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Deriving the absolute wave spectrum from an encountered distribution of wave energy spectral densities

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Abstract

The objective of ship motion-based wave spectrum estimation is to provide the distribution of wave energy densities in *absolute domain*. However, as a ship generally advances relative to the progressing waves, any spectrum estimate inherently dates back to the *encounter domain* and, consequently, the spectrum estimate must be transformed to absolute domain. In following sea conditions, spectrum transformation from encounter to absolute domain has no unique (mathematical) solution. This article presents an optimisation-based technique to carry out the particular transformation in following sea conditions. The optimisation relies on an object function established using (wave) spectral moments; calculated directly using the estimated encounter-wave spectrum on the one side and by using a parameterised wave spectrum valid in absolute domain on the other side. The simplicity of the transformation technique is a strength in itself as it leads to an insignificant computational effort in the transformation to absolute domain. Equally important, the specific technique proves capable to provide accurate results in the majority of cases, when comprehensive testings with numerically simulated data of following sea conditions are performed. Furthermore, the technique is tested successfully using experimental full-scale sea trials data.

Keywords:

Ocean wave spectrum, advancing ship, Doppler Shift, encounter and absolute domains, spectrum transformation, following waves

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1. Introduction

When a ship advances in its seaway (on deep water), the relative velocity and the angle between vessel and the progressing waves are the parameters that determine the observed period of the encountered waves. Mathematically, this phenomenon is described by the Doppler Shift which expresses that the *encountered wave frequency*, observed on a ship advancing relative to the inertial frame of reference, is different from the *absolute wave frequency*. Consequently, the Doppler Shift needs to be strictly taken into account when theoretical calculations of wave-induced responses are to be made since, otherwise, it is not possible to compare the result to any corresponding set of measurements. Note, in the remaining parts, deep-water conditions are assumed throughout. **Moreover, all operational parameters, including vessel advance speed and encounter-wave angle, are constant.**

It is a fact, see Section 2, that the Doppler Shift relates one - unique - absolute (wave) frequency to any one single encountered frequency when the velocity vectors of the ship and the waves have opposing projections on the ship's centreline, meaning that the angle between the pair of velocity vectors is larger than 90 degrees and smaller than 270 degrees; qualitatively expressed as the ship is in beam to head sea conditions. On the other hand, the Doppler Shift associates three absolute frequencies to any one single encountered frequency when two conditions are fulfilled, (1) the ship "follows" the progressive waves, (2) the encounter frequency takes a value less than a certain number, noting that the first condition implies that the angle between the pair of velocity vectors is either smaller than 90 degrees or larger than 270 degrees for the 1-to-3 relationship to be occurring.

The existing literature about the Doppler Shift, including its implications for ships sailing in waves, is fairly extensive when it comes to conceptual understanding and theoretical studies, see e.g. Lewandowski (2004); Price and Bishop (1974); Bhattacharyya (1978); Beck et al. (1989); Journée and Massie (2001). However, the situation is different when focus is on practical calculations related to real-case scenarios and applications. This is indeed so because of the 3-to-1 relationship, or mapping, between corresponding sets of frequencies in the absolute and the encounter domains when a ship advances in following waves*. Thus, any set of wave-induced measurements cannot be uniquely

*While it may be a 3-to-1 mapping in the one "direction", it will be a 1-to-3 mapping in the other.

31 transferred, or transformed, into the absolute domain, if comparisons with theoretical
 32 calculations are to be made. Quite often this "issue" is of no particular concern, since
 33 it is always possible, within theoretical calculations, to transform uniquely the other
 34 way, from absolute domain to encountered domain, when, or *if*, the wave conditions are
 35 known; although the encounter-wave spectrum may appear heavily distorted. In real-
 36 time, in-service scenarios, however, the wave conditions themselves need to be measured
 37 or estimated by *some* available means. This implies that the wave conditions need to be
 38 estimated from an *observation platform* that advances relative to the progressive waves,
 39 and, as a result, some practical approach to deal with the 1-to-3 relationship between
 40 encounter and absolute wave frequency must be introduced. As mentioned, there is a
 41 large literature related to the fundamental understanding and the conceptual, or theoret-
 42 ical, consequences of the Doppler Shift, whereas the practical complications are studied
 43 to a smaller degree. One interesting study, however, needs mentioning and, thus, it may
 44 be worthwhile to consult Lindgren et al. (1999) to obtain additional knowledge about
 45 the complications and consequences introduced because of the *stochastic* Doppler shift
 46 for "a ship traveling with constant speed on a Gaussian random sea with a directionally
 47 distributed frequency spectrum". This particular study (Lindgren et al., 1999) indicates,
 48 indeed, why it is by no means elementary to convert an encountered wave spectrum to a
 49 corresponding absolute one.

50 The present article deals specifically with a method to derive the absolute wave spec-
 51 trum from an encounter-spectrum, representing the encountered distribution of wave
 52 energy spectral densities. As such, the work has its origin in studies related to the wave
 53 buoy analogy (Iseki and Ohtsu, 2000; Tannuri et al., 2003; Pascoal et al., 2007, 2017;
 54 Nielsen, 2006, 2008; Brodtkorb et al., 2018), where measurements of wave-induced vessel
 55 motions are processed to yield an estimate of the on-site (absolute) wave energy spectrum.
 56 Some of the studies consider vessels with zero-advance speed, while others, with smaller
 57 and larger degrees of success, introduce non-zero forward speed by establishing the math-
 58 ematical equations directly in the absolute domain, and thus facilitating the wave energy
 59 spectrum as the very solution. However, one recent ship motion-based estimation algo-
 60 rithm (Nielsen et al., 2018; Nielsen and Brodtkorb, 2018) has proven computationally

Anyhow, the problem will generally from this point just be referred to as a *1-to-3* mapping problem.

61 very efficient and capable in providing accurate estimates of the *encounter*-wave spec-
62 trum together with the relative wave heading angle for ships with advance speed. It
63 is therefore highly relevant to address how to obtain the corresponding absolute wave
64 energy spectrum.

65 In a previous work by Nielsen (2017), a practical approach was suggested for the
66 transformation from encounter to absolute domain of wave spectral densities. The ap-
67 proach was tested comprehensively with numerical simulations, and the method has also
68 been studied together with full-scale experimental data for its purpose in relation to ship
69 motion-based wave spectrum estimation. Generally, the approach has been found to per-
70 form satisfactory but examples of the opposite also occur (Nielsen and Brodtkorb, 2018).
71 The present work is introducing a second alternative transformation technique which
72 relies on optimisation of a set of characteristic - absolute - wave parameters (e.g. signif-
73 icant wave height and zero-upcrossing period) based on an object function formulated
74 through the wave spectral moments. This particular approach makes the transformed
75 spectrum, in absolute domain, to have a shape of a parameterised wave spectrum such as
76 a generalised JONSWAP spectrum (Hasselmann et al., 1973). It is the overall objective
77 of this article to compare the new optimisation-based transformation technique with the
78 "old" transformation technique (Nielsen, 2017).

79 Besides the introduction, the article consists of five sections: Details and consequences
80 of the Doppler Shift set the problem faced when spectrum transformation from encounter
81 to absolute domain must be conducted, and this is explained in Section 2. The remedy
82 in terms of a (new) specific transformation algorithm is given in Section 3, while appli-
83 cation and testings on data are outlined in Sections 4 and 5 which contain performance
84 evaluations, including comparative studies between the old and the new transformation
85 techniques. Finally, Section 6 summarises and brings the conclusions and suggestions for
86 further work.

87 2. Problem formulation

88 The distribution of wave energy density $S(\omega)$, or simply the wave spectrum, in a
89 fixed point is considered, where ω is the absolute frequency. The wave energy in the
90 infinitesimal range $d\omega$ is proportional to $S(\omega)d\omega$ and, as a result, the total energy of

the wave system is proportional to $\int S(\omega)d\omega$. The distribution of wave energy density depends generally also on the physical location, say (x,y). However, in the present work, the variation with location is not considered, and throughout it is the distribution of wave energy density in a specific point which is addressed, noting that the point can be fixed or it can be advancing relative to the inertial frame of reference.

The Doppler Shift implies that the wave encounter-frequency, ω_e , is different from the absolute wave frequency ω_0 , and it can be shown that the mathematical relationship (on deep water) is given by,

$$\omega_e = \omega_0 - \omega_0^2 \psi, \quad \psi = \frac{U}{g} \cos \chi \quad (1)$$

where U is the speed of the vessel, and χ is the angle between the pair of velocity vectors of the vessel and the waves, respectively. Note that indices ₀ (absolute) and _e (encounter) are used to emphasise the particular domains in question.

Although the vessel encounters waves at a frequency which is different from the absolute one, the total energy of the wave system is the same in the two domains, expressed as

$$E(\omega_e)d\omega_e = S(\omega_0)d\omega_0 \quad (2)$$

where $E(\omega_e)$ refers to the encountered distribution of wave energy density, while $S(\omega_0)$ is the corresponding one in absolute domain. Consequently, wave energy spectral density can, in theory, be easily transformed from the one domain to the other, simply by multiplication with the derivative, $\frac{d\omega_0}{d\omega_e}$ or $\frac{d\omega_e}{d\omega_0}$, depending on the 'transformation direction';

$$E(\omega_e) = S(\omega_0) \frac{d\omega_0}{d\omega_e} \quad (3)$$

$$S(\omega_0) = E(\omega_e) \frac{d\omega_e}{d\omega_0} \quad (4)$$

As was indicated in the Introduction, wave spectrum transformation can be uniquely carried out when a ship sails "against" the waves (beam to head sea). In certain conditions related to following seas*, however, there exists no unique solution to the problem, since the Doppler Shift, Eq. (1), imposes a 1-to-3 relation between encounter frequency

*Onwards, "following waves" or "following seas" are used to indicate a situation in which the 1-to-3 mapping problem occurs, without having a specific relative wave heading in study.

111 and absolute frequency, that is, one 'encountered wave component' is the result of three
 112 (true) absolute wave components. Consequently, the energy density at the specific en-
 113 counter frequency cannot be assigned uniquely to the corresponding set of absolute fre-
 114 quencies as illustrated in Figure 1. This very problem has been comprehensively studied
 115 in Nielsen (2017), resulting in an elaborate, but at the same time practical, transforma-
 116 tion algorithm. Briefly said, the algorithm is based on a scaling approach that assures
 117 preservation of energy (density) at corresponding sets of encounter and absolute fre-
 118 quencies by mapping the set of absolute frequencies into energy densities of a standard
 119 parameterised wave energy spectrum (e.g., Bretschneider or JONSWAP). Thus, a set
 120 of scaling ratios applies to specific absolute frequencies, obtained through the Doppler
 121 Shift of given encounter frequencies, and multiplication between, respectively, the scaling
 122 ratios, the encounter-wave spectrum ordinate and the frequency-derivative will make the
 123 (transformed) absolute wave spectrum available.

Figure 1: In following waves, it cannot be uniquely determined how the one single *encountered* (discrete) wave energy density, say $E(\omega_{e,i})$, transforms into absolute domain because of the 1-to-3 relationship between frequencies. This problem occurs due to the Doppler Shift which is illustrated by the parabola inside the dashed box, showing that one encounter frequency is mapped into three absolute frequencies, whenever ω_e is smaller than $\frac{1}{4\psi}$, cf. Eq. (1).

124 A recent study on ship motion-based wave estimation (Nielsen et al., 2018) has shown
 125 that the transformation algorithm by Nielsen (2017) performs well in many cases related
 126 to following sea conditions. However, reports about the opposite also exist (Nielsen and
 127 Brodtkorb, 2018). Hence, having had the problem set forth, it is the purpose of this
 128 article to propose an alternative, but still practical, transformation technique which can
 129 be applied with any ship motion-based sea state estimation algorithm to transform a
 130 wave energy spectrum consistently from encounter domain to absolute domain, and at
 131 insignificant extra computational costs.

132 3. Transformation to absolute domain

133 In the formulation of a new transformation technique, the work should *not* make
 134 direct use of Eq. (4), since this was/is the solution strategy already applied by Nielsen
 135 (2017). Instead, the strategy is to take note that, despite of the fact that the individ-
 136 ual frequency components are related through a 1-to-3 mapping (in following waves),
 137 it is possible to transform integrated parameters (such as characteristic periods, total
 138 energy content, shape parameters) if it is assumed that the absolute spectrum has a
 139 parameterised spectral shape.

140 In the following, the overall assumption is that the vessel speed U , the relative heading
 141 χ between the directions of vessel course and wave propagation, and the wave encounter-
 142 spectrum $E(\omega_e)$ are given, equivalently said, exactly known. Secondly, it is assumed
 143 that the transformed wave spectrum $S(\omega_0)$ (in the absolute domain) has a spectral shape
 144 determined by the summation of two generalised JONSWAP spectra. The summation
 145 of two individual spectra has the potential to accommodate a true/generating (absolute)
 146 wave system consisting of swell and wind sea at the same time, while any single peaked
 147 generating spectrum should realise itself by fitting the two parametric spectra on top
 148 of each other. The analytical expression of the generalised JONSWAP spectrum follows
 149 from the parameterised formula (Hasselmann et al., 1973),

$$\hat{S}(\omega_0) = \alpha g^2 \frac{1}{\omega_0^5} \exp \left[-\frac{5}{4} \left(\frac{\omega_0}{\omega_p} \right)^{-4} \right] \gamma^P; \quad P = \exp \left[-\left(\frac{\frac{\omega_0}{\omega_p} - 1}{\sigma \sqrt{2}} \right)^2 \right] \quad (5)$$

where the absolute frequency ω_0 is the independent variable, while a number of (wave)
 parameters are conditioning variables, including the peakedness factor γ . The peak

frequency ω_p is derived from the peak period T_p , while the step function σ takes values $\sigma = 0.07$ for $\omega_0 < \omega_p$, and $\sigma = 0.09$ for $\omega_0 > \omega_p$ with $\omega_p = \frac{2\pi}{T_p}$. Note that the constant α should be determined to secure conservation of energy, which requires $\int \hat{S}(\omega_0) d\omega_0 = \frac{1}{16} H_s^2$, where H_s is the significant wave height. Furthermore, a Bretschneider spectrum is established by setting $\gamma = 1$; otherwise, the peakedness factor takes values $1 < \gamma \leq 7$. Thus, spectrum transformation results in a parameterised spectrum given by,

$$\hat{S}(\omega_0|\mathbf{\Omega}) = \hat{S}_1(\omega_0|\dots) + \hat{S}_2(\omega_0|\dots) \quad (6)$$

150 which is conditional on totally six parameters $\mathbf{\Omega} = \{\hat{H}_{s,1}, \hat{T}_{p,1}, \hat{\gamma}_1, \hat{H}_{s,2}, \hat{T}_{p,2}, \hat{\gamma}_2\}$, using
 151 the hat-symbol to indicate that the parameters relate to the transformed spectrum.
 152 Hence, the transformation problem has been converted into an optimisation problem with
 153 the objective to determine the optimum values of the six characteristic wave parameters,
 154 although it is yet unsaid what the cost function, or object function, is.

In spectral analysis, characteristic wave parameters can be calculated by the spectral moments, noting that, generally, the values of the spectral moments are (obviously!) not the same in absolute domain and in encounter domain. The n-th order spectral moment is defined by

$$m_{n,e} = \int_0^\infty \omega_e^n E(\omega_e) d\omega_e \quad (7)$$

using index 'e' to indicate that conditions in the encounter domain are considered. A variety of characteristic wave parameters can subsequently be calculated (e.g. Tucker, 1993; Krogstad et al., 1999), and examples are,

$$H_{s,e} = 4\sqrt{m_{0,e}} \quad \text{Significant wave height} \quad (8)$$

$$T_{m,e} = 2\pi \frac{m_{0,e}}{m_{1,e}} \quad \text{Mean period} \quad (9)$$

$$T_{z,e} = 2\pi \sqrt{\frac{m_{0,e}}{m_{2,e}}} \quad \text{Zero-crossing period} \quad (10)$$

$$S_{S,e} = 2\pi \frac{H_{s,e}}{gT_{z,e}^2} \quad \text{Significant wave steepness} \quad (11)$$

It should be clear that these parameters, in the given context, are readily available, because the encounter wave spectrum $E(\omega_e)$ is known. On the other hand, *estimates* of the particular parameters, Eqs. (8)-(11), can also be obtained by using the absolute

spectrum, given by Eq. (6), in the calculation of the n-th order spectral moment, cf. Eq. (7). Thus, estimates of the encounter-parameters can be *approximated* by,

$$m_{n,0 \rightarrow e} = \int_0^\infty \tilde{\omega}_e^n \hat{S}(\omega_0|\mathbf{\Omega}) d\omega_0 \quad (12)$$

where $E(\omega_e)d\omega_e = S(\omega_0)d\omega_0$ is introduced, cf. Eq. (2), and using index $0 \rightarrow e$ to denote that 'encounter-condition' is obtained by using the absolute spectrum. The calculation of $m_{n,0 \rightarrow e}$ can be done independently on the set of encounter frequencies ω_e (without the tilde on top), but instead any range of absolute frequencies ω_0 can be pre-specified, whereafter $\tilde{\omega}_e$ is easily - and uniquely - obtained by the Doppler Shift in Eq. (1). Consequently, a cost function can be established by minimising a difference-metric obtained by comparing the actual - known - encounter-characteristic wave parameters, Eqs. (8)-(11), with corresponding estimates that are calculated by the n-th order spectral moment,

$$m_{n,0 \rightarrow e} = \int_0^\infty \left(\omega_0 - \omega_0^2 \frac{U}{g} \cos \chi \right)^n \hat{S}(\omega_0|\mathbf{\Omega}) d\omega_0 \quad (13)$$

for any set of pre-specified absolute frequencies, using the absolute (parameterised) wave spectrum from Eq. (6). As such, the object function is defined according to,

$$\min_{\mathbf{\Omega}} \Xi^2 \equiv \min \sum_i (\beta_i - \hat{\beta}_i)^2 \quad , \quad \hat{\beta}_i \doteq \hat{\beta}_i(\hat{S}(\omega_0|\mathbf{\Omega})) \quad (14)$$

155 where the "observables" β_i respectively $\hat{\beta}_i$ is any one of the considered characteristic wave
 156 parameters, cf. Eqs. (8)-(11); emphasising that any β_i follows directly from the relevant
 157 expression in Eqs. (8)-(11) while $\hat{\beta}_i$ at the same time is conditioned on $\mathbf{\Omega}$. Altogether, the
 158 transformed spectrum is obtained by minimising the sum of the squared errors between
 159 β_i and $\hat{\beta}_i$. Note, in the formulation herein, cf. Eqs. (8)-(11), four observables are used for
 160 the estimation of the six (unknown) wave parameters. However, if relevant, additional
 161 observables could easily been included.

162 3.1. Constraints and initial values

163 It is noteworthy that energy preserves (it is a requirement!) and, hence, the optimi-
 164 sation problem is formulated with the following constraint,

$$\hat{H}_{s,1}^2 + \hat{H}_{s,2}^2 \equiv H_{s,e}^2 \quad (15)$$

where $\hat{H}_{s,1}$ and $\hat{H}_{s,2}$ are the single significant wave heights used in the parameterised absolute spectrum. Furthermore, a second type of constraint is introduced to reflect that the spectral model, i.e. the parameterised wave spectrum, should represent the expected spectral shape for a given combination of H_s and T_p . This has been studied in detail by Torsethaugen and Haver (2004) and, based on empirical findings therein, it is suggested to impose the following additional constraint on the one (wind-dominated) wave system,

$$\hat{\gamma}_1 = k_g s_p^{6/7}; \quad s_p = \frac{2\pi}{g} \frac{\hat{H}_{s,1}}{\hat{T}_{p,1}^2}; \quad k_g = 35.0 \quad (16)$$

171

Like in any other optimisation, a set of initial values needs to be assigned for $\mathbf{\Omega}$. The initial values for the periods $\hat{T}_{p,1}, \hat{T}_{p,2}$ are functions of $T_{z,e}$ following the proposal made by Nielsen (2017), while the significant wave heights $\hat{H}_{s,1}, \hat{H}_{s,2}$ initially take equal shares of $H_{s,e}$, that is, $\hat{H}_{s,1} = \hat{H}_{s,2} = \frac{1}{\sqrt{2}} H_{s,e}$, and the peakedness factors are initialised by the values $\gamma_1 = \gamma_2 = 2$.

177 3.2. Other optimisation-based conversion algorithms

It should be mentioned that the original work (Nielsen, 2017) also introduces the possibility to rely on an optimisation in the transformation of wave energy spectral densities from encounter to absolute domain. However, the fundamental solution strategy was different compared to the present work, since the cost function was established from a direct comparison of spectral densities in the two domains, leading to a non-robust and unstable solution that turned out to be sensitive to the initial guess of the wave parameters to be optimised. Likewise, a study by Montazeri et al. (2016) introduces the calculation of spectral moments as the very means for ship motion-based wave spectrum estimation. As a last note in this context, it deserves to be mentioned that attempts have also been made by Japanese researchers in the past to secure 'frequency conversions in the evaluation of wave spectra', cf. Saito and Maeda (1998), but, unfortunately, details of the work are accessible only in Japanese. Thus, any potential valuable information of this Japanese work is not clear to the author, but based on a screening of the actual paper, it appears that an optimisation strategy, like suggested in Nielsen (2017), is investigated.

192 3.3. *Evaluation of performance*

193 In the following sections, the transformation algorithms will be tested. In Section 4,
194 testings are made extensively on simulated data to have a solid indication of the perfor-
195 mance levels of the old and new transformation techniques and their 'mutual consistency'.
196 The reason to test on synthetic data is clear, as it allows complete knowledge about the
197 processes to be studied, whereas real ocean wave conditions can be extremely difficult to
198 accurately deal with because the truth is never exactly known. On the other hand, some
199 means must be established to evaluate, or validate, the outcome of the transformation
200 technique(s), in case the *true* generating conditions are unknown; which typically will
201 be the case when full-scale data is studied. In the present article use will be made of
202 full-scale data where "external" information *is* available by buoy measurements from a
203 free-floating wave buoy positioned at a fixed location. Thus, testings of the transforma-
204 tion techniques are also made on full-scale data, cf. Section 5.

205 4. **Testing with numerical simulations**

206 In this section, the transformation techniques will be applied to transform a given
207 encountered distribution of wave energy densities. Initially, the two algorithms are care-
208 fully compared in a study, Subsection 4.1, focused on conditions described by a unimodal
209 sea and based on moderate forward speed. Subsequently, testings are conducted using
210 more complex scenarios, cf. Subsections 4.2 and 4.3.

211 The numerical simulations will be entirely focused on following sea conditions **in short-**
212 **crested (directional) waves using a plus/minus 90 degrees spreading**. Realistic operational
213 scenarios are considered and, in the context of ship motion-based sea state estimation
214 (Nielsen et al., 2018), it means that stochastic simulations of the encountered wave
215 spectra will be the input to the transformation techniques. The (stochastic) realisations
216 of encounter wave spectra are obtained by simulating time histories of the encountered
217 wave elevation process itself, where a standard simulation procedure is used (e.g. Nielsen,
218 2017) for the generation of the time histories. Subsequently, spectral analysis provides
219 the "measured" wave spectrum, where for instance the WAFO toolbox (Brodtkorb et al.,
220 2000) or the function `cpsd` in MATLAB® can be used to make the FFT, but noting
221 that any standard approach can be used. The use of stochastic simulations should be

followed by a statistical analysis/evaluation to obtain the average performance of the transformation techniques. Thus, the evaluation is based on (sub)cases that, individually, consist of totally 20 single 30-minutes time history simulations of encountered wave elevation records.

4.1. Case study based on a unimodal sea

In the first case study, several subcases are studied. The subcases have as input a combination of 'generating-parameters', where the most relevant ones to study are the forward speed U of the vessel, the (relative) wave heading χ , and the characteristic period of the wave system, for instance the peak period T_p . Any variation in energy content, represented by significant wave height H_s , of the generating wave system is of no importance, since both the new transformation technique, Section 3, and the old one (Nielsen, 2017) preserve energy as a requirement, i.e. energy conservation is prescribed in the transformation process. The naming of the individual subcases follows a convention represented by ' $U.T_p.\chi$ ', and in this way combinations are defined in accordance with Table 1. Hence, it is understood that totally thirty ($= 2 \times 3 \times 5$) subcases exist and, for example, one specific subcase, say, B.a.IV is based on a generating-condition with $U = 12$ knots, $T_p = 8$ s, and $\chi = 60$ deg. Throughout, the significant wave height is 2.0 m but this will not be given any further attention; emphasising, and repeating, that H_s , by "default" preserves in the transformation.

Although the new and the old transformation techniques both allow for a double-peaked (absolute) wave spectrum, the initial case study is based on numerical simulations assuming a single-peaked spectrum. Specifically, the generating *point*-wave spectrum is taken as a Bretschneider spectrum, cf. Eq. (5), **assuming a fully developed sea.**

Table 1: Naming convention of test cases: Each subcase is named ' $U.T_p.\chi$ ' which represent a combination of the parameters of vessel speed U (A or B), wave peak period T_p (a or b or c), and relative wave heading χ (I or II or III or IV or V).

	U [knots]		T_p [s]			χ [deg.]				
Combination:	A	B	a	b	c	I	II	III	IV	V
Values:	9	12	8	10	12	0	20	40	60	80

Moreover, the optimisation-based results are (initially) achieved *without* enforcing the second constraint, cf. Eq. 16, related to the spectral shape of the (transformed) absolute wave spectrum.

The generating conditions are defined by combinations of the parameters specified in Table 1. Hence, any wave elevation record can be simulated, using a specific parameter-combination, and, subsequently, a corresponding encounter wave spectrum can be computed. It should be noted that the simulations are made with the wave spectrum truncated at $\omega_{high} = 4\pi$ (corresponding to 2 Hz). An example of a particular wave elevation realisation, as measured by an advancing ship, is shown in Figure 2 that also includes the corresponding encounter-wave spectrum. With this 'sequence' at hand, the transformation to absolute domain can be made by application of the relevant transformation technique; emphasising, as mentioned previously, that the transformation takes the relative wave heading and the vessel forward speed to be (exactly) known, with values as listed in Table 1 for any given subcase. It should be noted that, by nature, ocean wave spectra are smooth in their frequency-wise density distribution. In the spectral analysis (i.e. FFT) of the wave elevation record(s), smoothing is imposed by a Parzen window applied with a 50% overlap on the full range of frequencies from the FFT.

Figure 2: Top: Wave elevation record as encountered by an advancing ship ($U = 9.0$ knots, $\chi = 40$ deg.). Bottom: The corresponding encounter-wave spectrum, including the generating spectrum (Bretschneider: $H_s = 2.0$ m, $T_p = 8.0$ s).

262 4.1.1. Results and discussions

263 The pair of outcomes shown in Figure 2 makes up one single realisation of encounter-
 264 wave conditions, out of twenty, for one single of the thirty subcases found in Table
 265 1. Thus, a specific (transformed) absolute wave spectrum is associated with the given
 266 encounter-wave spectrum, and for the particular realisation the result is shown in Figure
 267 3. Note, from this point and onwards the results of the new transformation technique,
 268 Section 3, is named 'Optimisation', whereas the old technique (Nielsen, 2017) is named
 269 'Direct'.

270 The entire collection of absolute wave spectra, as produced by the transformation
 271 techniques, is presented in Figures 4-6, where the left- and right-side plots present the
 272 results of $U = 9$ knots and $U = 12$ knots, respectively, and any pair of left- and right-side
 273 plots applies to a specific relative heading $\chi = \{0, 20, 40, 60, 80\}$ deg., for the different
 274 (generating) peak periods $T_p = \{8, 10, 12\}$ s in Figures 4, 5, and 6, respectively. The
 275 general observation from the plots is that the optimisation-based transformation provides
 276 the most consistent and accurate results. The problem with the direct approach clearly
 277 manifests in some of the subcases, where energy densities are indeed "misplaced" on the
 278 transformed frequency range(s). The main concern with the optimisation-based results is
 279 the tendency that the spectral shape, in some the subcases, looks more like a "standard"
 280 JONSWAP spectrum with $\gamma \approx 2 - 3$, while in reality $\gamma = 1$ for the generating spectrum
 281 in all the (particular) subcases, as a fully developed sea is considered.

Figure 3: Transformation of encounter-wave spectrum (Figure 2) to absolute domain with results for the new optimisation-based technique (Sec. 3) and the former (more) 'direct' approach (Nielsen, 2017). The true, generating spectrum is included (Bretschneider: $H_s = 2.0$ m, $T_p = 8.0$ s), and the conditions correspond to subcase A.a.III.

It is not practically (feasible) to compare each and every single spectrum one-by-one with the true generating ones for all realisations making up the entire set of subcases. In the following, it is therefore decided to make a statistical evaluation of the transformation

Figure 4: Absolute wave spectra of subcases A.a.I-A.a.V (left-hand side plots) and subcases B.a.I-B.a.V (right-hand side plots).

techniques' performances, and the approach for doing this is explained below. The basis
for the evaluation is the entire collection of (transformed) absolute spectra, cf. Figures
4-6. Any given absolute spectrum associates a set of (transformed) absolute wave param-

Figure 5: Absolute wave spectra of subcases A.b.I-A.b.V (left-hand side plots) and subcases B.b.I-B.b.V (right-hand side plots).

eters which can be directly compared to the corresponding correct, or *generating*, set of values. It is decided to give focus to the three characteristic periods T_z (zero-upcrossing period), T_m (mean period), and T_p (peak period), that all are easily calculated when the

Figure 6: Absolute wave spectra of subcases A.c.I-A.c.V (left-hand side plots) and subcases B.c.I-B.c.V (right-hand side plots).

wave spectrum is available, cf. Eqs. (8)-(11). In the end, there will be twenty of such
 comparisons for each of the considered subcases, cf. Table 1, and a statistical evaluation,
 focused on the average value of relative deviations between the 'transformed value' and
 the 'mutual true value', can be made. Thus, statistics are produced for the considered
 set of wave parameters from any one subcase's twenty outcomes, which in themselves
 result in plots of 'performance measures' as shown in Figure 7. In the figure, it is under-
 stood that any single point indicates the relative deviation between the transformed and
 the true, generating one. The particular comparison between (estimates of) transformed
 wave parameters to true (generating) ones is feasible only because numerical simulations
 are in study. In real-case scenarios, this sort of comparison is generally not possible, as it
 requires the (true) absolute wave spectrum to be available, which it is not in most cases.
 The primary observation from Figure 7 is that the optimisation-based transformation
 technique consistently provides better estimates of the true absolute wave periods, where
 it should be kept in mind that the particular plots apply to a specific subcase, namely
 A.a.I, cf. Table 1 ($U = 9.0$ knots, $\chi = 0$ deg. for $T_p = 8.0$ s). The complete statistical
 evaluation for each and every single subcase is observed from Figure 8 that presents the
 performance of the two transformation techniques when they have been applied to all

Figure 7: Comparison between transformed wave periods, from encounter to absolute domain, and corresponding generating periods of all realisations making up subcase A.a.I (cf. Table 1). Relative deviation is considered for, respectively, zero-upcrossing period (upper), mean period (middle) and peak period (lower).

308 subcases. The individual points are measures of the average performance with respect
 309 to a particular characteristic wave period for any given subcase. Specifically, the single
 310 points yield the average relative deviations, relevant for a given wave parameter, with
 311 due account to all of the realisations of the given subcases. Thus, it is understood that
 312 the average values have been obtained from any set of twenty (stochastic) outcomes,
 313 like Figure 7, representing the data of one subcase. Moreover, it should be noted that
 314 the standard deviation is included as an "error-indicator" positioned at the center of
 315 the points as plus-minus one standard deviation. The reason to focus on (just) the
 316 wave periods (T_z, T_m, T_p) and not (all) other characteristic wave parameters is somewhat
 317 subjective, but it is clear that these periods yield a reasonable representation of how the
 318 energy density, on average, is distributed with wave frequency. The plots in Figure 8 are
 319 separated, column-wise, in the two speeds and, row-wise, in the characteristic periods,
 320 with any plot showing the results for the respective set of subcases (cf. Table 1). The

Figure 8: Statistics of relative deviation between transformed wave periods and true corresponding periods in **absolute domain**. Statistics are shown for the zero-upcrossing period (T_z , top), the mean period (T_m , middle), and peak period (T_p , bottom).

plots reveal that both transformation techniques provide reasonable results, although exceptions occur. For both speeds there is a similar tendency that the results of the two transformation techniques improve with account to two observations: (a) as heading becomes closer to beam sea, (b) as the wave period increases, where the findings are

Figure 9: Comparison of statistics produced by optimisation-based technique and direct approach (Nielsen, 2017) when relative deviation between transformed wave periods and true corresponding periods in **absolute domain** are considered, with a "decomposition", or dependence, on heading angles.

most notable for the direct approach. With a focus on the direct approach it can be seen that this method performs better for the lower speed ($U = 9$ knots vs. 12 knots). This observation does not show for the optimisation-based technique, where the results of the two speeds (9 knots vs. 12 knots) basically are equally good.

It may be valuable to evaluate the performance level of the transformation techniques with respect to the average accuracy considering all three characteristic periods, quantified as a *whole*, to have a better interpretation of a direct comparison between the two techniques. The comparison can therefore be organised more like a correlation-type-of-plot, see Figure 9. In this comparative illustration any point *not* located at origo represents an error relative to the exact generating value, and it should be realised that the distance from origo on the y-axis is the error of the optimisation-based technique whereas the distance away from origo on the x-axis is the error of the direct approach. Based on the plots in Figure 9 the most notable observations are the following: (a) the optimisation-based technique yields errors on the wave periods that are somewhat evenly distributed around zero, whereas the direct approach tends to produce wave periods which are too large; (b) the optimisation-based technique is capable to give the better results the closer the wave heading is to cases of strict following waves with $\chi = 0$ deg.; (c) as the wave heading becomes closer to beam sea, the direct approach produces equally good results. It is clear that these findings can be observed also by taking a closer look at the entire set of the individual absolute spectra obtained from all the realisations, cf. Figures 4-6.

4.1.2. Main findings

Some overall findings and remarks based on the preceding analyses can be formulated as:

- Comprehensive testings with numerical simulations indicate that both techniques can be used to transform an encountered wave spectrum to absolute domain. However,
- the optimisation-based transformation technique is superior to the direct approach almost conclusively; emphasising that this point relates to the specific set of numerical simulations and may not be generally true.

- Generally, the characteristic absolute wave parameters of the optimisation-based transformation agree with reasonable accuracy to the corresponding generating wave parameters; the exception is the spectral shape parameter (γ) that takes values on the higher side.

In the next subsections, additional testings are presented but a last note on the above set of results (e.g. Figures 4-6) concerns the spectral shape of the optimisation-based (transformed) spectrum. As mentioned in the last bullet point above, the value of the peakedness parameter is off, resulting in transformed spectra that look JONSWAP-like while in reality they should be Bretschneider-like. It is therefore interesting to impose the constraint in Eq. (16) that relates the spectral shape to H_s and T_p . Figure 10 illustrates the effect of the constraint when the two subcases B.a.I and B.b.I are re-considered, cf. Table 1. It is clear that the results improve in the sense that the (transformed) absolute spectra now have a shape which agrees better with the Bretschneider spectrum; and pointing out that this finding is generally representative to the other subcases listed in Table 1. On the other hand, it calls for additional tests to study fetch-limited conditions where the JONSWAP spectrum is usually assumed to be a better match. However, this is an exercise left for the future; where the referred work by Torsethaugen and Haver (2004) should be relevant to address also in such cases.

Figure 10: The effect of the constraint, cf. Eq. (16), can be seen by re-considering subcases B.a.I and B.b.I (upper plots in Fig. 5), which were previously found to have spectral shapes similar to a JONSWAP spectrum. Indeed, the introduction of the constraint leads to a better match, and this observation is identical for all remaining cases although not shown. Note, scale on y-axis is changed compared to Figs. 4-6.

373 4.2. Transformation at higher forward-speeds

374 The subcases in the previous tests were characterised by speeds 9 and 12 knots.
 375 These values do not necessarily reflect real, in-service conditions of merchant ships. On

Figure 11: Absolute (transformed) wave spectra for advance speed $U = 18$ knots (left-hand side plots) and $U = 21$ knots (right-hand side plots). True wave peak period and wave heading appear in text within the single plots. Note, scale on y-axis is changed compared to Figs. 4-6.

the other hand, an increase in advance speed is indeed resulting in more complex conditions due to the (stochastic) Doppler Shift, as also reported by Lindgren et al. (1999). From a theoretical viewpoint it is of interest to have efficient and accurate transformation techniques at *any* vessel-advance speed, but this concern is not fully applicable to practical interests. The reason is that very often the merchant ships of today rarely sail at speed levels beyond 18-20 knots, drawing attention on today's - and the future's - slow-steaming regime. And, in case a ship advances at such high speed, the severity of the wave condition can certainly not be a real issue/problem, for what reason estimates of the (absolute) wave spectrum most likely is not needed; at least not for safety reasons. Or, alternatively, the ship may spend 20-30 minutes at reduced speed (10-15 knots) to allow for (ship motion-based) wave spectrum estimation (Nielsen et al., 2018). Nonetheless, performance evaluations of the transformation algorithms have also been made on cases with higher forward-speed. Thus, transformations have been conducted at speeds 18 and 21 knots, but otherwise under similar conditions, see Table 1. The main finding from such tests is that it is possible to successfully make the transformation in many cases by the optimisation-based technique, although the performance level drops compared to the preceding subcases (9 and 12 knots). Sets of transformed spectra are shown in Figure 11 where the results for $U = 18$ knots and $U = 21$ knots are shown at the left-hand side and right-hand side, respectively, and showing the results just for $T_p = 10$ s. The most important observations are: (a) The optimisation-based results are fairly consistent for $U = 18$ knots *and* reasonable results are obtained, except for the case at $\chi = 80$ deg. The observations are somewhat identical for $U = 21$ knots although the performance is more "obscured"; (b) The direct approach fails to transform in almost all cases for both speeds; but, importantly, it yields reasonable results at $\chi = 80$ deg. for both speed cases; (c) The optimisation-based approach "predict" (erroneously) a transformed spectrum with two distinct peaks for nearly all realisations related to $\chi = 80$ deg. for both speeds. It is not fully understood why double-peaked (transformed) spectra manifest for $\chi = 80$ deg. but it should be kept in mind that the simulations are concerned with short-crested waves, based on a plus/minus 90 degrees spreading. Thus, the effect of the Doppler Shift becomes indeed (more) relevant/important for a wide range of directional wave components at higher advance speeds compared to situations at lower

407 speeds. Again, it may be useful to consult the referred work about "Stochastic Doppler
 408 shift and encountered wave period distributions in Gaussian waves" (Lindgren et al.,
 409 1999, p. 514) that supports this explanation.

410 4.3. Double peaked (bimodal) wave spectrum

411 The final testings are made on a more complex wave scenario where a bimodal wave
 412 spectrum is considered. The test conditions are given in Table 2 and, on this basis, nu-
 413 merical simulations of wave elevations have been computed exactly like in the preceding;
 414 emphasising that short-crested directional waves are considered also here. From the table
 415 it is seen that totally eight (4×2) subcases exist, where the main differences relate to
 416 heading, studied in subcases a versus b, and speed. Moreover, two different wave condi-
 417 tions are studied, where cases 1 and 2 consider $T_p = \{15, 10\}$ s for the peak periods of a
 418 swell and a wind sea system, respectively, while cases 3 and 4 consider $T_p = \{20, 10\}$ s
 419 for the peak periods of the bimodal wave system. The results are presented in Figures
 420 12 and 13.

421 The findings from the figures can be summarised as: (i) Overall, the best results
 422 are obtained for the lower speed $U = 9$ knots (independent on the algorithm), i.e. the
 423 left-hand side plots in both figures; (ii) The optimisation-based algorithm is capable to
 424 distinctly separate the swell and wind sea peaks at $U = 9$ knots for the heading $\chi = 0$ deg.
 425 (iii) At the higher advance speed, the results are more obscured, and it is difficult to point
 426 out some general observations; except that for case 4a (upper right plot in Figure 13)
 427 both algorithms tend to be off, although the optimisation-based algorithm "detects" the
 428 wind sea peak correctly.

Table 2: Operational conditions and generating sea state in cases with a bimodal wave spec-
 trum.

	U [knots]	β [deg.]	Swell		Wind sea	
			H_s [m]	T_p [s]	H_s [m]	T_p [s]
Case 1(a,b)	9.0	{0,80}	2.0	15.0	2.0	10.0
Case 2(a,b)	18.0	{0,80}	2.0	15.0	3.0	10.0
Case 3(a,b)	9.0	{0,80}	2.0	20.0	2.0	10.0
Case 4(a,b)	18.0	{0,80}	2.0	20.0	2.0	10.0

Figure 12: Comparison of transformed absolute wave spectra including the (true) generating spectrum in case of a bimodal (short-crested) sea state. Cases 1a and 1b as the left-hand side plots, and cases 2a and 2b as the right-hand side plots.

Figure 13: Comparison of transformed absolute wave spectra including the (true) generating spectrum in case of a bimodal (short-crested) sea state. Cases 3a and 3b as the left-hand side plots, and cases 4a and 4b as the right-hand side plots.

5. Testing with full-scale experimental data

5.1. Measurements from sea trials

Experimental data has been collected during sea trials with a smaller research vessel, *R/V Gunnerus* owned and operated by the Norwegian University of Science and Technology. The specific vessel is characterised by main dimensions: Length $L_{pp} = 28.9$ m, breadth $B = 9.6$ m, draught $T = 2.7$ m, and block coefficient $C_B = 0.56$. A more detailed description of the sea trials, and the studied vessel, is given by Steen et al. (2016), and

Figure 14: Vessel run paths during sea trials; IDs 03-06 at top and IDs 07-10 at bottom with a very short transition time in between all runs. The (visually) observed wave heading is included in parentheses; noting that the observed wave direction was nearly the same during all runs, with waves coming from North North-East. From Nielsen and Brodtkorb (2018).

here it suffices to say that the experimental data has been used in a study about ship
 motion-based sea state estimation (Nielsen and Brodtkorb, 2018) which makes the spe-
 cific set of data interesting. During the sea trials, the run pattern of the vessel was like

Figure 15: Pairs of encounter wave spectrum and corresponding absolute wave spectrum as obtained from experimental sea trials data (Nielsen and Brodtkorb, 2018).

illustrated in Figure 14, where it is noted that the analysed motion recordings have been obtained on the single straight-line paths. On each path, course and engine power were held constant to secure stationary conditions, and the durations of the runs were 25-30 minutes. The detailed outcome of the ship motion-based wave spectrum estimation will not be presented herein. Instead reference is given to Nielsen et al. (2018) and Nielsen and Brodtkorb (2018), pointing out that the outcome of the ship motion-based estimation includes the *encounter* wave spectrum. As part of the sea trials data, information was also available about the *absolute* wave spectrum, which was measured continuously by a free-floating wave buoy (Datawell BV) deployed at a fixed position in the test area.

5.2. Performance of transformation algorithms

The two transformation techniques have been applied to transform the *measured* encountered-wave energy density distributions to corresponding absolute wave spectra. The run pattern, cf. Figure 14, of the sea trials data shows that four particular run paths correspond to following sea conditions. The runs are ID03, ID04, ID07, and ID09, and it is noteworthy that the advance speed of the vessel has been reported as 10-12 knots in the specific cases. The outcomes of the analyses are shown in Figure 15, where the pairs of upper and lower plots show the estimated encounter-wave spectrum and, respectively, the (transformed) spectrum estimate in absolute domain together with a corresponding estimate by a free-floating wave buoy.

Inspection of the plots in Figure 15 reveals that the two transformation techniques give fairly consistent results in three of the cases, but in the one case, ID09, there is an inconsistency. The comparison with corresponding wave spectrum measurements by a free-floating wave buoy favours the optimisation-based transformation technique. Thus, it is evident that the absolute wave spectra produced by the optimisation-based technique are indeed consistent with corresponding estimates by the wave buoy.

6. Summary and suggested future work

In the context of ship motion-based wave spectrum estimation, one well-performing methodology (Nielsen et al., 2018) requires a means to transform an encountered distribution of wave energy densities to absolute domain. In this article, an optimisation-based

468 technique to transform encounter-wave spectra to absolute domain has been proposed.
 469 Comprehensive testings with simulated data, based on a unimodal wave spectrum, indi-
 470 cated a high level of performance of the specific transformation technique. Investigations
 471 of the performance were conducted at different advance speeds, in the order 9-21 knots,
 472 and at various 'generating conditions' of the wave system. Along the investigations, com-
 473 parisons were made to an existing transformation technique (Nielsen, 2017). It became
 474 clear from the studies of simulated data that the (new) optimisation-based technique led
 475 to more consistent and accurate absolute wave spectra, and the same finding was made
 476 in a study using experimental full-scale sea trials data. Testings at the higher advance
 477 speeds (18-21 knots), for numerically simulated data, indicated a decrease in the general
 478 performance level of the transformation algorithms. Additional tests, based on numerical
 479 simulations using a bimodal wave spectrum, confirmed that that the transformation al-
 480 gorithm(s) perform satisfactorily at lower speeds, while the performance drops at higher
 481 speeds (18 knots). In this connection, it needs, however, to be realised that, if the purpose
 482 of transformation relates to shipboard wave spectrum estimation (Nielsen et al., 2018),
 483 the particular vessel should be able to reduce speed in a shorter period of time (20-30
 484 minutes) to have consistent and reliable estimates of the - absolute - wave spectrum;
 485 pointing out also that very often the merchant ships of today rarely sail at speed lev-
 486 els beyond 18-20 knots, drawing attention on today's - and the future's - slow-steaming
 487 regime.

488 It is suggested that future work addresses proposals on the following list:

- 489 • Most of the analyses in this study were focused on numerically simulated data, while
 490 less focus was given to (full-scale) experimental data. Obviously, the transformation
 491 algorithm(s) should be tested more carefully with in-service and/or sea trial data.
- 492 • From the analyses it was observed, in some cases, that the optimisation-based algo-
 493 rithm failed to transform accurately, while, for the same cases, the direct algorithm
 494 (Nielsen, 2017) performed satisfactorily. Obviously, it should therefore be inter-
 495 esting to combine the two algorithms. Note, if such a combined algorithm were
 496 available, the use of a parameterised wave spectrum would also be relaxed, since
 497 the direct algorithm (Nielsen, 2017) is semi-nonparametric.

- The study of simulated data, or experimental data with external wave information available, e.g. by buoy measurements, allows direct validation of the transformation technique by comparing its outcome to the true "generating" conditions. However, in practice, estimates of the generating wave conditions will typically not be available and, hence, some other way to validate the (transformed) absolute wave spectrum should be developed.
- The particular transformation algorithms are (usually) applied for shipboard sea state estimation based on measured vessel motions. It should therefore be interesting to apply a given (transformed) absolute wave spectrum to make 'repeated response calculations', i.e. to simulate the real motion measurements. Thus, if the measurements can be successfully simulated by using the transformed absolute wave spectrum, in a given case, it is highly likely that the specific transformed spectrum is correct.

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