



## Timetable Design Theory and Practice

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# Timetable Design Theory and Practice

7<sup>th</sup> International Conference on Railway Operations  
Modelling and Analysis

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DTU Management Engineering

Danmarks Tekniske Universitet/Technical University of Denmark

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# Today's Program

- The Timetable Design Process
  - Current Practice
  - The stakeholders
  - Timeline
- Micro Optimization of the Train Path Plan
  - MISLP
  - BIOP
  - Hypergraph
  - PESP
- Future Opportunities

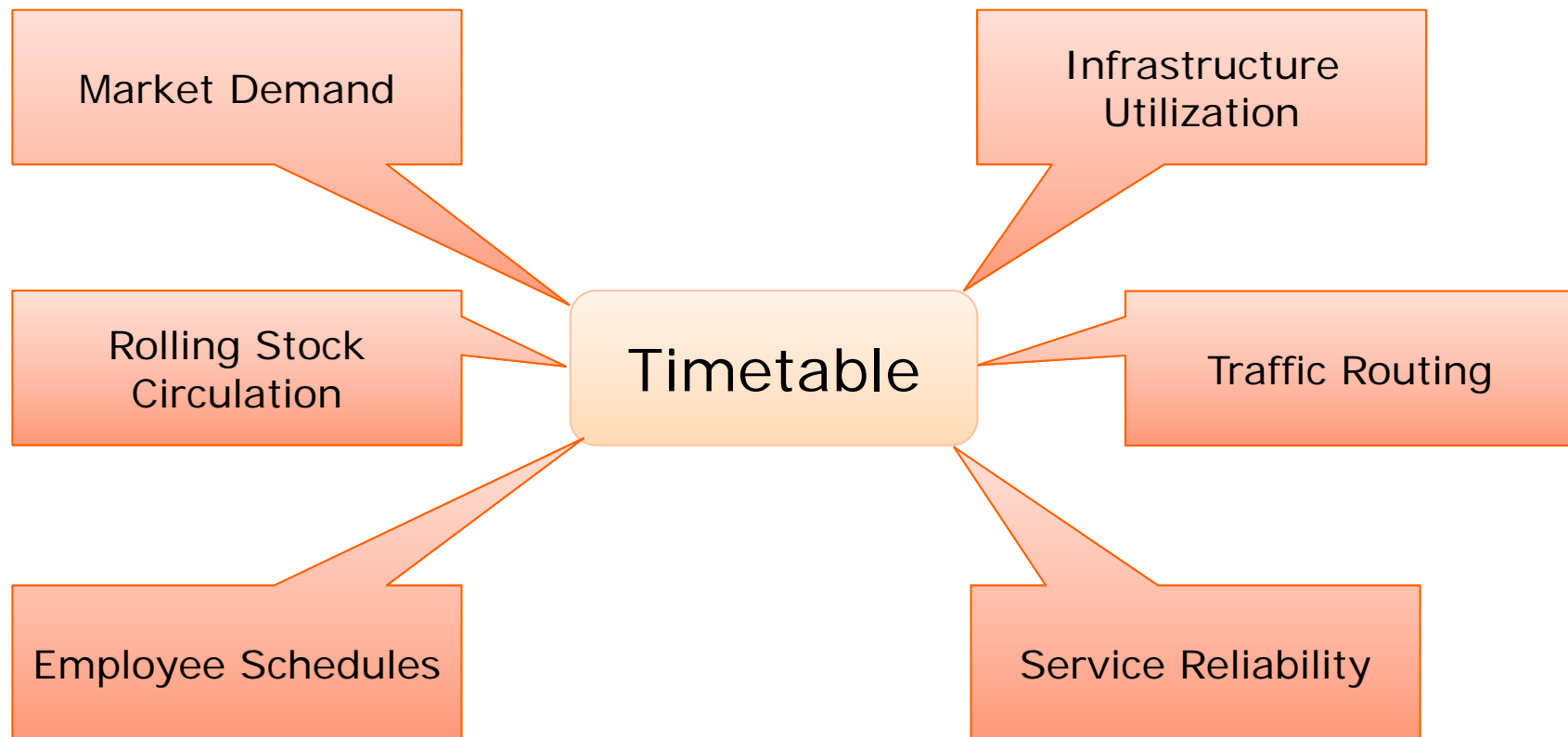


# Why Do We Need a Timetable?

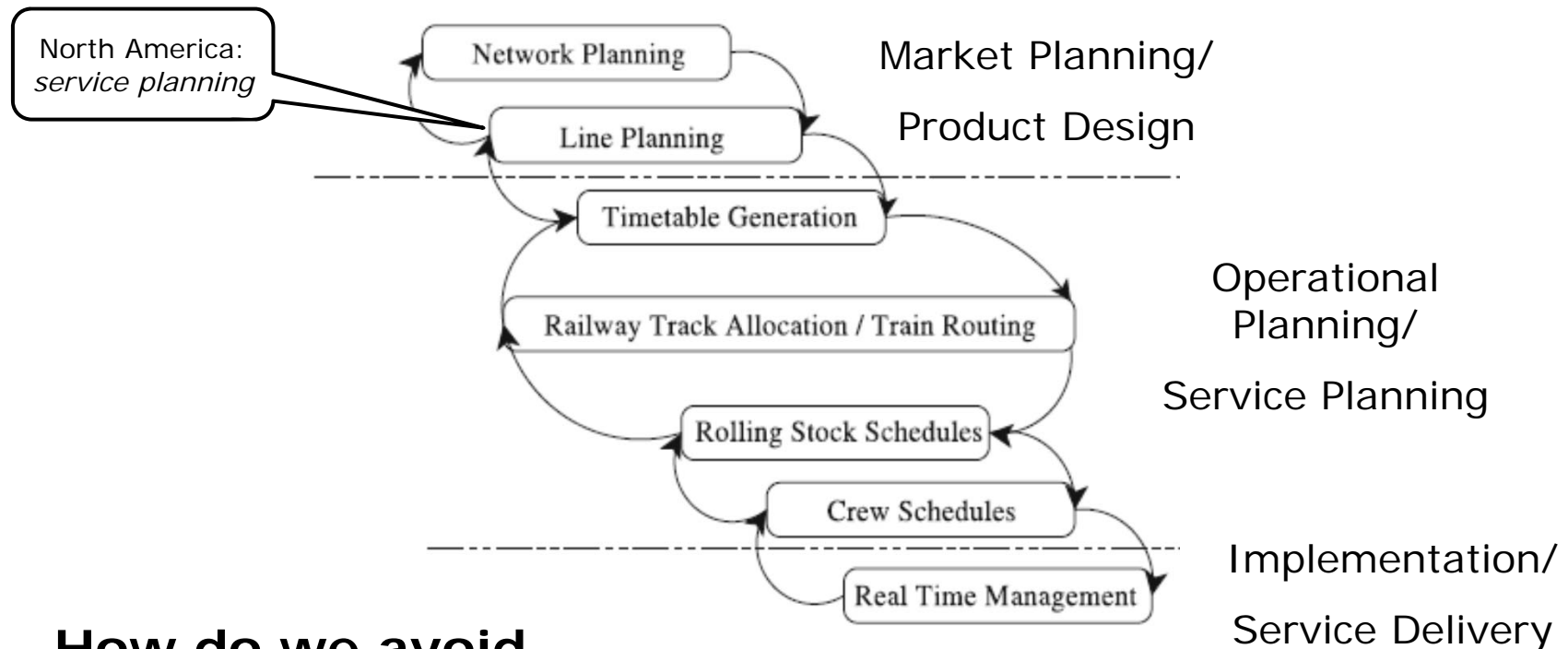
*Gør det selv!*



# The Timetable is Central to All



# The Timetable Design Process

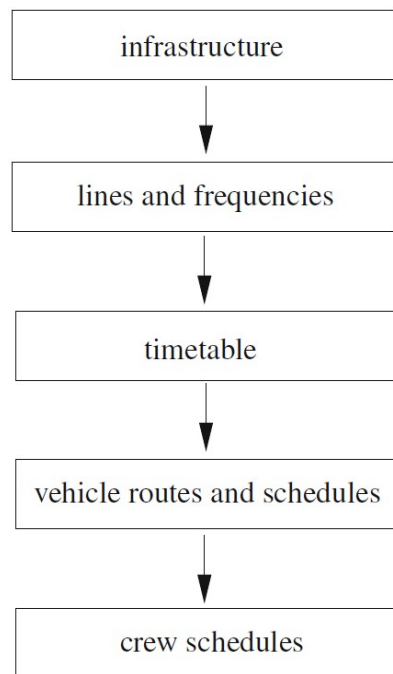


**How do we avoid the feedback loops?**

*The railway track allocation step does not exist in North American practice.*

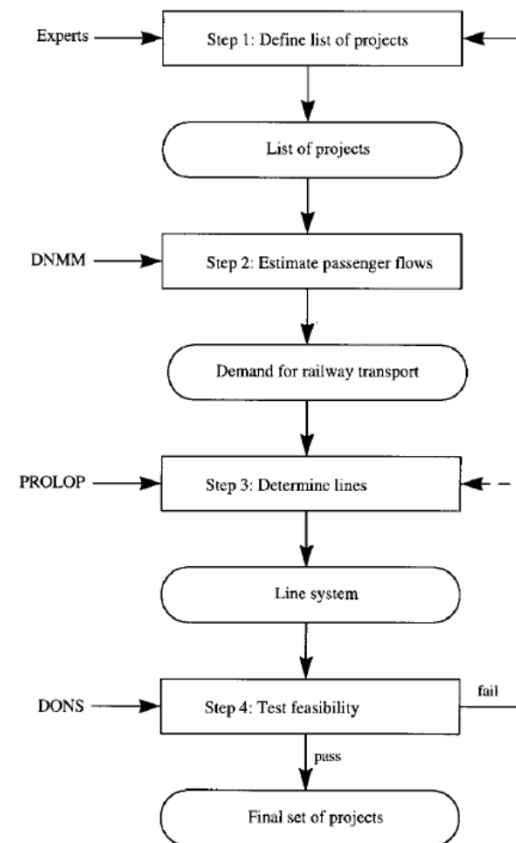
# The Application is Fairly Universal

## Danish - DSB



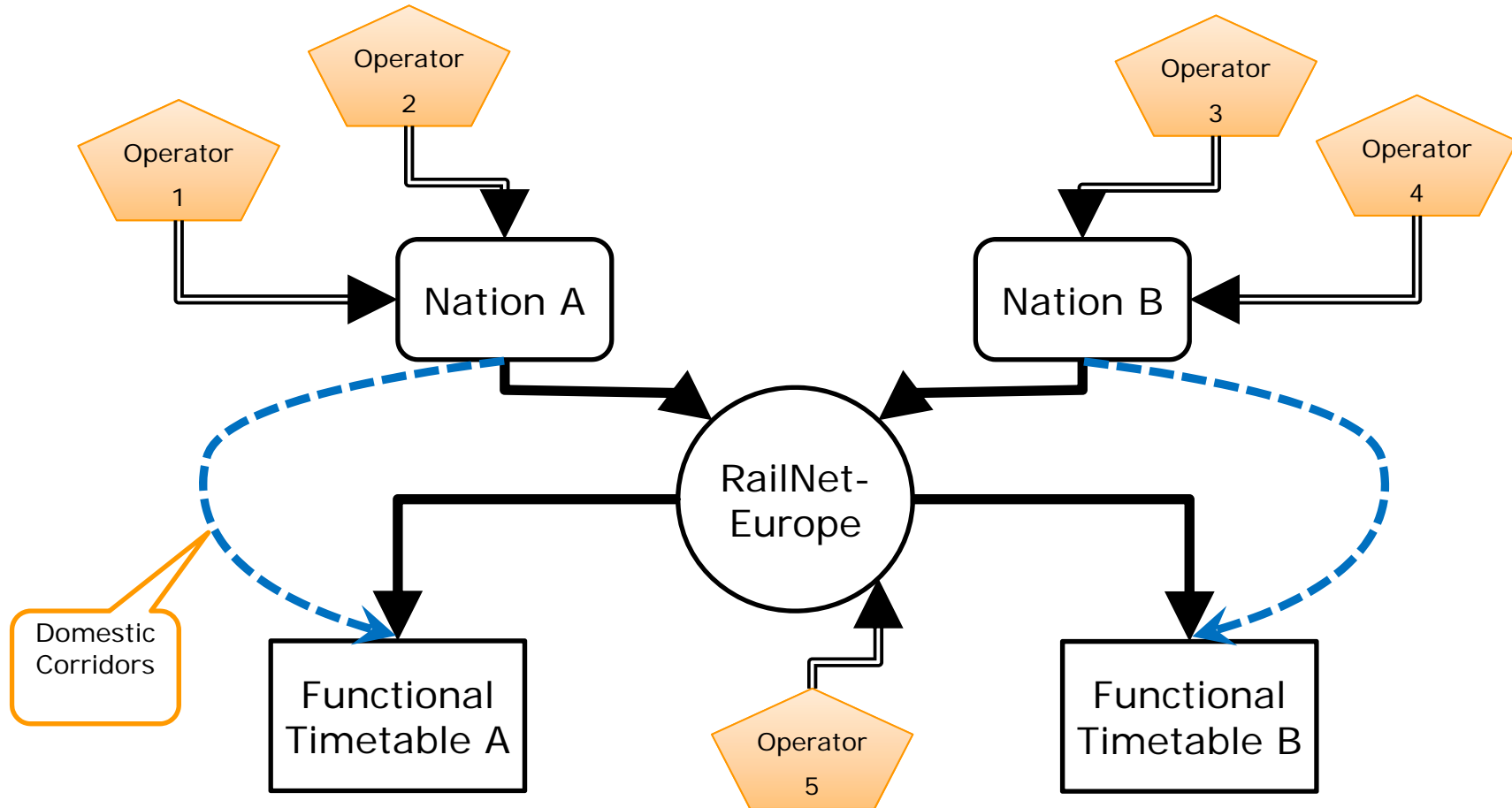
DSB (2015)

## Netherlands - NS



Hooghiemstra (1999)

# Complications Due to Jurisdiction, Competition



A separate planning process at *each* entity

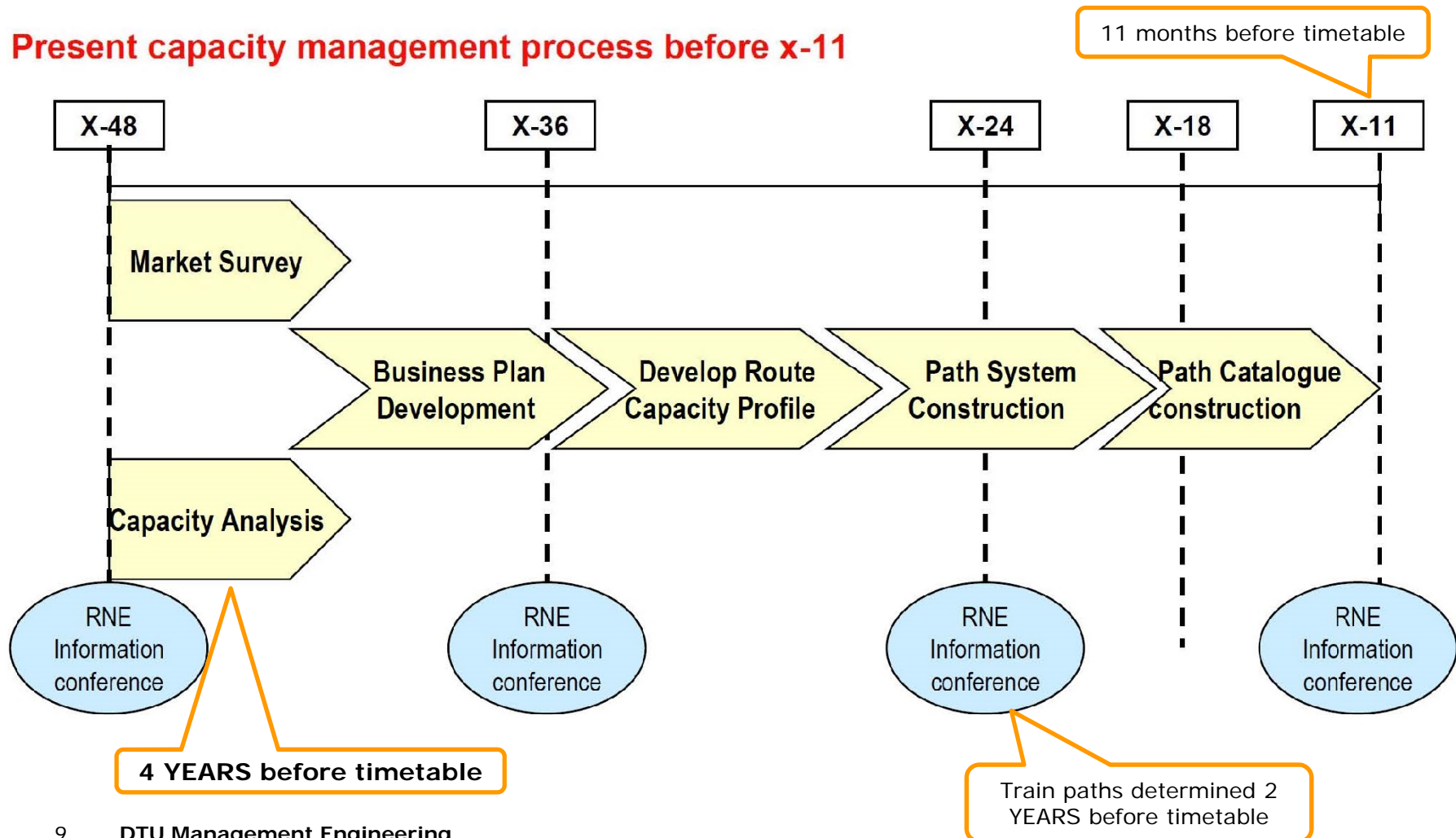


# Typical Planning Calendar - DSB

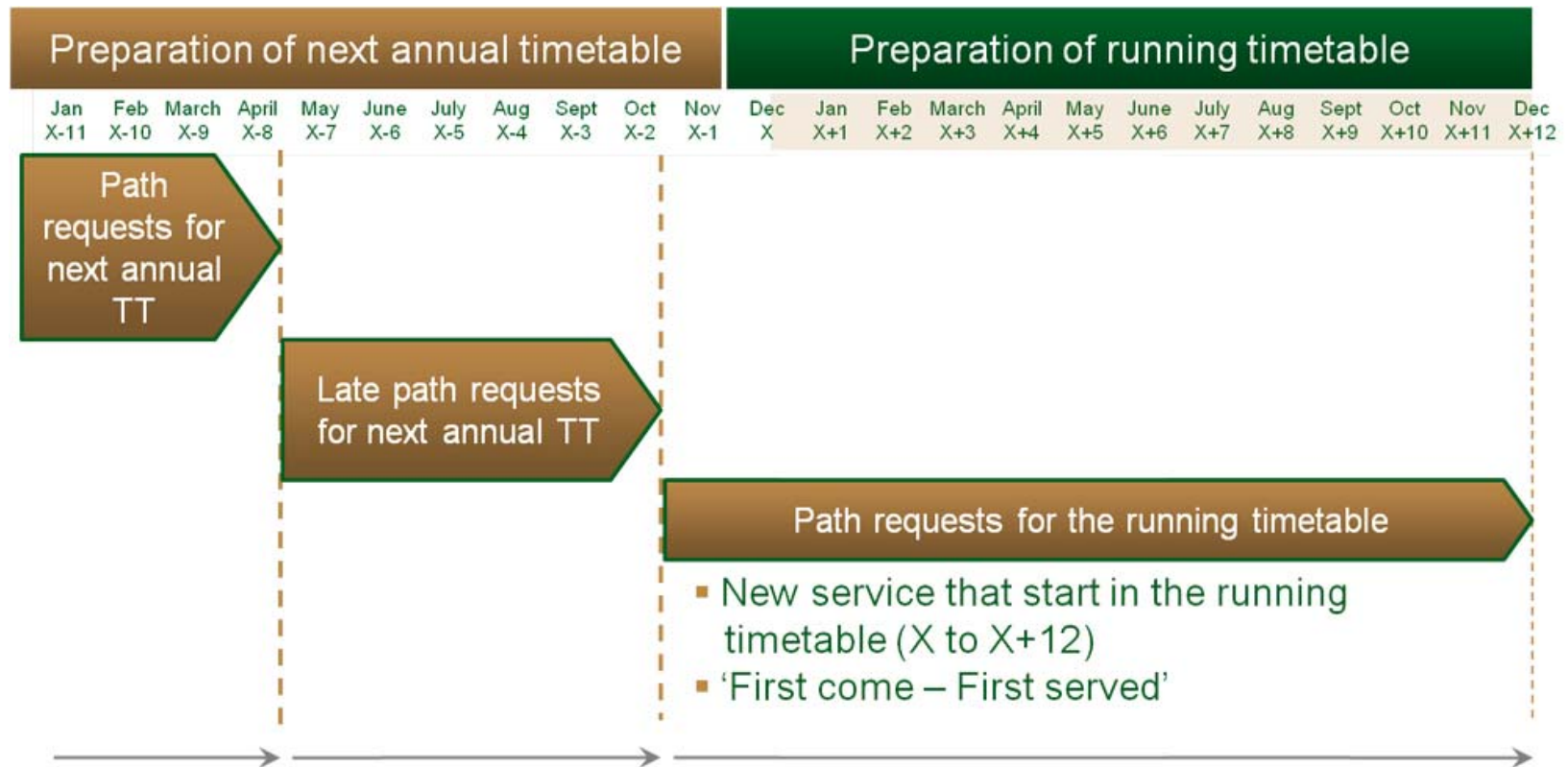
|               | 2+ years                 | 1 year                                    | 2-3 months                            | Now                                    |
|---------------|--------------------------|---|---------------------------------------|--|
| Timetable     | Commercial aspects       | Fixed departure and arrival times         | Trackwork and holidays                | Disturbances and delays                |
| Rolling Stock | Requirements and budget  | "Normal week" capacity, cycles, and tasks | Different capacities on specific days | Assignment of vehicles and maintenance |
| Maintenance   | Maintenance requirements | Planning of maintenance                   | Workshop plan                         | Carry out maintenance                  |
| Staff         | Recruiting, education    | Staff and unassigned trips                | Inform staff of assignments           | Dispatch personal to assignments       |

# Timetabling Process at Banedanmark (Danish Infrastructure)

## Present capacity management process before x-11

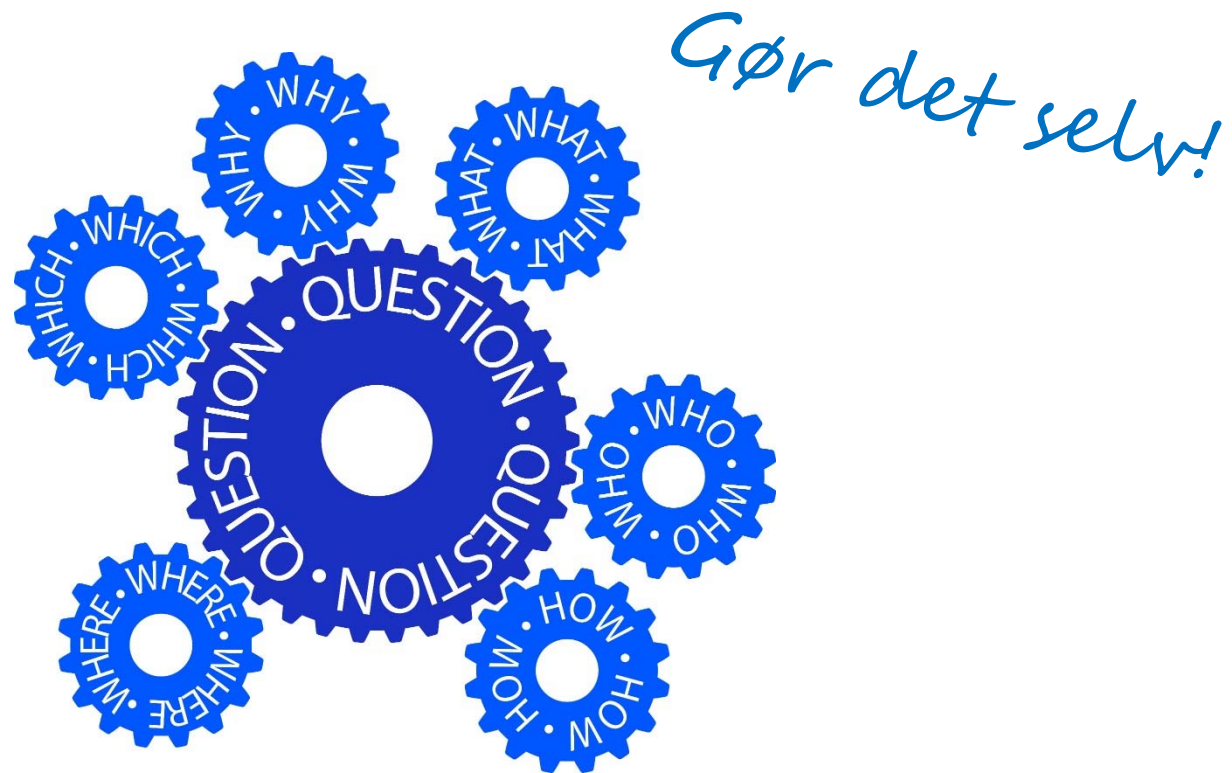


# Planning Calendar - EU



"X" is current timetable

# What Defines a Timetable?



# What Defines a Timetable?

In Europe, a timetable at a minimum defines:

- a) Individual paths for the trains to follow on the railway network (dispatching guide)
- b) Infrastructure commitments to individual trains (contracted train paths)
- c) No conflicts between trains (traffic coordination)
- d) It may also define tactical alternatives in case of disruption or changes in market demand

# Methods in Application

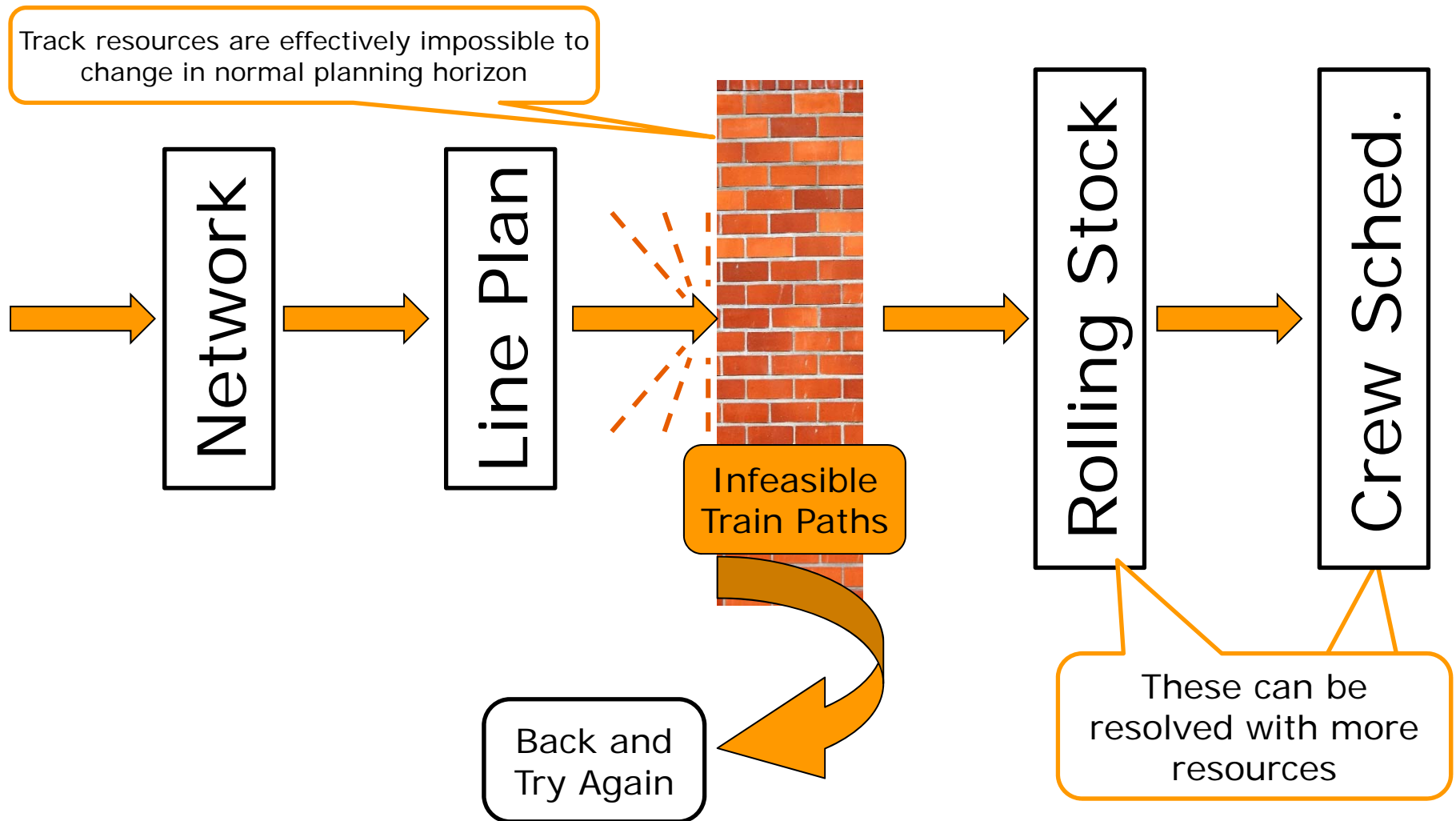
|                           | <b>Denmark – DSB</b>   | <b>Netherlands - NS</b>               |
|---------------------------|--|---------------------------------------|
| Demand Forecasting        | National Traffic Model<br>(Landstrafikmodellen)                                    | Dutch National Mobility<br>Model      |
| Train Path<br>Generation  | Timetable Planning<br>System<br>( <a href="http://www.hacon.de">www.hacon.de</a> ) | CADANS                                |
| Station Shunting          | - none -   | STATIONS                              |
| Line Planning             | ILP, Goossens & Kroon<br>(2006)  | ILP, "PROLOP",<br>Hooghiemstra (1999) |
| Validation,<br>Simulation | Railsys<br>( <a href="http://www.rmcon.de">www.rmcon.de</a> )                      | SIMONE                                |

# Some Timetable Definitions

- Feasible timetable: a timetable that is physically possible to operate if no delays occur
- Robust timetable: the ability to absorb small delays (doors held open, wet rail)
- Stable timetable: the ability to recover, to return to plan after a disruption has occurred
- Resilient timetable: a timetable that has options for tactical, ad hoc change to respond to disruptions
  - The ability to change train routes
  - The ability to modify the timing of a few select trains



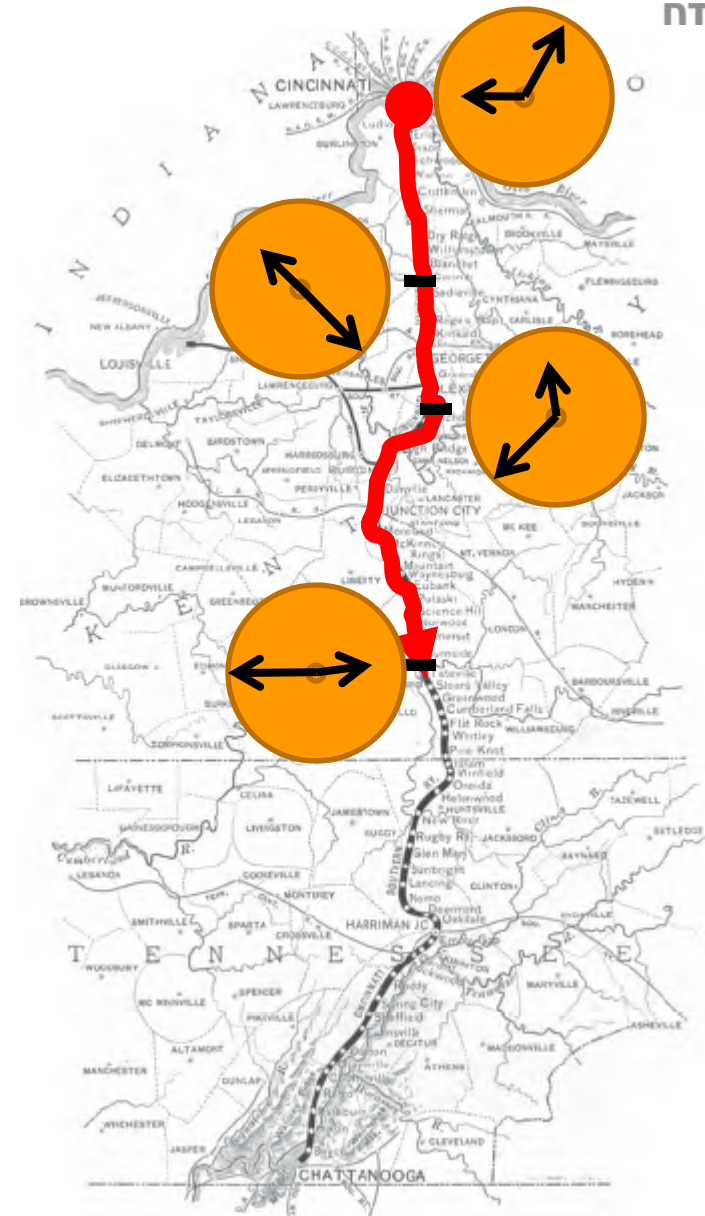
# Train Paths Are a Hard Constraint





# Timetabling

- A Train Path
  - Specific tracks, sidings
  - A contractual commitment
- Train Timings
  - Meet/pass plan
  - Safety separation
- Combinatorial Alternatives



# Historic Motivations

- European Land War
  - Harris & Ross (1955) [Ford and Fulkerson]
- High Speed Rail
  - Petersen (1987)
  - Harrod (2009)
- Dispatch Technology
  - Burlington Northern ARES (Jovanovich & Harker, 1990)
  - Southern Railway (Sauder & Westerman, 1983)
- Australia Iron Railways
  - Mees (1991)
  - Higgins (1997)
- Platform Assignment
  - Zwaneveld (1996)

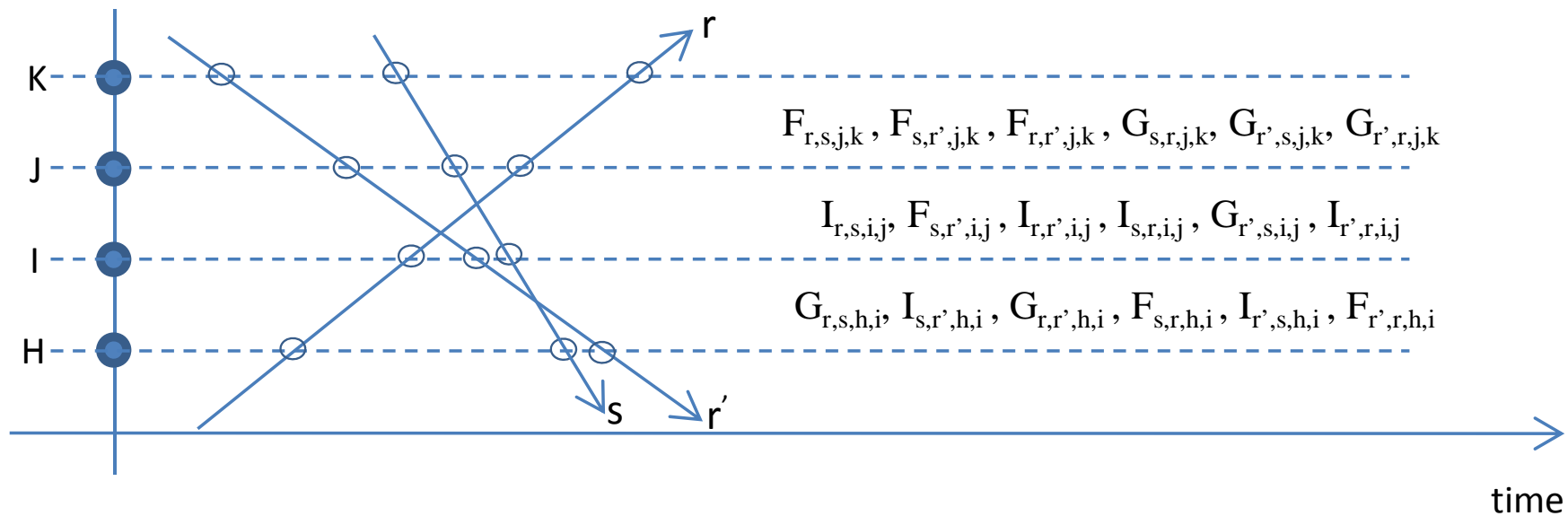
# Models

- MISLP
  - Originated at the University of Pennsylvania
  - Trials on Burlington Northern, not implemented
- BIOP
  - Motivated by Australian heavy haul trains
  - Single track focus
- Hypergraph
  - Desire for operationally feasible train paths in solution
  - “Zero basis” solution: no assumption of input set feasibility
- PESP
  - Used successfully in many single direction flows
  - Not a sequencing model, no track occupancies
  - Some challenges in overtake modelling

# Mixed Integer Sequencing Linear Program (MISLP)

- Binary Variables
  - Enforce sequencing of pairs of trains
  - I: indicates meet/pass interaction
  - F: first train precedes second train
  - G: first train follows second train
- Real Variables
  - Determine train timings
  - d: departure time from check point
  - a: arrival time at check point

# MISLP Sequencing Variables



Note that  $F_{r',s',j',k}$  implies  $G_{s',r',j',k}$

(MISLP)

$$\min \sum_{\substack{r \in R \\ i \in B_r}} [\beta_{r,i} |d_{r,i} - d_{r,i}^*|^\phi + \alpha_{r,i} |a_{r,i} - a_{r,i}^*|^\phi] \quad (1)$$

s.t.

$$d_{r,i} \geq a_{r,i} \quad \forall r \in R; i \in B_r \quad (2)$$

$$I_{r,r',i,j} + F_{r,r',i,j} + G_{r,r',i,j} = 1 \quad \forall r, r' \in R; i, j \in B_r \cap B_{r'} \quad (3)$$

$$a_{r,i} - d_{r,j} \geq s_{r,i,j} + \sum_{\substack{r' \in R \\ i,j \in B_r \cap B_{r'}}} f_{r,r',i,j} I_{r,r',i,j} \quad \forall r \in R; i, j \in B_r \quad (4)$$

$$d_{r,i} - a_{r',i} \geq M(1 - F_{r,r',i,j}) \quad \forall r, r' \in R; i, j \in B_r \cap B_{r'}$$

$$d_{r',j} - a_{r,j} \geq M(1 - G_{r,r',i,j}) \quad \forall r, r' \in R; i, j \in B_r \cap B_{r'}$$

$$a_{r',i} - d_{r,i} \geq M(1 - I_{r,r',i,j}) \quad \forall r, r' \in R; i, j \in B_r \cap B_{r'}$$

$$a_{r,j} - d_{r',j} \geq M(1 - I_{r,r',i,j}) \quad \forall r, r' \in R; i, j \in B_r \cap B_{r'} \quad (5)$$

$$a, d \geq 0 \quad I, F, G \in 0, 1$$

# MISLP Comments

- Note that all train interactions are pairwise
- Input train set must be feasible
- Not a capacity model
- No track resource variables, no pricing duals

# Binary Integer Occupancy Program (BIOP)

- All timings in discrete units
- Binary variables
- Multicommodity flow
- Network packing



(BIOP)

$$\min \sum_{\substack{r \in R \\ (i,t) \in B^r}} c_{i,t}^r x_{i,t}^r \quad (6)$$

s.t.

$$\sum_{(i,t) \in B^r | i=o^r} x_{i,t}^r = 1 \quad \forall r \in R \quad (7)$$

$$x_{i,t}^r = \sum_{(k,l) \in \Phi_{i,t}^r} x_{k,l}^r \quad \forall r \in R; (i,t) \in B^r | i \neq d^r \quad (8)$$

$$\sum_{(i,t) \in B^r | i=d^r} x_{i,t}^r = 1 \quad \forall r \in R \quad (9)$$

$$\sum_{\substack{m \in R \\ (k,l) \in B^m \cap (\Omega_{i,t} \cup \{(i,t)\})}} x_{k,l}^m \leq 1 \quad \forall (i,t) \in \bigcup_{r \in R} B^r \quad (10)$$

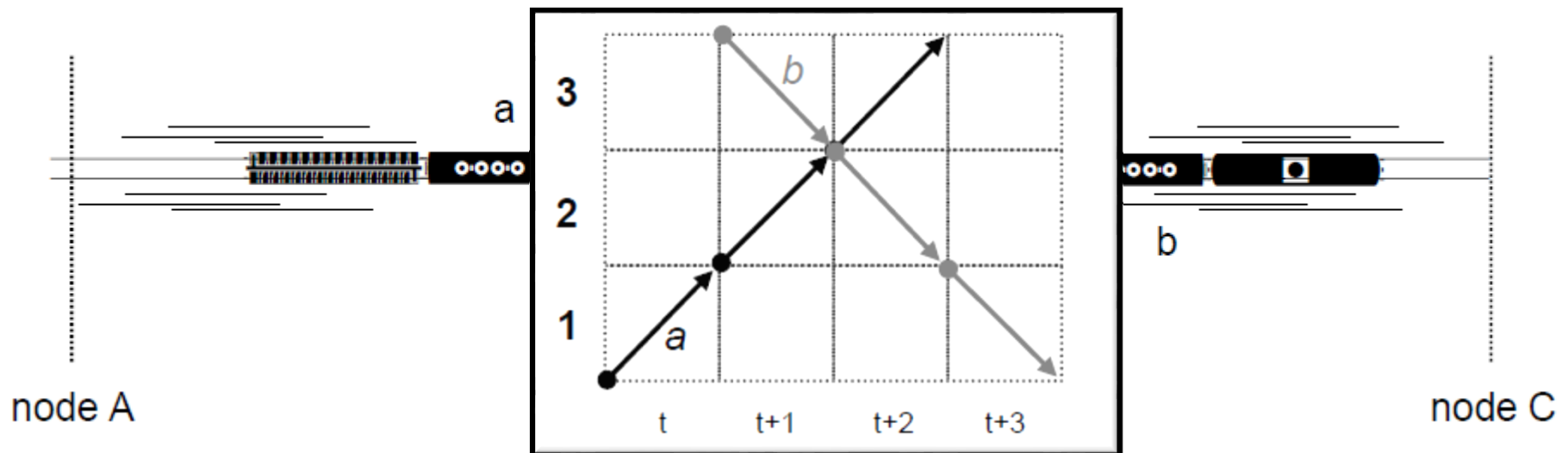
$$x \in \{0, 1\}$$

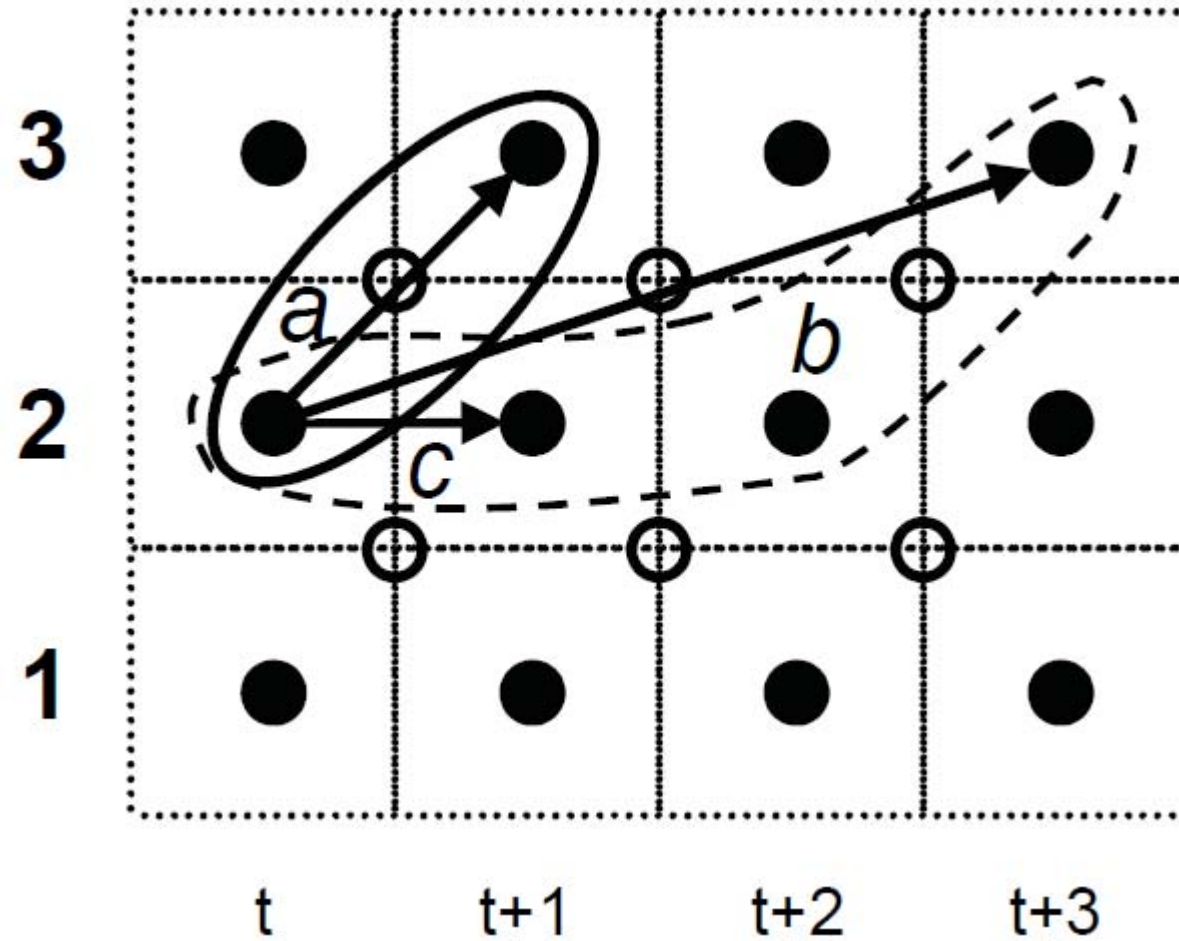
# BIOP Comments

- Explicit track constraints
- Pricing duals for infrastructure
- Input data set need not be wholly feasible
- Not “dispatch ready”

# Hypergraph

- Strict occupancy constraint is not operationally feasible
- Model pairs of track segments and their transitions





$$\begin{aligned}
\max \quad & \sum_{\substack{r \in R \\ (p_o^r, j, u, v) \in \Psi^r}} (c_p^r + c_e^r(u - p_e^r)) x_{p_o^r, j, u, v}^r \\
& + \sum_{\substack{r \in R \\ (p_d^r, e^r, u, v) \in \Psi^r}} c_l^r(p_l^r - v) x_{p_d^r, e^r, u, v}^r + \sum_{\substack{r \in R \\ (i, j, u, v) \in \Psi^r | i=j}} c_s^r x_{i, j, u, v}^r. \quad (11a)
\end{aligned}$$

Linear Network Constraints

$$\sum_{(p_o^r, j, u, v) \in \Psi^r} x_{p_o^r, j, u, v}^r \leq 1 \quad \forall r \in R \quad (11b)$$

$$\begin{aligned}
\sum_{(a, i, u, t) \in \Psi^r} x_{a, i, u, t}^r &= \sum_{(i, j, t, v) \in \Psi^r} x_{i, j, t, v}^r \\
\forall r \in R, \{i \in B | i \neq p_o^r\}, t \in T \quad (11c)
\end{aligned}$$

$$\sum_{(p_d^r, e^r, u, v) \in \Psi^r} x_{p_d^r, e^r, u, v}^r \leq 1 \quad \forall r \in R \quad (11d)$$

$$x \in \{0, 1\}. \quad (11e)$$

## Side Constraints

$$\sum_{\substack{r \in R \\ (i,j,u,v) \in \Psi^r | u \leq t < v}} x_{i,j,u,v}^r \leq b_t^i \quad \forall i \in B, t \in T \quad (12a)$$

$$\sum_{\substack{r \in R^N \\ v \in \{t+1-\epsilon, \dots, t+1+\delta\} \\ (i,j,u,v) \in \Psi^r | j=a+1, j \neq i}} x_{i,j,u,v}^r + \sum_{\substack{r \in R^S \\ v \in \{t+1-\epsilon, \dots, t+1+\delta\} \\ (i,j,u,v) \in \Psi^r | j=a, j \neq i}} x_{i,j,u,v}^r \leq v_t^a$$

$$\forall (a, t) \in \mathcal{Y} \quad (12b)$$

$$\sum_{\substack{r \in R^N | h^r \geq 1 \\ a \in \{i-h, \dots, i-1\} \\ (a,j,u,v) \in \Psi^r | u \leq t < v, a \neq j}} x_{a,j,u,v}^r + \sum_{\substack{r \in R^N \\ (i,j,u,v) \in \Psi^r | u \leq t < v}} x_{i,j,u,v}^r \leq b_t^i$$

$$\forall i \in B, t \in T \quad (12c)$$

$$\sum_{\substack{r \in R^S | h^r \geq 1 \\ a \in \{i+1, \dots, i+h\} \\ (a,j,u,v) \in \Psi^r | u \leq t < v, a \neq j}} x_{a,j,u,v}^r + \sum_{\substack{r \in R^S \\ (i,j,u,v) \in \Psi^r | u \leq t < v}} x_{a,j,u,v}^r \leq b_t^i$$

$$\forall i \in B, t \in T. \quad (12d)$$

# Hypergraph Comments

- Tractable for Applied Problem Sizes
- Competitive with other published timetabling models
- Indifferent to track structure
- Not pairwise in rulemaking
- Less Suitable for
  - Symmetric data sets (cyclical timetables)
  - Overbooked networks

# Periodic Event Scheduling Problem (PESP)

- Primarily formulated for directional, multi-track
- Actually in service
  - Netherlands Railways (2008 Edelman)
  - Berlin U-bahn
- Periodic, “clockface”, timetables



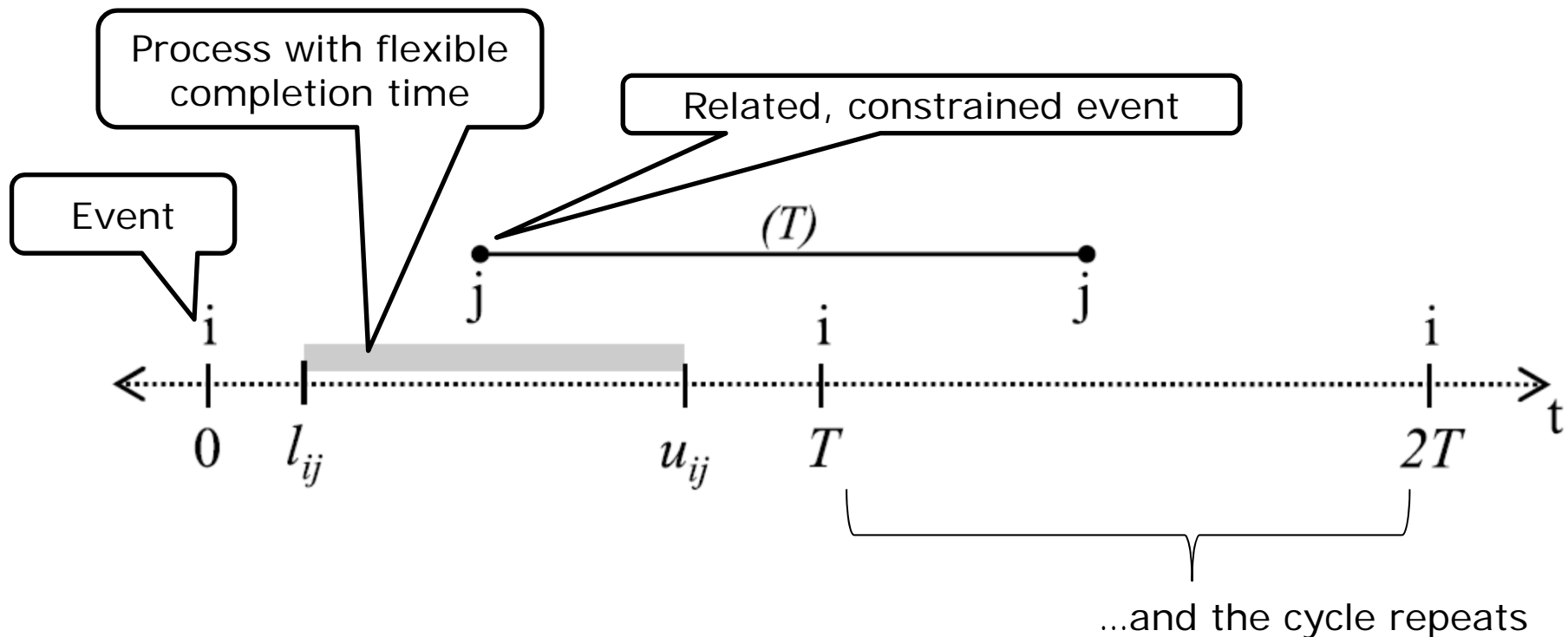
# PESP Constraint Structure

$$\tau_j = (\tau_i + x) \bmod T$$

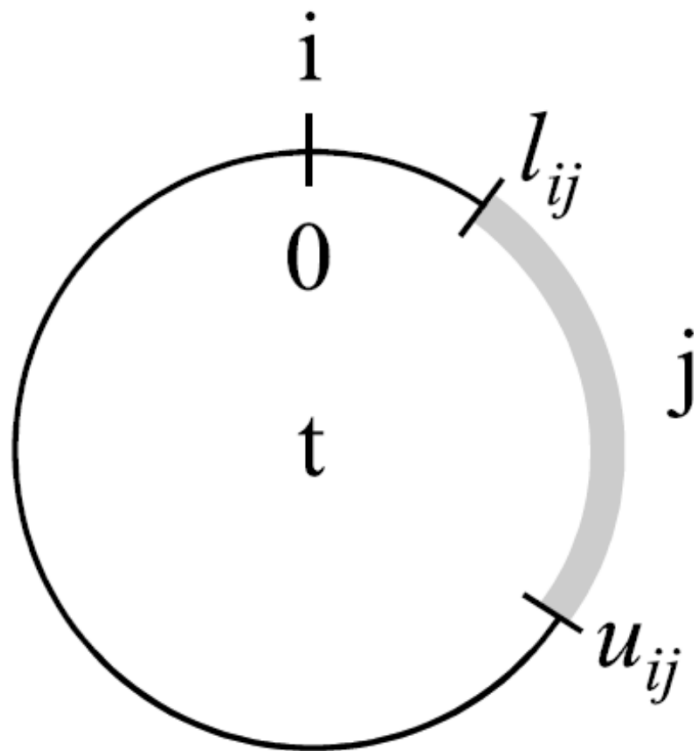
$$x \in [l_{ij}, u_{ij}]$$

$l$ : lower bound of activity time

$u$ : upper bound of activity time

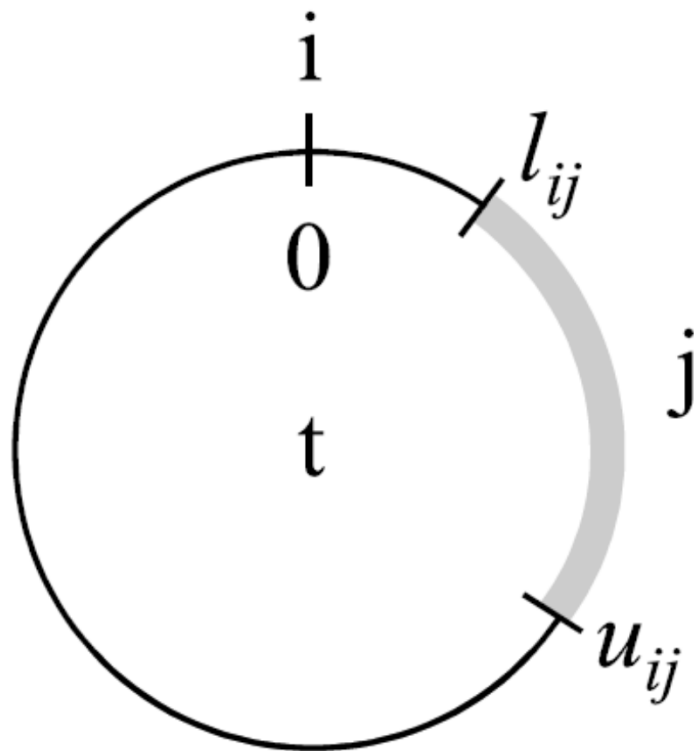


# Example PESP Constraint Headway on Line



1. Event  $i$ : train  $i$  passes track signal at time  $T_i$
2. Event  $j$ : train  $j$  passes same point
3.  $l$ : minimum headway between trains
4.  $u$ : maximum headway, typically headway between successive services of train  $i$ .

# Example PESP Constraint Across Platform Connection



1. Event  $i$ : train  $i$  arrives at platform 1a
2. Event  $j$ : train  $j$  departs platform 1b (center platform between tracks)
3.  $l$ : minimum walking time across platform
4.  $u$ : maximum desired waiting time on platform plus  $l$

# Typical PESP Formulation

$$\min \sum_{a \in A} w_a x_a \quad (15)$$

s.t.

$$\Gamma^T x = Tz \quad (16)$$

$$l \leq x \leq u \quad (17)$$

$$z \in \mathbb{Z}^v \quad (18)$$

$$x \in \mathbb{Z}^{|A|}. \quad (19)$$

# PESP Comments

- Similar to MISLP, train interactions are pairwise
- Modeling of junctions is challenging
- Input data must be feasible
  - OR- search algorithms must be implemented
- It can be seen that a MISLP with fixed sequences can be reduced to a PESP

# Status and Future

*Gør det selv!*

In 2013, DSB began the process of a major timetable revision for the year 2016, called "K16". Using all the resources of DSB over a two year period, the timetable was completely revised, with half as many time and stopping patterns.

The average passenger journey time in the K16 timetable was

- |                       |                        |
|-----------------------|------------------------|
| a) Unchanged          | b) 4 min, 02 sec. less |
| c) 1 min, 20 sec less | d) 6 min, 10 sec less  |

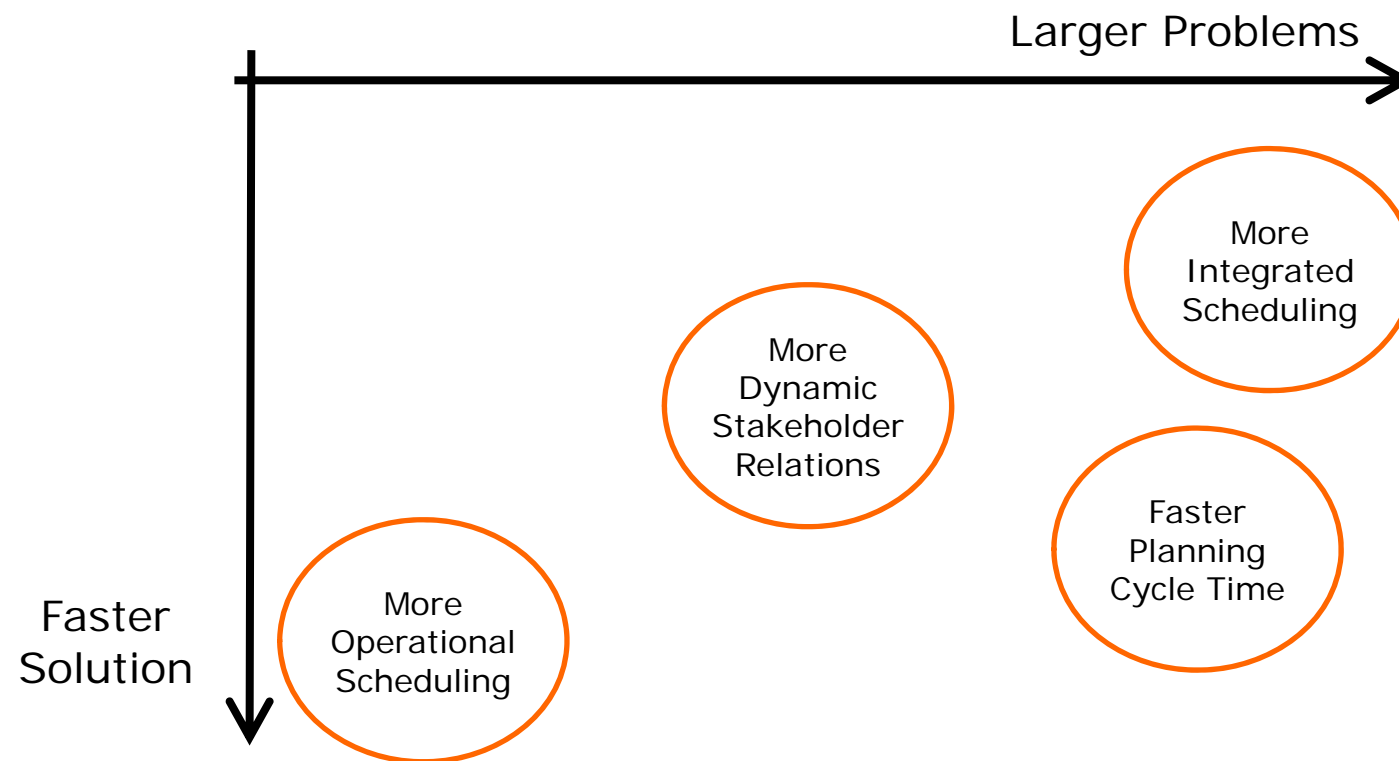
# The average passenger journey time in the K16 timetable was

SOLUTION

- a) Unchanged
- b) 4 min, 02 sec. less
- c) 1 min, 20 sec less
- d) 6 min, 10 sec less

*Improved timetabling, by itself, is unlikely to have a significant impact on the customer experience*

# Future Needs in Timetabling





# Conclusion

- The timetabling process is long, with many stakeholders
- Train pathing is a fundamental progress point
- There are a variety of train pathing models
- Each has different capabilities for micro optimization of train paths
- Future timetabling needs
  - Larger integrated models with micro train pathing
  - Faster micro train pathing for dispatch support



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