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Timetable Design Theory and Practice

Harrod, Steven

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Timetable Design
Theory and Practice

7th International Conference on Railway Operations Modelling and Analysis

Dr. Steven Harrod

4 April, 2017

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Today's Program

- The Timetable Design Process
  - Current Practice
  - The stakeholders
  - Timeline

- Micro Optimization of the Train Path Plan
  - MISLP
  - BIOP
  - Hypergraph
  - PESP

- Future Opportunities

Why Do We Need a Timetable?

Gør det selv!

The Timetable is Central to All

Market Demand

Infrastructure Utilization

Rolling Stock Circulation

Timetable

Traffic Routing

Employee Schedules

Service Reliability

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The Timetable Design Process

North America:
service planning

Network Planning

Line Planning

Market Planning/

Product Design

Timetable Generation

Railway Track Allocation / Train Routing

Operational
Planning/

Rolling Stock Schedules

Service Planning
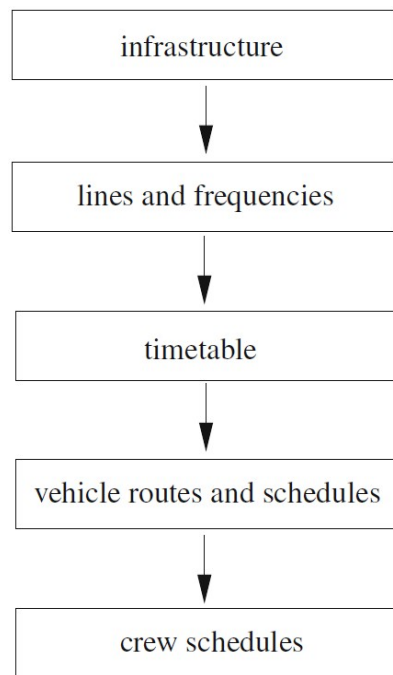
Crew Schedules

Real Time Management

Implementation/

Service Delivery

How do we avoid the feedback loops?

The railway track allocation step does not exist in North American practice.

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Lusby (2011)

The Application is Fairly Universal

Danish - DSB

infrastructure

↓

lines and frequencies

↓

timetable

↓

vehicle routes and schedules

↓

crew schedules

DSB (2015)

Netherlands - NS

Experts → Step 1: Define list of projects

↓

List of projects

↓

DNMM → Step 2: Estimate passenger flows

↓

Demand for railway transport

↓

PROLOP → Step 3: Determine lines

↓

Line system

↓

DONS → Step 4: Test feasibility — fail

↓ pass

Final set of projects

Hooghiemstra (1999)

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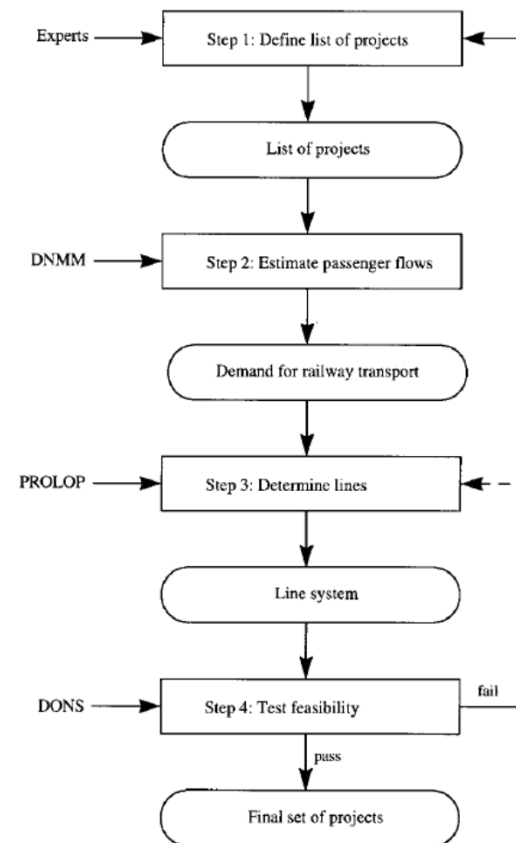
Complications Due to Jurisdiction, Competition
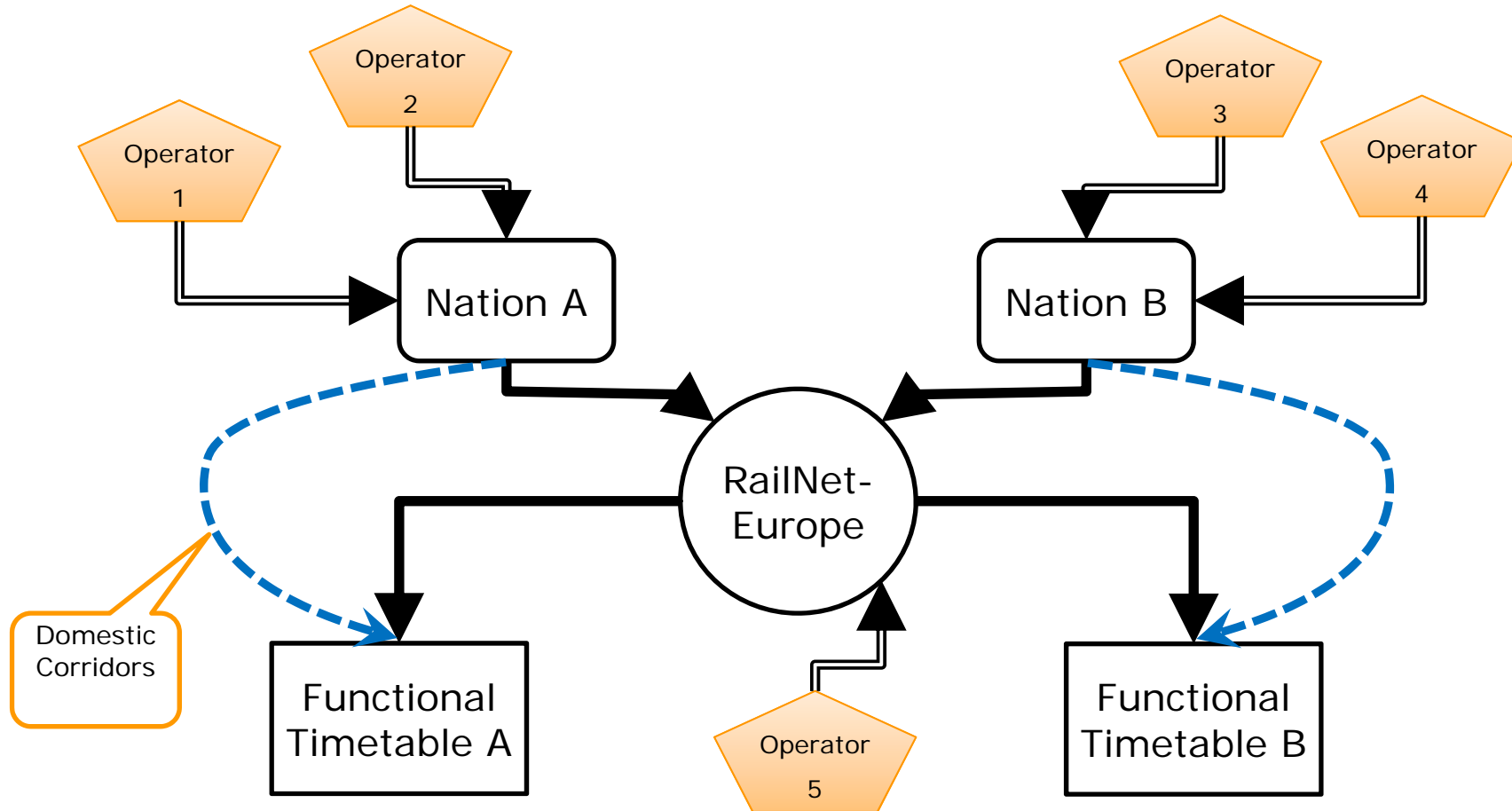
A separate planning process at *each* entity

Typical Planning Calendar - DSB

	2+ years	1 year	2-3 months	Now
Timetable	Commercial aspects	Fixed departure and arrival times	Trackwork and holidays	Disturbances and delays
Rolling Stock	Requirements and budget	"Normal week" capacity, cycles, and tasks	Different capacities on specific days	Assignment of vehicles and maintenance
Maintenance	Maintenance requirements	Planning of maintenance	Workshop plan	Carry out maintenance
Staff	Recruitting, education	Staff and unassigned trips	Inform staff of assignments	Dispatch personal to assignments

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Timetabling Process at Banedanmark (Danish Infrastructure)

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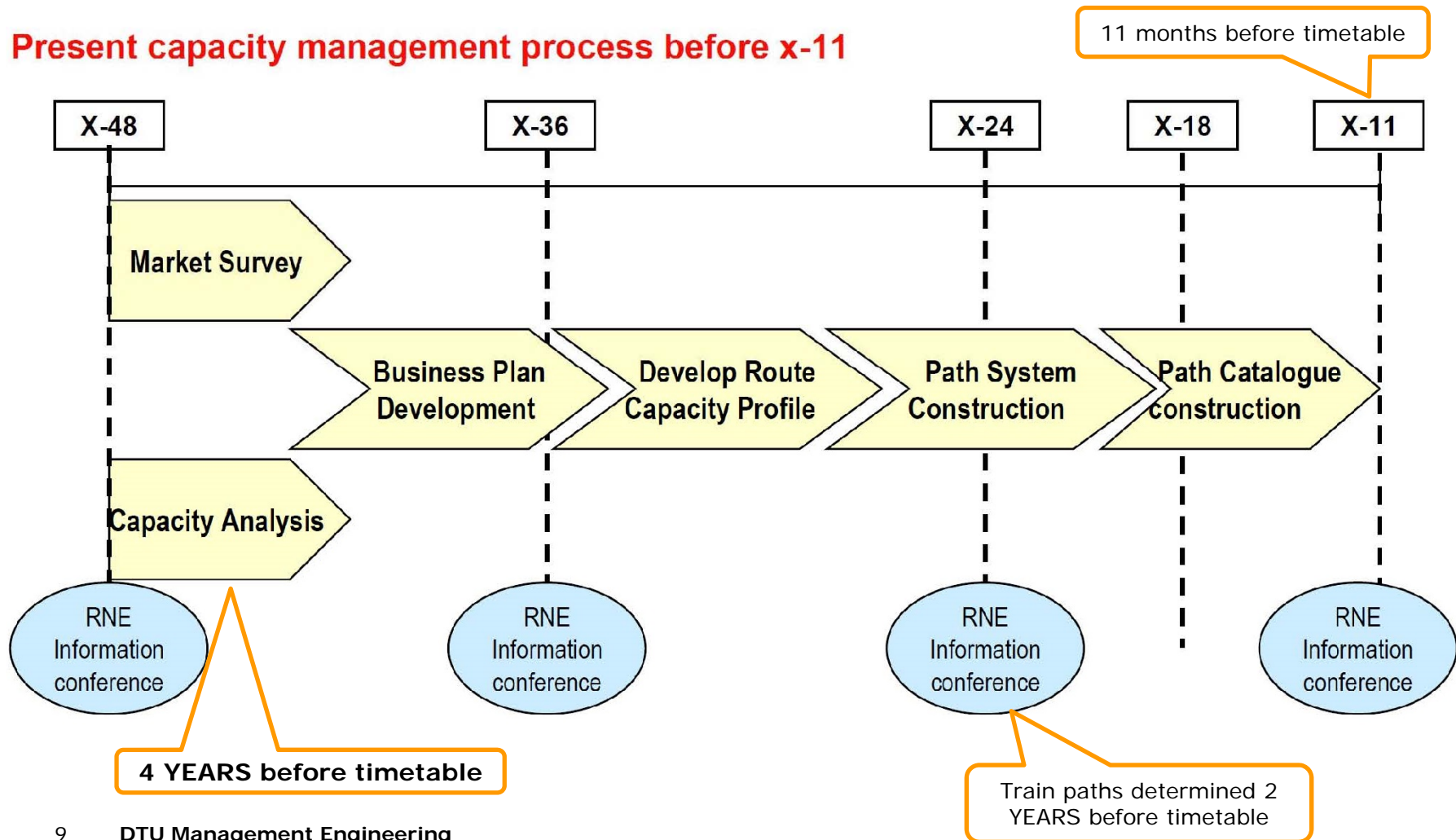
Planning Calendar - EU

"X" is current timetable

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What Defines a Timetable?

Gør det selv!
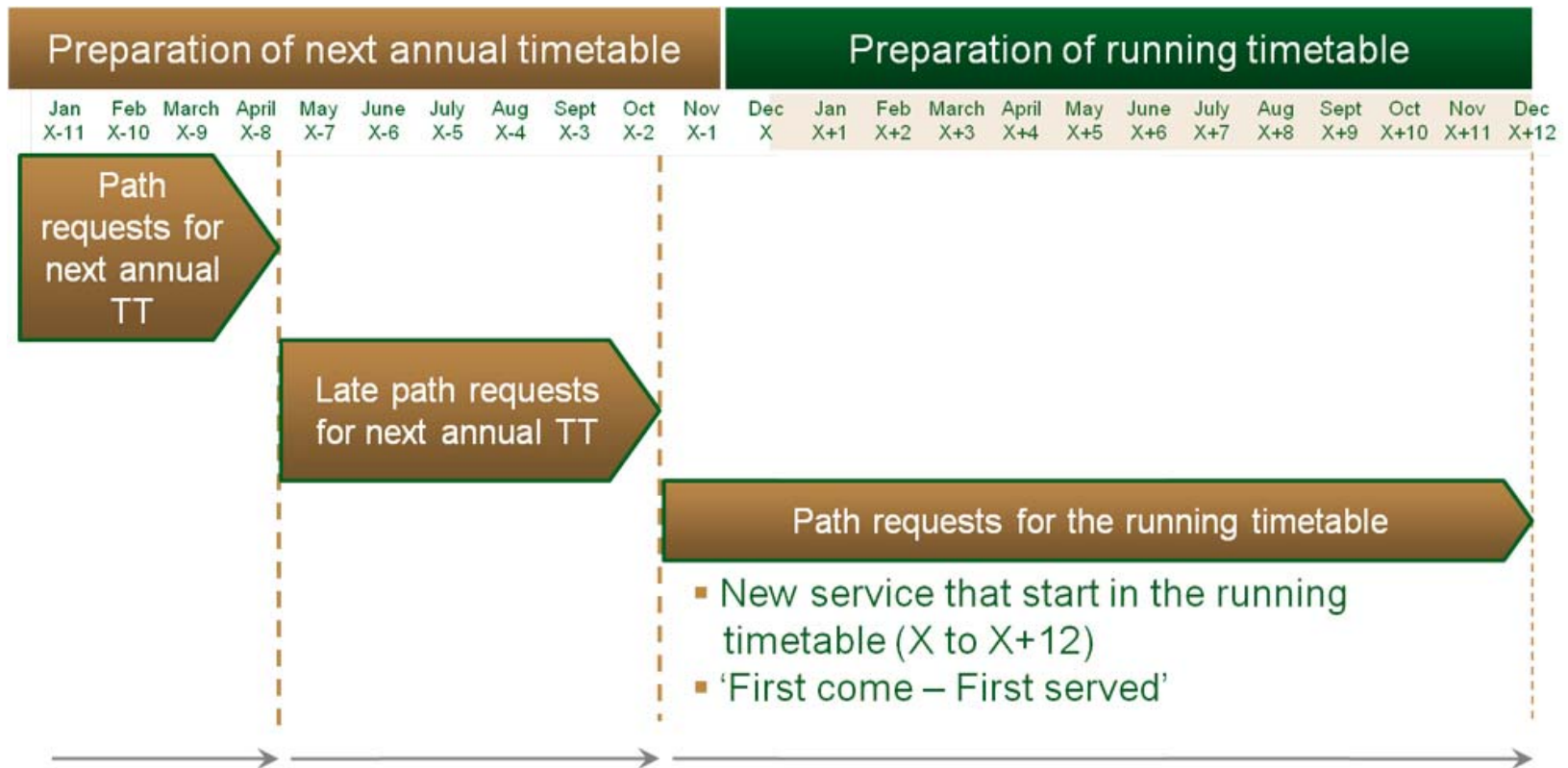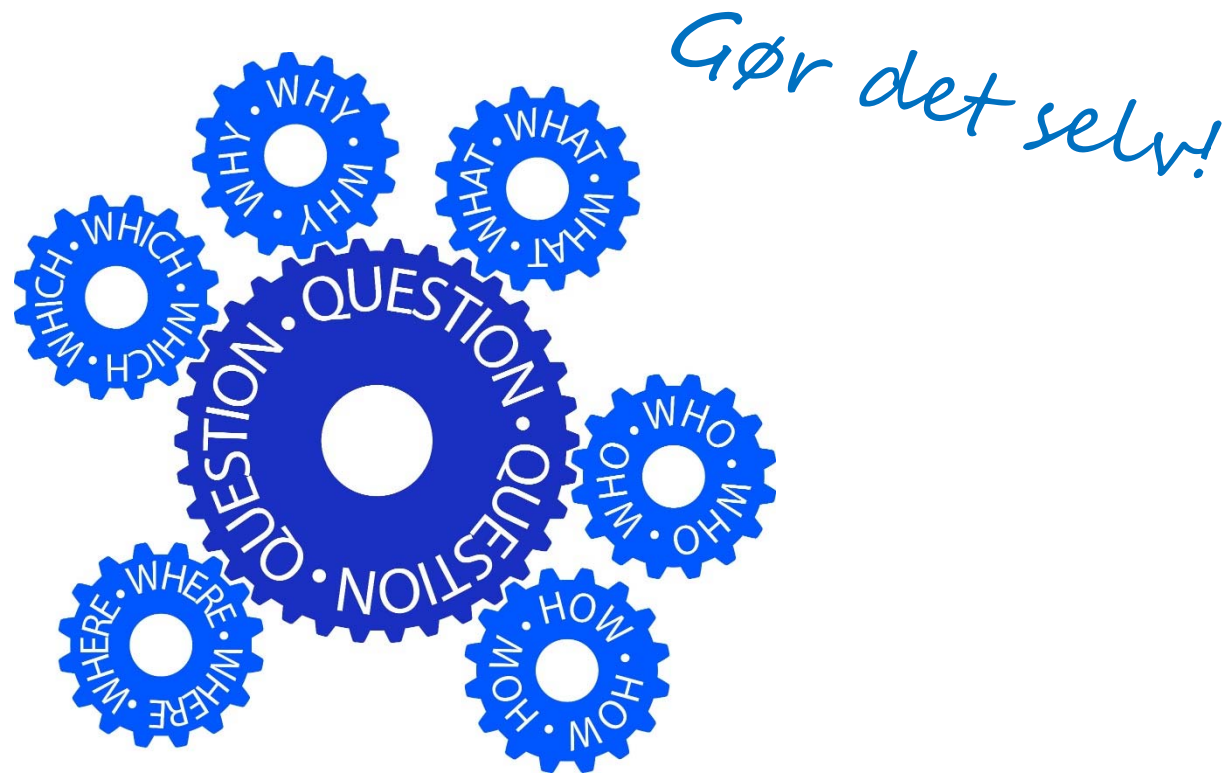
What Defines a Timetable?

In Europe, a timetable at a minimum defines:

a) Individual paths for the trains to follow on the railway network (dispatching guide)

b) Infrastructure commitments to individual trains (contracted train paths)

c) No conflicts between trains (traffic coordination)

d) It may also define tactical alternatives in case of disruption or changes in market demand

Methods in Application

	Denmark – DSB	Netherlands - NS
Demand Forecasting	National Traffic Model (Landstrafikmodellen)	Dutch National Mobility Model
Train Path Generation	Timetable Planning System (www.hacon.de)	CADANS
Station Shunting	- none -	STATIONS
Line Planning	ILP, Goossens & Kroon (2006)	ILP, "PROLOP", Hooghiemstra (1999)
Validation, Simulation	Railsys (www.rmcon.de)	SIMONE

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Some Timetable Definitions

- Feasible timetable: a timetable that is physically possible to operate if no delays occur
- Robust timetable: the ability to absorb small delays (doors held open, wet rail)
- Stable timetable: the ability to recover, to return to plan after a disruption has occured
- Resilient timetable: a timetable that has options for tactical, ad hoc change to respond to disruptions
  – The ability to change train routes
  – The ability to modify the timing of a few select trains

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Train Paths Are a Hard Constraint

Track resources are effectively impossible to change in normal planning horizon

Network → Line Plan → 🧱 Infeasible Train Paths → Rolling Stock → Crew Sched.

Back and Try Again

These can be resolved with more resources

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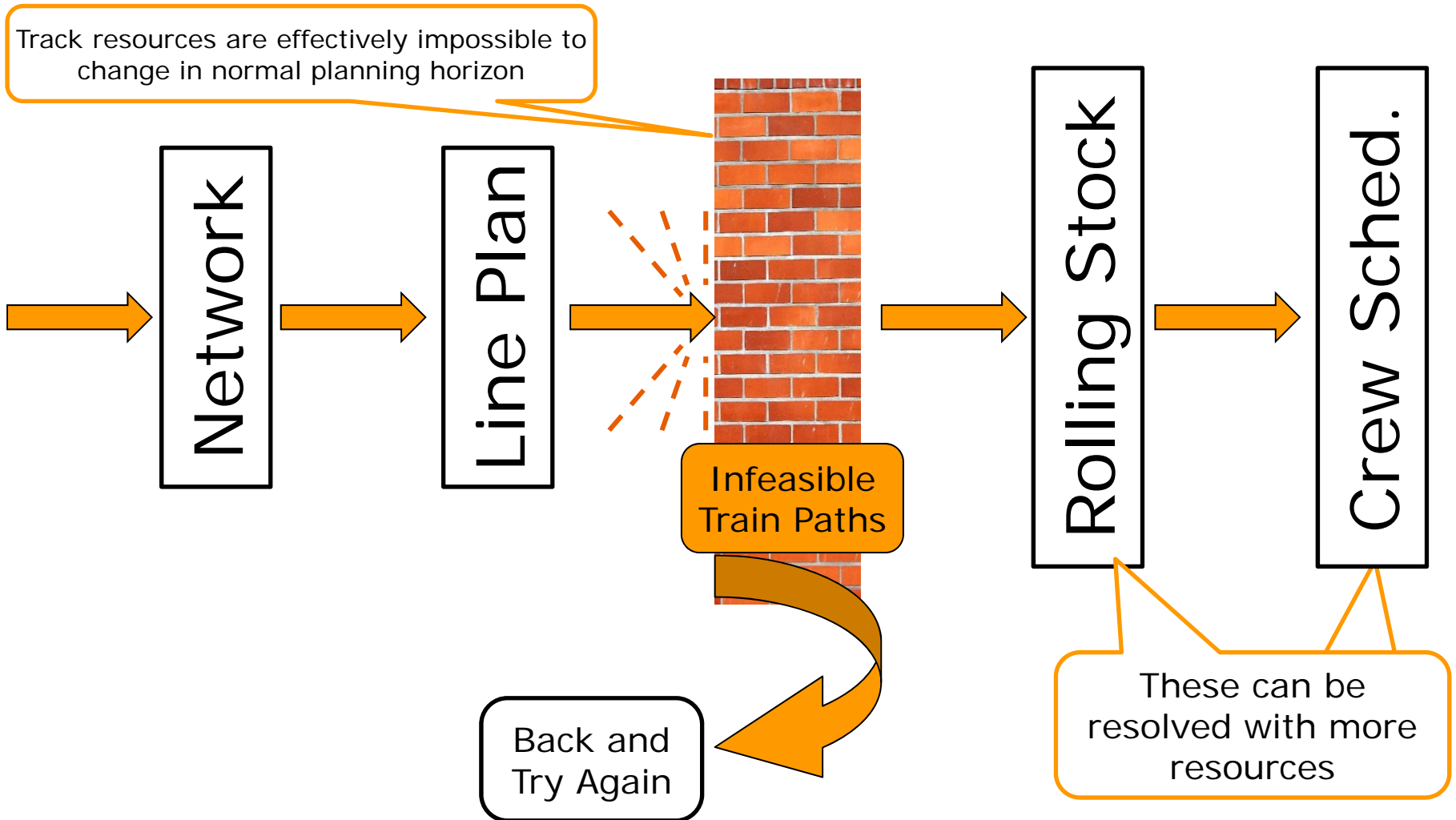
Timetabling

- A Train Path
  - Specific tracks, sidings
  - A contractual commitment
- Train Timings
  - Meet/pass plan
  - Safety separation
- Combinatorial Alternatives
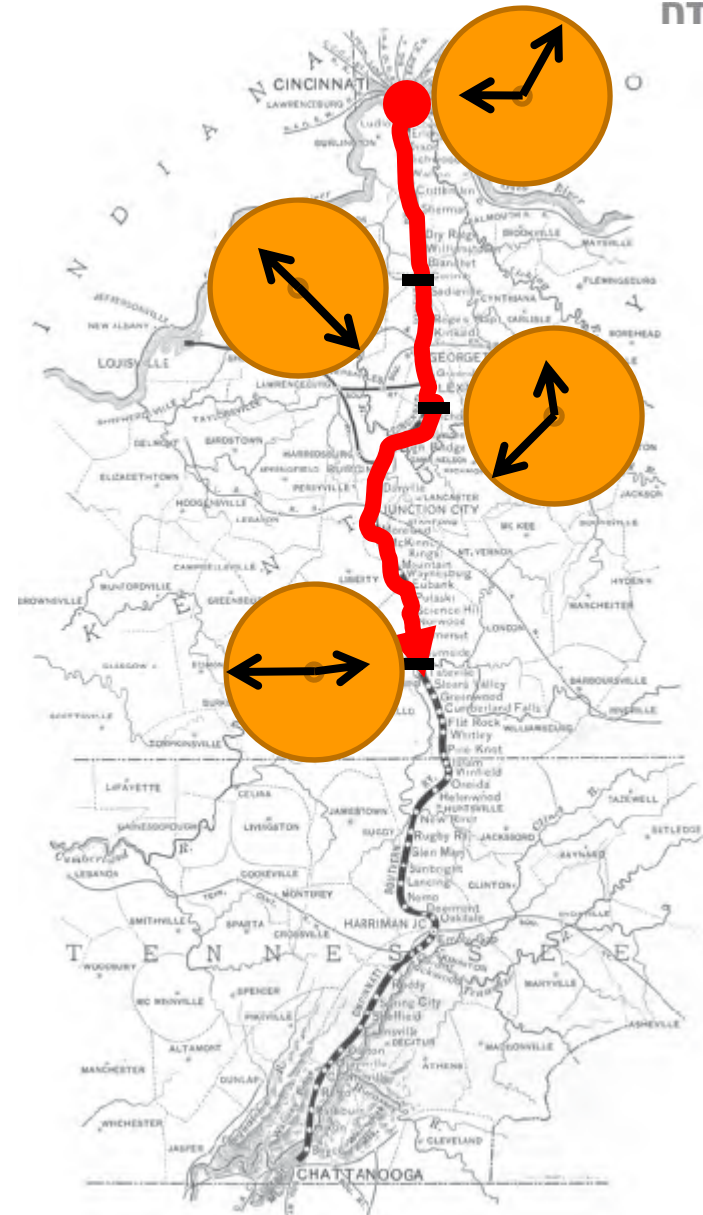
Historic Motivations

- European Land War
  - Harris & Ross (1955) [Ford and Fulkerson]
- High Speed Rail
  - Petersen (1987)
  - Harrod (2009)
- Dispatch Technology
  - Burlington Northern ARES (Jovanovich & Harker, 1990)
  - Southern Railway (Sauder & Westerman, 1983)
- Australia Iron Railways
  - Mees (1991)
  - Higgins (1997)
- Platform Assignment
  - Zwaneveld (1996)

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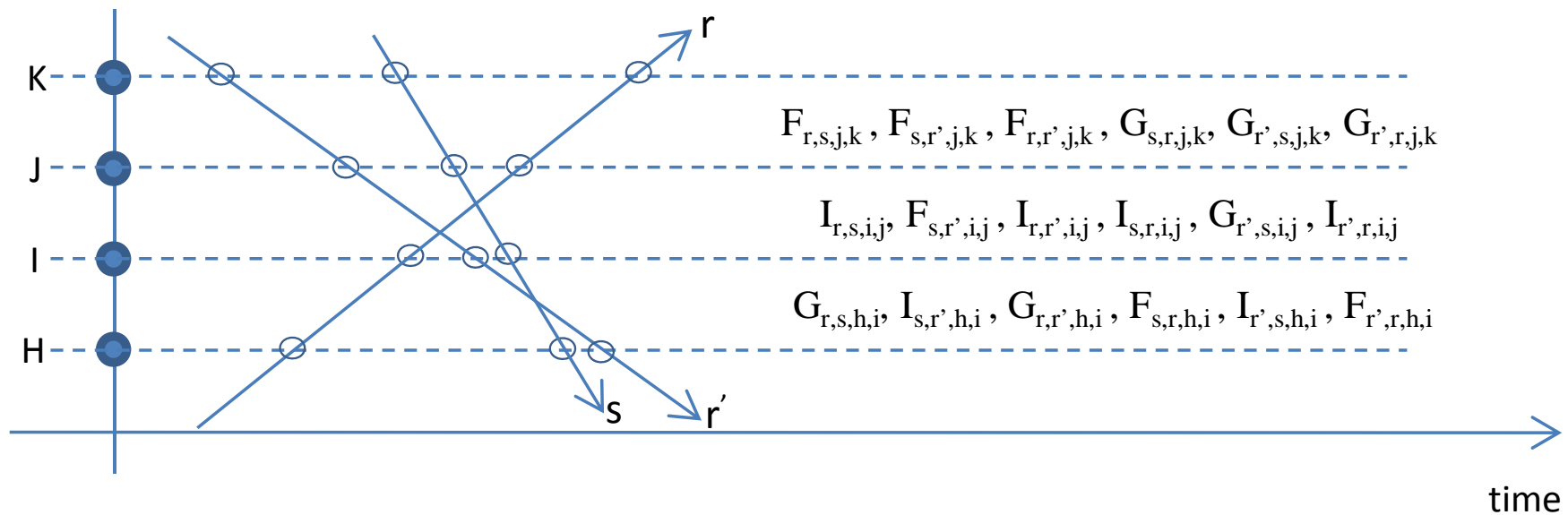Danmarks Tekniske Universitet**

Models

- MISLP
  - Originated at the University of Pennsylvania
  - Trials on Burlington Northern, not implemented
- BIOP
  - Motivated by Australian heavy haul trains
  - Single track focus
- Hypergraph
  - Desire for operationally feasible train paths in solution
  - "Zero basis" solution: no assumption of input set feasibility
- PESP
  - Used successfully in many single direction flows
  - Not a sequencing model, no track occupancies
  - Some challenges in overtake modelling

Mixed Integer Sequencing Linear Program (MISLP)

- Binary Variables
  - Enforce sequencing of pairs of trains
  - I: indicates meet/pass interaction
  - F: first train precedes second train
  - G: first train follows second train

- Real Variables
  - Determine train timings
  - d: departure time from check point
  - a: arrival time at check point

MISLP Sequencing Variables

$F_{r,s,j,k}$, $F_{s,r',j,k}$, $F_{r,r',j,k}$, $G_{s,r,j,k}$, $G_{r',s,j,k}$, $G_{r',r,j,k}$

$I_{r,s,i,j}$, $F_{s,r',i,j}$, $I_{r,r',i,j}$, $I_{s,r,i,j}$, $G_{r',s,i,j}$, $I_{r',r,i,j}$

$G_{r,s,h,i}$, $I_{s,r',h,i}$, $G_{r,r',h,i}$, $F_{s,r,h,i}$, $I_{r',s,h,i}$, $F_{r',r,h,i}$

Note that $F_{r,s,j,k}$ implies $G_{s,r,j,k}$

(MISLP)

$$min \sum_{\substack{r \in R \\ i \in B_r}} [\beta_{r,i} |d_{r,i} - d_{r,i}^*|^\phi + \alpha_{r,i} |a_{r,i} - a_{r,i}^*|^\phi] \qquad (1)$$

s.t.

$$d_{r,i} \geq a_{r,i} \qquad \qquad \forall r \in R; i \in B_r \qquad (2)$$

$$I_{r,r',i,j} + F_{r,r',i,j} + G_{r,r',i,j} = 1 \qquad \forall r, r' \in R; i, j \in B_r \cap B_{r'} \qquad (3)$$

$$a_{r,i} - d_{r,j} \geq s_{r,i,j} + \sum_{\substack{r' \in R \\ i,j \in B_r \cap B_{r'}}} f_{r,r',i,j} I_{r,r',i,j} \qquad \forall r \in R; i, j \in B_r \qquad (4)$$

$$d_{r,i} - a_{r',i} \geq M(1 - F_{r,r',i,j}) \qquad \forall r, r' \in R; i, j \in B_r \cap B_{r'}$$

$$d_{r',j} - a_{r,j} \geq M(1 - G_{r,r',i,j}) \qquad \forall r, r' \in R; i, j \in B_r \cap B_{r'}$$

$$a_{r',i} - d_{r,i} \geq M(1 - I_{r,r',i,j}) \qquad \forall r, r' \in R; i, j \in B_r \cap B_{r'}$$

$$a_{r,j} - d_{r',j} \geq M(1 - I_{r,r',i,j}) \qquad \forall r, r' \in R; i, j \in B_r \cap B_{r'} \qquad (5)$$

$$a, d \geq 0 \quad I, F, G \in 0,1$$

MISLP Comments

- Note that all train interactions are pairwise
- Input train set must be feasible
- Not a capacity model
- No track resource variables, no pricing duals

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Binary Integer Occupancy Program (BIOP)

- All timings in discrete units
- Binary variables
- Multicommodity flow
- Network packing

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(BIOP)

$$\min \sum_{\substack{r \in R \\ (i,t) \in B^r}} c_{i,t}^r x_{i,t}^r \qquad (6)$$

s.t.

$$\sum_{(i,t) \in B^r | i = o^r} x_{i,t}^r = 1 \qquad \forall r \in R \qquad (7)$$

$$x_{i,t}^r = \sum_{(k,l) \in \Phi_{i,t}^r} x_{k,l}^r \qquad \forall r \in R; (i,t) \in B^r | i \neq d^r \qquad (8)$$

$$\sum_{(i,t) \in B^r | i = d^r} x_{i,t}^r = 1 \qquad \forall r \in R \qquad (9)$$

$$\sum_{\substack{m \in R \\ (k,l) \in B^m \cap (\Omega_{i,t} \cup \{(i,t)\}}} x_{k,l}^m \leq 1 \qquad \forall (i,t) \in \bigcup_{r \in R} B^r \qquad (10)$$
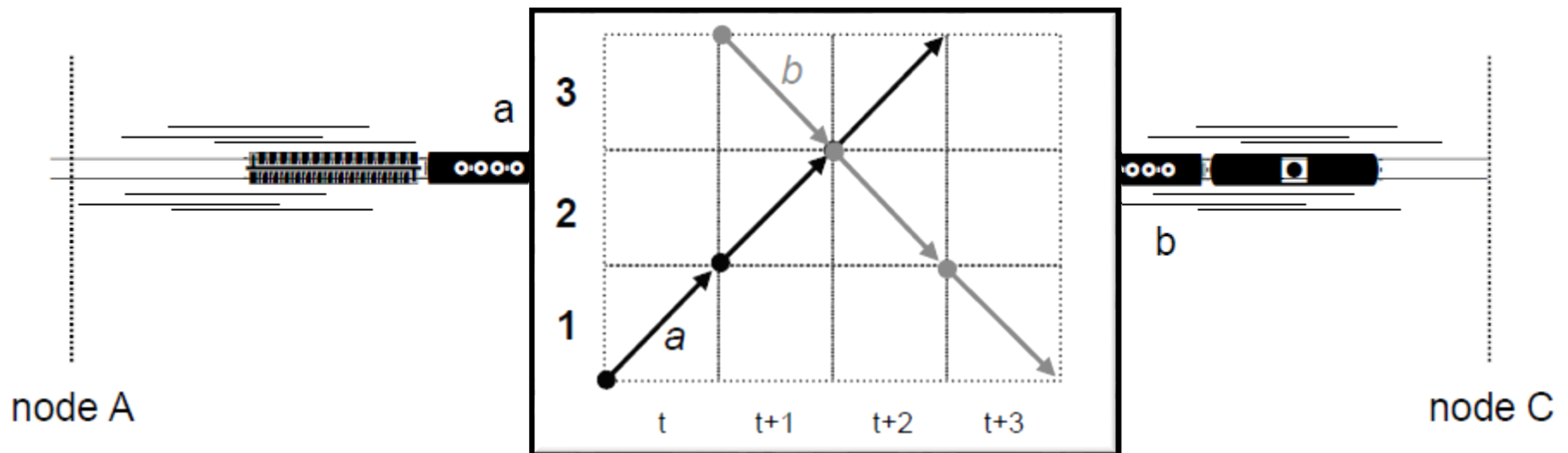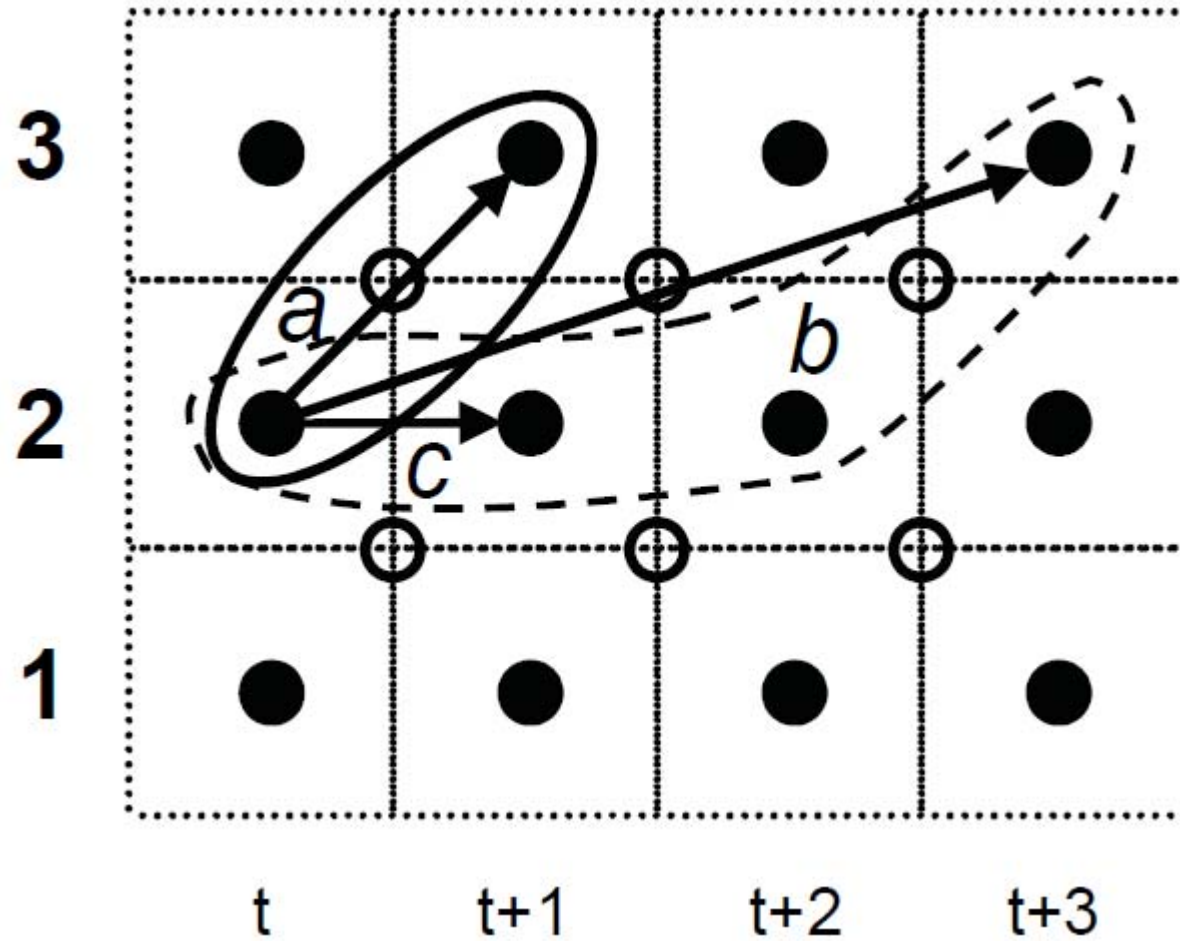
$$x \in \{0, 1\}$$

BIOP Comments

- Explicit track constraints
- Pricing duals for infrastructure
- Input data set need not be wholly feasible
- Not "dispatch ready"

Hypergraph

- Strict occupancy constraint is not operationally feasible
- Model pairs of track segments and their transitions

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$$\max \quad \sum_{\substack{r \in R \\ (p_o^r, j, u, v) \in \Psi^r}} (c_p^r + c_e^r (u - p_e^r)) x_{p_o^r, j, u, v}^r$$

$$+ \sum_{\substack{r \in R \\ (p_d^r, e^r, u, v) \in \Psi^r}} c_l^r (p_l^r - v) x_{p_d^r, e^r, u, v}^r + \sum_{\substack{r \in R \\ (i, j, u, v) \in \Psi^r \mid i=j}} c_s^r x_{i, j, u, v}^r. \quad (11a)$$

Linear Network Constraints

$$\sum_{(p_o^r, j, u, v) \in \Psi^r} x_{p_o^r, j, u, v}^r \le 1 \quad \forall r \in R \quad (11b)$$

$$\sum_{(a, i, u, t) \in \Psi^r} x_{a, i, u, t}^r = \sum_{(i, j, t, v) \in \Psi^r} x_{i, j, t, v}^r$$

$$\forall r \in R, \{i \in B \mid i \ne p_o^r\}, t \in T \quad (11c)$$

$$\sum_{(p_d^r, e^r, u, v) \in \Psi^r} x_{p_d^r, e^r, u, v}^r \le 1 \quad \forall r \in R \quad (11d)$$

$$x \in \{0, 1\}. \quad (11e)$$

Side Constraints

$$\sum_{\substack{r \in R \\ (i,j,u,v) \in \Psi^r \mid u \leq t < v}} x^r_{i,j,u,v} \leq b^i_t \quad \forall i \in B, t \in T \tag{12a}$$

$$\sum_{\substack{r \in R^N \\ v \in \{t+1-\epsilon,\dots,t+1+\delta\} \\ (i,j,u,v) \in \Psi^r \mid j=a+1, j \neq i}} x^r_{i,j,u,v} + \sum_{\substack{r \in R^S \\ v \in \{t+1-\epsilon,\dots,t+1+\delta\} \\ (i,j,u,v) \in \Psi^r \mid j=a, j \neq i}} x^r_{i,j,u,v} \leq \upsilon^a_t$$

$$\forall (a, t) \in \Upsilon \tag{12b}$$

$$\sum_{\substack{r \in R^N \mid h^r \geq 1 \\ a \in \{i-h,\dots,i-1\} \\ (a,j,u,v) \in \Psi^r \mid u \leq t < v, a \neq j}} x^r_{a,j,u,v} + \sum_{\substack{r \in R^N \\ (i,j,u,v) \in \Psi^r \mid u \leq t < v}} x^r_{i,j,u,v} \leq b^i_t$$

$$\forall i \in B, t \in T \tag{12c}$$

$$\sum_{\substack{r \in R^S \mid h^r \geq 1 \\ a \in \{i+1,\dots,i+h\} \\ (a,j,u,v) \in \Psi^r \mid u \leq t < v, a \neq j}} x^r_{a,j,u,v} + \sum_{\substack{r \in R^S \\ (i,j,u,v) \in \Psi^r \mid u \leq t < v}} x^r_{a,j,u,v} \leq b^i_t$$

$$\forall i \in B, t \in T. \tag{12d}$$

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Hypergraph Comments

- Tractable for Applied Problem Sizes
- Competitive with other published timetabling models
- Indifferent to track structure
- Not pairwise in rulemaking
- Less Suitable for
  - Symmetric data sets (cyclical timetables)
  - Overbooked networks

Periodic Event Scheduling Problem (PESP)

- Primarily formulated for directional, multi-track
- Actually in service
  - Netherlands Railways (2008 Edelman)
  - Berlin U-bahn
- Periodic, "clockface", timetables

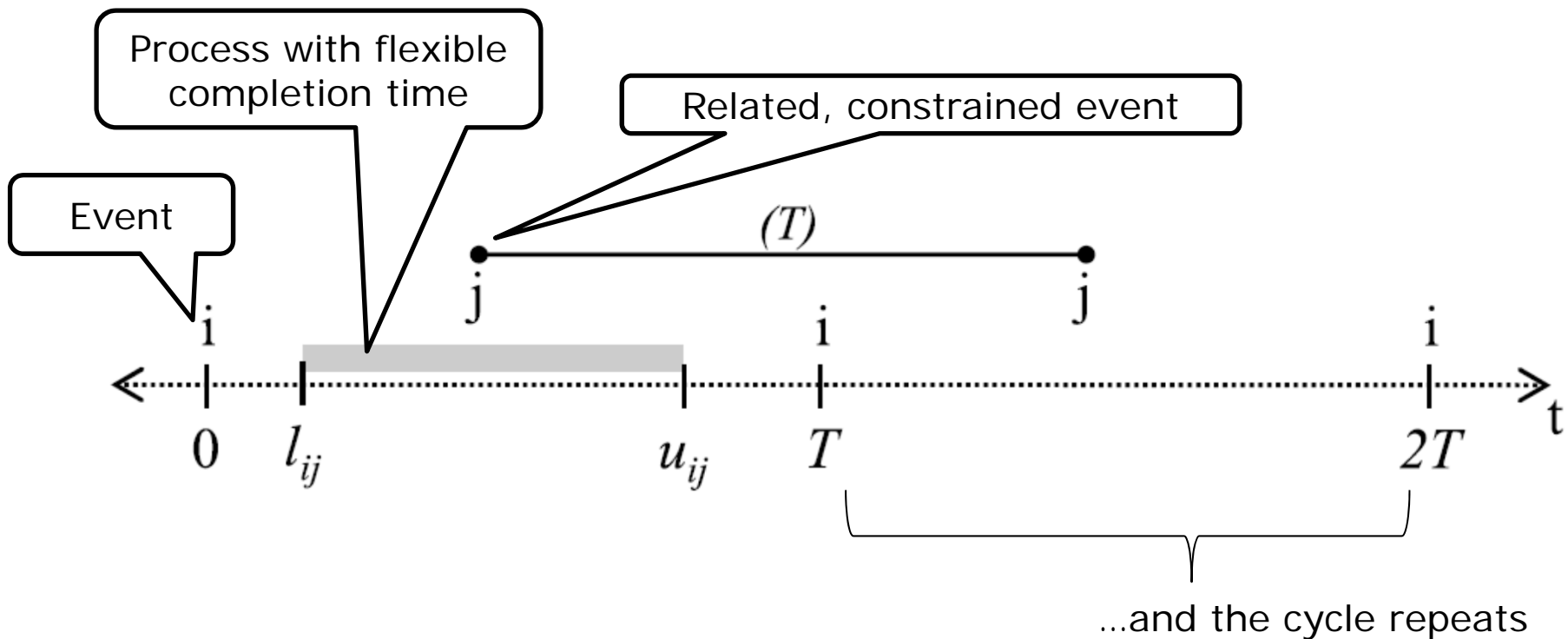**DTU Management Engineering**
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PESP Constraint Structure

$$\tau_j = (\tau_i + x) \bmod T$$

$$x \in [l_{ij}, u_{ij}]$$

l: lower bound of activity time

u: upper bound of activity time

Process with flexible completion time

Related, constrained event

Event

(T)

...and the cycle repeats

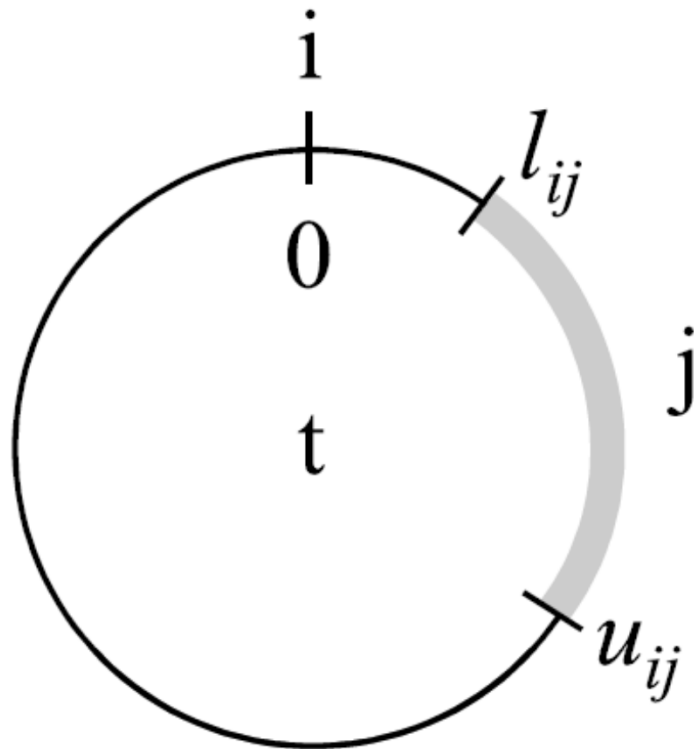# Example PESP Constraint Headway on Line
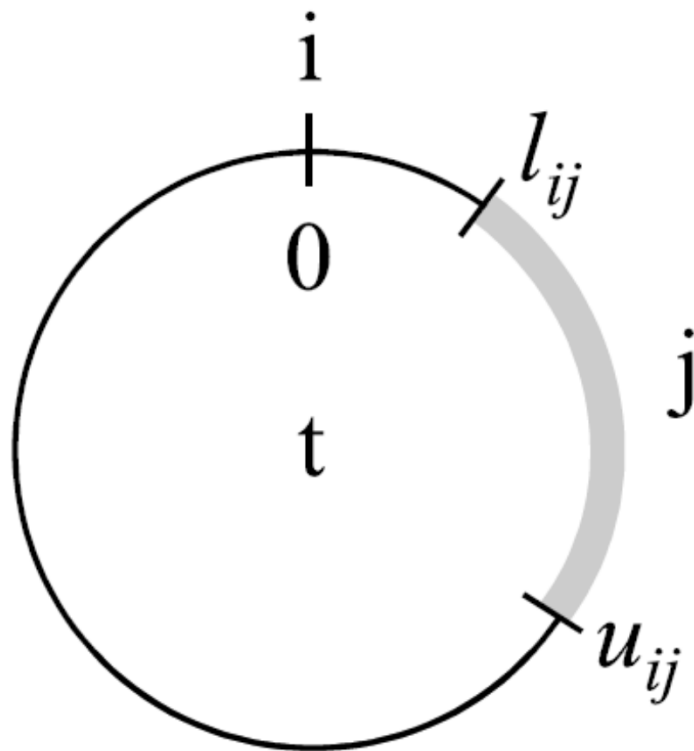
1. Event i: train i passes track signal at time Ti

2. Event j: train j passes same point

3. l: minimum headway between trains

4. u: maximum headway, typically headway between successive services of train i.

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Example PESP Constraint Across Platform Connection

1. Event i: train i arrives at platform 1a

2. Event j: train j departs platform 1b (center platform between tracks)

3. l: minimum walking time across platform

4. u: maximum desired waiting time on platform plus l

Typical PESP Formulation

$$\min \sum_{a \in A} w_a x_a \qquad (15)$$

s.t.

$$\Gamma^T x = Tz \qquad (16)$$

$$l \le x \le u \qquad (17)$$

$$z \in \mathbb{Z}^\nu \qquad (18)$$

$$x \in \mathbb{Z}^{|A|}. \qquad (19)$$

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PESP Comments

- Similar to MISLP, train interactions are pairwise
- Modeling of junctions is challenging
- Input data must be feasible
  -OR- search algorithms must be implemented
- It can be seen that a MISLIP with fixed sequences can be reduced to a PESP

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Status and Future

Gør det selv!

In 2013, DSB began the process of a major timetable revision for the year 2016, called "K16". Using all the resources of DSB over a two year period, the timetable was completely revised, with half as many time and stopping patterns.

The average passenger journey time in the K16 timetable was

a) Unchanged

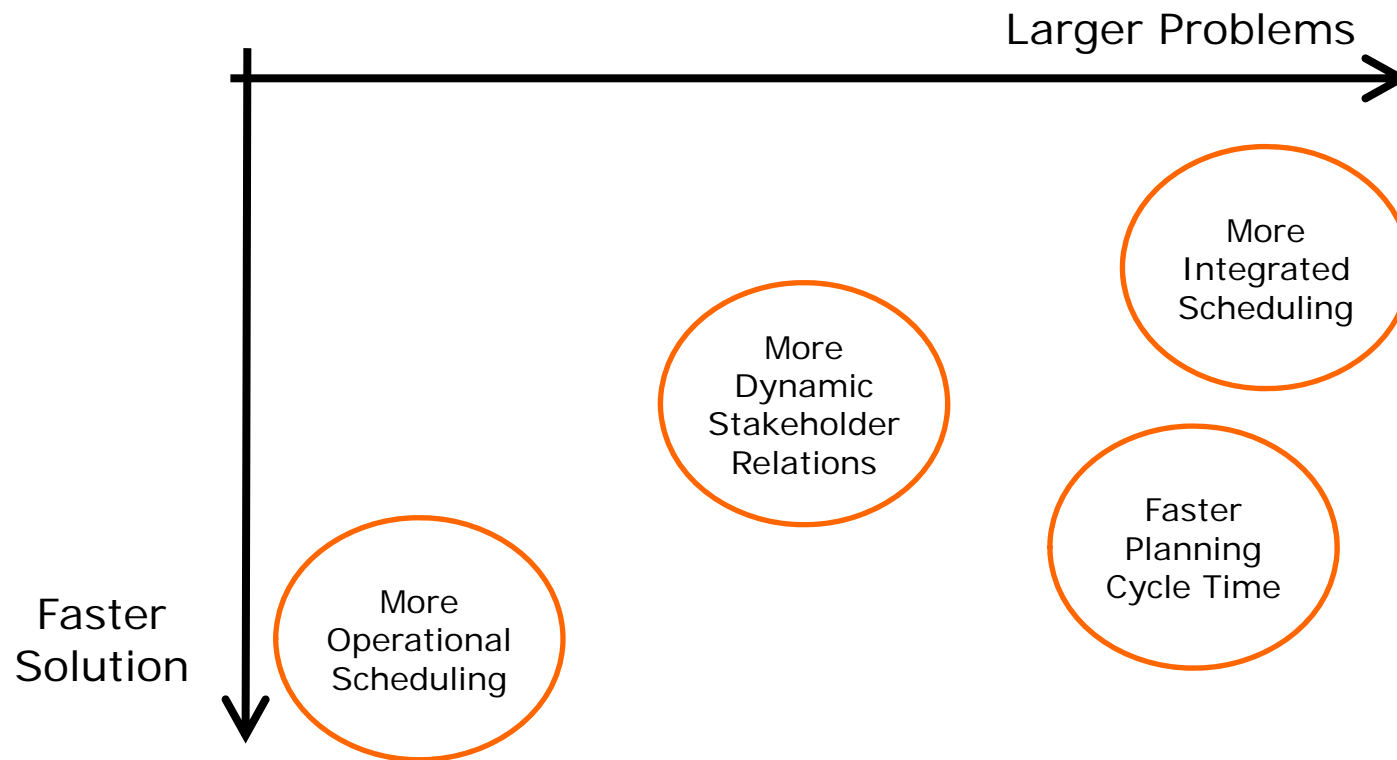b) 4 min, 02 sec. less

c) 1 min, 20 sec less

d) 6 min, 10 sec less

The average passenger journey time in the K16 timetable was

SOLUTION

a) Unchanged

b) 4 min, 02 sec. less

c) 1 min, 20 sec less

d) 6 min, 10 sec less

Improved timetabling, by itself, is unlikely to have a significant impact on the customer experience

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Future Needs in Timetabling

Larger Problems

Faster Solution

More Integrated Scheduling

More Dynamic Stakeholder Relations

Faster Planning Cycle Time

More Operational Scheduling

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Conclusion

- The timetabling process is long, with many stakeholders

- Train pathing is a fundamental progress point

- There are a variety of train pathing models

- Each has different capabilities for micro optimization of train paths

- Future timetabling needs
  - Larger integrated models with micro train pathing
  - Faster micro train pathing for dispatch support

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