-stage problems with many time periods or sources of uncertainty, solved with approximate dynamic programming (see Section 2.3). We use this second representation in Chapters 3, 4, and 5. Alternatively, the stochastic process could also be approximated with a scenario lattice (Löhndorf et al., 2013) but we do not use this structure in the thesis.

Several scenario reduction frameworks have been developed to reduce the size of a multi-stage problem (see, e.g., Heitsch and Römisch, 2009 and references therein). Given an input set of scenarios, these methods typically select the subset that best represents the original scenario set under some metric, with an iterative or recursive procedure. We use a similar scenario reduction technique in Chapter 6.

2.1.3 Challenges in uncertainty modeling: An example

Modeling the uncertainty in a principled manner is in general a complex procedure that requires multiple steps. These steps consist of selecting an appropriate statistical model/stochastic process to describe the evolution of the uncertain factors, collecting and processing historical data, and calibrating the parameters of the model using this data. Each step has its own challenges. Typical challenges include dealing with seasonality in the data, dealing with a long time horizon for which market contracts are not traded, and obtaining a statistically sound model calibration.

In this section, we illustrate an example of such challenges that we encountered in one of our applications and which involves the modeling of power forward contract prices. Specifically, in Chapter 3 we define a multi-commodity stochastic process to capture the joint evolution of power prices as well as prices of other commodities including e.g. aluminum (see Section 3.5.2 for details). To calibrate this model, we collected forward contract data with multiple maturities for these commodities but realized that they were not directly comparable across commodities. Indeed, power contracts deliver electricity continuously during an interval of time, and the delivery period of contracts of different lengths can overlap. For a sample trading date, Figure 2.4 shows Nord Pool power forward contracts with monthly, quarterly, and yearly delivery extending out to 6 months, 8 quarters, and 3 years, respectively. In contrast, forward contracts of the other considered commodities delivers only at maturity, that is, at a fixed point in time.

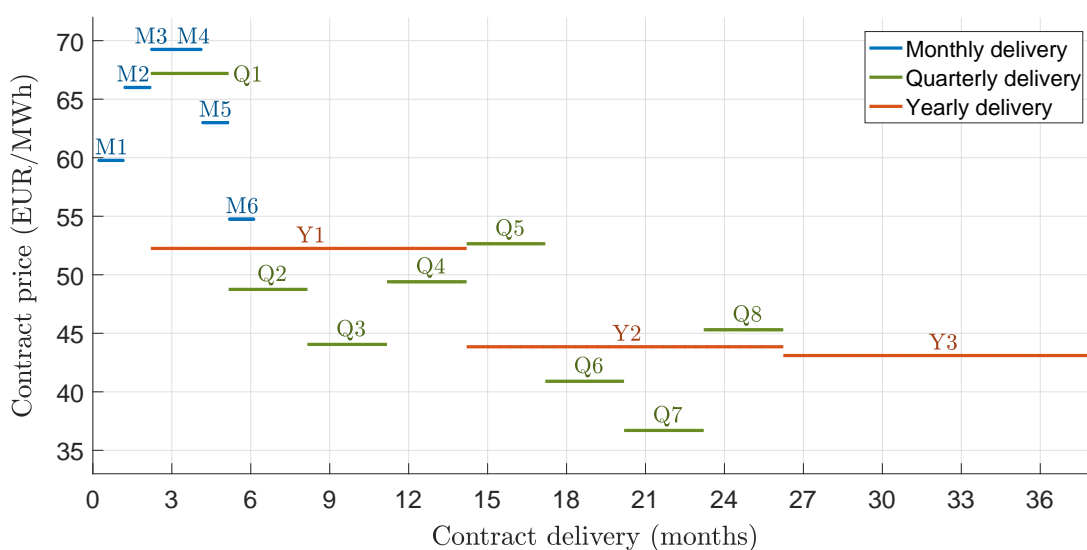


Figure 2.4: Nord Pool power forward contracts traded on October 26th, 2006 with delivery of power during monthly, quarterly, and yearly periods.

Inconsistent contract data across commodities would prevent us from calibrating a multi-commodity stochastic process. To resolve this issue, we employed a *smoothing* technique to obtain an implied power forward curve that models settlements at fixed time and replicates the market average-based forward contracts. This synthetic curve contains power forward contract prices that are consistent with the ones for the other considered commodities. Specifically, we used the approach of Benth et al. (2007) that we do not discuss in the paper for brevity but illustrate instead here.

Suppose at time 0 (today) we observe M average-based forward contracts, where each contract m has price P_m and delivers power in the interval $[T_m^s, T_m^e] \subset \mathbb{R}_+$. These intervals can overlap as shown in Figure 2.4. Based on these contracts, our goal is to determine a smooth forward curve $F(0, t) = \epsilon(t) + s(t)$ with delivery at a fixed time $t \geq 0$, where $s(t)$ is a given trigonometric function that captures annual seasonality in the power forward curve, and $\epsilon(t)$ is a fourth-degree smooth polynomial spline that models deviations from this seasonality (the fourth-degree property is needed to ensure a smoothness property; see Benth et al., 2007 for details). Let $\{t_0, t_1, \dots, t_n\}$ be the list of sorted dates constructed by ordering the intervals boundary dates $\{T_m^s, T_m^e, m = 1, \dots, M\}$ and removing duplicates. The spline $\epsilon(t)$ is defined for each interval $t \in [t_{j-1}, t_j]$, for $j = 1, \dots, n$ as

$$\epsilon(t) = a_j t_j^4 + b_j t_j^3 + c_j t_j^2 + d_j t_j + e_j.$$

Considering the full domain $[t_0, t_n]$, the spline thus contains $5n$ parameters that we denote by $x^\top = [a_1, b_1, c_1, d_1, e_1, \dots, a_n, b_n, c_n, d_n, e_n]$. We call the parametrized spline $\epsilon(t; x)$ and determine x by imposing the following set of conditions.

- *Arbitrage-free.* We require that the synthetic forward curve replicates each average-based contract m using an arbitrage-free pricing argument. In other words, we require the average-based contract price P_m to match the average spot price over the contract delivery interval $[T_m^s, T_m^e]$, which means

$$P_m = \int_{T_m^s}^{T_m^e} \mathbb{E}_0[S(t)] dt = \int_{T_m^s}^{T_m^e} F(0, t) dt = \int_{T_m^s}^{T_m^e} [\epsilon(t; x) + s(t)] dt, \quad (2.1)$$

where $S(\cdot)$ denotes the power spot price and \mathbb{E}_0 is the risk-neutral time 0 expectation (for clarity, condition (2.1) was simplified here and the effect of the risk-free interest rate is not considered). Equation (2.1) is linear in x .

- *Connectivity.* We want the spline to be continuous and smoothly connected at junction points t_1, \dots, t_{n-1} . Specifically, we impose $\epsilon(t; x)$, $\epsilon'(t; x)$, and $\epsilon''(t; x)$ to

match in adjacent spline components:

$$\begin{cases} (a_{j+1} - a_j)t_j^4 + (b_{j+1} - b_j)t_j^3 + (c_{j+1} - c_j)t_j^2 + (d_{j+1} - d_j)t_j + e_{j+1} - e_j = 0 \\ 4(a_{j+1} - a_j)t_j^3 + 3(b_{j+1} - b_j)t_j^2 + 2(c_{j+1} - c_j)t_j + d_{j+1} - d_j = 0 \\ 12(a_{j+1} - a_j)t_j^2 + 6(b_{j+1} - b_j)t_j + 2(c_{j+1} - c_j) = 0, \end{cases}$$

for $j = 1, \dots, n - 1$. We also require the curve to end flat: $\epsilon'(t_n) = 0$.

- *Smoothness.* We want the spline to be as smooth as possible, where smoothness is defined as mean square value of the second derivative. In other words, we solve

$$\min_x \int_{t_0}^{t_n} [\epsilon''(t; x)]^2 dt = \min_x x^\top H x, \quad (2.2)$$

where H is a $5n \times 5n$ matrix (see Benth et al., 2007 for its full expression).

The problem is thus a quadratic program with objective given by (2.2) and linear equality constraints $Ax = b$ given by the arbitrage-free and connectivity conditions. It is possible to reformulate this problem as an unconstrained quadratic program using the Lagrange Multiplier method as

$$\min_x x^\top H x + \lambda(Ax - b),$$

and solve it as a system of linear equations

$$\begin{bmatrix} 2H & A^\top \\ A & 0 \end{bmatrix} \begin{bmatrix} x \\ \lambda \end{bmatrix} = \begin{bmatrix} 0 \\ b \end{bmatrix}. \quad (2.3)$$

The solution of (2.3) contains the parameter vector x which defines the spline $\epsilon(t)$. For the same trading date used in the example of Figure 2.4, we display the synthetic forward curve $F(0, t)$ resulting after the described smoothing approach in Figure 2.5.

The forward curve (red curve) is smooth and captures the clear annual season price pattern. Moreover, the curve graphically satisfies the arbitrage-free integral condition (2.1) as it is never entirely above nor entirely below each of the traded contracts (blue segments). From this curve, we can then extract maturities (black dots) matching those traded for the other commodities. Finally, to obtain a complete and consistent cross-commodity dataset, we repeat this smoothing technique to each trading date in our dataset. Only at this point we are able to calibrate the stochastic process.

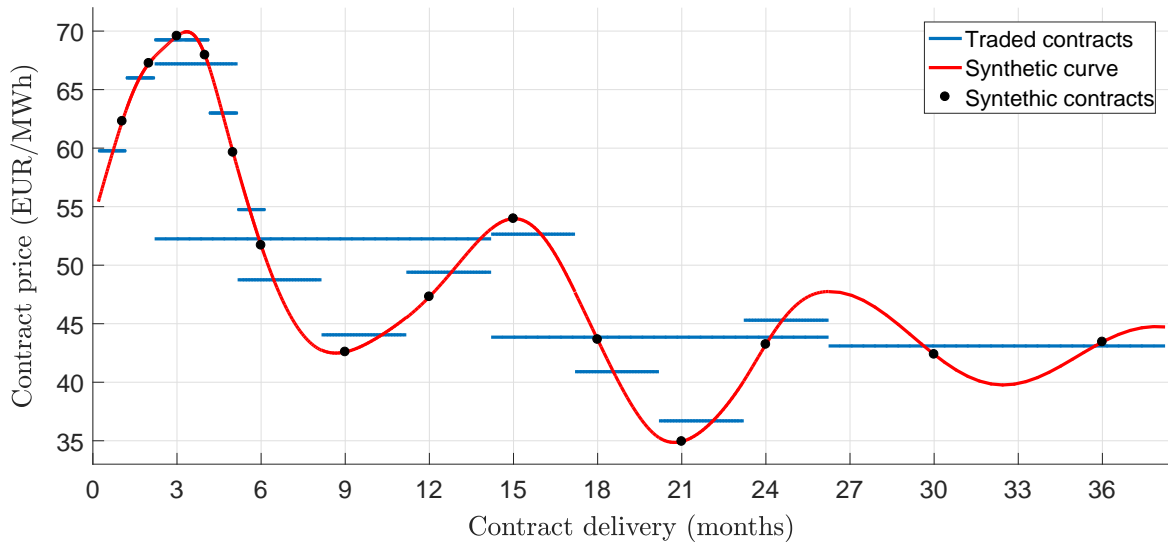


Figure 2.5: Synthetic power forward curve and extracted maturities obtained after the spline smoothing process.

2.2 Stochastic programming

One of the important mathematical approaches used in this thesis for modeling optimization problems that include uncertainty is stochastic programming (SP; Birge and Louveaux, 2011; Kall and Wallace, 1994). SP has been extensively used in the energy optimization literature, for example, in electricity market bidding problems (Boomsma et al., 2014; Ottesen et al., 2016, Chapter 6 of this thesis and references therein), unit commitment (Takriti et al., 1996; Takriti et al., 2000), power generation capacity expansion (Ahmed et al., 2003; López et al., 2007), hydropower production planning (Fleten and Kristoffersen, 2008), electricity procurement (Carrión et al., 2007), and power portfolio optimization (Fleten et al., 2002; Sen et al., 2006). See also Wallace and Fleten (2003) for an overview of energy problems solved with SP including traditional oil and gas applications. Stochastic programs have two or more stages, where stages represent discrete points in time when the value of some uncertain parameters become known to the decision maker. In Section 2.2.1, we introduce the two-stage SP with recourse. Then, we generalize it to a multi-stage setting in Section 2.2.2.

2.2.1 Two-stage stochastic programs

Consider the decision making problem represented in Figure 2.6 with two periods: $i = 0$ (now) and $i = 1$ (future). The known time 0 and random time 1 information is denoted by w_0 and w_1 , respectively. A first-stage decision $x_0 \in \mathcal{X}_0$ is made at time 0 knowing only

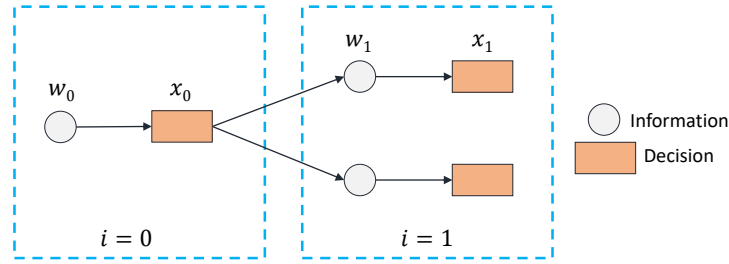


Figure 2.6: Decision making process in a two-stage SP with recourse.

w_0 and resulting in a reward $r_0(x_0, w_0) \in \mathbb{R}$. Subsequently, a random event w_1 occurs and a recourse decision $x_1 \in \mathcal{X}_1$ is made at the second stage to compensate the effect of this event. This decision results in a reward $r_1(x_1, w_1) \in \mathbb{R}$. The goal of the decision maker is to determine a feasible *policy* that maximizes the total expected reward over the two periods, where a policy is given by a first-stage decision x_0 and a collection of recourse decisions $x_1(w_1)$ to be taken in response to each random outcome of w_1 . This problem can be formulated as

$$\max_{x_0 \in \mathcal{X}_0} r_0(x_0, w_0) + \mathbb{E} \left[\max_{x_1 \in \mathcal{X}_1(x_0, w_1)} r_1(x_1, w_1) \middle| w_0 \right], \quad (2.4)$$

and is denoted two-stage stochastic program with recourse. The decision variables of this model are thus divided into two groups:

- The first-stage, or *here-and-now*, decision variables (x_0) are made before the realization of the uncertainty w_1 ;
- The second-stage, or *wait-and-see*, decision variables (x_1) are made after the uncertainty w_1 reveals to compensate the outcome of this random event.

In the most common linear case, the reward functions r_0 and r_1 are linear functions of the decision variables, and the feasible sets \mathcal{X}_0 , and \mathcal{X}_1 are polytopes, i.e., they are defined by sets of linear constraints. In this case, the two-stage SP with recourse (2.4) can be formulated as

$$\max r_0^\top x_0 + \mathbb{E} [\max r_1(w_1)^\top x_1(w_1) | w_0] \quad (2.5a)$$

$$\text{s.t. } Ax_0 = b \quad (2.5b)$$

$$T(w_1)x_0 + Wx_1(w_1) = h(w_1) \quad (2.5c)$$

$$x_0 \geq 0, x_1(w_1) \geq 0, \quad (2.5d)$$

where constraints (2.5b) and (2.5c) define the feasible region, respectively, for the first-stage variables x_0 and for the second-stage variables x_1 given x_0 and the uncertainty

realization w_1 . The optimization problem (2.5) can then be translated in the so called *deterministic equivalent*. Consider a discrete and finite set of scenarios \mathcal{K} describing the distribution of the random variables w_1 , where each scenario $k \in \mathcal{K}$ has probability $\pi_k \in [0, 1]$ and $\sum_{k \in \mathcal{K}} \pi_k = 1$. We index the recourse decision by k , i.e. $x_{1,k}$, since this decision adapts to the uncertainty outcome. We then write the expectation in (2.5a) and the constraints (2.5c) with their discrete expressions using the scenarios $k \in \mathcal{K}$, resulting in the following model

$$\max_x r_0^\top x_0 + \sum_{k \in \mathcal{K}} \pi_k r_{1,k}^\top x_{1,k} \quad (2.6a)$$

$$\text{s.t. } Ax_0 = b \quad (2.6b)$$

$$T_k x_0 + W x_{1,k} = h_k, \quad \forall k \in \mathcal{K} \quad (2.6c)$$

$$x_0 \geq 0, x_{1,k} \geq 0, \quad \forall k \in \mathcal{K}, \quad (2.6d)$$

which indeed is a linear program and can be solved with an LP solver. The formulation of (2.5) and (2.6) with integer or mixed-integer variables is analogous. Note that the size of model (2.6) increases with the number of scenarios $|\mathcal{K}|$ (both the number of variables and constraints). Thus, this model is often a large-scale model.

2.2.2 Multi-stage stochastic programs

The approach described in the previous section can be generalized to a multi-stage setting with periods $i = 0, \dots, I$, by allowing a recourse decision in each periods based on the uncertainty realized by that period. We illustrate this setting in Figure 2.7 and formulate a generic multi-stage SP with recourse using the following nested formulation.

$$\begin{aligned} & \max_{x_0 \in \mathcal{X}_0} r_0(x_0, w_0) + \mathbb{E} \left[\max_{x_1 \in \mathcal{X}_1(x_0, w_1)} r_1(x_1, w_1) + \mathbb{E} \left[\max_{x_2 \in \mathcal{X}_2(x_1, w_2)} r_2(x_2, w_2) \right. \right. \\ & \left. \left. \dots + \mathbb{E} \left[\dots + \mathbb{E} \left[\max_{x_I \in \mathcal{X}_I(x_{I-1}, w_I)} r_I(x_I, w_I) \mid w_{I-1} \right] \dots \mid w_i \right] \dots \mid w_1 \right] \mid w_0 \right]. \quad (2.7) \end{aligned}$$

Similarly to two-stage SPs, multi-stage SPs can be reformulated using a deterministic equivalent model based on the underlying scenario tree. If functions $r_i(\cdot, w_i)$ in formulation (2.7) are linear and sets \mathcal{X}_i are polytopes for $i = 0, \dots, I$, then the resulting deterministic equivalent is a linear program. We omit this model for brevity (see, e.g., Birge and Louveaux, 2011, ch. 3). The number of variables and constraints of this model grows quickly (exponentially) with the number of stages. Therefore, as the number of stages

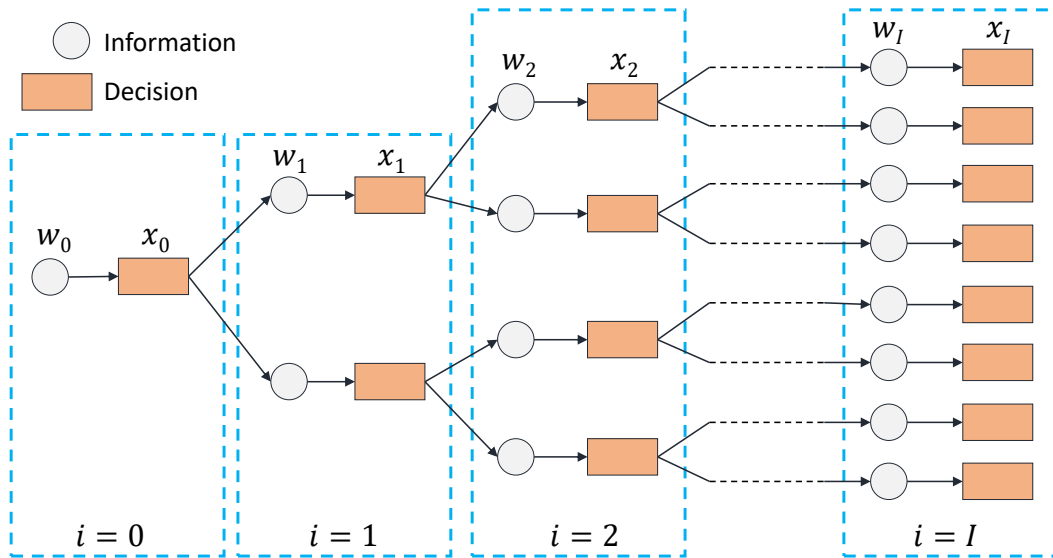


Figure 2.7: Decision making process in a multi-stage SP with recourse.

increases, multi-stage SPs become intractable. In practice, the majority of literature in SP deals with programs with only a few stages (typically two or three). In Chapter 6, we use two- and three-stage linear mixed-integer stochastic programs to describe the electricity market offering strategies of particular types of power producers.

Many stochastic programming problems, including the models that we developed in Chapter 6, can be successfully solved by directly using a general purpose LP or MIP solver on their deterministic equivalent translation. If such a direct attempt fails, the use of specialized scenario decomposition techniques is also a popular approach in stochastic programming. For example, the progressive hedging and Uzawa methods decompose the original problem by means of parallel smaller subproblems defined on each scenario $k \in \mathcal{K}$, coordinated and updated by a master algorithm that increasingly enforces non-anticipativity (see e.g. Leclere, 2014). Stochastic dynamic programming (see next section 2.3) can also be seen as a decomposition of multi-stage optimization problems over time periods instead, that is, the problem is solved by smaller sequential subproblems defined on each period i .

2.3 Stochastic dynamic programming

Sequential decision making problems under uncertainty can often be modeled by what is known as stochastic dynamic programming (SDP; Puterman, 2005; Bertsekas, 2011). In this section, we introduce the SDP framework and focus on some specific aspects and solution methods employed in this thesis. In Section 2.3.1, we formulate a generic stochastic

dynamic program to maximize the cumulative reward of a system subject to uncertainty, and highlight some of the challenges that frequently arise when dealing with these models. In Sections 2.3.2 and 2.3.3 we introduce two approximate dynamic programming methods frequently used to solve complex SDPs known as the least squares Monte Carlo and the reoptimization heuristic, respectively. We conclude in Section 2.3.4 by presenting techniques to compute dual bounds on the optimal solution value, thus an optimality gap.

2.3.1 Formulation and challenges

Consider an optimization problem involving a system (for instance, a power plant or an electricity storage) in which decisions are made sequentially over a finite time horizon composed of stages $i \in \mathcal{I} := \{0, \dots, I - 1\}$ (see, e.g., Puterman, 2005, ch. 5, for infinite-horizon models). The system at stage i is represented by a *state* that contains the minimally dimensioned function of history necessary and sufficient to model the system from stage i onward (Powell, 2011). The state of the system at stage i is denoted by $(x_i, w_i) \in \mathcal{X}_i \times \mathcal{W}_i$ and is composed of an endogenous and an exogenous part of the state. The endogenous (“physical”) state $x_i \in \mathcal{X}_i$ is controllable, while the exogenous (“information”) state $w_i \in \mathcal{W}_i$ are random variables that affect the cash flows and/or the evolution of the system. We assume that the uncertainties are driven by stochastic factors that evolve in a Markovian manner, that is, w_i carries all the history of information needed to describe w_{i+1} . To illustrate these concepts, consider the following situations.

- Operating a hydro power system composed by N connected reservoirs that participate in a wholesale electricity market. At a given stage i , the endogenous state space $\mathcal{X}_i \subset \mathbb{R}_+^N$ is the set of all possible reservoir states, that is, $x_i \in \mathcal{X}_i$ is a vector of N continuous variables $x_{i,n} \in [L_n^{\min}, L_n^{\max}]$, $n = 1, \dots, N$, each representing the water level in one reservoir. The exogenous state \mathcal{W}_i is given by the stochastic environmental and market factors driving the wholesale electricity prices and natural water inflows (Löhndorf et al., 2013).
- Operating an aluminum production facility that takes power as input and produces aluminum to sell to the wholesale market. At each stage i , this plant can: produce full load (O), temporarily suspend production for maximum three consecutive periods (S), or permanently shut down (C). In this case, the endogenous state space is discrete and composed by five states $\mathcal{X}_i = \{O, S_1, S_2, S_3, C\}$, while the exogenous state \mathcal{W}_i is composed by the market factors driving the evolution of aluminum and electricity prices (Trivella et al., 2018).

- Valuing the investment timing and capacity choice of a wind energy project subject to uncertainty in (i) the price of power to sell to the wholesale market through the project lifetime, and (ii) the price of steel needed to build the turbines. Assuming a discrete investment time horizon, the endogenous state $x_i \in \mathcal{X}_i \subset \mathbb{R}_+$ is a continuous variable representing the project capacity that is installed at stage i , and the exogenous state \mathcal{W}_i is given by electricity and steel spot prices (Boomsma et al., 2012).

The stage 0 state (x_0, w_0) is known. At stage $i \in \mathcal{I}$ and state $(x_i, w_i) \in \mathcal{X}_i \times \mathcal{W}_i$, a decision, or action, a_i from the feasible action set $\mathcal{A}_i(x_i)$ is executed, resulting in an immediate reward $r_i(x_i, w_i, a_i) \in \mathbb{R}$ and a transition to a stage $i + 1$ operating state $x_{i+1} = f_i(x_i, w_i, a_i)$ in set \mathcal{X}_{i+1} . We define a terminal stage I where no action is allowed and a terminal reward $r_I(x_I, w_I)$ is received, with (x_I, w_I) belonging to a terminal-stage state space $\mathcal{X}_I \times \mathcal{W}_I$.

Making decisions in such setting requires a *policy* π , that is, a collection of decision rules mapping states to actions. At a given stage i , a decision rule A_i^π associates the state $(x_i, w_i) \in \mathcal{X}_i \times \mathcal{W}_i$ to a feasible action $a_i \in \mathcal{A}_i(x_i)$. The policy π is then the collection $\{A_i^\pi, i \in \mathcal{I}\}$. We denote by Π the set of all feasible policies. An optimal risk-neutral policy belongs to Π and maximizes the expected cumulative cash flow from applying this policy over the finite problem horizon. More formally, this policy solves the following Markov decision process (MDP):

$$\max_{\pi \in \Pi} \mathbb{E} \left[\sum_{i \in \mathcal{I}} \delta^i r_i(x_i^\pi, w_i, A_i^\pi(x_i^\pi, w_i)) + \delta^I r_I(x_I^\pi, w_I) \mid (x_0, w_0) \right], \quad (2.8)$$

where x_i^π is the endogenous state reached in stage i by following policy π , and $\delta \in (0, 1]$ is the per-stage discount factor. (For finite horizon problems, note that δ could be embedded directly into the reward function, thus, not considered explicitly in (2.8).) It is well known that in theory the optimal policy of MDP (2.8) could be obtained with stochastic dynamic programming, that is, by solving the Bellman equations:

$$V_I(x_I, w_I) = r_I(x_I, w_I), \quad \forall (x_I, w_I) \in \mathcal{X}_I \times \mathcal{W}_I, \quad (2.9a)$$

$$V_i(x_i, w_i) = \max_{a_i \in \mathcal{A}_i(x_i)} \left\{ r_i(x_i, w_i, a_i) + \delta \mathbb{E}[V_{i+1}(f_i(x_i, w_i, a_i), w_{i+1}) \mid w_i] \right\}, \quad \forall (i, x_i, w_i) \in \mathcal{I} \times \mathcal{X}_i \times \mathcal{W}_i. \quad (2.9b)$$

Here $V_i(\cdot, \cdot)$ is the *value function* at stage $i \in \mathcal{I} \cup \{I\}$, representing the optimal utility of being in a stage i state when following an optimal policy from stage i to I . At the terminal stage I , the value function is specified in (2.9a) using the terminal reward $r_I(\cdot, \cdot)$.

In a backward recursion, the value function at stage $i \in \mathcal{I}$ and state $(x_i, w_i) \in \mathcal{X}_i \times \mathcal{W}_i$ is determined using (2.9b). Specifically, (2.9b) chooses the best feasible action $a_i \in \mathcal{A}_i(x_i)$ by maximizing the sum of the immediate reward, $r_i(x_i, w_i, a_i)$, and a *continuation function* that represents the expected utility of being in the new $i + 1$ state $f_i(x_i, w_i, a_i)$ induced by action a_i . The optimal objective value of (2.8) is given by $V_0(x_0, w_0)$.

Determining an optimal policy to SDP (2.9) is intractable in most real-life applications due to three possible curses of dimensionality (Powell, 2011):

1. The state space $(\mathcal{X}_i, \mathcal{W}_i)$ can be high dimensional making it difficult or impossible to compute and store the value functions in (2.9) for all states. The state can be high-dimensional in its endogenous component \mathcal{X}_i , exogenous component \mathcal{W}_i , or both. For example, if $x_i = (x_{i,1}, \dots, x_{i,N})$ has N dimensions and $x_{i,n}$ can take on H possible values, then we might have up to N^H different states. If x_i is continuous, even if scalar, then we cannot even enumerate all the states. This situation indeed arise in Chapter 5 in which \mathcal{X}_i is a vector of 25 continuous variables. Instead, our numerical study of Chapter 3 presents a high-dimensional exogenous state \mathcal{W}_i composed by eight continuous random variables.
2. The expectations needed to evaluate the continuation function in (2.9b) can require solving high-dimensional integrals and thus can be hard to compute.
3. The optimization problem in (2.9b) can be hard to solve due to a high-dimensional action space $\mathcal{A}_i(x_i)$ or a difficult problem class. For example, finding the best action in our numerical study of Chapter 5 requires solving a mixed-integer program.

Developing methods to overcome these curses of dimensionality is an active area of research under the umbrella of approximate dynamic programming (ADP). ADP methods exploit different types of approximations in SDP (2.9), for instance, of the value function, to overcome its intractability. A large variety of ADP strategies have been proposed in the literature. We do not attempt to summarize these strategies here but refer to Powell (2011) for a classification of policies determined with ADP into four classes: policies based on (i) policy function approximations, (ii) cost function approximations, (iii) value function approximations, and (iv) direct lookahead.

In the following, we describe two methods that are used in this thesis to approximate high-dimensional and intractable SDPs. The first method approximates the SDP value or continuation function (Chapters 3 and 4), the second is based on iteratively solving deterministic approximation of the model over time (Chapters 3 and 5).

2.3.2 Least squares Monte Carlo methods

Least squares Monte Carlo (LSM) is a state-of-the-art ADP technique which overcomes some of the curses of dimensionality by computing heuristic policies based on low-dimensional approximations of the SDP continuation or value function. Pioneered by Longstaff and Schwartz (2001) and Tsitsiklis and Van Roy (2001), LSM is a popular approach in both academia and industry for solving financial American options and real options. LSM has been also applied to energy real option problems, for example, in renewable energy investments (Boomsma et al., 2012), energy storage management (Arvesen et al., 2013; Mazières and Boogert, 2013; Nadarajah et al., 2017), and hydropower-reservoir management (Carmona and Ludkovski, 2010; Denault et al., 2013). There are two versions of LSM that we introduce below, mainly following the discussion in Nadarajah et al. (2017) where these two variants are compared. We assume that at each stage $i \in \mathcal{I} \cup \{I\}$ the endogenous state space \mathcal{X}_i is finite and there is no randomness in endogenous state transitions, that is, $x_{i+1} = f(x_i, a_i)$.

A standard version denoted regress-now LSM (LSMN; Longstaff and Schwartz, 2001; Tsitsiklis and Van Roy, 2001) approximates the SDP continuation function using basis functions. Consider at each stage $i \in \mathcal{I}$ a set of B_i basis functions of the information state $\Phi_i := \{\Phi_{i,b}(w_i), b \in \mathcal{B}_i\}$, where $\mathcal{B}_i := \{1, \dots, B_i\}$. These basis functions can be, for instance, polynomials, radial functions, or Laguerre polynomials (Longstaff and Schwartz, 2001; Stentoft, 2004; Mazières and Boogert, 2013). LSMN computes a continuation function approximation (CFA) as a linear combination of these basis functions:

$$\widehat{C}_i(x_{i+1}, w_i) := \sum_{b=1}^{B_i} \beta_{i,x_{i+1},b}^N \Phi_{i,b}(w_i) \simeq \mathbb{E}[V_{i+1}(x_{i+1}, w_{i+1}) | w_i],$$

where $\beta_{i,x_{i+1},b}^N$ denotes the b -th weight of the linear combination. We denote the vector of CFA weights by $\beta_{i,x_{i+1}}^N := (\beta_{i,x_{i+1},1}^N, \dots, \beta_{i,x_{i+1},B_i}^N)$. At a high level, LSMN generates sample paths of the uncertainty using Monte Carlo, and fits the CFA weights using least squares regression in a backward recursive fashion. The LSMN procedure is illustrated in Algorithm 1.

The outputs of LSMN are the CFA weight vectors $\beta_{i,x_{i+1}}^N$ for each stage $i \in \mathcal{I}$ and endogenous state $x_{i+1} \in \mathcal{X}_{i+1}$. Given such weights, the action $a^N(x_i, w_i)$ taken at stage i and state (x_i, w_i) is obtained by substituting the CFA in SDP (2.9) and maximizing over the feasible actions:

$$a^N(x_i, w_i) = \arg \max_{a \in \mathcal{A}_i(x_i)} \left\{ r_i(x_i, w_i, a) + \delta \sum_{b=1}^{B_i} \beta_{i,f_i(x_i,a),b}^N \Phi_{i,b}(w_i) \right\}. \quad (2.10)$$

Algorithm 1 LSMN

Inputs: Initial state $(x_0, w_0) \in \mathcal{X}_0 \times \mathcal{W}_0$, set of information state sample paths $\{w_i^p, (i, p) \in \mathcal{I} \cup \{I\} \times \mathcal{P}\}$, and set of basis function vectors $\{\Phi_i, i \in \mathcal{I}\}$.

Initialization: For each $x_I \in \mathcal{X}_I$, compute CFA $\widehat{C}_{I-1}(x_I, w_{I-1}^p) := r_I(x_I, w_I^p)$ estimates for $p \in \mathcal{P}$ and perform a least squares regression on these CFA using basis functions Φ_{I-1} to determine the vector of CFA weights β_{I-1, x_I}^N .

For each $i = I - 2$ to 0 **do**:

For each $x_i \in \mathcal{X}_i$ **do**:

1. **For** each $p \in \mathcal{P}$ **do**: Compute the CFA estimate

$$\widehat{C}_i(x_{i+1}, w_i^p) = \max_{a \in \mathcal{A}_i(x_{i+1})} \left\{ r_{i+1}(x_{i+1}, w_{i+1}^p, a) + \delta \sum_{b=1}^{B_{i+1}} \beta_{i+1, f_{i+1}(x_{i+1}, a), b}^N \Phi_{i+1, b}(w_{i+1}^p) \right\}.$$

2. Perform a least squares regression on the CFA estimates $\{\widehat{C}_i(x_{i+1}, w_i^p), p \in \mathcal{P}\}$ using basis functions Φ_i to determine the vector of CFA weights $\beta_{i, x_{i+1}}^N$.

Outputs: Vectors of CFA weights $\beta_{i, x_{i+1}}^N$ for each $(i, x_i) \in \mathcal{I} \times \mathcal{X}_{i+1}$.

To estimate a lower bound on the optimal policy value, that is, the objective of MDP (2.8), we generate a second set of evaluation sample paths in Monte Carlo and simulate the policy along each sample paths according to (2.10). A lower bound estimate is then computed as a sample average of the time 0 sum of discounted rewards gained along these sample paths.

A non-standard LSM version approximates instead the value function and is denoted regress-later LSM (LSML; Glasserman and Yu, 2004; Nadarajah et al., 2017). Consider for each stage $i \in \{1, \dots, I\}$ a set of B_i basis functions $\Phi_i := \{\Phi_{i, b}(w_i), b \in \mathcal{B}_i\}$, where $\mathcal{B}_i := \{1, \dots, B_i\}$. LSML computes a value function approximation (VFA) as a linear combination

$$\widehat{V}_i(x_i, w_i) := \sum_{b=1}^{B_i} \beta_{i, x_i, b} \Phi_{i, b}(w_i) \simeq V_i(x_i, w_i).$$

where $\beta_{i, x_i, b}^L$ denotes the b -th VFA weight. LSML is appealing when the basis functions expectations can be computed in closed form, that is: $\mathbb{E}[\Phi_{i+1, b}(w_{i+1}) | w_i] =: \bar{\Phi}_{i, i+1, b}(w_i)$. In this case, sample averages are not needed to evaluate expectations of the next stage

VFAs because

$$\begin{aligned}\mathbb{E}[\widehat{V}_{i+1}(x_{i+1}, w_{i+1}) | w_i] &= \mathbb{E}\left[\sum_{b=1}^{B_{i+1}} \beta_{i+1, x_{i+1}, b} \Phi_{i+1, b}(w_{i+1}) \middle| w_i\right] \\ &= \sum_{b=1}^{B_{i+1}} \beta_{i+1, x_{i+1}, b} \mathbb{E}[\Phi_{i+1, b}(w_{i+1}) | w_i] \\ &= \sum_{b=1}^{B_{i+1}} \beta_{i+1, x_{i+1}, b} \bar{\Phi}_{i, i+1, b}(w_i).\end{aligned}$$

This property is valid, for instance, when using polynomial and call/put option type basis functions on elements of common log-normal or term structure models. We describe the LSML procedure in Algorithm 2 for the case this closed-form property holds.

Algorithm 2 LSML

Inputs: Initial state $(x_0, w_0) \in \mathcal{X}_0 \times \mathcal{W}_0$, set of information state sample paths $\{w_i^p, (i, p) \in \mathcal{I} \cup \{I\} \times \mathcal{P}\}$ and set of basis function vectors $\{\Phi_i, i \in \{1, \dots, I\}\}$.

Initialization: For each $x_I \in \mathcal{X}_I$, compute estimates $\widehat{V}_I(x_I, w_I^p) = r_I(x_I, w_I^p)$ for $p \in \mathcal{P}$ and perform a least squares regression on these VFAs using basis functions Φ_I to determine the vector of VFA weights β_{I, x_I}^L .

For each $i = I - 1$ to 1 **do**:

For each $x_i \in \mathcal{X}_i$ **do**:

 1. **For** each $p \in \mathcal{P}$ **do**: Compute the VFA estimate

$$\widehat{V}_i(x_i, w_i^p) = \max_{a \in \mathcal{A}_i(x_i)} \left\{ r_i(x_i, w_i^p, a) + \delta \sum_{b=1}^{B_{i+1}} \beta_{i+1, f_i(x_i, a), b}^L \bar{\Phi}_{i, i+1, b}(w_{i+1}^p) \right\}.$$

 2. Perform a least squares regression on the VFA estimates $\{\widehat{V}_i(x_i, w_i^p), p \in \mathcal{P}\}$ using basis functions Φ_i to determine the vector of VFA weights β_{i, x_i}^L .

Outputs: Vectors of VFA weights β_{i, x_i}^L for each $(i, x_i) \in \{1, \dots, I\} \times \mathcal{X}_i$.

Given the LSML VFA weights, the action $a^L(x_i, w_i)$ taken at stage $i \in \mathcal{I}$ and state (x_i, w_i) is obtained by substituting the VFA in SDP (2.9) and maximizing over the feasible actions

$$a^L(x_i, w_i) = \arg \max_{a \in \mathcal{A}_i(x_i)} \left\{ r_i(x_i, w_i, a) + \delta \sum_{b=1}^{B_{i+1}} \beta_{i+1, f_i(x_i, a), b}^L \bar{\Phi}_{i, i+1, b}(w_i) \right\}. \quad (2.12)$$

A lower bound can then be estimated analogously to LSMN but using (2.12) to simulate the policy along the set of evaluation sample paths.

As shown by Nadarajah et al. (2017), LSMN and LSML usually produce equally accurate (and often near-optimal) lower bounds in similar running time. However, these methods can be also used to estimate dual bounds (see Section 2.3.4), and estimating a dual bound with an LSML VFA can be orders of magnitude faster than with an LSMN CFA, which supports the use of LSML to compute dual bounds. Moreover, LSML can be adapted to more sophisticated settings where the expectation in the SDP continuation function is replaced by other operators such as risk measures. The use of LSML in fact enabled us to approximate the shutdown-averse SDPs of Chapters 3–4.

2.3.3 Reoptimization heuristic

The reoptimization heuristic (RH) is based on iteratively solving deterministic approximations of the original stochastic model (2.8) over time (Wu et al., 2012; Secomandi, 2015; Löhndorf and Wozabal, 2017; Nadarajah and Secomandi, 2018). At each stage $i \in \mathcal{I}$, this heuristic replaces the future uncertainty $w_j, j > i$ by a forecast $E[w_j|w_i]$, solves a deterministic model which embeds such forecast, and implements the action pertaining to this stage alone. At stage $i + 1$, new information w_{i+1} becomes available and the process is repeated. RH thus accounts for the uncertainty in a reactive manner by reoptimizing at each stage using updated information. RH can also be applied to models with high-dimensional endogenous state space \mathcal{X}_i . In the following we assume deterministic state transitions, i.e. $x_{i+1} = f(x_i, a_i)$ but RH variants that handle stochastic state transitions can also be defined.

At stage i and state $(x_i, w_i) \in \mathcal{X}_i \times \mathcal{W}_i$, the RH model is formulated as

$$\max_{x_j, a_j} \sum_{j \in \mathcal{I}, j \geq i} \delta^j r_j(x_j, \mathbb{E}[w_j|w_i], a_j) + \delta^I r_I(x_I, \mathbb{E}[w_I|w_i]) \quad (2.13a)$$

$$\text{s.t.}: x_j = x_i, \quad j = 1 \quad (2.13b)$$

$$x_{j+1} = f_j(x_j, a_j), \quad \forall j \in \mathcal{I}, j \geq i, \quad (2.13c)$$

$$\text{var.}: a_j \in \mathcal{A}_j(x_j), \quad \forall j \in \mathcal{I}, j \geq i, \quad (2.13d)$$

$$x_j \in \mathcal{X}_j, \quad \forall j \in \mathcal{I} \cup \{I\}, j \geq i. \quad (2.13e)$$

The decision variables are given by endogenous states and actions for stages i to I . The objective function (2.13a) is the discounted sum of rewards obtained between i and I using expected values of uncertainty. Constraint (2.13b) initializes the stage i state to the current state x_i . Constraints (2.13c) model the state transitions and constraints (2.13d)–(2.13e) ensure the policy is feasible. The model (2.13) is a math program that can arise, for instance, as a linear, mixed-integer, or non-linear program depending on the

application. The full RH heuristic is shown in Algorithm 3.

Algorithm 3 RH

Inputs: Initial state $(x_0, w_0) \in \mathcal{X}_0 \times \mathcal{W}_0$ and sample path of the uncertainty $\{w_i, i \in \mathcal{I} \cup \{I\}\}$ conditioned on w_0 .

Initialization: Define sum of discounted rewards $R = 0$.

For each $i = 0$ to $I - 1$ **do**:

1. Solve deterministic model (2.13) formulated at stage i and state (x_i, w_i) .
2. Implement optimal stage i decision a_i^* at stage i alone, which means:
 - Add stage i reward $R \leftarrow R + \delta^i r_i(x_i, w_i, a_i^*)$;
 - Compute new state $x_{i+1} = f_i(x_i, a_i^*)$.

Add terminal reward $R \leftarrow R + \delta^I r_I(x_I, w_I)$.

Outputs: Sum of discounted rewards R gained along the sample path.

Estimating the value of the RH policy requires generating a set of H sample paths of the uncertainty conditioned on w_0 and simulating the policy according to Algorithm 3. Each sample path $h = 1, \dots, H$ results in the cumulative reward R_h , and the lower bound is then computed as a sample average $\sum_{h=1}^H R_h / H$.

The RH heuristic is popular in contexts such as the merchant management of commodity storage, where reoptimization policies (also known as rolling intrinsic in this context) are used in practice and known to be near optimal (Lai et al., 2010; Secomandi, 2015). In contrast, RH can be significantly suboptimal in MDPs with irreversible decisions such as the decision to shutdown a production plant (Chapter 3) or to invest in a long term power contract (Chapter 5). In the former case, for instance, the RH might incorrectly choose the permanent decision at a stage i state as a result of not capturing the evolution of the uncertainty, whereas the optimal policy chooses a different action. This mistake is irreversible and can result in significant revenue losses. In commodity storage applications the RH policy can instead offset a potentially incorrect energy injection/withdrawal decision at stage i using future injection/withdrawal decisions. Another reason why RH is popular in some practical applications is the following. Suppose the uncertainty is in market prices and that market information, as, e.g., forward prices, can be used to evaluate the expectations in the RH model objective function (2.13a). Then, a stochastic model for the evolution of market factors is not needed to use the RH policy and this policy is thus not affected by errors incurred in modeling the uncertainty (Secomandi et al., 2015); otherwise, it is subject to such errors. In any case, a stochastic model of the uncertainty is necessary to estimate the value of the RH policy.

As mentioned, the RH model can be viewed as a deterministic approximation of the original MDP. In some applications where the endogenous state space is low dimensional and finite, the RH objective function (2.13a) can be replaced by

$$\max_{x_j, a_j} \sum_{j \in \mathcal{I}, j \geq i} \delta^j \mathbb{E}[r_j(x_j, w_j, a_j) | w_i] + \delta^I \mathbb{E}_i[r_I(x_I, w_I) | w_i], \quad (2.14)$$

where the expectation is taken to the reward function instead of the uncertainty. This objective is intuitively preferable as it avoids one approximation step (we indeed use (2.14) in Chapter 3). We use RH variants or extensions in both Chapters 3 and 5.

2.3.4 Information relaxations

Evaluating the performance of a policy is important, but depending on the problem, one strategy may be more appropriate than the other. For example one could: (i) consider a smaller or simplified (but still meaningful) problem that can be solved exactly and compare the policy value estimate with the optimal solution, (ii) compare the policy value estimate with simpler policies, e.g. myopic policies which only optimize the current reward without modeling future decisions, and (iii) assess dual bounds on the optimal policy value, that is, upper bounds in case of profit maximization as in (2.9). In this section, we give a brief introduction to the latter strategy.

Typically, the computation of dual bounds relies on relaxing the non-anticipativity constraints embedded in the SDP, that is, some future uncertainty is known before making a decision. Giving to the decision maker more information than what is truly available, the solution to the relaxed problem represents a dual bound on the optimal value. The simplest dual bound assumes perfect information of the future. In this case, we consider H Monte Carlo samples of uncertainty $\{w_i^h, (i, h) \in \mathcal{I} \cup \{I\} \times \{1, \dots, H\}\}$ and solve the following deterministic dynamic program on each sample path h :

$$\begin{aligned} U_I^h(x_I) &= r_I(x_I, w_I^h), \quad \forall x_I \in \mathcal{X}_I, \\ U_i^h(x_i) &= \max_{a_i \in \mathcal{A}_i(x_i)} \left\{ r_i(x_i, w_i^h, a_i) + \delta U_{i+1}^h(f(x_i, w_i^h, a_i)) \right\}, \quad \forall (i, x_i) \in \mathcal{I} \times \mathcal{X}_i. \end{aligned}$$

A dual bound is then obtained as the sample average $\sum_{h=1}^H U_0^h(x_0)/H$. The dual bound based on perfect information is often rather weak and tighter bounds are generally needed. An alternative choice is to consider imperfect information relaxations, that is, the decision maker only knows in advance some of the uncertainty. This approach can be useful when the resulting problem is significantly easier to solve, for example, when a multi-dimensional

uncertainty model is reduced to a single stochastic factor.

Another way of computing efficient dual bounds is based on the information relaxation and duality framework (Brown et al., 2010), which we use in Chapters 3–5. This approach also relies on relaxing the non-anticipativity constraints, but penalizes knowledge of future information at time $i \in \mathcal{I}$ using a penalty function $q_i(f(x_i, a_i), w_i, w_{i+1})$. In a maximization problem, a feasible dual penalty q_i satisfies $\mathbb{E}[q_i(f(x_i, a_i), w_i, w_{i+1}) | w_i] \leq 0$. Using Monte Carlo samples of uncertainty $\{w_i^h, (i, h) \in \mathcal{I} \cup \{I\} \times \{1, \dots, H\}\}$, we solve the following deterministic dynamic program:

$$\begin{aligned} U_I^h(x_I) &= r_I(x_I, w_I^h), \quad \forall x_I \in \mathcal{X}_I, \\ U_i^h(x_i) &= \max_{a_i \in \mathcal{A}_i(x_i)} \left\{ r_i(x_i, w_i^h, a_i) - q_i(f(x_i, w_i^h, a_i), w_i^h, w_{i+1}^h) \right. \\ &\quad \left. + \delta U_{i+1}^h(f(x_i, w_i^h, a_i)) \right\}, \quad \forall (i, x_i) \in \mathcal{I} \times \mathcal{X}_i, \end{aligned}$$

for all $h \in \{1, \dots, H\}$, where q_i is a feasible penalty. A dual bound is then obtained as the sample average $\sum_{h=1}^H U_0^h(x_0)/H$.

Regarding the dual penalty choice, it is well known (Brown et al., 2010) that given a VFA $\hat{V}_i(\cdot)$, a feasible dual penalty can be defined for $(x_{i+1}, h) \in \mathcal{X}_{i+1} \times \{1, \dots, H\}$ as follows:

$$q_i(x_{i+1}, w_i^h, w_{i+1}^h) = \delta \left\{ \hat{V}_{i+1}(x_{i+1}, w_{i+1}^h) - \mathbb{E}[\hat{V}_{i+1}(x_{i+1}, w_{i+1}) | w_i^h] \right\}. \quad (2.15)$$

The VFA used in (2.15) can be determined, for example, using an LSM method as we do in Chapters 3–4. In Chapter 5, obtaining a VFA is hard and we use a linear dual penalty instead. In the same chapter, we leverage the described information relaxation approach and combine it with simulation to determine non-anticipative policies.

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Part II

Research work

CHAPTER 3

Managing Shutdown Decisions in Merchant Commodity and Energy Production: A Social Commerce Perspective

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Abstract: Merchant commodity and energy production assets operate in markets with volatile prices and exchange rates. Producers can choose in each period between production, production suspension, and permanent shutdown. Plant closures, however, adversely affect societal entities beyond the specific plant being shutdown such as the parent company and the local community. Motivated by an aluminum

producer, we study if mitigating these hard-to-assess broader impacts of a shutdown is financially viable using the plant's operating flexibility. Our social commerce perspective towards managing shutdown decisions deviates from the commonly used asset value maximization objective in merchant operations. We formulate a high-dimensional constrained Markov decision process to manage shutdown decisions. We approximate this intractable model using unconstrained stochastic dynamic programs and define operating policies that account for preferences to delay and reduce shutdowns. Our first policy leverages anticipated regret theory in behavioral psychology while the second policy generalizes production margin heuristics used in practice using machine learning. We compute the former and latter policies using a least squares Monte Carlo method and combining this method with binary classification, respectively. We also propose a reoptimization heuristic to simplify the anticipated-regret policy. We show that anticipated-regret policies possess desirable asymptotic properties absent in classification- and reoptimization-based policies. On instances created using real data, anticipated-regret and classification-based policies outperform practice-based production margin strategies and, to a lesser extent, reoptimization. Specifically, the former policies decrease the shutdown probability by 25% and, in addition, delay shutdown decisions by an average of 4 years for a 4% asset value loss. Our operating policies show that unaccounted social costs amounting to a few percent of the maximum asset value can justify delaying or avoiding the use of a plant's shutdown option by adapting its operating flexibility in our application. Thus, taking a social commerce perspective towards managing a plant's operating flexibility appears financially viable

Keywords: Commodity and energy operations · social commerce · real options · shutdown option · approximate dynamic programming

3.1 Introduction

Commodity and energy production assets play a critical role in ensuring the supply of commodities such as ethanol, aluminum, copper, iron, and steel, and power from coal, natural gas, and renewable sources. In 2015, iron ore, copper, and aluminum were respectively \$225, \$130, and \$90 billion US dollars annual industries (IMF, 2015), and roughly \$10 trillion US dollars will be invested in new power generation by 2040 (BNEF, 2017). Based on work with a major base metal producer, we study the management of permanent shutdown decisions in a merchant commodity and energy production asset. For illustration, consider an aluminum production facility that takes as inputs bauxite, carbon, and

electricity, and produces aluminum to be sold into the wholesale market. Production is economical when the conversion spread between the aluminum price and the input prices is positive after accounting for production costs. If this spread is negative the merchant may temporarily suspend production. If a combination of production and suspension leads to losses it may be economical to permanently shut down the plant.

A merchant commodity and energy producer can maximize the market value of the plant by adapting production, suspension, and shutdown decisions over time to the evolution of uncertain market factors such as prices of commodity and energy sources and exchange rates (Secomandi and Seppi, 2014). While this pure asset value perspective is popular, the cost of a permanent plant shutdown is hard to assess as it may impact societal entities outside the specific plant being shut down, which could include the parent company owning the plant and the local community. For example, aluminum producers often own both a production plant and an electricity source, such as a dam, due to the large amounts of power consumed during the production process. It is common for licenses to operate plants and dams to include social and economic criteria that the producer must meet (e.g., maintaining local jobs). Abandoning a plant can make the producer default on these obligations and the government to deny the renewal of operating licenses for the dam as well as jeopardize licenses for assets in other regions. In addition, such a shutdown can cause loss of employment and adverse publicity and result in political resistance from powerful unions and interest groups (see Kasa, 2000 for evidence of institutionalized links between unions and government).

Given the gravity of a plant shutdown on society, companies deviate from a pure asset value perspective: several years of severely challenged profitability are typically needed to justify shutdown to a labor union, but waiting to incur such losses before shutting down may be at odds with maximizing asset value. This accounting of social impact is evident, for instance, from the large number of aluminum producers who are members of the Aluminum Stewardship Initiative, which defines quality and social standards (ASI, 2017). Public web pages of major aluminum producers also signal their focus on “social commerce” (Alcoa, 2016; Rio Tinto, 2016; Hydro, 2018; see also Kleindorfer et al., 2005; Lee and Tang, 2017, and references therein, for other examples of socially responsible operations). In sum, producers would benefit from models and methods to manage asset value and the broader impact of plant closures in a principled manner under evolving market factors, which include overproduction, low commodity prices, and volatility in power prices and exchange rates (The Telegraph, 2012; BI, 2016; The Australian, 2016).

We define the shutdown probability of a plant’s operating policy at a given time as the probability of it closing down by this time. Each policy can thus be associated with a shutdown profile, that is, its shutdown probability at each time period over a finite

planning horizon. From the perspective of mitigating any adverse societal impact of a shutdown, we prefer an operating policy and an associated shutdown profile that (i) delays shutdown decisions to later parts of the planning horizon, and (ii) reduces the shutdown probability at the end of this horizon. The operating policy that maximizes the plant's asset value may understandably have an undesirable shutdown profile. If we can improve the shutdown profile of this policy while incurring only minor losses in asset value, then producers may be able to justify the operations of the plant for a longer period of time on the basis of unaccounted social costs.

In this paper, we focus on finding operating policies that trade off asset value for a more desirable shutdown profile. We formulate a constrained Markov decision process (MDP) to maximize the plant's market value subject to restrictions on the shutdown profile of its operating policy that capture preferences for delaying and reducing the likelihood of shutdowns. This MDP is a constrained version of the well-known risk-neutral MDP formulation (henceforth referred to as the shutdown-neutral MDP) that maximizes the asset value alone (Guthrie, 2009). Solving our constrained MDP is significantly more challenging than the shutdown-neutral MDP, especially because the MDP state space is high-dimensional when using realistic models for the evolution of uncertain factors such as prices and exchange rates. Practical methods to tackle the solution of large scale constrained MDPs are severely limited (Dufour and Prieto-Rumeau, 2013; Dufour and Prieto-Rumeau, 2014). We develop two strategies to approximate our constrained MDP that strive to maintain the familiarity of the shutdown-neutral MDP while accounting for societal preferences over the shutdown profile of an operating policy.

Our first strategy incorporates the desire for a favorable shutdown profile by modifying the plant closure cost of the shutdown-neutral MDP in a manner that is consistent with anticipated regret theory in behavioral psychology (Loomes and Sugden, 1982 and Zeelenberg, 1999). This theory and subsequent empirical validation posit that a decision maker chooses among multiple decisions by accounting for the anticipated regret from future scenarios where one of the unselected decisions provides higher utility than the chosen one. Additional experimental research also shows that this regret is large for irreversible decisions and small for reversible decisions (see, e.g., Tsiros and Mittal, 2000), which supports our inflation of only the shutdown cost – the sole irreversible decision in our setting. Intuitively, the anticipated regret associated with a shutdown decision at a given stage and state is the expected increase in some utility along scenarios where a non-shutdown decision is preferred to a plant closure. We formulate a stochastic dynamic program (SDP), dubbed the anticipated-regret (AR) SDP, by inflating the shutdown cost by an anticipated-regret term that includes parameters for controlling the preferences to delay and reduce shutdowns. We establish that the optimal policy of this SDP is consistent with our constrained MDP in asymptotic regimes where the social preferences

are either shutdown-neutral or extremely shutdown-averse (i.e., require zero shutdowns). Computing an AR optimal policy is however challenging due to the high dimensionality of the AR SDP state space. We thus adapt a least squares Monte Carlo (LSM) approximate dynamic programming heuristic (Longstaff and Schwartz, 2001; Glasserman and Yu, 2004a; Nadarajah et al., 2017) to compute AR policies. We also define a simpler policy using a shutdown-averse reoptimization heuristic (RH) that efficiently computes operating decisions by solving a deterministic version of the AR SDP as a shortest path problem.

Our second solution strategy focuses on production margin-based policies, which choose to shut down when a sum of current and expected-future production margins are below a threshold. This structure facilitates showing to external stakeholders the challenged profitability leading to a potential shutdown decision. We formalize production margin policies used in practice and also combine approximate dynamic programming and machine learning to improve them. Specifically, we create new policies using binary classification (Bishop, 2006, chapter 4) where thresholds are determined on training data (i.e. operating decisions) generated by simulating a shutdown-neutral policy obtained using LSM. These thresholds are subsequently modified to account for shutdown profile preferences. We refer to the resulting policy as the classification-based margin (CM) policy.

We perform a numerical study involving the operations of a real aluminum producer over a forty-year time horizon. Our case study uses operational data from this producer and market data from the Nord Pool, London Metal Exchange (LME), and FOREX markets. We calibrate an eight-factor stochastic model to capture the evolution of uncertainty, which includes electricity and aluminum prices and exchange rates (Farkas et al., 2017). We use an LSM method to obtain a near optimal Monte Carlo simulation estimate of the maximum asset value benchmarked against a dual upper bound estimate (Brown et al., 2010). We also employ LSM to obtain a collection of AR and CM policies for different preferences over delaying and reducing shutdowns. We find that both policies can substantially improve the shutdown profile compared to that of the shutdown-neutral policy for small asset value losses. For instance, AR and CM policies can decrease the shutdown probability by 25% and, in addition, delay shutdown decisions by an average of 4 years for a 4% loss in asset value. We also find that the shutdown-averse RH policy improves both the asset value and shutdown profile compared to an existing shutdown-neutral version of this policy. Nevertheless, the shutdown-averse RH policy exhibits less stable performance than AR and CM policies for small asset value losses (e.g. 0-5%) that are most likely to be acceptable in practice. The CM policies dominate the practice-based margin policies for managing the trade-off between shutdowns and asset value, even though the latter rules exhibit good performance from a pure asset value maximization standpoint.

Our findings inform the management of shutdown decisions in merchant production assets.

Producers could justify significantly delaying and/or reducing the use of the shutdown option in the presence of unaccounted social and political costs amounting to a few percent of the maximum asset value. Operating flexibility emerges as a promising lever to achieve these shutdown profile improvements with the AR and CM policies providing effective ways to manage the trade-off between shutdown decisions and asset value. The insights and operating policies in this paper are relevant beyond aluminum production to other commodity producers, refineries and hydrocarbon crackers in the petro-chemical industry, and power plants (Brennan and Schwartz, 1985; Tseng and Barz, 2002; Méndez et al., 2006; Cortazar et al., 2008; Adkins and Paxson, 2011; Boyabatli et al., 2011; Kazaz and Webster, 2011; Boyabatli, 2015; Nadarajah et al., 2016; Hekimoğlu et al., 2016; Boyabatli et al., 2017). Managing shutdown decisions is also important in renewable energy production, for example in biogas plants, where minimizing its likelihood without significant asset value loss is important to remain economical and to uphold the plant as a source of power (Di Corato and Moretto, 2011; Hochloff and Braun, 2014).

Our work on managing the trade-off between shutdown probability and asset value adds to the literature on merchant commodity and energy operations (Markland, 1975; Geman, 2005; Boogert and De Jong, 2008; Lai et al., 2010; Berling and Martínez-de-Albéniz, 2011; Devalkar et al., 2011; Wu et al., 2012; Löhndorf et al., 2013; Mazières and Boogert, 2013; Nadarajah et al., 2015; Secomandi, 2015; Thompson, 2016; Devalkar et al., 2017; Nadarajah et al., 2017), and socially responsible operations (Kleindorfer et al., 2005; Lee and Tang, 2017). Models of production assets as switching options have been considered, for example, by Kulatilaka and Trigeorgis (2001); Tseng and Barz (2002), and Adkins and Paxson (2011). These papers model a temporary shutdown option. Brennan and Schwartz (1985), Cortazar et al. (2008), Guthrie (2009, ch. 17), Nadarajah and Secomandi (2017b), and Yang et al. (2017) consider switching options with permanent shutdown but focus on risk-neutral valuation and operations. The growing literature on socially responsible operations incorporates social costs into operations and supply chain models but taking a social commerce view on shutdown decisions in merchant energy production appears new (see Lee and Tang, 2017 and references therein). We also complement research in this literature and merchant operations by introducing methods that are grounded in anticipated regret theory and practitioner heuristics, as well as applying them to an aluminum case study based on real data.

Our approach for approximating large-scale MDPs by combining LSM and machine learning classification methods contributes to the extant research that obtains heuristic policies to high-dimensional real option problems (see, e.g., Glasserman and Yu, 2004b, Chapter 8, Cortazar et al., 2008; Boogert and De Jong, 2008; Carmona and Ludkovski, 2010; Mazières and Boogert, 2013; Nadarajah et al., 2015; Nadarajah et al., 2017) and the literature on decision rule approximations (Ben-Tal et al., 2004; Kuhn et al., 2011; Georghiou et al.,

2015). The LSM approach has regress-now (Longstaff and Schwartz, 2001) and regress-later (Glasserman and Yu, 2004a; Nadarajah et al., 2017) variants that approximate the SDP continuation and value functions, respectively. The computation of risk-neutral policies in a commodity production application with a permanent shutdown decision has been approached by Cortazar et al. (2008) using regress-now LSM and by Nadarajah and Secomandi (2017b) and Yang et al. (2017) using regress-later LSM. Unlike these papers, we adapt LSM to account for the shutdown profile of operating policies. Our combination of regress-later LSM with binary classification to learn margin-based policies, that is the CM policy, and its modification to satisfy policy constraints are both new and add to the decision rule approximations literature. Prior work in this literature has considered affine and piece-wise affine decision rules for approximating stochastic optimization problems (see, e.g., Georghiou et al., 2011). We leverage the regress-later LSM approach from Nadarajah et al. (2017) to compute dual upper bounds (Haugh and Kogan, 2004; Brown et al., 2010) in a merchant production setting as also done in Nadarajah and Secomandi (2017b) and Yang et al. (2017).

RH has been studied in the merchant operations literature due to its ease of implementation and applied to operate commodity storage, electricity transport, swing, and tolling assets (Deng et al., 2001; Keppo, 2004; Wu et al., 2012; Secomandi, 2015; Mahoney, 2016; Nadarajah and Secomandi, 2017a; Löhndorf and Wozabal, 2017). These applications do not include irreversible decisions such as permanent shutdown nor consider features of the policy. Our shutdown-averse RH approach addresses both these gaps in the merchant operations literature in a novel manner by observing that penalizing irreversible decisions can improve the RH policy performance in terms of both asset value and shutdown profile. Moreover, our efficient shortest path approach to compute the RH policy decisions is new and facilitates the estimation of the value of this policy in Monte Carlo simulation.

The long term planning nature (e.g. 30-40 years) of shutdown decisions in our application entails additional challenges in modeling the evolution of uncertainty because traded contracts are unavailable in the Nord Pool, LME, and FOREX markets over this long time horizon. For example, single-factor processes such as geometric Brownian motion and Ornstein-Uhlenbeck, which are easy to calibrate, do not capture the short term and long term dynamics exhibited by power prices and exchange rates. Multi-factor market models (Eydeland and Wolyniec, 2003, p. 205, Secomandi et al., 2015; Nadarajah and Secomandi, 2017b; Nadarajah and Secomandi, 2017a) may be viable for representing the long-term dynamics of a single commodity under certain parameter assumptions (Thompson, 2016) but extending these assumptions to our multi-commodity setting is difficult. We thus adapt a state-of-the-art multi-commodity and multi-factor stochastic process from Farkas et al. (2017), which is an extension of the single-commodity two-factor model by Schwartz and Smith (2000). Farkas et al. (2017) apply their stochastic process to a

number of commodities, including power, but do not consider aluminum price and exchange rates. We show that this process represents the evolution of aluminum and power prices and exchange rates in a statistically sound manner and is suitable for long term operations planning problems.

The rest of this paper is organized as follows. We formulate a model of a merchant production plant and formalize the notion of shutdown profile in Section 3.2. We introduce anticipated-regret policies in Section 3.3. We describe production margin-based policies in Section 3.4. We discuss our numerical study in Section 3.5 followed by conclusions in Section 3.6. Additional details regarding proofs, model calibration, and algorithms are provided in the appendix.

3.2 Merchant commodity and energy production operations

In this section we discuss the merchant operations of commodity/energy production assets. We present in Section 3.2.1 a model for the operations of such assets that embed a permanent shutdown option. To identify a specific operating policy, we describe the known asset value maximization perspective in Section 3.2.2 and then introduce a social commerce perspective in Section 3.2.3.

3.2.1 Operating model

We consider a commodity and energy production facility operating over I time periods (stages). This plant produces output and sells it into the wholesale market when prices are favorable, that is, it operates in a merchant fashion. We model the plant's operating problem as an MDP, where at each stage $i \in \mathcal{I} := \{0, \dots, I - 1\}$ the plant could be open or permanently shutdown. Examples of open states include in production, ramping up/down to a higher/lower production rate, and temporary suspension. We denote the finite set of open states at stage i by \mathcal{O}_i and assume that this set includes at least the state corresponding to full production labeled O . The permanent shutdown state is represented by C . The stage i operating status belongs to set $\mathcal{X}_i := \mathcal{O}_i \cup \{C\}$.

The full state of the production asset is composed of the operating (endogenous) state component described above as well as a market information (exogenous) state component that affects the operating cash flows. Let w_i denote the vector of stochastic market factors

at stage i driving the evolution of market information (e.g., commodity/energy prices and exchange rates). We assume the vector w_i evolves in a Markovian manner and denote its support by \mathcal{W}_i .

We assume that the plant is initially in an open state, that is, the known stage 0 state represented by (x_0, w_0) satisfies $x_0 \in \mathcal{O}_0$. A decision a_i from the feasible action set $\mathcal{A}_i(x_i)$ executed at stage $i \in \mathcal{I}$ and state $(x_i, w_i) \in \mathcal{X}_i \times \mathcal{W}_i$ results in an immediate reward $r_i(x_i, w_i, a_i)$ and a transition to a stage $i + 1$ operating state $f_i(x_i, a_i)$ in set \mathcal{X}_{i+1} . We define a terminal stage I where no action is allowed and a terminal reward $r_I(x_I, w_I)$ is received; the pair (x_I, w_I) belongs to the terminal-stage state space $\mathcal{X}_I \times \mathcal{W}_I$. Since $f_i(x_i, a_i)$ is an element of \mathcal{X}_{i+1} we use the next stage operating state as action labels, that is, taking action $a_i = x_{i+1}$ at stage i and operating state x_i results in a transition to the stage $i + 1$ operating status $f_i(x_i, x_{i+1}) = x_{i+1}$. To account for the permanent nature of a shutdown decision, we impose the following conditions on the action set and transition function: $\mathcal{A}_i(\mathbf{C}) = \{\mathbf{C}\}$ and $f_i(\mathbf{C}, \mathbf{C}) = \mathbf{C}$ for all $i \in \mathcal{I}$. A shutdown decision at stage $i \in \mathcal{I}$ incurs a one time fixed cost, which we model by requiring $r_i(x_i, w_i, \mathbf{C}) = -K^{(x_i, \mathbf{C})}$ for all $(x_i, w_i) \in \mathcal{O}_i \times \mathcal{W}_i$ and $r_i(\mathbf{C}, w_i, \mathbf{C}) = 0$, where $K^{(x_i, \mathbf{C})}$ is the strictly positive fixed cost of shutting down at an open state $x_i \in \mathcal{O}_i$.

This operating model can be specialized to capture a fairly broad class of production assets including coal and natural gas power plants, ethanol and biogas plants, and metal smelters. An example application of this model can be found in Section 3.5, where we use it to describe the merchant operations of a real aluminum plant as part of our numerical study.

3.2.2 Asset value maximization perspective

To make decisions in the operating model of Section 3.2.1, the plant manager requires a policy, which is a collection of stage-specific decision rules. At a given stage i , a decision rule A_i^π specifies a feasible operating decision in $\mathcal{A}_i(x_i)$ for each state $(x_i, w_i) \in \mathcal{X}_i \times \mathcal{W}_i$. A policy π is then the collection $\{A_i^\pi, i \in \mathcal{I}\}$, and the set of all feasible policies is denoted by Π . The value of the production asset when using a policy $\pi \in \Pi$ is the discounted sum of expected cash flows from using this policy over the finite problem horizon. The policy that maximizes the asset value thus solves

$$\max_{\pi \in \Pi} \mathbb{E}_0 \left[\sum_{i \in \mathcal{I}} \delta^i r_i(x_i^\pi, w_i, A_i^\pi(x_i^\pi, w_i)) + \delta^I r_I(x_I^\pi, w_I) \right], \quad (3.1)$$

where \mathbb{E}_0 denotes expectation with respect to a Markovian stochastic process describing the distribution of w_i given the stage 0 information w_0 (see (3.9) for an example of such a process), $\delta \in (0, 1]$ is the discount factor, and x_i^π the random endogenous state reached in stage i by following policy π . We refer to the operating policy solving (3.1) as the *shutdown-neutral* optimal policy and name it π^{SN} . It is well known that this policy is the solution of the following stochastic dynamic program (SDP):

$$V_I^{\text{SN}}(x_I, w_I) = r_I(x_I, w_I), \quad \forall (x_I, w_I) \in \mathcal{X}_I \times \mathcal{W}_I, \quad (3.2a)$$

$$V_i^{\text{SN}, \mathcal{O}}(x_i, w_i) = \max_{a_i \in \mathcal{A}_i(x_i) \setminus \{C\}} \left\{ r_i(x_i, w_i, a_i) + \delta \mathbb{E}_i [V_{i+1}^{\text{SN}}(f_i(x_i, a_i), w_{i+1})] \right\},$$

$$\forall (i, x_i, w_i) \in \mathcal{I} \times \mathcal{X}_i \times \mathcal{W}_i, \quad (3.2b)$$

$$V_i^{\text{SN}}(x_i, w_i) = \max \left\{ V_i^{\text{SN}, \mathcal{O}}(x_i, w_i), -K^{(x_i, C)} \right\}, \quad \forall (i, x_i, w_i) \in \mathcal{I} \times \mathcal{X}_i \times \mathcal{W}_i. \quad (3.2c)$$

Here $V_i^{\text{SN}}(\cdot, \cdot)$ denotes the shutdown-neutral value function at stages i in set $\mathcal{I} \cup \{I\}$. The value function at the terminal stage is specified in (3.2a) using the terminal reward $r_I(\cdot, \cdot)$. In (3.2b), we define an intermediate value function $V_i^{\text{SN}, \mathcal{O}}(x_i, w_i)$ that excludes the shutdown action at stage i . Here \mathbb{E}_i denotes expectation given stage i information w_i . We define $V_i^{\text{SN}}(x_i, w_i)$ as the maximum of $V_i^{\text{SN}, \mathcal{O}}(x_i, w_i)$ and the shutdown cost in (3.2c), that is, this equation models the choice between the shutdown action and the best open action selected in (3.2b).

Computing an optimal shutdown-neutral policy by solving SDP (3.2) is in general intractable due to its high-dimensional state space and potentially hard to compute expectations (Powell, 2011). These curses of dimensionality indeed arise when considering the aluminum production application described in Section 3.5.1 as the exogenous state that we model is eight dimensional (see Section 3.5.2 for details on the price model). LSM is a popular approximate dynamic programming method for computing policies in real option applications (Longstaff and Schwartz, 2001). We discuss LSM in Appendix 3.7.2 for approximating the shutdown-neutral SDP and other high-dimensional SDPs later in this paper.

3.2.3 Social commerce perspective

For reasons detailed in Section 3.1, managing plant shutdown decisions is of strategic importance to a business because closing down a production asset has both direct and indirect effects on entities beyond the plant. As a result, delaying and reducing shutdowns is desirable from a social commerce standpoint. We model these social preferences by associating a shutdown profile with each policy $\pi \in \Pi$, which is the following collection of shut-

down probabilities: $\{\Pr(x_i^\pi = C | x_0, w_0), i \in \mathcal{I} \cup I\}$. Reducing the shutdown probability entails finding an operating policy π where the probability at the end of the planning horizon $\Pr(C; \pi) := \Pr(x_I^\pi = C | x_0, w_0)$ is smaller than the analogous probability $\Pr(C; \pi^{\text{SN}})$ under the shutdown-neutral policy. Delaying shutdowns on the other hand requires emphasizing shutdown probability decreases at early stages more than at later stages, where delay could be measured by the expected time to shutdown along sample paths of uncertainty where a shutdown decision is chosen. Examples 3.1 and 3.2 illustrate the notions of shutdown probability reduction and shutdown delay, also highlighting that achieving a balance between asset value and shutdown profile can be non-trivial. Specifically, Example 3.1 shows that multiple policies can result in the same shutdown probability but different asset value, while Example 3.2 presents a case where multiple policies have the same combination of asset value and shutdown probability but delay shutdown differently.

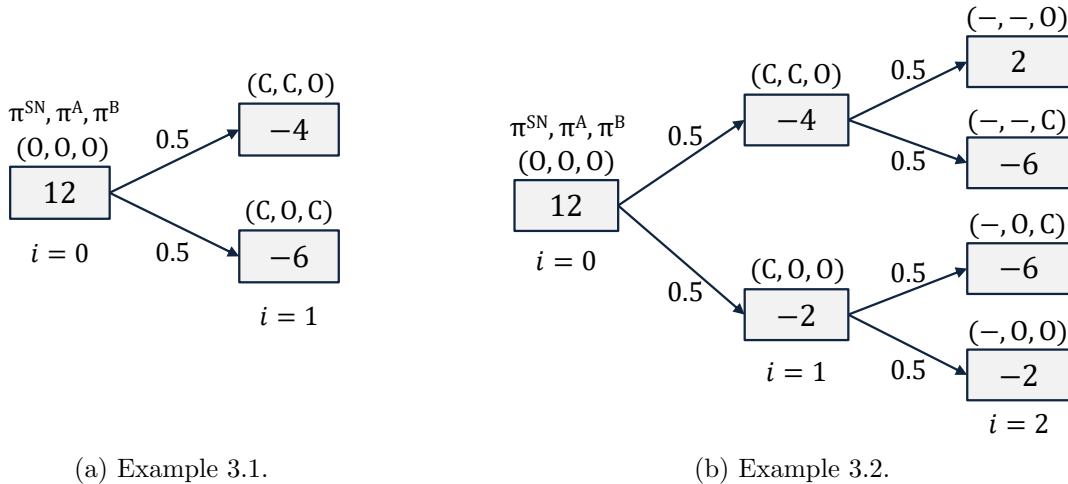


Figure 3.1: Production rewards, probabilities, and policies for examples 3.1 and 3.2.

Example 3.1 (Shutdown probability reduction). Consider the operations of a stylized plant that can either produce or shut down over two periods $i \in \mathcal{I} = \{0, 1\}$. The cost of shutdown is equal to 2 at each stage. Figure 3.1(a) displays the reward (inside the rectangular box) from producing at time 0 (now) as well as the two equally likely random rewards from producing at period 1 (future). We ignore discounting for simplicity. The optimal shutdown-neutral policy π^{SN} produces in period 0 but shuts down in both states in period 1 as indicated by the first element of the triples in Figure 3.1(a). The value of this policy is 10 ($= 12 - 2 \cdot 0.5 - 2 \cdot 0.5$) and its shutdown probability is 100%. Next consider the two policies π^A and π^B defined by the second and third coordinates, respectively, of the triples in Figure 3.1(a). Both policies have a shutdown probability of 50% and the values of π^A and π^B are 8 ($= 12 - 2 \cdot 0.5 - 6 \cdot 0.5$) and 9 ($= 12 - 4 \cdot 0.5 - 2 \cdot 0.5$), respectively. Thus, it is possible to reduce shutdown probability by 50% for a 10% decrease in asset value using π^B , but choosing π^A instead entails a 20% decrease in asset value to achieve

the same shutdown probability reduction.

Example 3.2 (Shutdown delay). Consider a plant operating for three-periods with production rewards and probabilities summarized in Figure 3.1(b). The shutdown cost at each stage equals 2. The coordinates of the triples in Figure 3.1(b) contain the decisions taken by the shutdown-neutral policy π^{SN} and two heuristic policies π^{A} and π^{B} . The value of π^{SN} is 10 and its shutdown probability is 100%. For both π^{A} and π^{B} , the shutdown probability and asset value are 50% and 8, respectively, but these policies shutdown at different periods. The expected time to shutdown on sample paths where π^{A} chooses to shutdown is equal to 1 ($= [0.5 \cdot 1]/0.5$). The analogous measure for π^{B} evaluates to 2 ($= [0.25 \cdot 2 + 0.25 \cdot 2]/0.5$). Therefore, for a 20% decrease in asset value, both π^{A} and π^{B} reduce shutdown probability by 50% but the latter policy delays shutdown by one more period on average compared to the former policy.

Finding operating policies to reduce and/or delay shutdowns without losing significant asset value is critical to the financial viability of partaking in social commerce. Conceptually, social preferences on the shutdown profile of a policy can be modeled by adding bounds on the shutdown probabilities at each stage to MDP (3.1):

$$\max_{\pi \in \Pi} \mathbb{E}_0 \left[\sum_{i \in \mathcal{I}} \delta^i r_i(x_i^\pi, w_i, A_i^\pi(x_i^\pi, w_i)) + \delta^I r_I(x_I^\pi, w_I) \right] \quad (3.3a)$$

$$\text{s.t. } \Pr(x_i^\pi = \text{C} \mid x_0, w_0) \leq \xi^{I-i} U, \quad \forall i \in \mathcal{I} \cup I. \quad (3.3b)$$

Here U is an upper bound on the shutdown probability at the terminal stage and ξ is a parameter that deflates this bound at earlier stages. The parameters U and ξ control the preferences to reduce and delay shutdown decisions, respectively. If both U and ξ are equal to 1, then MDP (3.3) reduces to the shutdown-neutral MDP (3.1) which only focuses on maximizing asset value. The opposing extreme of this trade-off includes policies with zero shutdowns. Among them, the one maximizing asset value can be obtained by setting $U = 0$ in (3.3), that is, removing the shutdown action C. This policy, labeled $\pi^{\text{SN} \setminus \{\text{C}\}}$, can be determined by solving the following modified version of SDP (3.2):

$$V_i^{\text{SN} \setminus \{\text{C}\}}(x_i, w_i) = \max_{a_i \in \mathcal{A}_i(x_i) \setminus \{\text{C}\}} \left\{ r_i(x_i, w_i, a_i) + \delta \mathbb{E}_i[V_{i+1}^{\text{SN} \setminus \{\text{C}\}}(f_i(x_i, a_i), w_{i+1})] \right\},$$

$$\forall (i, x_i, w_i) \in \mathcal{I} \times \mathcal{X}_i \times \mathcal{W}_i. \quad (3.4)$$

We do not write the boundary condition of this SDP and others in the rest of the paper as they are analogous to (3.2a).

Although the shutdown-neutral and zero-shutdown policies can be characterized, what is needed in practice is the ability to define a collection of policies that offer different

trade-offs between shutdown profile and asset value so that managers can choose one that is acceptable, possibly taking into account the non-financial costs of shutdown and other strategic considerations. To obtain this collection of policies, ideally we would solve our constrained MDP (3.3) for different choices of U and ξ . However, the solution of constrained MDPs is in general considerably more challenging than MDPs (Dufour and Prieto-Rumeau, 2013; Dufour and Prieto-Rumeau, 2014). The challenge of solving our constrained MDP is more acute as it is also high-dimensional under realistic models for the evolution of the uncertainty such as the one used in our numerical study and described in Section 3.5.2. Therefore, in Section 3.3 and Section 3.4 we approximate our constrained MDP using SDPs that stay close in structure to the shutdown-neutral SDP and develop approximate dynamic programming approaches to tackle them.

3.3 Operating policies based on modified shutdown costs

In this section, we present operating policies that embed preferences for reducing and delaying shutdowns by modifying the shutdown fixed cost of the shutdown-neutral SDP. In other words, we use this cost modification to indirectly capture the effect of the shutdown profile constraints in the constrained MDP (3.3). In Section 3.3.1, we describe a shutdown-averse SDP based on the aforementioned shutdown cost modification and an LSM approach to compute shutdown-averse policies. In Section 3.3.2, we introduce a reoptimization heuristic that solves deterministic versions of the shutdown-averse SDP.

3.3.1 Anticipated-regret SDP

We model aversion to a shutdown action consistent with anticipated regret theory in behavioral psychology (Loomes and Sugden, 1982; Zeelenberg, 1999). According to this theory, when choosing among multiple decisions, one accounts for the anticipated regret from the chosen decision being worse than an alternative under realizations of future uncertainty. This regret has also been shown to be significantly higher for irreversible decisions (Tsiros and Mittal, 2000), which is intuitive and also supports our focus on shutdowns in our merchant production setting. To formalize these ideas, let $X(w_{i+1})$ denote a random variable defined at stage i as a function of the random stage $i + 1$ information state w_{i+1} . Given a $\kappa \in \mathbb{R}_+$, we define anticipated regret of $X(w_{i+1})$ as

$$\mathbb{E}_i [\max\{X(w_{i+1}) + \kappa, 0\}].$$

To build some intuition, suppose that κ represents the shutdown cost and $X(w_{i+1})$ is the random utility from not shutting down the plant at a stage i operating state. The term $X(w_{i+1}) + \kappa$ is then the excess of this utility over the shutdown cost κ , which is the positive regret from shutting down on this specific realization of uncertainty. The maximum of this term and zero captures the realizations of w_{i+1} where regret is strictly positive. Thus, anticipated regret is the expected regret from shutting down along sample paths where regret is positive. Its usefulness for modeling aversion to shutdown decisions is illustrated in Example 3.3.

Example 3.3. Consider a production asset operating over 2 stages, that is, $\mathcal{I} \equiv \{0, 1\}$. This asset has a single non-shutdown decision and its shutdown cost κ is 11. We consider two cases for the random stage 1 utility when the decision at stage 0 is not to shut down. These cases are characterized by discrete reward distributions $X^1(w_1)$ and $X^2(w_1)$ defined as

$$X^1(w_1) := \begin{cases} -12 & \text{with probability } 0.5, \\ -28 & \text{with probability } 0.5; \end{cases} \quad X^2(w_1) := \begin{cases} +4 & \text{with probability } 0.25, \\ -28 & \text{with probability } 0.75. \end{cases}$$

The expected utility from not shutting down is the same in each case, that is, $\mathbb{E}[X^1(w_1)] = \mathbb{E}[X^2(w_1)] = -20$. Thus, the two cases are equivalent from a shutdown-neutral perspective. Since the expected utility of -20 is smaller than the shutdown cost of -11 , the optimal shutdown-neutral decision at stage 0 is to shut down in both cases. Suppose the producer now accounts for anticipated regret. The anticipated regret with respect to distribution $X^1(w_1)$ is zero because the reward in each scenario of case 1 is lower than $-\kappa$. Instead, this regret for distribution $X^2(w_1)$ is $(4 + 11) \cdot 0.25 = 3.75$. A decision maker that heavily weights anticipated regret would switch to a non-shutdown decision in case 2 but never switch in case 1.

We next use anticipated regret to modify the shutdown cost in the shutdown-neutral SDP (3.2). Let Θ denote the triple $\{\lambda, \tilde{\xi}, \eta\}$, which is feasible when $\lambda \geq 0$, $\tilde{\xi} \in (0, 1]$, and $\eta \geq 1$. The AR SDP is defined for each $(i, x_i, w_i) \in \mathcal{I} \times \mathcal{X}_i \times \mathcal{W}_i$ as

$$V_{\Theta, i}^{\text{A}, \mathcal{O}}(x_i, w_i) = \max_{a_i \in \mathcal{A}_i(x_i) \setminus \{C\}} \left\{ r_i(x_i, w_i, a_i) + \delta \mathbb{E}_i [V_{\Theta, i+1}^{\text{A}}(f_i(x_i, a_i), w_{i+1})] \right\}, \quad (3.5a)$$

$$\text{AR}_{\Theta, i}(x_i, w_i) = \max_{a_i \in \mathcal{A}_i(x_i) \setminus \{C\}} \mathbb{E}_i \left[\max \{ r_i(x_i, w_i, a_i) + \delta V_{\Theta, i+1}^{\text{A}}(f_i(x_i, a_i), w_{i+1}) + \eta \cdot K^{(x_i, C)}, 0 \} \right], \quad (3.5b)$$

$$V_{\Theta, i}^{\text{A}}(x_i, w_i) = \max \left\{ V_{\Theta, i}^{\text{A}, \mathcal{O}}(x_i, w_i), -K^{(x_i, C)} - \lambda \tilde{\xi}^i \text{AR}_{\Theta, i}(x_i, w_i) \right\}. \quad (3.5c)$$

The structure of the optimization used to compute $V_{\Theta, i}^{\text{A}, \mathcal{O}}(x_i, w_i)$ from $V_{\Theta, i+1}^{\text{A}}$ in (3.5a) is

identical to the one used in the shutdown-neutral setting, that is, (3.2b). However, comparing (3.5c) and (3.2c) shows that the shutdown cost is increased by $\lambda \tilde{\xi}^i \text{AR}_{\Theta,i}(x_i, w_i)$ in the former case, where the term $\text{AR}_{\Theta,i}(x_i, w_i)$ is based on a maximization of the anticipated regret across non-shutdown decisions. Specifically, the anticipated regret of a non-shutdown action $a_i \in \mathcal{A}_i(x_i) \setminus \{C\}$ is computed with respect to the sum of the immediate reward and the random discounted next stage value function, $r_i(x_i, w_i, a_i) + \delta V_{\Theta,i+1}^A(f_i(x_i, a_i), w_{i+1})$, and the shutdown cost $K^{(x_i, C)}$ scaled by η . The role of $\text{AR}_{\Theta,i}(x_i, w_i)$ can be interpreted as a way to dynamically increase the shutdown cost in a state dependent fashion using the value function.

Parameters λ and $\tilde{\xi}$ model the preferences to reduce and delay shutdowns, respectively. Increasing λ leads to a larger inflation of the shutdown cost due to the anticipated-regret term $\text{AR}_{\Theta,i}$. Reducing $\tilde{\xi}$ results in discounting $\text{AR}_{\Theta,i}$ more heavily at later stages than at earlier stages, which amounts to modeling the preference to delay shutdown. The effects of changing these parameters indirectly capture the impact of the shutdown profile constraints in MDP (3.3). The constant η controls the set of states where $\text{AR}_{\Theta,i}(x_i, w_i)$ is positive and can thus cause a reversal of a shutdown decision. As η (≥ 1) increases, the term $\text{AR}_{\Theta,i}(x_i, w_i)$ will become positive at states where it was previously zero due to the shutdown cost $K^{(x_i, C)}$ in its definition being multiplied by η .

Proposition 3.1 establishes properties of the AR value function $V_{\Theta,i}^A(x_i, w_i)$ and policy π_{Θ}^A .

Proposition 3.1. (a) For any feasible Θ , $V_i^{\text{SN} \setminus \{C\}}(x_i, w_i) \leq V_{\Theta,i}^A(x_i, w_i) \leq V_i^{\text{SN}}(x_i, w_i)$, $\forall (i, x_i, w_i) \in \mathcal{I} \times \mathcal{X}_i \times \mathcal{W}_i$;

(b) If \mathcal{W}_i is compact for each stage $i \in \mathcal{I} \cup \{I\}$, then for each $\tilde{\xi} \in (0, 1]$, it holds:

$$\lim_{(\lambda, \eta) \rightarrow \infty} \Pr(C; \pi_{\Theta}^A) = 0 \text{ and } \lim_{(\lambda, \eta) \rightarrow \infty} V_{\Theta,i}^A(x_i, w_i) = V_i^{\text{SN} \setminus \{C\}}(x_i, w_i).$$

Part (a) of Proposition 3.1 shows that the AR value function is bounded below and above, respectively, by the shutdown-neutral value functions that incorporate and exclude the shutdown action. Part (b) establishes that the shutdown probability associated with policy π_{Θ}^A becomes zero as η and λ are increased to sufficiently large values. In addition, it shows that the value of the latter policy converges to the value of the policy $\pi^{\text{SN} \setminus \{C\}}$, which is optimal among the set of all policies with zero shutdown probability. Therefore, the anticipated-regret policy exhibits desirable asymptotic behavior with respect to both shutdown probability and asset value. (We assume a compact set of information states \mathcal{W}_i , $i \in \mathcal{I} \cup \{I\}$, in part (b) of Proposition 3.1 to avoid the technicalities that arise when dealing with unbounded distributions.) The asymptotic behavior of the AR policy with respect to $\tilde{\xi}$ is intuitive. For $\tilde{\xi} = 1$, the policy is neutral to shutdown delay. As $\tilde{\xi}$ decreases, the shutdown cost is inflated lesser at later stages compared to earlier stages.

When $\tilde{\xi} \rightarrow 0$, there is no preference to delay shutdowns at stages other than stage 0, which translates to inflating only the shutdown cost at this initial stage.

The optimal policies of the AR SDP (3.5) for different triples Θ provides a family of shutdown-averse policies. However, solving this SDP is challenging as it suffers from the curses of dimensionality discussed of the shutdown-neutral SDP in Section 3.2.2, and in addition, embeds the anticipated regret term that contains more involved expectations. The regress-later least squares Monte Carlo (LSML) method computes heuristic policies based on low-dimensional approximations of the SDP value function (Glasserman and Yu, 2004a; Nadarajah et al., 2017). Specifically, we adapt an LSML approach to compute heuristic shutdown-averse policies by approximating the AR value function of SDP (3.5).

LSML computes a value function approximation (VFA) at each stage $i \in \{1, \dots, I\}$ that is a linear combination of a given set of B_i basis functions $\Phi_i := \{\Phi_{i,b}(w_i), b \in \mathcal{B}_i\}$, where $\mathcal{B}_i := \{1, \dots, B_i\}$. The VFA at stage i and state (x_i, w_i) is $\sum_{b \in \mathcal{B}_i} \beta_{i,x_i,b} \Phi_{i,b}(w_i)$, where $\beta_{i,x_i,b}$ denotes the b -th weight of this linear combination. Possible choices for basis functions include polynomials, radial basis functions, and Laguerre polynomials of the information state (Longstaff and Schwartz, 2001; Mazières and Boogert, 2013). At a high level, the LSML approach generates samples of the information state in Monte Carlo simulation and then combines backward recursion with regression to compute the VFA weights. Hard-to-compute expectations are replaced by sample average approximations in this procedure. For a given Θ , the output of LSML is the VFA weight vectors $\beta_{i,x_i}^A := (\beta_{i,x_i,1}^A, \dots, \beta_{i,x_i,B_i}^A)$ for each stage $i \in \{1, \dots, I\}$ and operating state $x_i \in \mathcal{X}_i$, which approximate $V_{\Theta,i}^A(x_i, w_i)$ by $\sum_{b \in \mathcal{B}_i} \beta_{i,x_i,b}^A \Phi_{i,b}(w_i)$. An AR policy decision can be computed at stage i and state (x_i, w_i) by first substituting the aforementioned VFA for $V_{\Theta,i}^A(x_i, w_i)$ in the right hand side of the AR SDP (3.5) and then solving the maximization over actions. The appendix 3.7.2 contains the details of the LSML algorithm and describes its use for approximating the AR SDP (3.5) as well as the shutdown-neutral SDPs (3.2) and (3.4).

By varying our choices of λ and $\tilde{\xi}$ in the triple Θ , LSML can be used to obtain a family of policies offering different trade-offs between asset value and shutdown profile in a tractable manner. We set a value for the parameter η in Θ based on Proposition 3.1(b), that is, we remove η as a policy parameter by fixing it before computing any AR policy. Specifically, we employ the following steps to choose a large enough value for this parameter so that increasing λ eventually leads to a zero-shutdown policy: (i) we compute the shutdown-neutral policy π^{SN} , (ii) we simulate the inflation term $\text{AR}_{\Theta,i}$ in (3.5b) for different η values, and measure at each stage the percentage of states where this term is positive, and (iii) we select the smallest η value so that the coverage is 100% at each stage. Under this choice, the AR policy converges very close to a zero-shutdown policy as λ is increased.

3.3.2 Reoptimization heuristic

We focus here on computing shutdown-averse policies based on a static modification of the shutdown cost, that is, this modification, unlike in the AR policy, does not depend on the state. Given the parameters $\lambda \geq 1$ and $\tilde{\xi} \in (0, 1]_2$, we define a modified reward function $r_i^{\lambda, \tilde{\xi}}(\cdot, \cdot, \cdot)$ by increasing only the shutdown cost: $r_i^{\lambda, \tilde{\xi}}(x_i, w_i, C) := -\lambda \tilde{\xi}^i K^{(x_i, C)}$ and $r_i^{\lambda, \tilde{\xi}}(x_i, w_i, a_i) := r_i(x_i, w_i, a_i)$ for all $(x_i, w_i, a_i) \in \mathcal{X}_i \times \mathcal{W}_i \times \mathcal{A}_i(x_i) \setminus \{C\}$. We embed this static cost modification in a reoptimization heuristic (RH) that solves at each stage $i \in \mathcal{I}$ a deterministic version of the AR SDP (3.5). At stage i and open state $(x_i, w_i) \in \mathcal{O}_i \times \mathcal{W}_i$, our RH policy solves

$$\max_{\{(x'_j, a_j), j \in \mathcal{I}, j \geq i\}} \sum_{j \in \mathcal{I}, j \geq i} \delta^j \mathbb{E}_i[r_j^{\lambda, \tilde{\xi}}(x'_j, w_j, a_j)] + \delta^I \mathbb{E}_i[r_I^{\lambda, \tilde{\xi}}(x'_I, w_I)] \quad (3.6a)$$

$$x'_i = x_i, \quad (3.6b)$$

$$x'_{j+1} = f_j(x'_j, a_j), \forall j \in \mathcal{I}, j \geq i, \quad (3.6c)$$

$$a_j \in \mathcal{A}_j(x'_j), \forall j \in \mathcal{I}, j \geq i, \quad (3.6d)$$

$$x_j \in \mathcal{X}_j, \forall j \in \mathcal{I} \cup \{I\}, j \geq i. \quad (3.6e)$$

The optimization variables are the operating states and actions for stages between i and I . The objective function (3.6a) is the discounted expected sum of rewards over these stages conditioned on the market information at stage i . Constraint (3.6b) initializes the stage i operating status decision variable to the current operating state. Constraints (3.6c) capture the operating state transition rules. Constraints (3.6d) and (3.6e) restrict decision variables to their respective discrete feasible domains.

Using the RH policy involves reactively accounting for market information. Specifically, at a given stage i and open state (x_i, w_i) , this policy solves the optimization problem (3.6) and implements the optimal decision corresponding to stage i alone. If this decision is to shut down, then the process ends. Otherwise, the plant remains in an open state in stage $i + 1$ and new market information becomes available at this stage. Then a stage $i + 1$ optimization model is formulated using the updated state information and the process is repeated. Estimating the value of the RH policy involves executing its decisions along sample paths of market information generated in Monte Carlo simulation, which requires solving the RH optimization model for each stage and sample path. Thus, being able to solve this optimization efficiently is important. Given the discrete nature of states and actions, the optimization problem (3.6) is in general an integer program but Proposition 3.2 shows that it can be cast as an efficiently solvable shortest path problem. The proof of this proposition contains the details of a directed acyclic graph representation of the RH optimization model (3.6) and the associated shortest path problem. Let $\bar{\mathcal{X}}$ be the

maximum cardinality of the endogenous state sets \mathcal{X}_j , $j \in \{i, \dots, I\}$.

Proposition 3.2. *An optimal solution to the RH optimization model (3.6) formulated at stage $i \in \mathcal{I}$ can be obtained by solving a shortest path problem in $\mathcal{O}(I \cdot \bar{\mathcal{X}}^2)$ time.*

3.4 Operating policies based on production margins

In this section, we describe policies that determine operating decisions based on production margins. We first formalize a version of such policies used in practice and then improve on them by combining approximate dynamic programming and machine learning classification methods.

Stakeholders often consider poor production margins as a factor when making shut down decisions. Production margin-based policies are appealing as they choose to shut down when production margins are below a certain threshold. For example, a myopic margin-based policy would (i) shut down the plant when the immediate production margin is less than the shutdown cost and (ii) continue to produce otherwise. More formally, assuming that the plant is producing at stage i and exogenous state w_i , a shut down decision is selected if $r_i(\text{O}, w_i, \text{O}) < -K^{(\text{O}, \text{C})}$. The shortsighted nature of this policy can be overcome by modifying it to consider the sum of current and discounted future production margins. A forward-looking policy that considers T expected-future margins chooses to shut down at stage i and state (O, w_i) if

$$\widehat{r}_i^T(w_i) := r_i(\text{O}, w_i, \text{O}) + \sum_{j=1}^{\min\{T, I-i\}} \delta^j \mathbb{E}_i [r_{i+j}(\text{O}, w_{i+j}, \text{O})] < -K^{(\text{O}, \text{C})}, \quad (3.7)$$

and continues to produce otherwise.

The aforementioned simple production margin policies use the fixed cost $K^{(\text{O}, \text{C})}$ as the threshold to switch between full production and shutdown. This threshold choice is somewhat adhoc and, in addition, does not incorporate the effect of shutdown profile constraints in (3.3b). Moreover, it is unclear how these policies can be extended to account for the full flexibility of the plant, that is, incorporate operating states other than full production. We address these issues next by presenting a more principled approach that first learns thresholds in an attempt to “mimic” the shutdown-neutral policy and then modifies them to account for the shutdown profile constraints.

Suppose momentarily that the only operating state is full production O . For each stage $i \in \mathcal{I}$, we define a threshold $\Upsilon_i \in \mathbb{R}$ on the cumulative margin $\widehat{r}_i^T(w_i)$ below which the de-

cision switches from full production to shutdown. This threshold is computed as follows. First, we simulate the shutdown-neutral policy π^{SN} defined in Section 3.2.2 along sample paths $\{w_i^p, (i, p) \in \mathcal{I} \cup \{I\} \times \mathcal{P}\}$ of the uncertain information state generated in Monte Carlo simulation (π^{SN} can be approximated using LSML as discussed in Section 3.3.1 and Appendix 3.7.2). Second, at each stage $i \in \mathcal{I}$, we compute for each of the samples $p \in \mathcal{P}$ the cumulative margin $\widehat{r}_i^T(w_i^p)$ and an action label from the shutdown-neutral policy π^{SN} , which is either to continue full production (i.e. O) or shutdown (i.e. C). Finally, based on these two labels, we partition the sample paths at each stage into two classes. The value of the threshold Υ_i is chosen as the cumulative margin value that best discriminates between (or separates) these two classes. Hence, identifying the shutdown threshold is equivalent to solving a binary classification problem where $\widehat{r}_i^T(w_i) \in \mathbb{R}$ is the explanatory variable and $x_{i+1} \in \{O, C\}$ is the outcome class (Bishop, 2006). In general, it may not be possible to find a threshold that perfectly separates the “produce” and “shutdown” classes, in which case, it is standard to minimize misclassification error to compute the threshold, where error is measured using a loss function. Common choices in machine learning include hinge, squared, and logarithmic loss functions (Rosasco et al., 2004).

The above classification procedure can be extended to handle multiple operating states. To ease exposition, we focus on the case of two operating states, that is, $\mathcal{O} := \{O, S\}$, where S denotes a temporary suspension of production. The ideas discussed below extend to handle more than two operating states with straightforward modifications. Suppose the plant is open at stage i ($x_i = O$) and can transition to state $x_{i+1} \in \{O, S, C\}$. We simulate the shutdown-neutral policy π^{SN} at stage i over multiple sample paths of the uncertain information state $p \in \mathcal{P}$ and compute the cumulative margin $\widehat{r}_i^T(w_i^p)$ as before, but divide the sample paths using three action labels corresponding to the shutdown-neutral policy choosing to continue full production O, suspend production S, and shutdown C. Implementing a production margin policy entails finding two thresholds $\Upsilon_i(O, S)$ and $\Upsilon_i(O, C)$ to determine when the plant switches from full production to suspension and shutdown, respectively. We proceed to define these thresholds by converting the multi-class classification problem over the three action labels in set $\{O, S, C\}$ into several binary classification problems between pairs of such classes, which are $\{O, S\}$, $\{S, C\}$, and $\{O, C\}$. This is a well-known strategy in machine learning (see, e.g., Bishop, 2006, ch. 4). Applying a binary classification procedure to each pair would result in three different thresholds, that is, one more than the required number of thresholds. We resolve this issue by assuming an adjacency structure over the cumulative margins associated with action labels as illustrated in Figure 3.2(a). Specifically, cumulative margins associated with the action labels O and S as well as S and C are assumed adjacent, while those associated with O and C are non-adjacent. In other words, the action labels have a weak ordering with respect to the cumulative margin. Then applying binary classification to $\{O, S\}$ and $\{S, C\}$ gives us

the thresholds $\Upsilon_i(O, S)$ and $\Upsilon_i(O, C)$, respectively.

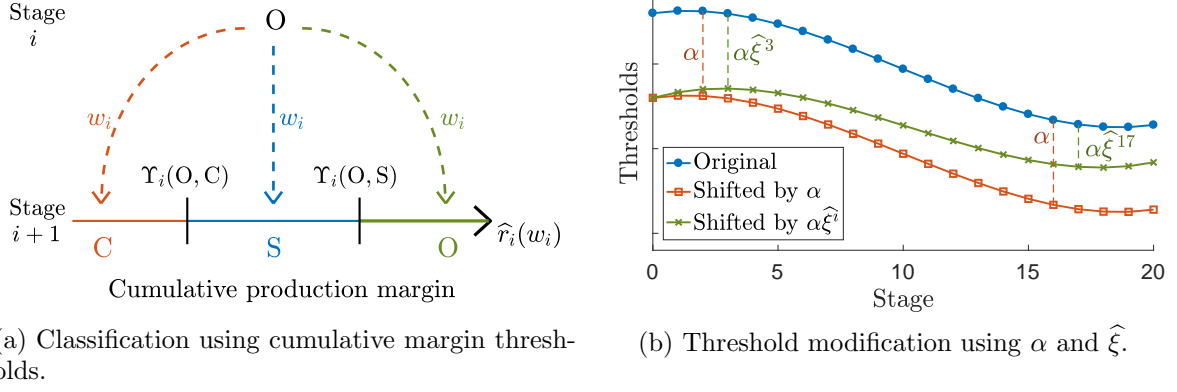


Figure 3.2: Illustration of threshold computation using binary classification with states in set $\{O, S, C\}$ and subsequent modification of thresholds to model shutdown profile preferences.

Finally, we modify the shutdown thresholds $\{\Upsilon_i(O, C), \Upsilon_i(S, C), i \in \mathcal{I}\}$, which mimic the shutdown-neutral policy, to account for shutdown profile preferences as illustrated in Figure 3.2(b). Decreasing the value of these thresholds (blue curve) by a constant at each stage results in a set of downward-shifted thresholds (orange curve), which will result in a policy with a smaller shutdown probability. In other words, the threshold levels at each stage are proportional to the shutdown probability at that stage when using a production margin policy, thus providing a lever to shape the shutdown profile. As a consequence, preferences to delay shutdown decisions can be emphasized by decreasing thresholds by a larger value at earlier stages in the horizon and less aggressively at later periods, which involves modifying the original shutdown probability thresholds (blue curve) and obtaining a shifted and tilted set of thresholds (green curve). Formally, given a “shift” parameter $\alpha \geq 0$ and a “tilt” parameter $\hat{\xi} \in (0, 1]$, we define the modified shutdown thresholds as follows:

$$\begin{aligned}\Upsilon_i^{\alpha, \hat{\xi}}(O, C) &:= \Upsilon_i(O, C) - \alpha \hat{\xi}^i, \quad \forall i \in \mathcal{I}, \\ \Upsilon_i^{\alpha, \hat{\xi}}(S, C) &:= \Upsilon_i(S, C) - \alpha \hat{\xi}^i, \quad \forall i \in \mathcal{I},\end{aligned}$$

and leave unchanged the non-shutdown thresholds, e.g. $\Upsilon_i(O, S)$. The parameter α represents the magnitude of a downward-shift in the thresholds related to shutdown actions, while $\hat{\xi}$ tilts the threshold curve to account for shutdown delay. We vary the values of α and $\hat{\xi}$ in Monte Carlo simulation to obtain a family of policies with different shutdown profile preferences. We first fix $\hat{\xi}$ equal to a constant ξ and then perform a line search on α to identify its smallest value for which the shutdown probability of the corresponding production margin policy satisfies the bound U in (3.3b) at the last stage. Modifications to this procedure are possible, for instance, one could do a grid search over both α and $\hat{\xi}$,

if needed. Overall, we have generalized simple practice-based production margin policies to use thresholds computed in a principled manner while accounting for shutdown profile preferences and maintaining their intuitive margin structure.

3.5 Numerical study

We numerically evaluate the performance of our methods in this section. In Section 3.5.1, we introduce a case study of a real aluminum producer, which serves as our application for this evaluation. In Section 3.5.2, we describe the stochastic process used for modeling the evolution of uncertainty. In Section 3.5.3, we define the aluminum production instances used for our experiments and the associated computational setup. In Section 3.5.4, we discuss our findings.

3.5.1 Aluminum production case study

Our case study is based on a real aluminum producer. Shutting down an aluminum production plant (often referred to as a smelter) or temporarily suspending its production are strategic decisions that are re-evaluated on an annual basis. Assessing the shutdown profile thus requires planning over a long time horizon. Our instance considers a forty-year horizon (i.e., $I = 40$) where each stage i corresponds to a year and decisions are made from $i = 0$ (present) to $I - 1 = 39$.

Aluminum production relies on an energy intensive electrolysis process that takes as inputs alumina from bauxite, carbon, and electricity, and produces aluminum as its main output. The aluminum producer is vertically integrated and owns bauxite mines and carbon plants. Therefore, capturing the uncertainty in alumina and carbon prices is not critical, whereas modeling the volatile electricity and aluminum prices is important. In addition to price risk, the producer faces exchange rate risk because aluminum is sold globally in US dollars (USD), electricity is purchased in Euro (EUR) from the Nord Pool electricity market, and the operating costs are incurred in Norwegian Krone (NOK), that is, the local currency. Assuming cash flows are measured in USD, the production spread is exposed to volatile EUR-USD and NOK-USD exchange rates. Thus, we model four sources of uncertainty in the set $\{P_i^{\text{AL}}, P_i^{\text{EL}}, P_i^{\text{EUR-USD}}, P_i^{\text{NOK-USD}}\}$ denoting, respectively, aluminum price, electricity price, and the two exchange rates. We capture the dynamics of prices and exchange rates using eight stochastic factors w_i discussed in Section 3.5.2.

The specific aluminum plant we model has a shutdown state (C) and open states corresponding to (i) production at full load (O), which we normalize to an output of 1 metric tonne of aluminum; and (ii) temporary suspension. We denote by S_m the m -th consecutive year of suspension, where m is restricted to at most 3, that is, $m \in \{1, 2, 3\}$. Intermediate production capacity options are not modeled in this case since the electrolysis cells within a smelter operate in a near continuous fashion, that is, these cells can only be turned off for a few hours. A prolonged shut off results in cell damage and expensive repairs before a restart (see, e.g., Øye and Sørli, 2011). Thus, $\mathcal{O}_i = \{O, S_1, S_2, S_3\}$ for each $i \in \mathcal{I} \setminus \{0\}$. We assume that the plant must be either shutdown or in production at the end of the planning horizon.

Table 3.1: State-action set and reward function in the aluminum production case study.

State $[x_i]$	Decision $[a_i]$	Reward $[r_i(x_i, w_i, a_i)]$
O	O	$r_O(w_i)$
	S_1	$-K^{(O,S_1)}$
	C	$-K^{(O,C)}$
$S_m, m \in \{1, 2, 3\}$	O	$r_O(w_i) - K^{(S_m,O)}$
	S_{m+1} (if $m < 3$)	$-K^{(S_m,S_{m+1})}$
	C	$-K^{(S_m,C)}$
C	C	0

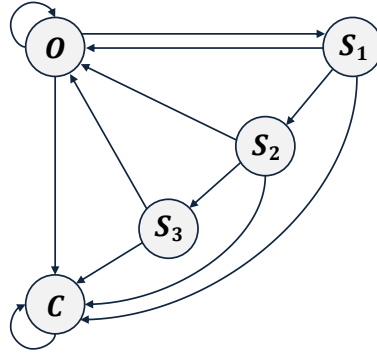


Figure 3.3: State transitions in the aluminum production case study.

Table 3.1 and Figure 3.3 illustrate the set of feasible actions $\mathcal{A}_i(x_i)$ at a given operating state $x_i \in \mathcal{X}_i$, the resulting reward, and a diagram indicating the next stage operating state. Here $K^{(x_i, x_{i+1})}$ represents the fixed operating and/or maintenance cost associated with a state transition from x_i to x_{i+1} . The function $r_O(w_i)$ denotes the profit from producing aluminum and is defined as

$$r_O(w_i) := (1 - \tau) [P_i^{\text{AL}}(1 + \gamma^{\text{AL}}) - c^{\text{USD}} - c^{\text{NOK}} P_i^{\text{NOK-USD}} - \rho P_i^{\text{EL}} P_i^{\text{EUR-USD}}], \quad (3.8)$$

where τ is the corporate tax rate; γ^{AL} the aluminum premium, which is the surcharge a buyer pays for taking delivery of aluminum; c^{USD} and c^{NOK} the production fixed costs incurred in USD and NOK, respectively; and ρ the rate of electricity consumption in the aluminum production process. We have fixed costs in both USD and NOK because they include components of the electrolysis and casting costs in each of these currencies.

3.5.2 Model of price and exchange rate dynamics

As discussed in Section 3.5.1, the exogenous market information in our SDP consists of the factors driving four sources of uncertainty: Aluminum price, electricity price, and two exchange rates (EUR-USD and NOK-USD). A stochastic process to model the evolution of these sources of uncertainty can be calibrated using information from financially traded contracts. Futures contracts on (primary) aluminum are traded at the LME in US dollars with monthly maturities going out to 10 years. The aluminum contract with a 3-month maturity is the most liquid. Currency futures are traded in the FOREX market with monthly maturities up to 3 years. In contrast to aluminum and currencies, power is traded in regional markets. In the Nord Pool market, which covers Scandinavia and part of northern Europe, power forward contracts are denominated in Euro and extend out to 10 years. Negative prices present in the electricity spot markets are not a feature seen in these longer term contract prices. Moreover, none of the aforementioned contracts have maturities that cover our planning horizon of 40 years.

We capture the dynamics of the four sources of uncertainty using an eight-factor continuous-time price model, where each source is driven by short term and long term factors. All the eight factors are correlated. This choice addresses the lack of traded contracts for the entire planning horizon and captures the clear short and long term trends observed in market data (see page 55 of Section 3.1 for a related discussion), which we illustrate in Figure 3.4 using only the front month contracts from the broader data set used for calibration. In particular, we denote the short and long term factors by the vectors $Y(t) = [Y_t^{\text{EL}}, Y_t^{\text{AL}}, Y_t^{\text{EUR-USD}}, Y_t^{\text{NOK-USD}}]$ and $Z(t) = [Z_t^{\text{EL}}, Z_t^{\text{AL}}, Z_t^{\text{EUR-USD}}, Z_t^{\text{NOK-USD}}]$, respectively. These factors evolve according to the following stochastic differential equations:

$$d \begin{bmatrix} Y(t) \\ Z(t) \end{bmatrix} = \begin{bmatrix} -K_y(Y(t) - Z(t)) \\ \mu_z - K_z Z(t) \end{bmatrix} dt + \begin{bmatrix} \Sigma_y^{1/2} & 0 \\ \Sigma_{yz}^{1/2} & \Sigma_z^{1/2} \end{bmatrix} d \begin{bmatrix} W_y(t) \\ W_z(t) \end{bmatrix}, \quad (3.9)$$

where K_y and K_z represent the speeds of mean reversion and are assumed to be diagonal (4×4) matrices, μ_z is the long term drift, $\Sigma_y^{1/2}$, $\Sigma_z^{1/2}$, and $\Sigma_{yz}^{1/2}$ are diffusion matrices, and W_y and W_z are each independent four-dimensional standard Brownian motions. The

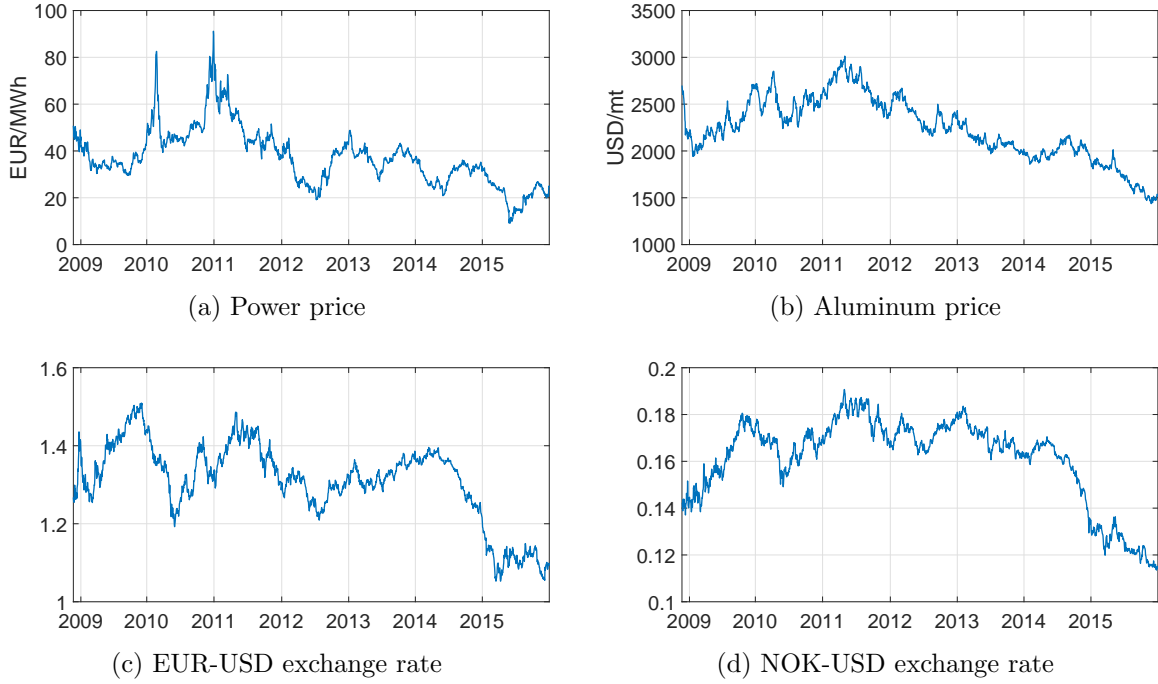


Figure 3.4: Front month futures prices for power and aluminum and exchange rates for EUR-USD and NOK-USD between 2009 and 2015.

short term factors in this model revert to the long term factors. Moreover, the short term factors are related to the prices and exchange rates by $P_t^k = \exp(Y_t^k - \psi^k(t))$, for $k \in \mathcal{J}$, where $\psi^{(\cdot)}(t)$ is a function that captures seasonality (see Appendix 3.7.3 for details). Our model can be seen as a multi-commodity extension of the one in Schwartz and Smith (2000), where we added a cross-commodity correlation structure, and a special case of the process in Farkas et al. (2017). Under this specification, the information state is $w_i = (Y_{i,k}, k \in \mathcal{J}) \cup (Z_{i,k}, k \in \mathcal{J})$, where $Y_{i,k}$ and $Z_{i,k}$, respectively, denote the k -th short and long term factor values at the beginning of stage i . Moreover, $\mathcal{W}_i = \mathbb{R}_+^8$.

Calibrating our multi-asset and multi-factor stochastic process requires several steps (see Appendix 3.7.3 for details). We collected futures contract data with multiple maturities for aluminum, power, and exchange rates from the LME, Nord Pool, and FOREX markets, respectively. We used interpolation and spline smoothing to ensure that data across different sources were consistent. We then employed a multi-stage Kalman filter process to estimate parameters. Our calibrated model provides a statistically sound representation of market data and allowed us to generate sample paths of the sources of uncertainty in Monte Carlo simulation needed to implement our algorithms as well as estimate the asset value and shutdown profile of a policy.

3.5.3 Instances and computational setup

We define our reference instance using the stochastic process calibration described in Section 3.5.2 and real operational data from an aluminum producer, which includes parameter values of the reward function (3.8), the operations and maintenance costs, and the discount factor. We summarize this information in Table 3.2.

Table 3.2: Parameters for the aluminum production reference instance.

Name	Parameter description	Value	Unit
τ	Corporate tax rate	25%	-
γ^{AL}	Aluminum premium as a percentage of aluminum price	5%	-
c^{USD}	USD fixed production cost	520	USD/mt
c^{NOK}	NOK fixed production cost	6110	NOK/mt
ρ	Electricity consumption rate	14	MWh/mt
δ	Discount factor	0.971	-
$K^{(\text{O},\text{S}_1)}$	Switching cost from production to suspension	600	USD/mt
$K^{(\text{O},\text{C})}$	Switching cost from production to shutdown	1200	USD/mt
$K^{(\text{S}_m,\text{C})}$	Switching cost from suspension to shutdown	600	USD/mt
$K^{(\text{S}_m,\text{O})}$	Switching cost from suspension to production	600	USD/mt
$K^{(\text{S}_m,\text{S}_{m+1})}$	Cost to remain in suspension	0	USD/mt

We define the terminal reward function at a production state by extending the problem horizon beyond the actual planning horizon, which is standard. Specifically, we choose $r_I(\text{O}, w_I)$ to represent the value of a plant that operates for 20 years starting from stage I and state (O, w_I) using an LSM approach. We employ the same terminal condition across all methods for consistency.

By modifying the reference instance described above, we created an extended instance set to study the performance of our methods when operational and market parameters change. We increased/decreased the costs c^{USD} and c^{NOK} , which are the only two parameters in the reward function (3.8) that are directly related to plant operations. Adjusting these two parameters result in different effects on the reward function because the first cost c^{USD} appears on its own while the second cost c^{NOK} multiplies the exchange rate $P_i^{\text{NOK-USD}}$ (see reward function (3.8)). We changed these costs by $\pm 15\%$ to obtain instances OP1-OP4 in Table 3.3 because these perturbations resulted in asset value changes of $\pm 40\%$, which seem reasonable upper bounds on the changes that can be expected in practice. We also considered significantly changing the volatility estimates of both the short and long term factors for aluminum and power by $\pm 30\%$ to obtain instances MP1-MP4 in Table 3.3. Power price volatilities vary by region while LME is the primary market for assessing aluminum volatility. Our aluminum price volatility change is consistent with

historic data: Brunetti and Gilbert (1995) examined the monthly volatility of LME-traded metals, including aluminum, over a 24-year period and $\pm 30\%$ is representative of the maximum variations they observe.

Table 3.3: Changes in operational and market parameters of the reference instance to obtain the extended instance set.

Label	Instance description
OP1	Cost c^{USD} is raised by 15%
OP2	Cost c^{USD} is reduced by 15%
OP3	Cost c^{NOK} is raised by 15%
OP4	Cost c^{NOK} is reduced by 15%
MP1	Power volatility is raised by 30%
MP2	Power volatility is reduced by 30%
MP3	Aluminum volatility is raised by 30%
MP4	Aluminum volatility is reduced by 30%

We next describe our computational setup. To compute AR (i.e., anticipated-regret) policies, we used LSML as described in Section 3.3.1. We also used LSML to compute heuristic versions of the shutdown-neutral policy π^{SN} and the zero-shutdown policy $\pi^{\text{SN}\setminus\{C\}}$ which include and exclude the shutdown option, respectively. We find the switching thresholds of the classification based margin policies (i.e., CM policies) while computing the LSML VFA for π^{SN} . We chose the LSML basis functions to be the following set of third-degree polynomials of the eight stochastic factors discussed in Section 3.5.2:

$$\{\Phi_{i,b}(w_i)\}_{b=1}^{B_i} = \{1, Y_{i,k}, Z_{i,k}, Y_{i,k_1}Y_{i,k_2}, Z_{i,k_1}Z_{i,k_2}, Y_{i,k}Z_{i,k}, Y_{i,k_1}Y_{i,k_2}Y_{i,k_3}, Z_{i,k_1}Z_{i,k_2}Z_{i,k_3}, Y_{i,k}Z_{i,k}^2, Y_{i,k}^2Z_{i,k} \mid k, k_1, k_2, k_3 \in \mathcal{J}\}.$$

Based on experimentation, we chose the number of regression samples and inner samples in LSML (see Algorithm 4 in Appendix 3.7.2) equal to 20,000 and 200, respectively. We estimated the asset value under each policy in Monte Carlo simulation using 20,000 sample paths. For consistency, we ensured that the same sample paths were used across policy evaluations. We also estimated (dual) upper bounds in Monte Carlo simulation using the LSML VFA (see Appendix 3.7.4 for details). We implemented the binary classification scheme needed to obtain CM policies with both hinge and squared loss functions, and found them to perform almost identically. We thus only report results related to the squared loss case.

We implemented all the algorithms using Matlab R2016b and executed them on a server equipped with two Intel Xeon E5-2660v3 processors with 10 cores each and a shared memory of 128 GB RAM. Obtaining families of policies offering different asset value and

shutdown profile trade-offs required significant computation. We distributed the computational load across the 20 cores in our server to significantly reduce the run time to obtain trade-off curves. Specifically, when using 20,000 sample paths on a single core, estimating the LSML VFA weights consumed about 18 minutes for each of the two shutdown-neutral policies and 22 minutes on average for the AR policy for each value of λ and $\tilde{\xi}$. Computing the asset value estimate of one of these policies required roughly the same running time as estimating the VFA, and estimating a dual bound took about 16 minutes on average. The RH asset value estimation took 40 minutes on average, mainly due to the time to compute expected margins. Computing the expected margins in the production margin-based heuristics used in practice and our extensions thereof required 4 minutes. Fitting the CM thresholds (including the LSML run) as discussed in Section 3.4 entailed an additional 18 minutes of CPU time. Updating these thresholds for a given U and $\hat{\xi}$ took less than 10 seconds on average.

3.5.4 Results

We start by considering the reference instance described in Section 3.5.3. The asset value estimates for the policies π^{SN} and $\pi^{\text{SN} \setminus \{C\}}$ are within 2.0% and 2.6% of their respective dual bound estimates (standard errors of asset value and dual bound estimates are at most 1.23% and 0.10%, respectively). In other words our LSML shutdown-neutral policies are near optimal. The asset value estimate of π^{SN} will henceforth be referred to as maximum asset value.

To evaluate the shutdown profile of a policy, we used two metrics: (i) the shutdown probability, that is, the total percentage of shutdowns at the end of the planning horizon, and (ii) the shutdown delay, that we measure by computing the expected time to shutdown (over sample paths where a shutdown decision is made). We begin by assessing the performance of different policies along the shutdown probability dimension only, that is, we temporarily neglect the preference to delay shutdown by fixing $\tilde{\xi} = 1$ in LSML and $\hat{\xi} = 1$ in the threshold shifting procedure at the end of Section 3.4.

Figure 3.5 displays the trade-offs between asset value and shutdown probability for the policies discussed in Sections 3.3-3.4. Each asset value estimate is expressed as a percentage of the maximum asset value (standard errors of the asset value and shutdown probability are at most 1.24% and 0.35%, respectively). The asterisk in Figure 3.5 labeled SN corresponds to the maximum asset value and its shutdown probability is 39.2%. The asset value estimate of $\pi^{\text{SN} \setminus \{C\}}$ is only 75.4% of the maximum asset value (i.e., the asterisk labeled SN \setminus {C} in this figure). Thus, completely removing the shutdown option

is not a practically viable strategy to reduce shutdown probability as the associated asset value loss is substantial.

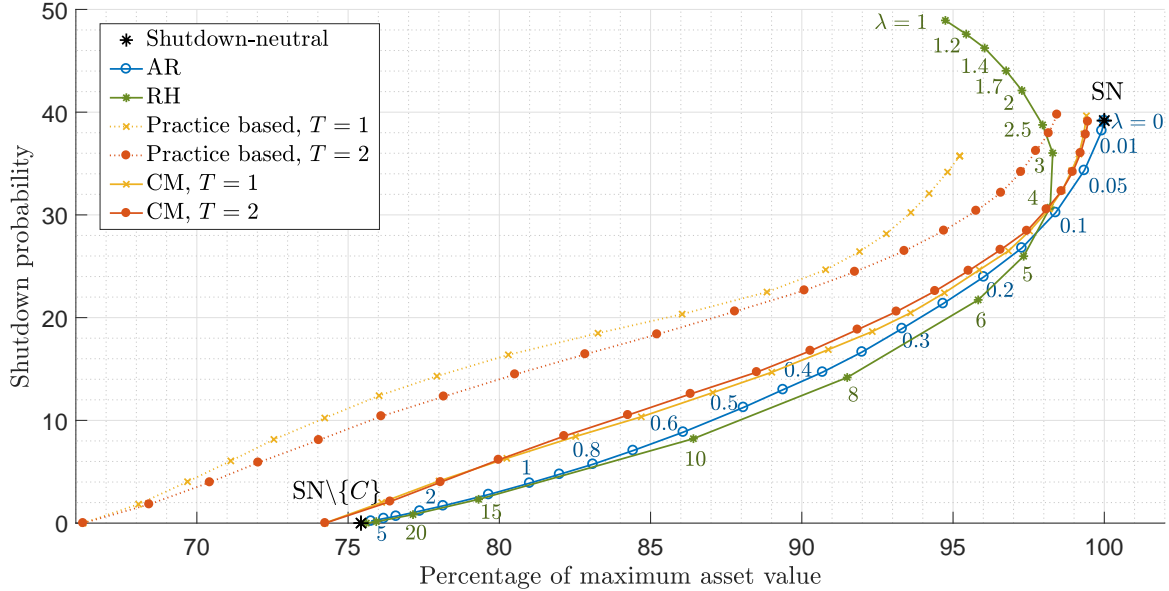


Figure 3.5: Trade-offs in asset value and shutdown probability.

The AR policies were computed by varying λ from 0 to 10 to obtain policies that range between the shutdown-neutral policy and the zero-shutdown policy. We find that the AR approach performs well and achieves substantial shutdown probability reductions for small asset value losses. For example, the shutdown probability is reduced by 26% and 40% for asset value losses of 2% and 4%, respectively. Consistent with Proposition 3.1, the AR policies converge to $\pi^{\text{SN}\{C\}}$, which is the optimal zero-shutdown asset value, when λ is large enough and η is chosen as discussed at the end of Section 3.3.1.

We tested the RH policy for different λ values ranging from 1 to 30, and found that the standard RH policy ($\lambda = 1$) is suboptimal in terms of both asset value and shutdown probability which are, respectively, 5.3% lower and 25% higher than the same measures for the shutdown-neutral policy π^{SN} . However, our shutdown-averse RH extension ($\lambda > 1$) not only reduces shutdown probability but also captures additional asset value, and the asymptotic RH behavior for large asset value losses is efficient.

The production margin-based policies were tested using one or two expected-future margins, in addition to the current margin, which entails choosing $T \in \{1, 2\}$. The shutdown probability U was varied from 40% to 0%. The best asset value achieved by the practice-based policies (dotted lines) when T equals 2 is only around 1.3% less than the maximum asset value, but it is 4.8% when T equals 1. Both practice-based policies decrease shutdown probability in an inefficient manner. We instead find that the CM method

outperforms the practice-based methods and is more robust, that is, the asset value and shutdown probability trade-off curves produced by CM for T equals one and two are almost identical. The performance of CM is comparable to AR for small asset value losses but slightly worse for larger losses, which suggests that the favorable asymptotic properties of AR policies translate into good performance of these policies in non-asymptotic settings as well. For example, the shutdown probability is reduced by 25% in the CM and AR policies for asset value losses of 2.2% and 1.8%, respectively.

Overall, the methods we compare exhibit different performance profiles. From a practical standpoint, the performance for small asset value losses (e.g., right portion of the graph from 0-5%) is most relevant as these losses have a higher likelihood of being accepted by the producer when managing shutdown decisions. In this region, practice-based heuristics are not competitive with AR and CM while the performance of RH is non-monotonic in λ , which may lead to an increase in shutdown probability when using it. Since the performance of AR and CM policies are similar for small asset value losses, we focus on them in the remainder of this section when discussing results pertaining to both the reduction and delay of shutdowns.

Intuitively, one would expect that delaying shutdown decisions also reduces the shutdown probability at the end of the horizon. However, this intuition may not be true because delaying/reducing shutdowns also affects asset value. Therefore, in Figure 3.6 we display the trade-off between shutdown probability reduction and shutdown delay for fixed asset value losses. To obtain this figure, we considered $\tilde{\xi} \in \{1, 0.99, 0.98, 0.97, 0.95\}$ for AR policies and $\hat{\xi} \in \{1, 0.98, 0.95, 0.93, 0.9\}$ for CM policies with T equal to 2. For each $\tilde{\xi}$ and $\hat{\xi}$, we computed profiles akin to the ones in Figure 3.5 so that we could choose AR and CM policies that matched a target asset value within 0.3%.

Figure 3.6 suggests that shutdown delay and shutdown reduction are substitutes for a fixed asset value and cannot be improved simultaneously. For example, for an asset value loss of 6%, AR could delay shutdowns by 6 years on average and decrease the shutdown probability of 23%, or delay shutdowns by an average of only 3 years and capture a larger shutdown probability decrease of 44%. Furthermore, the concavity of most trade-off curves in Figure 3.6 suggests diminishing returns: Higher average shutdown delays imply a higher marginal increment in shutdown probability, and vice versa. The trade-offs originating from AR slightly dominate the CM ones, that is, for the same asset value and shutdown probability, AR policies delay shutdown by roughly one additional year on average relative to the CM policies. As one would expect, it appears harder to manage the preference to reduce and to delay shutdown decisions substantially for smaller asset value losses.

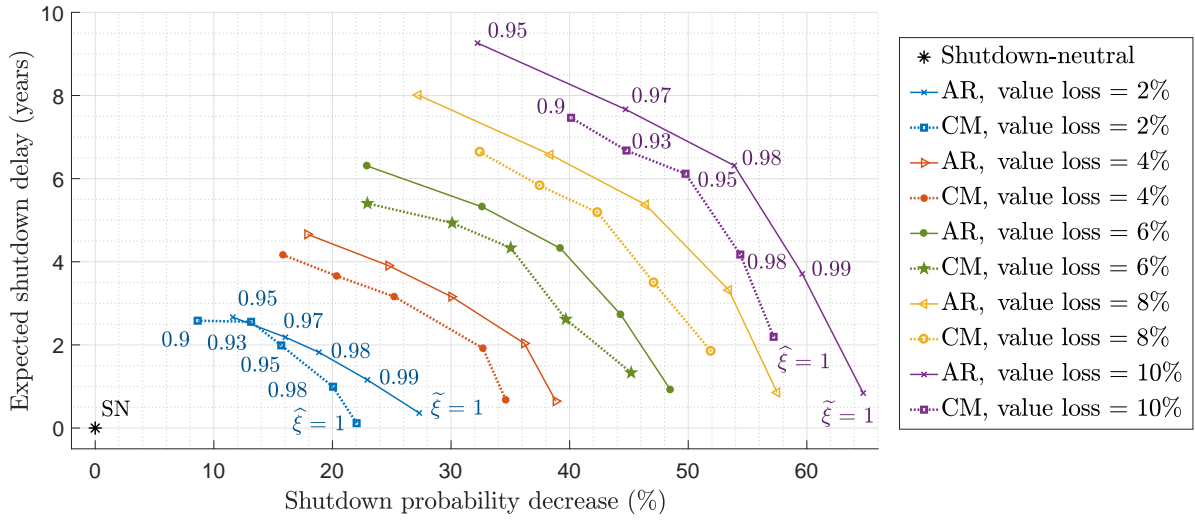


Figure 3.6: Trade-offs in shutdown probability decrease and shutdown delay for AR and CM policies.

We now consider the performance of AR and CM on the extended instance set, that is, OP1-OP4 and MP1-MP4 defined in Table 3.3 by varying operational costs and market volatilities of the reference instance (labeled REF). Table 3.4 reports results that examine the asset value and shutdown probability trade-off for policies with $\tilde{\xi} = \hat{\xi} = 1$ similar to Figure 3.5. Specifically, for decreases in shutdown probability equal to 10%, 20%, 30%, and 40%, this table shows the asset value loss expressed as a percentage of the maximum asset value. We find that AR and CM incur similar asset value losses to achieve a target shutdown probability reduction on the extended instance set, with the former method performing slightly better than the latter method. For example, both methods reduce shutdown probability by 10% and 20% in most instances for asset value losses, respectively, below 1% and 2%. A larger shutdown probability reduction of 40% entails average losses of 4.7% and 5.7% when using AR and CM, respectively. Changing the fixed cost c^{NOK} leads to the largest fluctuation in the asset value loss incurred to achieve a shutdown probability reduction across the extended instance set. For example, reducing the shutdown probability by 40% requires a roughly 9-12% asset value loss in OP3 while the analogous loss is less than 2% for OP4. Changing the power (an input) and aluminum (an output) price volatility have opposite effects. Specifically, the asset value loss to achieve a target shutdown-probability reduction becomes smaller as power and aluminum volatilities, respectively, decrease (compare MP2 with MP1) and increase (compare MP3 with MP4).

Table 3.5 analyzes both shutdown reduction and shutdown delay on the extended instance set. For fixed asset value losses, we determined trade-off curves for AR and CM analogous to those displayed in Figure 3.6. In particular, we consider the two extreme ends of each trade-off curve, which correspond to policies computed with (i) $\tilde{\xi} = \hat{\xi} = 1$, and (ii)

Table 3.4: Asset value loss as a percentage of the maximum asset value for 10%, 20%, 30%, and 40% decreases in shutdown probability when using AR and CM with T equal to 2.

Instance	Shutdown probability decrease							
	10%		20%		30%		40%	
	AR	CM	AR	CM	AR	CM	AR	CM
REF	0.7	0.9	1.1	1.6	2.7	3.0	4.0	5.0
OP1	1.0	1.3	1.6	2.8	4.4	5.3	6.4	8.3
OP2	0.5	0.5	0.8	1.0	1.9	2.1	2.7	3.2
OP3	1.2	1.7	3.0	3.9	5.3	7.4	9.3	12.3
OP4	0.2	0.4	0.6	0.6	1.2	1.2	1.8	2.0
MP1	0.7	1.0	1.8	1.9	3.1	3.9	5.2	6.2
MP2	0.3	0.7	1.0	1.4	2.0	2.5	3.7	4.2
MP3	0.2	0.7	0.8	1.2	1.7	2.0	2.6	3.2
MP4	0.8	1.2	1.9	2.2	3.3	4.0	6.2	7.3
Average	0.6	0.9	1.4	1.9	2.8	3.5	4.7	5.7

$\tilde{\xi} = 0.95$ for AR and $\hat{\xi} = 0.90$ for CM (as in Figure 3.6). We refer to the former and latter policies as shutdown delay-neutral and delay-averse, respectively. The shutdown probability reduction is expressed as a percentage of the shutdown-neutral probability computed for the same instance and the average shutdown delay is reported in number of years and displayed within parenthesis. For a fixed asset value, shutdown probability reduction and shutdown delay are substitutes on all instances in the extended instance set. The AR policies manage this substitution slightly better than CM policies on average. The sensitivity of results to changes in fixed costs and market volatilities are analogous to those observed in Table 3.4. For asset value losses of 3% and 6%, shutdown decisions can be delayed by 4-5 years on average across the instances by incurring a larger shutdown probability later in the horizon. While shutdown probability reduction and shutdown delay are indeed sensitive to operational and market parameters, our findings underscore that shutdown decisions can be significantly delayed or made less likely for small asset value losses. AR and CM emerge as robust methods to obtain these desired shutdown profile changes.

Table 3.5: Shutdown probability reduction (%) and average shutdown delay (years) for fixed asset value losses when using AR and CM policies with delay-neutral and delay-averse preferences.

Instance	Asset value loss							
	3%				6%			
	AR		CM		AR		CM	
delay-neutral	delay-averse	delay-neutral	delay-averse	delay-neutral	delay-averse	delay-neutral	delay-averse	
REF	31.6 (0.5)	16.7 (4.2)	30.0 (0.4)	13.2 (3.7)	48.5 (0.9)	22.9 (6.3)	45.2 (1.3)	22.9 (5.4)
OP1	27.3 (0.9)	13.0 (3.2)	20.0 (0.4)	7.3 (2.6)	39.0 (1.2)	19.7 (5.4)	32.5 (1.2)	13.8 (4.6)
OP2	42.9 (0.5)	19.8 (4.9)	40.0 (0.4)	20.5 (4.0)	60.6 (0.4)	31.2 (8.0)	57.5 (1.2)	39.7 (6.3)
OP3	20.4 (1.0)	12.5 (3.2)	15.9 (0.6)	6.0 (2.4)	32.2 (1.3)	15.8 (4.3)	26.0 (1.1)	9.8 (3.8)
OP4	54.6 (0.2)	25.3 (5.7)	49.2 (0.2)	32.6 (4.3)	76.6 (-0.5)	51.2 (8.4)	71.1 (2.3)	60.6 (6.0)
MP5	30.3 (0.6)	14.7 (3.7)	25.6 (0.3)	10.0 (3.2)	43.8 (0.9)	21.3 (6.1)	40.4 (1.1)	19.1 (5.2)
MP6	36.5 (0.6)	17.3 (4.1)	31.9 (0.5)	14.9 (3.7)	53.3 (0.8)	25.1 (7.0)	47.4 (1.4)	29.2 (5.9)
MP7	42.6 (1.1)	21.2 (4.9)	36.7 (1.2)	19.5 (4.5)	62.3 (1.4)	31.7 (8.5)	55.1 (2.3)	39.5 (7.1)
MP8	28.8 (0.4)	14.0 (3.5)	24.6 (0.2)	10.5 (3.3)	41.4 (0.7)	19.1 (5.3)	36.1 (0.7)	16.0 (4.6)
Average	35.0 (0.6)	17.2 (4.2)	30.4 (0.5)	14.9 (3.5)	50.8 (0.8)	26.4 (6.6)	45.7 (1.4)	27.8 (5.4)

3.6 Conclusions

Motivated by an aluminum producer, we studied the management of permanent shutdown decisions in merchant commodity and energy production assets from a social commerce perspective, which deviates from the popular asset value maximization approach. We formulated a constrained MDP to maximize the asset value subject to constraints on the shutdown profile, that is, we imposed bounds on the shutdown probability of an operating policy at each period to capture the preferences to delay and reduce the likelihood of a plant shutdown. We approximated this intractable constrained MDP by defining two policies: (i) AR policies grounded in anticipated regret theory and determined using LSML, and (ii) CM policies, which extend production margin-based heuristics used in practice, computed by combining LSML and binary classification methods in machine learning. We found on realistic aluminum production instances that both policies can improve the shutdown profile substantially for small asset value losses. Production margin heuristics used by practitioners instead incur larger asset value losses to achieve these improvements. We also developed a reoptimization policy that simplifies the AR policy and solves a shortest path problem at each period to compute decisions. This reoptimization policy performed well for high asset value losses but was dominated by AR and CM policies for smaller losses. Our results highlight that social commerce appears financially viable, that is, shutdown decisions can be significantly delayed or avoided when unaccounted social costs amount to a few percent of the plant's maximum asset value. Moreover, adapting a plant's operating flexibility emerges as an effective lever to achieve socially-responsible shutdown decisions.

3.7 Appendix

3.7.1 Proofs

Proof of Proposition 3.1. Part (a): Our proof is based on backward induction. At the last stage I , the following inequalities trivially hold as equalities because the terminal conditions are the same in each SDP:

$$V_I^{\text{SN}\setminus\{\text{C}\}}(x_I, w_I) = V_{\Theta, I}^A(x_I, w_I) = V_I^{\text{SN}}(x_I, w_I) = r_I(x_I, w_I), \quad \forall (x_I, w_I) \in \mathcal{X}_I \times \mathcal{W}_I. \quad (3.10)$$

Assume that analogous inequalities hold from stages $i + 1$ through stage I . Consider stage i and the inequalities $V_{i+1}^{\text{SN}\setminus\{\text{C}\}}(x_{i+1}, w_{i+1}) \leq V_{\Theta, i+1}^A(x_{i+1}, w_{i+1}) \leq V_{i+1}^{\text{SN}}(x_{i+1}, w_{i+1})$ for all $(x_{i+1}, w_{i+1}) \in \mathcal{X}_{i+1} \times \mathcal{W}_{i+1}$, which hold by the induction hypothesis. Using these

inequalities, we obtain for each state $(x_i, w_i) \in \mathcal{X}_i \times \mathcal{W}_i$ and non-shutdown action $a_i \in \mathcal{A}_i(x_i) \setminus \{C\}$, the following relationships:

$$\begin{aligned} r_i(x_i, w_i, a_i) + \delta \mathbb{E}_i \left[V_{i+1}^{\text{SN} \setminus \{C\}}(f(x_i, a_i), w_{i+1}) \right] &\leq r_i(x_i, w_i, a_i) + \delta \mathbb{E}_i \left[V_{\Theta, i+1}^A(f(x_i, a_i), w_{i+1}) \right] \\ &\leq r_i(x_i, w_i, a_i) + \delta \mathbb{E}_i \left[V_{i+1}^{\text{SN}}(f(x_i, a_i), w_{i+1}) \right]. \end{aligned}$$

Taking a maximum over the non shutdown actions preserves this ordering, that is,

$$V_i^{\text{SN} \setminus \{C\}}(x_i, w_i) \leq V_{\Theta, i}^{A, \mathcal{O}}(x_i, w_i) \leq V_i^{\text{SN}, \mathcal{O}}(x_i, w_i), \quad \forall (x_i, w_i) \in \mathcal{X}_i \times \mathcal{W}_i.$$

Using $V_i^{\text{SN} \setminus \{C\}}(x_i, w_i) \leq V_{\Theta, i}^{A, \mathcal{O}}(x_i, w_i)$ we get for all $(x_i, w_i) \in \mathcal{X}_i \times \mathcal{W}_i$:

$$V_i^{\text{SN} \setminus \{C\}}(x_i, w_i) \leq \max \left\{ V_{\Theta, i}^{A, \mathcal{O}}(x_i, w_i), -K^{(x_i, C)} - \lambda \tilde{\xi}^i \text{AR}_{\Theta, i}(x_i, w_i) \right\} = V_{\Theta, i}^A(x_i, w_i).$$

To prove $V_{\Theta, i}^A(x_i, w_i) \leq V_i^{\text{SN}}(x_i, w_i)$ we notice that the penalty term $\text{AR}_{\Theta, i}(x_i, w_i)$ is always positive. This implies that

$$-K^{(x_i, C)} - \lambda \tilde{\xi}^i \text{AR}_{\Theta, i}(x_i, w_i) \leq -K^{(x_i, C)}, \quad \forall (x_i, w_i) \in \mathcal{X}_i \times \mathcal{W}_i. \quad (3.11)$$

Combining $V_{\Theta, i}^{A, \mathcal{O}}(x_i, w_i) \leq V_i^{\text{SN}, \mathcal{O}}(x_i, w_i)$ with (3.11) results in

$$\begin{aligned} V_{\Theta, i}^A(x_i, w_i) &= \max \left\{ V_{\Theta, i}^{A, \mathcal{O}}(x_i, w_i), -K^{(x_i, C)} - \lambda \tilde{\xi}^i \text{AR}_{\Theta, i}(x_i, w_i) \right\} \\ &\leq \max \left\{ V_i^{\text{SN}, \mathcal{O}}(x_i, w_i), -K^{(x_i, C)} \right\} = V_i^{\text{SN}}(x_i, w_i), \quad \forall (x_i, w_i) \in \mathcal{X}_i \times \mathcal{W}_i. \end{aligned}$$

Part (b): We fix $\tilde{\xi} \in (0, 1]$. To prove the limiting results, it suffices to show that the shutdown-neutral value function with no shutdown and the anticipated regret value function coincide for any strictly positive lambda λ and large enough η . This result implies that for a given $\lambda > 0$, $\lim_{\eta \rightarrow \infty} V_{\Theta, i}^A(x_i, w_i) = V_I^{\text{SN} \setminus \{C\}}(x_i, w_i)$. Moreover, since the value functions coincide, the associated optimal policies are also equal, which implies that the shutdown probability under the AR policy is zero, that is, $\lim_{\eta \rightarrow \infty} \Pr(C; \pi_{\Theta}^A) = 0$ for any $\lambda > 0$.

We proceed to show that for every $\lambda > 0$, there exists $\eta_i^* \geq 1$ (we suppress the dependence of η^* on λ) such that $V_{\Theta, i}^A(x_i, w_i) = V_i^{\text{SN} \setminus \{C\}}(x_i, w_i)$ for all $(x_i, w_i) \in \mathcal{X}_i \times \mathcal{W}_i$ and $\eta \geq \eta_i^*$. The statement is proved by backward induction. At stage I , the equality trivially holds for any η and λ as in (3.10). Assuming that analogous equalities and values for η exist for stages $i+1$ to I , we establish this property at stage i . The equality $V_{\Theta, i+1}^A(x_{i+1}, w_{i+1}) = V_{i+1}^{\text{SN} \setminus \{C\}}(x_{i+1}, w_{i+1})$ for all $(x_{i+1}, w_{i+1}) \in \mathcal{X}_{i+1} \times \mathcal{W}_{i+1}$ is true by the induction hypothesis for $\eta \geq \eta_{i+1}^*$. Note that $V_{\Theta, i+1}^A(x_{i+1}, w_{i+1}) = V_{i+1}^{\text{SN} \setminus \{C\}}(x_{i+1}, w_{i+1})$ im-

plies that $V_{\Theta,i+1}^A(x_{i+1}, w_{i+1}) = V_{\Theta,i+1}^{A,\mathcal{O}}(x_{i+1}, w_{i+1})$ because the shutdown decision does not determine the value function. Based on these relationships we replace $V_{\Theta,i+1}^A(x_{i+1}, w_{i+1})$ in the right hand side of the anticipated regret SDP step (3.5c) with $V_i^{\text{SN}\setminus\{C\}}(x_i, w_i)$ and also use the definition of $\text{AR}_{\Theta,i}(x_i, w_i)$ to obtain

$$V_{\Theta,i}^A(x_i, w_i) = \max \left\{ V_i^{\text{SN}\setminus\{C\}}(x_i, w_i), -K^{(x_i,C)} - \lambda \tilde{\xi}^i \max_{a_i \in \mathcal{A}_i(x_i) \setminus \{C\}} \mathbb{E}_i \left[\max \{ r_i(x_i, w_i, a_i) + \delta V_{i+1}^{\text{SN}\setminus\{C\}}(f_i(x_i, a_i), w_{i+1}) + \eta \cdot K^{(x_i,C)}, 0 \} \right] \right\}. \quad (3.12)$$

Since \mathcal{W}_i is compact at each stage by assumption, there exists an $L \in \mathbb{R}_+$ such that $r_i(x_i, w_i, a_i) \geq -L$ for all $(i, x_i, w_i, a_i) \in \mathcal{I} \times \mathcal{X}_i \times \mathcal{W}_i \times \mathcal{A}_i(x_i)$ and $r_I(x_I, w_I) \geq -L$ for all $(x_I, w_I) \in \mathcal{X}_I \times \mathcal{W}_I$. An immediate consequence is that the optimal value function is bounded below, that is, $V_i^{\text{SN}\setminus\{C\}}(x_i, w_i) \geq -L(I - i + 1)$ for all $(i, x_i, w_i) \in \mathcal{I} \times \mathcal{X}_i \times \mathcal{W}_i$. To prove that $V_{\Theta,i}^A(x_i, w_i) = V_i^{\text{SN}\setminus\{C\}}(x_i, w_i)$ are equal if η is large enough, we show that shutdown will not be chosen in equation (3.12) by using the following chain of inequalities for all $(x_i, w_i) \in \mathcal{X}_i \times \mathcal{W}_i$:

$$\begin{aligned} & -K^{(x_i,C)} - \lambda \tilde{\xi}^i \max_{a_i \in \mathcal{A}_i(x_i) \setminus \{C\}} \mathbb{E}_i \left[\max \{ r_i(x_i, w_i, a_i) + \delta V_{i+1}^{\text{SN}\setminus\{C\}}(f_i(x_i, a_i), w_{i+1}) + \eta \cdot K^{(x_i,C)}, 0 \} \right] \\ & \leq -\lambda \tilde{\xi}^i \max_{a_i \in \mathcal{A}_i(x_i) \setminus \{C\}} \mathbb{E}_i \left[r_i(x_i, w_i, a_i) + \delta V_{i+1}^{\text{SN}\setminus\{C\}}(f_i(x_i, a_i), w_{i+1}) + \eta \cdot K^{(x_i,C)} \right] \end{aligned} \quad (3.13a)$$

$$\leq -\lambda \tilde{\xi}^i (-L - \delta L(I - i) + \eta \underline{K}) \quad (3.13b)$$

$$< -L - \delta L(I - i) \quad (3.13c)$$

$$\leq \max_{a_i \in \mathcal{A}_i(x_i) \setminus \{C\}} \left\{ r_i(x_i, w_i, a_i) + \delta \mathbb{E}_i \left[V_{i+1}^{\text{SN}\setminus\{C\}}(f(x_i, a_i), w_{i+1}) \right] \right\} \quad (3.13d)$$

$$= V_i^{\text{SN}\setminus\{C\}}(x_i, w_i). \quad (3.13e)$$

The first inequality (3.13a) is obtained by dropping the first term $-K^{(x_i,C)} \leq 0$ and employing the relation $\mathbb{E}[\max\{X, 0\}] \geq \mathbb{E}[X]$; the second (3.13b) by replacing the reward function and the value function terms by their lower bounds based on the compactness of \mathcal{W}_i , and the shutdown cost by $\underline{K} = \min\{K^{(x_i,C)} : x_i \in \mathcal{X}_i, i \in \mathcal{I}\} > 0$; the third (3.13c) by choosing $\eta > \eta_i^* = (\lambda \tilde{\xi}^i + 1)(L + \delta L(I - i)) / (\lambda \tilde{\xi}^i \underline{K})$; the fourth (3.13d) from noticing that $r_i(x_i, w_i, a_i) \geq -L$ and $V_{i+1}^{\text{SN}\setminus\{C\}}(f(x_i, a_i), w_{i+1}) \geq -L(I - i)$; and the final equality (3.13e) by using the definition of $V_i^{\text{SN}\setminus\{C\}}(x_i, w_i)$. Our claim thus holds at stage i and it is also true at all stages by the principle of mathematical induction. \square

Proof of Proposition 3.2. Consider the RH optimization problem (3.6) formulated at stage i where $i \in \{0, \dots, I - 1\}$. We establish that this problem can be formulated as a shortest

path problem on a directed weighted graph $G_i := (\mathcal{V}_i, \mathfrak{E}_i, \psi_i)$, where \mathcal{V}_i and \mathfrak{E}_i denote the node and edge sets, respectively, and $\psi_i \in \mathbb{R}^{|\mathfrak{E}_i|}$ denotes the vector of edge weights. We construct this graph as follows (see Figure 3.7 for an illustration).

We define the node set as $\mathcal{V}_i := \{\nu_{j,x_j} : (j, x_j) \in \{i, \dots, I\} \times \mathcal{X}_j\} \cup \{\nu_{I+1}\}$, where ν_{j,x_j} denotes the node corresponding to stage j and endogenous state x_j in the set \mathcal{X}_j , and ν_{I+1} represents an artificial sink node. There is a singleton node ν_{i,x_i} at stage i denoting the initial operating status with $x_i \in \mathcal{O}_i$. We use $e(\nu_{j,x_j}, \nu_{j+1,x_{j+1}})$ to denote a directed edge from node ν_{j,x_j} to $\nu_{j+1,x_{j+1}}$. A directed edge $e(\nu_{j,x_j}, \nu_{j+1,x_{j+1}})$ belongs to \mathfrak{E}_i if and only if the transition from x_j to x_{j+1} is feasible, that is, there exists a_j in the action set $\mathcal{A}_j(x_j)$ such that $f_j(x_j, a_j) = x_{j+1}$. Moreover $e(\nu_{I,x_I}, \nu_{I+1}) \in \mathfrak{E}_i$ for each $x_I \in \mathcal{X}_I$. The weights are defined as $\psi_i(\nu_{j,x_j}, \nu_{j+1,x_{j+1}}) := \delta^j \mathbb{E}[r_j^{\lambda, \xi}(x_j, w_j, x_{j+1}) | w_i]$ and corresponds to the stage j expected reward from the transition between x_j and x_{j+1} discounted back to stage zero. Each edge connecting the node ν_{I,x_I} and the sink node ν_{I+1} has a weight of zero.

By construction, each path from the node ν_{i,x_i} to the sink node ν_{I+1} represents a feasible set of actions at each stage. In addition, the sum of edge weights on this path is the stage j discounted sum of expected rewards resulting from executing these actions. Thus, a longest path of G_i is an optimal solution to the RH optimization problem (3.6). Given that G_i is a directed acyclic graph (arrows follow the direction of time), it is well known that the longest path in this graph coincides with the shortest path in the augmented graph $G_i^- := (\mathcal{V}_i, \mathfrak{E}_i, -\psi_i)$ with negated weights $-\psi_i$ and this shortest path can be found in $\mathcal{O}(|\mathcal{V}_i| + |\mathfrak{E}_i|)$ (see, e.g., Sedgewick and Wayne, 2011, Chapter 4). Since nodes correspond to endogenous states, the time complexity can be represented in terms of the numbers of stages and endogenous states, that is, $\mathcal{O}(I \cdot \bar{\mathcal{X}}^2)$. \square

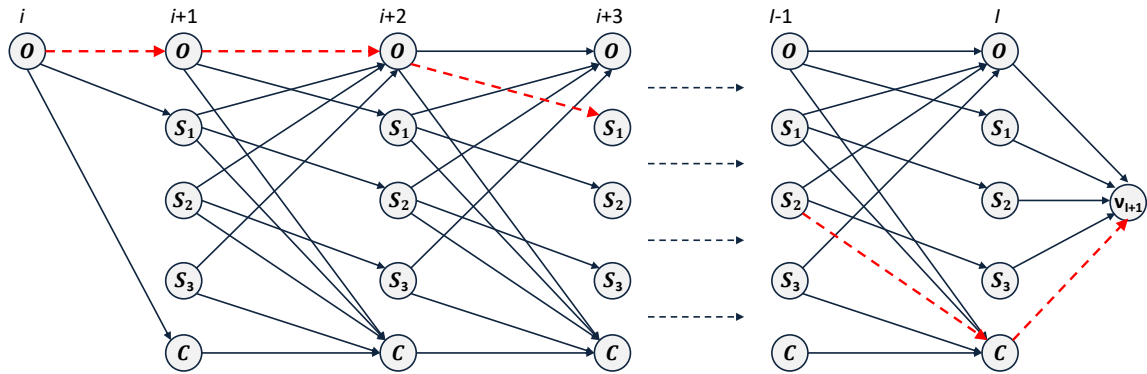


Figure 3.7: Illustration of the network underlying the shortest path reformulation of the RH optimization model corresponding to the aluminum production case study of Section 3.5. The red dashed path is an example of shortest path solution of this model.

To provide an illustration of the graph, consider the aluminum case study of Section 3.5 where the operating states of the production asset correspond to the set $\mathcal{X}_i = \{O, S_1, S_2, S_3, C\}$ (see Section 3.5.1 for details). In this setting, the graph underlying the RH shortest path problem is shown in Figure 3.7.

3.7.2 LSML algorithm

We describe in this section the LSML procedure briefly discussed in Section 3.3.1 of the paper. We primarily focus on tackling the anticipated-regret SDP (3.5) but point out at the end of this section minor changes that can be made to approximate the shutdown-neutral SDPs (3.2) and (3.4).

Algorithm 4 summarizes the LSML steps to approximate the anticipated-regret SDP (3.5). Let $\beta_{i,x_i}^A := (\beta_{i,x_i,1}^A, \dots, \beta_{i,x_i,B_i}^A)$ be the VFA weight vector for stage $i \in \{1, \dots, I\}$ and operating state $x_i \in \mathcal{X}_i$. To ease notation, we do not subscript terms in LSML by Θ . The inputs to LSML are a set of P information state sample paths $\{w_i^p, (i, p) \in \mathcal{I} \cup \{I\} \times \mathcal{P}\}$ generated in Monte Carlo simulation, where $\mathcal{P} := \{1, \dots, P\}$; the number of samples N used to construct sample average approximations of expectations; and a set of VFA basis functions. LSML initializes the terminal stage VFA weight vector β_{I,x_I}^A by regressing the basis functions Φ_I on evaluations of the terminal reward function for each $x_I \in \mathcal{X}_I$. At each stage $i \in \mathcal{I}$, starting from stage $I - 1$ and moving backward to stage 1, and for each operating state $x_i \in \mathcal{X}_i$ it executes Steps 1 and 2.

- In Step 1(a), for each sample path $p \in \mathcal{P}$, LSML generates N stage $i + 1$ information state samples conditioned on w_i^p . We denote the n -th such sample by $\bar{w}_{i+1}^{p,n}$ and the set of these samples by $\{\bar{w}_{i+1}^{p,n}, n \in \mathcal{N}\}$, where $\mathcal{N} := \{1, \dots, N\}$.
- In Step 1(b), it computes estimates $v_i^A(x_i, w_i^p)$ of the stage i AR value function $V_{\Theta,i}^A(x_i, w_i^p)$ by applying to the right hand sides of (3.5a) and (3.5c) the known stage $i + 1$ VFA and sample average approximations of expectations based on the samples generated in the previous step. Specifically, LSML computes $v_i^{A,\mathcal{O}}(x_i, w_i^p)$ by replacing $\mathbb{E}_i[V_{\Theta,i+1}^A(f_i(x_i, a_i), w_{i+1})]$ in the right hand side of (3.5a) by the sample average approximation

$$\sum_{b \in \mathcal{B}_{i+1}} \beta_{i+1, f_i(x_i, a_i), b}^A \left[\frac{1}{N} \sum_{n \in \mathcal{N}} \Phi_{i+1, b}(\bar{w}_{i+1}^{p,n}) \right],$$

and obtains $v_i^A(x_i, w_i^p)$ by substituting for $V_{\Theta,i}^{A,\mathcal{O}}(x_i, w_i)$ and $\text{AR}_{\Theta,i}(x_i, w_i^p)$ in the right hand side of (3.5c), respectively, with $v_i^{A,\mathcal{O}}(x_i, w_i^p)$ and the sample average

approximation

$$\widehat{\text{AR}}_{\Theta,i}(x_i, w_i^p) := \max_{a_i \in \mathcal{A}_i(x_i) \setminus \{C\}} \frac{1}{N} \sum_{n \in \mathcal{N}} \max \left\{ \hat{Q}(x_i, w_i^p, a_i, \bar{w}_{i+1}^{p,n}) + \eta \cdot K^{(x_i, C)}, 0 \right\}, \quad (3.14)$$

where

$$\hat{Q}(x_i, w_i^p, a_i, \bar{w}_{i+1}^{p,n}) := r_i(x_i, w_i^p, a_i) + \delta \sum_{b \in \mathcal{B}_{i+1}} \beta_{i+1, f_i(x_i, a_i), b}^A \Phi_{i+1, b}(\bar{w}_{i+1}^{p,n}).$$

- In Step 2, LSML performs a least squares (2-norm) regression on these estimates to determine the vector of VFA weights β_{i, x_i}^A .

Algorithm 4 Shutdown-averse LSML

Inputs: Set of information state sample paths $\{w_i^p, (i, p) \in \mathcal{I} \cup \{I\} \times \mathcal{P}\}$, number of sample average approximation samples N , and set of basis function vectors $\{\Phi_i, i \in \{1, \dots, I\}\}$.

Initialization: For each $x_I \in \mathcal{X}_I$, compute estimates $v_I^A(x_I, w_I^p) := r_I(x_I, w_I^p)$ for $p \in \mathcal{P}$ and perform a least squares regression on these VFA estimates using basis functions Φ_I to determine the vector of VFA weights β_{I, x_I}^A .

For each $i = I - 1$ to 1 **do:**

For each $x_i \in \mathcal{X}_i$ **do:**

 1. **For** each $p \in \mathcal{P}$ **do:**

 (a) Sample N stage $i + 1$ information state samples conditional on w_i^p : $\{\bar{w}_{i+1}^{p,n}, n \in \mathcal{N}\}$.

 (b) Compute the VFA estimates

$$v_i^{A, \mathcal{O}}(x_i, w_i^p) := \max_{a_i \in \mathcal{A}_i(x_i) \setminus \{C\}} \left\{ r_i(x_i, w_i^p, a_i) + \delta \sum_{b \in \mathcal{B}_{i+1}} \beta_{i+1, f_i(x_i, a_i), b}^A \left[\frac{1}{N} \sum_{n \in \mathcal{N}} \Phi_{i+1, b}(\bar{w}_{i+1}^{p,n}) \right] \right\};$$

$$v_i^A(x_i, w_i^p) := \max \left\{ v_i^{A, \mathcal{O}}(x_i, w_i^p), -K^{(x_i, C)} - \lambda \tilde{\xi}^i \widehat{\text{AR}}_{\Theta, i}(x_i, w_i^p) \right\}.$$

 2. Perform a least squares regression on the VFA estimates in set $\{v_i^A(x_i, w_i^p), p \in \mathcal{P}\}$ using basis functions Φ_i to determine the vector of VFA weights β_{i, x_i}^A .

Outputs: Vectors of VFA weights β_{i, x_i}^A for each $(i, x_i) \in \{1, \dots, I\} \times \mathcal{X}_i$.

The outputs of LSML are the vectors of VFA weights β_{i, x_i}^A for each stage $i \in \{1, \dots, I\}$ and operating state $x_i \in \mathcal{X}_i$. Given such VFA weights, the action $a_i^A(x_i, w_i)$ taken by the

anticipated regret policy at stage i and state (x_i, w_i) is defined as

$$a_i^A(x_i, w_i) := \begin{cases} C, & \text{if } v_i^{A,\mathcal{O}}(x_i, w_i) < -K^{(x_i,C)} - \lambda \tilde{\xi}^i \widehat{\text{AR}}_{\Theta,i}(x_i, w_i), \\ \arg \max_{a_i \in \mathcal{A}_i(x_i) \setminus \{C\}} \left\{ r_i(x_i, w_i, a_i) + \delta \sum_{b \in \mathcal{B}_{i+1}} \beta_{i+1, f_i(x_i, a_i), b}^A \left[\frac{1}{N} \sum_{n \in \mathcal{N}} \Phi_{i+1, b}(w_{i+1}^n) \right] \right\}, & \\ \text{otherwise,} & \end{cases}$$

where the sample average approximations are constructed using N next stage information state samples $(w_{i+1}^n, n \in \mathcal{N})$ conditioned on w_i .

Algorithm 4 can also be used with minor changes to approximate the shutdown-neutral SDPs (3.2) and (3.4). Specifically, computing a sample average approximation of the shutdown cost inflation $\widehat{\text{AR}}_{\Theta,i}(x_i, w_i^p)$ as in (3.14) is not needed for the shutdown-neutral SDPs, thus, this term is replaced by zero.

3.7.3 Calibration of the price and exchange rate dynamics

For all commodities, we considered futures prices for weekly trading dates from November 18th 2008 to December 30th 2015. Aluminum and exchange rate futures contracts have physical/financial delivery at the end of the contract duration (that is, at maturity). We collected aluminum futures prices from Bloomberg for maturities extending out 1, 3, 6, 9, 12, 15, 18, 21, 24, and 36 months. Exchange rate futures for EUR-USD and NOK-USD on Bloomberg were only available for the months of March, June, September, and December. We applied standard linear interpolation to these rates and obtained a term structure curve with maturities that match the ones for aluminum contracts (Guthrie, 2009, §12). Nord Pool power contracts were obtained from the information provider Montel (see www.montel.no). The power delivery duration of these contracts were monthly, quarterly, or yearly with trading dates extending out to 6 months, 8 quarters, and 3 years, respectively. In contrast to aluminum and exchange rate futures contracts, power contracts deliver electricity continuously during an interval of time. For example, the first and second nearest quarterly contracts traded in August 2015 delivered power during, respectively, the entire 4th quarter (Oct-Dec) of 2015 and 1st quarter (Jan-Mar) of 2016. Power futures contracts of different lengths can overlap, for instance, the nearest monthly and quarterly contracts. To obtain implied power futures prices of contracts that deliver only at maturity, we used the smoothing approach of Benth et al. (2007) (see Fleten and Lemming, 2003 for an alternative smoothing technique), which determines a smooth polynomial spline that replicates the observed market prices for each trading date. This synthetic curve contains power futures contract prices that are consistent with the ones

for aluminum and currency exchange rates.

We calibrated the parameters of the stochastic process (3.9) by applying a Kalman filter (Hamilton, 1994, Chapter 13) to match the model-implied log-futures prices with the data by maximizing the log-likelihood function. The transition and measurement equations correspond to Farkas et al. (2017) equations (6)-(8) and (15)-(16), respectively. The integral terms involving matrix exponentials were evaluated numerically using the reductions to new matrix exponentials described in Carbonell et al. (2008). Due to the large number of unknown parameters, we followed a multi-step calibration process where parameters estimated in a given step are kept fixed in future steps, which is common (see, e.g., Farkas et al., 2017). These steps are the following:

1. We estimated power seasonality $\psi^{\text{El}}(t)$ by regressing the function $\chi_1 \cos(2\pi t) + \chi_2 \sin(2\pi t)$ on the log-futures data (Paschke and Prokopczuk, 2009; Farkas et al., 2017). We set the seasonality for aluminum and exchange rates, that is $\psi^{\text{Al}}(t)$, $\psi^{\text{EUR-USD}}(t)$ and $\psi^{\text{NOK-USD}}(t)$, to zero as we observed no empirical evidence of seasonal effects.
2. We calibrated single-commodity 2-factor versions of model (3.9) for each commodity with a single correlation between short and long term factors. We estimated μ_z , the short and long term risk premia λ_y and λ_z , respectively, the diagonal terms of K_y , K_z and $\rho_{y,z}$, and the variance of the measurement error. The value of μ_z for currencies was not statistically significant and was set to zero. We noticed that the volatility estimates were unrealistically high. Hence σ_y and σ_z were replaced with data estimates directly from historical data.
3. We calibrated the cross-commodity correlation structure, that is, matrices ρ_y , ρ_z and $\rho_{y,z}$ keeping all previous estimates fixed. Doing this reduced the number of free variables to 24, which we could handle in the maximum likelihood estimation.

Below we report the parameter estimates where statistical significance at 1%, 5% and 10% are indicated by superscripts ^{***}, ^{**} and ^{*}, respectively. The order of the commodities in the vectors and matrices is power, aluminum, EUR-USD rate and NOK-USD rate.

$$\hat{K}_y = \begin{bmatrix} 1.904^{***} & 0 & 0 & 0 \\ 0 & 0.067^{***} & 0 & 0 \\ 0 & 0 & 0.011^{***} & 0 \\ 0 & 0 & 0 & 0.010 \end{bmatrix}, \quad \hat{K}_z = \begin{bmatrix} 0.055^{***} & 0 & 0 & 0 \\ 0 & 0.184^{***} & 0 & 0 \\ 0 & 0 & 0.001 & 0 \\ 0 & 0 & 0 & 0.005 \end{bmatrix}$$

$$\begin{aligned}
\hat{\mu} &= \begin{bmatrix} 0.19^* \\ 1.41^{***} \\ 0 \\ 0 \end{bmatrix}, \quad \hat{\sigma}_y = \begin{bmatrix} 0.26 \\ 0.12 \\ 0.08 \\ 0.13 \end{bmatrix}, \quad \hat{\sigma}_z = \begin{bmatrix} 0.11 \\ 0.09 \\ 0.18 \\ 0.15 \end{bmatrix}, \quad \hat{\chi}_1 = \begin{bmatrix} 0.21 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \hat{\chi}_2 = \begin{bmatrix} 0.03 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\
\hat{\rho}_y &= \begin{bmatrix} 1 & 0.79^{***} & 0.63^{***} & 0.65^{***} \\ 0.79^{***} & 1 & 0.72^{***} & 0.79^{***} \\ 0.63^{***} & 0.72^{***} & 1 & 0.83 \\ 0.65^{***} & 0.79^{***} & 0.83 & 1 \end{bmatrix}, \quad \hat{\rho}_z = \begin{bmatrix} 1 & 0.80^{***} & 0.68^{***} & -0.73^* \\ 0.80^{***} & 1 & 0.75^{***} & -0.72 \\ 0.68^{***} & 0.75^{***} & 1 & -0.79^{***} \\ -0.73^* & -0.72 & -0.79^{***} & 1 \end{bmatrix} \\
\hat{\rho}_{yz} &= \begin{bmatrix} 0.25^{***} & 0.76^{***} & 0.71^{***} & -0.68^{***} \\ 0.81^{***} & 0.27^{***} & 0.77^{***} & -0.73^* \\ 0.61^{***} & 0.73 & 0.02^{***} & -0.55 \\ 0.65^{***} & 0.83^{***} & 0.65^{***} & -0.09^{***} \end{bmatrix}, \quad \hat{\lambda}_y = \begin{bmatrix} -0.79^{***} \\ -0.59^{***} \\ -0.28^{***} \\ -0.50^{***} \end{bmatrix}, \quad \hat{\lambda}_z = \begin{bmatrix} 0.74^{***} \\ -0.14 \\ 3.49^{**} \\ -3.24^* \end{bmatrix}
\end{aligned}$$

Most of the estimates above are statistically significant. There is strong positive correlation between the short term factors of all commodities and some negative correlation in the long term factors, in particular between the first three assets and NOK-USD. The estimated covariance matrix Σ was computed as

$$\Sigma = \begin{bmatrix} \text{diag}(\sigma_y) & 0 \\ 0 & \text{diag}(\sigma_z) \end{bmatrix} \begin{bmatrix} \rho_y & \rho_{yz} \\ \rho_{yz}^T & \rho_z \end{bmatrix} \begin{bmatrix} \text{diag}(\sigma_y) & 0 \\ 0 & \text{diag}(\sigma_z) \end{bmatrix}.$$

This matrix is in general not positive semi-definite (PSD), which is a property needed to generate multivariate random draws. Nonetheless, after our calibration Σ was very close to being PSD (the negative eigenvalues are smaller than 10^{-2}). Therefore, to gain the desired PSD property, we computed the nearest PSD matrix to Σ by minimizing the Frobenius norm of the difference (Higham, 1988). This can be seen as a small perturbation of Σ . Finally, the initial values for the eight factors used in the analysis are based on representative spot and 1-year futures prices observed during the first quarter of 2017 and are, respectively, $Y_0 = \log [35, 1800, 1.10, 0.12]^\top$ and $Z_0 = \log [33, 1900, 1.25, 0.15]^\top$.

3.7.4 Dual bound on the asset value

To obtain a dual (upper) bound on the shutdown-neutral asset value, we use the information relaxation and duality framework of Brown et al. (2010). This approach relies on relaxing the non-anticipativity constraints embedded in the SDP, and penalizing knowledge of future information at time $i \in \mathcal{I}$ using a penalty function $q_i(f(x_i, a_i), w_i, w_{i+1})$. A feasible dual penalty q_i satisfies $\mathbb{E}[q_i(f(x_i, a_i), w_i, w_{i+1}) | w_i] \leq 0$. The dual bound estimation process relies on H Monte Carlo samples of uncertainty $\{w_i^h, (i, h) \in \mathcal{I} \cup \{I\} \times \{1, \dots, H\}\}$.

We solve the following deterministic dynamic program on each sample path h :

$$U_I^h(x_I) = r_I(x_I, w_I^h), \quad \forall x_I \in \mathcal{X}_I,$$

$$U_i^h(x_i) = \max_{a_i \in \mathcal{A}_i(x_i)} \left\{ r_i(x_i, w_i^h, a_i) - q_i(f(x_i, a_i), w_i^h, w_{i+1}^h) + \delta U_{i+1}^h(f(x_i, a_i)) \right\},$$

$$\forall (i, x_i) \in \mathcal{I} \times \mathcal{X}_i,$$

for all $h \in \{1, \dots, H\}$, where q_i is a feasible penalty. A dual bound on the option value is then obtained as the sample average $\sum_h U_0^h(x_0)/H$. It is well known (Brown et al., 2010) that given a VFA $\hat{V}_i(\cdot)$, a feasible dual penalty can be defined for $(x_{i+1}, h) \in \mathcal{X}_{i+1} \times \{1, \dots, H\}$ as follows:

$$q_i(x_{i+1}, w_i^h, w_{i+1}^h) = \delta \left\{ \hat{V}_{i+1}(x_{i+1}, w_{i+1}^h) - \mathbb{E}[\hat{V}_{i+1}(x_{i+1}, w_{i+1}^h) | w_i^h] \right\}.$$

We employed this penalty function in the computation of the dual bounds.

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Managing Shutdown Decisions in Merchant Commodity and Energy Production: The Performance of Popular Strategies

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Abstract: This paper complements the previous chapter by developing additional methodology and providing findings that could potentially be relevant to researchers and practitioners. Specifically, we investigate two alternative strategies to manage shutdown decisions that are popular in academia and industry. The first strategy incorporates in our dynamic merchant operations framework the popular conditional

value-at-risk (CVaR) and an intuitive variant thereof that we define (RCVaR). The second strategy exploits the firm's financial flexibility of entering into long term forward contracts, which is common practice to hedge cash flow volatility and we assess its impact on shutdown probability. We performed numerical experiments considering the same real aluminum production instances and computational setting from the previous chapter but, for simplicity, we focused on shutdown reduction neglecting the preference to delay shutdowns. We computed CVaR and RCVaR policies using a regress-later least squares Monte Carlo (LSML) method, and compared them with the anticipated-regret (AR) policies. We found that AR policies significantly outperforms CVaR-based policies and provide more efficient trade-offs between asset value and shutdown probability. We computed forward contracting strategies also using LSML and found that financial flexibility has little impact on shutdown probability. Our findings suggest caution in using methods that are not tailored to manage shutdown decisions as they might result in under-performing policies and incorrect shutdown decisions. Moreover, financial flexibility emerges as a less efficient lever to manage shutdown decisions compared to operating flexibility.

Keywords: Commodity and energy operations · conditional value-at-risk · forward contracts · least squares Monte Carlo

4.1 Introduction

Balancing the asset value perspective with a shutdown perspective is of strategic importance in commodity/energy production. A permanent shutdown decision in fact carries not only financial losses but also adverse social and political consequences to entities beyond the specific plant being shut down such as the parent company and the local community (for details, see Chapter 3 and in particular Section 3.1).

In Chapter 3 we have developed operating policies, namely anticipated-regret (AR) policies and classification-based margin (CM) policies, that directly trade off asset value for a more desirable shutdown profile and we have shown that these policies exhibit excellent numerical performance. In this paper, we investigate two alternative strategies to manage shutdown decisions that are commonly used in academia and industry: the first is based on dynamic risk measures and the second on the use of forward contracts to reduce the uncertainty. We analyze these two strategies by developing models and theory, and perform numerical experiments based on the same real aluminum production case study from the previous chapter. Specifically, we use the same operating model, instances data,

model of the evolution of the uncertainty and its calibration. We thus refer to Chapter 3 for the definition of these modeling elements and the related notation that we do not redefine entirely here. We evaluate the new strategies under a simplified shutdown metric and focus on the trade-off between asset value and shutdown probability. In other words, we only consider shutdown probability reductions here and neglect the preference to delay shutdowns to later parts of the planning horizon.

Our first approach considers the use of dynamic risk measures to manage shutdown decisions. Modeling risk aversion in multi-stage decision making applications is an active area of research (see, e.g., Smith and Nau, 1995; Ruszczyński, 2010; Philpott et al., 2013; Devalkar et al., 2017; Jiang and Powell, 2017b; Löhndorf and Wozabal, 2017). Multi-stage risk measures are typically constructed by nesting one-step coherent risk measures that satisfy axiomatic conditions to ensure the consistency of optimal decisions made over time. Dynamic conditional value-at-risk (CVaR) is a time-consistent risk measure (see, e.g., Ruszczyński, 2010; Philpott et al., 2013, and reference therein), and is a popular approach for modeling risk aversion in multi-stage optimization problems (Philpott et al., 2013; Jiang and Powell, 2017a; Jiang and Powell, 2017b; Devalkar et al., 2017). Dynamic CVaR is typically used to model financial risk. In a commodity operations setting, Devalkar et al. (2017) successfully incorporate dynamic CVaR to capture financial distress costs. When modeling shutdown decisions, we find that policies based on dynamic CVaR favor shutdown as opposed to modeling aversion to this decision. However, policies based on a simple modification of dynamic CVaR, which we dub reverse CVaR (RCVaR), are averse to shutdown decisions and coincide with the shutdown-neutral policy in the case of zero risk aversion. Although CVaR/RCVaR are intuitive measures for capturing cash flow risk, they are harder to interpret in the context of shutdown decisions. Our use of dynamic CVaR and RCVaR thus adds to this line of work by evaluating whether these measures are effective for modeling shutdown decisions.

We employ a regress-later least squares Monte Carlo (LSML; see Appendix 3.7.2 in Chapter 3) to obtain a collection of dynamic CVaR-based operating policies for different risk aversion levels. Our use of LSML builds on the extant literature that uses least squares Monte Carlo methods to obtain risk-neutral policies to high-dimensional real option problems. We compare CVaR/RCVaR policies with AR policies (see Chapter 3 for details on AR policies) and find that the latter policies provide a significantly better trade-off between shutdown probability and asset value than the former ones, which is consistent with our theory of asymptotic AR regimes (see Proposition 3.1). Specifically, AR policies reduce shutdown probability by 25% and 50%, respectively, for 2% and 6% losses in asset value, whereas RCVaR policies lose roughly 5% and 14% of the asset value to achieve the same shutdown probability reductions.

Second, we formalize a strategy that reduces uncertainty by using financial contracts. Opposed to all approaches previously introduced, this strategy involves adapting the firm's financial flexibility rather than the operating flexibility of the production asset. In particular, it is common practice to hedge cash flow volatility by entering into long term forward contracts. However, hedging all the sources of uncertainty may be undesirable. Consider for example our numerical study of Chapter 3 that involves the operations of a real aluminum producer, where uncertainty is in power price, aluminum price, and currency exchange rates. This producer does not hedge the selling price of aluminum (that is, the output) as the entire aluminum market bears this risk (see McKinsey, 2010 for a similar strategy in oil production). Hedging exchange rate risk is also not done at the level of an individual plant but rather by the holding company that aggregates risk across several business units (see, e.g., McKinsey, 2015). In contrast, long term contracts are used to hedge power price and secure supply. There is a preconceived notion that such forward contracting will help decrease the probability of shutdown as it reduces input price uncertainty. Although this expectation may be intuitive, the impact of power price uncertainty on shutdown decisions and shutdown probability is a priori unclear.

To assess this impact, we model fixed sourcing strategies that use long term power contracts with fixed length. This entails modifying the shutdown-neutral SDP to account for long term power purchases and to allow the sale of power into the wholesale market in case the production is suspended or shut down before completing the tenure of existing contract. We solve this modified SDP using LSML on our realistic aluminum instances, and find that the forward contracting strategy seems to have little impact on reducing the shutdown probability. Specifically, the shutdown probability varies only marginally compared to no forward purchasing: It decreases and increases by at most 0.8% and 0.5%, respectively.

Our findings suggest some caution in using methods, such as RCVaR polices and practice based forward sourcing strategies, that are not tailored to directly manage shutdown decisions as they may lead to the incorrect conclusion that a significant shutdown probability reduction is not possible or implies a large asset value loss. Furthermore, given our results, operating flexibility using shutdown-specific methods as AR and CM emerges as a more efficient lever to manage shutdown decisions than financial flexibility.

The rest of this paper is organized as follows. In Section 4.2, we present CVaR and RCVaR polices and discuss their performance. We describe fixed forward sourcing strategies in Section 4.3. In Section 4.4, we conclude by analyzing the value of the shutdown option as further motivation to the work that has been conducted in relation to the management of shutdown decisions.

4.2 CVaR-based policies

In this section, we present operating policies that are obtained by leveraging the conditional value-at-risk (CVaR), which is a popular measure for modeling risk aversion in academia and industry. Specifically, here we explore its suitability for mitigating shutdown decisions. Let $X(w_{i+1})$ be a random variable defined at stage i as a function of the random stage $i+1$ information state w_{i+1} . CVaR is defined with respect to a risk aversion level $\alpha \in [0, 1)$ as

$$\text{CVaR}_{\alpha,i}[X(w_{i+1})] := \mathbb{E}_i[X(w_{i+1}) | X(w_{i+1}) \leq \text{VaR}_{1-\alpha,i}(X(w_{i+1}))], \quad (4.1)$$

where $\text{VaR}_{1-\alpha,i}(X) := \inf\{x | F_X(x) \geq 1-\alpha\}$ and $F_X(\cdot)$ is the cumulative distribution function of $X(w_{i+1})$ conditioned on the information state w_i . Intuitively, $\text{CVaR}_{\alpha,i}[X(w_{i+1})]$ computes the expected value of the worst $1-\alpha$ proportion of realizations of the random variable X . If $\alpha = 0$, then the $\text{CVaR}_{\alpha,i}$ operator (4.1) reduces to the expectation operator, that is, $\text{CVaR}_{0,i}(\cdot) \equiv \mathbb{E}_i[\cdot]$.

Dynamic CVaR is a nested extension of CVaR used in multi-stage decision making applications (see, e.g., Ruszczyński, 2010; Philpott et al., 2013; Devalkar et al., 2017; Jiang and Powell, 2017a). For a given $\alpha \in [0, 1)$, a risk-averse CVaR optimal policy can be found by solving the SDP

$$V_{\alpha,i}^{\text{CV},\mathcal{O}}(x_i, w_i) = \max_{a_i \in \mathcal{A}_i(x_i) \setminus \{C\}} \left\{ r_i(x_i, w_i, a_i) + \delta \text{CVaR}_{\alpha,i} [V_{\alpha,i+1}^{\text{CV}}(f_i(x_i, a_i), w_{i+1})] \right\}, \quad \forall (i, x_i, w_i) \in \mathcal{I} \times \mathcal{X}_i \times \mathcal{W}_i, \quad (4.2a)$$

$$V_{\alpha,i}^{\text{CV}}(x_i, w_i) = \max \left\{ V_{\alpha,i}^{\text{CV},\mathcal{O}}(x_i, w_i), -K^{(x_i, C)} \right\}, \quad \forall (i, x_i, w_i) \in \mathcal{I} \times \mathcal{X}_i \times \mathcal{W}_i, \quad (4.2b)$$

where we omit the terminal (stage I) condition in this and the following SDPs as it is analogous to the shutdown-neutral terminal reward (3.2a). $V_{\alpha,i}^{\text{CV},\mathcal{O}}(x_i, w_i)$ and $V_{\alpha,i}^{\text{CV}}(x_i, w_i)$ are value functions that represent utilities when using the CVaR measure. Contrasting the shutdown-neutral value function (3.2b) and (4.2a) shows that $V_{\alpha,i}^{\text{CV},\mathcal{O}}(x_i, w_i)$ is computed using the left tail of the next-stage value function distribution instead of its expectation. Because $\text{CVaR}_{\alpha,i}[X(w_{i+1})] \leq \mathbb{E}_i[X(w_{i+1})]$, the value of $V_{\alpha,i}^{\text{CV},\mathcal{O}}(x_i, w_i)$ is smaller than $V_i^{\text{SN},\mathcal{O}}(x_i, w_i)$ and this difference generally increases with α . In addition, (3.2c) and (4.2b) compare $V_i^{\text{SN},\mathcal{O}}(x_i, w_i)$ and $V_{\alpha,i}^{\text{CV},\mathcal{O}}(x_i, w_i)$, respectively, with the same shutdown cost $-K^{(x_i, C)}$. Therefore, intuitively, as α is increased, the CVaR-based policy favors the shutdown decision more relative to the shutdown-neutral policy, which is at odds with the desired aversion to this decision. We verify this intuition by evaluating numerically the performance of CVaR policies in Section 4.2.3.

4.2.1 Reverse CVaR (RCVaR)

Overcoming this issue requires an intuitive modification of the standard CVaR definition to focus on the $1 - \alpha$ proportion of the *best* realizations of a random variable. This modification, which we denote reverse CVaR (RCVaR), is defined as

$$\text{RCVaR}_{\alpha,i}[X(w_{i+1})] := \mathbb{E}_i[X(w_{i+1}) | X(w_{i+1}) \geq \text{VaR}_{\alpha,i}(X(w_{i+1}))]. \quad (4.3)$$

For illustration, in Figure 4.1 we contrast the CVaR operator (4.1) with the RCVaR operator (4.3).

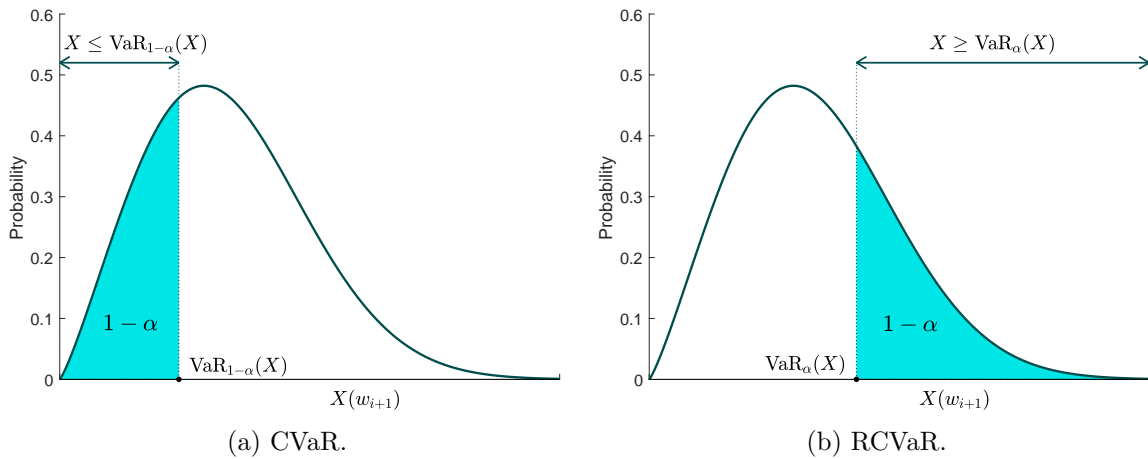


Figure 4.1: Sample profit distribution $X(w_{i+1})$ of which $\text{CVaR}_{\alpha,i}[X(w_{i+1})]$ and $\text{RCVaR}_{\alpha,i}[X(w_{i+1})]$ consider, respectively, the left tail (i.e. the worst realizations) and the right tail (i.e. the best realizations).

The definition of RCVaR presented in equation (4.3) holds for continuous distributions. Following Rockafellar and Uryasev (2002), we provide a more general definition that also handles both continuous and discrete distributions. For consistency with the above discussion, we consider a random variable $X(w_{i+1})$ representing a profit function, rather than a loss function as in Rockafellar and Uryasev (2002). For $\alpha \in [0, 1)$, let us define:

$$\begin{aligned} \text{VaR}_{\alpha,i}(X(w_{i+1})) &:= \inf \{x | F_X(x) \geq \alpha\}, \\ \text{RCVaR}_{\alpha,i}^+[X(w_{i+1})] &:= \mathbb{E}_i [X(w_{i+1}) | X(w_{i+1}) > \text{VaR}_{\alpha,i}(X(w_{i+1}))]. \end{aligned}$$

Then, $\text{RCVaR}_{\alpha,i}[X(w_{i+1})]$ is computed as a weighted average of the two terms

$\text{VaR}_{\alpha,i}(X(w_{i+1}))$ and $\text{RCVaR}_{\alpha,i}^+[X(w_{i+1})]$:

$$\text{RCVaR}_{\alpha,i}[X(w_{i+1})] := \begin{cases} \phi_{\alpha,i}(X(w_{i+1}))\text{VaR}_{\alpha,i}(X(w_{i+1})) + (1 - \phi_{\alpha,i}(X(w_{i+1}))) \cdot \\ \quad \text{RCVaR}_{\alpha}^+[X(w_{i+1})], & \text{if } F_X(\text{VaR}_{\alpha}(X(w_{i+1}))) < 1; \\ \text{VaR}_{\alpha,i}(X(w_{i+1})) & \text{if } F_X(\text{VaR}_{\alpha,i}(X(w_{i+1}))) = 1, \end{cases} \quad (4.5)$$

where $\phi_{\alpha,i}(X(w_{i+1})) := (F_X(\text{VaR}_{\alpha,i}(X(w_{i+1}))) - \alpha) / (1 - \alpha)$. We will show an application of this general definition to a discrete distribution later in this section.

We use $\text{RCVaR}_{\alpha,i}$ to define a set of shutdown-averse policies π_{α}^{RC} parameterized by α via the following SDP.

$$V_{\alpha,i}^{\text{RC},\mathcal{O}}(x_i, w_i) = \max_{a_i \in \mathcal{A}_i(x_i) \setminus \{C\}} \left\{ r_i(x_i, w_i, a_i) + \delta \text{RCVaR}_{\alpha,i} [V_{\alpha,i+1}^{\text{RC}}(f_i(x_i, a_i), w_{i+1})] \right\}, \quad \forall (i, x_i, w_i) \in \mathcal{I} \times \mathcal{X}_i \times \mathcal{W}_i, \quad (4.6a)$$

$$V_{\alpha,i}^{\text{RC}}(x_i, w_i) = \max \left\{ V_{\alpha,i}^{\text{RC},\mathcal{O}}(x_i, w_i), -K^{(x_i, C)} \right\}, \quad \forall (i, x_i, w_i) \in \mathcal{I} \times \mathcal{X}_i \times \mathcal{W}_i. \quad (4.6b)$$

Analogously to CVaR, also the RCVaR operator reduces to the expectation when $\alpha = 0$, that is, $\text{RCVaR}_{0,i}(\cdot) \equiv \mathbb{E}_i[\cdot]$. However, in contrast to CVaR, for any stage $i \in \mathcal{I}$ it holds that

$$\text{RCVaR}_{\alpha_1,i}[X(w_{i+1})] \geq \text{RCVaR}_{\alpha_2,i}[X(w_{i+1})] \geq \mathbb{E}_i[X(w_{i+1})]$$

for $\alpha_1 \geq \alpha_2$. Proposition 4.1 establishes that this monotonicity property holds in the general multi-stage setting, which confirms that the RCVaR policy π_{α}^{RC} models aversion to shutdown decisions when α is increased. Let us recall the notation $\Pr(C; \pi)$ to indicate the shutdown probability at the end of the horizon for the policy π .

Proposition 4.1. *Given $\alpha_1, \alpha_2 \in [0, 1)$ such that $\alpha_1 \geq \alpha_2$, it holds that*

$$\Pr(C; \pi_{\alpha_1}^{\text{RC}}) \leq \Pr(C; \pi_{\alpha_2}^{\text{RC}}).$$

Proof. It suffices to show that the value function $V_{\alpha,i}^{\text{RC},\mathcal{O}}(x_i, w_i)$ excluding shutdown is increasing in α because the policies $\pi_{\alpha_1}^{\text{RC}}$ and $\pi_{\alpha_2}^{\text{RC}}$ are determined by comparing this value function with the same shutdown cost $K^{(x_i, C)}$. We thus focus on showing that

$$V_{\alpha_1,i}^{\text{RC},\mathcal{O}}(x_i, w_i) \geq V_{\alpha_2,i}^{\text{RC},\mathcal{O}}(x_i, w_i), \quad \forall (i, x_i, w_i) \in \mathcal{I} \times \mathcal{X}_i \times \mathcal{W}_i \quad (4.7)$$

for $\alpha_1, \alpha_2 \in [0, 1)$ such that $\alpha_1 \geq \alpha_2$. The proof of (4.7) is based on backward induction. At stage I , the terminal rewards match: $V_{\alpha_1,I}^{\text{RC}}(x_I, w_I) = V_{\alpha_2,I}^{\text{RC}}(x_I, w_I) = r_I(x_I, w_I)$ for all $(x_I, w_I) \in \mathcal{X}_I \times \mathcal{W}_I$. Assume that analogous inequalities to (4.7) hold from stages $i + 1$ through stage I and consider stage i . We use the following chain of inequalities for all

$(x_i, w_i) \in \mathcal{X}_i \times \mathcal{W}_i$:

$$\begin{aligned} & r_i(x_i, w_i, a_i) + \delta \text{RCVaR}_{\alpha_1, i} [V_{\alpha_1, i+1}^{\text{RC}}(f_i(x_i, a_i), w_{i+1})] \\ & \geq r_i(x_i, w_i, a_i) + \delta \text{RCVaR}_{\alpha_1, i} [V_{\alpha_2, i+1}^{\text{RC}}(f_i(x_i, a_i), w_{i+1})] \\ & \geq r_i(x_i, w_i, a_i) + \delta \text{RCVaR}_{\alpha_2, i} [V_{\alpha_2, i+1}^{\text{RC}}(f_i(x_i, a_i), w_{i+1})]. \end{aligned}$$

We obtain the first inequality using the monotonicity property of $\text{RCVaR}_{\alpha}[\cdot]$ with respect to its argument and the condition $V_{\alpha_1, i+1}^{\text{RC}}(f_i(x_i, a_i), w_{i+1}) \geq V_{\alpha_2, i+1}^{\text{RC}}(f_i(x_i, a_i), w_{i+1})$ which follows from the inequality $V_{\alpha_1, i+1}^{\text{RC}, \mathcal{O}}(x_{i+1}, w_{i+1}) \geq V_{\alpha_2, i+1}^{\text{RC}, \mathcal{O}}(x_{i+1}, w_{i+1})$ that holds by the induction hypothesis. We derive the second inequality by leveraging the monotonicity of the $\text{RCVaR}_{[\cdot]}[X]$ operator with respect to α . Maximizing over actions in $\mathcal{A}_i \setminus \{C\}$ on both sides of this inequality implies equation (4.7) at stage i . Our claim holds at all stages by the principle of mathematical induction. \square

Despite this property, the shutdown probability under π_{α}^{RC} does not necessarily converge to zero when $\alpha \rightarrow 1$. The reasoning behind this statement is the following. RCVaR inflates the value of the non-shutdown actions by a term $\text{RCVaR}_{\alpha, i} [V_{\alpha, i+1}^{\text{RC}}(f_i(x_i, a_i), w_{i+1})]$ which depends on the next-stage value function distribution shown in Proposition 4.1 to increase with α . However, even if α is high, this inflation may not be sufficiently high to eliminate shutdown as an optimal action. For example, given a particular state (x_i, w_i) and action a_i , suppose the distribution of $V_{\alpha, i+1}^{\text{RC}}(f_i(x_i, a_i), w_{i+1})$ with respect to the random next-stage information state w_{i+1} is bounded by a constant C . Then the term $\text{RCVaR}_{\alpha, i}$ is also bounded above by C . Therefore, if shutdown is optimal and $r_i(x_i, w_i, a_i) + C < -K^{(\mathcal{O}, C)}$ holds for every non-shutdown action a_i , then shutdown will remain optimal under an RCVaR policy for any value of α . There is consequently no guarantee that the RCVaR policy will converge to zero shutdown probability for $\alpha \rightarrow 1$. We observed this behavior in our numerical experiments as well. Moreover, even if this policy reaches zero shutdown probability, it is in general suboptimal, that is, its corresponding asset value may be smaller than under the policy $\pi^{\text{SN} \setminus \{C\}}$ as a result of a possible incorrect use of the non-shutdown flexibility. Indeed, the RCVaR inflation term is different across actions and can alter the order preference of the non-shutdown actions compared to $\pi^{\text{SN} \setminus \{C\}}$ (this instead does not occur in anticipated regret decision rules as proven by Proposition 3.1).

Both RCVaR and the anticipated regret policies model aversion to a shutdown decision and are defined through a shutdown-averse SDP. However, in Example 4.1, we highlight the differences between these two policies showing that they can make opposite decisions.

Example 4.1. Consider a production asset operating over 2 stages (that is, $\mathcal{I} \equiv \{0, 1\}$), which has a single non-shutdown decision and shutdown cost κ equal to 11. We consider two cases for the random stage 1 utility when the decision at stage 0 is not to shut down.

These cases are characterized by discrete utility distributions $X^1(w_1)$ and $X^2(w_1)$ defined as

$$X^1(w_1) := \begin{cases} -10 & \text{with probability } 0.5, \\ -30 & \text{with probability } 0.5; \end{cases} \quad X^2(w_1) := \begin{cases} +4 & \text{with probability } 0.25, \\ -28 & \text{with probability } 0.75. \end{cases}$$

The expected utility is the same in each case, that is, $\mathbb{E}[X^1(w_1)] = \mathbb{E}[X^2(w_1)] = -20$. Since the expected utility from not shutting down of -20 is smaller than the shutdown cost of -11 , the optimal shutdown-neutral decision at stage 0 is to shut down in both cases. Next consider the anticipated regret. We have $\mathbb{E}_0[\max\{X^1(w_1) + 11, 0\}] = (-10 + 11) \cdot 0.5 = 0.5$ and $\mathbb{E}_0[\max\{X^2(w_1) + 11, 0\}] = (4 + 11) \cdot 0.25 = 3.75$. The anticipated regret is clearly larger in case 2 than case 1. Thus a producer that accounts for anticipated regret would favor switching to a non-shutdown decision in case 2 compared to case 1. We show next that an RCVaR policy may do the opposite by choosing α equal to 0.5 for our illustration, that is, we compute the expected value of 50% ($1 - \alpha = 0.5$) of the best outcomes. An intuitive RCVaR computation is as follows. In the distribution $X^1(w_1)$ the event with the higher utility of -10 has mass exactly equal to 0.5, which implies that $RCVaR_{0.5,0}[X^1(w_1)] = -10$. In contrast, the distribution $X^2(w_1)$ has a mass of only 0.25 associated with the outcome 4, and the residual mass of 0.25 needs to be associated with the outcome -28 . Therefore, $RCVaR_{0.5,0}[X^2(w_1)] = (4 \cdot 0.25 - 28 \cdot 0.25)/0.5 = -12$. More formally, let us apply the general RCVaR definition of (4.5). We start with $X(w_1)$ and determine $VaR_{0.5,0}(X^1(w_1)) = -30$, $RCVaR_{0.5,0}^+[X^1(w_1)] = -10$, $F_{X^1}(VaR_{0.5,0}(X^1(w_1))) = 0.5$, and $\phi_{\alpha,0}(X^1(w_1)) = (0.5 - 0.5)/(1 - 0.5) = 0$. These computations imply that $RCVaR_{0.5,0}[X^1(w_1)] = RCVaR_{0.5,0}^+[X^1(w_1)] = -10$. Similarly, we compute $VaR_{0.5,0}(X^2(w_1)) = -28$, $RCVaR_{0.5,0}^+[X^2(w_1)] = 4$, $F_{X^2}(VaR_{0.5,0}(X^2(w_1))) = 0.75$, and $\phi_{\alpha,0}(X^2(w_1)) = (0.75 - 0.5)/(1 - 0.5) = 0.5$, which gives $RCVaR_{0.5,0}[X^2(w_1)] = 0.5 \cdot (-28) + 0.5 \cdot 4 = -12$. The RCVaR of $X^1(w_1)$ and $X^2(w_1)$ are larger and smaller, respectively, than the shutdown cost of -11 . Hence, the RCVaR policy switches out of the shutdown decision in case 1 but not in case 2.

Proposition 4.2 establishes that the discrepancy between anticipated regret and RCVaR shown in Example 4.1 is possible at any shutdown aversion level α .

Proposition 4.2. *For a given shutdown aversion level $\alpha \in (0, 1)$, shutdown cost $\kappa > 0$, and constant τ , such that $\tau < -\kappa$, let $\{X(w_1; \zeta) | \zeta \in \mathbb{N}^+, \zeta > -\tau\}$ be a sequence of distributions where*

$$X(w_1; \zeta) := \begin{cases} \zeta & \text{with probability } \frac{(1-\alpha)(\zeta+\tau)}{2\zeta}, \\ -\zeta & \text{with probability } 1 - \frac{(1-\alpha)(\zeta+\tau)}{2\zeta}. \end{cases}$$

It holds that $\text{RCVaR}_{\alpha,0}[X(w_1; \zeta)] = \tau$ and $\mathbb{E}_0[\max\{X(w_1; \zeta) + \kappa, 0\}]$ (i.e., the anticipated regret) strictly increases with ζ for $\zeta > -\tau$.

Proof. Let $p_\zeta := (1 - \alpha)(\zeta + \tau)/(2\zeta)$. This probability is strictly positive since $\zeta > -\tau$ by assumption. Moreover, $p_\zeta < 1 - \alpha$ follows from $\tau < 0$. Since $p_\zeta \in (0, 1 - \alpha)$ we have

$$\text{RCVaR}_{\alpha,0}[X(w_1; \zeta)] = \frac{1 - p_\zeta - \alpha}{1 - \alpha}(-\zeta) + \left(1 - \frac{1 - p_\zeta - \alpha}{1 - \alpha}\right)\zeta = \frac{2\zeta p_\zeta}{1 - \alpha} - \zeta = \tau,$$

where the first equality follows from the definition of RCVaR (4.5), the second from simplifying, and the last from substituting for p_ζ using its definition and further simplifying. Using the definition of anticipated regret and re-arranging terms gives

$$\begin{aligned} \mathbb{E}_0[\max\{X(w_1; \zeta) + \kappa, 0\}] &= (\zeta + \kappa)p_\zeta = \frac{(\zeta + \kappa)(1 - \alpha)(\zeta + \tau)}{2\zeta} \\ &= \frac{1 - \alpha}{2} \left(\zeta + \tau + \kappa + \frac{\kappa\tau}{\zeta} \right). \end{aligned}$$

Therefore, as ζ tends to infinity, the anticipated regret also tends to infinity. \square

Specifically, this proposition constructs a sequence of utility distributions where the RCVaR value as a function of ζ equals a constant that is strictly less than the shutdown cost, but the anticipated regret strictly increases with ζ and tends to infinity. Hence, the RCVaR policy always chooses the shutdown action even when the anticipated regret is arbitrarily large.

4.2.2 Computing CVaR-based policies with LSML

To obtain CVaR and RCVaR policies we used LSML to approximate the SDPs (4.2) and (4.6), respectively. The computation of these policies is similar to the computation of AR policies. We thus refer to Algorithm 4 in Appendix 3.7.2 for a detailed description of the LSML algorithm to approximate the AR shutdown-averse SDP, and only report the changes needed to compute RCVaR policies here. For a given $\alpha \in [0, 1)$, the VFA estimates to approximate the RCVaR SDP (4.6) are obtained with

$$\begin{aligned} v_{\alpha,i}^{\text{RC},\mathcal{O}}(x_i, w_i^p) &:= \max_{a_i \in \mathcal{A}_i(x_i) \setminus \{C\}} \left\{ r_i(x_i, w_i^p, a_i) + \delta \text{RCVaR}_{\alpha,i} \left[\sum_{b \in \mathcal{B}_{i+1}} \beta_{i+1, f_i(x_i, a_i), b}^{\text{RC}\alpha} \right. \right. \\ &\quad \left. \left. \cdot \Phi_{i+1, b}(\bar{w}_{i+1}^{p,n}) : n \in \mathcal{N} \right] \right\}; \end{aligned}$$

$$v_{\alpha,i}^{\text{RC}}(x_i, w_i^p) := \max \left\{ v_{\alpha,i}^{\text{RC},\mathcal{O}}(x_i, w_i^p), -K^{(x_i,\text{C})} \right\},$$

where $\Phi_{i,b}$ and $\beta_{i,x_i,b}^{\text{RC}\alpha}$ denote, respectively, the b -th VFA basis function at stage i and its associated coefficient in the linear combination, and $\text{RCVaR}_{\alpha,i}$ is computed using the definition (4.5). Given VFA weight vectors $\beta_{i,x_i}^{\text{RC}\alpha}$ for each $(i, x_i) \in \{1, \dots, I\} \times \mathcal{X}_i$, the action $a_{\alpha,i}^{\text{RC}}(x_i, w_i)$ taken by the RCVaR_{α} policy at stage i and state (x_i, w_i) is

$$a_{\alpha,i}^{\text{RC}}(x_i, w_i) := \begin{cases} \text{C}, & \text{if } v_{\alpha,i}^{\text{RC},\mathcal{O}}(x_i, w_i) < -K^{(x_i,\text{C})}, \\ \arg \max_{a_i \in \mathcal{A}_i(x_i) \setminus \{\text{C}\}} \left\{ r_i(x_i, w_i, a_i) + \delta \text{RCVaR}_{\alpha,i} \left[\sum_{b \in \mathcal{B}_{i+1}} \beta_{i+1,f_i(x_i,a_i),b}^{\text{RC}\alpha} \cdot \Phi_{i+1,b}(w_{i+1}^n) : n \in \mathcal{N} \right] \right\}, & \text{otherwise.} \end{cases}$$

Computing CVaR policies is analogous to RCVaR policies but the operator $\text{RCVaR}_{\alpha,i}[\cdot]$ is replaced by $\text{CVaR}_{\alpha,i}[\cdot]$.

4.2.3 Results

We used LSML to compute CVaR and RCVaR policies and assess their performance. The setting used in LSML—including the choice of basis functions and the number of regression and inner sample paths of the uncertainty—matches the one employed in Chapter 3 (see the computational setup in Section 3.5.3). For consistency, we also used the same 20,000 sample paths for policy evaluation used in Section 3.5.3.

Figure 4.2 displays the trade-off between asset value and shutdown probability for CVaR and RCVaR policies corresponding to a range of aversion levels $\alpha \in [0, 0.35]$ and $\alpha \in [0, 0.9]$, respectively. The figure also reports the trade-off achieved with the shutdown-neutral policies π^{SN} and $\pi^{\text{SN} \setminus \{\text{C}\}}$, and the delay-neutral AR policies, that is, AR policies with $\tilde{\xi} = 1$ and varying λ values (as in Figure 3.5). Each asset value estimate is expressed as a percentage of the maximum asset value, that is, the asset value estimate for π^{SN} . The CVaR and RCVaR standard errors of the asset value and shutdown probability are at most 1.02% and 0.35%, respectively.

As expected, the trade-off curve corresponding to the standard CVaR is incorrect as the shutdown probability inflates as α is increased. In contrast, our intuitive RCVaR modification provides correct trade-offs and the shutdown probability decreases when α is increased. Contrasting RCVaR and AR, however, highlights that AR policies significantly outperform RCVaR policies and provide more efficient trade-offs between asset value and

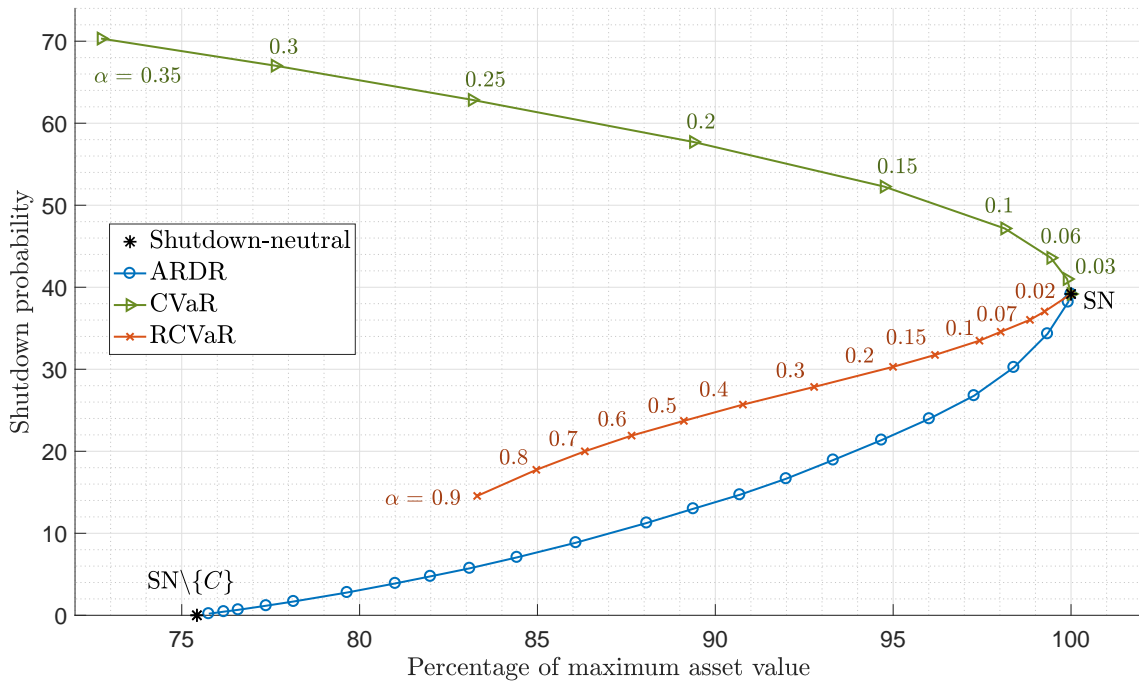


Figure 4.2: Trade-offs in asset value and shutdown probability when using shutdown-neutral, CVaR, RCVaR, and AR policies.

shutdown probability. For example, a shutdown probability decrease of 50% is achieved in the former and latter policies, respectively, for asset value reductions of roughly 6% and 14%. Moreover, we know from Proposition 3.1 that AR policies present desirable asymptotic properties and the shutdown probability converges to zero when λ is large enough. From Figure 4.2, we notice that the same property is instead not true for CVaR-based policies. Specifically, while both CVaR and RCVaR policies coincide to π^{SN} when $\alpha = 0$, the opposite shutdown-averse asymptotic behaviour does not converge to zero shutdowns, consistently with the discussion in Section 4.2.1.

The algorithms for these experiments were implemented using Matlab R2016b. For a given α value, estimating the LSML VFA for CVaR and RCVaR policies took on average 24 minutes, and estimating the value of a given policy required roughly the same running time as estimating the VFA. We employed parallel computing to distribute the load across multiple cores and drastically reduce the total running time (see Section 3.5.3 for details on the computational resources used and the parallel computing setting).

This application represents a case in which the popular CVaR measure performs badly, that is, it does not capture shutdown aversion. Our intuitive adaptation, RCVaR, captures shutdown aversion but exhibits poor performance. The reason behind this poor performance might be that CVaR is a metric used for capturing risk aversion but our

work focuses on aversion to a decision in our control as opposed to the standard risk aversion in the literature. Shutdown is indeed an option the firm can exercise in response to movements in costs, prices and exchange rates. Rather than considering risk aversion, in this work we are optimizing over two metrics: asset value and shutdown probability. Moreover, CVaR is a generic metric for managing risk aversion while other methods that we have developed in Chapter 3, especially anticipated regret (AR) and classification margin (CM) policies, are designed to directly manage shutdown decisions and can thus have an advantage. We conclude by suggesting caution in using methods that are not tailored to specifically manage shutdown decisions, even though such methods are popular in other related contexts as classical financial risk aversion.

4.3 Forward purchasing of power

To assess the impact of power price uncertainty on shutdown probability, we consider modifications of the shutdown-neutral aluminum production model with front-year procurement of electricity (that is, SDP (3.2) with the reward function (3.8)). Specifically, we use forward power contracts of fixed length of d years, with $d \in \mathbb{N}$, $d > 0$, for power purchases and allow the sale of power into the wholesale market when the plant is shut down before completing the tenure of existing contracts. For example, a duration of $d = 10$ years means that 10-years power contracts are entered at stages 0, 10, 20, and 30, that is, the contract is renewed every 10 years provided that the plant is not shut down.

The price of a forward power contract with duration d years entered at the beginning of stage i is denoted $P_{i,d}^{\text{EL}}$. This price is calculated so that arbitrage opportunities are precluded from trading in the forward and front-year markets, which corresponds to the condition

$$\sum_{j=0}^{d-1} \delta^j P_{i,d}^{\text{EL}} = (1 + \gamma_d^{\text{EL}}) \sum_{j=0}^{d-1} \delta^j \mathbb{E}_i [P_{i+j}^{\text{EL}}], \quad (4.8)$$

where γ_d^{EL} represents the risk premium to enter a forward power contract with duration d years. To intuitively understand this relationship, suppose that there is no discounting and no risk premium, that is, $\delta = 1$ and $\gamma_d^{\text{EL}} = 0$. In this case, the no arbitrage condition reduces to the familiar equality

$$P_{i,d}^{\text{EL}} = \frac{1}{d} \sum_{j=0}^{d-1} \mathbb{E}_i [P_{i+j}^{\text{EL}}],$$

where the left-hand side is the contract price and the right-hand side is the average of

the expected power prices over the contract duration. When discounting is present, we introduce δ^j and obtain the summation in the left-hand side of (4.8). The risk premium is incorporated in (4.8) using the term $(1 + \gamma_d^{\text{EL}})$.

We next define the modifications to the reward function in SDP (3.2) to model forward contracting. Let $\xi(w_i) := (1 - \tau)\rho P_i^{\text{EUR-USD}}$. Denote by $i' = i'(i, d) := i - \text{mod}(i, d)$ the starting year of the forward contract and by $l_i = l(i, d) := i' + d - i$ the number of years of delivery remaining before renewal. For example, if $i = 24$ and $d = 10$, then $i' = 20$ and $l_i = 6$. The production cash flow at time $i \in \mathcal{I}$ under a d -year power contract entered at $i' \leq i$ is

$$r_{\text{O}}^d(w_i) := (1 - \tau) [P_i^{\text{AL}}(1 + \gamma^{\text{AL}}) - c^{\text{USD}} - c^{\text{NOK}} P_i^{\text{NOK-USD}} - \rho P_{i',d}^{\text{EL}} P_i^{\text{EUR-USD}}]. \quad (4.9)$$

Expression (4.9) can be obtained from (3.8) by replacing P_i^{EL} with $P_{i',d}^{\text{EL}}$. If the production is suspended before reaching the tenure of a forward contract, the power for only that specific suspension year is sold back into the wholesale market for a revenue of $\xi_i(w_i) \cdot (P_i^{\text{EL}} - P_{i',d}^{\text{EL}})$. When the plant is reopened after temporary suspension, the residual portions of existing contracts entered prior to suspension are still available. In contrast, if the plant is shut down before the end of a contract tenure, the remaining years of power delivery are sold back into the wholesale market for a revenue equal to $\xi_i(w_i) \cdot (P_i^{\text{EL}} - P_{i',d}^{\text{EL}}) l_i$. The complete immediate reward function is defined in Table 4.1. The possibility of selling

Table 4.1: Aluminum production asset immediate reward function with power forward contracts.

State $[x_i]$	Action $[a_i]$	Reward $[r_i^d(x_i, w_i, a_i)]$
O	O	$r_{\text{O}}^d(w_i, P_{i',d}^{\text{EL}})$
	S ₁	$-K^{(\text{O}, \text{S}_1)} + \xi_i(w_i) \cdot (P_i^{\text{EL}} - P_{i',d}^{\text{EL}})$
	C	$\begin{cases} -K^{(\text{O}, \text{C})} & \text{if } i = i' \\ -K^{(\text{O}, \text{C})} + \xi_i(w_i) \cdot (P_i^{\text{EL}} - P_{i',d}^{\text{EL}}) l_i & \text{if } i \neq i' \end{cases}$
S _{m} , $m \in \{1, 2, 3\}$	O	$r_{\text{O}}^d(w_i, P_{i',d}^{\text{EL}}) - K^{(\text{S}_m, \text{O})}$
	S _{$m+1$} (if $m < 3$)	$-K^{(\text{S}_m, \text{S}_{m+1})} + \xi_i(w_i) \cdot (P_i^{\text{EL}} - P_{i',d}^{\text{EL}})$
	C	$\begin{cases} -K^{(\text{S}_m, \text{C})} & \text{if } i = i' \\ -K^{(\text{S}_m, \text{C})} + \xi_i(w_i) \cdot (P_i^{\text{EL}} - P_{i',d}^{\text{EL}}) l_i & \text{if } i \neq i' \end{cases}$
C	C	0

power back into the wholesale market affects the smelter operating decisions: It favors the suspension and shutdown actions when the spread $P_i^{\text{EL}} - P_{i',d}^{\text{EL}}$ is positive and the open decision when negative.

We employed LSML to compute heuristic operating policies of the SDP (3.2) variant

based on the reward function of Table 4.1 for $d \in \{1, 2, 4, 5, 10, 20\}$. We used the same computational setup from Section 3.5.4 and report the results in Table 4.2. The policy for $d = 1$ coincides with π^{SN} . In the experiments, we used a risk premium that grows linearly with the contract duration d , for $d > 1$. Specifically, we tested two values: $\gamma_d^{\text{EL}} = d \cdot 0.1\%$ and $\gamma_d^{\text{EL}} = d \cdot 0.2\%$. The latter choice, for example, corresponds to a premium of 2% for entering into a 10-year power contract. For comparison, we also tested the model where no sale of power is allowed. This model helps us assess the impact of forward contracts without the sale of power. The asset value estimates are expressed as percentages of the estimates for $d = 1$.

Table 4.2: Estimates of asset value and shutdown probability with forward power contracts.

Power procurement model		1-Y	2-Y	4-Y	5-Y	10-Y	20-Y
Power sales allowed (Table 4.1)	Value (% of $d = 1$)	100.0	100.4	101.1	101.4	102.6	104.5
Risk premium $\gamma_d^{\text{EL}} = d \cdot 0.1\%$	Shutdown prob. (%)	39.2	39.2	39.3	39.3	39.2	38.9
Power sales allowed (Table 4.1)	(% of $d = 1$)	100.0	100.1	100.4	100.5	100.7	100.6
Risk premium $\gamma_d^{\text{EL}} = d \cdot 0.2\%$	Shutdown prob. (%)	39.2	39.2	39.4	39.4	39.3	39.1
No power sales (Table 3.1)	(% of $d = 1$)	100.0	100.3	101.0	101.3	102.0	102.5
Risk premium $\gamma_d^{\text{EL}} = d \cdot 0.1\%$	Shutdown prob. (%)	39.2	39.2	39.5	39.5	39.8	40.3
No power sales (Table 3.1)	(% of $d = 1$)	100.0	100.0	100.3	100.4	100.4	99.3
Risk premium $\gamma_d^{\text{EL}} = d \cdot 0.2\%$	Shutdown prob. (%)	39.2	39.3	39.6	39.6	39.6	40.6

We find that the asset value when using forward power contracts can increase by at most 4.5% relative to no forward purchasing. Moreover, the shutdown probability varies only marginally compared to no forward purchasing: It decreases and increases by at most 0.8% and 0.5%, respectively. The standard error is at most 0.74% for asset value estimates and 0.35% for shutdown probability estimates.

The increase in asset value and shutdown probability, although unexpected, is determined by two characteristics of the model. First, when the power price at period i is higher than the price of the forward contract entered at time $i' < i$ (that is, the spread $P_i^{\text{EL}} - P_{i',d}^{\text{EL}}$ is positive), the possibility of selling power makes shutting down a more attractive option to increase the value of the asset. Second, even if power sales is not allowed, asset value and shutdown probability can increase when using forward power contracts because of the quadratic term $P_i^{\text{EUR-USD}} P_{i',d}^{\text{EL}}$ in the reward function (4.9) (we verified that the asset value is constant when keeping the EUR-USD rate deterministic, that is, using a linear reward function and a zero risk premium, in which case it is known that forward contracting adds no value).

Our analysis shows that forward purchasing of power has little impact on shutdown prob-

ability on our realistic aluminum instances. Moreover, financial hedging in this context seems to be complex to set up due to a reward function that is non-linear as it includes products between commodity/energy prices and exchange rates.

4.4 Conclusions: The value of the shutdown option

Reducing shutdown probability using the plant's operating flexibility involves managing the use of its shutdown option. A conceptually simple, albeit still computationally challenging, approach to eliminate shutdown completely involves restricting an operating policy to choose only among the set of non-shutdown decisions. In other words, we can solve the shutdown-neutral SDP (3.4), for instance using LSML, to obtain a near optimal zero-shutdown-probability operating policy. We refer to the difference between the value of this policy and the maximum asset value (that is, the asset value of the plant including shutdown action) as the shutdown option value. When this option value is small, managing the trade-off between asset value and shutdown probability may not be as critical because it will be economically feasible to use an operating policy that does not consider the shutdown decision. In similar spirit, if the shutdown probability under a near optimal shutdown-neutral policy obtained from SDP (3.2) is small, then the benefit of reducing this probability further is likely to be low. In other words, shutdown-averse models and operating policies are not needed in the preceding two cases.

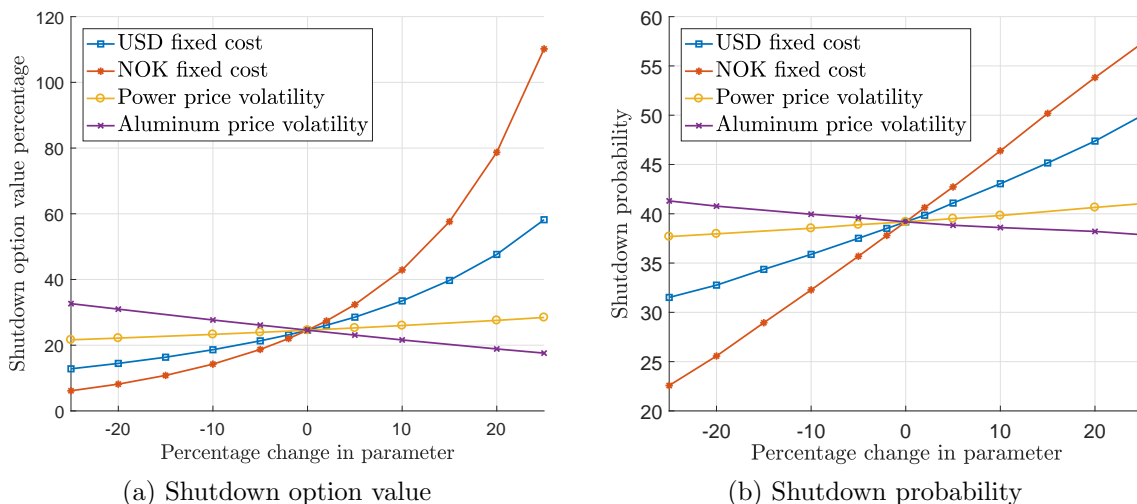


Figure 4.3: Shutdown option value percentage and probability as a function of percentage changes in NOK and USD fixed costs and electricity and aluminum volatilities relative to the reference instance.

Figure 4.3(a) displays the shutdown option value as a percentage of the maximum asset

value for changes of the USD and NOK operating fixed costs (c^{USD} and c^{NOK}) and the volatilities of power and aluminum price. Specifically, we increase and decrease by up to 25% the fixed costs and calibrated volatilities relative to the reference instance (see Section 3.5.3 for details on the instances). The shutdown option value percentage is 25% for no parameter change, that is, the reference instance. This option value percentage remains large for all considered volatility changes ($> 18\%$) and increases with the fixed costs. For large increases in the NOK fixed cost, the shutdown option value exceeds the maximum asset value because the asset value without the shutdown decision becomes negative. Decreasing the fixed cost in USD reduces the shutdown option value percentage, but it still remains significant and greater than 13%. An analogous statement holds for small to moderate decreases in the NOK fixed cost, but the shutdown option value percentage reduces to about 6% when this fixed cost is reduced by 25%. Overall, the shutdown option value remains large under almost all realistic perturbations of parameters that we considered in our aluminum production case study.

Figure 4.3(b) reports shutdown probability as a function of the same parameter changes considered in Figure 4.3(a). As expected, shutdown probability increases with fixed costs and the volatility of the power price, but surprisingly decreases with the volatility of the aluminum price. Nevertheless, shutdown probability remains large in all cases.

Based on the above findings, we conclude that managing shutdown decisions is significant in our aluminum production application for realistic parameter settings, and shutdown-neutral policies alone are unlikely to provide a practical approach to manage these decisions given the substantial shutdown option value. Therefore, shutdown-averse policies are needed to manage the trade-off between shutdown probability and asset value. The policies we have developed, in particular AR and CM, efficiently manage this trade-off as observed in Chapter 3. On the other hand, we found that popular methods used in academia and industry such as CVaR-based policies and forward electricity sourcing do not manage shutdown decisions well or do not reduce shutdown probability significantly. Managerial implications of our results from this chapter and the previous one include that (i) adapting the plant's operating flexibility is effective at reducing shutdown probability—more in general, at improving the shutdown profile—for small losses in asset value on realistic instances, but (ii) methods that are not tailored to manage shutdown decisions should be used with caution as their intuitive link with shutdown might be imprecise and result in under-performing policies and incorrect shutdown decisions. Finally, (iii) financial flexibility emerges as a less efficient lever to manage shutdown decisions compared to operating flexibility.

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Meeting Corporate Renewable Power Targets using a Dual Reoptimization Scheme

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Abstract: Large companies have recently started to incorporate renewable energy standards in their corporate sustainability goals. In this paper, we consider the problem of a company constructing a dynamic power procurement portfolio to satisfy a renewable power target, i.e. to procure a specific percentage of its electricity demand from renewable sources, at minimum expected cost. To meet this target, the company has the option at each stage of a finite planning horizon to enter into different short and long term power contracts with renewable generators and to purchase renewable energy certificates in the spot market. We formulate the problem as a stochastic dynamic program (SDP) which is high dimensional in both the endogenous and exogenous components of its state. Approximate dynamic programming methods to tackle this highly intractable formulation are very limited. To overcome this intractability, we consider the information relaxation approach that is

typically used to obtain dual bounds, and use it to develop a novel dual reoptimization scheme (DRH) that extracts non-anticipative decision rules from sample action distributions. We find that our DRH approach outperforms commonly used primal reoptimization methods and simple heuristics on realistic instances. Our DRH framework is applicable beyond the specific application presented in this paper and emerges as a promising approach to tackle high-dimensional SDPs.

Keywords: Renewable energy targets · power purchase agreements · energy real options · approximate dynamic programming · information relaxations · reoptimization

5.1 Introduction

During the last decade, a large number of companies have started to incorporate renewable energy standards into their corporate sustainability goals. Half of the Fortune 500 companies have announced commitments in or before 2016 to sustainability and climate targets which include targets on greenhouse gas emissions reduction, energy efficiency, and renewable energy procurement (CDP et al., 2017). One of the main reasons why companies are setting these targets is the increasing environmental attention and regulations over the past few years. Climate change has become a source of concern for companies and shareholders which consequently try to implement policies to hedge against possible regulator expectations and risks (CDP et al., 2017). The second main reason is the increasing competitiveness of renewable energy sources. The levelized cost of the energy from wind and solar generators has in fact decreased in the recent years, and in some regions this happened so fast that renewable energy generation is now even slightly cheaper than conventional energy generation (Wiser and Bolinger, 2017).

In this paper, we focus on companies that have committed to a renewable energy target. Meeting a renewable energy target for a company means procuring a specific percentage of its electricity demand from renewable power sources by a future date. To illustrate, assume that a company commits to a 50% renewable energy target by 2025. Then, before 2025 the company has no restrictions on its power procurement but it can enter into long term contracts with renewable generators that will deliver renewable energy also beyond 2025. Instead, for each year from 2025 onward, at least 50% of the electricity demand of the company must be supplied by renewable energy sources. To give some examples, Procter & Gamble, Intel, and Nike committed to reach a 30%, 75%, and 100% renewable target, respectively, by 2020, 2020, and 2025. The number of companies that are

setting this kind of target is rapidly increasing and ten percent of Fortune 500 companies have committed to a renewable target in or before 2016 (CDP et al., 2017; precisely, 53 companies in 2016 compared to 42 companies in 2014), with several of them striving to achieve 100% renewable power usage. Regarding the future date by which the target has to be met, around half of these companies chose the year 2020, which also corresponds to the expiration date of the production tax credit for renewable energy sources in the U.S. (DOE, 2016).

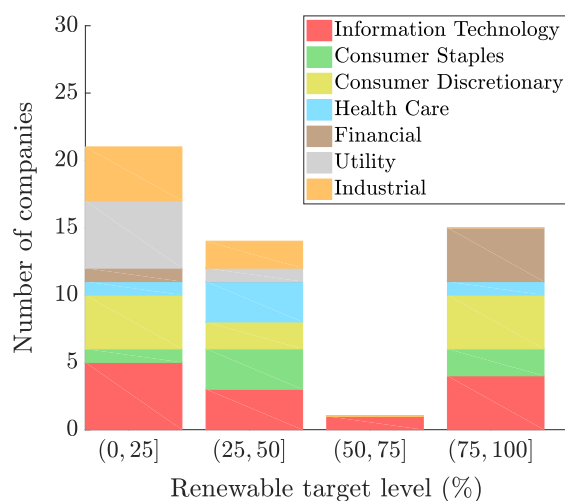


Figure 5.1: Renewable energy targets in the Fortune 500 companies.

Figure 5.1 reports the renewable power target distribution among the Fortune 500 divided into four intervals. Although many companies have chosen a target which is lower than 25% of the power demand, the figure shows that there is also a significant number of companies that have set a target higher than 75%. The histogram then illustrates how companies from distinct sectors value the procurement of renewable energy differently, for instance, financial corporations often have higher targets than industrial corporations. Companies with higher targets value the sustainability more than those with low targets and this benefit is clearly diverse across the sectors, resulting in various choices of the target level.

Despite the increasing trend of companies setting a climate target, most companies have not committed to one such target yet. According to PWC (2016), the lack of strategic knowledge about the renewable energy procurement is the main reason that drives companies back from pursuing a renewable energy target. Based on a CDP questionnaire, 44% of the companies without any climate target (that is, on greenhouse gas emissions reduction, energy efficiency, or renewable energy) were in the process of investigating how to set up a target and, in case of renewable targets, an associated procurement strategy (CDP et al., 2017). In this paper, we try to narrow this knowledge gap by constructing in a principled manner renewable energy procurement strategies over a long-time planning

horizon to meet a renewable target.

Constructing a multi-year procurement portfolio to meet a renewable target is challenging because of the long term planning horizon and also the number of buying options in the market. Thus, navigating the different renewable power procurement options and understanding their benefits and potential disadvantages is practically important. To meet a renewable target, companies can use a variety of approaches including unbundled renewable energy certificates (RECs), onsite installations, and offsite purchases via bilateral contracts known as power purchase agreements (PPAs). These options can provide companies with diversified renewable energy supply sources and can ensure long term power price stability. In particular, companies have recently paid attention to the PPAs due to the drop in renewable energy cost. In this analysis, we consider two dominant renewable energy procurement options (BNEF, 2018) that corporations use to satisfy their renewable power target: (i) buying power from the utility (i.e., akin to a spot purchase) and supplementing it with RECs and (ii) entering into PPAs to receive power directly from a renewable generator for a predefined number of years. Companies can also combine these two options. Corporate PPAs and in particular synthetic PPAs (i.e., contracts without physical delivery of power) are becoming increasingly popular around the world, with 1.6 GW of renewable capacity—mostly wind and solar—contracted through corporate PPAs in 2015 in the U.S. alone (McKenzie, 2015). Power under such PPAs is purchased at a fixed and predetermined price per megawatt hour, called strike price, which helps the firm hedging against the risk of fluctuating power and RECs prices. On the other hand, long term contracts expose the company to stochastic demand risk which might result in over procurement. We do not consider electricity storage options in our analysis due to the long planning horizon of our problem and because electricity storage capacity is still too expensive for widespread use and very limited, for instance, in the U.S. it is only about 0.15% (excluding pumped hydro) of the total production capacity (EPA, 2018).

In this paper, we study how firms can manage the spot and PPA purchasing options to reach a renewable target and then sustain this level of renewable procurement in the future. Within the PPA options, companies have the flexibility to choose size and length of the contract. For example, in the period 2010–2016 Google entered into 20 different PPAs with capacity ranging from 26 to 407 MW (Google, 2017), and the duration of a PPA usually varies between 5 and 30 year (Wiser and Bolinger, 2017). While the problem of energy and commodity procurement has been studied in the presence of short and long term contracts/spot and forward markets (Li and Kouvelis, 1999; Kleindorfer and Wu, 2003; Nascimento and Powell, 2009; Boyabatli et al., 2011; Secomandi and Kekre, 2014), our work is differentiated by the pricing structure of PPAs and the presence of a renewable energy target. Our research is also related to the literature on corporate social and environmental responsibility, in that, it also considers deviating from a pure financial

objective by adding a social/environmental component (Kleindorfer et al., 2005; Atasu, 2016; Lee and Tang, 2017; Trivella et al., 2018; see also Hoejmose and Adrien-Kirby, 2012 for a review on socially and environmentally responsible procurement), but our renewable energy application and methodological approach are new.

We formulate a multi-period stochastic dynamic program (SDP) to minimize the expected procurement cost. The company can decide at each stage of a finite planning horizon whether to enter into new PPAs of varying size and length. We define a PPA strike price model that accounts for the generator perspective in a way that is consistent with publicly available software (NREL, 2017). The horizon is divided into two parts: a reach period where the renewable target does not have to be fulfilled (but contracts can be signed), and a sustain period where the target must be fulfilled. Computing an optimal policy of this SDP is intractable because its state space has a high-dimensional endogenous component corresponding to the pipeline of power inventories from PPAs, and another high-dimensional exogenous component containing the stochastic factors driving the evolution of power and RECs prices and demand.

Approximate dynamic programming (ADP) methods are used to solve high-dimensional SDPs. To understand which ADP options are available in the literature to tackle our problem, in Table 5.1 we categorize SDPs into four problem classes depending on the dimensionality of their state, and report popular ADP approaches that can deal with these problem classes. In problem class 1, the state space is low dimensional and the SDP

Table 5.1: Popular methods for solving different SDP problem classes.

Class	SDP state		Methods
	Endogenous	Exogenous	
1	low-dim.	low-dim.	stochastic dynamic programming
2	low-dim.	HIGH-dim.	least squares Monte Carlo
3	HIGH-dim.	low-dim.	approximate linear programming, stochastic dual dynamic programming
4	HIGH-dim.	HIGH-dim.	limited: complex policy iteration techniques

can be solved exactly (Puterman, 2005). Several well-studied ADP methods exist in the literature to handle problems with high-dimensional exogenous state but low-dimensional endogenous state, or vice versa (classes 2–3). These methods include the well-know least squares Monte Carlo (Longstaff and Schwartz, 2001; Tsitsiklis and Van Roy, 2001; Carmona and Ludkovski, 2010; Boomsma et al., 2012; Arvesen et al., 2013; Nadarajah et al., 2017), approximate linear programming (De Farias and Van Roy, 2003; De Farias and Van Roy, 2004; Desai et al., 2012; Nadarajah et al., 2015; Veatch, 2015), and stochastic

dual dynamic programming (Pereira and Pinto, 1991; Shapiro, 2011; Shapiro et al., 2013; Löhndorf et al., 2013). However, ADP options to tackle problems with high dimensionality in both endogenous and exogenous state components are limited, and we identified only a few papers that deal with this latter problem class by developing sophisticated policy iteration procedures. These papers include Nadarajah and Secomandi (2018) that consider the merchant operations of a network of gas storage and transport assets with high-dimensional term structure prices models, and Salas and Powell (2017) that develop a control algorithm for multiple storage devices under wind, demand, and electricity price uncertainty.

The standard ADP framework to solve classes 2–4 first computes a value function approximation (VFA), then uses this VFA to compute a control policy which gives an upper (primal) bound, finally embeds the VFA in the information relaxation and duality scheme (Brown et al., 2010) to obtain a lower (dual) bound. However, computing a good-quality VFA is very hard for class 4 and methods in the literature are limited. Additionally, our SDP is non-convex when using realistic constraints on the PPA buying options, which makes it harder or not possible to apply existing methods which rely on value function approximations or exploit convexity. For these reasons, we overcome the intractability in our problem by developing a novel ADP framework which does not rely on VFAs to derive bounds and instead reverse the standard ADP framework. We first consider the information relaxation approach to estimate lower bounds as done in the literature, where we adapt linear dual penalty functions (Brown and Smith, 2014; Secomandi et al., 2015; Nadarajah and Secomandi, 2018). Then, we develop a novel dual reoptimization heuristic (DRH) that extracts non-anticipative decision rules from distributions of actions obtained by generating sample paths of the uncertainty in Monte Carlo from the current state, and solving a dual model with information relaxation on each sample path. Our DRH method provides an upper bound on the optimal procurement value, and constitutes a rather general ADP framework which relies on (i) Monte Carlo simulation and (ii) solving deterministic dual math programs with a dual penalty on individual sample paths. If the latter math programs can be solved efficiently, then DRH is tractable also for problem class 4.

We conduct numerical experiments on realistic instances with PPA contract lengths ranging from 5 to 25 years and a planning horizon of 40 years. We define a PPA pricing model that is consistent with publicly available software (NREL, 2017), and calibrate to data a mean reverting process with monthly seasonality, a Jacobi process, and a geometric Brownian motion for modeling the evolution of power prices, REC prices, and demand, respectively. Using these models, we benchmark our new DRH policy against three procurement strategies. The first strategy only relies on spot procurement of power and RECs, that is, it does not consider PPAs. The second strategy uses a single PPA and

renews this contract each time it expires. The last benchmark is the popular primal reoptimization technique in the literature that iteratively solves deterministic approximations of the SDP at each stage (Wu et al., 2012; Secomandi et al., 2015; Löhndorf and Wozabal, 2017; Nadarajah and Secomandi, 2018). We find that the lower bound and procurement policy computed by our dual reoptimization method outperforms the benchmark heuristics on most instances. Specifically, the optimality gap achieved with DRH and the standard reoptimization policy are, respectively, 3.9% and 9.2% on average. Moreover, the spot procurement strategy results in a very large optimality gap, meaning that the long term price hedging effect of PPAs is beneficial to contain the overall power procurement costs of companies meeting a renewable target.

These findings bode well for the use of our dual approach to make multi-period procurement decisions that meet and sustain a renewable target in the presence of multiple PPA contracts and spot power purchases. Furthermore, being broadly applicable beyond the specific application presented in this paper, our dual reoptimization framework emerges as a promising approach to tackle large-scale SDPs, especially those with high-dimensional endogenous and exogenous state space for which existing methods are limited or rely on specific problem structure.

The rest of this paper is organized as follows. We formulate a power procurement model to reach and sustain a renewable target in Section 5.2. We develop methods to obtain both primal and dual bounds on the optimal procurement cost in Section 5.3. We define three procurement strategies that we use as benchmark in Section 5.4. We discuss our numerical study in Section 5.5 followed by conclusions in Section 5.6. Additional details regarding extended mathematical programming formulations are provided in the appendix.

5.2 Procurement model

In this section, we formulate a multi-period procurement model for a company to meet a renewable power target at minimum cost. Companies typically consider power procurement strategies over a long planning horizon (e.g., 30–40 years) and have access to multiple PPA options. We thus consider a finite decision horizon $\mathcal{I} = \{0, \dots, I - 1\}$ with discrete stages corresponding to years (we use stages and years interchangeably in the rest of the paper). In Section 5.2.1, we develop a PPA pricing model that is consistent with practice. In Section 5.2.2, we formulate the power procurement model as an SDP.

5.2.1 PPA strike price model

The strike price is one of the main features of a PPA, hence, employing a realistic and accurate pricing model is critical. We define a PPA strike price model which accounts for the generator perspective, that is, the incentive of the generator when entering into a contract.

For an investment in a renewable project to be profitable, the net present value (NPV) of the project must be positive after accounting for a desired return on investment. Consider a PPA of length equal to m years signed at the beginning of stage i . We define $K_{i,m}^{\text{NPV}}$ (USD/MWh) to represent the smallest price of this contract such that the renewable power generator would recoup its investment cost. We compute this value in a manner that is consistent with the one used in the System Advisor Model (SAM; NREL, 2017), an open source performance and financial tool designed by the National Renewable Energy Laboratory (NREL) to assess the feasibility of renewable energy projects (e.g., wind, solar, or biomass) under different operating and financial conditions. Specifically, assuming that the generator will receive a fixed “baseline” strike price \hat{K}_i (USD/MWh) during its entire lifetime, then the NPV corresponding to 1 MW of installed capacity at stage i is computed as follows:

$$\text{NPV}_i = \sum_{l=1}^{L_P} r^l \theta \hat{K}_i + \sum_{l=1}^{L_{T,i}} r^l \theta T_i - C_i^{\text{Inv}}. \quad (5.1)$$

In (5.1), $r \in (0, 1]$ captures the investor’s desired return on investment (internal rate of return; IRR), L_P (years) is the expected lifetime of the renewable project, and T_i (USD/MWh) is the production tax credit for the renewable project in place at year i , that is, when the contract is signed, and effective for a predetermined number of years $L_{T,i}$. (Currently in the U.S., for example, only renewable projects which start construction before 2020 are eligible for this credit, and the duration of the support is 10 years after the date the facility is in service; DOE, 2016.) The parameter θ is the annual capacity factor for the project representing the average number of hours of operations in a year. In case of wind and solar power, for a given technology (e.g., a wind turbine type and size), θ mainly depends on the geographical location of the generator (Wiser and Bolinger, 2017). Since our analysis is based on a region served by a single electricity market, for example PJM, we can consider θ to be a fixed parameter computed as the average capacity factor for that region. Finally, we define C_i^{Inv} (USD) to be the investment cost for acquiring 1 MW of energy generation capacity at stage i . This cost includes material, installation, operations and administrative costs over the project lifetime. Following NREL (2010), we use a functional form for C_i^{Inv} that decreases over time by a fixed percentage ξ ; in other words, it evolves according to a learning model $C_i^{\text{Inv}} = C_0^{\text{Inv}}(1 - \xi)^i$, where C_0^{Inv} is the investment cost at stage 0. Other model choices for C_i^{Inv} include, for instance,

exponential learning curve models such as $C_i^{\text{Inv}} = C_0^{\text{Inv}} (M_i/M_0)^{-\xi}$ where M_0 and M_i denote, respectively, the current and the forecasted stage i installed capacity of a given generation technology (Adler and Clark, 1991).

We now modify (5.1) by incorporating the PPA contract length in the strike price model. In particular, we define $K_{i,m}^{\text{NPV}} := \hat{K}_i \cdot K_m^+$, where the parameter $K_m^+ \geq 1$ is a risk factor of the contract duration m , and $K_{i,m}^{\text{NPV}}$ is the implied contract-dependent strike price. We assume $K_m^+ = 1$ if $m = L^P$ (that is, the PPA finances the project for its entire lifetime), and increases as the contract length m decreases to represent the dominant risk a generator incurs when entering into a contract with shorter duration, which results in an uncertain cash flow for a number of operating years. Substituting $\hat{K}_i = K_{i,m}^{\text{NPV}}/K_m^+$ in equation (5.1), and denoting $r' := \sum_{l=1}^{L^P} r^l$ and $r'' := \sum_{l=1}^{L_{T,i}} r^l$, the condition $\text{NPV}_{i,m} = 0$ (we introduce the dependency of the NPV from m) is solved for $K_{i,m}^{\text{NPV}}$ so that the break-even would determine a floor on the strike price from a generator perspective:

$$r'\theta (K_{i,m}^{\text{NPV}}/K_m^+) + r''\theta T_i - C_i^{\text{Inv}} = 0 \iff K_{i,m}^{\text{NPV}} = \left[\frac{C_i^{\text{Inv}}}{r'\theta} - \frac{r''}{r'} T_i \right] K_m^+.$$

To summarize, in line with the SAM tool we specify a minimum target IRR r required for the project, and calculate the price $K_{i,m}^{\text{NPV}}$ as a result, keeping into account the main cash flow components (SAM handles a more detailed representation of the project costs C_i^{Inv}) and the contract duration.

Then, we define a second term, $\underline{K}_{i,m}$ (USD/MWh), representing the average expected power price over the contract duration and is computed as $\underline{K}_{i,m} := (\sum_{l=1}^m \gamma^l \mathbb{E}[P_{i+l}|P_i]) / (\sum_{l=1}^{L^m} \gamma^l)$, where P_i (USD/MWh) is the power price at stage i and $\gamma \in (0, 1]$ is the per-stage discount factor corresponding to the risk-free interest rate. Developers consider the average wholesale market price when pricing PPAs (Wiser and Bolinger, 2017). Therefore, it is expected that the cost of PPAs will not fall significantly below the expected power price in the future, which motivates the use of this term as a lower bound on the strike price. We thus define the effective strike price $K_{i,m}$ of a PPA of length equal to m years signed at the beginning of stage i to be the maximum between the two terms, $K_{i,m} := \max \{ K_{i,m}^{\text{NPV}}, \underline{K}_{i,m} \}$.

5.2.2 Stochastic dynamic program

We start by formulating the model as a Markov decision process (MDP), and then we reformulate it as an SDP. At each stage of the decision horizon $i \in \mathcal{I}$, the company can decide whether to enter into new PPAs of different length and size. We assume that a

renewable power target $\alpha \in (0, 1]$, expressed as a the fixed percentage of the (future) power demand, must be reached within I^R stages and must be kept at least at the same percentage α afterwards, until the end of the problem horizon. The horizon is thus divided into two terms: a *reach* period $\mathcal{I}^R = \{0, \dots, I^R - 1\}$ where the renewable target α does not have to be fulfilled (but PPA contracts can be signed), and a *sustain* period $\mathcal{I}^S = \{I^R, \dots, I - 1\}$ where the target must be fulfilled, hence $\mathcal{I} = \mathcal{I}^R \cup \mathcal{I}^S$. There is a terminal stage I where PPAs cannot be entered.

We consider a set of PPAs, where each contract is identified by its length $m \in \mathbb{N}$ (number of stages). We call the set of lengths \mathcal{M} , and the length of the longest contract $M = \max\{m \in \mathcal{M}\}$. We assume the PPAs to have a one-period lead time, meaning that a PPA m signed at stage i will deliver power from stages $i + 1$ to $i + m$. The lead time is introduced in the model to reflect the installation time of a renewable project once the PPA is signed. At each stage $i \in \mathcal{I}$, the firm can enter into new PPAs from the set \mathcal{M} , and can decide on the contract size. Thus, an action corresponds to a continuous-valued vector $z_i = \{z_{i,m}, m \in \mathcal{M}\}$, where $z_{i,m}$ is the size in megawatt (MW) of the PPA of length m signed at stage i . PPA contracting involves complex administrative and legal processes so that only PPAs above a certain size are typically signed. For example, in the period 2010 to 2016 Google entered into 20 PPAs with average and minimum capacity, respectively, equal to 130 MW and 26 MW (Google, 2017). Therefore, we impose a lower bound on the project capacity z^{\min} (MW) to avoid entering into very small contracts which are not found in reality. The feasible action set at each stage is thus given by the non-convex set $\mathcal{Z} = (\{0\} \cup [z^{\min}, +\infty))^{|M|}$, meaning that the decision regarding each individual contract m must either be $z_{i,m} = 0$ (the contract is not entered) or has to satisfy the condition: $z_{i,m} \geq z^{\min}$.

The MDP state at stage $i \in \mathcal{I} \cup \{I\}$ is described by the pair $(x_i, w_i) \in \mathcal{X}_i \times \mathcal{W}_i$. The endogenous (controllable) state \mathcal{X}_i characterizes all possible PPA portfolio configurations at the beginning of stage i , that is, the capacity (MW) of each owned PPA $m \in \mathcal{M}$ for each of the remaining stages of delivery $l \in \{0, \dots, m - 1\}$. The element x_i of this space is naturally written as $x_i = \{x_{i,m,l}\} \in \mathbb{R}_+^{|\mathcal{M}| \sum\{m \in \mathcal{M}\}}$. We notice, however, that a smaller space is actually sufficient to convey all information needed to model the portfolio from i onward. In fact, we can use the contract *pipeline* $x_i = \{x_{i,l}, l \in \{0, \dots, M - 1\}\} \in \mathbb{R}_+^M$, where $x_{i,l}$ is the summed capacity of the owned PPAs with delivery in exactly l stages. This state space reduction is possible by noting that the total cost of a PPA, i.e. the cost of all future delivery periods, can be accounted for at the stage i in which the PPA is ordered because the strike price is fixed and hence this cost is known at i . In this way, we do not need to keep track of individual PPA quantities (and their strike price) but only of the inventory pipeline. The exogenous (information) state $w_i \in \mathcal{W}_i$ is given by the stochastic factors $w_i = \{w_{i,j}, j \in \mathcal{J}\}$ driving the evolution of the uncertainties and

affecting the cash flows. We assume that the factors w_i evolve in a Markovian manner and that the sources of uncertainty represented by power price P_i (USD/MWh), REC price R_i (USD/MWh), and electricity demand D_i (MWh/year), are functions of w_i but we omit this explicit dependence in the rest of the paper. Specifically, in our numerical study we use the stochastic models of the uncertainty discussed in Section 5.5.2.

The stage 0 state is known and is denoted by (x_0, w_0) . At stage $i \in \mathcal{I}$ and state $(x_i, w_i) \in \mathcal{X}_i \times \mathcal{W}_i$, the execution of a decision $z_i \in \mathcal{Z}$ results in an immediate cost $c_i(x_i, w_i, z_i) \in \mathbb{R}$ and a transition to a stage $i + 1$ state $x_{i+1} = f(x_i, z_i)$ in set \mathcal{X}_{i+1} . Each state (x_I, w_I) in the terminal-stage state space $\mathcal{X}_I \times \mathcal{W}_I$ is associated with a terminal cost $c_I(x_I, w_I) \in \mathbb{R}$. The cost function at stage i includes the investment cost of the newly purchased PPAs as well as the spot power and RECs needed to supply possible residual demand at stage i . Specifically, if a PPA of size $z_{i,m}$ is signed at strike price $K_{i,m}$, then the company will receive $\theta z_{i,m}$ MWh/year of power from the generator at fixed price $K_{i,m}$ for the entire contract duration, where θ (hours/year) is the annual capacity factor (see Section 5.2.1 for details). The PPA size in MW is converted through θ into the annual power output in MWh/year that is actually produced. Consequently, the fixed value $\theta K_{i,m} z_{i,m}$ (USD/year) is the per-stage power cost before discounting associated with this contract. The situation can be more complex as some companies owning PPAs buy power at fixed price $K_{i,m}$ from the generator but then sell this power back to the wholesale market and buy it again from a utility (Google, 2016). This process—buy twice and sell once—is necessary to ensure a reliable power supply which is not affected by the stochastic production from the renewable project (for instance, the flat amount of power needed to run a data center 24/7 cannot be supplied directly from a wind farm). The wholesale and utility prices are not always correlated, thus, the exposure to the market price is not entirely eliminated by PPAs. However, large corporations and utilities have recently entered into agreements where the utility price is set to match the wholesale market price (Amazon, 2016). Following this trend, we assume that a PPA comprehensively results in a flat cost based on the strike price $K_{i,m}$. The cost function is so calculated with:

$$c_i(x_i, w_i, z_i) = \sum_{m \in \mathcal{M}} \sum_{l=1}^m \gamma^l \theta K_{i,m} z_{i,m} + \begin{cases} P_i u_i + P_i v_i & \text{if } i \in \mathcal{I}^R; \\ (P_i + R_i) u_i + P_i v_i & \text{if } i \in \mathcal{I}^S, \end{cases} \quad (5.2a)$$

$$c_I(x_I, w_I) = (P_I + R_I) u_I + P_I v_I. \quad (5.2b)$$

The first part of (5.2a) is the PPAs investment cost, $u_i := \max\{\alpha D_i - \theta x_{i,0}, 0\}$ is the residual power needed to fulfill the renewable target αD_i , which has to be supplied with RECs during \mathcal{I}^S , and $v_i := \max\{D_i - \theta x_{i,0} - u_i, 0\}$ is the residual power needed to meet the total demand D_i . Due to the presence of the electricity spot market and the RECs market, the company always meets 100% of both its power demand and renewable target.

The terminal cost (5.2b) is analogous to (5.2a) but no PPAs are entered at I . Notice that the cost (5.2) depends not only on the power and RECs prices but also on the demand through u_i and v_i . When an action $z_i \in \mathcal{Z}$ is executed at state $x_i \in \mathcal{X}_i$, the l -th pipeline element is updated according to

$$x_{i+1,l} = f_i(x_i, z_i)_l = \begin{cases} x_{i,l+1} + \sum_{m>l} z_{i,m} & \text{if } l \in \{0, \dots, M-2\}; \\ z_{i,M} & \text{if } l = M-1. \end{cases} \quad (5.3)$$

In Figure 5.2, we provide an illustration of states, actions, and state transitions. Figure 5.2(a) shows a PPA pipeline x_i at stage i . The first element of this pipeline represents the total owned PPAs that will deliver power at stage i (15 MW). As the contracts expire over time, the pipeline elements becomes smaller. In Figure 5.2(b) a decision is made to enter into three PPAs of duration 5, 10, and 15 years with size represented by the height of the bars. Figure 5.2(c) shows the total power delivery profile resulting from the stage i PPAs purchase, and accounts for the lead time of one period. Finally, the new x_{i+1} pipeline is given by adding the stage i purchases to the pipeline x_i shifted one year ahead, as shown in Figure 5.2(d).



Figure 5.2: Example of PPA inventory pipeline, PPA investment decision, and inventory update.

Making decisions in such a setting requires a *policy* π , that is, a collection of decision rules mapping states to actions. At a given stage i , a decision rule A_i^π associates the

state $(x_i, w_i) \in \mathcal{X}_i \times \mathcal{W}_i$ to a feasible action $z_i \in \mathcal{Z}$; the policy π is then the collection $\{A_i^\pi, i \in \mathcal{I}\}$. We denote by Π the set of all feasible policies. An optimal policy belongs to Π and minimizes the expected cumulative costs from applying this policy over the finite problem horizon. More formally, this policy solves the MDP:

$$\min_{\pi \in \Pi} \mathbb{E} \left[\sum_{i \in \mathcal{I}} \gamma^i c_i(x_i^\pi, w_i, A_i^\pi(x_i^\pi, w_i)) + \gamma^I c_I(x_I^\pi, w_I) \mid (x_0, w_0) \right], \quad (5.4)$$

where x_i^π is the endogenous state reached in stage i when following the policy π . It is well known that in theory the optimal policy of (5.4) could be obtained with stochastic dynamic programming, that is, by introducing a value function $V_i(\cdot, \cdot)$ and solving the Bellman equations:

$$V_I(x_I, w_I) = c_I(x_I, w_I), \quad \forall (x_I, w_I) \in \mathcal{X}_I \times \mathcal{W}_I, \quad (5.5a)$$

$$V_i(x_i, w_i) = \min_{z_i \in \mathcal{Z}} \left\{ c_i(x_i, w_i, z_i) + \gamma \mathbb{E} [V_{i+1}(f_i(x_i, z_i), w_{i+1}) \mid w_i] \right\}, \\ \forall (i, x_i, w_i) \in \mathcal{I} \times \mathcal{X}_i \times \mathcal{W}_i. \quad (5.5b)$$

Equation (5.5a) specifies the terminal cost. Equation (5.5b) is applied recursively from stage $i = I - 1$ backward to $i = 0$, and for each state $(x_i, w_i) \in \mathcal{X}_i \times \mathcal{W}_i$ determines the best feasible action, that is, the action $z_i \in \mathcal{Z}$ minimizing the sum of the immediate cost resulting from applying this action and the discounted expected utility from transitioning to the new state $x_{i+1} = f_i(x_i, z_i)$.

The SDP (5.5) is intractable as it suffers from well-known curses of dimensionality (Bertsekas, 2011; Powell, 2011). In particular, our model has endogenous and exogenous components of the state which are both high dimensional: The endogenous state \mathcal{X}_i is a vector of M continuous variables constituting the PPA pipeline (e.g., 25 variables in our instances), and the exogenous state \mathcal{W}_i is composed by the stochastic factors driving multiple sources of uncertainty including prices and demand. In our study we consider a three-dimensional continuous exogenous state \mathcal{W}_i (see Section 5.5.2 for details) but in general higher-dimensional price models such as term structure models are also encountered in similar applications.

Furthermore, because of the presence of a minimum PPA size requirement, the value function our SDP is not convex in the endogenous state x_i which makes it harder to determine a good value function approximation. We show that the value function is non-convex using a counter-example in Appendix 5.7.1.

5.3 Dual reoptimization heuristic and bounds

In this section we present methods to obtain lower and upper bounds on the exact solution of SDP (5.5). In Section 5.3.1, we introduce the information relaxation and duality framework that is commonly used to compute dual bounds. In Section 5.3.2, we present a reoptimization heuristic that extracts non-anticipative decision rules from sample action distributions.

5.3.1 Information relaxation and duality

The information relaxation and duality framework (Brown et al., 2010) consists of relaxing the non-anticipativity constraints of SDP (5.5) and penalizing the knowledge of future information at stage $i \in \mathcal{I}$ using a real-valued penalty function $q_i(x_{i+1}, \underline{w}_i)$, where $\underline{w}_i := (w_i, w_{i+1}, \dots, w_I)$. In a minimization problem, a penalty function is said to be dual-feasible if it satisfies $\mathbb{E}[q_i(x_{i+1}, \underline{w}_i) | w_i] \geq 0$. Using such function, one can estimate a dual bound by generating H Monte Carlo samples of uncertainty $\{w_i^h, (i, h) \in \mathcal{I} \cup \{I\} \times \mathcal{H}\}$ and solving the following deterministic dynamic program on h :

$$V_{I,h}^{\text{IR}}(x_I) = c_I(x_I, w_I^h), \quad \forall x_I \in \mathcal{X}_I, \quad (5.6a)$$

$$V_{i,h}^{\text{IR}}(x_i) = \min_{z_i \in \mathcal{Z}} \left\{ c_i(x_i, w_i^h, z_i) - q_i(f_i(x_i, z_i), \underline{w}_i^h) + \gamma V_{i+1,h}^{\text{IR}}(f_i(x_i, z_i)) \right\}, \quad \forall (i, x_i) \in \mathcal{I} \times \mathcal{X}_i. \quad (5.6b)$$

The sample average of the stage 0 values $\sum_{h \in \mathcal{H}} V_{0,h}^{\text{IR}}(x_0)/H$ gives a lower bound estimate of the optimal policy value. In our problem, given the high-dimensional endogenous state space, instead of the dynamic program (5.6) we solve the math programming formulation (5.7). (We report a compact/general formulation here and relegate an extended model to Appendix 5.7.2.)

$$V_{0,h}^{\text{IR}}(x_0) = \min_{x_i, z_i} \sum_{i \in \mathcal{I}} \gamma^i \left[c_i(x_i, w_i^h, z_i) - q_i(x_{i+1}, \underline{w}_i^h) \right] + \gamma^I c_I(x_I, w_I^h) \quad (5.7a)$$

$$\text{s.t.}: \quad x_i = x_0, \quad i = 0 \quad (5.7b)$$

$$x_{i+1} = f_i(x_i, z_i), \quad \forall i \in \mathcal{I} \quad (5.7c)$$

$$\text{var:} \quad x_i \in \mathcal{X}_i, \quad \forall i \in \mathcal{I} \cup \{I\} \quad (5.7d)$$

$$z_i \in \mathcal{Z}, \quad \forall i \in \mathcal{I}. \quad (5.7e)$$

The decision variables in this model are the endogenous states and actions for stages from 0 to I . The objective function (5.7a) is the discounted sum of costs over these stages with

known future uncertainty and correction penalty terms q_i . Constraints (5.7b) initialize the stage 0 state to the current state, (5.7c) enforce feasibility of the state transitions, and (5.7d)–(5.7e) restrict the decision variables to their respective feasible domains. In our power procurement case, (5.7) corresponds to a mixed-integer program where binary variables are necessary to model the non-convex action set \mathcal{Z} .

The next step is defining a dual penalty that can be used in (5.6)/(5.7). If the penalty is set to zero, i.e. $q_i(\cdot, \cdot) \equiv 0$, then the model assumes perfect information of the future uncertainty but this generally produces loose lower bounds. It is known (Brown et al., 2010) that an ideal penalty can be defined for $x_{i+1} \in \mathcal{X}_{i+1}$ using the exact value function $V_i(\cdot, \cdot)$ of (5.5) as:

$$q_i(x_{i+1}, \underline{w}_i) := \gamma \{V_{i+1}(x_{i+1}, w_{i+1}) - \mathbb{E}[V_{i+1}(x_{i+1}, w_{i+1}) | w_i]\}. \quad (5.8)$$

Obviously, the exact value function $V_i(\cdot, \cdot)$ is not known in reality. Nevertheless, if we are given a VFA $\hat{V}_{i+1}(\cdot, \cdot)$, $i \in \mathcal{I} \cup \{I\}$, then a dual-feasible penalty function (Brown et al., 2010) can be defined for $x_{i+1} \in \mathcal{X}_{i+1}$ as:

$$q_i(x_{i+1}, \underline{w}_i^h) := \gamma \{\hat{V}_{i+1}(x_{i+1}, w_{i+1}) - \mathbb{E}[\hat{V}_{i+1}(x_{i+1}, w_{i+1}) | w_i]\}. \quad (5.9)$$

Such VFA-based penalty is common and has been successfully used in previous works to obtain dual bounds (Brown et al., 2010; Brown and Smith, 2011; Nadarajah et al., 2017). However, as discussed in Section 5.1, obtaining a VFA is hard for our problem. To overcome this issue, we instead use a linear dual penalty function at stage $i \in \mathcal{I}$ of the form:

$$q_i(z_i, \underline{w}_i) := \sum_{m \in \mathcal{M}} z_{i,m} \sum_{l=1}^m \sum_{j \in \mathcal{J}} \gamma^l \theta \left(w_{i+l,j} - \mathbb{E}[w_{i+l,j} | w_i] \right) \psi_{i,j}, \quad (5.10)$$

where $\{\psi_{i,j}, i \in \mathcal{I}, j \in \mathcal{J}\}$ are stage- and factor-dependent weights. This penalty is trivially dual-feasible since each inner term $w_{i+l,j} - \mathbb{E}[w_{i+l,j} | w_i]$ has null expectation. Linear dual penalties on high-dimensional states have been successfully used in Brown and Smith (2014) and Nadarajah and Secomandi (2018). In our implementation, based on experimentation we select simple weights $\{\psi_{i,j}, i \in \mathcal{I}, j \in \mathcal{J}\}$ equal to 1 for power and 0 for RECs and demand for all stages. Future work could determine such weights in a more principled two-step process which first uses an initial set of simple weights but then perform a linear regression over the policy values obtained from sample paths to refine them. In the following, we show that the discussed information relaxations and duality framework can be leveraged to determine non-anticipative policies.

5.3.2 Dual reoptimization heuristic

Assume a decision has to be made at stage $i \in \mathcal{I}$ and state $(x_i, w_i) \in \mathcal{X}_i \times \mathcal{W}_i$, and that the exact value functions $V_j(\cdot, \cdot)$ for $j \geq i$ are known. Proposition 5.1 states that if we choose any sample path starting at w_i and solve the dual model with ideal penalty, then the resulting decision is optimal regardless of which sample path is chosen.

Proposition 5.1 (IDEAL PENALTY). *Given $(i, x_i) \in \mathcal{I} \times \mathcal{X}_i$ and any information state sample path conditioned on w_i : $\underline{w}_i = \{w_j, j \in \mathcal{I} \cup \{I\}, j \geq I\}$, if the ideal penalty (5.8) is used in the dual model (5.6), then it holds that:*

$$V_j^{\text{IR}}(x_j) = V_j(x_j, w_j), \quad \forall j \in \mathcal{I} \cup \{I\}, j \geq i, \forall x_j \in \mathcal{X}_j, \quad (5.11)$$

and the stage i decision $z_i^* = \arg \min_{z_i \in \mathcal{Z}} \left\{ c_i(x_i, w_i, z_i) - q_i(f_i(x_i, z_i), \underline{w}_i) + \gamma V_{i+1}^{\text{IR}}(f(x_i, z_i)) \right\}$ is optimal at state (x_i, w_i) .

Proof. Equation (5.11) trivially holds at stage I , in fact: $V_I^{\text{IR}}(x_I) = c_I(x_I, w_I) = V_I(x_I, w_I)$ for each $x_I \in \mathcal{X}_I$. By backward induction, we assume (5.11) is true for $j + 1$ and prove it for j . Substituting the penalty (5.8) into equation (5.6) for stage i gives:

$$V_j^{\text{IR}}(x_j) = \min_{z_j \in \mathcal{Z}} \left\{ c_j(x_j, w_j, z_j) - \gamma V_{j+1}(f(x_j, z_j), w_{j+1}) + \gamma \mathbb{E}[V_{j+1}(f(x_j, z_j), w_{j+1}) | w_j] + \gamma V_{j+1}^{\text{IR}}(f(x_j, z_j)) \right\}, \quad \forall x_j \in \mathcal{X}_j. \quad (5.12a)$$

$$= \min_{z_j \in \mathcal{Z}} \left\{ c_j(x_j, w_j, z_j) + \gamma \mathbb{E}[V_{j+1}(f(x_j, z_j), w_{j+1}) | w_j] \right\}, \quad \forall x_j \in \mathcal{X}_j. \quad (5.12b)$$

$$= V_j(x_j, w_j), \quad \forall x_j \in \mathcal{X}_j, \quad (5.12c)$$

where (5.12b) follows from the induction hypothesis, and (5.12c) from (5.5b). The relation (5.11) thus holds at the generic stage $j \in \mathcal{I} \cup \{I\}$, $j \geq i$ for the principle of mathematical induction. The optimality of the action is immediate from (5.11). \square

Since the exact value function is unknown, the result from Proposition 5.1 is not directly applicable. Therefore, similarly to Proposition 5.1, we show in Proposition 5.2 that if we have a VFA which is uniformly close to exact value function, then the value function of the dual model is close to the optimal value function. As a consequence, decisions made along any sample path of the uncertainty are near optimal regardless of the selected sample path.

Proposition 5.2 (VFA-BASED PENALTY). *Consider a stage $i \in \mathcal{I}$ and a VFA $\hat{V}_{[i]}(\cdot, \cdot)$ such that*

$$|V_j(x_j, w_j) - \hat{V}_j(x_j, w_j)| \leq \epsilon, \quad \forall j \in \mathcal{I} \cup \{I\}, j \geq i, \forall (x_j, w_j) \in \times \mathcal{X}_j \times \mathcal{W}_j. \quad (5.13)$$

Given any information state sample path conditioned on w_i : $\underline{w}_i = \{w_j, j \in \mathcal{I} \cup \{I\}, j \geq I\}$, if the VFA-based penalty (5.9) is used in the dual model (5.6), then it holds that:

$$|V_j^{\text{IR}}(x_j) - V_j(x_j, w_j)| \leq 2(I - j)\epsilon, \quad \forall j \in \mathcal{I} \cup \{I\}, j \geq i, \forall x_j \in \mathcal{X}_j. \quad (5.14)$$

Proof. Equation (5.14) trivially holds at stage I as both values equal the terminal cost function. By backward induction, we assume (5.14) is true for stage $j + 1$ and prove it for j . Consider a state $x_j \in \mathcal{X}_j$, an action $z_j \in \mathcal{Z}$, and the resulting $j + 1$ state $x_{j+1} = f(x_j, z_j)$. The difference in absolute value of the terms inside the optimization problems determining the value functions in (5.14) for z_j is:

$$\begin{aligned} & \left| c_j(x_j, w_j, z_j) - \gamma \hat{V}_{j+1}(x_{j+1}, w_{j+1}) + \gamma \mathbb{E}[\hat{V}_{j+1}(x_{j+1}, w_{j+1}) | w_j] \right. \\ & \quad \left. + \gamma V_{j+1}^{\text{IR}}(x_{j+1}) - c_j(x_j, w_j, z_j) - \gamma \mathbb{E}[V_{j+1}(x_{j+1}, w_{j+1}) | w_j] \right| \\ = & \gamma \left| V_{j+1}^{\text{IR}}(x_{j+1}) - \hat{V}_{j+1}(x_{j+1}, w_{j+1}) + \mathbb{E}[\hat{V}_{j+1}(x_{j+1}, w_{j+1}) - V_{j+1}(x_{j+1}, w_{j+1}) | w_j] \right| \end{aligned} \quad (5.15a)$$

$$\begin{aligned} \leq & \gamma \left| V_{j+1}^{\text{IR}}(x_{j+1}) - V_{j+1}(x_{j+1}, w_{j+1}) \right| + \gamma \left| V_{j+1}(x_{j+1}, w_{j+1}) - \hat{V}_{j+1}(x_{j+1}, w_{j+1}) \right| \\ & + \gamma \mathbb{E} \left[\left| \hat{V}_{j+1}(x_{j+1}, w_{j+1}) - V_{j+1}(x_{j+1}, w_{j+1}) \right| | w_j \right] \end{aligned} \quad (5.15b)$$

$$\leq 2(I - j - 1)\epsilon + \epsilon + \epsilon \quad (5.15c)$$

$$= 2(I - j)\epsilon. \quad (5.15d)$$

where (5.15a) follows from canceling the costs c_j out, (5.15b) from adding and removing the exact value function, and (5.15c) follows from the induction hypothesis, the VFA assumption (5.13), and from $\gamma \leq 1$. Given that the inequality (5.15) holds for any action $z_j \in \mathcal{Z}$, it also holds for the pair of optimal actions determining the value functions in equation (5.14). The proposition is then true by mathematical induction at all stages $j \in \mathcal{I} \cup \{I\}, j \geq i$. \square

We introduce a new scheme to obtain non-anticipative policies in a tractable manner for our SDP with high-dimensional endogenous and exogenous state. This scheme does not rely on the construction of a VFA; Instead, it is based on the following three steps:

1. Obtain a distribution of actions at a given stage and state by generating sample

paths of the uncertainty in Monte Carlo and solving a dual model with information relaxation on each sample path;

2. Extract non-anticipative decision rules from the action distribution;
3. Embed the decision rules in a reoptimization scheme to obtain feasible policies.

Let us consider a decision to be made at stage i and state (x_i, w_i) , and generate a sample path of the uncertainty in Monte Carlo simulation from i to the end of the planning horizon I conditioned on the current state w_i . Knowing the evolution of the uncertainty along this sample path does not violate non-anticipativity because this sample path does not carry true future information. Thus, we can solve the dual program (5.7) with information relaxations defined on the generated sample path to determine a non-anticipative stage i action z_i . As discussed, this action z_i is optimal if the dual program exploits the ideal penalty (see Proposition 5.1), but such penalty is unknown. Without ideal penalty, z_i is likely to be suboptimal and dependent on the generated sample path. To reduce the dependency of z_i from the chosen sample path and the consequent potential large decision error, we proceed by repeating this process over N Monte Carlo sample paths of the uncertainty $n \in \mathcal{N}$ to obtain an action distribution $\{z_i^n, n \in \mathcal{N}\}$. Then, we can extract a feasible decision rule from the action distribution, that is, we define a map \mathfrak{R}_i as

$$\begin{aligned} \mathfrak{R}_i : \mathcal{Z}_i(x_i)^N &\longrightarrow \mathcal{Z}_i(x_i) \\ \mathfrak{R}_i(\{z_i^n, n \in \mathcal{N}\}) &= z_i^{\mathfrak{R}}, \end{aligned} \tag{5.16}$$

where we use a general stage- and state-dependent action set $\mathcal{Z}_i(x_i)$ even though in our case this set is simply $\mathcal{Z}_i(x_i) = \mathcal{Z}$. Finally, we embed this process and the decision rules \mathfrak{R}_i in a reoptimization scheme. This means that the described process is performed at stage i , and the decision $z_i^{\mathfrak{R}}$ corresponding to stage i alone is implemented. Afterwards, new market information becomes available at stage $i + 1$, and the stage $i + 1$ process is repeated using the updated state information. We denote the resulting feasible policy by *dual reoptimization heuristic* (DRH) and detail it in Algorithm 5.

The applicability of the framework outlined in Algorithm 5 transcends our specific application and can be used to derive non-anticipative decision rules to MDPs in other application contexts. Because the algorithm uses inner simulation and solves math programs on sample paths, the main condition for the algorithm to be applicable is that these math programs can be solved efficiently, which is usually the case for small to medium-size linear programs, for instance. Also, the algorithm relies on the definition of a dual penalty (we could be trivially set to zero).

Algorithm 5 DRH

Input: Initial state $(x_0, w_0) \in \mathcal{X}_0 \times \mathcal{W}_0$, information state sample path conditioned on w_0 : $\{w_i, i \in \mathcal{I} \cup \{I\}\}$, number of inner samples N , dual feasible penalties $q_i(\cdot, \cdot)$ for $i \in \mathcal{I}$.

Initialization: Set the initial cost $C(x_0, w_0) = 0$.

For each $i = 0$ to $I - 1$ **do**:

1. Generate N information samples conditional on w_i : $\{w_j^n, (j, n) \in \{i, \dots, I\} \times \mathcal{N}\}$.
2. **For** each $n \in \mathcal{N}$ **do**:
 - (i) Define a stage i dual model (5.7) on inner sample n using initial state x_i .
 - (ii) Solve the model and save the optimal stage i action $z_i^{n,*}$.
3. Extract a decision rule from the action distribution $z_i^{\mathfrak{R}} = \mathfrak{R}_i(\{z_i^{n,*}, n \in \mathcal{N}\})$.
4. Implement the stage i decision $z_i^{\mathfrak{R}}$, which means:
 - (i) Add the stage i cost $C(x_0, w_0) \leftarrow C(x_0, w_0) + \delta^i c_i(x_i, w_i, z_i^{\mathfrak{R}})$.
 - (ii) Compute the new state $x_{i+1} = f_i(x_i, z_i^{\mathfrak{R}})$.

Add the terminal cost $C(x_0, w_0) \leftarrow C(x_0, w_0) + \delta^I c_I(x_I, w_I)$.

Output: Cumulative discounted cost $C(x_0, w_0)$ incurred from stage 0 to I .

In general, different non-anticipative decision rules \mathfrak{R}_i can be extracted from the sample action distribution at Step 3 of the algorithm. For example, we can use a naive decision rule \mathfrak{D}_i which computes the element-wise average decision, that is, $z_i^{\mathfrak{D}} = \{z_{i,m}^{\mathfrak{D}}, m \in \mathcal{M}\}$, where

$$z_{i,m}^{\mathfrak{D}} := \frac{1}{N} \sum_{n \in \mathcal{N}} z_{i,m}^{n,*}, \quad \forall m \in \mathcal{M}. \quad (5.17)$$

Under ideal dual penalty (5.8), this naive decision rule is optimal as a consequence of Proposition 5.1. More precisely, with the ideal penalty, the decision $z_i^{n,*}$ made along each inner sample path $n \in \mathcal{N}$ at Step 2 of the algorithm is optimal, thus, the action distribution collapses to a single action which is the optimal action. Without ideal penalty, i.e. with an actual distribution, the average decision rule defined by (5.17) may be infeasible in our application due to the minimum PPA quantity $z_{i,m} \geq z^{\min}$ that each signed contract has to meet. This constraint is satisfied by the dual solutions $z_i^{n,*}$ for $n \in \mathcal{N}$ but not necessarily by $z_i^{\mathfrak{D}}$ which averages samples where $z_{i,m}^{n,*} = 0$ with samples where $z_{i,m}^{n,*} \geq z^{\min}$. Therefore, if the average action $z_i^{\mathfrak{D}}$ violates the size constraint for at least one contract $m \in \mathcal{M}$, we compute a “small” perturbation of $z_i^{\mathfrak{D}}$ to satisfy the PPA size requirements. Specifically, we define a second mapping $\mathfrak{L}_i : \mathbb{R}_+^{|\mathcal{M}|} \rightarrow \mathcal{Z}$ which, given an action $z_i^{\mathfrak{D}}$, returns a feasible action $z_i^{\mathfrak{L}}$ such that the generated power delivery profile is as close as possible to the original one and the constraints on PPA size are fulfilled (see Figure 5.3 for an example). We define this mapping as the solution to the following optimization

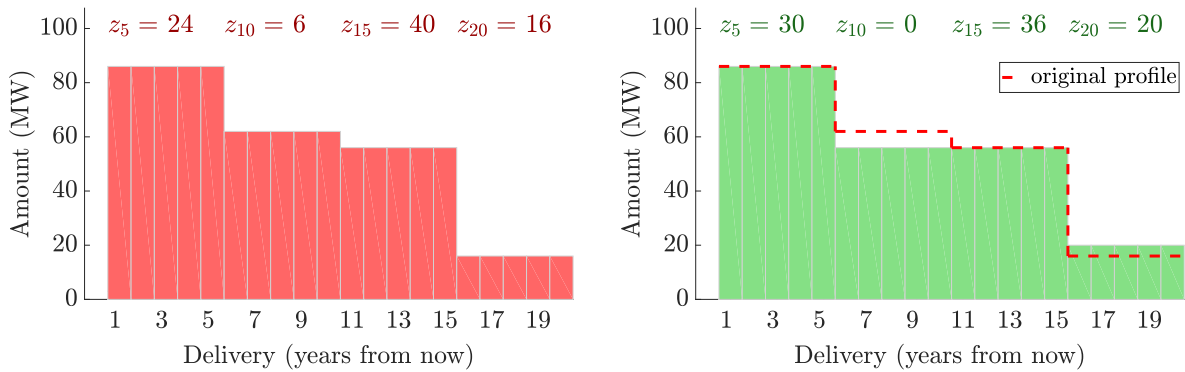
problem (5.18).

$$z_i^{\mathcal{L}} = \arg \min_{\Xi} \sum_{m \in \mathcal{M}} \left| \sum_{m' \geq m} (z_{i,m'}^{\mathcal{R}} - z_{i,m'}^{\mathcal{L}}) \right| \quad (5.18a)$$

$$\text{s.t.}: \xi_m z^{\min} \leq z_{i,m}^{\mathcal{L}} \leq \xi_m z^{\max}, \quad \forall m \in \mathcal{M} \quad (5.18b)$$

$$\text{var: } \Xi = \{z_{i,m}^{\mathcal{L}} \geq 0, \xi_m \in \{0, 1\}, \forall m \in \mathcal{M}\}. \quad (5.18c)$$

The objective function (5.18a) represents the difference in absolute value of the power delivery profiles and can be easily linearized with standard modeling techniques. The variables ξ_m and constraints (5.18b) enforce the minimum contract size. The model (5.18) is easy to solve as it only contains $|\mathcal{M}|$ binary variables and a few continuous variables. We provide an illustration of this perturbation in Figure 5.3. We eventually select our non-anticipative and feasible DRH decision rule (5.16) as the composition between average and perturbation mapping, i.e. $\mathcal{R}_i := \mathcal{L}_i \circ \mathcal{D}_i$.



(a) Power delivery profile from average action $z_i^{\mathcal{D}}$. (b) Power delivery profile from perturbed action $z_i^{\mathcal{L}}$.

Figure 5.3: Action perturbation using contracts from the set $\mathcal{M} = \{5, 10, 15, 20\}$ and $z^{\min} = 20$ MW.

Estimating the value of the DRH policy relies on generating H Monte Carlo sample paths of the uncertainty conditioned on w_0 : $\{w_i^h, (i, h) \in \mathcal{I} \cup \{I\} \times \mathcal{H}\}$, where $\mathcal{H} = \{1, \dots, H\}$. For each sample path $h \in \mathcal{H}$, decisions are made according to Algorithm 5 with decision rule from (5.17)–(5.18) which returns the total cost $C^h(x_0, w_0)$ incurred on sample path h by executing the DRH policy. An upper bound on the expected power procurement cost is then given by the sample average $\sum_{h \in \mathcal{H}} C^h(x_0, w_0)/H$. Overall, evaluating the DRH policy can be expensive as it requires–neglecting (5.18)–solving $H \times I \times N$ mixed-integer programs, that is, solving model (5.7) for each evaluation sample path $h \in \mathcal{H}$, stage $i \in \mathcal{I}$, and inner sample $n \in \mathcal{N}$. Each program in particular has a number of binary variables to the order of $\mathcal{O}(I \cdot |\mathcal{M}|)$ that originate from the minimum contract size

requirement. To substantially speed up the DRH policy evaluation, we can remove the PPA size constraints from the dual model (5.7) and the associated binary variables (see the extended formulation (5.23) in Appendix 5.7.2 for details), and solve a linear program instead. The perturbation map (5.18) will then ensure that the solution from the linear programming action distribution fulfills the minimum contract quantity. We do not use this simplification when using (5.7) to compute a dual bound in order not to decrease the quality of this bound and because estimating a dual bound only requires solving H mixed-integer programs.

5.4 Benchmark policies

We test our DRH method against three benchmark strategies: (i) a spot procurement strategy that ignores PPAs, (ii) a “block” heuristic that uses a single PPA and renews it at expiration, and (iii) a standard reoptimization heuristic. These strategies are described in this order in the following.

The simplest power procurement policy satisfying the renewable target consists of procuring the entire power demand D_i spot in each stage $i \in \mathcal{I} \cup \{I\}$, and supplementing with RECs the percentage corresponding to the target αD_i in the sustain period $\mathcal{I}^S \cup \{I\}$. This policy has no demand risk but is fully exposed to volatile spot market prices, thus, it may result in high costs of securing power.

Our second benchmark strategy uses a single PPA of length m and enters into a new contract every m years, that is, each time a contract expires a new one of the same length is signed. Precisely, we define this procurement policy such that the first contract is entered at the last stage of the reach period, I^{R-1} , and delivers power during the first m stages of the sustain period $\mathcal{I}^S \cup \{I\}$, contributing so to meeting the renewable target. The second contract is then ordered one year before the first contract expires to ensure a continuous delivery of power from PPAs, and so on. Assuming that a new contract has to be ordered at stage i , the contract quantity $z_{i,m}$ is chosen by solving a deterministic model that minimizes the total procurement cost defined using a forecast of the uncertainty over the delivery period of m stages (this model is a small mixed-integer program that we report in Appendix 5.7.2). If the owned contract is not large enough to meet the target or satisfy the power demand, then additional power and RECs must be purchased from the spot market. We call this policy *block* heuristic (BH_m) due to the block structure of the PPA purchasing.

For illustration, we provide an example of the spot and block procurement strategies

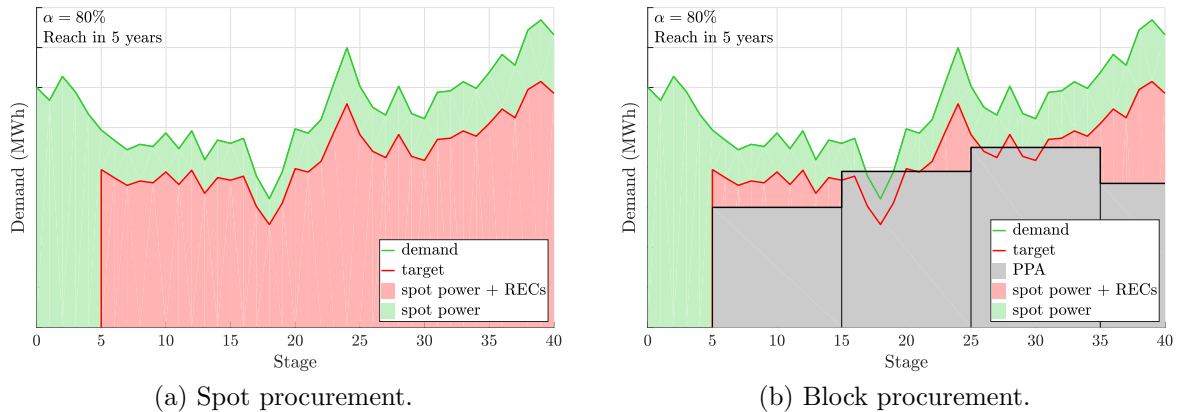


Figure 5.4: Example of spot and 10-year block procurement strategies for a demand sample path.

in Figure 5.4. The green and red lines represent a trajectory for the company's power demand and a renewable target of 80% of the demand to reach in 5 years, respectively. The height of the gray bars in Figures 5.4(b), i.e. the size of the purchased PPA, is chosen based on expected prices and demand and there is a risk of over procurement (see e.g. the second contract in the figure).

Our third benchmark policy is a standard reoptimization heuristic (RH) that solves a deterministic approximation of the MDP (5.4) at each stage. At stage i and state $(x_i, w_i) \in \mathcal{X}_i \times \mathcal{W}_i$, the RH policy solves model (5.19) (see Appendix 5.7.2 for an extended formulation).

$$\min_{x_j, z_j} \sum_{j \in \mathcal{I}, j \geq i} \gamma^j c_j(x_j, \mathbb{E}[w_j | w_i], z_j) + \gamma^I c_I(x_I, \mathbb{E}[w_I | w_i]) \quad (5.19a)$$

$$\text{s.t.} \quad x_j = x_i, \quad j = i \quad (5.19b)$$

$$x_{j+1} = f_j(x_j, z_j), \quad \forall j \in \mathcal{I}, j \geq i \quad (5.19c)$$

$$\text{var:} \quad x_j \in \mathcal{X}_j, \quad \forall j \in \mathcal{I} \cup \{I\}, j \geq i \quad (5.19d)$$

$$z_j \in \mathcal{Z}, \quad \forall j \in \mathcal{I}, j \geq i \quad (5.19e)$$

The objective function (5.19a) represents a deterministic approximation of the future expected costs in which the uncertainty is replaced by a forecast. The constraints and decision variables in (5.19b)–(5.19e) are analogous to those discussed in (5.7). To speed up the RH value estimation process, the same DRH linear programming simplification scheme could be used here coupled with the perturbation model (5.18) to restore feasibility when needed. Although both RH and DRH policies are based on a reoptimization scheme, the respective decision rules are defined using different models as illustrated in Figure 5.5 and are conceptually very different.

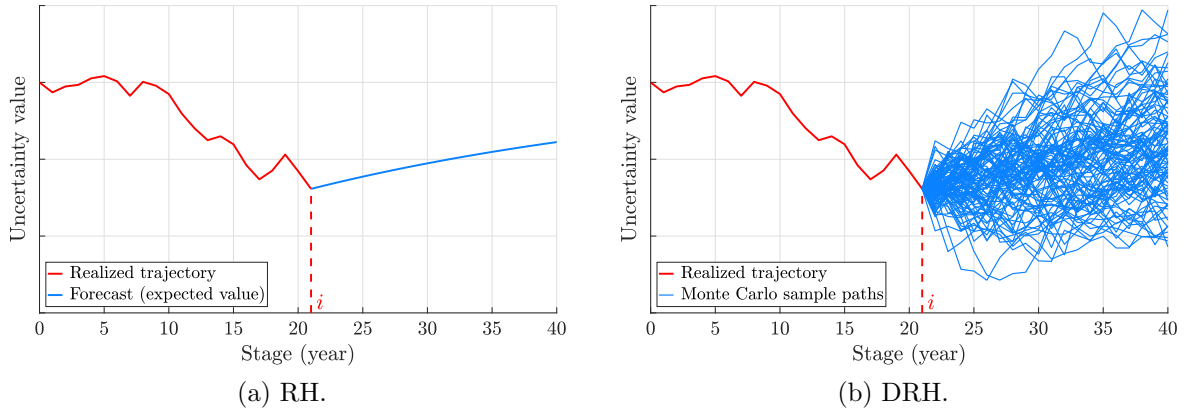


Figure 5.5: *RH* and *DRH* decision rules: The *RH* decision is based on a forecast of the uncertainty while the *DRH* decision on an action distribution over Monte Carlo sample paths of the uncertainty.

5.5 Numerical study

In this section, we numerically assess the performance of the dual bounds and procurement strategies discussed in Sections 5.3–5.4. In Section 5.5.1, we define the reference PPA instance used in the numerical analysis. In Section 5.5.2, we introduce the stochastic processes used to describe the evolution of the market factors and their calibration. In Section 5.5.3, we summarize the methods that we test and discuss the computational setting. In Section 5.5.4, we present and discuss the numerical results, including a sensitivity analysis of the performance of the methods with respect to changes in market and instance parameters.

5.5.1 PPA instances

In Table 5.2, we define our reference PPA instance. The table contains parameters of the strike price model in Section 5.2.1 and of the SDP model in Section 5.2.2. The production tax credit parameters are taken from DOE (2016). In particular, we use $T_i = 0$ USD/MWh for $i \geq 5$, that is, the tax credit is only granted to those PPAs that are ordered within the first 5 years of the horizon. The hours of θ correspond to a capacity factor of 35% which is representative of the average capacity factor from wind farms in the U.S. (EIA, 2018) and common in the wind energy investment literature (Boomsma et al., 2012). The project cost C_0^{Inv} is representative of new wind projects in the U.S. (Wiser and Bolinger, 2017), and the learning rate value ξ is taken from NREL (2010). The set \mathcal{M} and the minimum quantity z^{\min} are chosen by looking at the PPA portfolio of Google (Google,

Table 5.2: Parameters defining the reference PPA instance.

Name	Value	Unit	Name	Value	Unit
$L_{T,i}$	10	years	I	40	years
T_i	23	USD/MWh	I^R	5	years
L_P	30	years	α	90%	-
θ	3066	hours/year	\mathcal{M}	{5, 10, 15, 20, 25}	years
C_0^{Inv}	1.7	ml.USD/MW	z^{min}	20	MW
ξ	1%	-	γ	0.97	-
K_+^5	1.2	-	r	0.94	-

2017). We use a maximum risk factor $K_+^m = 1.2$ for $m = 5$, i.e. a 20% premium for the 5-year PPAs, which decreases linearly to 1 as m is increased. We use γ corresponding to a risk-free interest rate of approximately 3%, which equals the 10-year U.S. treasury rate in May 2018 (Bloomberg, 2018). We choose the IRR to be $r = 0.94$ corresponding to an investor return on investment which is approximately twice as large as the risk-free interest rate.

Starting from this reference instance, in our numerical experiments in Section 5.5.4 we define an extended data set where we vary some of these parameters to assess the robustness of the methods and the resulting effect on the procurement strategies.

5.5.2 Model of market dynamics

In this section we present the stochastic processes used to model the three sources of uncertainty in our procurement model: Power prices, REC prices, and company's power demand. We also discuss the calibration of these models and report the parameter estimates.

The evolution of electricity prices has been studied in the field of financial engineering using different types of stochastic processes. These models often capture features such as seasonality (Lucia and Schwartz, 2002), mean-reverting behavior and long-term trends (Schwartz and Smith, 2000 and references therein), and jumps (Weron, 2014; Weron, 2007; Cartea and Figueroa, 2005; Seifert and Uhrig-Homburg, 2007; Escibano et al., 2011). To obtain a power price model that captures the main features of spot electricity prices, we construct a mean-reverting stochastic process with jumps and seasonality. We use a continuous-time process for the power price $\{P_t, t \in \mathbb{R}_+\}$, and then consider in our decision model discrete-time values $\{P_i, i \in \mathcal{I} \cup \{I\}\}$, which are the values taken by this process at the beginning of stages $i \in \mathcal{I} \cup \{I\}$ (analogously we do the same for the other

uncertainties). The power price model is written as:

$$\begin{cases} \ln(P_t) = \chi_t + g(t) & (5.20a) \\ d\chi_t = (\nu_P - \mathcal{K}_P \chi_t) dt + \sigma_P dW_t + J(\mu_J, \sigma_J) d\Pi(\lambda) & (5.20b) \\ g(t) = \phi_0 + \sum_{k=1}^{12} \phi_k \hat{P}_t^k. & (5.20c) \end{cases}$$

Equation (5.20a) describes the log power price as the sum of a stochastic component χ_t and a deterministic component $g(t)$. In the stochastic component of the process (5.20b), \mathcal{K}_P is the speed of mean reversion, ν_P models the drift, σ_P is the volatility, and W_t is a standard Brownian motion. We capture spikes in monthly prices by a jump diffusion model in which the jump size follows a normal distribution $J(\mu_J, \sigma_J)$ and the jump frequency a Poisson distribution $\Pi(\lambda)$ (Cartea and Figueroa, 2005). The deterministic function $g(t)$ in (5.20c) models the monthly price seasonality by using a constant ϕ_k for each month k , and binary values \hat{P}_t^k equal to one if time t falls in month k and zero otherwise. We calibrated the parameters of model (5.20) using historical monthly power price data from the PJM market in the interval January 2010–August 2017. Analyzing power prices, we found that the jump frequency and intensity are small when considering the monthly prices. We thus tested the number of jumps in the monthly power prices using the algorithm presented in Weron (2014, p. 1047), and found the jump diffusion parameters to be not significant. Therefore, we eventually removed jumps from the model and only focused on a mean-reverting process with seasonality. We first estimated the seasonality function $g(t)$ directly from the data using linear regression, resulting in coefficients $\{\phi_k, k = 0 \dots, 12\} = \{0.258, 0.163, 0.078, -0.026, 0.003, 0.039, 0.174, 0.013, 0.009, -0.037, -0.038, 0.000, 3.519\}$. Then, we calibrated the mean reverting coefficients using a maximum likelihood estimation, which is common and resulted in $\mathcal{K}_P = 0.295$, $\sigma_P = 0.178$ (both with a p-value below 0.001), and $\nu_P = 0$. We use the initial point of the process $P_0 = 31.5$ USD/MWh which is the average price observed in 2017. When plotting the scenarios, we noticed that the mean reversion effect was too strong and so we decreased \mathcal{K}_P to 0.04 for our numerical study.

The dynamics of REC prices has been less studied in the literature. Renewable portfolio standards (RPS) represent one of the prominent support programs for renewable energy sources. Under RPS, the regulators require producers, distributors, and consumers to purchase certificates from the REC market. The price of these certificates is stochastic but is bounded from below by zero and from above by what is called the alternative compliance payment (ACP), which is the penalty that the regulator imposes if the RPS is not met. To forecast the evolution of REC prices consistently with these bounds, following Zeng et al. (2015) we (i) use a Jacobi diffusion process to generate numbers between zero

and one, and (ii) obtain REC prices as the product between the output of this stochastic process and the ACP threshold. The stochastic process is defined by:

$$\begin{cases} dr_t = (\nu_R - \mathcal{K}_R r_t)dt + \sigma_R \sqrt{r_t(1-r_t)} dW_t, & (5.21a) \\ R_t = r_t \bar{R}. & (5.21b) \end{cases}$$

In the Jacobi diffusion equation (5.21a), ν_R and \mathcal{K}_R are the mean-reverting parameters, σ_R is the volatility, and W_t is a standard Brownian motion. The process (5.21a) generates values $r_t \in [0, 1]$, which are then scaled in the second equation (5.21b) by the ACP value \bar{R} to produce the REC price $R_t \in [0, \bar{R}]$. There are different RECs markets, in which the average price can vary from a few U.S. dollars to more than two hundred in the case of expensive solar RECs. Thus we chose an ACP upper bound of $\bar{R} = 60$ USD/MWh that is more representative of wind RECs. We then estimated the parameters of the REC price model (5.21a) using monthly data of New Jersey REC prices between May 2015 and December 2017, and used an adaptation of the maximum likelihood estimation method for Jacobi diffusion processes from Gouriéroux and Valéry (2004). We obtained the parameter estimates $\mathcal{K}_R = 0.448$, $\nu_R = 0.080$, and $\sigma_R = 0.109$ (p-values are below 0.001). We use the average price in our time series $R_0 = 17.5$ USD/MWh as initial value of the process. Using this model, however, all the generated sample paths showed a downward slope, so we adjusted the parameters to $\nu_R = 0.03$, and $\sigma_R = 0.10$ to obtain more realistic scenarios presenting both downward and upward slope.

The electricity demand of a company is uncertain due to various factors including technology change, company expansion programs, energy efficiency programs, and environmental conditions that can cause potentially large shifts in the demand. We chose to model power demand uncertainty using a geometric Brownian motion because this process is often used for modeling commodity demand in procurement problems (Heath and Jackson, 1994; Hausman, 1969; Berling and Rosling, 2005; Kouvelis et al., 2013; Secomandi and Kekre, 2014). This process is formulated as:

$$dD_t = \mu_D D_t dt + \sigma_D D_t dW_t, \quad (5.22)$$

where μ_D and σ_D represent, respectively, the drift and volatility of the process, and W_t is a standard Brownian motion. To estimate the process parameters, we use as reference the power consumption of a Google data center in the U.S. Considering the total Google annual power consumption (Google, 2016) and the number of its data centers, we can say that the consumption of one such facility is approximately $3 \cdot 10^5$ MWh/year. Given that we consider major power consumers with possibly multiple facilities in a market zone, we use $6 \cdot 10^5$ MWh/year corresponding to the consumption of two Google data centers as the initial point D_0 of our stochastic process. We then assume that there

is no drift in data centers power consumption (i.e. $\mu_D = 0$). In fact, on one hand, the increasing size and demand for such centers would suggest a positive drift. On the other hand, improving technology and energy efficiency reduce consumption. Our choice thus accounts for these two opposite effects assuming that they balance each other out. Regarding volatility, it is reasonable for industrial levels of natural gas consumption to use an annual coefficient of variation of 0.05 (Secomandi and Kekre, 2014). Even though we deal with electricity, the proximity of these commodities provides some support for the choice of this parameter. This translates in a volatility for the geometric Brownian process of $\sigma_D = \sqrt{\ln(1 + 0.05^2)} = 0.05$.

The calibrated models for power price, RECs, and demand allowed us to generate sample paths of the uncertainty in Monte Carlo simulation which is needed to estimate the value of the policies.

5.5.3 Summary of methods and computational setup

In Table 5.3, we summarize the methods previously described to obtain lower and upper bounds on the optimal policy value and that we use in the numerical study. In the upper part of the table, we report two information relaxations used for deriving lower bounds. In the lower part of the table, we display the feasible procurement policies divided into benchmark heuristics and our new DRH method. We consider two DRH versions that solve dual programs on the inner sample paths based the same two information relaxations used to obtain lower bounds.

Table 5.3: Dual bounds and procurement policies

	Method	Description
Dual bounds	PI	Perfect information
	LDP	Information relaxation with linear dual penalty
Feasible policies	Spot	Spot purchase of power and RECs
	BH _m	Block heuristic with single PPA <i>m</i> (benchmark)
	RH	Reoptimization heuristic
	DRH-PI	Dual reoptimization, PI on inner samples (new)
	DRH-LDP	Dual reoptimization, LDP on inner samples

All algorithms were implemented using C++ with GUROBI as solver. For RH and DRH, we used a cross-sample and cross-inner-sample implementation, that is, at each stage

$i \in \mathcal{I}$ we built only one math program from scratch for the first sample path. Then, for all the evaluation samples $h \in \mathcal{H}$ and inner samples $n \in \mathcal{N}$, we changed the related coefficients in the objective function and right-hand-side of the constraints, and solved an updated stage i model. Due to GUROBI's automatic warm-start, this approach resulted in a computation several times faster than solving the policy over the entire horizon sample path by sample path. Based on experimentation, we chose a number of evaluation sample paths $H = 200$ because this number was sufficient to obtain a standard error of the lower and upper bound estimates around 1% of the mean. For the DRH method, we used a number of inner samples $N = 25$ since the policy value did not improve (or it did but very marginally) when this number was increased.

5.5.4 Results

In this section we report the results from our numerical analysis. We start by varying the required IRR from $r = 0.91$ to $r = 0.96$ to test the impact that changes on the power producers side have on the company procurement strategy. We keep fixed the other instance parameters as in Table 5.2. In Figure 5.6, we display the optimality gap for these instances with respect to the best lower bound, which corresponds to LDP in all instances. Specifically, the LDP bound is much tighter than PI with an improvement of 6.2% on average. To maintain readability in the graph, we do not display the spot

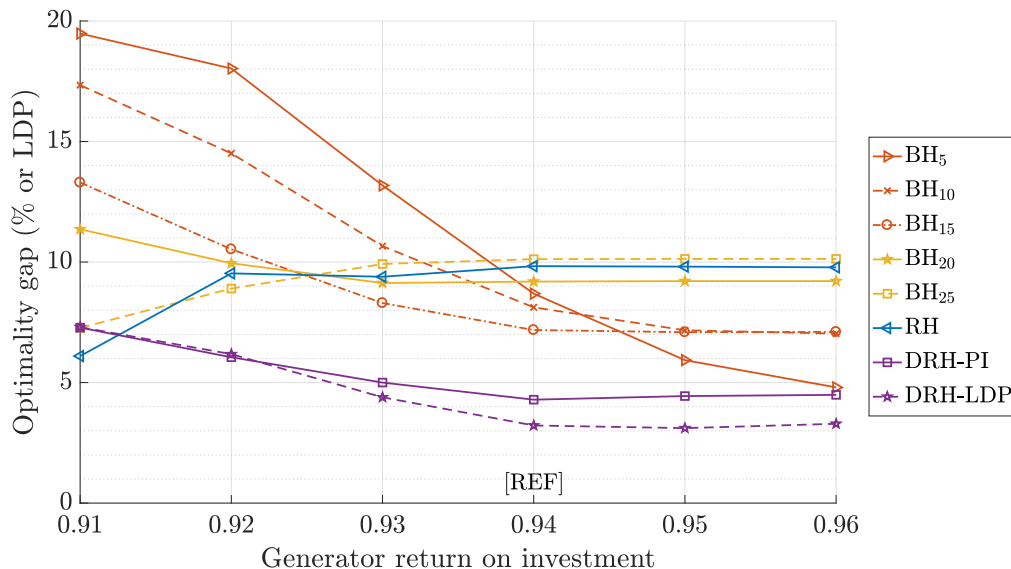


Figure 5.6: Optimality gap (%LDP) when changing PPA return on investment

strategy because it results in a very high optimality gap of 33% on average. This means that a portfolio which includes PPAs helps hedging against power and REC price volatility,

resulting in a significantly lower power procurement cost when a renewable target has to be met. Overall, the DRH policy achieves the best performance, with the DRH-LDP variant being 0.7% better than the DRH-PI one on average. It is interesting to see that DRH also without dual penalties performs well, which is the result of the action averaging scheme. Despite its simplicity, the BH_m heuristic with a single contract m performs well in some instances but its performance is unstable as it varies significantly across instances. For example, BH_5 is only 0.3% and 1.5% worse, respectively, than DRH-PI and DRH-LDP when $r = 0.96$, but it is up to 12.2% worse of these policies for $r = 0.91$. Similarly, the performance of BH_{25} is comparable to DRH (both versions) when $r = 0.91$, but it is about 7% worse for $r \in \{0.94, 0.95, 0.96\}$. The standard RH is overall substantially worse than DRH with the exception of one instance.

Second, we vary the level of the renewable energy target α from 50% to 100% to test the impact of different corporate sustainability goals on power procurement. We report the optimality gap of the methods in Figure 5.7. As before, the LDP bound is substantially

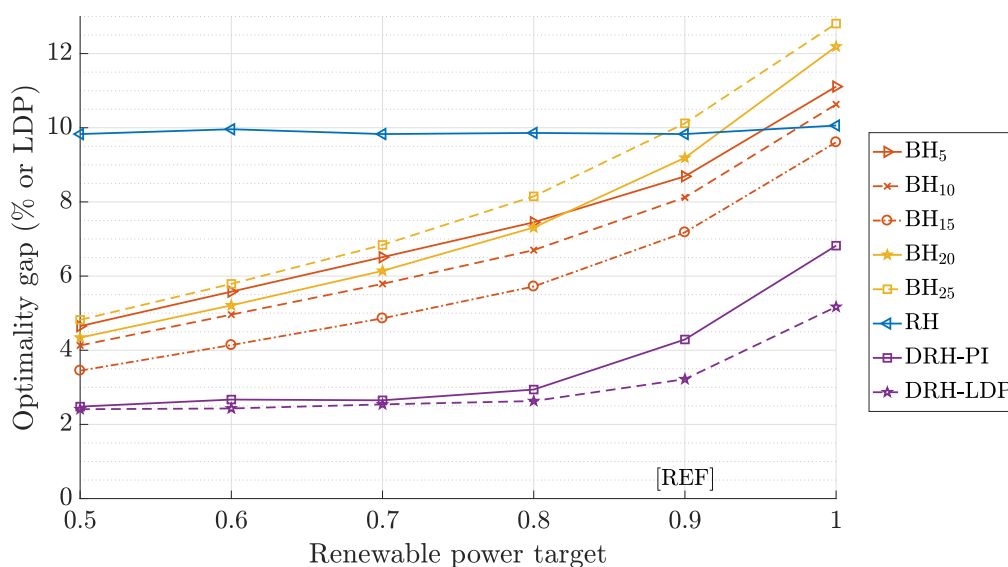


Figure 5.7: Optimality gap (%LDP) when changing the renewable energy target

tighter than PI (9.1% on average) and the spot procurement strategy performs the worst with an average gap of 28.2%. The standard reoptimization (RH) does not perform well and the optimality gap is about 10% in all instances. The DRH-LDP achieves the lowest gap also on these set of instances, which is 3.1% on average. From these results, it appears harder for the company to obtain policies with low optimality gap when the renewable target is higher and close to 100%.

Finally, we remove the production tax credit to test the effect of regulatory changes in the procurement strategies. We show the optimality gap in Table 5.4 and the average power

portfolio resulting under the DRH-LDP method in Figure 5.8.

Table 5.4: Optimality gap (%LDP) with and without production tax credit

PTC	Spot	BH ₅	BH ₁₀	BH ₁₅	BH ₂₀	BH ₂₅	RH	DRH-PI	DRH-LDR
0 USD/MWh	32.88	10.16	10.59	9.42	10.47	10.14	6.00	5.55	4.33
23 USD/MWh	34.27	8.69	8.12	7.18	9.19	10.12	9.83	4.29	3.22

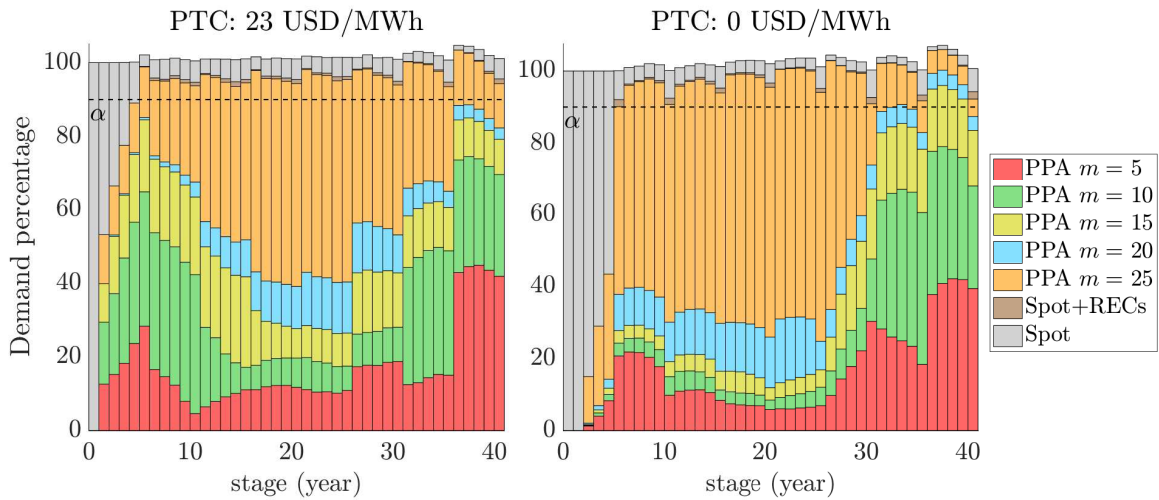


Figure 5.8: Average power portfolio inventory with and without production tax credit

When the PTC is removed, the optimality gap does not change significantly in the methods, except for RH which improves by about 4% and outperforms the BH methods. However the effect of the production tax credit is evident on the procurement strategy from Figure 5.8. In fact, in the presence of a PTC (left side of the figure) the company enters into PPAs more aggressively in the years in which the PTC is effective (5 years). Without PTC (right side of the figure), fewer/smaller PPAs are entered in the first 5 years. Yet, some PPAs are signed during the first 5 years in this second case too because the reach period also corresponds to 5 years. In the long run, the effect of this initial different purchase reduces and the procurement policies are more similar.

Regarding running times of the methods, evaluating the spot policy on 200 sample paths took a fraction of a second and the block heuristic less than 5 seconds on average. Evaluating the RH (the linear programming version with perturbation; see Section 5.4) took on average 1 minute while DRH (both PI and LDP variants) took on average 40–45 minutes. DRH is obviously computationally more demanding than RH due to the inner sampling dimension. However, evaluating such policy takes less than one hour which does not seem problematic for this strategic procurement problem where decision epochs are years. To

reduce the DRH the running time, one could reoptimize every second year instead of every year if the solution quality does not decrease. In any case, implementing the policy, that is, evaluating the single DRH action takes usually less than a second.

5.6 Conclusion

Motivated by the recent global trends in corporate energy procurement, we studied the problem of companies that committed to satisfy a renewable energy target by a future date. We formulated a multi-period power procurement model with short and long term procurement options as an SDP with endogenous and exogenous components of its state which are both high-dimensional. This model is particularly hard to approach by existing ADP techniques. Thus, we developed a novel framework (DRH) that uses action distributions and information relaxations to obtain non-anticipative decision rules, and found it to outperform standard reoptimization methods and other simpler heuristics on realistic instances. We currently use simple linear dual penalties in the information relaxations but further work includes determining the dual penalties in a more principled manner. The DRH method is broadly applicable beyond our specific power procurement context as it only requires (i) simulating the evolution of the uncertainty in Monte Carlo, and (ii) solving deterministic dual math programs on each individual sample path. Consequently, DRH emerges as a promising approach to tackle high-dimensional SDPs.

5.7 Appendix

5.7.1 Non-convexity of the value function

We show that the value function $V_i(\cdot, w_i)$ of the SDP (5.5) is not convex in the state x_i by using a simple counter-example with two periods 0 and 1, in which a PPA can be entered at stage 0 with delivery at stage 1. We consider a setting where the demand is constant $D_0 = D_1 = 10$ MWh, the power and RECs prices have values $P_0 = R_0 = 10$ USD/MWh and are martingales (i.e. $\mathbb{E}[P_1|P_0] = 10$ and $\mathbb{E}[R_1|R_0] = 10$), the strike price is $K = 11$ USD/MWh, $\alpha = 0.8$, and $z^{\min} = 6$ MWh. To ease intuition, we consider the power units of actions and states in MWh in this example to avoid the conversion from MW to MWh through θ . Proceeding backward, the terminal value function (stage 1) is trivially convex

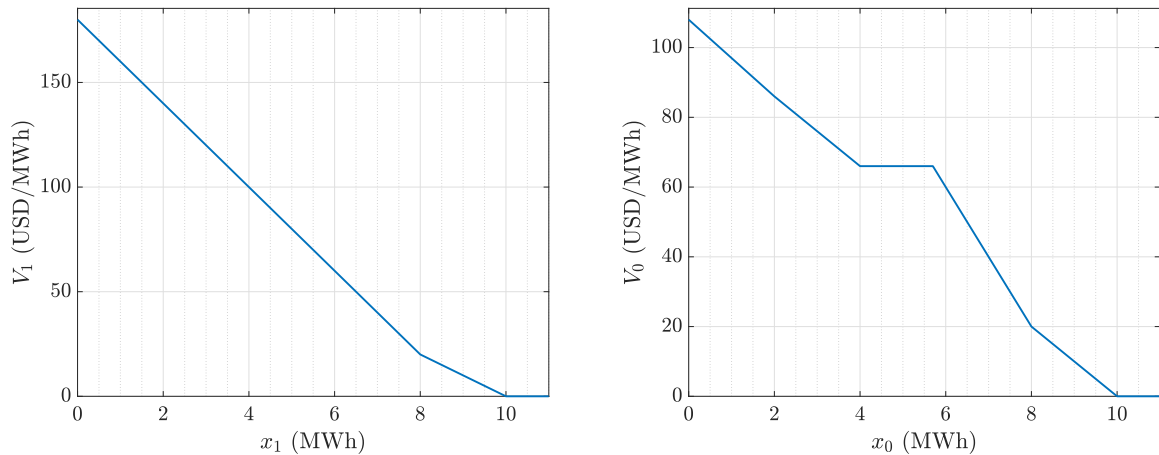
in the state x_1 as it is equal to

$$V_1(x_1, w_1) = P_1 \max\{D_1 - x_1, 0\} + R_1 \max\{\alpha D_1 - x_1, 0\},$$

which has higher slope $P_1 + R_1$ for $x_1 \in [0, \alpha D_1]$, smaller slope P_1 for $x_1 \in [\alpha D_1, D_1]$, and is zero if $x_1 \geq D_1$ (this property is always true). Instead, the stage 0 value function is given by

$$\begin{aligned} V_0(x_0, w_0) &= \min_{z_0 \in \mathcal{Z}} \{K z_0 + \mathbb{E}[P_1(D_1 - x_0 - z_0)\mathbf{1}_{\{D_1 > x_0 + z_0\}} \\ &\quad + R_1(\alpha D_1 - x_0 - z_0)\mathbf{1}_{\{\alpha D_1 > x_0 + z_0\}} | w_0]\} \\ &= \min_{z_0 \in \mathcal{Z}} \{K z_0 + P_0(D_0 - x_0 - z_0)\mathbf{1}_{\{D_0 > x_0 + z_0\}} \\ &\quad + R_0(\alpha D_0 - x_0 - z_0)\mathbf{1}_{\{\alpha D_0 > x_0 + z_0\}}\}, \end{aligned}$$

and can be non convex as illustrated in Figure 5.9. In particular, in figure Figure 5.9(a), we show the value function $V_1(\cdot, w_1)$ given the realization of the uncertainty $P_1 = R_1 = 10$, and in Figure 5.9(b) we show $V_0(\cdot, w_0)$. The plateau in the latter function originates because of a region for x_0 where it is optimal to enter into a PPA of size z^{\min} which cause over-procurement at stage 0.



(a) Value function $V_1(\cdot, w_1)$ with $w_1 = (10, 10, 10)$. (b) Value function $V_0(\cdot, w_0)$ with $w_0 = (10, 10, 10)$.

Figure 5.9: Stage-1 and stage-0 SDP value functions of the endogenous state.

5.7.2 Mathematical programs

In this section, we describe the extended math programming formulations of the optimization models presented in the paper: the information relaxation model with linear

dual penalty (5.7), the decision rule of the block heuristic, and the decision rule of the reoptimization heuristic (5.19).

Given the initial inventory $x_0 \in \mathcal{X}_i$ and a sample path of the uncertainty $\{w_i, i \in \mathcal{I} \cup \{I\}\}$, the information relaxation model which includes linear dual penalties defined on this sample path is formulated with the following mixed-integer linear program:

$$V_{0,h}^{\text{IR}}(x_0) = \min_{\Xi} \sum_{i \in \mathcal{I}} \sum_{m \in \mathcal{M}} \sum_{l=1}^m \gamma^{i+l} \theta z_{i,m} \left[K_{i,m} - \sum_{j \in \mathcal{J}} (w_{i+l,j} - \mathbb{E}[w_{i+l,j} | w_i]) \psi_{i,j} \right] \\ + \sum_{i=0}^I \gamma^i [(P_i + R_i)u_i + P_i v_i] \quad (5.23a)$$

$$\text{s.t.}: u_i + \theta x_{i,0} \geq \alpha D_i, \quad \forall i \in \mathcal{I}^S \cup \{I\} \quad (5.23b)$$

$$u_i + v_i + \theta x_{i,0} \geq D_i, \quad \forall i \in \mathcal{I} \cup \{I\} \quad (5.23c)$$

$$x_{i+1,l} = x_{i,l+1} + \sum_{m>l} z_{i,m}, \quad \forall i \in \mathcal{I}, l \in \{0, \dots, M-2\} \quad (5.23d)$$

$$x_{i+1,l} = z_{i,m}, \quad \forall i \in \mathcal{I}, l = M-1, m = M \quad (5.23e)$$

$$x_{i,l} = x_{0,l}, \quad i = 0, \forall l \in \{0, \dots, M-1\} \quad (5.23f)$$

$$\xi_{i,m} z^{\min} \leq z_{i,m} \leq \xi_{i,m} z^{\max}, \quad \forall i \in \mathcal{I}, m \in \mathcal{M} \quad (5.23g)$$

$$\text{var: } \Xi = \{z_{i,m}, u_i, v_i, x_{i,l} \geq 0, \xi_{i,m} \in \{0, 1\}\}. \quad (5.23h)$$

The decision variable $z_{i,m}$ denotes the size of the PPA of length m years entered at stage i . The variable u_i represents the amount of spot power and RECs purchase needed to meet the renewable target αD_i at stage i (if part of the sustain period), while v_i models the additional spot power to meet the company's demand D_i at stage i . Moreover, the collection of variables $x_i = \{x_{i,l}, l \in \{0, \dots, M-1\}\}$ model the stage i PPA pipeline. In other words, $x_{i,l}$ denotes the total PPAs capacity the firm holds at stage i with delivery at stage $i+l$. The objective function (5.23a) minimizes the total perfect foresight power procurement cost incurred on the sample path corrected with the linear dual penalty. Due to the PPAs lead time of one period, the spot cost at stage i is a constant that could be removed from the objective function of this models and the others in the paper. The constraints (5.23b)-(5.23c) are used to model the amount of power to be procured spot, and the constraints (5.23d)-(5.23e) ensure that the pipeline transitions are consistent with the PPA investment decisions made. The constraints (5.23f) set the initial inventory equal to the initial stage 0 inventory level x_0 , and (5.23g) enforces the minimum contract size, which requires introducing the binary variables $\xi_{i,m}$ and a parameter z^{\max} acting as a "big-M". Finally, (5.23h) establishes the variables domain.

The block heuristic that uses PPAs of length m , i.e. BH_m , enters into a new contract at

stage i according to the following rule:

$$z_{i,m} = \begin{cases} z^* & \text{if } \text{mod}(i+1 - I^R, m) = 0; \\ 0 & \text{otherwise,} \end{cases} \quad (5.24)$$

where z^* is obtained from solving the deterministic mixed-integer linear program (5.25) defined at stage i and state $(0, w_i)$, and which uses a look-ahead of m periods.

$$\min_{\Xi} \sum_{l=1}^m \gamma^l \left(\theta K_{i,m} z + \mathbb{E}[P_{i+l} + R_{i+l}|w_i] u_{i+l} + \mathbb{E}[P_{i+l}|w_i] v_{i+l} \right) \quad (5.25a)$$

$$\text{s.t.}: u_{i+l} + \theta z \geq \alpha \mathbb{E}[D_{i+l}|w_i], \quad \forall l : i+l \in \mathcal{I}^S \cup \{I\} \quad (5.25b)$$

$$u_{i+l} + v_{i+l} + \theta z \geq \mathbb{E}[D_{i+l}|w_i], \quad \forall l : i+l \in \mathcal{I} \cup \{I\} \quad (5.25c)$$

$$\xi z^{\min} \leq z \leq \xi z^{\max} \quad (5.25d)$$

$$\text{var: } \Xi = \{z, u_{i+l}, v_{i+l} \geq 0, \xi \in \{0, 1\}\}. \quad (5.25e)$$

The decision variable z represents the size of the PPA while u_{i+l} and v_{i+l} denote the amount of power purchased in the spot market at stage $i+l$, respectively, supplemented with and detached from RECs. The objective function (5.25a) minimizes the power procurement cost over the delivery period $i+1, \dots, i+m$ and includes the PPAs cost (variable z) as well as the spot costs to fulfill the renewable target (variables u_{i+l}) and the residual power demand (variables v_{i+l}). The constraints (5.25b)-(5.25c) ensure that both the renewable target and the total electricity demand are satisfied in expectation during the delivery period, and (5.25d) enforces the minimum contract size. Notice that the use of a single PPA and the block structure makes the BH_m policy simple to implement as the contract pipeline x_i does not need to be stored nor modeled. Moreover, (5.25) contains only one binary variable and is thus very quick to solve.

Finally, the RH decision model at stage i and state (x_i, w_i) is formulated as follows.

$$\min_{\Xi} \sum_{j \in \mathcal{I}, j \geq i} \sum_{m \in \mathcal{M}} \sum_{l=1}^m \gamma^{j+l} \theta K_{j,m} z_{j,m} + \sum_{j=i}^I \gamma^j \left(\mathbb{E}[P_j + R_j|w_i] u_j + \mathbb{E}[P_j|w_i] v_j \right) \quad (5.26a)$$

$$\text{s.t.}: u_j + \theta x_{j,0} \geq \alpha \mathbb{E}[D_j|w_i], \quad \forall j \in \mathcal{I}^S \cup \{I\}, j \geq i \quad (5.26b)$$

$$u_j + v_j + \theta x_{j,0} \geq \mathbb{E}[D_j|w_i], \quad \forall j \in \mathcal{I} \cup \{I\}, j \geq i \quad (5.26c)$$

$$x_{j+1,l} = x_{j,l+1} + \sum_{m>l} z_{j,m}, \quad \forall j \in \mathcal{I}, j \geq i, l \in \{0, \dots, M-2\} \quad (5.26d)$$

$$x_{j+1,l} = z_{j,m}, \quad \forall j \in \mathcal{I}, j \geq i, l = M-1, m = M \quad (5.26e)$$

$$x_{j,l} = x_{i,l}, \quad j = i, \forall l \in \{0, \dots, M-1\} \quad (5.26f)$$

$$\xi_{j,m} z^{\min} \leq z_{j,m} \leq \xi_{j,m} z^{\max}, \quad \forall j \in \mathcal{I}, j \geq i, m \in \mathcal{M}, \quad (5.26g)$$

$$\text{var: } \Xi = \{z_{j,m}, u_l, v_l, x_{j,l} \geq 0, \xi_{j,m} \in \{0, 1\}\} \quad (5.26h)$$

The model (5.26) has essentially the same structure of (5.23) and the decision variables $z_{j,m}, u_j, v_j, x_{j,l}$ keep the same interpretation. The main difference with respect to (5.23) is that the objective function (5.26a) and the constraints (5.26b)–(5.26c) are formulated using a forecast for the uncertainty instead of the true values on the sample path (the RH decision rule is in fact non-anticipative), and that there is no dual penalty correction term.

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CHAPTER 6

Enabling Active/Passive Electricity Trading in Dual-Price Balancing Markets

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Abstract: In electricity markets with a dual-pricing scheme for balancing energy, controllable production units typically participate in the balancing market as “active” actors by offering regulating energy to the system, while renewable stochastic units are treated as “passive” participants that create imbalances and are subject to less competitive prices. Against this background, we propose an innovative market framework whereby the participant in the balancing market is allowed to act as an active agent (i.e., a provider of regulating energy) in some trading intervals and as a passive agent (i.e., a user of regulating energy) in some others. To illustrate and evaluate the proposed market framework, we consider the case of a virtual power plant (VPP) that trades in a two-settlement electricity market composed of a day-ahead and a dual-price balancing market. We formulate the optimal market offering

problem of the VPP as a three-stage stochastic program, where uncertainty is in the day-ahead electricity prices, balancing prices and the power output from the renewable units. Computational experiments show that the VPP expected revenues can increase substantially compared to an active-only or passive-only participation, and we discuss how the variability of the stochastic sources affects the balancing market participation choice.

Keywords: Electricity markets · balancing market · virtual power plant · offering strategy · stochastic programming

6.1 Introduction

Society is moving towards using more renewable energy sources to decrease the dependency on fossil fuels. Governments, seeking to increase the share of renewable energy, typically support stochastic power sources such as wind and solar power by means of subsidies. However, with the steep growth and decreasing cost of renewable energy generation experienced in the recent years, stochastic producers are increasingly required to be financially responsible for the imbalances created in the real-time. Accordingly, renewable energy producers access the balancing electricity market as “passive” actors by settling the deviation from the day-ahead contracted schedule at a less favorable power price.

The imbalance of the system, often caused by forecast errors of power demand and renewable generation, is restored by rescheduling the market position of the “active” participants in the balancing market. Such producers offer to the Transmission System Operator (TSO) the flexibility to upward or downward adjust their day-ahead contracted schedule, provided to be remunerated at a more convenient price. However, to qualify as regulators in the balancing market, generators must fulfill specific requirements from the TSO, which include the ability to always meet the contracted schedule except for unpredictable unit failures. As a consequence, only conventional generators can currently be qualified as active balancing market participants. In this context, it is quite straightforward to distinguish between passive participants (i.e., stochastic producers) that regularly deviate from their contracted schedule, and active participants (i.e., conventional producers) that can consistently respect their market position and offer additional regulating energy to the TSO.

Against the current balancing market setup, we propose an innovative and more flexible framework where the TSO’s requirements needed for the active balancing market partic-

ipation are loosened. Specifically, our idea is to allow the market participants to actively sell regulating energy in some trading intervals and passively deviate from their schedule in others. However, when a market participant commits to sell regulating energy during a trading interval, it is prevented from generating an imbalance in the same interval.

To illustrate and evaluate the proposed market framework, we analyze the case of a virtual power plant (VPP), i.e., a cluster of combined generating units, storage systems and flexible loads that acts as a single participant in the electricity market (Morales et al., 2013). Under the current market setup, a VPP with both stochastic units (e.g., wind and solar units) and dispatchable technologies (e.g., conventional generators and batteries) can hardly fulfill the qualification procedure for being a regulator in the balancing market. However, such VPP may actually be able to actively provide regulating energy in some trading intervals. A VPP composed of a PV solar unit and a dispatchable unit, for instance, can offer regulating energy at night when the output of the stochastic unit is known with certainty. Differently, during the day it may not be able to internally handle the PV solar unit fluctuations and will passively deviate from its contracted schedule. Therefore, a VPP is a natural market participant who could benefit from our proposed *Active/Passive* market framework since a more flexible balancing market participation may translate into higher profits. Nonetheless, this added flexibility leads to a rethinking of the VPP offering model that we thus examine in this paper.

Indirectly, this new setup can facilitate the development and integration of sustainable energy through VPPs. Moreover, the TSO may benefit from this innovative market structure by having more regulating energy available in the real time.

6.1.1 Literature review

The problem of determining the optimal market offer for a stochastic power producer has been widely studied in the literature. In Bremnes (2004), Pinson et al. (2007) and Bitar et al. (2012), the optimal quantity to be submitted in the day-ahead market is derived as a quantile of the probability distribution of the future wind or solar power production. In Morales et al. (2010), the optimal offering strategy for a wind power producer is solved using stochastic programming. These models consider the stochastic producer as a passive actor in the balancing market, i.e., the balancing stage is only used to settle deviations from the day-ahead contracted schedule.

Similarly, several models have been developed to derive the optimal offering strategy for a conventional production unit. In Arroyo and Conejo (2000), Arroyo and Conejo (2004),

and Conejo et al. (2004), the feasible operating region of a thermal unit is formulated using a mixed-integer linear program (MILP). Other papers combine such operating region with an electricity market trading problem obtaining different offering strategies (Conejo et al., 2002; Baillo et al., 2004; Conejo et al., 2010; Maenhoudt and Deconinck, 2014). In contrast to stochastic units, conventional units are modeled as active participants in the balancing market, i.e., they access the balancing market to offer regulating energy to the system operator.

The optimal participation of a VPP in an electricity market has been less investigated. In Ruiz et al. (2009), a direct load control algorithm is used for managing an aggregate of controllable loads. Mashhour and Moghaddas-Tafreshi (2011a) and Mashhour and Moghaddas-Tafreshi (2011b) study the bidding problem of a VPP in a joint market for energy and reserve. The authors consider a deterministic setting and formulate a mixed-integer non-linear program solved using a genetic algorithm. In Peik-Herfeh et al. (2013), a VPP offering strategy is formulated as a unit commitment problem where point estimates are used to model the uncertainty in market prices and power generation. Pandžić et al. (2013a) proposes a stochastic MILP to derive the optimal self-scheduling of a VPP, considering a weekly time horizon and including long-term bilateral contracts and technical constraints of the units. Subsequently, Pandžić et al. (2013b) develop a two-stage stochastic offering model to maximize the expected profit of a VPP with uncertainty in electricity prices and power production. Other works, e.g., Kardakos et al. (2016), include the electricity market clearing process within the optimal offering strategy, resulting in a hierarchical stochastic optimization model. Finally, we refer to Morales et al. (2013) for a general VPP modeling approach in which different combinations of generating units, flexible loads, and storage systems are examined.

6.1.2 Approach and contributions

The VPP offering strategies from the extant literature discussed above model the VPP as a passive balancing market actor which solely settles the real-time deviations that cannot be self-balanced by the cluster. In this paper, we propose a novel *Active/Passive* balancing market model where agents, and in particular VPPs, can extend their decision space and offer regulating energy in some trading intervals.

To evaluate the impact of this market setup, we consider a VPP trading in a two-settlement electricity market composed of a day-ahead and a dual-price balancing market, and formulate the optimal *Active/Passive* market offering problem of the VPP as a three-stage stochastic program. We test the model using 300 scenarios and two VPPs composed

of a conventional production unit, an electricity storage, and a stochastic production unit represented by (i) a wind farm or (ii) a PV solar unit. We benchmark the proposed *Active/Passive* model against a *Passive* (i.e., passive-only) and *Active* (i.e., active-only), and show that under the former model the expected VPP profit can increase substantially and up to 8% compared to the other two. We eventually discuss the choice for the VPP of being active vs. passive participant depending on the uncertainty in market prices and renewable power production.

6.1.3 Paper structure

The rest of this paper is organized as follows. We start in Section 6.2 by presenting the electricity market framework, the VPP structure, and characterizing the uncertainty. In Section 6.3, we formulate the *Active/Passive* offering strategy as a three-stage stochastic program. In Section 6.4, we compare this strategy with the *Passive* and *Active* strategies for different VPP configurations. Conclusions are drawn in Section 6.5. We summarize the nomenclature used in the models in the appendix.

6.2 Market framework and modeling assumptions

6.2.1 Electricity market framework

We consider a two-settlement electricity market composed of a day-ahead and a balancing market. The day-ahead market is cleared at noon for all 24 hourly trading intervals of the following day. The accepted day-ahead market offers are settled under a uniform pricing scheme. Subsequently, closer to the real-time operation, a separate balancing market is cleared for each hourly interval, one hour before operation. At the balancing stage the active participants submit their offers, in the form of non-decreasing offer curves, for the provision of regulating energy to the TSO. The accepted offers are priced under a uniform pricing scheme. Passive participants inform the TSO of their deviations from the contracted schedule. Such deviations are priced under a dual-price imbalance settlement scheme, i.e., a different price for positive (extra-production) and negative (under-production) deviations (Morales et al., 2013; Morales et al., 2010).

This balancing market structure (i.e., uniform pricing for settling active offers and dual-pricing for passive deviations) is widely used across Europe, e.g., in Spain, Portugal, and

Denmark among other countries (Wang et al., 2015). Our novel *Active/Passive* market model thus adapts to one of the major market contexts.

6.2.2 VPP structure

We consider a VPP composed of a stochastic power unit (either wind or solar), a conventional thermal unit, and an electric energy storage. The structure of the VPP is illustrated in Figure 6.1. The power production of the stochastic unit and of the ther-

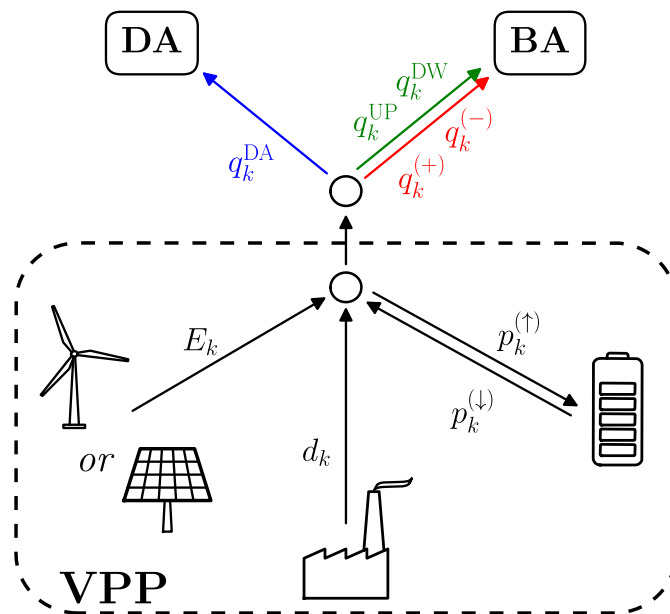


Figure 6.1: Illustration of the VPP structure highlighting the energy exchanged with the electricity market platform.

mal unit is denoted by E_k and d_k , respectively. The storage unit produces energy when discharging and consumes energy during the charging phase; the amount of charging and discharging power is denoted by $p_k^{(\uparrow)}$ and $p_k^{(\downarrow)}$, respectively. The total amount of energy production (or consumption) of the VPP has to match the amount of energy exchanged with the electricity market platform. The energy quantity contracted in the day-ahead market is denoted by q_k^{DA} . We then indicate with q_k^{UP} and q_k^{DW} the upward and downward adjustments in the balancing market, respectively, which are associated with an active participation at the balancing stage. Alternatively, the VPP can create a positive $q_k^{(+)}$ or negative $q_k^{(-)}$ deviation in the real-time. Thus, under the *Active/Passive* participation model, the “deterministic” energy balance at the hourly interval k between the VPP production (or consumption) and the quantity exchanged with the market platform can be

expressed by

$$q_k^{\text{DA}} + q_k^{\text{UP}} - q_k^{\text{DW}} + q_k^{(+)} - q_k^{(-)} = E_k + d_k + p_k^{(\downarrow)} - p_k^{(\uparrow)}.$$

The VPP is assumed price-taker in both the day-ahead and the balancing market. Accordingly, the market prices within its offering strategy are exogenous and uncertain, and modeled by means of a set of scenarios.

6.2.3 Scenario generation

To derive the offering strategy, the VPP is provided with an input set of scenarios describing the evolution of uncertain market prices and power production from the wind or PV power unit. We generate such scenarios starting from probabilistic forecasts. The probabilistic forecasts for the day-ahead and the balancing market prices are obtained through the fundamental market model proposed in Mazzi et al. (2017), where parametrized supply and demand curves are used to simulate the market clearing mechanism. Then, by introducing uncertainty in one or more parameters of the two curves, we obtain probabilistic forecasts of the day-ahead and balancing market prices. For wind and PV power production we instead use the dataset of probabilistic forecasts, respectively, from Mazzi et al. (2017) and Pierro et al. (2016).

Probabilistic forecasts describe an estimate of the random variable density function for each look-ahead time, without any inter-temporal correlation. To include temporal dependencies, starting from the probabilistic forecasts we generate a set of trajectories following the methodology presented in Pinson et al. (2009) and Pinson and Girard (2012). In brief, series of forecast errors are converted into a multivariate Gaussian random variable, and a unique covariance matrix is used to describe the interdependence structure. We model this covariance matrix using an exponential covariance function (Pinson and Girard, 2012) where the exponential parameter controls the correlation among the lead times. Accordingly, the day-ahead market prices λ_k^{DA} are represented by the set of trajectories $\{\lambda_{ik}^{\text{DA}} : i \in I, k \in K\}$. Then, for each day-ahead scenario i , the balancing market prices λ_k^{BA} are modeled using a set J of scenarios $\{\lambda_{ijk}^{\text{BA}} : i \in I, j \in J, k \in K\}$. Finally, the uncertain power production E_k from the stochastic unit is represented by the set of trajectories $\{E_{\omega k} : \omega \in W, k \in K\}$.

The number of scenarios needed to accurately represent continuous random variables or stochastic processes is usually large, leading to intractable stochastic programs. Therefore, we use the technique of Growe-Kuska et al. (2003) to reduce the number of scenarios while preserving most of the stochastic information.

6.3 Optimal offering strategy through multi-stage stochastic programming

At noon, the VPP submits its day-ahead market offers to the market operator, aiming to maximize its total expected profit. While determining the optimal day-ahead market offers, the VPP also takes into account the uncertainty, and endogenously models the future decisions in the balancing market. This results in a three-stage stochastic programming framework that we illustrate in Figure 6.2. The day-ahead quantity offers, q_k^{DA} , are modeled as first-stage decisions. Then, the up- and down-regulation adjustments q_{ik}^{UP} and q_{ik}^{DW} are chosen after the day-ahead market prices λ_k^{DA} realize, and are hence second-stage decisions. The positive and negative deviations $q_{i\omega k}^{(+)}$ and $q_{i\omega k}^{(-)}$ are third-stage decisions that have to be made after the disclosure of the uncertain power production E_k . Finally, the balancing market prices $\lambda_{ijk}^{\text{BA}}$ realize.

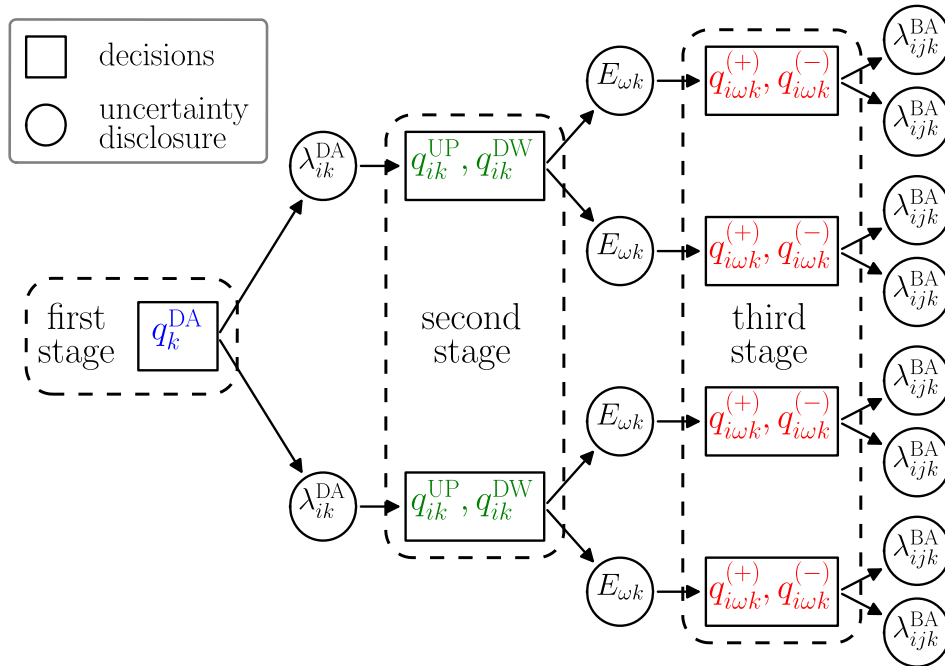


Figure 6.2: Representation of the multi-stage stochastic programming setup.

Following the methodology presented in Conejo et al. (2010) and Mazzi et al. (2017), we make the day-ahead quantities q_k^{DA} scenario i dependent (i.e., $q_k^{\text{DA}} \rightarrow q_{ik}^{\text{DA}}$) to build the non-decreasing offer curves. Despite being built using scenario-dependent price-quantity offers, the curves adapt to any realization of the uncertainty and are, in fact, scenario-independent (the non-anticipativity structure of our stochastic program is not violated). Similarly, the up- and down-regulation adjustments q_{ik}^{UP} and q_{ik}^{DW} are made index j dependent (i.e., $q_{ik}^{\text{UP}} \rightarrow q_{ijk}^{\text{UP}}$ and $q_{ik}^{\text{DW}} \rightarrow q_{ijk}^{\text{DW}}$) to derive the offer curves in the balancing

market. Finally, the operational variables of the VPP are made dependent on indices i , j , and ω (e.g., $d_k \rightarrow d_{ij\omega k}$ and $p_k^{(\uparrow)} \rightarrow p_{ij\omega k}^{(\uparrow)}$). To derive its day-ahead market offers, the VPP solves the following optimization model

$$\max_{\Xi} \sum_k \hat{\rho}_k^{\text{DA}} + \hat{\rho}_k^{\text{Act}} + \hat{\rho}_k^{\text{Pas}} - \hat{c}_k \quad (6.1a)$$

$$\text{s.t. } q_{ik}^{\text{DA}} + q_{ijk}^{\text{UP}} - q_{ijk}^{\text{DW}} + q_{i\omega k}^{(+)} - q_{i\omega k}^{(-)} = \quad (6.1b)$$

$$E_{\omega k} + d_{ij\omega k} + p_{ij\omega k}^{(\uparrow)} - p_{ij\omega k}^{(\downarrow)}, \quad \forall i, \forall j, \forall \omega, \forall k$$

$$(\hat{\rho}_k^{\text{DA}}, q_{ik}^{\text{DA}}) \in \Pi^{\text{DA}}, \quad \forall i, \forall k \quad (6.1c)$$

$$(\hat{\rho}_k^{\text{Act}}, q_{ijk}^{\text{UP}}, q_{ijk}^{\text{DW}}) \in \Pi^{\text{Act}}, \quad \forall i, \forall j, \forall k \quad (6.1d)$$

$$(\hat{\rho}_k^{\text{Pas}}, q_{ijk}^{(+)}, q_{ijk}^{(-)}) \in \Pi^{\text{Pas}}, \quad \forall i, \forall \omega, \forall k \quad (6.1e)$$

$$(d_{ij\omega k}, p_{ij\omega k}^{(\uparrow)}, p_{ij\omega k}^{(\downarrow)}) \in \Omega, \quad \forall i, \forall j, \forall \omega, \forall k \quad (6.1f)$$

$$\hat{c}_k = h(\{d_{ij\omega k}, \forall i, \forall j, \forall \omega\}), \quad \forall k \quad (6.1g)$$

$$(q_{ijk}^{\text{UP}}, q_{ijk}^{\text{DW}}, q_{i\omega k}^{(+)}, q_{i\omega k}^{(-)}) \in \Gamma, \quad \forall i, \forall j, \forall \omega, \forall k \quad (6.1h)$$

$$\text{where } \Xi = \{\hat{\rho}_k^{\text{DA}}, \hat{\rho}_k^{\text{Act}}, \hat{\rho}_k^{\text{Pas}}, \hat{c}_k, q_{ik}^{\text{DA}}, q_{ijk}^{\text{UP}}, q_{ijk}^{\text{DW}}, q_{i\omega k}^{(+)}, q_{i\omega k}^{(-)}, d_{ij\omega k}, p_{ij\omega k}^{(\uparrow)}, p_{ij\omega k}^{(\downarrow)}\}.$$

The objective function (6.1a) maximizes the VPP expected revenues considering both the day-ahead and balancing markets. Constraint (6.1b) imposes the energy balance between the VPP production (or consumption) and the energy exchanged with the electricity market. The sets of constraints (6.1c)-(6.1h) are expressed and discussed in detail in the following.

6.3.1 Linear formulation of Π^{DA}

The set of constraints (6.1c), denoted by Π^{DA} , computes the expected profit from the day-ahead market $\hat{\rho}_k^{\text{DA}}$ and includes constraints on the day-ahead offer curve. It is written as

$$\hat{\rho}_k^{\text{DA}} = \sum_i \pi_i^{\text{DA}} \lambda_{ik}^{\text{DA}} q_{ik}^{\text{DA}}, \quad \forall k \quad (6.2a)$$

$$q_{ik}^{\text{DA}} \geq q_{i'k}^{\text{DA}} \quad \text{if } \lambda_{ik}^{\text{DA}} \geq \lambda_{i'k}^{\text{DA}}, \quad \forall i, \forall i', \forall k \quad (6.2b)$$

$$q_{ik}^{\text{DA}} = q_{i'k}^{\text{DA}} \quad \text{if } \lambda_{ik}^{\text{DA}} = \lambda_{i'k}^{\text{DA}}, \quad \forall i, \forall i', \forall k \quad (6.2c)$$

$$-\bar{P}^{(\uparrow)} \leq q_{ik}^{\text{DA}} \leq \bar{D} + \bar{E} + \bar{P}^{(\downarrow)}, \quad \forall i, \forall i', \forall k. \quad (6.2d)$$

Constraint (6.2a) yields the expected income of the VPP associated with the day-ahead market offer curves. Constraints (6.2b) and (6.2c) force the offer curves to be, respectively, non-decreasing and non-anticipative. Finally, constraints (6.2d) restrict the day-ahead offer quantities to the VPP capacity.

6.3.2 Linear formulation of Π^{Act}

The constraints (6.1d), denoted by Π^{Act} , yield the expected profit $\hat{\rho}_k^{\text{Act}}$ from an active participation in the balancing market, and comprises constraints on the offer curves in the balancing market. They are formulated as

$$\hat{\rho}_k^{\text{Act}} = \sum_{ij} \pi_i^{\text{DA}} \pi_j^{\text{BA}} \lambda_{ijk}^{\text{BA}} (q_{ijk}^{\text{UP}} - q_{ijk}^{\text{DW}}), \quad \forall k \quad (6.3a)$$

$$q_{ijk}^{\text{UP}} \geq q_{ij'k}^{\text{UP}} \quad \text{if} \quad \lambda_{ijk}^{\text{BA}} \geq \lambda_{ij'k}^{\text{BA}}, \quad \forall i, \forall j, \forall j', \forall k \quad (6.3b)$$

$$q_{ijk}^{\text{UP}} = q_{ij'k}^{\text{UP}} \quad \text{if} \quad \lambda_{ijk}^{\text{BA}} = \lambda_{ij'k}^{\text{BA}}, \quad \forall i, \forall j, \forall j', \forall k \quad (6.3c)$$

$$q_{ijk}^{\text{DW}} \leq q_{ij'k}^{\text{DW}} \quad \text{if} \quad \lambda_{ijk}^{\text{BA}} \geq \lambda_{ij'k}^{\text{BA}}, \quad \forall i, \forall j, \forall j', \forall k \quad (6.3d)$$

$$q_{ijk}^{\text{DW}} = q_{ij'k}^{\text{DW}} \quad \text{if} \quad \lambda_{ijk}^{\text{BA}} = \lambda_{ij'k}^{\text{BA}}, \quad \forall i, \forall j, \forall j', \forall k \quad (6.3e)$$

$$q_{ijk}^{\text{UP}} = 0 \quad \text{if} \quad \lambda_{ijk}^{\text{BA}} \leq \lambda_{ik}^{\text{DA}}, \quad \forall i, \forall j, \forall k \quad (6.3f)$$

$$q_{ijk}^{\text{DW}} = 0 \quad \text{if} \quad \lambda_{ijk}^{\text{BA}} \geq \lambda_{ik}^{\text{DA}}, \quad \forall i, \forall j, \forall k \quad (6.3g)$$

$$q_{ijk}^{\text{UP}}, q_{ijk}^{\text{DW}} \geq 0, \quad \forall i, \forall j, \forall k. \quad (6.3h)$$

Constraint (6.3a) evaluates the expected revenue from the submission of offer curves in the balancing market as an active participant. Constraints (6.3b) and (6.3c) ensure, respectively non-decreasing shape and non-anticipativity of the up-regulation offer curve. Similarly, constraints (6.3d) and (6.3e) do the same for the down-regulation offer curve. Constraints (6.3f) and (6.3g) restrict the possibility to offer regulating energy to the scenarios in which it is required. Finally, constraint (6.3h) enforces q_{ijk}^{UP} and q_{ijk}^{DW} to be non-negative variables.

6.3.3 Linear formulation of Π^{Pas}

The constraints (6.1e), denoted by Π^{Pas} , give the expected profit $\hat{\rho}_k^{\text{Pas}}$ associated with a passive participation in the balancing market. They are formulated as

$$\hat{\rho}_k^{\text{Pas}} = \sum_{ij\omega} \pi_i^{\text{DA}} \pi_j^{\text{BA}} \pi_\omega^E \left(\lambda_{ijk}^{(+)} q_{i\omega k}^{(+)} - \lambda_{ijk}^{(-)} q_{i\omega k}^{(-)} \right), \quad \forall k \quad (6.4a)$$

$$q_{i\omega k}^{(+)}, q_{i\omega k}^{(-)} \geq 0, \quad \forall i, \forall \omega, \forall k \quad (6.4b)$$

where $\lambda_{ijk}^{(+)} = \min(\lambda_{ijk}^{\text{BA}}, \lambda_{ik}^{\text{DA}})$ and $\lambda_{ijk}^{(-)} = \max(\lambda_{ijk}^{\text{BA}}, \lambda_{ik}^{\text{DA}})$, in accordance with the dual-price imbalance settlement scheme (Morales et al., 2013; Morales et al., 2010). Constraint (6.4a) computes the expected income from a passive participation in the balancing stage and accounts for the imbalances created. Constraint (6.4b) ensures that $q_{i\omega k}^{(+)}$ and $q_{i\omega k}^{(-)}$ are non-negative variables.

6.3.4 MILP formulation of Ω

The constraints (6.1f), denoted by Ω , establish the feasible operating region of the VPP and are formulated as

$$\ell_{ij\omega k} = \ell_{ij\omega(k-1)} + \eta p_{ij\omega k}^{(\uparrow)} - p_{ij\omega k}^{(\downarrow)}, \quad \forall i, \forall j, \forall \omega, \forall k \quad (6.5a)$$

$$\underline{L} \leq \ell_{ij\omega k} \leq \bar{L}, \quad \forall i, \forall j, \forall \omega, \forall k \quad (6.5b)$$

$$0 \leq p_{ij\omega k}^{(\uparrow)} \leq \bar{P}^{(\uparrow)}, \quad \forall i, \forall j, \forall \omega, \forall k \quad (6.5c)$$

$$0 \leq p_{ij\omega k}^{(\downarrow)} \leq \bar{P}^{(\downarrow)}, \quad \forall i, \forall j, \forall \omega, \forall k \quad (6.5d)$$

$$u_{ij\omega k} \underline{D} \leq d_{ij\omega k} \leq u_{ij\omega k} \bar{D}, \quad \forall i, \forall j, \forall \omega, \forall k \quad (6.5e)$$

$$d_{ij\omega k} - d_{ij\omega(k-1)} \leq R^{\text{UP}}, \quad \forall i, \forall j, \forall \omega, \forall k \quad (6.5f)$$

$$d_{ij\omega(k-1)} - d_{ij\omega k} \leq R^{\text{DW}}, \quad \forall i, \forall j, \forall \omega, \forall k \quad (6.5g)$$

$$u_{ij\omega k} \in \{0, 1\}, \quad \forall i, \forall j, \forall \omega, \forall k. \quad (6.5h)$$

Constraint (6.5a) represents the energy balance of the storage unit. Constraint (6.5b) forces the level of energy in the storage $\ell_{ij\omega k}$ to lie between its minimum and maximum limits. Similarly, constraints (6.5c) and (6.5d) do the same for $p_{ij\omega k}^{(\uparrow)}$ and $p_{ij\omega k}^{(\downarrow)}$, respectively. Constraint (6.5e) imposes the thermal unit to operate within its minimum output and its capacity when on-line (i.e., $u_{ij\omega k} = 1$) and not to produce when off-line (i.e., $u_{ij\omega k} = 0$). Constraints (6.5f) and (6.5g) enforce, respectively, the upward and downward ramping limitations of the thermal unit. Finally, constraint (6.5h) sets the commitment status $u_{ij\omega k}$ of the thermal unit as a binary variable. Constraint (6.5a) requires the initial level of the storage as input, and constraints (6.5f)-(6.5g) need the initial production level of the thermal unit.

Richer models for the feasible operating region of a dispatchable unit exist in the literature. However, to keep the focus on the *Active/Passive* offering strategy and its formulation intuitive, we chose to capture the main operating constraints of the unit limiting the level of details of the model.

6.3.5 MILP formulation of $h(\cdot)$

Constraint (6.1g) computes the expected production cost \hat{c}_k associated with the thermal unit. A possible mixed-integer linear programming formulation of this cost function is (6.6)

$$\hat{c}_k = \sum_{ij\omega} \pi_i^{\text{DA}} \pi_j^{\text{BA}} \pi_\omega^{\text{E}} (C_0 u_{ij\omega k} + C d_{ij\omega k}), \quad \forall k, \quad (6.6)$$

where C_0 is the fixed cost incurred when the unit is on, and C is the marginal production cost of the unit.

6.3.6 MILP formulation of Γ

The set of constraints (6.1h), denoted by Γ , enforces complementarity between the active and passive participation in the balancing market. Γ can be formulated as

$$q_{ijk}^{\text{UP}} + q_{ijk}^{\text{DW}} \leq \epsilon_{ik} \bar{M}, \quad \forall i, \forall j, \forall k \quad (6.7a)$$

$$q_{i\omega k}^{(+)} + q_{i\omega k}^{(-)} \leq (1 - \epsilon_{ik}) \bar{M}, \quad \forall i, \forall \omega, \forall k \quad (6.7b)$$

$$\epsilon_{ik} \in \{0, 1\}, \quad \forall i, \forall k. \quad (6.7c)$$

Constraints (6.7a) and (6.7b) force the VPP to be in only one state between active (i.e., $\epsilon_{ik} = 1$) and passive (i.e., $\epsilon_{ik} = 0$) in the balancing market, through the so-called *big-M* approach. A natural and sensible choice for the parameter \bar{M} can be

$$\bar{M} := \bar{E} + \bar{D} + \bar{P}^{(\uparrow)} + \bar{P}^{(\downarrow)}.$$

Finally, constraint (6.7c) forces the variables ϵ_{ik} to be binary.

6.4 Case study

Next we present a case study to test the offering strategy of Section 6.3. The aim is to analyze whether the proposed *Active/Passive* balancing participation setup may drive the VPP to offer its flexibility when available.

The scenarios provided as input to the offering model are generated as described in Section 6.2.3. First, we generate 300 scenarios for the day-ahead market price λ_{ik}^{DA} and keep the ten most representative ones. Then, for each day-ahead scenario i , we randomly sample

300 scenarios of the balancing market price λ_{ik}^{BA} and select the six most significant. Finally, we generate 300 trajectories of the renewable energy production $E_{\omega k}$ (wind or solar power) and keep the five most representative. This results in a scenario tree with 300 branches ($10 \times 6 \times 5$). The parameters of the thermal unit are shown in Table 6.1. Similarly, the characteristics of the storage unit are presented in Table 6.2. The programs are modeled in PYTHON environment and solved with GUROBI.

Table 6.1: Parameters of the thermal unit.

\underline{D} (MW)	\overline{D} (MW)	R^{UP} (MW/h)	R^{DW} (MW/h)	C_0 (EUR)	C (EUR/MWh)
0	70	30	30	0	45

Table 6.2: Parameters of the electric storage unit.

\underline{L} (MWh)	\overline{L} (MWh)	$\overline{P}^{(\uparrow)}$ (MW)	$\overline{P}^{(\downarrow)}$ (MW)	η
0	80	30	30	0.81

The *Active/Passive* offering strategy is compared against two benchmarks: a *Passive* and an *Active* offering strategy. Based on the *Passive* approach, the VPP is assumed to be always a passive participant in the balancing market. Differently, under the *Active* strategy, the VPP is an active actor in the balancing stage for the entire trading horizon. These two alternative models can be derived from the optimization model (6.1) by fixing the binary variables $\epsilon_{ik} = 0, \forall i, \forall k$ for the *Passive* strategy, or $\epsilon_{ik} = 1, \forall i, \forall k$ for the *Active* one.

6.4.1 VPP with wind farm

Figure 6.3 shows the ten selected trajectories for the day-ahead market price λ_{ik}^{DA} , the six chosen balancing price scenarios $\lambda_{ijk}^{\text{BA}}$ for a sample day-ahead trajectory i , and the five selected trajectories for the wind power production $E_{\omega k}$ (in p.u.).

The wind farm capacity \overline{E} is initially set to 50 MW. We solve the *Active/Passive* offering model (6.1) using as input the scenarios shown in Figure 6.3. The complementarity between the active/passive choice is enforced through the binary variables ϵ_{ik} . If $\epsilon_{ik} = 1$, the VPP is predicting to act as an active participant during the interval k of the balancing stage, provided that the day-ahead price scenario i realizes. For the same scenario i and interval k , if $\epsilon_{ik} = 0$, then the VPP is expecting to behave passively. Being ϵ_{ik}^* the optimal

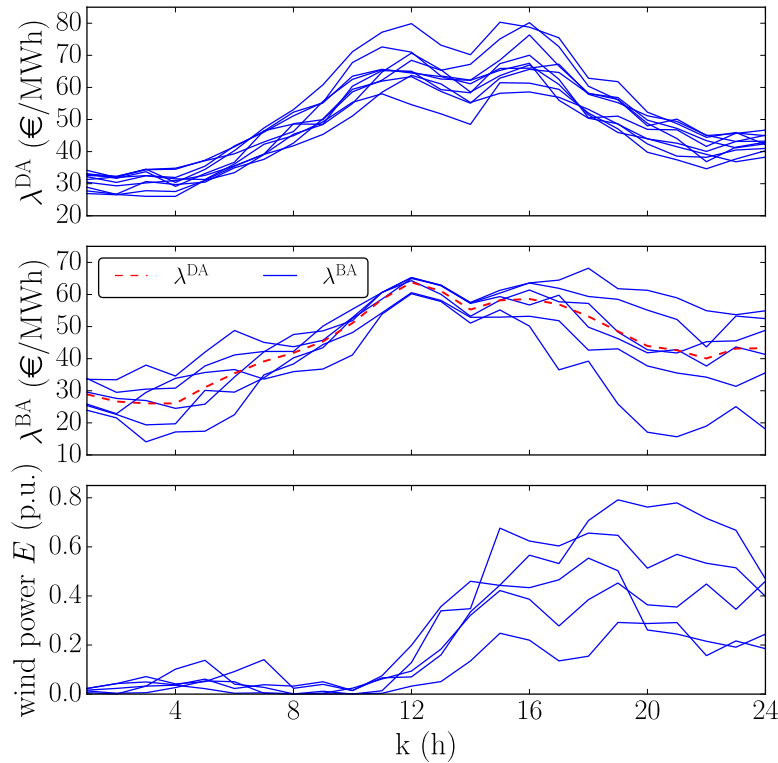


Figure 6.3: Input day-ahead market price (top), balancing price (middle), and wind power production scenarios (bottom) to the offering model.

solution, then the probability that the VPP will be active is computed as $\sum_i \pi_i^{\text{DA}} \epsilon_{ik}^*$, and the probability that it will be passive as $\sum_i \pi_i^{\text{DA}} (1 - \epsilon_{ik}^*)$. These probabilities are illustrated in Figure 6.4.

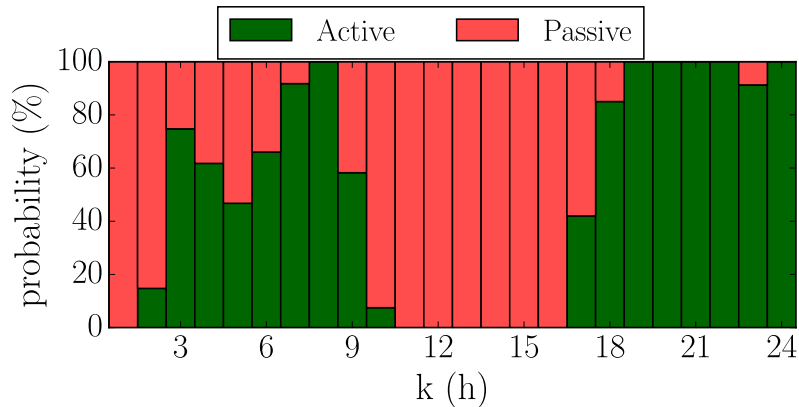


Figure 6.4: Probability of being active vs. passive ($\bar{E} = 50$ MW).

From midnight to 10 a.m., the VPP will decide to be either active or passive depending on the day-ahead price realization i . On one hand, the uncertainty in the wind power production is limited in this trading interval, which would benefit an active approach as the flexibility of the controllable units could be used to offer regulating energy. On the

other hand, even if uncertain, the spread between the balancing price scenarios and the day-ahead prices is also limited, leading consequently to low additional profits resulting from an active participation. Then, from 10 a.m. to 4 p.m., the VPP decides to be a passive participant for each realization i of the day-ahead price. Indeed, the uncertainty in the balancing market prices is limited while the amount of wind power production is very uncertain. Finally, from 6 p.m. to midnight, the VPP is almost sure to sell regulating energy in the balancing market. This translates into internally handling the wind power production fluctuations, which is highly uncertain in these time intervals as it can vary from 20% to almost 80% of the wind farm capacity. However, the balancing market price scenarios are going to be far-off the day-ahead market price with high probability; accordingly, passive deviations from the day-ahead schedule may result in heavy penalties, while selling regulating energy can be very profitable.

As an example, Figure 6.5 illustrates the day-ahead market offer curves for the interval $k = 15$ obtained when using the *Active/Passive* strategy (left), *Active* strategy (middle), and *Passive* strategy (right). The VPP is willing to produce 91.1 MWh (*Active/Passive* and *Passive*) or 92.1 MWh (*Active*) provided that $\lambda_{15}^{\text{DA}} \geq 58.2$. Then, if $\lambda_{15}^{\text{DA}} \geq 65.3$ the *Active/Passive* and the *Active* approach contract additional 6.8 MWh and 13.3 MWh, respectively. Differently, the *Passive* approach increases its production to 106.5 MWh if $\lambda_{15}^{\text{DA}} \geq 67.4$. Finally, if $\lambda_{15}^{\text{DA}} \geq 75.1$ the *Active/Passive* and the *Passive* approach further increase the production level to 121.1 MWh. From Figure 6.5, we note that the *Active* approach appears to be less “reactive” to the day-ahead market price compared to the other strategies, and no additional quantity is scheduled for high values of the day-ahead market price. Indeed, the position of the VPP after the day-ahead market affects its capability to internally compensate for the wind power fluctuations. Therefore, the VPP position is more constrained and driven by feasibility limitations compared to the other two strategies.

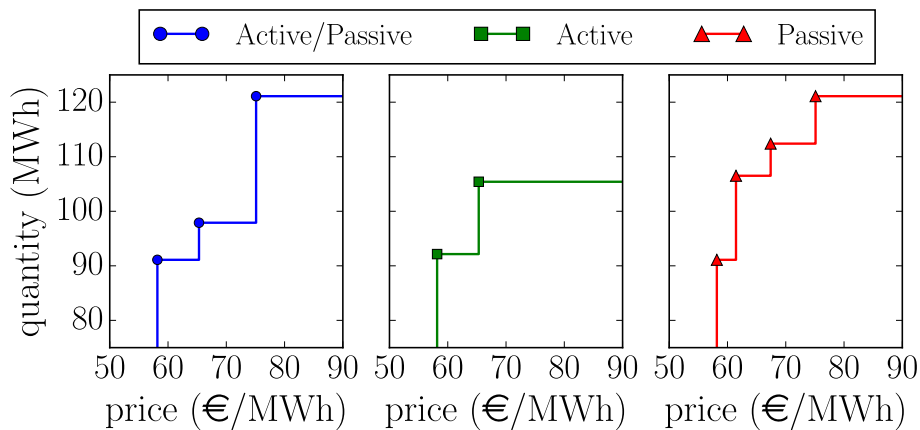


Figure 6.5: Day-ahead market offer curves at $k = 15$ ($\bar{E} = 50$ MW).

Finally, in Table 6.3 we compare the expected profit of the three offering models for different values of the wind farm capacity \bar{E} , ranging from 10 to 90 MW. When $\bar{E} = 10$ MW, the expected profit increment under the *Active/Passive* approach is 0.6% and 8.1% compared to the *Active* and *Passive* strategies, respectively. In effect, if the capacity of the stochastic unit is small, then the VPP can internally handle most of the wind power deviations and offer its regulating energy into the balancing market. Accordingly, the increase in profit compared to the *Active* strategy is limited whereas the *Passive* strategy is strongly outperformed. This trend progressively changes as the wind farm capacity \bar{E} increases. As \bar{E} grows, the VPP based on an *Active/Passive* participation is more likely to settle deviations in the balancing stage and has less flexibility to offer in the balancing market. When $\bar{E} = 90$ MW, the increase in profit is 4.6% compared to the *Active* strategy and 2.1% compared to the *Passive* one.

Table 6.3: Expected profit for the three offering strategies for different values of the wind farm capacity \bar{E} .

\bar{E} (MW)	Expected profit (10^3 EUR)		
	<i>Active/Passive</i>	<i>Active</i>	<i>Passive</i>
10	18.16	18.06	16.81
30	22.89	22.48	21.69
50	27.59	26.85	26.57
70	32.35	31.16	31.45
90	37.07	35.44	36.32

6.4.2 VPP with PV solar

Figure 6.6 shows the ten selected trajectories for the day-ahead market price λ_{ik}^{DA} , the six chosen balancing price scenarios $\lambda_{ijk}^{\text{BA}}$ for a sample day-ahead trajectory i , and the five selected trajectories for the solar power production $E_{\omega k}$ (in p.u.).

The PV solar unit is initially considered with capacity $\bar{E} = 50$ MW. We run the *Active/Passive* offering model (6.1), and in Figure 6.7 show the probabilities of the VPP being active and passive in the trading horizon, computed as in Section 6.4.1. From midnight to 6 a.m. and from 8 p.m. to midnight, the VPP decides to be active in the balancing market for each day-ahead scenario i . Indeed, these time intervals are before and after the sunset, and the VPP is certain that the output of its PV solar unit will be zero. Differently, from 10 a.m. to 5 p.m., the VPP is almost sure that it will passively deviate from its contracted schedule to compensate for the fluctuations from the PV solar unit. In this time horizon, the PV power production is very uncertain (e.g., at 2 p.m. it can

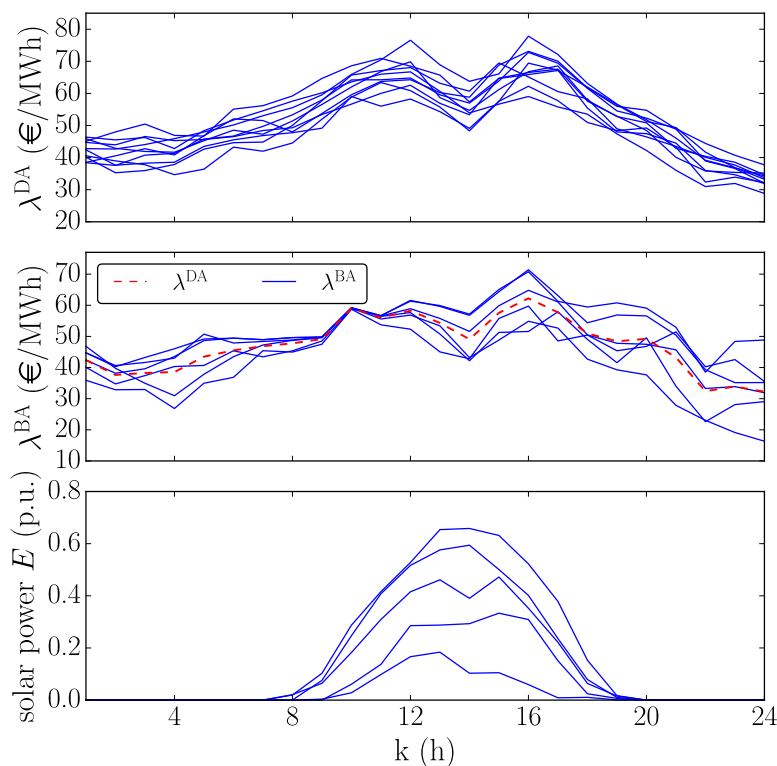


Figure 6.6: Input day-ahead market price (top), balancing price (middle), and PV solar power production scenarios (bottom) to the offering model.

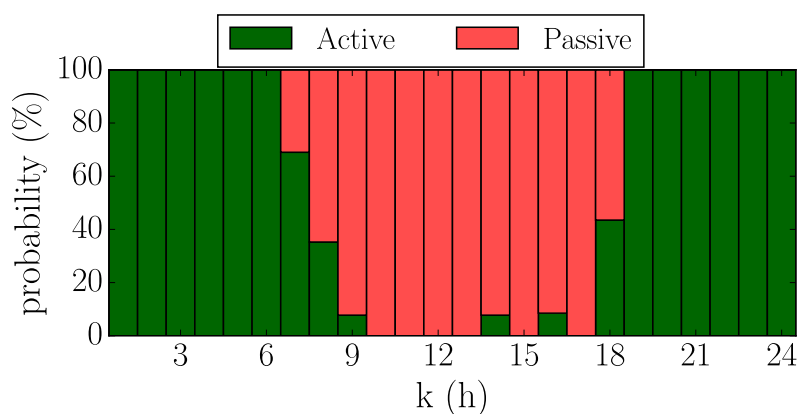


Figure 6.7: Probability of being active vs. passive ($\bar{E} = 50$ MW).

vary from 20% to 70% of the unit capacity), and it is more convenient to settle deviations in the balancing market. Finally, from 6 a.m. to 9 a.m., the VPP will decide whether the active or the passive participation is more profitable depending on the day-ahead price realization i , and the associated amount of energy contracted. In this time interval, the uncertainty of the PV solar production is in fact limited, which would suggest that an active participation may be preferable. However, the possibility of gaining extra profits from the balancing market is low as the balancing market price scenarios are very close

to the day-ahead price. Differently, from 6 p.m. to 8 p.m., an active participation is more attractive since the balancing market price scenarios give the opportunity to gain extra profits.

In Figure 6.8, we plot the day-ahead market offer curves from the three VPP strategies *Active/Passive* (left), *Active* (middle), and *Passive* (right) at $k = 5$. In this interval, the offer curves derived for the *Active/Passive* and *Active* strategy are equivalent, which is consistent with the results shown in Figure 6.7 where the VPP decides to be always active from midnight to 6 a.m.. Under these two models, the VPP is willing to consume (buy) 14.4 MWh provided that $\lambda_5^{\text{DA}} \leq 36.4$. This energy is used to charge the electric storage unit. Then, the VPP schedules to produce 40.0 MWh when $\lambda_5^{\text{DA}} \geq 46.7$ and it does not produce if $36.4 < \lambda_5^{\text{DA}} < 46.7$. Differently, the *Passive* strategy suggests to schedule 40.0 MWh when $\lambda_5^{\text{DA}} \geq 43.5$ and to increase the production by extra 30.0 MWh if $\lambda_5^{\text{DA}} \geq 48.0$. Thus, the *Passive* strategy schedules more energy in the day-ahead as the VPP is less driven by feasibility constraints.

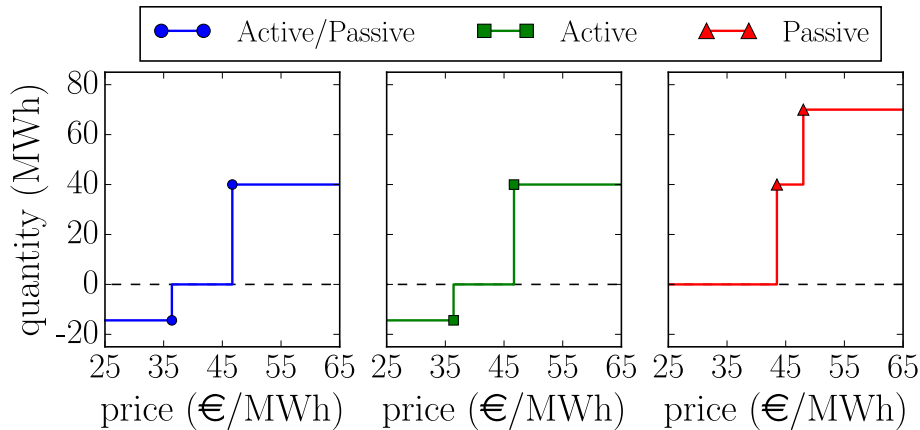


Figure 6.8: Day-ahead market offer curves at $k = 5$. ($\bar{E} = 50$ MW).

Lastly, we compare the expected profit obtained with the three offering models for increasing capacity of the PV unit, \bar{E} , from 10 to 90 MW. We show the results in Table 6.4. When $\bar{E} = 10$ MW, the expected profit of the *Active/Passive* and the *Active* approach are similar. Indeed, as the capacity of the PV unit is small, the VPP can easily handle the PV solar fluctuations internally and offer the remaining flexibility in the balancing market. The expected profit increase under the *Active/Passive* approach is 1.7% and 6.0% compared to the *Active* and the *Passive* strategies, respectively. As the PV unit capacity increases, the *Passive* strategy becomes more competitive as it can benefit from a more flexible VPP operation, and can consequently contract more profitable positions in the day-ahead market. Instead, as \bar{E} grows, the *Active* approach becomes increasingly constrained in its operation. First, it has less flexibility to offer in the balancing stage as

it needs to allocate it to balance the PV unit fluctuations. Second, the day-ahead position is more constrained by ensuring a feasible real-time operation, thus it is less driven by the market prices.

Table 6.4: Expected profit for the three offering strategies for different values of the PV solar unit capacity \bar{E} .

\bar{E} (MW)	Expected profit (10^3 EUR)		
	<i>Active/Passive</i>	<i>Active</i>	<i>Passive</i>
10	18.59	18.27	17.54
30	22.19	21.28	21.15
50	25.79	24.24	24.76
70	29.39	27.17	28.36
90	32.98	30.08	31.97

6.5 Conclusions

In this paper we proposed an innovative participation model for the balancing market, denoted by *Active/Passive*, aimed to increase the flexibility of market participants as well as the amount of regulating energy available to the TSO in the real-time. Specifically, we suggest to allow market agents such as VPPs to actively offer regulating energy in time intervals where they can ensure to internally handle the eventual fluctuations from the stochastic energy sources, while passively deviating from their day-ahead schedule in other intervals. We enforced these two participation modes (active and passive) to be complementary, and agents submitting regulating energy offers for a specific trading interval are prevented from creating an imbalance in the same interval.

To analyze this novel participation model, we took the perspective of a VPP that includes both controllable and stochastic generation units, and that trades in a two-settlement electricity market. The *Active/Passive* offering strategy arises as a three-stage decision making problem. We formulated this problem as a MILP in which binary variables are introduced to model the feasible operating region of conventional production units and the complementarity between active and passive participation in the balancing stage. Compared to an *Active* and a *Passive* strategy, computational experiments showed that an *Active/Passive* approach can result in a significantly higher VPP expected profit (up to 8% higher). The analysis reveals that the active participation is more attractive for the VPP in the hourly intervals with limited production uncertainty from the stochastic sources and profitable balancing market price scenarios, and the passive one when highly

uncertain renewable energy production is combined with narrow balancing market price scenarios (i.e., close to the day-ahead price).

The proposed framework is potentially relevant from a system operator perspective which would benefit from more flexible regulating energy to schedule in real-time, and can be also seen as a lever to facilitate the integration of renewable power in the system through the aggregation into VPPs.

The focus of this work was to provide useful insights on the proposed *Active/Passive* participation model. Therefore, we presented a case study with 300 scenarios and a simplified operating region of the dispatchable generators to keep the model intuitive and solvable in about 30 minutes. Further research may be in the direction of developing more efficient algorithms capable of solving the model for a larger number of scenarios or including more operating details of the units.

Nomenclature

Indices and Sets

$i, i' \in I$	Indices of day-ahead market price scenarios
$j, j' \in J$	Indices of balancing market price scenarios
$\omega \in W$	Index of renewable energy generation scenarios
$k \in K$	Index of time intervals
Π^{DA}	Feasible region of the day-ahead market offers
Π^{Act}	Feasible region of the active participation
Π^{Pas}	Feasible region of the passive participation
Ω	Feasible region of the VPP's operation
Γ	Feasible region of <i>Active/Passive</i> participation

Parameters

λ_{ik}^{DA}	Day-ahead market price (EUR/MWh)
$\lambda_{ijk}^{\text{BA}}$	Balancing market price (EUR/MWh)
$E_{\omega k}$	Wind (or solar) power generation (MWh)
\bar{E}	Capacity of the wind (or solar) unit (MW)
\bar{D}	Capacity of the thermal unit (MW)
\underline{D}	Minimum power limit of the thermal unit (MW)
$R^{\text{UP}}, R^{\text{DW}}$	Thermal unit ramp-up and -down limits (MW/h)
$\bar{P}^{(\uparrow)}, \bar{P}^{(\downarrow)}$	Charging/discharging power limits (MW)

\underline{L}, \bar{L}	Minimum/maximum level of the storage (MWh)
η	Round-trip efficiency of the energy storage
C	Marginal cost of the thermal unit (EUR/MWh)
C_0	Fixed cost of the thermal unit (EUR)
π_i^{DA}	Probability of day-ahead price scenario i
π_{ij}^{BA}	Probability of balancing price scenario j , provided that day-ahead price scenario i realizes
π_ω^{E}	Probability of renewable energy production scenario ω

Variables

q_{ik}^{DA}	Quantity offer at day-ahead market (MWh)
$q_{ijk}^{\text{UP}}, q_{ijk}^{\text{DW}}$	Up/down regulation quantity offer (MWh)
$q_{i\omega k}^{(+)}, q_{i\omega k}^{(-)}$	Positive/negative real-time deviation (MWh)
$d_{ij\omega k}$	Thermal unit energy production (MWh)
$p_{ij\omega k}^{(\uparrow)}, p_{ij\omega k}^{(\downarrow)}$	Charging/discharging quantities (MWh)
$\ell_{ij\omega k}$	Energy storage level (MWh)
$\hat{\rho}_k^{\text{DA}}$	Expected day-ahead market profit (EUR)
$\hat{\rho}_k^{\text{Act}}$	Expected profit of the active participation (EUR)
$\hat{\rho}_k^{\text{Pas}}$	Expected profit of the passive participation (EUR)
\hat{c}_k	Expected operational cost (EUR)
$u_{ij\omega k}$	Commitment (binary) status of the thermal unit
ϵ_{ik}	Auxiliary binary variables to enforce complementarity of the <i>Active/Passive</i> participation

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Modelling of Electricity Savings in the Danish Households Sector: From the Energy System to the End-User

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Abstract: In this paper we examine the value of investing in energy-efficient household appliances from both an energy system and end-user perspectives. We consider a set of appliance categories constituting the majority of the electricity consumption in the private household sector, and focus on the stock of products which need to be replaced. First, we look at the energy system and investigate whether investing in improved energy efficiency can compete with the cost of electricity supply from existing or new power plants. To assess the analysis, Balmorel, a linear optimization model for the heat and power sectors, has been extended in order to endogenously determine the best possible investments in more efficient home appliances. Second, we propose a method to relate the optimal energy system solution to the end-user choices by incorporating consumer behaviour and electricity price addition due to taxes. The model is non-exclusively tested on the Danish energy system

under different scenarios. Computational experiments show that several energy efficiency measures in the household sector should be regarded as valuable investments (e.g. an efficient lighting system) while others would require some form of support to become profitable. The analysis quantifies energy and economic savings from the consumer side and reveals the impacts on the Danish power system and surrounding countries. Compared to a business-as-usual energy scenario, the end-user attains net economic savings in the range of 30–40 EUR per year, and the system can benefit of an annual electricity demand reduction of 140–150 GWh. The paper enriches the existing literature about energy efficiency modelling in households, contributing with novel models, methods, and findings related to the Danish case.

Keywords: Energy efficiency · Household appliances · Consumer investments · Energy system modelling · Emissions reduction

7.1 Introduction

In compliance with the recent international effort towards the climate change mitigation (European Commission, 2010), Denmark has set its goals for the year 2020 and is working to fulfil the targets concerning renewable energy (RE) integration in the system and energy efficiency (EE) improvements. Compared to the 1990 levels, Denmark has reduced its greenhouse gas emissions by more than 30% and, according to the current policies and trends, the Danish Energy Agency forecasts that the reduction will reach almost 40% by 2020 (Breum, 2015), thus exceeding the legally binding EU commitment of 34%. Denmark can vaunt one of the highest contributions of renewables in any energy system worldwide (excluding hydro-power), with a 56% contribution in 2014. In particular, in 2015, more than 40% of the Danish electricity demand was satisfied by wind energy, and this figure is expected to increase up to 50% by 2020 (Breum, 2015). Besides the effort in integrating renewables in the energy system, the Danish government has set a number of targets for the further development of EE measures. According to the National 2020 Energy Efficiency Targets, Denmark is aiming to reduce the primary and final energy consumption by 12.6 and 7.2%, respectively, compared to 2006 (Danish Energy Agency, 2014).

Both RE and EE measures have been identified by the European Commission as the most suitable options to evolve the national energy systems towards greener configurations (European Commission, 2012). Nevertheless, if not properly enforced, the simultaneous implementation of RE and EE can lead to suboptimal investment planning and missed

cost-saving opportunities (Baldini and Klinge Jacobsen, 2016). The challenge is to identify the optimal trade-off between EE levels and power system configurations while exploring future scenarios, i.e. understanding where to invest in order to obtain the most cost-effective energy system given a target on emissions reduction. Several studies, for instance, have shown that enhancing EE is likely the most cost-effective way to reduce carbon emissions in the medium term (Enkvist et al., 2007; López-Peña et al., 2012).

The literature has then considered the modelling of EE in households along two main lines: the heat and electricity sectors. Available literature presents many examples from the Danish heating context, while EE literature from the electricity sector is lacking, whereby we broaden our perspective.

On the heat sector side, Zvingilaite (2013) models heat savings in the Danish building sector using a heat and power optimization model, showing that the attainable level of heat savings can reach up to 11% of the projected heat demand in 2025. At the time of publication, the study represented the front-runner implementation of heat savings as endogenous investment variables in an energy system model, thus providing a first estimation of the cost-effective heat savings level from a socio-economic perspective. Several studies target environmental goals as CO₂ emission reduction, stressing the need to identify the trade-off between heat savings and heat supply. Connolly et al. (2014) examine the joint role of district heating and heat savings to decarbonise the EU energy system, and conclude that coupling the two measures can help reducing primary energy supply and CO₂ emissions at the lowest costs compared to other alternatives. Zvingilaite and Klinge Jacobsen (2015) investigate the trade-off between heat savings and heat generation technologies in the Danish energy system, focusing on the residential investment behaviour and including health costs. The study reveals that savings up to 24% of the heat demand can be achieved with an optimal configuration of investments in heat savings and heat generation technologies. Hansen et al. (2016) estimate the optimal heat savings investment levels within various European countries. This level is identified in investments aimed to reduce the projected heat demand of about 30–40%, while supplying the remaining demand with sustainable heat technologies.

On the electricity side, the literature suggests that disaggregating the household electricity demand into different appliances is the starting point for modelling EE measures and the attitude of consumers towards them (Lefebvre and Desbiens, 2002; Evora et al., 2011; Batih and Sorapipatana, 2016). Rodríguez Fernández et al. (2016) propose the use of machine learning techniques to identify individual electrical devices in households based on power consumption, so that specific appliances can be targeted for efficiency improvement. Numerous authors then focused on the trade-off between electric energy savings in households and power supply with interesting examples, close to the direction of

our work, in an Asian context. Parikh and Parikh (2016) examine the potential energy and emission savings from choosing energy-efficient home appliances in India. Based on the 5-star-rating EE promotion programme, the authors modelled the attitude of consumers (poor and rich) in adopting more efficient appliances. The results show that, given the awareness of consumers concerning the various options of efficient appliances, a demand and emission reduction from households exceeding 30% can be reached in 2030. Batih and Sorapipatana (2016) analyse the electricity consumption of urban households and its saving potential in Indonesia. Similar to the Indian's case, the results illustrate how implementing specific EE improvements can lead to a reduction of 21% of both power demand and CO₂ emissions from households by 2030. Xie et al. (2016) prove that energy management strategies in the Chinese household sector should include investments in energy-efficient home appliances. The policy recommendation is thus in terms of subsidies driving customers to purchase a higher share of energy-labelled appliances. Mizobuchi and Takeuchi (2016) examine the influence of an increase in purchasing energy-efficient home appliances on the power system in Japan. The conclusions are in line with previous studies, showing that households with new energy-efficient appliances can save a large amount of electricity, but also that the rebound effect may cancel part of the savings out due to a more intense use of the appliance. Finally, a few studies consider the contribution of appliances to the household electricity use with a global scope, illustrating the huge potential of energy efficiency improvements in the global residential sector (Wada et al., 2012; Cabeza et al., 2014).

As indicated by the consistent amount of literature, in the residential sector lies a large potential for EE improvements. In Denmark, electricity consumption from private households exceeds 20% of the total load (Klinge Jacobsen and Juul, 2015). This figure is also expected to increase in the next years due to the upcoming electrification of the household facilities, and should then be balanced with improvements in energy efficiency measures (Bartiaux and Gram-Hanssen, 2005). The electricity consumption in the household sector is mainly related to the different home appliances. Therefore, if electricity savings could be targeted to the different appliance categories, then lower consumption profiles associated to the households could lead to savings for the system in terms of necessary power plants, capacity investments and emissions. Furthermore, the electricity savings may have different effects on the power system depending on the hourly consumption profile of the appliance category whose demand is reduced.

Using a bottom-up approach (Swan and Ugursal, 2009), the analysis proposed in this paper will make use of hourly consumption profiles of home appliances determined in previous studies (Klinge Jacobsen and Juul, 2015) to investigate the effect of EE improvements in the Danish energy system. In particular, the aim of this paper is threefold:

1. to evaluate from a system perspective whether it is worth to invest in more energy-efficient appliances rather than install new power plants, and observe the effects on the energy mix;
2. to assess from an end-user perspective which energy-efficient appliance should be regarded as a profitable investment, taking into account the behavioural dimension of the consumer;
3. to compare the investment choices of the model according to the system and consumer perspectives.

The paper enriches the existing literature about EE modelling in households, contributing with new models, methods, and findings related to the Danish case.

7.2 Methodology

7.2.1 Overview of Balmorel

Balmorel is a linear programming-based optimization model for the energy sector, originally developed in 2001 to analyse the Baltic system (Balmorel, 2017). The model finds economically efficient dispatches and optimal capacity investments for the energy system. The emphasis is on the electricity and combined heat and power (CHP) sectors, and the major technologies for electricity, heat generation and storage are included in the model.

The model consists of a set of neighbouring countries that participate in various electricity markets. Each country is then split into one or more regions, depending on the market features, where electricity can be traded with constraints. Denmark, for instance, is modelled using two electricity zones, Denmark East and Denmark West (in the following DK-E and DK-W), according to the NordPool system. The electricity transmission between adjacent zones is limited by a given transmission capacity. Moreover, to model the CHP sector, each electricity region is further divided into several district heating areas.

Time in Balmorel is organized into three step categories: years, seasons (weeks), and individual time units (hours). Each year is composed of 52 weeks and each season is, in turn, composed of 168 time units. The time is however flexible and the user can decide how many seasons and time units to use in the model. The choice depends on the needs for the specific investigation and typically ranges from weeks, when the focus is operational,

to years, common for investment analyses. The running time of the model is influenced by the time aggregation used, and varies from minutes to several hours. The main output is, among others, electricity and heat production levels, electricity prices, system costs, electricity transmission, and emissions.

Despite being used in the industry (Balmorel, 2017), Balmorel has been applied by the research community to several energy systems worldwide and for a wide range of purposes, from the integration of renewable technologies in the energy mix, to the analysis of market conditions, policies implementation, and future role of district heating in energy systems (Ball et al., 2007; Jensen and Meibom, 2008; Karlsson and Meibom, 2008; Münster et al., 2012; Münster and Meibom, 2010). Balmorel has also been used to integrate heat savings and residential investment behaviour into the energy systems (Zvingilaite, 2013; Zvingilaite and Balyk, 2014; Zvingilaite and Klinge Jacobsen, 2015).

7.2.2 Modelling investments in household appliances

Consider a set of home appliances $i \in \{1, \dots, I\}$, and a set of electricity zones $r \in \{1, \dots, R\}$ where we allow investments in energy-efficient appliances (in our study, DK-E and DK-W). To extend Balmorel with EE investments, we need to introduce first the following group of parameters. The assumptions behind data and how data is collected will be topic of the next section.

- ξ_i^{\max} = maximum consumption reduction for appliance i with respect to a baseline new, non-EE appliance of the same type and functionality (kWh/year). For example, assume that the average consumption for new, non-EE refrigerators is 300 kWh/year, and the average consumption of the most efficient refrigerators, of same type and functionality, available in the market is 180 kWh/year, then the maximum annual electricity saving from a refrigerator is $\xi_i^{\max} = 300 - 180 = 120$ kWh/year.
- c_i = additional cost of investing in a single appliance i with maximum saving of ξ_i^{\max} (EUR) with respect to the cost of a baseline consumption class. For example, assume that the baseline refrigerator efficiency class is A with average cost of EUR 650, and the most efficient is $A+++$ with average cost of EUR 1000, then $c_i = \text{EUR } 350$.
- ρ = discount rate, used to annuitize the investment cost of new appliances. More comments on the discount rate are provided in the case study.
- L_i = average lifetime of appliance i (years). The lifetime is used to annuitize the

investment cost and to approximate the annual substitution rate of the appliances, by computing $1/L_i$.

- N_{ir} = estimated number of appliances i in region r . It can be approximated by multiplying the share of an appliance with the number of households; for example, if the share of washing machines is 0.80 (items/household) and the number of households in DK-W is 1.4 mln., then N_{ir} is $0.80 \times 1.4 = 1.12$ mln. Our construction of N_{ir} applies if the total stock of appliances is fixed over time, as it is for the Danish market where household growth is very low. For developing economies, such as China or India, N_{ir} should be time-dependent.
- $n_{ir} = N_{ir}/L_i$ = estimated number of appliances i in region r which are replaced on average every year (e.g. because they are too old and not well-functioning anymore). For instance, if the average lifetime of a dishwasher is $L_i = 10$ years and the existing stock in DK-E is $N_{ir} = 1$ mln., then approximately $n_{ir} = 0.1$ mln. dishwashers are expected to be purchased in DK-E during a year.
- d_{irt} = gross electricity consumption (MWh) in region r due to the appliance category i at hour t of the year. We also define the total annual consumption of appliance i in region r as $D_{ir} = \sum_t d_{irt}$, and we will refer to the collection $\{d_{irt}\}_t$ as the yearly *consumption profile* of appliance i in region r .

We summarize the set of parameters necessary to implement the model in Table 7.1.

Table 7.1: Data required to implement the model extension

Name	Description	For each
ξ_i^{\max}	Max. consumption reduction	Appliance
c_i	Extra cost of more efficient appliance	Appliance
L_i	Lifetime of appliance	Appliance
N_{ir}	Stock of existing home appliances	Appliance and region
ρ	Discount rate	-
d_{irt}	Hourly consumption profile	Appliance and region

It is now possible to compute the annuitized extra investment cost of a new EE appliance, c_i^a [EUR], as

$$c_i^a = \frac{\rho c_i}{1 - 1/(1 + \rho)^{L_i}}.$$

Then, we define the decision variables $x_{ir} \in [0, 1]$ as the percentage of new appliances of type i that are replaced with the most energy-efficient version in region r . In particular,

$x_{ir} = 0$ means that there is no investment in more efficient appliances of category i , while $x_{ir} = 1$ means that the full amount n_{ir} of appliances i in the region is upgraded. In this case, the system will benefit of an annual electricity saving of $\xi_i^{\max} n_{ir}$ for the lifetime of the appliance.

The introduction of investments in EE has two main effects in the energy system model. First, the investment cost represents a new contribution in the objective function, given by

$$\min : \text{SysCost} + \sum_{i=1}^I \sum_{r=1}^R c_i^a n_{ir} x_{ir} \quad (7.1)$$

where SysCost is the original objective function in Balmorel representing the total cost of the energy system, and includes the cost of fuel consumption, operation and maintenance cost for the different technologies, investment cost in new generation and storage capacity, emission and fuel taxes, etc. Second, the demand profile is reduced according to the saving associated with x_{ir} . The saving is spread over the whole year and applies with the same percentage across the consumption profile of the appliance. We can consequently work hour by hour and, denoting with d_{rt} the electricity demand in region r and time t , we define a new power balance equation

$$(\text{electricity supply in } r \text{ at } t) = d_{rt} - \sum_{i=1}^I \frac{d_{irt} \xi_i^{\max} n_{ir} x_{ir}}{D_{ir}}. \quad (7.2)$$

For instance, if there is no investment in EE, meaning $x_{ir} = 0$ for all appliances i , then the summation term (i.e. the saving) is zero and the equation reduces to the original one. If the investment is maximum for appliance i , i.e. $x_{ir} = 1$, then the demand is reduced by a factor $d_{irt} \xi_i^{\max} n_{ir} / D_{ir}$. This amount corresponds to the annual saving $\xi_i^{\max} n_{ir}$ from appliance i , scaled with the fraction of total demand D_{ir} occurring in hour t , d_{irt} / D_{ir} . In line with the other investments in Balmorel, in (7.2) it is implicitly assumed that new appliances are purchased and installed in the first hour of the year.

In addition, several studies suggest that the gains achieved from new energy saving measures are usually slightly lower than what initially expected, due to the so-called rebound effect (Khazzoom, 1980; Buluş and Topalli, 2011; Shrestha and Marpaung, 2006; Galvin, 2010; Farinelli et al., 2005; Galarraga et al., 2013; Carnall et al., 2015). This happens because the consumer typically responds to new EE measures in a way that tends to offset the effects of the changes. In more practical words, if we have a more efficient appliance or service, we tend to use it more because its use is cheaper, and we may also purchase additional appliances of the same type. We include the rebound effect in our model and characterize it as a linear response. Introducing $\beta_{ir} \in [0, 1]$ and indicating with D_r the

total yearly electricity demand in region r , we extend (7.2) with

$$\begin{aligned} (\text{electricity supply in } r \text{ at } t) = & d_{rt} - \sum_{i=1}^I \frac{d_{irt} \xi_i^{\max} n_{ir} x_{ir}}{D_{ir}} \\ & + d_{rt} \frac{\sum_{i=1}^I D_{ir}}{D_r} \frac{\sum_{i=1}^I \xi_i^{\max} \beta_{ir} x_{ir}}{\sum_{i=1}^I \xi_i^{\max}}. \end{aligned} \quad (7.3)$$

Even though the magnitude of the effect might change depending on appliance and region, in the following we set all variables to be the same ($\beta_{ir} = \beta$).

To summarize, investing in efficient household appliances reduces the electricity consumption as in (7.3). Less demand implies that less production technologies to operate or install are needed to supply electricity, which in turn implies lower costs for the system. The optimization process will then implicitly compare this economic saving with the investment cost added to (7.1) and, if convenient, will endogenously trigger the investment.

7.2.3 From the energy system to the end-user

The model presented optimizes investments from a system perspective. It is a socio-economic analysis and does not include taxes on the consumer side. This means that the solution resulting from the optimization process should be interpreted as the least expensive solution for the whole energy system, and investments in energy-efficient appliances implicitly compete with the supply of electricity at the system price, i.e. the wholesale market price. However, the analysis currently disregards a representation of the end-user choices, which are relevant since in practice investments in home appliances are made by end-users. The consumer pays a higher price for electricity due to additional taxes on e.g. transmission, distribution, and policy costs for promotion of renewables. In Denmark, the tax addition to the electricity price is a fixed additive amount that makes the consumer's price up to ten times higher than the system price (Energitilsynet, 2016). As a consequence, investments which are not worth for the society might be actually profitable for the single user, who individually evaluates an EE investment.

To include the consumer utility in the analysis, we propose the following sequential approach. First, the consumer observes the annual electricity price profile generated from the system model and estimates the consumer price by considering an average overpricing factor. Second, the consumer determines whether investing in more energy-efficient appliances is profitable by comparing the extra investment cost with the economic saving implied by the consumption reduction. Third, the energy system model is solved for the

second time embedding the investment decisions of the consumer. New electricity prices are generated, and the actual saving on the consumer side is determined together with possible changes in the energy system. Figure 7.1 summarizes the sequential process.

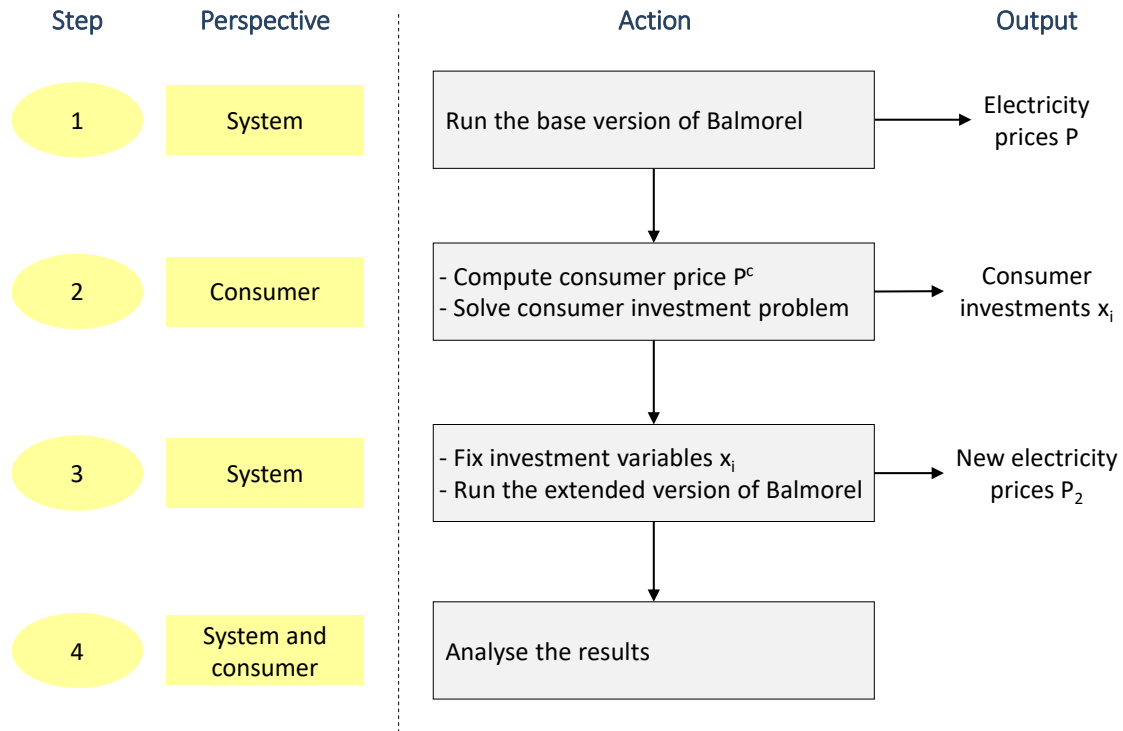


Figure 7.1: Sequential process to analyse investment decisions for end-users

Let us focus on the consumer model. When should a consumer purchase a new energy-efficient appliance, e.g. a refrigerator? If the refrigerator is well-functioning, one would generally need some strong incentive to replace it with a more efficient product. However, as discussed earlier, by introducing a substitution rate we limit the analysis to the subgroup already needing to replace the given appliance due to capital depreciation. Thus, the question we try to answer is more specific: I need to purchase a new refrigerator, should I invest in a very energy-efficient product, paying an extra cost but having an annual energy saving, or should I buy an average refrigerator similar to what I had before? A rational consumer would compare the extra investment cost of the more efficient product with the expected economic saving resulting from the consumption reduction throughout the appliance lifetime, and would undertake the EE investment in case of positive net present value (NPV) of cash flows. In particular, we denote with p_{rt} the system price of electricity, which in Balmorel corresponds to the dual value of the power balance equation, and with γ the average price overcharge on the consumer side. The consumer price is then estimated by $p_{rt}^c = p_{rt} + \gamma$, and the NPV of an EE investment is computed for every appliance i and

region r with

$$\text{NPV}_{ir} = -c_i + \sum_{y=1}^{L_i} \frac{\alpha_y}{(1 + \rho)^{y-1}} \left(\sum_{t=1}^T p_{rt}^c d_{irt} \xi_i^{\max} / D_{ir} \right). \quad (7.4)$$

Equation (7.4) represents the trade-off between extra investment cost and cumulative annual saving. The expression inside brackets is the economic saving for the current year, calculated by multiplying the consumer price at a given hour t with the consumption reduction achieved in t , then summing over the whole year ($T = 8760$ is the number of hours in a year). This expression is then summed over a number of years corresponding to the lifetime of the appliance L_i , discounted, and multiplied by a factor α_y indicating the expected change (increase or decrease) of electricity prices for year y .

In practice, however, a consumer does not act in a fully economically rational way, and there are behavioural aspects that may influence the investment decision. The consumer behaviour is difficult to capture and model since it is by definition subjective. Previous research tried to quantify the correlation between the propensity to invest in EE (intended as both housing renovation and the purchase of energy-efficient appliances) and factors like income, age and education (Hausman, 1979; Mills and Schleich, 2010; Ward et al., 2011; Murray and Mills, 2011; Allcott, 2011b; Davis and E. Metcalf, 2014; Houde, 2014; Newell and Siikamäki, 2013; Schaffrin and Reibling, 2015; Bartiaux and Gram-Hanssen, 2005). Most of the studies agree on a positive correlation between household's income and investments level. In contrast, conclusions regarding other factors (age, education etc.) often show ambiguity and there is generally no statistical significance in the correlation with investment.

In line with these studies, we include in the model a *behavioural uncertainty* related to the household's income level. A low-income household might not be willing to pay a high up-front cost for relatively small annual electricity savings. Consequently, even though the EE investment turns out to be profitable according to (7.4), it may not be undertaken because the payback period is too long. The choice also depends on the other expenses of the households in the same period, i.e. your overall liquidity constraints. On the other hand, the up-front investment cost for a high-income household is typically not a constraint, and, if the EE investment is profitable, then it will be undertaken. It can be seen as a sort of budget constraint and a linear probability model is used to describe it. Moreover, as suggested by some authors (Allcott, 2011b; Ward et al., 2011; Cooper, 2011), the opposite phenomenon is also possible: a high-income consumer may invest in an efficient appliance 'just' because it is the green option, also when the choice is not profitable from a strictly economic perspective. Thus, similarly as before, we assign a probability of purchasing the energy-efficient appliance when the investment is not

profitable.

The curves in Figure 7.2 represent the probability of purchasing an energy-efficient product when economically profitable and when not. They are constructed partly based on the results from Allcott (2011b); Ward et al. (2011); Cooper (2011) and partly by using data about income and annual expenditure in appliances by households from Statistics Denmark (2016). The curves are employed as model assumptions as no empirical evidence for the functional slope is available in the literature. We also assume that the curves are not static but dependent on the specific appliance: if the NPV is positive but the payoff takes many years, then the blue curve shifts down, and vice versa. In the analysis, we are not incorporating possible variations of the number of appliances and replacement rate by income class, and we equally split the stock among the classes.

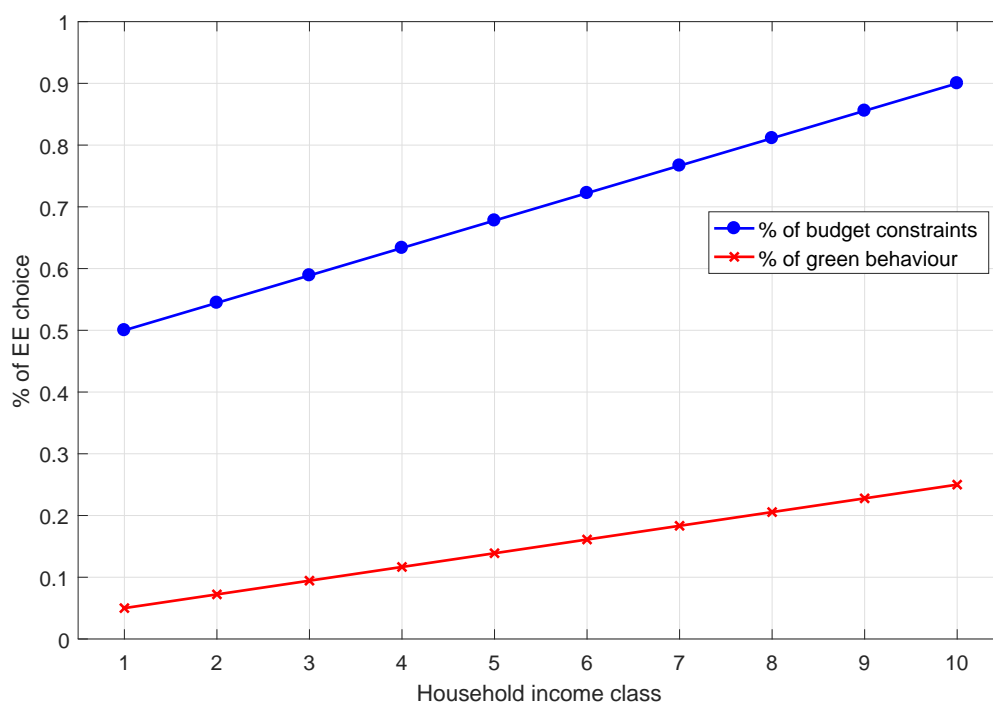


Figure 7.2: Probability of purchasing an energy-efficient appliance when economically profitable (blue line) and when not profitable (red line). On the x-axis are the deciles of the income distribution of Danish households.

In reality, the consumer choice is also subject to uncertainty regarding the information available (e.g. electricity prices and products on the market) and errors in computing the economic convenience. This uncertainty is already included in the consumer model, indeed, for instance, the adoption rate of profitable products by the highest income class is lower than 100%.

Coming back to the sequential approach, notice that also other authors have incorporated consumer classes (income deciles) with different behavioural profiles into a model which ultimately solves as a system optimization, for example Bunch et al. (2015). We conclude the section with a few remarks.

1. After new electricity prices are generated, the end-user's model could be executed once again leading to a potentially different investment decision. This new decision could be plugged into the system model, and the sequential approach iterated until convergence (i.e. when there are no changes in electricity prices between two iterations). However, in all our experiments the model converged after the first iteration, thus we neglect the convergence topic in the following discussion.
2. In (7.4) savings are modelled using flexible electricity pricing. Even though most of the Danish households currently pay electricity based on a flat tariff (Energitilsynet, 2016), in the last few years smart meters have been spreading, reaching almost 50% of the of the Danish households in 2015 and aiming at 100% for 2020 (Danish Ministry of Energy Utilities and Climate, 2013; Danish Ministry of Energy Utilities and Climate, 2014). With smart meters and exposition to real-time rates, the adoption of flexible pricing is expected to quickly increase (Allcott, 2011a; Katz et al., 2016; Katz, 2014; Krishnamurti et al., 2012; Broman Toft et al., 2014; Faruqui et al., 2010).

7.3 Case study

The proposed model extension has been tested on the Danish energy system. However, the test is non-exclusive, and the same analysis could be performed on a different system, provided that all the input data needed to run the model is available.

7.3.1 Scenario description

We characterize the scenarios based on three main elements on the input side: simulation year, fuel price forecast, and fuel availability. Two different simulation years are considered:

- **2015:** serves as an ex-post analysis to understand how the known energy system would have changed if consumer (or society) had invested in EE in an optimal

way. For this case, the system is fully determined exogenously and we do not allow investments in new power plants. Thus, the model is in an operational simulation mode.

- **2025:** to assess the saving analysis on a future energy system. For this case, the energy system is also allowed to evolve by endogenously investing in new power plants and decommissioning the old and unproductive ones.

To cope with the uncertainty in fuel and emission prices in 2025, following Zvingilaite (2013) we identify a range of price values presented in Table 7.2: from a *low price* scenario to a *high price* scenario. The low price scenario has been constructed with the guidelines of the Danish Energy Agency for future socio-economic analyses (Danish Energy Agency, 2016b). The high price scenario is based on the oil price development in Oilprice.com (2016) and IEA (2016), with the assumption that the high prices for other fuels follow the price of oil with certain elasticity, as indicated e.g. in Karlsson and Meibom (2008). The cost of municipal waste is assumed to be negative and constant, since in Denmark, the waste incineration plants are paid to treat the waste (Münster, 2009). Regarding CO₂, the low price scenario is based on the carbon trading price, which in fall 2009 was around 15 EUR/t (Reuters, 2016), whereas the high price scenario is based on the IPCC considerations (IPCC, 2007). In the table, we also report the average price scenario.

Table 7.2: Prices of fuels and emissions in 2025 according to different scenarios. Prices for renewable sources, e.g. wind, sun and hydro, are assumed to be zero.

	Low price (EUR/GJ)	Average price (EUR/GJ)	High price (EUR/GJ)
Fuel oil	13.33	17.24	21.14
Natural gas	12.01	15.02	18.02
Municipal waste	-3.60	-3.60	-3.60
Coal	5.05	6.97	8.89
Wood pellets	12.25	13.03	13.82
Straw	7.69	8.47	9.25
CO ₂ [EUR/t]	18.02	39.04	60.07

In addition, we model availability constraints on the main input fuel sources for 2025. The limitations are decided according to the *4 degree scenario* proposed by the IEA in the Nordic Energy Technology Perspective (IEA, 2016). Table 7.3 reports the most relevant values.

Table 7.3: Fuel availability for 2025 (fuel input for power, heat and CHP plants), NETP (IEA, 2016)

	DK	SE	NO	FI
Coal (PJ)	99.2	9.4	0.0	87.8
Oil (PJ)	1.9	4.4	0.1	1.7
Gas (PJ)	21.4	3.6	0.0	34.8

The scenarios are tested using four representative weeks of the year (weeks 09, 22, 32, and 51), where each week is composed by the full hourly resolution (168 h), giving a total of 672 time steps for the simulation. In this way, we are able to obtain sufficiently accurate results, keeping the size of the model and its running time limited. Even though we deal with an investment analysis, the hourly resolution is needed here to entirely capture the differences of consumption profiles of the various home appliances.

7.3.2 Relevant parameters

A set of input data for each of the two Danish electricity zones must be collected. In Table 7.4 we report some of the most relevant parameters along with the reference.

Table 7.4: Relevant model parameters: values and references

Data	Zone	Value	Source
Electricity demand (TWh)	DK-E	13.70	NordPoolSpot (2016)
Electricity demand (TWh)	DK-W	20.44	NordPoolSpot (2016)
Number of households (mln.)	DK-E	1.15	Statistics Denmark (2016)
Number of households (mln.)	DK-W	1.41	Statistics Denmark (2016)
Electricity tax addition (EUR/MWh)	DK	265	Energitilsynet (2016)
Discount rate (%)	DK	3	Danmark NationalBank (2016)
Rebound effect (%)	DK	3	Nässén and Holmberg (2009)

Nowadays, the risk-free investment rate in Denmark is very close to zero (Danmark NationalBank, 2016). However, in our analysis we also account for the expected uncertainty from EE investments (given e.g. fuel price volatility and regulatory uncertainty). Therefore, ρ is increased and set equal to 3%, like the value used in Zvingilaite (2013) for heat saving investments in Denmark. The magnitude of the rebound effect related to EE, a debated topic in the literature, can vary from moderate to negligible levels depending on the analysis. In our model, we use a rebound effect level of 3% related to household electric appliances (Nässén and Holmberg, 2009). The tax addition to the electricity system

price in Denmark is estimated to be 265 EUR/MWh according to Energitilsynet (2016), and is expected to remain stable in the near future.

7.3.3 Appliances data

We selected the subset of 11 home appliance categories listed in Table 7.5. This set is chosen for several reasons. First of all these devices, together, constitute approximately 80% of the electricity demand of the private household sector in Denmark; hence, they are the most interesting to study from an energy consumption perspective. The electricity demand of residential air-conditioning systems, for instance, is negligible in the Danish context, and such appliance is therefore excluded from the study. Second, given the high energy consumption of the chosen appliances, the price of purchasing a new product reflects in a good extent its efficiency: when buying e.g. a new refrigerator of a given volume, the energy use of the product is typically the main factor driving the choice. On the other hand, for more high-tech appliances such as desktops, laptops, and printers, this is generally not true, and price difference between two products or brands is given by the functionalities rather than the consumption. Third, most of the selected appliances fall under the EU energy labelling programme, therefore it is easier to collect the relevant data and assess the relationship between price and efficiency.

Table 7.5: List of household appliances considered in the analysis. The second and third columns refer to the annual consumption reduction and extra cost with respect to an average consumption class.

Appliance category	Saving [kWh/y]	Δ cost [EUR]	lifetime [years]
Stand-alone refrigerator	50	413	15
Stand-alone freezer	88	138	20
Refrigerator-freezer	152	605	17
Washing machine	109	242	12
Dish washer	65	572	10
Dryer	118	605	13
Lighting living room	29	9	6
Lighting secondary rooms	25	9	7
Cooker	52	435	12
TV LCD	24	243	7
Vacuum cleaner	11	130	7

The saving and cost data in Table 7.5 are averages over different products and brands, but with same volume or size, taken from some of the major producers and retailers

active in Denmark (Bosch, Siemens, Electrolux, Miele, Aeg etc.). In the table, cooker refers to both electric hobs and electric baking oven. The lighting system is split in two components to account for the different use: one main room (living room) with an higher usage and the other secondary rooms. The extra cost is generally rather high since we model investments in appliances with the highest available efficiency class (e.g. A++ or A+++). Limiting the investment analysis to Denmark, we assume there are no differences in the performance or cost characteristics of existing or new appliances between the two regions DK-E and DK-W.

The presented model uses linear cost-efficiency relations for appliances, i.e. the purchasing cost of an appliance grows linearly with the consumption reduction. In practice, there may be differences between the appliances and more complex cost-efficiency relations. However, the data collected supports the assumption that a linear fitting describes the relation sufficiently well for our purposes. Figure 7.3 illustrates the cost-efficiency relation for two sample appliances.

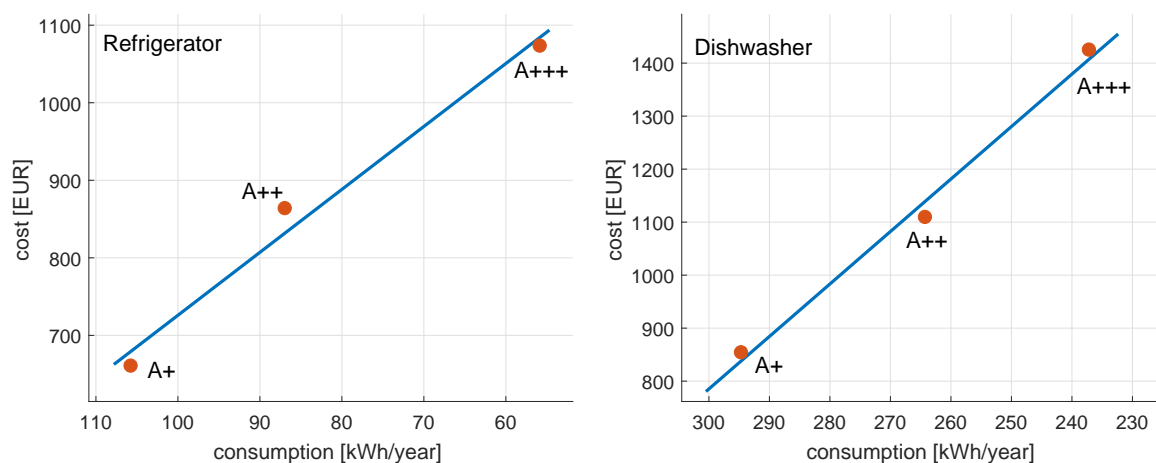


Figure 7.3: Cost-efficiency relations for refrigerators (left) and dishwashers (right), classes A+ to A+++ . The dots represent average cost and consumption of a number of products of the same efficiency class; the blue line is the linear interpolation between them.

Regarding demand profiles, we rely on the results from Klinge Jacobsen and Juul (2015) who investigated the electricity consumption of a typical Danish household and determined consumption profiles for each appliance category. The profiles of the 11 appliances included in the analysis are illustrated in Figure 7.4, summing DK-E and DK-W.

As expected, the profile changes considerably between the different categories. For example, the *cold appliances* (refrigerator, freezer) manifest a fairly flat profile while other appliances like lighting more contribute to the peaks, especially during the evening hours. Differences can be also found between working days and weekend: in the weekend the

kitchen equipment is used more, in particular during lunch hours, and the use of the vacuum cleaner is higher too.

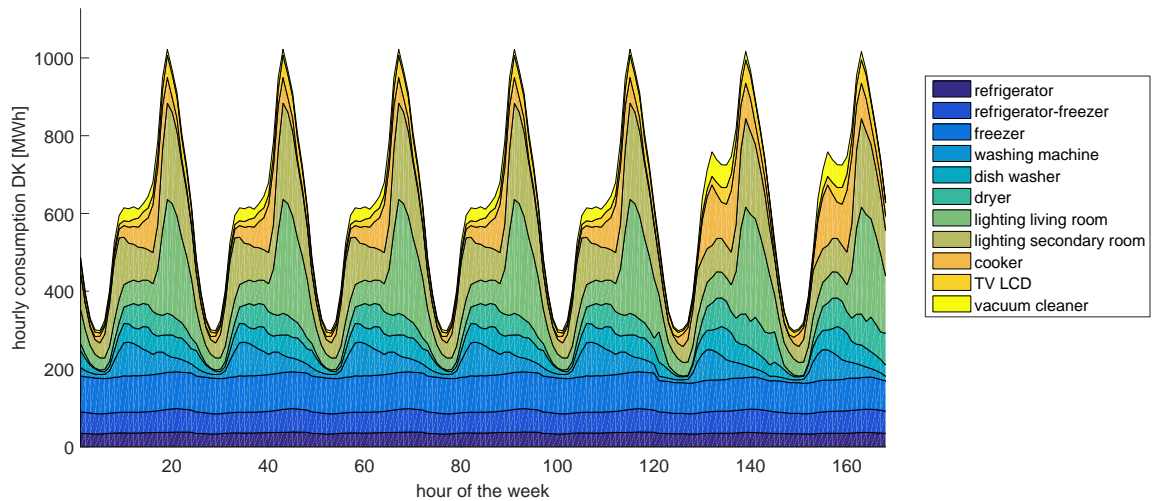


Figure 7.4: Electricity consumption profile during a sample week (week 09) of the 11 home appliances included in the analysis.

In Figure 7.5, we show how the aggregated profile of the 11 appliances contributes to the total electricity demand of households and of all sectors in Denmark.

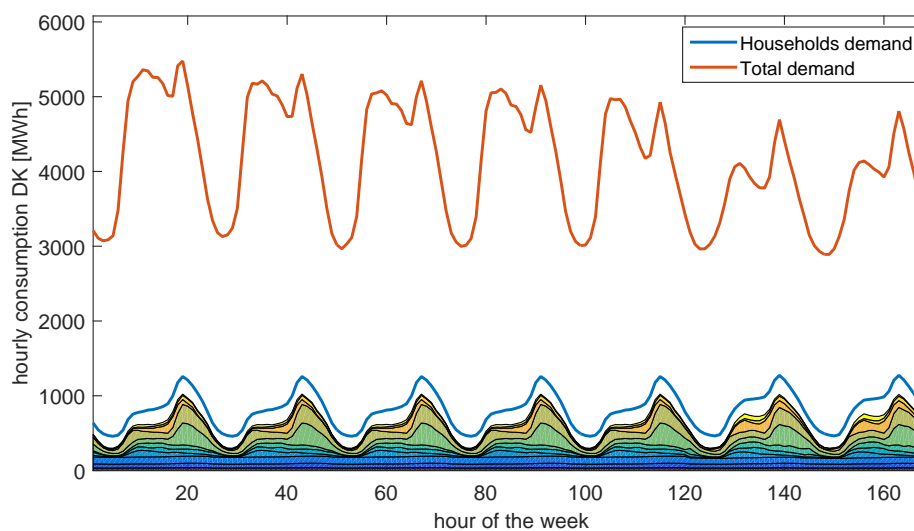


Figure 7.5: Aggregated profile of the 11 appliances compared to the total electricity demand in a sample week (week 09)

7.4 Results and discussion

A sensitivity analysis of the model using fuel and CO₂ costs for 2025 reported in Figure 7.2 was made, resulting in similar electricity prices p_{rt} (although a different capacity mix is installed). This limited local sensitivity to scenario prices occurs because, given the replacement rate, only a small component of the energy demand is affected by the EE investments. As a consequence, we noticed no or very little change in the consumer choices (but different CO₂ implications) and, throughout the section, we will present the results for the average cost scenario for 2025.

7.4.1 Preliminary check

The driver for the investment choice lies in the economic profitability of adopting a particular appliance, based on the cumulative savings achieved during its entire lifetime. To get a first idea of the potential of investing in the different appliances, in Figure 7.6, we compute the amount of energy per unit that could be saved if an EE investment of 1 EUR is made in one of the examined appliances.

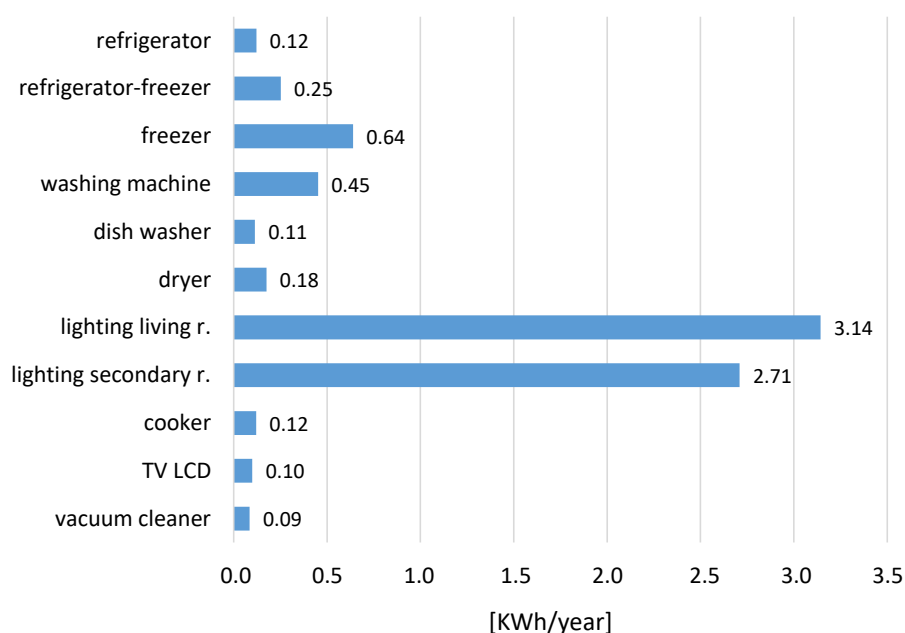


Figure 7.6: Annual electricity saving per 1-EUR investment.

As can be seen, the gap between lighting and other appliances is large: investing 1 EUR in lights results in an annual saving of around 3 kWh, while for other appliances it ranges

from 0.1 to 0.6 kWh, i.e. an order of magnitude lower. Excluding lighting, from the picture, it emerges that freezers and washing machines provide the best saving per unit investment, compared to the rest of the stock. Although Figure 7.6 gives a picture of the potential benefit of investing in the different devices, the final investment choices also depend on the hourly electricity price and the consumption profile of each specific appliance.

7.4.2 EE investments

The investments in EE appliances resulting from the simulations are shown in Figure 7.7. The left graph illustrates the optimal levels when the system model with endogenous investments is used, whereas the right graph represents the consumer choices after the sequential model is run. In the following, the values for DK-E and DK-W are presented as merged, even though they are separate zones from a model logic.

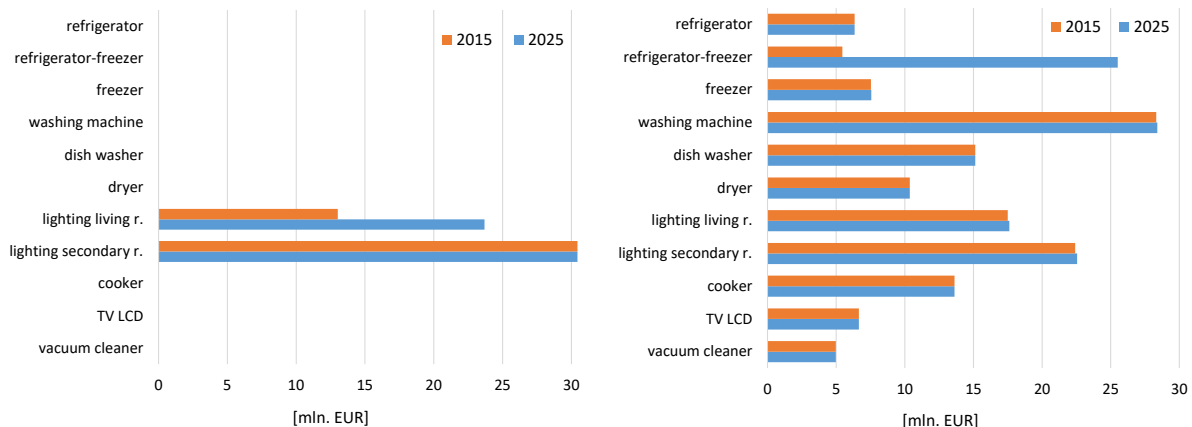


Figure 7.7: Investments in efficient appliances with the system model (left) and consumer model (right). The amount on the x-axis corresponds only to the extra cost with respect to the baseline efficiency class, and not to the overall investment cost in new appliances.

When the system is considered, the economic/energy saving criterion shows that the only EE investments worth doing are efficient lighting replacements. For 2025, the investment level in lighting for the living room is higher than 2015 due to the corresponding higher system prices of electricity for that year. This price difference develops because the system prices for the future energy system include long-term investments in renewable technologies and other system adjustments. Finally, given the lower saving per unit cost, no investment in other EE appliances is triggered during the optimization process.

In contrast, the end-user economic convenience is based on the consumer electricity price

and more diversified investments occur. Due to the consumer's behavioural dimension and the incompleteness of information, however, not all investments with positive NPV are undertaken, and vice versa, some investments in appliances with negative NPV occur. For instance, the investment level in lighting for secondary rooms ($NPV > 0$) are lower than 100% (as they are in the system model), and some investments in EE refrigerators ($NPV < 0$) take place. The consumer investments in EE exceed the system investments by 95 mln. EUR in 2015 and 105 mln. EUR in 2025.

Overall, the two years investigated show small differences in consumer choices, and investments in 2025 are only slightly higher than those in 2015. Indeed, even if the system prices of electricity are higher in 2025, the additive nature of the tax component makes the difference perceived by consumers less pronounced. The combined refrigerator-freezer represents an exception; in fact, the NPV becomes positive for some consumer classes between the two years, leading to a substantial increase for 2025.

To better understand the results, in Figure 7.8, we compare the lifetime of a new and more efficient household electric device with the discounted payback period (DPP) of its extra investment cost, i.e. the time needed for the EE investment to break even. As can

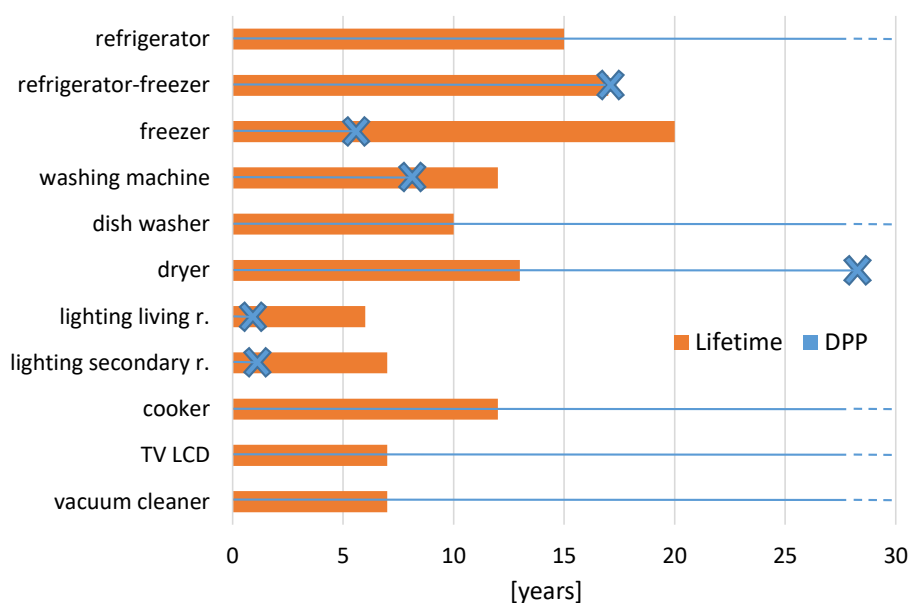


Figure 7.8: Lifetime versus discounted payback period (discount rate 3%) in the consumer perspective.

be seen, the DPP of an efficient lighting is approximately 1 year, for freezer and washing machine, it is about 5 and 8 years, respectively, and for all other appliances, it is longer than 15 years. For a rational consumer with no liquidity constraints, an investment is deemed worthy if the DPP is lower than the lifetime of the appliance, meaning that the

appliance will be paid-off before the end of its expected lifetime (this is the same to having a positive NPV). The analysis shows that this criterion applies for lights, stand-alone freezers, washing machines, and combined refrigerator-freezers which are at the borderline. Specifically, one can notice that, although similar savings can be achieved with efficient washing machines and dryers (Table 7.5), the investment profitability differs substantially. Indeed, the most energy-efficient dryers are still very expensive, and the extra investment cost is higher than in washing machines. Moreover, among the cold appliances, we notice that the profitability of EE freezers is higher than that of EE refrigerators.

In Table 7.6, we report the details of the investments, quantifying the adoption and effectiveness of energy-efficient appliances. Consider for instance 2015: with an up-front extra cost of 138 mln. EUR, the resulting energy and economic savings is 141 GWh and 41 mln. EUR per year respectively. Including the lifetime of the appliances and the discounting, this translates into revenues of 222 mln. EUR, i.e. a net discounted saving equal to 84 mln. EUR for Danish consumers investing in energy-efficient appliances (similar for 2025).

Table 7.6: Summary table for the scenarios analysis, 2015 and 2025

Appliance	Investments [K units]		Investments [mln. EUR]		Economic savings [mln. EUR]		Electricity savings [GWh/y]	
	2015	2025	2015	2025	2015	2025	2015	2025
Refrigerator	15.4	15.4	6.3	6.3	0.22	0.25	0.77	0.77
Refr.-freez.	9.0	42.2	5.5	25.5	0.40	2.13	1.37	6.42
Freezer	54.6	54.7	7.5	7.6	1.39	1.60	4.80	4.82
Wash.mach.	117.0	117.3	28.3	28.4	3.71	4.33	12.76	12.79
Dish washer	26.5	26.5	15.1	15.1	0.50	0.58	1.72	1.72
Dryer	15.4	15.4	10.4	10.4	0.53	0.61	1.81	1.81
Light L.R.	1892	1904	17.5	17.6	16.00	19.03	54.87	55.23
Light S.R.	2423	2438	22.4	22.6	17.67	20.97	60.57	60.95
Cooker	31.4	31.4	13.6	13.6	0.47	0.56	1.63	1.63
TV LCD	27.4	27.4	6.7	6.7	0.19	0.22	0.66	0.66
Vacuum cl.	38.4	38.4	5.0	2.2	0.12	0.14	0.42	0.42
Total	4650	4711	138.3	156.0	41.2	50.4	141.4	147.2

In Table 7.7, we report the analysis of the benefits on the consumer side, highlighting the annual economic and energy savings resulting from the investments. The saving for 2025 is slightly higher because of the higher electricity prices. Notice that the saving is spread over the entire Danish population, disregarding the fact that only a portion of it is actually replacing a given appliance.

Table 7.7: Average electricity and economic saving for Danish households

Year	Extra-investment costs (EUR)	Annual electricity saving (kWh/year)	Annual economic saving (EUR/year)	Net economic saving (EUR/year)
2015	54.0	55.2	16.1	32.5
2025	62.0	57.5	19.7	44.2

In the methodology section, we discussed the ability to afford investments according to the income class. In Figure 7.9, we illustrate the investment levels for each appliance disaggregated per class. The graph shows that the higher the income, the higher the share of the investment, reflecting the trends defined in the linear consumer model of Figure 7.2.

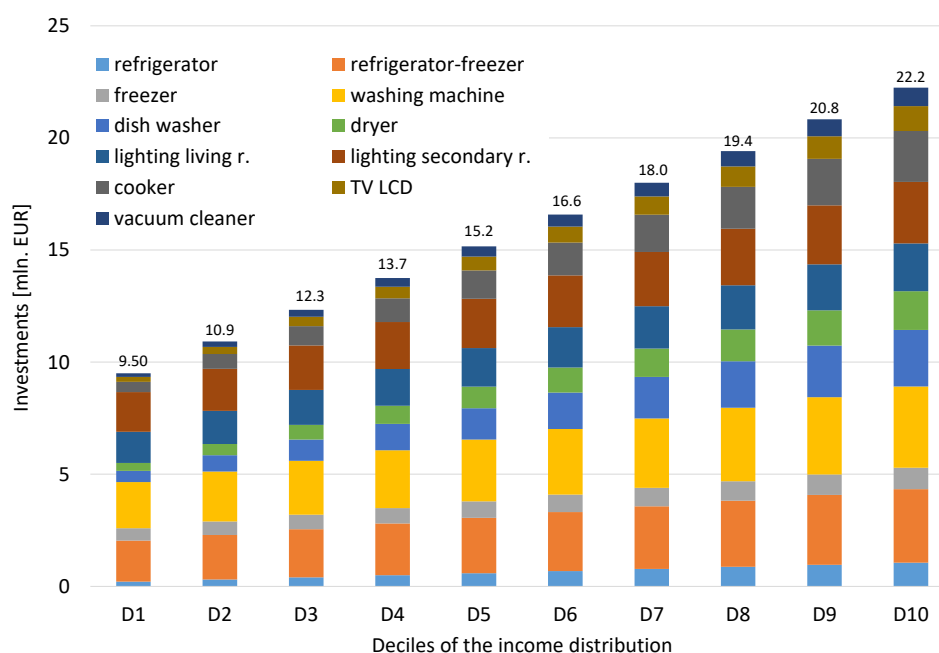


Figure 7.9: Investments in energy-efficient appliances according to the income class (2025)

7.4.3 System changes and comparison of perspectives

One of the main purposes of modelling the consumer behaviour is to determine its impact on the energy system. To assess the changes, we focus on two key parameters, CO₂ emissions and electricity demand reduction, and summarize the results in Table 7.8.

For 2015, we notice that introducing a consumer model leads to higher electricity savings compared to the optimal system investments (141 GWh vs. 123 GWh). With the imple-

Table 7.8: Total electricity and CO₂ savings.

	Electricity savings			CO ₂ savings		
	Amount (GWh)	% households DK	% DK	Amount (Kton CO ₂)	% system	% DK
2015 Sys	123	1.88	0.38	83.7	0.020	0.34
2015 Cons	141	2.15	0.43	117.2	0.030	0.48
2025 Sys	157	2.44	0.49	32.8	0.017	0.87
2025 Cons	147	2.29	0.46	19.2	0.010	0.51

mented savings, Denmark could cut its CO₂ emissions of almost 0.48% according to the consumer model. Although this percentage seems small, the reader should keep in mind that a similar saving will occur in the years succeeding the investment. Considering the lifetime of the appliances and the substitution rate of the yearly stock, the cumulative savings will result higher in the long term.

A different configuration emerges for 2025 where the level of electricity savings achieved in both models is higher than that in 2015. Nevertheless, the total amount of CO₂ reduction is lower. Indeed, the future energy system in 2025 will be highly based on renewable energy sources, especially wind, and several fossil fuels power plants will be decommissioned by then. Although the emissions reduction is lower, we notice that in percentage we obtained a CO₂ cut of almost 1%, implying a larger impact of the savings on the system. Moreover, for 2025, the savings achieved are higher in the system perspective. Indeed, in the system model, more investments in lights take place which, as shown in Figure 7.6, contribute more effectively to the electricity demand reduction.

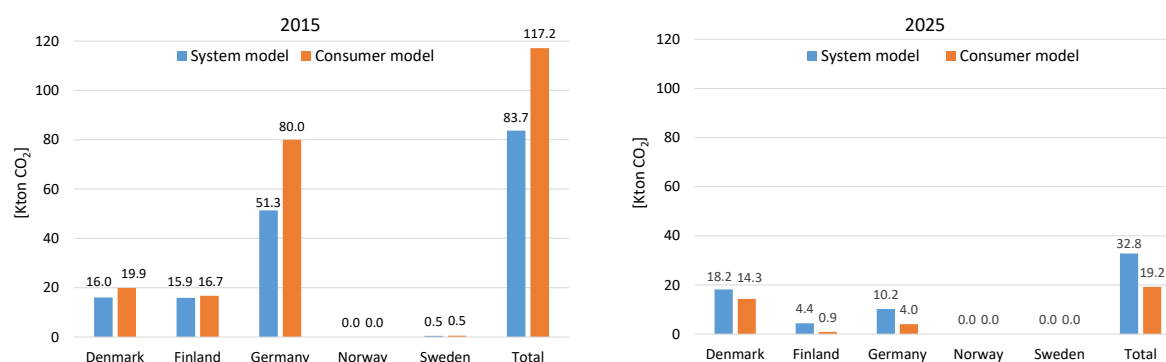
Figure 7.10: CO₂ emissions reduction in 2015 (left) and 2025 (right).

Figure 7.10 provides a graphical representation of the emissions reduction divided by country. It is interesting to see that, although a demand reduction via EE was implemented in the model only for Denmark, the decrease in CO₂ emission takes place in several other

countries connected with Denmark. This highlights the influence of the interconnections between countries and proves that changes occurring in the Danish system have an impact on the electricity production not only of Denmark itself but also of the other countries. For 2015, the largest emissions reduction occurs in Germany, where the simulation shows that future use of nuclear, natural gas, coal, and lignite decreases while the power production from wind, wood pellets, and municipal waste increases. Denmark comes after together with Finland; energy mix highly based on hydro and nuclear power, as Norway and Sweden, is not greatly influenced by small changes in the demand of a surrounding country. For 2025, instead, Denmark contributes more to the total CO₂ emissions reduction with 55 and 74% for the system and consumer perspectives, respectively.

The EE investments also affects the electricity consumption profile, as reported in Figure 7.11 for a sample week. The two different models, system and consumer, influence the demand in diverse ways. As can be noticed, the investments in the system model are entirely based on lights and mainly contributes to reducing the peaks. This is also in line with results from previous studies (Klinge Jacobsen and Juul, 2015). In contrast, being the consumer's investments more diversified, the demand is reduced homogeneously through the year, including hours outside peak loads.

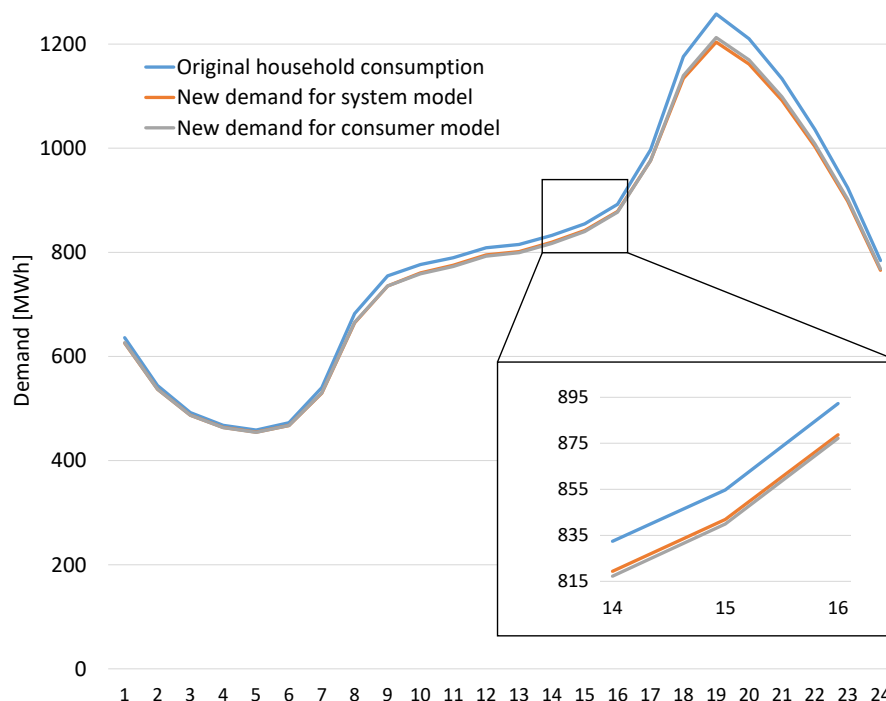


Figure 7.11: Electricity demand after system and consumer investment models (week 9, day 1).

Even though investments are generally higher and more variegated for the consumer

model, the overall demand reduction is similar in the two cases. In fact, the slightly higher investment in efficient lighting for the system model results in total savings comparable to that of all the other appliances chosen by consumers together.

7.5 Conclusions

The goal of this paper was to investigate the value of investments in more energy-efficient home appliances compared to a business-as-usual electricity supply scenario.

Two different perspectives have been examined: energy system and end-user. When the system is given the possibility to invest in efficient appliances, only investments in the lighting sector take place. In contrast, when the consumer has the choice, the investments are more diversified and generally higher. This highlights the different selection criteria for the two models: the system considers purely economical convenience, whereas for consumers a behavioural dimension comes into play. Moreover, two main factors have been considered when modelling the choices of the end-users: economic profitability and ‘green investments’ propensity according to the income class. This last component, together with the different electricity prices, represents the reason for the diverse investments compared to the system perspective.

The findings presented in the paper are the result of a soft-linking between a well-known energy system model and a consumer-behaviour model designed for the study. The interactions between the two models is the key for understanding the impact of the consumer choices on the energy system. When compared to a business-as-usual energy scenario, with the investment solution resulting from the model, the end-user ends up on average with a net economic savings in the range of 30–40 EUR per year. Moreover, the system benefits of a total electricity savings of 141 GWh in 2015 and 147 GWh in 2025, and CO₂ emission reduction of 117 Kton in 2015 and 19 Kton in 2025. Because of the international interconnections and energy markets, changes in the energy system (e.g. in installed capacity, fuel consumption, emissions) occur not only in the country the consumer belongs to, but also in the surrounding countries. The decision of a single consumer, thus, contributes to the diversification and transformation of the global energy system.

The study also reveals the potential appliances that will be attractive from a system perspective and, despite the simplicity of the consumer choice model, it provides a first indication of the profitability of investments for private consumers. The closing considerations have highlighted the relevance of this analysis for a country that is aiming at important targets in terms of environmental issues. Therefore, this study should be

pushed forwards.

7.5.1 Future work

The presented study could be extended in several key directions. One way is to include a more sophisticated consumer behaviour into the investment decision function. The data from a survey conducted from the Danish Energy Agency¹ over a representing set of houses serve as the starting point for the new thorough analysis. Using this dataset, an exclusive, latent class logistic model could be employed to categorize the consumers into subsets with respective propensities to purchase (Shen and Saijo, 2009; Murray and Mills, 2011; Mills and Schleich, 2010). This could also help to better assess the functional slope of the purchase propensity by income class proposed in this paper. Using different discount rates could also be a natural way to incorporate several of the behavioural differences that are noted between consumer income classes. Of additional interest, Danish specific data and appliance purchasing behaviour is currently under investigation by UserTEC (2016) and could potentially be included. The end goal intended is then to incorporate the consumer categories into Balmorel to compute a more realistic energy savings scenario.

Additionally, this analysis can be extended to re-examine the efficacy of Denmark's imposed policies (i.e. EU driven energy labelling programme and overall energy efficiency targets). Analyses in 2013 (Danish Energy Agency, 2016a) predicted savings of 5640 GWh/year by the year 2020 as a result of ecodesign requirements and the labelling programme. With updated data on actual adoption, these projections can be re-examined. Additionally, these propensity estimates can inform investigation into the potential benefits of energy-efficient appliance support schemes.

Another avenue could be to explore the interaction and/or trade-off between reduced consumption and smart consumption. Indeed, in a decentralized system, EE means not only energy consumption reduction anymore, but also smart energy consumption. Denmark is still committed to equipping every household with a smart electricity meter by 2020. Despite much interest in intelligent demand response, such a sporadic system could diminish the service aspect of energy use. Thus, a comparative analysis into the savings provided by smart use versus efficient investment could be explored via Balmorel.

¹At the time of the article writing (October 2016), this data is not available and is expected to be released in the upcoming months.

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The Impact of Socioeconomic and Behavioural Factors for Purchasing Energy Efficient Household Appliances: A Case Study for Denmark

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Abstract: Increasing the share of evermore energy efficient household electric appliances is one strategy to address environmental impacts arising from residential electricity demand. Hence, governments and energy actors are interested in the determining factors behind the consumer choice of conventional versus high efficiency labelled appliances. This study employs empirical survey data from the Danish Energy Agency to model influential factors behind Danish consumer choice of energy efficient appliances. To estimate consumer propensities, we use a logistic regression model over a set of socioeconomic, demographic, and behavioural variables. The study regresses over this unique combination of end-use behavioural variables by creating an energy efficiency index. Statistical results show that housing type, quantity

of inhabitants, age, and end-use behaviour are strong predictors for choosing energy efficient appliances. Interestingly, income is a weaker predictor. Despite a relatively wealthy national income and well-educated population, information campaigns have been largely ineffective in driving high efficiency investments. In light of this study's results and exogenous factors such as urbanising demographics and shifting Danish housing stock towards apartments, the study suggests improved information campaigns by targeting key demographics.

Keywords: Consumer behaviour · energy efficiency · household appliances · purchase propensity · regression model

8.1 Introduction

Like other Western European nations, Danish household electricity consumption accounts for more than 20% of total electricity demand (Gaspar and Antunes, 2011). Electric devices such as dishwashers, washing machines, cooking hobs, microwaves, fridges and freezers account for 50% of this figure (FEHA, 2017). The quantity of household appliances, due to rising wealth and access to technology, has increased dramatically over the last decades according to the Danish Association for Suppliers of Electrical Domestic Appliances (FEHA).

In 1992, the European Union (EU) addressed rising household electricity demand and its environmental impacts with the EU Directive 92/75/EC establishing the energy consumption labelling scheme for most white goods and light bulbs (EU, 1992). Its aim was to increase consumer awareness of energy consumption by demanding clearly visible labels classifying electric devices from the most energy efficient (Class A) to the least (Class G). Since 1995, EU consumers have been exposed to this letter-grade labelling system. Given increasing appliance energy efficiency, the EU extended the labelling system with Directive 2010/30/EU by introducing classes A+, A++ and A+++, and is planning to rescale the metric to the A-G scale in the future (see EU, 2015 and EU, 2017 for details).

In light of EU energy saving efforts, the purchasing propensity for EE (energy efficient) appliances, coupled with efficient end-use, now carry greater importance. The drivers of appliance purchasing are diverse: short and long run household economics, attitudes towards the environment, and casual choice, among others. By addressing these factors, governments may improve efficiency standards and labelling campaigns. Due to their diverse nature, though, leveraging these factors and estimating their effect on appliance

purchasing might be challenging. Data from surveys assessing consumer preferences could represent an initial valuable source of information while tools like consumer choice models, for instance, could help drawing considerations about purchasing choices.

Our study is motivated by the following research questions. Which socioeconomic characteristics best predict the consumer selection of high-labelled household appliances? What impact has the end-user behaviour, in relation to energy use and savings, on purchasing such household appliances? Which energy end-use daily actions are more relevant for predicting the purchase of appliances? Accordingly, which policies can increase consumer consciousness of energy efficiency so as to adopt high-labelled household appliances thus reducing CO₂ emissions?

To address these research questions, we considered the results of a Danish Energy Agency (DEA) survey over a representative housing sample, and developed a logistic regression model to predict the propensity of the Danish consumer to choose a new, highest-labelled household appliance. In this model, we employed socioeconomic and demographic variables (e.g., income, age, job of the consumer, housing type, size, year built, and number of inhabitants) as well as a behavioural energy efficiency variable (EE-index) calculated from a set of consumer energy end-use behavioural questions (e.g., turning off the power sockets during the night and adapting the heating system to the seasons). Based on the model results, we estimated propensities of Danish consumers to choose more efficient appliances at the moment of purchase. Eventually, we drew policy recommendations, relevant beyond the Danish context, to foster energy efficient behaviours and increase the purchase of EE appliances in the residential sector.

This paper contributes to the field by developing novel methodology resulting in practical findings that can be useful for policy makers and governmental institutions. On the methodological side, the contributions of the paper consist of (i) the construction of the EE-index that gathers and synthesizes a rich set of consumer behavioural characteristics and daily actions regarding energy end-use and energy savings, and (ii) the integration of such index in a consumer choice model to study the joint effect of socioeconomic, demographic, and behavioural variables on consumers energy efficiency investment choices. Finally, unlike previous studies, (iii) we performed an extensive investigation of a behavioural index through correlation matrices and by examining interrelations between its constituent parts.

On the practical side, we find from our statistical results that socioeconomic and behavioural characteristics are highly significant when explaining the choice of purchasing EE appliances. Specifically, income, housing type, quantity of inhabitants, age, and end-use behaviour are predictors for choosing energy efficient appliances, with EE-index and

housing type being the strongest of these predictors while income is weaker. From our analysis of the EE-index, we identify that specific daily actions are correlated with investment in efficient household appliances. Furthermore, by analyzing the correlations within the EE-index, we found that respondents generalize their EE behaviour by appliance type and that efficient end-use behaviours are related with particular living conditions, e.g., housing type.

By providing empirical results on the influence of both socioeconomic and behavioural variables on consumer choice, the paper narrows the knowledge gap on household energy consumption behaviour and broadens knowledge on the drivers of purchasing high-labelled household appliances.

The remainder of the paper is organized as follows. In Section 8.2, we review the literature on household energy consumption behaviour. In Section 8.3, we introduce the survey data and describe the consumer investment model based on logistic regression. In Section 8.4, we present the model estimation results and discuss the effect of different socioeconomic, demographic, and behavioural variables in the choice of EE appliances. We conclude in Section 8.5 by drawing practical policy suggestions based on our findings.

8.2 Literature review

The study of household energy consumption behaviour focuses on understanding the reasons why end-users adopt particular consumption patterns. Four key questions are the focus of debate: (1) What is driving energy consumption; (2) How does lifestyle and habits influence the use of energy; (3) Which models can closely describe the consumer behaviour; and (4) Which policies can be proposed to decrease total energy use.

Socioeconomic characteristics are often cited as significant drivers of household energy consumption. Global research programs, conducted via household surveys, suggest that demographic and socioeconomic factors, such as income level, ownership, dwelling type and number of inhabitants, are correlated with the energy use (De Almeida et al., 2011; Bedir et al., 2013; Wyatt, 2013; Zhou and Teng, 2013; Hayn et al., 2014; Huebner et al., 2015; Murphy, 2014; Jones and Lomas, 2016; Zhou and Yang, 2016; Girod et al., 2017).

Beyond these factors, researchers stress the focus on energy consumers' end-use behaviour. Lifestyle and habits impact the final use of energy, most often in an unpredictable way (Zhou and Teng, 2013; Gram-Hanssen, 2014; Frederiks et al., 2015). Empirical research indicates that behaviour (or comfort preference) is related to the socioeconomic charac-

teristics, including income (Vassileva et al., 2012), household type (Bedir et al., 2013; Huebner et al., 2015; Jones and Lomas, 2016; Girod et al., 2017), family age composition (Mills and Schleich, 2012), and employment (Hayn et al., 2014). Additionally, ulterior motives influence behaviour such as environmental consciousness (Gram-Hanssen, 2014; Zhou and Yang, 2016), environmental innovation intention (Long et al., 2017b) and attitude towards environmental behaviour (Long et al., 2017a) which, ultimately, has an impact on consumer's intentions (Ajzen, 1991; Abrahamse and Steg, 2009).

The aforementioned socioeconomic and behavioural characteristics are also studied as relevant reasons prompting consumers to choose high-labelled appliances. The results from a 2014 Organisation for Economic Co-operation and Development (OECD) survey on household environmental behaviour and attitudes identified potential factors behind consumer choices on energy efficiency investments as home ownership, income, social context, and household energy conservation practices (Ameli and Brandt, 2015). Various analyses, based on different surveys in an international context, resulted in similar conclusions (Mills and Schleich, 2010b; Gaspar and Antunes, 2011; Qiu et al., 2014; Jacobsen, 2015). In the Danish context, although previous studies have used survey results to assess the factors influencing household electricity consumption (Bartiaux and Gram-Hanssen, 2005), efficient utilization of household appliances (Nielsen, 1993), and patterns of domestic electricity use (Gram-Hanssen et al., 2004), to the best of our knowledge none has focused on purchase propensities in relation to energy efficient household appliances. Moreover, while other studies made use of energy-related behaviours and habits in consumer models (Gaspar and Antunes, 2011; Kavousian et al., 2013; Krishnamurthy and Kriström, 2015; Ameli and Brandt, 2015), none has performed an extensive investigation of such energy end-use behaviours. In fact, in this paper we analyze interrelations among various behavioural components to investigate which actions make the consumer more likely to invest in EE appliances and if specific end-use behaviours are related to particular living conditions.

The science of consumer behaviour and energy literacy—that is, the ability of consumers to make rational decisions on EE investments (Brounen et al., 2013)—adopts and employs energy efficient behavioural measures, equipment, intentions and planned behaviour (Abrahamse and Steg, 2009; Ajzen, 1991; Long et al., 2017a). Often, when designing appropriate tools, the economic theories on consumer's choices are based on rational maximizing models describing how consumers should choose (normative theories) rather than how they do choose (descriptive theory). Results from orthodox-economic models where the consumer is depicted as a robot-like expert, can thus be a poor prediction of the actual behaviour of the average consumer (Thaler, 1980). Realistic empirical studies provide evidence that consumers don't always act rationally and their choices are influenced by a myriad of non-rational influences. Thus, consumer behaviour models, if wrongly formu-

lated, can lead to misleading outcomes (Thaler, 1981). Realistic consumer behaviour is crucial when designing proper tools for predicting or describing consumer choices. With this in mind, in this paper we built a logistic regression model—validated using different statistical tests—that accounts for socioeconomic and demographic variables as well as behaviours, trying to capture non-rational influences on consumer choices. This model provided us with interesting insights on the characteristics influencing the decision process of the consumers when purchasing high-labelled household appliances.

Studies investigating the success of policies implemented, such as the ENERGY STAR in the U.S. or A-G energy labels in Europe, show that financial incentives (subsidies), energy audits, minimum energy performance standards (MEPS), energy literacy and reduced value added taxes for EE technologies contribute positively to the uptake of energy efficient appliances and replacement of old equipment (De Almeida et al., 2011; Mills and Schleich, 2012; Brounen et al., 2013; OECD, 2013; Murphy, 2014; Krishnamurthy and Kriström, 2015; Datta and Filippini, 2016; Zhou and Yang, 2016; Girod et al., 2017). Similar to MEPS, mandated energy efficiency measures (for new equipment) coupled with properly designed and implemented public awareness campaigns results in legitimate energy savings (Wyatt, 2013; Frederiks et al., 2015; Young, 2008). A recent analysis on the Danish market, for example, showed that new labelling schemes lead to a notable increase in the sales of EE appliances (Bjerregaard and Framroze Møller, 2017). However, in contrast, some of the literature on the efficacy of policies and information campaigns showed that a large portion of the population is still unaware of energy labelling (De Almeida et al., 2011; McMichael and Shipworth, 2013; OECD, 2013; Zhou and Yang, 2016) or energy conservation behaviour measures (Brounen et al., 2013). Finally, recent research has shown that policies and actions need be tailored to specific households, tenants and technologies since a generalized approach might not work as efficiently and lead to less than desirable outcomes (Vassileva et al., 2012; Frederiks et al., 2015; Krishnamurthy and Kriström, 2015; Jones and Lomas, 2016; Chai and Samatha, 2017; Girod et al., 2017). Following this literature, in this paper we suggest improved energy efficiency policies that indeed target key demographics identified through our purchase propensity analysis.

8.3 Data and model

The primary dataset analysed in this study is the DEA's bi-annual survey "El-model Bolig", the goal of which was to collect information about consumers' purchasing and use of household appliances. Although the survey is performed every two years, the 2012 set was chosen over the most recent dissemination because the 2012 survey uniquely contains questions on the efficiency labelling of major household appliances. The total number

of survey respondents, or observations, was 2053; however, we removed 337 observations due to missing values giving a final sample size of $n = 1716$. The survey comprises about 340 questions in total. The number of questions for each respondent, though, depends on logical operators and reported ownership—for instance, a respondent without a freezer will not be asked questions about its usage. The sampling was conducted under random block design as to approximately represent Denmark’s geographic and housing category distributions (apartments, farmhouses etc.), and was not stratified with respect to other socioeconomic and demographic variables.

8.3.1 Socioeconomic, demographic, and behavioural variables

The primary variables of interest from the survey are the socioeconomic and demographic variables listed below, chosen with the intention of predicting investment in the highest EE labelling.

- Age: an ordered categorical variable whereby Age 1 = 18–29 years, Age 2 = 30–39 years, Age 3 = 40–49 years, Age 4 = 50–59 years, and Age 5 = 60 years or older.
- Quantity of inhabitants: recorded as a continuous variable in the original survey dataset, counting the total number of adults and children living in the respondents’ household.
- Housing type: four choice levels given by apartment, farmhouse, single/detached (referred to as “single” henceforth), and townhouse.
- House size: an ordered categorical variable with 8 levels from less than $39 m^2$ to over $200 m^2$ interior floor space.
- Year built: an ordered categorical variable with 6 levels for the year a house/apartment was constructed, ranging from before 1900 to 2001 or newer.
- Income: gross household income (before taxes).
- Investments in EE appliances, that is, the labelling of most recent purchased appliance.

Beyond questions about appliance investment and ownership, the survey contains a wealth of questions regarding end-use behaviour for appliances and heating systems. Several of these questions can capture whether the consumer performs daily activities classifiable as energy efficient behaviour. Questions like “How full do you fill your clothes/washing

machine on a normal use” or “Do you turn off the power socket during the night” have thus been used (see Appendix 8.6.1 for the full list of questions included in the index). We incorporated these unique responses by computing a behavioural energy efficiency index, abbreviated throughout the paper as EE-index. The combination of these variables in the index represents a level of energy consciousness and intent to save energy for both electricity and heating. For example, managing heating between night and day (turn heat down at night) or removing power sockets after use are all positive EE indicators.

To compute the EE-index, we assigned each question an equally weighted point: 1 for positive energy saving behaviour, 0 for poor behaviour. Although in theory different actions can result in different levels of energy savings, the survey does not contain detailed appliance and action characteristics (e.g., appliance type, capacity, consumption, time of use) that enable directly quantified savings nor define action-specific weights. All questions are weighted equally with scores normalised per each respondent’s appliance portfolio. Of course, not all respondents own oil or natural gas heating, for instance. Thus, to compare respondents with differing levels of appliance ownership, the individual scores were standardized by their individually maximum possible score (see Appendix 8.6.1 for the percentage of respondents eligible for each question). The score is defined for each consumer $j \in \{1, \dots, n = 1716\}$ in the sample as

$$\text{EE-index}_j = \frac{1}{Q_j} \sum_{i=1}^{\bar{Q}} Z_{ij},$$

where \bar{Q} is the total number of questions, Q_j is the count of eligible questions for respondent j , and Z_{ij} equals 1 for a point awarded to respondent j for question i . Eligible questions Q_j are counted according to the appliance ownership profile of respondent j . For example, a respondent without a washing machine will not be scored nor counted in Q_j for questions pertaining to washing machine use. The index is on a $[0, 1]$ scale. Of course, more appliances (greater summed Q_j) will decrease the marginal weight of each point, that is, the index is less sensitive to those with many appliances or eligible questions.

The survey also contains additional questions regarding profession of respondent and spouse, lighting system and electricity consumption. In addition to the EE-index, we thus calculated:

- a “job index” whereby the respondents’ professions were ranked per average years of training or education on a scale from 1 to 10 for the job categories included in the survey. This job index was then considered a potential predictor of EE appliance investment.

- a “light score” assessing the respondents’ ownership of EE lighting. The light score is calculated as the ratio of reported saving light bulbs, or EE lighting (for instance, LEDs and compact fluorescent lamps), to the total sum of both EE lighting and traditional incandescent light bulbs. Thus, the score is normalised on a $[0, 1]$ scale.
- the “know el.”, representing a non-socioeconomic binary variable equalling 1 if the respondent reports to currently know her annual electricity consumption, and 0 if the respondent reports not knowing.

Table 8.1: Explanatory variable name, type, and description

Explanatory variable	Type	Description
Qty. inhabitants	continuous	Number of household inhabitants, from 1 to ≥ 8
House type	categorical	4 levels: apartment, farmhouse, single house, townhouse
House size	categorical	8 levels, from less than $39 m^2$ to over $200 m^2$
Year-built	categorical	6 levels, from < 1900 to ≥ 2001
Age	categorical	5 levels: 18–29, 30–39, 40–49, 50–59, 60 or older
Income	continuous	Gross household income, in $[0, +\infty]$
EE-index	continuous	Behavioural energy efficiency index, in $[0, 1]$
Job index	continuous	Average years of education/training, in $[1, 10]$
Light score	continuous	Energy efficiency lighting ownership, in $[0, 1]$
Know el.	categorical	Knowledge of own electricity consumption, in $\{0, 1\}$

In Table 8.1 we present a summary of the explanatory variables used in the model, and their characteristics. Lastly, we are interested in the investments in EE labelled appliances. The survey asked each respondent to state the energy labelling of a given appliance they report to own or which they had recently purchased. The full set of appliances in the survey are: combination washer-dryer, washing machine (standalone), dryer (standalone), dishwasher, combination fridge/freezer, fridge with integrated box freezer, fridge (standalone), chest-freezer, and a standing freezer. For some of the appliances (e.g., chest-freezer) too few respondents reported ownership, not allowing us to make a meaningful analysis per each individual appliance. Thus, the set is aggregated to a singular latent variable: “for her most recent purchase in any one of these appliances, has the consumer invested in the rating A+ or higher?” Because of such aggregation, 68% of respondents reported EE investment while 32% did not. The rest of the paper focuses on identifying which of the explanatory variables best distinguish these two groups of consumers. We first present descriptive statistics for the modelling sample and compare them against national statistics.

8.3.2 Dataset validation

To verify that our dataset provides a good representation of Denmark, we compared the distribution of the socioeconomic factors in our modelling sample against the 2012 national statistics from Statistics Denmark (DS; see DS, 2017b).

The age distributions of the survey sample and DS are displayed in Figure 8.1. The distribution of the survey sample is slightly skewed towards middle and elder ages since, typically, it is the head of the household who is answering the survey. This explains why age level 1 is only 7% of the survey sample while age level 4 is 32%. The remaining classes are similar to those of DS.

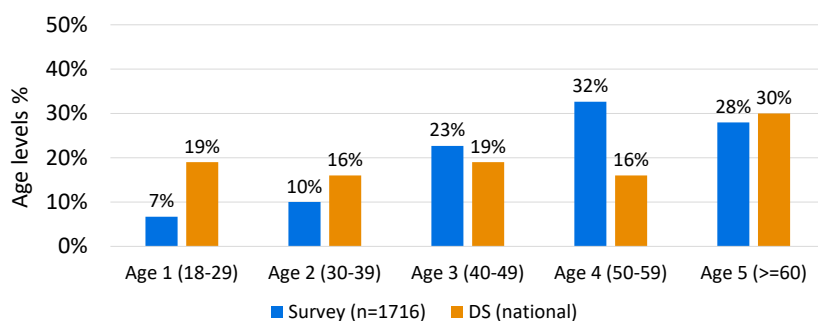


Figure 8.1: Age of respondents: survey sample and national statistics

The survey distribution of the number of inhabitants is displayed in Figure 8.2 and is deemed fairly representative. Some differences compared to the national statistics hold for one and two inhabitants per household, but overall are acceptable.

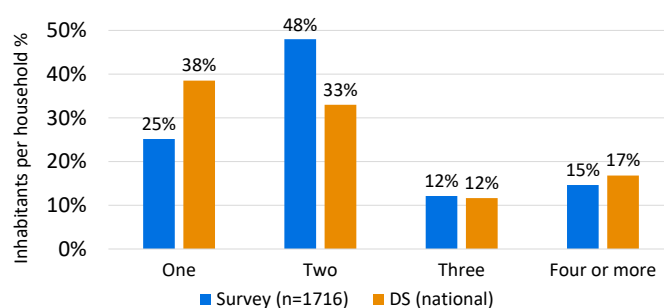


Figure 8.2: Quantity of inhabitants: survey sample and national statistics

Regarding housing age, Table 8.2 shows that the 2012 distribution of year built in the survey sample closely matches that of official registries, thus, it is representative of Denmark.

The variables for which a comparison was not possible include housing type and income. Regarding housing type, the categories used in the survey diverge from those recorded

Table 8.2: Housing year built: survey sample and DS.

Year built	Sample	DS
Before 1900	8%	9%
1900–1925	11%	12%
1926–1950	15%	16%
1951–1975	34%	32%
1976–2000	25%	23%
2001 or newer	7%	8%

in the official statistics. For example, DS includes student housing and cottages, which are ignored by the survey. Moreover, DS includes some detached housing types in its farmhouse category, whereas the survey farmhouse category explicitly pertains to properties with land holding. As a consequence, a comparison between DS and the survey housing type distributions would be misleading. Regarding income, the survey originally reported the total household income before taxes, whereas DS reported the “disposable equivalised income”, which is the household income after taxation divided by a weighted number of adults and dependents living in the given household (DS, 2015). Therefore, any comparison would be inaccurate due to the different income calculation and the inability to assume taxation rates on the survey’s gross incomes and convert gross incomes into disposable incomes.

8.3.3 Consumer investment model

Consumer behaviour in relation to investments in household energy efficient appliances is evaluated with a discrete choice model. The merit of this modelling framework is the ability to empirically test the predictive strength of the survey’s explanatory variables. Specifically, we use a logistic regression model that is constructed as follows. The EE investment is considered as a binary outcome Y ($1 = \text{investment}$, $0 = \text{no investment}$) and the model assumes that

$$\begin{aligned} \text{logit}(P(Y = 1 | X_1 = x_1, \dots, X_n = x_n)) &= \log \frac{P(Y = 1 | X_1 = x_1, \dots, X_n = x_n)}{1 - P(Y = 1 | X_1 = x_1, \dots, X_n = x_n)} \\ &= \beta_0 + \beta_1 x_1 + \dots + \beta_n x_n, \end{aligned}$$

where $X = [X_1, \dots, X_n]$ represents the vector of all explanatory variables discussed in Section 8.3.1 (age, income, type of house, EE-index etc.) and $\beta = [\beta_0, \dots, \beta_n]$ the weight vector. The dependent variable Y represents a single investment in an A+ or higher labelled appliance among a set of nine appliances listed in the survey (the set of appliances

is considered as aggregated to maintain an adequate sampling size and distribution, as discussed in Section 8.3.1).

To estimate the model, the weights β are fitted through logistic regression on the survey data via the logit maximum likelihood function. Then, given the estimates $\hat{\beta} = [\hat{\beta}_0, \dots, \hat{\beta}_n]$ and the characteristics of a consumer $x = [x_1, \dots, x_n]$, the resulting predicted joint-probability of EE appliance investment π , or the probability that $Y = 1$, is computed as:

$$\pi = P(Y = 1 | X_1 = x_1, \dots, X_n = x_n) = \frac{\exp(\hat{\beta}_0 + \hat{\beta}_1 x_1 + \dots + \hat{\beta}_n x_n)}{1 + \exp(\hat{\beta}_0 + \hat{\beta}_1 x_1 + \dots + \hat{\beta}_n x_n)}.$$

8.4 Results and discussion

8.4.1 Model estimation

Table 8.3 reports the outcome of the multivariate regression consumer investment model, computed with the software R. The final regressors are chosen, according to common practice, using a backwards elimination process until the model only contains statistically significant explanatory variables (Derksen and Keselman, 1992). The factor levels age

Table 8.3: Consumer investment model estimates. Significance codes for p-values: 0.001 ‘***’, 0.01 ‘**’, 0.05 ‘*’, 0.1 ‘.’

Explanatory variable	$\hat{\beta}$ estimate	Std. error	p-value	Significance level
Intercept	-2.001	0.295	< 0.001	***
Income	0.076	0.030	0.013	*
Light score	0.480	0.180	0.007	**
EE-index	0.762	0.303	0.010	*
Know el.	0.221	0.127	0.082	*
Qty. inhabitants	0.198	0.066	0.002	**
Farmhouse	0.673	0.230	0.003	**
Single house	0.550	0.142	< 0.001	***
Townhouse	0.304	0.173	0.079	.
Age group 2	0.674	0.267	0.011	**
Age group 3	0.683	0.245	0.005	**
Age group 4	0.712	0.242	0.003	**
Age group 5	0.849	0.244	< 0.001	***

group 1 and apartment are considered model reference levels and thus do not respectively have model terms. The joint probability for age group 1 and apartment is considered

to be the estimate of the model intercept, or the probability of investing when all other variables are set to 0. One can see that the explanatory variables positively affect the total probability of EE investment choice. For example, assuming all other variables constant, by increasing income of one unit (100,000 DKK), the expected odds of choosing an EE appliance will be 1.079 times greater (since $\exp(0.076) = 1.079$).

The values in Table 8.3 represent the outcome of the final model only. Other explanatory variables, as house size or job of the respondent, were included in a previous larger model but discarded in the backward elimination process. Table 8.4 reports the dropped explanatory variables (that is, with p-values higher than 0.1) along with their $\hat{\beta}$ estimates. The dropped model estimates show that the year in which the building was built, the size of the households and the job of the respondent appear not to be relevant characteristics to predict selection of EE appliances.

Table 8.4: Consumer investment model estimates for the dropped explanatory variables.

Explanatory variable	$\hat{\beta}$ estimate	Std. error	p-value
Year-built	0.045	0.044	0.313
House size	0.005	0.032	0.867
Job index	0.014	0.017	0.410

The final model adapted for the analysis has been validated to prove the consistency of the findings and assess the reliability of the model. Different criteria have been used for model diagnostics:

1. The Hosmer-Lemeshow's Goodness of Fit test is widely used in logistic regression to prove the fit between the model and the data (Hosmer and Lemeshow, 1980). It tests against the null hypothesis H_0 of observed investment rates matching the predicted ones, and returns a p-value. A p-value lower than 0.05 suggests that the model does not adequately predict the binary outcome of Y and should be rejected. The outcome of the test was a p-value of 0.33, meaning there is no evidence to reject the model.
2. The McFadden R-squared test is similar to the R-squared test but based on the rho-squared measure (McFadden, 1977). The test returns a value representing the predictive ability of the fitted model compared to the null model, that is, a model with only an intercept and no covariates. According to the test, any result between 0.2 and 0.4 represents an excellent fit. The outcome for our model was a value of 0.21.

8.4.2 EE-index and light score

We summarise the most important variables in the EE-index composition in the form of a heatmap in Figure 8.3. The EE-index variables are divided by housing type and, for brevity, they are listed in their coded format (see the Appendix for full description). The graph reports the ratio r whereby the numerator is the total sum of points for question i for all of respondents in housing type k , and the denominator is the sum of eligible respondents per question, per housing type. The ratio r allows for relative comparisons within and between each housing type: a block at 100% indicates that all respondents of housing type k received points for that particular question. For example, question X587 has one of the greatest relative importance for farmhouses (indicating whether or not the respondent turns her natural gas heating to summer mode).

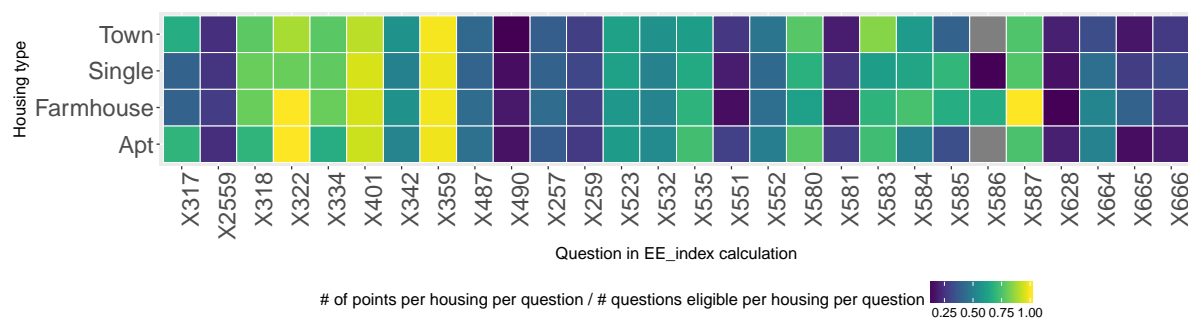


Figure 8.3: Heatmap with ratio r showing relative percentage of EE-index points by housing type.

We noticed that the housing types differ with respect to the EE-index and its composition. Some questions as X359 and X401 are relevant for all housings types with scoring close to 100% (one point awarded if the dishwasher and washing machine, respectively, are filled to over 50% per average use). In contrast, question X334 is relatively less important for apartments (one point awarded if owners of standalone washing machines normally wash at the highest RPM setting). Some questions carry little weight in the final score calculations since they pertain to specific heating technology behaviours such as X666 (a point awarded if the respondent applies a normal step circulation pump in her radiant heating system).

Figure 8.4 shows the correlation between each of the EE-index questions. The purpose of the graph is (i) to assess whether performing a specific energy end-use action is correlated to other actions, and (ii) to identify overall trends in end-use from the survey sample data.

An examination of these correlations reveals that there are several clusters of positively correlated variables, indicated by the dark blue colours. For example, one cluster is for

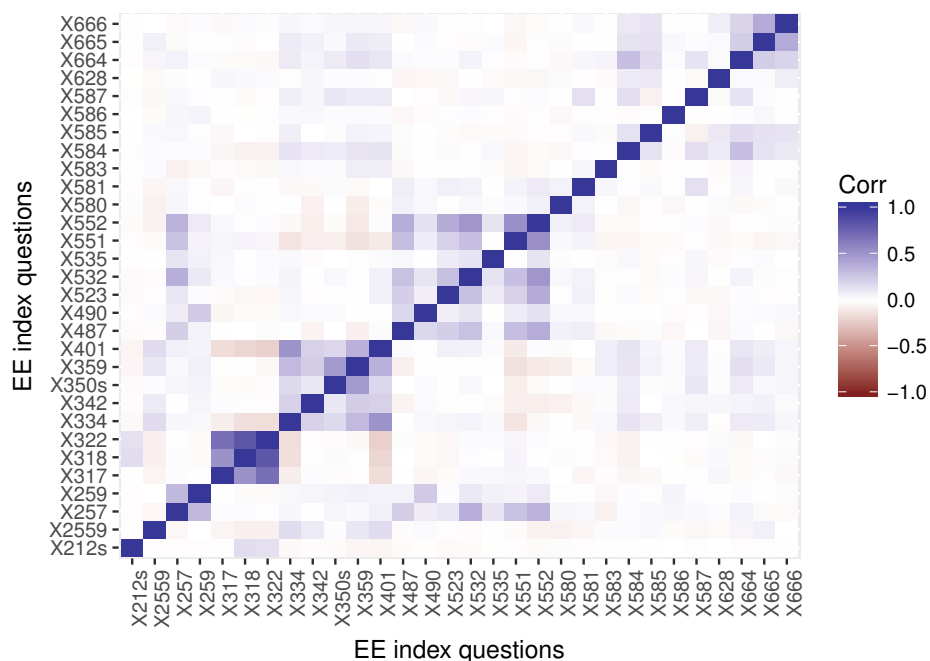


Figure 8.4: Correlation matrix of all EE-index questions

variables X317, X318, and X322, all related to dryer usage. Another positive cluster includes X487, X490, X523, X532, X551, and X552 which pertain to whether or not the respondent removes a specific appliance from the power socket after use. Also, a cluster includes variables X664, X665, and X666, pertaining to behaviour with heating technologies, such as turning your circulation pump to summer mode. The prevalence of these positively correlated clusters suggests that consumers generalize their behaviour by appliance. For example, a consumer who normally washes clothes at a low temperature is more likely to report EE conscious behaviour on remaining washing machine questions. The correlation analysis also shows that there are few negatively correlated variables.

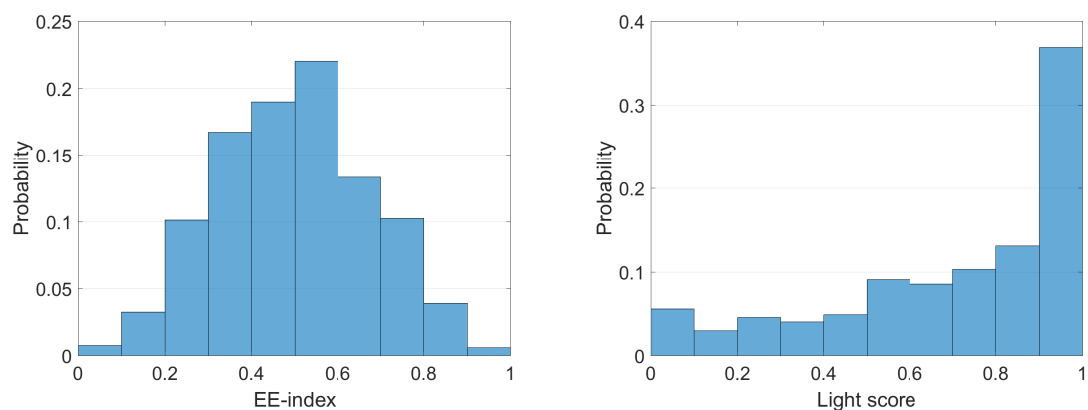


Figure 8.5: Probability distributions for EE-index (left) and light score (right).

The EE-index scores, shown in Figure 8.5 (left), present a distribution with a mean of

0.49 and a standard deviation of 0.17 which resembles a normal distribution. In Figure 8.5 (*right*) we also display the distribution of the light score. The score is highly left tail skewed. Moreover, more than 30% of respondents reported having only EE lighting (no incandescent lights), explaining the peak corresponding to an index value in the interval $[0.9, 1]$.

8.4.3 Purchase propensity curves

Propensity curves have been computed to study how the predicted probabilities in EE appliance choice change per variations in the explanatory variables. The curves are evaluated varying one variable at a time, while keeping the others fixed to the following values: income is kept fixed to 400,000 DKK, EE-index, light score and know el. are kept fixed to 0.5, number of inhabitants to 2 and age to class 3.

Figure 8.6 shows the development of the expected probabilities for different levels of income. The trends suggest that the higher the income, and consequently wealth, of the respondent, the higher is the probability that the same respondent will choose more efficient household appliances when investing. The curves are reported for the different type of dwellings to simplify the understanding of the analysis. The differing levels (intercepts) of the curves illustrates the importance of the house type factor: the propensity curves for choosing energy efficient appliances for farmhouses and single houses are on average more than 10% higher than apartments, and up to 15-17% higher for low income levels.

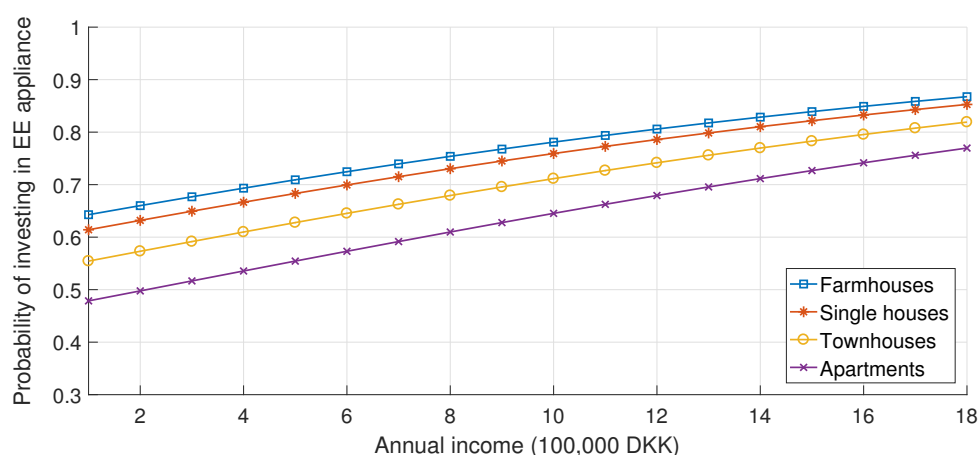


Figure 8.6: Predicted probability of investing in EE appliance by income.

Figure 8.7 and 8.8 report the development of the probabilities for the number of inhabitants and EE-index, respectively. The figures show that a higher number of people living

in the dwelling, as well as a higher EE-index, results in a greater predicted propensity for choosing energy efficient appliances. The curve levels for different housing types are consistent with those of Figure 8.6, with farmhouses and single houses being substantially higher than apartments.

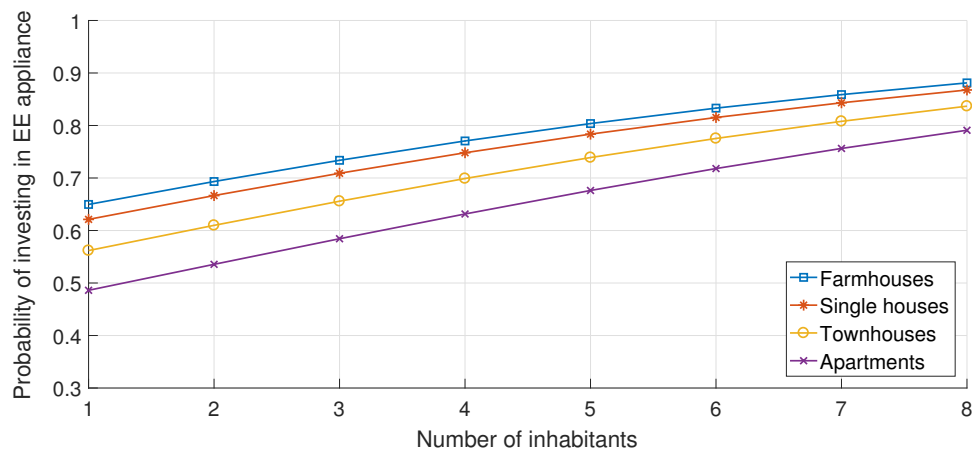


Figure 8.7: Predicted probability of investing in EE appliance by inhabitants.

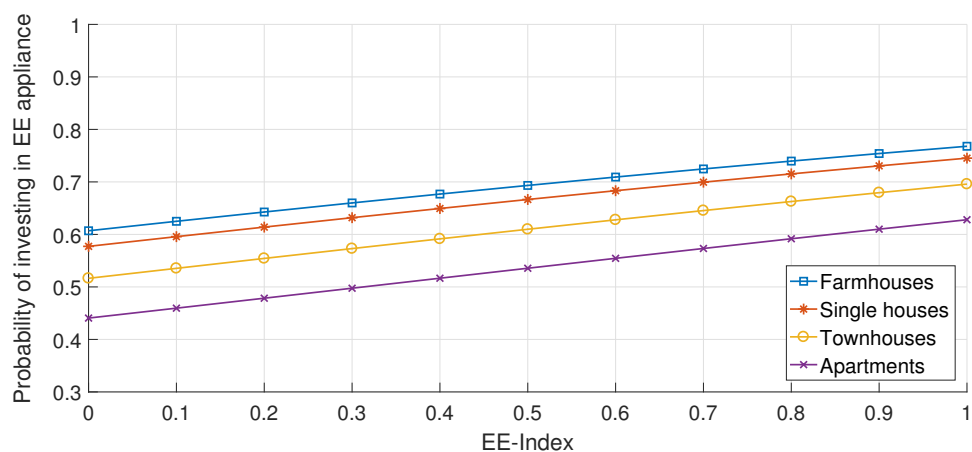


Figure 8.8: Predicted probability of investing in EE appliance by EE-index

Figures 8.9 and 8.10 illustrate respectively, the point-wise estimated probability for varying age and housing type, along with 95% confidence intervals. The results suggest that older respondents have a higher propensity to choose energy efficient appliances, as only the groups 2 through 5 differ significantly from group 1. Likewise, the range of the confidence intervals varies for the different type of dwellings. The apartments and single family houses present larger variation compared to other housing types. Also, the predicted probability is the highest for farmhouses and the lowest for apartments.

The probabilities of choosing EE investments resulting from the model can be perceived as generally high (e.g., the average rates are above 50–60%). This is explained by the original distribution of the reported survey data. Finally, we assessed the robustness of

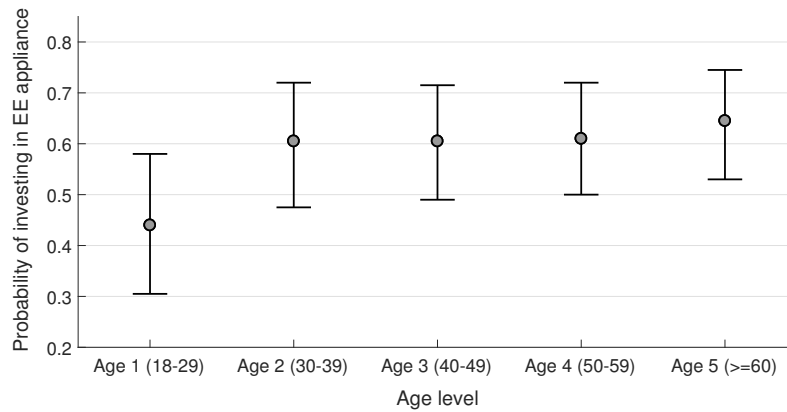


Figure 8.9: Predicted probability of investing in EE appliance by age.

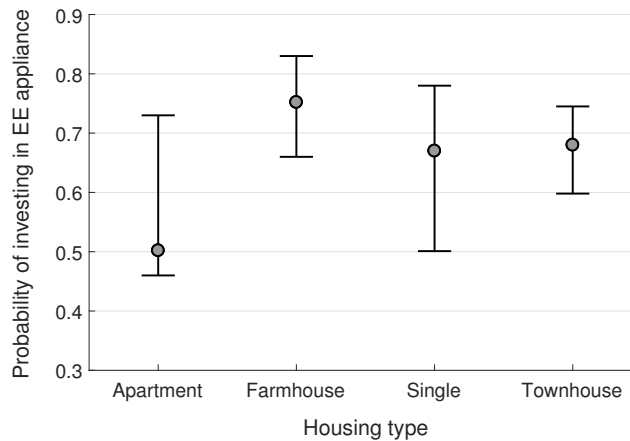


Figure 8.10: Predicted probability of investing in EE appliance by housing type.

the model using a marginal effect plot with a bootstrap error, displayed in Figure 8.11 for the different variables.

This analysis is employed to assess the sensitivity of the originally computed model estimates to statistical assumptions. The bootstrap method draws 1000 random samples from the original survey data, recalculating model estimates 1000 different times. If the bootstrapped estimates and standard errors deviate substantially from the original values, there is evidence of major violations of statistical assumptions (that is, collinearity or low predicting power resulting from few observations). Like the original coefficients, the marginal effects can be seen as partial derivatives of total joint-probability function. The average of the re-sampled marginal effects is the midpoint, while the tails illustrate the 95% confidence interval. The bootstrapping shows that the income is hardly significant and casts some doubt about the strength of income to predict EE investment choice, compared to the more qualitative EE-index and house setting. Given the results, farmhouses are more likely to choose EE appliances when compared to other house types.

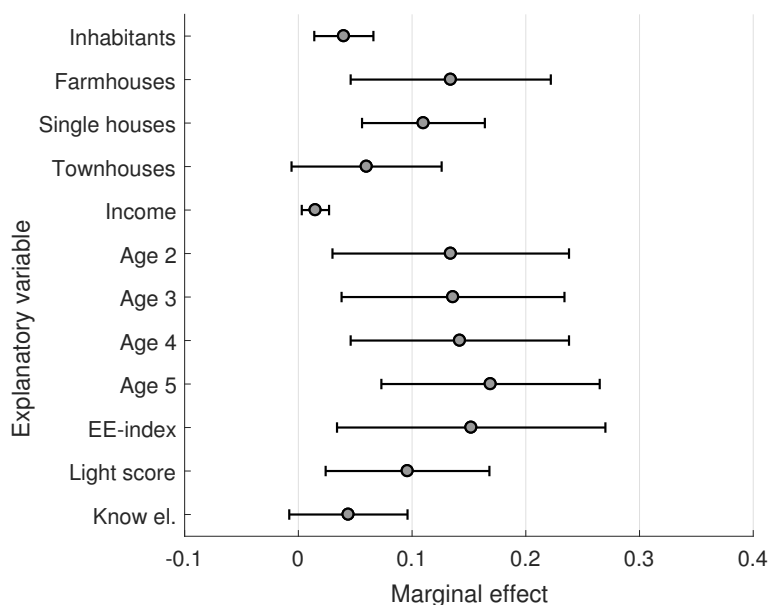


Figure 8.11: Marginal effects with bootstrap errors of the explanatory variables

8.4.4 Discussion of the results

The positive correlation between household income and EE appliances adoption concurs with previous studies (Long, 1993; Mills and Schleich, 2010a; Sardianou and Genoudi, 2013; Ameli and Brandt, 2015). However, our results show that income is not one of the strongest predictors to EE appliance purchases when compared to other variables considered. This finding might be specific to Denmark, a country with relatively high income and social welfare; in other countries, household income could possibly reveal to be the strongest predictor. Moreover, given the available data and logistic modelling assumptions, there is no convergence to 100% probability of investment for the highest income classes. In fact, even the highest earning consumers are unpredictable in their choices, and as mentioned, driving factors extend beyond energy efficiency to include cost, quality, brand, and functionality (Gaspar and Antunes, 2011; Baldini and Trivella, 2017).

The building type is one of the strongest predictors of EE appliance choice. In particular, the values of the estimates in Table 8.3 show that farmhouses and single family homes residents are significantly more likely to choose EE investments than apartment residents. The related purchase propensity curves in Figures 8.6–8.8 also highlight the relevance of the household type for this analysis. The curves vary because consumers living in different housing types, on average, own a different number of household appliances and have different levels of wealth and lifestyle. This translates into an energy end-use and attitude towards energy efficiency and environment that can vary substantially among

these groups. Apartments, for example, are associated with a lower probability of purchase because they are often rented out, and renters are less sensitive to energy-efficiency investments due to the short length of the stay (see our related discussion Section 8.5.2). In contrast, farmhouse dwellers typically own the property. Moreover, they are in general more sensitive towards energy-efficiency because farmhouses are, on average, larger than apartments and contain more appliances thus incurring higher expenses for electricity and heating. This leads to a higher purchase propensity as also confirmed by our results. Consistently with this discussion, single houses and townhouses lie somewhere in between as shown in Figures 8.6–8.8. Previous studies focusing on more specific investments (heat pumps, EE windows) agree with such correlation (Mills and Schleich, 2009; Michelsen and Madlener, 2012; Ameli and Brandt, 2015). More technical housing variables such as house size or year of construction appear instead to be insignificant.

Regarding age, respondents younger than 30 years are significantly less likely to invest in EE appliances. Other studies suggest that age, as a predictor, is sensitive to specific technologies: older consumers are more likely to invest in EE light bulbs (Mills and Schleich, 2010a; Mills and Schleich, 2010b), renewable energy technologies as wind mills and solar photovoltaic (Willis et al., 2011), but not heat pumps (Mills and Schleich, 2009; Willis et al., 2011; Michelsen and Madlener, 2012).

On the quantity of inhabitants, the estimates confirm the positive relationship: the odds of investing in EE appliances increase with inhabitants. Several other studies achieved a similar conclusion (Mills and Schleich, 2010b; Mills and Schleich, 2012; Ameli and Brandt, 2015). A larger household inhabitancy results in greater and more intensive energy consumption; reasonably, these households would have a greater incentive to invest in energy savings assuming rational economic behaviour (Bartiaux and Gram-Hanssen, 2005).

The variables light score and know el. result in comparatively strong, positive parameter estimates. Respondents with more EE lighting and those who report knowing their own consumption choose more efficient appliances at the moment of purchase; this suggests that one EE conscious behaviour begets the next.

The EE-index's high significance (and especially large parameter estimate) shows how daily energy conservation actions such as turning off the power socket by night and adapting the heating system to the seasons strongly predict the choice of investing in EE appliances. The positive relationship could be expected since it alludes indirectly to environmental stewardship and energy savings attitudes (and also economic savings). Nevertheless, this paper provides empirical evidence that energy end-use daily actions are correlated with EE investment. Furthermore, the correlation matrix of all the EE-

index questions, showing the correlation between the pertinent energy-savings end-uses, has highlighted that particular EE conscious behaviour begets some others. Thus, another practical finding from the EE-index analysis is that respondents generalize their EE behaviour by appliance group.

The correlation between overall high EE-index scores and A+ label investment poses a future research question: do respondents generalize their appliance specific behaviour because they purchased an A+ label (i.e., I buy green therefore I act green), or do respondents seek A+ appliances because they perceive their previous behaviours as green and efficient. One avenue for future research could be to test this relationship through a combination of surveying and direct end-use observations. Observational data is now possible through advanced metering infrastructure and smart appliances. Though there are privacy concerns, observational data would greatly complement a survey sampling which are inherently prone to bias and response errors.

8.5 Conclusions and policy implications

The study aimed to understand which characteristics lead consumers to choose energy efficient appliances at the moment of purchase. Using data from a DEA survey and a statistically sound logistic regression model, socioeconomic, behavioural, and housing characteristics were found to be highly significant when explaining the choice of investments in EE appliances, with housing type and EE-index being the strongest of these predictors. Particular focus was given to the EE-index, combining all behavioural characteristics pertinent to energy savings, and proving that consumers who performed energy conservation actions regularly were more likely to choose EE appliances.

The outcomes of the study spark suggestions about relevant policy measures. Even though energy efficiency continues to rise among most appliances (Barbieri and Palma, 2017), there are still large groups of the population that for many reasons do not invest in EE appliances. Given the importance of socioeconomic characteristics highlighted by our results, existing labelling directives should be assisted by product designs and promotion targeting citizen with such characteristics, for instance, using subsidised rebates and discounts for consumers who are least likely to undertake the investment. Following the results of this work, we identified three major points that should be addressed while outlining energy saving policies: (i) future development of appliance ownership and population housing, (ii) building ownership versus renting, and (iii) evolution of information campaigns.

8.5.1 Trends of appliance ownership and population housing

As policies are meant to be effective in the long term, it is fundamental to consider the future evolution trends of the appliances. The online tool *El-model Bolig - prognose* (El-model buildings - prognosis, in English; DEA, 2017b), developed by DEA, provides forecasts of appliance ownership based on the same 2012 El-model Bolig survey data employed in this analysis, as well as other survey editions (2006, 2008, 2010, 2012 and 2014). The tool allows user-specified inputs and can produce either linear or Gompertz forecasts of appliances' characteristics such as lifetime, sales, quantity, energy use and sales number. Figure 8.12 reports Gompertz forecasts for the sales of five of the major energy intensive household appliances for apartments (*left*) and detached houses (*right*), for the period 2017–2050. The forecasts do not contain labelling information, but provide a projection based on simple historical ownership data.

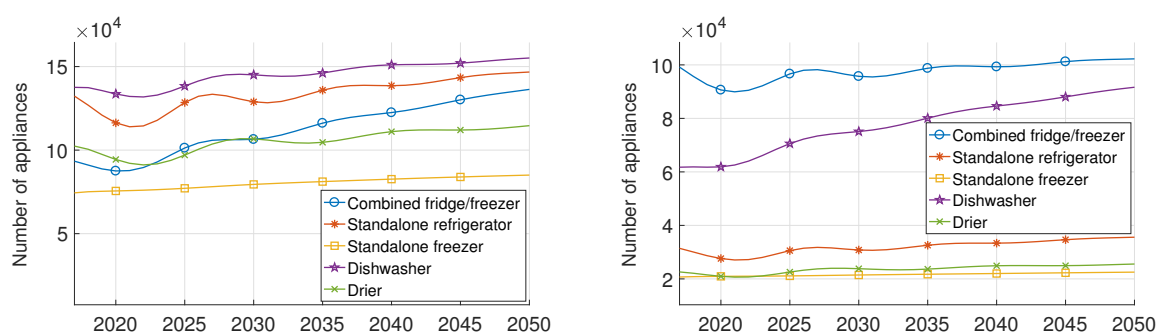


Figure 8.12: Sales forecasts of different appliance categories for apartments (*left*) and detached houses (*right*).

The projections for the apartments and detached housing illustrate increasing ownership, suggesting that residents of apartments and detached houses should be targeted for energy efficiency policy related actions. The raising trend of appliances for these housing types is largely due to an underlying increase in Danish urban populations and housing centres (Trading Economics, 2017). Broadening the scale, this trend is consistent with the recent global trends showing that world's population is increasingly urban with more than half living in urban areas (UN, 2014). As urban populations and the number of urban centres continue to grow, rural populations are expected to decrease. These trends entail a shift of housing conditions to more apartments, implying a change in the energy consumption.

Considering these trends and the results of our study, the authors suggest that policy makers emphasise energy efficiency awareness campaigns for urban citizens, for example, by establishing energy audits to sensitise these users on the contribution of each appliance to total household energy consumption and on the benefits that specific energy efficiency investments would bring in the short and long run. With the underlying assumptions that

the population does not choose EE appliances partly due to lack of knowledge regarding the benefits of energy saving, subsidies should thus be directed to increase the awareness of energy efficient appliance choice with additional focus on end-use behaviour. This should lead to more conscious energy use and savings, which in turn, as suggested by our findings, is correlated to a higher uptake of energy efficient appliances.

8.5.2 Building ownership versus renters

The status of home ownership should also be considered for targeted information campaigns. Intuitively, renters are less likely to choose EE appliances as it is improbable that such investments will break-even; in other words, renters would not enjoy the long run economic benefits of investing in energy efficiency. In fact, the payback time of investments in EE appliances is usually in the range of 5–25 years (Baldini and Trivella, 2017) while the average stay for a renter is shorter. Also, the lifetime of new appliances generally overruns the stay of renters within the building or even in the city and furthermore, particularly for large appliances such as fridge or dishwasher, the transfer to a new location implies logistic challenges. Empirical analyses have also found that renters were significantly less likely to invest in EE refrigerators, clothes washers, dishwashers, and lighting, for example (Davis, 2012; Krishnamurthy and Kriström, 2015). Special focus should be on designing subsidies for short term renters, like students, who are usually the least likely to undertake high upfront investment. In addition to living in rented apartments, students have low or no income and are generally younger than 30, all being socioeconomic factors leading to a low adoption rate of EE investments, as indicated by this study. To this end, dwelling owners could benefit from discount rebates when purchasing high-labelled household appliances for renting purposes. This would consequently help short term renters and in particular students who would enjoy EE appliances without bearing high investment cost.

In Denmark, home ownership levels are geographically disparate, being higher in countryside municipalities (50–65%) and lower in the main cities (e.g., only 20% in Copenhagen) (Kristensen, 2007). Moreover, population forecasts project growth rates of 5–10% across Danish urban centres contrasted to decreasing population rates in rural Western areas (DS, 2017a), supporting our overall recommendation of information campaigns directed especially towards apartment renters.

8.5.3 Evolution of information campaigns

In the past years, Denmark has been active concerning energy efficiency awareness campaigns. Beyond the EU labelling scheme, Ecodesign requirements, and other broad-stroke energy savings targets, there are several Denmark-specific examples pertinent to this study. *SparEnergi.dk* (DEA, 2017c) represents Denmark government's most advanced platform for helping consumers to make energy savings decisions. Launched in November 2013 by DEA, the website contains a wide range of information on how to interpret the current energy labelling for appliance groups (washing machines, dryers, fridge and freezers, lights), along with minimum labelling recommendations (e.g., A for combined washing machine/dryers, A++ for standalone dryers). The guidelines also focus on the size of the appliances and the monetary and energy savings resulting from the choice of a more efficient appliance compared to another. The platform provides suggestions about consumers' end-use behaviour; for example "fill the machine completely", "turn down the temperature", "short program", "clean filters", "leave room for ventilation" are all listed as means to reduce the energy consumption and achieve savings.

Therefore, the problem seems to be not the information itself, or lack of, but rather dissemination. Policy makers should thus improve the means of communication regarding energy efficiency. With respect to the degree of labelling influence, a recent analysis of EU-member residential energy efficiency policy over the period 1974–2016 casts doubt (Filippini et al., 2014). The study indicated that information campaigns such as labelling did not have a significant effect in promoting energy efficiency improvements over that time period. This result, combined with the findings of our study, suggests that the focus for future policies should extend beyond developing the labelling metric itself, to considering what that metric actually means to the consumer in the moment of purchase.

A personalised app or a feature on a merchant website could convey a simplified trade-off between energy efficiency and cost, such as payback times on a price-premium, for instance, going from a dryer label A to A++++. On the matter, *SparEnergi.dk* has just recently started operating free counselling services through popular social medias (Facebook) and call centres (DEA, 2017a; Viegand Maagøe, 2017), to answer questions related to household energy consumption. Given the recent progress and broad access to technology, information campaigns should extend to technology-based platforms such as mobile apps or social media so that they can reach a broader population. For example, local administrations could create and manage a municipality-based social media page, explaining the main factors contributing to the local household energy consumption and providing recommendations on how to reduce it. After the first sparks, the "neighbours effect" should induce the learning process and the dissemination of knowledge through families

and networked communities, leading to a likelihood increase of adoption rates (McMichael and Shipworth, 2013). Also, information and communication technologies can facilitate the transition towards a smarter use of energy by increasing consumer awareness on the impacts related to the number of devices as well as the importance of energy efficiency (Røpke et al., 2010; De Almeida et al., 2011; Zhou and Yang, 2016). For example, smart meters can play an important role by providing visual information about the disaggregated consumption of household appliances or suggesting the consumer to conserve energy during peak hours (Allcott, 2011).

8.6 Appendix

8.6.1 EE-index composition

Table 5 details the rationale and summary of all the variables included in the EE-index. In particular, the table reports variable name, code, and the total number of respondents that are eligible for scoring, meaning that they own the appliance the question refers to and thus can be scored accordingly.

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Table 5: Summary of EE-index questions and scoring rationale

Code	Scoring rationale	Eligible	% eligible
X212	Washing machine temp 29°C or less on average (combi washer-dryer)	52	3%
X2559	Washes clothes at 70°C less 1 time/wk	1442	84%
X257	Remove PC from power socket	1057	62%
X259	PC set to automatically shut down	1057	62%
X317	Dryer used at highest RPM on average	52	3%
X318	Air dries clothes in summer more than using electric dryer (combi washer-dryer)	52	3%
X322	Dryer filled over 50% on average	52	3%
X334	Washing machine used at highest RPM	1442	84%
X342	Air dries clothes in summer more than using electric dryer (standalone dryer)	940	55%
X350	Washing machine temp 29°C or less on average (standalone dryer)	1299	76%
X359	Dishwasher filled over 50% on average	1298	76%
X401	Washing machine filled over 50% on average (standalone washer)	1442	84%
X487	Removes TV from power socket after use	1689	98%
X489	TV set to automatically shut down	1689	98%
X523	Removes laptop from power socket after use	1448	84%
X532	Removes printer from power socket after use	1539	90%
X535	Removes scanner from power socket after use	173	10%
X551	Removes router from power socket after use	1716	100%
X552	Removes other PC/misc electric equipment from power socket after use	1716	100%
X580	Temperature setpoint at 21°C or less	1687	98%
X581	Temperature setpoint regulated night/day	1649	96%
X583	Turns off electric floor heating in summer	175	10%
X584	Turns off radiant floor heating in summer	345	20%
X585	Turns oilfloor to summer-mode	135	8%
X586	Turns oil/wood heating to summer-mode	11	1%
X587	Turns natural gas heating to summer-mode	192	11%
X628	Uses air-to-air heat pump for cooling	72	4%
X664	Changes circulation pump's step in summer	760	44%
X665	Regulates (up/down) circulation pump	522	30%
X666	Has a normal step circulation pump	256	15%

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