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On the Interdependence of Loudspeaker Motor Nonlinearities

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ABSTRACT

Two of the main nonlinearities in the electrodynamic loudspeaker are the position dependence of the force factor, Bl, and the voice coil inductance, L_e. Since they both are determined by the geometry of the motor structure, they cannot be independent. This paper investigates this dependence both analytically and via FEM simulations. Under certain simplifying assumptions the force factor can be shown to be proportional to the spatial derivative of the inductance. Using FEM simulations the implications of this relation is illustrated for drivers with more realistic geometry and material parameters.

1 Introduction

The electrodynamic loudspeaker contains several nonlinearities that degrade sound quality at higher levels [1]. Such nonlinearities are often modelled as a polynomial expansion of voice coil displacement or current, and nonlinear loudspeaker models may contain a number of such nonlinear parameters. Two of the most important nonlinearities are the position dependence of the force factor and the voice coil inductance. When measuring these nonlinearities either via direct measurement on the motor or using system identification algorithms on a complete loudspeaker one can observe that these two nonlinearities vary a lot and no direct link between them seems to always exist.

However, these two nonlinearities are both determined by the geometry and magnetic properties of the materials used and therefore they cannot be completely independent. This paper investigates the connection between the two nonlinearities.

In a recent paper [2] it was demonstrated that the nonlinear Cunningham force [3] is indeed the result of the B-field created by the voice coil, $B_i(x)$, acting on the voice coil itself via a Lorentz force (Bli). Thus the relation between the force factor due to the field created by the voice coil and its inductance is as simple as:

$$Bl_i(x) = \frac{1}{2}\frac{dL_e(x)}{dx} \tag{1}$$

This relation holds for any kind of geometry of the magnetic system and voice coil. The equation may be interpreted as follows: the only way the voice coil can change its inductance with position is if the field lines it creates cross the windings of the voice coil, thus creating a Lorentz force.

However the static force factor, $Bl_0(x)$, generated by the permanent magnet is different from the one induced by the voice coil current and the relation to the voice coil inductance, $L_e(x)$, is not obvious in the general case. These two nonlinearities in loudspeaker drivers are often fitted [4] to measurements by adjusting the (polynomial) coefficient describing the transducers parameters as a function of position, e.g.

$$Bl_0(x) = \sum_{n=0}^{N} b_n x^n \tag{2}$$

and

$$L_e(x) = \sum_{n=0}^{N} l_n x^n. \tag{3}$$

However, since both nonlinearities are determined solely by the geometry and magnetic properties of the motor they cannot be completely independent, and therefore it should be possible to reduce the number of parameters describing the nonlinearities or obtain a better model fit when utilising the inherent connection between these two motor nonlinearities.

The paper is organized as follows: first the case of a (infinitely thin) single winding voice coil is examined, and the relation between force factor and voice coil inductance is derived for a simplified motor structure. Then the influence of voice coil length is examined. Finite element simulations are presented to illustrate how accurate the proposed relation between force factor and voice coil inductance is under more realistic conditions.

2 Analysis of electromagnetic motor nonlinearities

This section explains how the inductance of a loudspeaker voice coil and the force factor can be derived analytically for a simple loudspeaker motor model. The first part of this derivation looks into a short coil with only one winding. This prepares for a more general model with an extended voice coils in the second part.

2.1 Single winding voice coil

The motor unit of the loudspeaker is modelled in the most simple way, as shown in figure 1. It consists of a metal core, a permanent magnet and a coil loop. It is assumed, that the core and magnet material follow a linear magnetization curve with permeabilities μ and μ_m, respectively. The cross sectional area A of the core is rectangular, such that $A = wb$, where w and b are the width and depth of the core, respectively. The core has a slit on one side, the air gap, with a width g' and a cross sectional area of A. The slit is filled with air of permeability μ_0. A single loop of wire is wound around the core in such a way, that it can move from the core's inside to the outside trough the air gap. This loop represents a coil with only one winding. The position of the coil relative to the inside edge of the air gap is denoted by x. The position $x = 0$ indicates that the coil is positioned at the inside edge of the air gap and a position $0 < x < w$ describes that the coil is inside the air gap. The permanent magnet creates a constant magnetic field H_0, while the current i flowing trough the coil will create a magnetic field $H_i(r)$ dependent on the current, both of which are guided trough the core by the high permeability, where r is the observing position inside the gap, measured the same way as x.

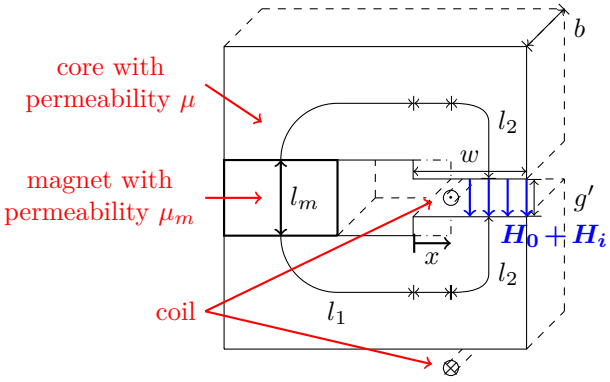

Fig. 1: Model of the core and a single coil. A current i trough the coil creates a magnetic field inside the air gap H_i. The currents direction is out of plane for the coil segment inside the air gap and in plane for the wire outside the core.

H_0 is determined by the strength of the permanent magnet. The following assumptions are made to determine $H_i(r)$:

1. The only place where magnetic flux exits and enters the core material is inside the air gap. Otherwise it is completely contained inside the core, because of its high permeability. This means that flux leakage and free air self-inductance of the coil are neglected.

2. The gap width g' is so small, that the spreading of flux lines at the edges of the gap can be neglected, i.e. the magnetic field outside the air gap is always zero and the magnetic field intensity inside is always perpendicular to the x-axis.

3. The magnetic field in every cross section of the core and air gap is homogeneous, as long as the coils stays completely inside the motor system, i.e. left of the the air gap, i.e. $x < 0$. We assume that the cross sectional area of the core does not change. This means that the magnitude of the magnetic field intensity has only one value inside the core material.

4. Every closed loop going around in the circuit which does not contain the wire does not contain any currents. Ampere's law therefore suggests, that the coil is not able to generate

a magnetic field in the air gap for positions $r < x$. The magnetic flux generated by the coil always passing the coil wire on it's outer side. The effect of the moving coil is therefore an effective reduction of the cross sectional area of the air gap. This is indicated in figure 1 by the dash-dotted line.

Using these assumptions, one can find the magnetic field intensity inside the air gap. Let's assume first that the coil is inside the core, i.e. $x < 0$. Ampere's law states then for a closed loop that follows the form of the core

$$\oint \boldsymbol{H} \cdot d\boldsymbol{l} = -H_m l_m - H_1(l_1 + 2l_2) - H_i g' = i, \quad (4)$$

where H_1 is the magnitude of the magnetic field inside the core material and H_m inside the magnet and l_1, l_2, l_m are the lengths of the integration path as seen in figure 1. From conservation of magnetic flux density, $\oint_S \boldsymbol{B} \cdot d\boldsymbol{s} = 0$, it follows that the magnetic flux has to stay constant

$$\mu A H_1 = \mu_0 A H_i = \mu_m A H_m. \quad (5)$$

We can insert $H_1 = \mu_0/\mu H_i = \mu_0/\mu_m H_m$ into (4) and get that magnetic field intensity inside the air gap due to the current in the coil

$$H_i = -i\left(\frac{\mu_0}{\mu}(2l_1 + 2l_2) + \frac{\mu_0}{\mu_m} l_m + g'\right)^{-1}. \quad (6)$$

We now define g as an effective air gap width such that

$$H_i = -\frac{i}{g}. \quad (7)$$

Let's look at the case when the coil enters the air gap, i.e. $0 < x < w$. The magnetic field intensity of the inner side of the coil, $0 < r < x$, will be zero due to assumption 4 and the magnetic flux will be forced to go around the coil through a smaller area, since the Ampere's law requires the H-field to the left of the coil to be zero. The magnitude of the magnetic field intensity is now changing with x and the magnetic field intensity in the air gap is therefore

$$H_i(r) = -\begin{cases} \frac{i}{g} & x < r < w \\ 0 & else \end{cases}. \quad (8)$$

The flux trough the air gap and through the single winding coil is the magnetic flux density multiplied with the area of the air gap that is on the right hand side of the coil

$$\Phi_i = -\mu_0 \frac{i}{g} A \begin{cases} 1 & x < 0 \\ \left(1 - \frac{x}{w}\right) & 0 \le x \le w \\ 0 & w < x \end{cases}. \quad (9)$$

Equation 8 shows that the magnetic field intensity inside the air gap is behaving like a step function with respect to the observing position r, i.e. $H_i(r) = -\frac{i}{g}\Theta(r - x)$. The step occurs at the position of the coil, as illustration in figure 2.

2.1.1 Voice coil inductance

Since all of the air gap flux goes through the single winding of the voice coil, its inductance may be found simply by dividing the flux by the voice coil current (a minus sign appears due to the sign convention of the magnetic field in the airgap)

$$L(x) = \frac{-\Phi_i}{i} = \frac{\mu_0}{g} A \begin{cases} 1 & x < 0 \\ \left(1 - \frac{x}{w}\right) & 0 \le x \le w \\ 0 & w < x \end{cases}. \quad (10)$$

2.1.2 Force factor

The static force factor generated by the permanent magnet may be calculated in a similar way. We will simplify the analysis by only representing the permanent magnet by its magnetomotive force (an equivalent loop current), I_0. This means that

$$Bl_0(x) = B_0(x) b = \begin{cases} \frac{\mu_0 b}{g} I_0 & 0 < x < w \\ 0 & else \end{cases}. \quad (11)$$

Note the similarity between the static force factor and the B field created when the coil is completely inside the motor system, as illustrated in figure 2.

Here we see that under the simplifying assumptions there is indeed a simple relation between static force factor and voice coil, as combining equations 10 and 11 leads to

$$Bl_0(x) = -I_0 \frac{dL(x)}{dx}. \quad (12)$$

The force factor is proportional to the spatial derivative of the voice coil inductance and the factor of proportionality is minus the magnetomotive force of the permanent magnet.

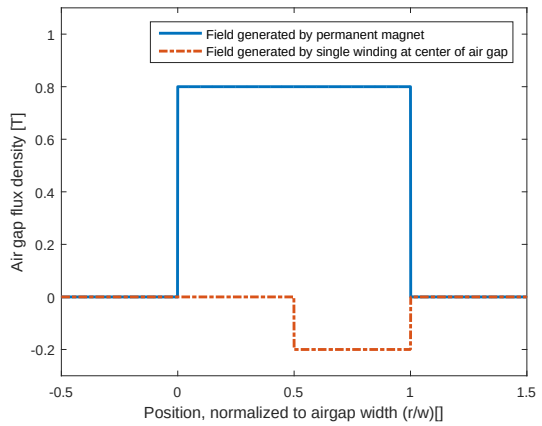

Fig. 2: Air gap flux density created by permanent magnet (solid line) and by single winding located in the middle of the air gap (dashed line)

2.2 Extended voice coil

We now extend the model by looking at a long coil. The coil is modelled by a continuous current sheet with current density per length σ and with a length l_c in x-direction, as seen in figure 3. This means that $i = \sigma l_c$. The current at the inside sheet is defined as coming out of the plane and the current through the part that is outside the core is flowing in to the plane, so the current directions are the same as in the last subsection. It is convenient for the following derivations to divide the coil into coil segments of length $\mathrm{d}x'$ and denote their position by x'. The position x is now defined as the position of the outer end of the coil, e.g. $x = 0$ denotes that the coil is completely inside the core, with its tip at the inner edge of the air gap and $x = w$ is the position, in which the coil starts to exit the air gap on the outer side.

2.2.1 Magnetic field intensity

The magnetic field intensity inside the air gap is needed for the derivation of the flux that passes through the coil. We can use the result (8) from a single loop and use it for the coil element $\mathrm{d}x'$. The current element $\sigma \mathrm{d}x'$ will produce a magnetic field inside the gap that depends on the observing position r and the position of the current element

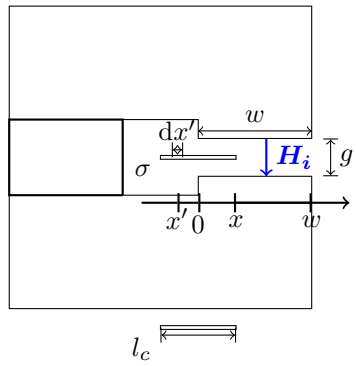

Fig. 3: Schematics of model with coil in form of current sheet with current density σ.

x'

$$\mathrm{d}H_i(r,x') = -\begin{cases} \frac{\sigma \mathrm{d}x'}{g} & x' < r < w \\ 0 & \text{else} \end{cases} . \tag{13}$$

To get the complete field at r one has to integrate this over the length of the coil, i.e. $H_i(r,x) = \int_{x-l_c}^{x} \mathrm{d}H_i(r,x')$. The result for both over- and underhung coil is

$$H_i(r,x) = -\frac{\sigma}{g} \times \begin{cases} l_c & x < 0 \\ \min(x,r) - (x - l_c) & 0 \leq x < r + l_c \\ 0 & \text{else} \end{cases} . \tag{14}$$

Figure 4 shows the magnetic field intensity inside the air gap for several positions of a overhung coil. If the coil is inside the core ($x < 0$) the magnetic field is constant. When the coil's tip is inside the air gap (0 < x < w), the field is constant for $r > x$ and linearly decreasing for $r < x$. The linear decrease for smaller r comes from the fact, that only coil elements with $x' < r$ contribute to the field at r. The amount of such segments decreases linearly with decreasing r and $H_i(r)$ therefore decreases linearly, too. At $r > x$ all elements contribute and the field intensity is therefore constant.

2.2.2 Voice coil inductance

To find the inductance one has to derive the total flux through the coil. First, we want to know the flux through the coil segment at x', which helps us then to find the total flux through the coil. If

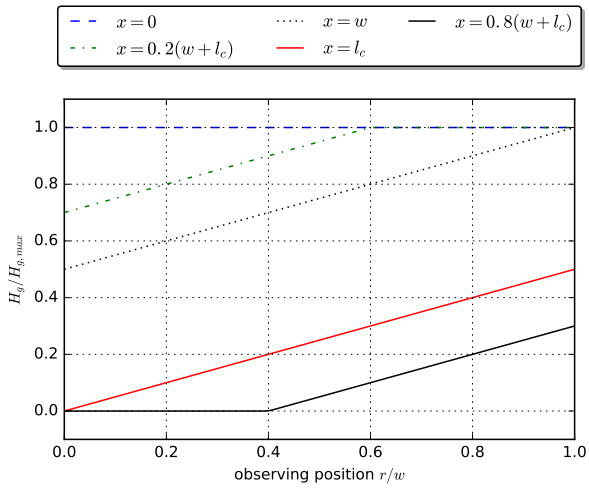

Fig. 4: Magnetic field H_i inside the air gap as a function of coil position x and observing position r for a coil of length $l_c = 2w$

the winding is inside the core ($x' < 0$), then the total flux through the air gap will go through the winding. If the winding is inside the air gap, then only the flux to its right hand side will flow trough it, i.e. the flux passing between x' and w. This flux is

$$\begin{cases} \int_{max(0,x')}^{w} -\mu_0 H_i(r,x) b \mathrm{d}r & x' < w \\ 0 & w \leq x' \end{cases}, \tag{15}$$

where $b\mathrm{d}r$ is the surface segment. The case for $w \leq x'$ states, that the loop does not create any flux outside the air gap. This is of course an approximation. The current loop is also creating an magnetic field when outside the air gap, which causes some flux to go through the loop. But this flux should be considerably smaller compared to the flux that is created inside the air gap, because the permeability of the metal is supposed to be much greater than that of air. Neglecting this flux means that we take the free air inductance of the coil as zero.

The total flux through a coil segment $\mathrm{d}x'$ is

$$\mathrm{d}\Phi_c(x,x') = \begin{cases} \left[\int_{max(0,x')}^{w} -\mu_0 H_i(r,x) b \mathrm{d}r\right] \frac{1}{l_c} \mathrm{d}x' & x' < w \\ 0 & w \leq x' \end{cases}. \tag{16}$$

The total flux through the whole coil is the sum of the flux trough every coil segment. Because we have a continuous case here, we integrate over all the windings so the total flux is

$$\Phi_c(x) = \int_{x-l_c}^{x} \mathrm{d}\Phi_c(x,x'). \tag{17}$$

The inductance of the coil is $L(x) = \Phi_c(x)/i$. Calculating the integral of equation 17 for our case, we get for the inductance of an overhung coil as a function of coil position x

$$L_{e,OH}(x) = \mu_0 \frac{1}{l_c^2} \frac{b}{g} \times \begin{cases} w l_c^2 & x < 0 \\ \frac{1}{3}x^3 - l_c x^2 + l_c^2 w & 0 \leq x < w \\ w(l_c - x)^2 + w^2(l_c - x) + \frac{1}{3}w^3 & w \leq x < l_c \\ \frac{1}{3}(l_c + w - x)^3 & l_c \leq x < w + l_c \\ 0 & w + l_c \leq x \end{cases} \tag{18}$$

Figure 5 shows the dependence of the inductance on the coil position. The inductance is maximal for $x < 0$ and decreases as soon as the coil enters the air gap. The inductance changes more rapidly for shorter coils. The inductance is zero if the coil is outside the air-gap ($x > w + lc$), which is a consequence of neglecting the free air self inductance of the coil.

2.2.3 Force factor

The force factor from the permanent magnet can be found from integrating the flux density in the air gap

$$Bl_0(x) = \frac{F}{i} = \frac{b}{i} \int_{x-l_c}^{x} B_0(x_1) \sigma \mathrm{d}x_1 \tag{19}$$

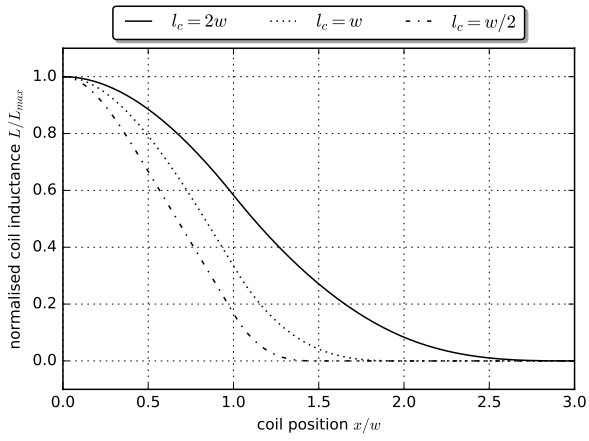

Fig. 5: Coil inductance L as a function of coil position x for three different coil lengths.

which for the overhung coil ($l_c > w$) gives

$$Bl_{0,OH}(x) = \mu_0 \frac{I_0}{l_c} \frac{b}{g} \times \begin{cases} 0 & x < 0 \\ x & 0 \leq x < w \\ w & w \leq x < l_c \\ w + l_c - x & l_c \leq x < w + l_c \\ 0 & w + l_c \leq x \end{cases} \tag{20}$$

When equations 18 and 20 are compared it is clear that the simple relation in equation 12 is not true anymore as the voice coil inductance contains x to the third power and the force factor only contains x to the first power. The equation that links force factor and inductance is more complex for the longer voice coils as will be investigated in the next section.

2.3 Voice coil length

In the previous subsection an analytical expression for the force factor and voice coil inductance was derived for the simplified geometry case. In real driver designs the stray field lines may not always be small enough to be ignored and the simplified model does not capture the full behaviour of the motor system. However, a connection between the force factor and the voice coil inductance remains since the magnetic flux from the permanent magnet and the voice still runs through the same system. Rearranging the equation for the force factor gives :

$$Bl_0(x) = b \int_{x-l_c}^{x} \frac{B(x_1)}{l_c} \mathrm{d}x_1 \tag{21}$$

i.e. the total force factor is found as the average over the entire length of the voice and each position is weighted equally.

For the inductance the total inductance can also be found by averaging over the length of the voice but the weighting is not uniform as will be shown in the following.

For a short coil in free air it can be assumed that the total flux goes through all the windings of the voice coil and hence the inductance increases with the square of the number of windings. However, in the speaker this is not the case and therefore the total inductance of the voice coil will have a different dependence on coil length. As discussed earlier the inner most segments of the coil are fully exposed to the field generated by the outer segments, however the influence in the reverse direction is less strong. The total flux of the coil can be found as the average over the flux going through each segment of the coil:

$$\Phi_i(x) = \frac{1}{l_c} \int_{x-l_c}^{x} \Phi_{i,1}(x_1) \mathrm{d}x_1 \tag{22}$$

where the flux going through each part of the coil is found as integrating over the contributions of all positions of the coil:

$$\Phi_{i,1}(x_1) = \int_{x-l_c}^{x} \Phi_{i,12}(x_1, x_2) \mathrm{d}x_2. \tag{23}$$

$\Phi_{i,12}(x_1, x_2)$ is the flux contribution in the segment at x_1 from the segment at x_2 and depends on the mutual inductance,

$$\Phi_{i,1}(x_1) = \int_{x-l_c}^{x} M_{12}(x_1, x_2) \sigma \mathrm{d}x_2 \tag{24}$$

and due to the asymmetric coupling between the difference parts, i.e. the inner parts see all the flux generated by the outer part, but not vice versa, the mutual inductance is the minimum of the (single winding) inductance value at x_1 and x_2:

$$M_{12}(x_1, x_2) = min(L(x_1), L(x_2)) \tag{25}$$

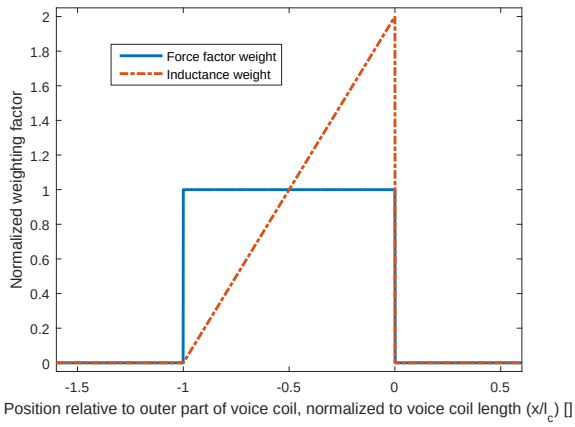

Fig. 6: Weighting function for averaging of force factor (solid line) and voice coil inductance (dashed)

this leads to

$$L_e(x) = \frac{\Phi_i(x)}{i} = \frac{1}{l_c}\int_{x-l_c}^{x}\int_{x-l_c}^{x} M_{12}(x_1,x_2)\mathrm{d}x_1\mathrm{d}x_2 \tag{26}$$

which reduces to

$$L_e(x) = \frac{1}{l_c}\int_{x-l_c}^{x} \frac{2(x_1-(x-l_c))}{l_c} L(x_1)\mathrm{d}x_1, \tag{27}$$

i.e. the total inductance of an extended voice is found as an average over the length of the voice coil but with a non uniform sawtooth weigthing function as illustrated in figure 6. Thus due to the mutual coupling between the different segments the total inductance actually decreases as the length of the voice is increased (for constant number of windings), this is because the outer most parts of the voice coil, which have a lower inductance, dominate the coupling.

3 Finite element analysis

The last section introduced a simple model for the motor system of a dynamic loudspeaker. The model enabled us to find analytic expressions for the position dependent coil inductance and flux modulation force at DC. Multiple assumptions had to be made to solve the problem analytically. The purpose of this section is to test the analytical model by comparing it to a finite element method (FEM) simulation to check, if our choice of assumptions was reasonable.

The simple motor geometry is implemented as seen in figure 7 with parameters given in 1. The core (3) is modelled to represent soft iron. The voice coil (5, 6) and the coil creating the static magnetic field (2,4) are modelled as copper and the rest of the domain is modelled as air (1). In order to exclude eddy currents the conductivity of the iron was set to approx. zero.

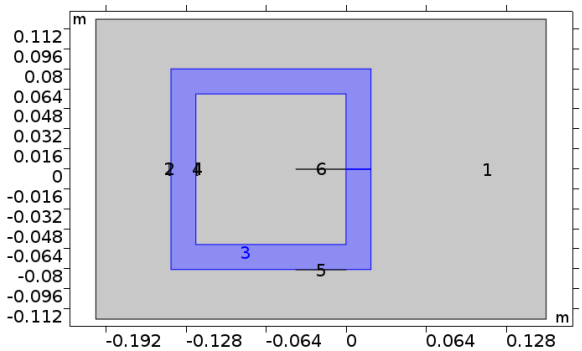

Fig. 7: Simplified geometry of FEM simulation

Figure 8 compares the analytical result of equation 20 with the FEM simulation. The agreement is good, indicating that the connection between force factor and inductance is accurate and the assumptions are reasonable. The analytical result for the overhung coil ($l_c = 0.04\,\mathrm{cm}$) underestimates the force factor slightly as the real coil is also affected by the magnetic field outside the air gap.

The same comparison for the voice coil inductance is shown in figure 9. The analytical solution (eq. 18) is underestimating the inductance slightly: at large x, we neglected the self inductance of the coil, while for small x the fringing field around the air-gap adds to flux going though the coil.

Figure 10 is comparing the voice coil inductance obtained directly through the FEM simulation with the inductance obtained from the simulated permanent magnetic field $B_0(x)$ through the use of Eqs. 12 and 27. For both values of the permeability of the iron The two quantities agree well, apart from a constant offset of approx. $1\times10^{-6}\,\mathrm{H}$, which could be attributed to the neglected free air inductance of the voice coil, e.g. by an appropriate choice of the integration constant in the solution of Eq. 12.

Name	Value	Description
g	0.5 mm	width of air gap
w	2 cm	length of air gap
x	$0-6$ cm	coil position
i	100 mA	current through coil
n	1	coil windings
l_c	1 mm, 2 cm, 4 cm	length of coil
d_c	0.1 mm	thickness of coil
b	1 m	dimension of magnet in out-of-plane direction
d	8 cm	magnet outer dimension
I_0	1 A	current in magnet coil with 100 windings
ϵ	1	relative permittivity of air, coil, and core domain
μ_{ac}	1	relative permeability of air and coil domain
μ	2×10^5	relative permeability of core material
σ_{core}	1×10^{-6} S/m	conductivity of core material (iron)
σ_{coil}	6×10^7 S/m	conductivity of coil material (copper)
σ_{air}	0 S/m	conductivity of air

Table 1: Simulation parameters

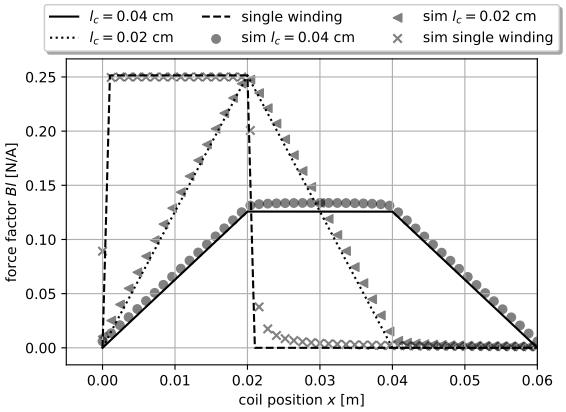

Fig. 8: Simulated and analytical force Factor Bl_0 as a function of coil position x for three different coil lengths.

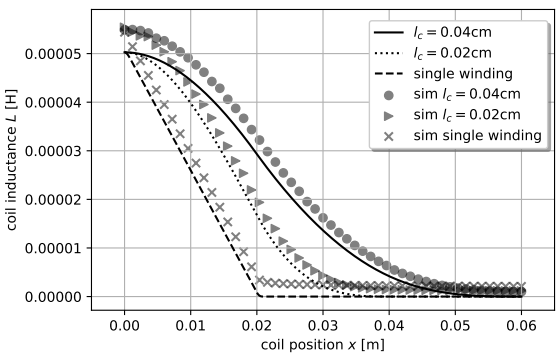

Fig. 9: Simulated and analytical inductance L as a function of coil position x for three different coil lengths.

4 Discussion

Real world speakers will have a distribution of the magnetic field which wont be as strictly confined to the air gap as assumed in the previous analysis but as long as the relation between the single winding inductance and the permanent field is valid, the connection between the force factor and the voice coil inductance should be preserved, so the presence of for example fringing effects is not expected to change the coupling.

However other effect may have an influence, for example if parts of the iron structure are heavily saturated, the voice coil field will see a quite different permeability of the iron structure than the

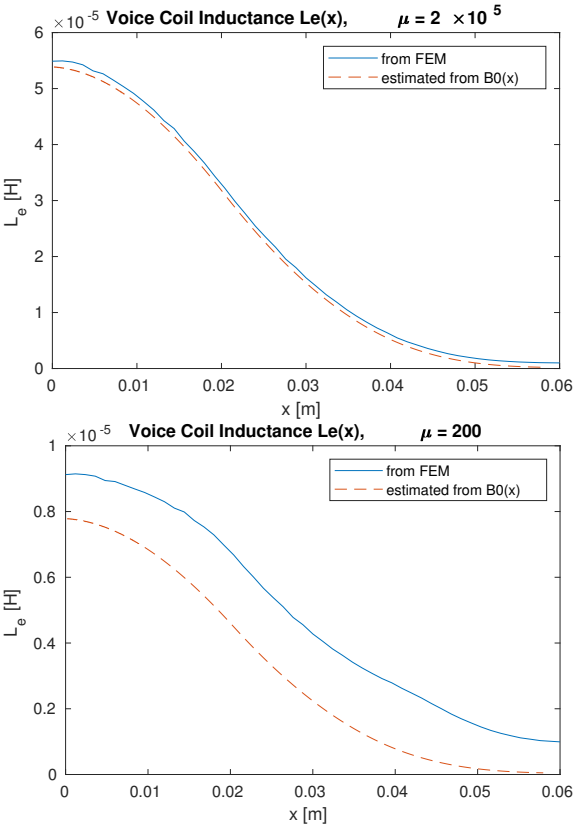

Fig. 10: Voice coil inductance $L_e(x)$ directly obtained from FEM simulation and derived from the simulated permanent magnetic field $B_0(x)$ through equations 12 and 27 for two different relative permeabilities μ of the core material and an overhung coil of length $l_c = 4\,\text{cm}$

static field of the permanent magnet does and in such cases the link between force factor and voice coil inductance is expected to be weaker.

Another factor that can significantly change the behaviour of the magnetic structure are eddy currents [5]. At medium and high frequencies eddy current in the iron parts or in shorting rings added to the structure will significantly reduce the effective permeability of the iron structure, which could significantly change the effective inductance nonlinearity; indeed the purpose of adding shorting rings is to make the inductance more linear. Thus the relation between force factor and inductance described in this paper will only be valid at low frequencies for most drivers.

For nonlinear inductance models such as the L2R2 model, this means that since R2 is short circuited at low frequencies then the relation between force factor and inductance only applies to the effective inductance at low frequencies, which is the sum of L_e and L_2. The position dependence of R2 is determined by the eddy current pattern in the iron parts and the number and locations of the any shorting rings and is not related to the force factor.

5 Conclusion

For a single winding coil a simple relation between the force factor and the voice coil inductance was derived analytically for a simplified magnetic circuit: The force factor is proportional to the spatial derivative of the voice coil inductance. It was also shown that when the voice coil is long the effective force factor and voice coil inductance can be found as averages of the narrow single winding values over the length of the coil, but the weightings are different, the force factor is found using a uniform weighting, whereas the voice coil inductance is derived using a asymmetric sawtooth weighing, which makes the connection between the two more complicated. The predicted interdependence of the two nonlinearities was investigated for more realistic motor configurations using FEM and the simulations confirmed the relation predicted by the analytical model. This connection may be used to improve modelling of nonlinear transducers as fewer parameters are needed in order to model the two nonlinearities.

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