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Publication date:
2018

Document Version
Publisher's PDF, also known as Version of record

Citation (APA):
Okulov, V., Sørensen, J. N., Fukumoto, Y., & Wood, D. H. (2018). Real history of helical vortices: theory and applications. Annex: comments on “motion of a helical vortex by o. Velasco fuentes”. 71-72. Abstract from 4th International Retreat on Vortical Flow and Aerodynamics (IRVA4) , Novosibirsk, Russian Federation.

BOOK OF ABSTRACTS

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REAL HISTORY OF HELICAL VORTICES: THEORY AND APPLICATIONS. Annex: Comments on "Motion of a helical vortex by O. Velasco Fuentes"

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Keywords: *helical vortex, vortex precession, self-induced motion*

Research on helical vortices has a long history, which dates back to the famous work of Lord Kelvin in 1880 on the helical perturbations of the columnar vortex. Helical vortices are of fundamental importance to fluid mechanics because they describe one of the main states of swirling flows. An accurate estimation of velocities in the rotor plane requires knowing the velocity field induced by the helical vortices. Therefore, starting with the inspirational work of Joukowsky, the theory of helical vortices has been actively studied as a prerequisite to understand and analyze rotor aerodynamics [1]. At present the fundamentals are based on various analytical components [1, 2], which holds true for all values of the helix pitch, such as (i) the 2D Biot-Savart law for helical filaments represented by Kapteyn series or in a form with singularity separation; (ii) solutions of helical vortex tubes with finite core, governed by series expansion of helical multipoles; (iii) relations between the induction of vortex filaments and the self-induced velocity of helical vortex tubes resulting in a closed analytical solution of the helix motion; (iv) analytical representation of Goldstein's solution for the circulation of a helical vortex sheet in equilibrium; (v) Kelvin's N-gon stability problem of point vortices generalized to multiple helical vortices.

Annex: Comments on "Motion of a helical vortex by O. Velasco Fuentes"

In the recent paper on the motion of helical vortices [VF2018] in J. Fluid Mech. 836, R1, doi:10.1017/jfm.2017.845 the model with both tangential and binormal velocities including in the helical motion was used in contrast to the classical approach when only the bi-normal component included [3]. A large difference between both solutions was found. We demonstrate here that conflicts of VF2018 were based on a bias in the representation of the velocities in different coordinate systems and that the motion of a helical vortex is uniquely dictated by the bi-normal component of the velocity as it was stated in the classical approach and only it correlates with the Betz-Joukowsky limit well [1].

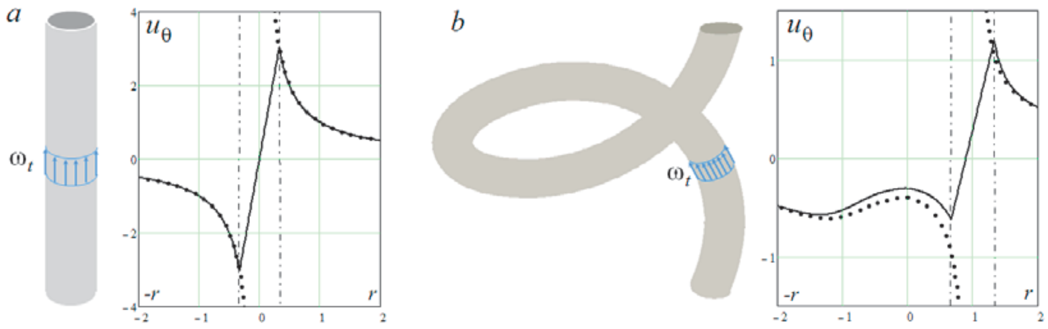

Figure 1: Correlations of azimuthal velocities u_θ inducing by infinite thin vortex line (dotted lines) and vortex with the finite cores of uniform vorticity ω_t distributions (solid lines): (a) rectilinear vortices and (b) helical vortices [4].

VF2018 also erroneously expand a property of the point vortex dynamics to the helical vortex (Fig. 1). He used the vortex-line solutions (dotted lines) instead of one for the finite core (solid lines) for the helical vortex with a finite core in which even the uniform vorticity concludes to non-uniform velocity profiles analytically proven by Eq. (3.6) of [5].

A comparison of the Ω of [3] with an axial motion of fluid particles along the helix axis

The main error of the VF2018 is his comparison of the angular velocities (red VF2018 and green [3] *curves on fig 2) because both ones were derived in different coordinate systems.* The vortex system in [3] p. 328, 335 moves with the fixed velocity $\Gamma N/(2\pi R\tau)$. For a partial case (Fig. 2) when the fluid particles move with the u_a only in the axial direction we will show that the classical approach of [3] can explain both axial u_a and azimuthal $\Delta\theta/\Delta t$ displacement of the helical vortex when the VF2018 gives only one of that, u_a (or $\Omega^*(\tau_0) = 0$).

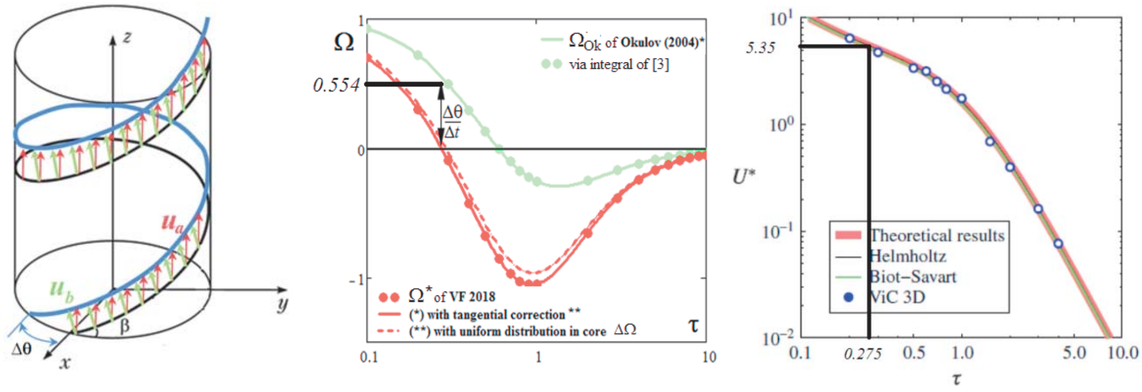

Figure 2: Sketch of the helix displacement with the axial translation of the fluid particles ua and the plots of the angular and axial displacements of the helix (green lines - [3]) and fluid particles (red lines –VF2018).

The root of red curve of VF2018 gives the $\tau_0 \approx 0.275$ and the correspondence value of the absolute velocity in this point $u_a(\tau_0) \approx U^*(\tau_0) = 5.35$. The angular velocity of [3] (green curve) has non zero value $\Omega_{Ok}(\tau_0) \approx 0.554$ at $\tau_0 = 0.275$. The coordinate system for the single vortex in [3] p. 328, 335, moves with the fixed velocity $\Gamma/(2\pi R\tau_0) = 4\pi/(2\pi\tau_0) = 2/\tau_0$. If we put the coordinate translation it in the formula $U_{Ok} = (\Gamma/(2\pi R) - R\Omega_{Ok}(\tau_0))/\tau_0$ of [3] for the axial velocity the value $U_{Ok} \approx (2 - 0.554)/0.275 = 5.28$. The correction to the non-uniform flow (fig 1b) with add-on $\Delta\Omega = 0.25\tau/(1+\tau^2)^{3/2} = 0.25 \cdot 0.75/(1+0.75^2)^{3/2} = 0.062$. Summing both lines: 5.28+0.062=5.342 which coincides with the axial movement of the fluid particles $U^*(\tau_0) = 5.35$. So the classical approach of [3] permits to estimate both real displacements of the helix in axial, $U_{Ok} \equiv u_a$, and azimuthal, $\Delta\theta/\Delta t = \Omega_{Ok}(\tau_0)$, directions when the by fluid-particle motion of VF2018 approximates only the axial displacement $U^* \approx u_a$ without rotation $\Omega^* = 0$.

Acknowledgements

The work was supported in part by Grant JSPS no. S17119.

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