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Quantum coherent absorption of squeezed light

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We investigate coherent perfect absorption (CPA) in quantum optics, in particular when pairs of squeezed coherent states of light are superposed on an absorbing beam splitter. First, by employing quantum optical input–output relations, we derive the absorption coefficients for quantum coherence and for intensity, and reveal how these will differ for squeezed states. Second, we present the remarkable properties of a CPA gate: two identical but otherwise arbitrary incoming squeezed coherent states can be completely stripped of their coherence, producing a pure entangled squeezed vacuum state that with its finite intensity escapes from an otherwise perfect absorber. Importantly, this output state of light is not entangled with the absorbing beam splitter by which it was produced. Its loss-enabled functionality makes the CPA gate an interesting new tool for continuous-variable quantum state preparation.

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1. INTRODUCTION

Coherent perfect absorption (CPA) of light [1] is an interference-assisted absorption process that in its simplest form can take place when two coherent beams impinge on the opposite sides of an absorbing beam splitter. With input light from only one side, some light would leave the beam splitter, but no light emerges if there is equal input from both sides. While scattering theory can provide a rigorous mathematical description, in essence, the reflected part of one of the incident beams interferes destructively with the transmitted part of the other (and vice versa), forming an artificial trap for the light that is subsequently dissipated [1]. So unlike the usual absorption, the coherent absorption of light is an emergent property [2] that arises from a specific interplay of interference and dissipation [1,3].

Many realizations of CPA have been proposed, including homogeneously broadened two-level systems [4], epsilon-near-zero metamaterials [5], graphene [6], and heterogeneous metal–dielectric composite layers [7]. CPA has been successfully demonstrated in many setups, e.g., in a silicon cavity with two counterpropagating waves [8], using a pair of resonators coupled to a transmission line [9], and using graphene to observe CPA of optical [10] and of terahertz radiation [11]. Achieving CPA under single-beam illumination with perfect magnetic conductor surfaces has also been reported [12]. A recent study revealed that CPA of light can be used as an unconventional tool to strongly couple light to surface plasmons in nanoscale metallic systems [13]. The fields in which CPA may play a central role include, but are not limited to, photodetection [14,15], sensing [16], photovoltaics [17], and cloaking [18,19]. For further details and examples of CPA realizations, refer to the excellent recent review by Baranov et al. [3].

Investigations of CPA in the quantum regime (also known as quantum CPA [3]) have only recently begun, notably with entangled few-photon input states [20–23]. Here instead, we consider squeezed coherent states of light [24] and report the effects of quadrature squeezing on the absorption profile of the system, and vice versa the effects of the absorbing beam splitter on the generated output states of light. To the best of our knowledge, this important class of continuous-variable quantum states of light has not yet been considered in the context of CPA.

Squeezed states of light have no classical analogues. They reduce noise in one field quadrature at the expense of larger noise in the other, such that Heisenberg's uncertainty relation for their product holds. First realized decades ago [25], squeezed states of light continue to attract attention, due mainly to their indispensable roles in quantum information, communication, and optics protocols. In particular, squeezed states are key for quantum teleportation [26–28], quantum key distribution [29], quantum metrology [30], quantum cryptography [31], quantum dense coding [32], quantum dialogue protocols [33], quantum laser pointers [34], and quantum memories [35,36]. They can be used to distinguish quantum states by enhancing quantum interference [37] and for robust electromagnetically induced transparency [38]. They are used to increase the sensitivity of gravitational wave detectors [39] as well. While usually produced in macroscopic setups [25], squeezed light may also be produced by (pairs of) individual emitters in optical nanostructures [40]. Recently, squeezed vacuum was proposed to engineer interactions between electric dipoles [41].

In this contribution, we distinguish between the usual absorption of intensity and that of quantum coherence, the latter measured as the coherent degree of freedom in Glauber's sense [42]. The latter measure is conveniently chosen such that the two measures are equivalent for coherent states, but for squeezed coherent input states, we show that they differ. For the latter case, we will...
show that under CPA conditions, a one- and two-mode combined squeezed vacuum state [43–46] is produced, a finding that does not rely on our definition of coherence. Meanwhile, all coherence of the input states is transferred to the internal modes of the beam splitter. Importantly, we will show that in the output state, the light is not entangled with the lossy beam splitter. These and further intriguing properties may make the lossy CPA beam splitter a useful element in continuous-variable protocols.

The interference of waves depends on their statistical properties. With quantum states of light as inputs, it is then natural to look for connections between CPA and quantum statistics. Here, we express the coherent absorption coefficient in terms of the fidelity [47] between two incoming coherent states. The requirement for perfect absorption of coherence becomes the complete indistinguishability of the incoming fields. This requirement also holds for squeezed coherent states of light as input.

This paper is organized as follows. In Section 2, we revisit CPA from first principles and distinguish between the absorption of quantum coherence and intensity, and we identify a statistical connection between the coherent absorption coefficient and the fidelity of the input states. In Section 3, we present coherent absorption of squeezed coherent input states. In Section 4, we derive and discuss the remarkable quantum state transformation that is performed by the CPA beam splitter, and we conclude in Section 5.

### 2. CPA REVISITED: COHERENT STATES

#### A. Basic Setup and Key Concepts

Let us consider a lossy beam splitter in free space that superposes two incident quantized modes of light and creates two outgoing modes, as shown in Fig. 1. The incident modes are described by the discrete annihilation operators $\hat{a}_1$ and $\hat{a}_2$. The field operators $\hat{b}_1$ and $\hat{b}_2$ of the outgoing modes are then given by the relations [48–51]

$$\hat{b}_1 = r\hat{a}_1 + r\hat{a}_2 + \hat{L}_1,$$

$$\hat{b}_2 = r\hat{a}_1 + r\hat{a}_2 + \hat{L}_2,$$

(1b)

where $t$ and $r$ are, respectively, the beam splitter's transmission and reflection amplitudes that in the case of loss satisfy $|r|^2 + |t|^2 < 1$ and $r^* r + t^* t \neq 0$. The $\hat{L}_1$ and $\hat{L}_2$ describe Langevin-type noise operators corresponding to device modes of the beam splitter in which the light absorption takes place. Both the input and output operators for the optical modes satisfy the standard bosonic input–output relations, so $[\hat{a}_i, \hat{a}_j^\dagger] = \delta_{ij}$ and $[\hat{a}_i, \hat{a}_j] = 0$ for the input operators. The output operators satisfy the analogous standard relations, and consistency is ensured by the Langevin operators for which $[\hat{L}_1, \hat{L}_2] \neq 0$. We discuss this quantum noise further in Section 4. We adopted the discrete-mode representation of the quantized fields, but there would be ways to generalize this to full continuum [52].

In the lossy beam splitter, the light typically loses part of its coherence and also part of its intensity, both due to a combination of destructive interference and dissipation. It will be enlightening to distinguish between the absorption of intensity and of coherent amplitudes (our distinction).

We define the total intensities of incoming and outgoing fields in the standard way as

$$I_{\text{in}} \equiv \langle \hat{a}_1^\dagger \hat{a}_1 \rangle + \langle \hat{a}_2^\dagger \hat{a}_2 \rangle \quad \text{and} \quad I_{\text{out}} \equiv \langle \hat{b}_1^\dagger \hat{b}_1 \rangle + \langle \hat{b}_2^\dagger \hat{b}_2 \rangle,$$

(2)

and the lost intensity as $\Delta I \equiv I_{\text{in}} - I_{\text{out}}$. The coefficient of absorption of intensity is

$$A_{\text{coh}}^I \equiv 1 - I_{\text{out}} / I_{\text{in}},$$

(3)

being the fraction of intensity that gets lost. Analogously, we choose to quantify the input coherences through the quantity

$$C_{\text{in}} \equiv |\langle \hat{a}_1 \rangle|^2 + |\langle \hat{a}_2 \rangle|^2,$$

(4)

i.e., as the sum of the absolute values squared of the expectation values $\langle \ldots \rangle$, the latter taken with respect to the initial quantum state $|\psi\rangle_{\text{in}}$. This state describes both the quantum state of light in both arms and of the internal states of the beam splitter. We give a rationale for this measure of coherences below. Correspondingly, we quantify the output coherences, $C_{\text{out}} \equiv |\langle \hat{b}_1 \rangle|^2 + |\langle \hat{b}_2 \rangle|^2$, and the net loss in coherent amplitudes as $\Delta C \equiv C_{\text{in}} - C_{\text{out}}$. We define the coefficient of absorption of coherences as

$$A_{\text{coh}}^C \equiv \Delta C / C_{\text{in}} \equiv 1 - C_{\text{out}} / C_{\text{in}}.$$  

(5)

This definition is state independent and well defined for systems for which the input–output relations (1) hold, and the input coherence is non-vanishing. We will be especially interested in two specific situations: CPA, corresponding to $A_{\text{coh}}^C \equiv 1$ (no output intensity), as opposed to perfect absorption of coherence, or $A_{\text{coh}}^C \equiv 0$ (vanishing output coherences).

Let us elucidate our measure of coherences (5), and let us first state what it is not. It does not measure quantum correlations between different optical ports. Such correlations of the form $\langle \hat{a}_1^\dagger \hat{a}_2 \rangle$ are already encoded in the coefficients $C_{\text{in, out}}$ through the relations (1) of our quantum optical input–output formalism. Second, it does not measure the most general case of quantum coherent resources introduced by arbitrary quantum states [53]. Such generality is not needed here, since we focus on coherent and squeezed coherent states.

So what does Eq. (5) measure? Throughout this paper, we consider coherent states $|\alpha\rangle$ as introduced by Glauber [42], and quantum states that can directly be defined in terms of them. The well-known coherent states are eigenstates of the annihilation
operator $a(\alpha) = a(\alpha)$, with the eigenvalue $\alpha$ being the complex coherent amplitude. Therefore, the magnitude of the coherent amplitude of the fully coherent states $|\alpha\rangle$ [42,54] and of quantum states that can directly be obtained through them can be quantified by the absolute expected value $|\langle \tilde{a} \rangle|$ of the corresponding bosonic annihilation operator $\tilde{a}$ [54]. We intentionally choose to measure the square of $|\langle \tilde{a} \rangle|$, so that for coherent states, our measure will numerically coincide with that of intensity (see Section 2B). This enables us to witness the breakdown of this equality in the case of squeezed states of light and thus provides a genuine new perspective to quantum CPA.

Written out in the photon-number state basis, the average intensity depends only on populations (diagonal elements) of the density matrix describing the state of light: $\langle \tilde{a}^\dagger \tilde{a} \rangle = \sum_\alpha \rho_{\alpha \alpha} n$. In contrast, the expectation value of the annihilation operator depends only on off-diagonal matrix elements (also known as quantum coherences [53]) of the density matrix: $\langle \tilde{a} \rangle = \sum_\alpha \rho_{\alpha \beta} \sqrt{n}$. In particular, it depends only on the one-photon coherences, being the coherences between states that differ in photon number by one. It follows that the intensity absorption coefficient $A_{\text{coh}}^C$ of Eq. (5) depends only on the photon-number populations of the density matrix, whereas the absorption coefficient of coherences $A_{\text{coh}}^C$ in Eq. (6) depends only on its one-photon coherences. In the following, for simplicity, we refer to $A_{\text{coh}}^C$, as the absorption of (quantum) coherence. The coherences are bounded from above by the average photon number as described by the inequality $|\langle \tilde{a} \rangle|^2 \leq \langle \tilde{a}^\dagger \tilde{a} \rangle$, which follows from the generalized Cauchy–Schwarz inequality [54].

**B. Coherent Absorption of Coherent States**

Throughout this paper, we make the common assumption that initially the internal device modes of the beam splitter are in their ground states, denoted as $|\rangle_{\text{BS}}$. For the incident optical modes, let us first consider that both are prepared in coherent states, also known as displaced vacuum states [55]. A coherent state (e.g., of mode 1) is defined as $|\alpha\rangle_1 = D_1(\alpha)|\rangle$ in terms of the optical vacuum state $|\rangle$ and the displacement operator

$$D_1(\alpha) \equiv \exp(-|\alpha|^2/2) \exp(a_1^+ \alpha \hat{a}_1).$$

The total input can then be written as $|\psi\rangle_{\text{in}} = |\alpha\rangle_1 \otimes |\beta\rangle_2 \otimes |\rangle_{\text{BS}}$, with $\alpha = |\alpha| \exp(i\theta_1)$ and $\beta = |\beta| \exp(i\theta_2)$. For the input coherence of these coherent states, we then find

$$C_{\text{in}} = |\alpha|^2 + |\beta|^2,$$

and for the output coherence,

$$C_{\text{out}} = (|\alpha|^2 + |\beta|^2)(|\alpha|^2 + |\beta|^2) + 2 \cos(\theta) |\alpha| |\beta|(r\alpha^* + r\beta^*),$$

where we used the input–output relations (1) and defined $\theta \equiv \theta_2 - \theta_1$ as the phase difference between the coherent states. The coefficient of absorption of quantum coherence then reads

$$A_{\text{coh}}^C = 1 - \left[|\alpha|^2 + |\beta|^2 + \frac{2 |\alpha| |\beta|(r\alpha^* + r\beta^*)}{|\alpha|^2 + |\beta|^2} \cos(\theta)\right],$$

which reduces to the conventional expression for absorption $A = 1 - (|\alpha|^2 + |\beta|^2)$ when replacing $\cos(\theta)$ by its average value of zero.

For two coherent incident states, the quantum coherence lost is in fact equal to lost intensity, because the identities $C_{\text{in}} = I_{\text{in}}$ and $C_{\text{out}} = I_{\text{out}}$ hold in that case. This then immediately implies $\Delta C = \Delta I$ and $A_{\text{coh}}^C = A_{\text{coh}}^T$. At this point, it may seem pedantic that we first distinguished between these two coefficients, but as we shall see in Section 3, this equality is not true for incident squeezed states of light.

**C. Fidelity and CPA**

Next, we relate the expression (9) for coherent absorption of coherent states to their quantum fidelity. As our starting point, we recall that the inner product of any pair of coherent states $|\alpha\rangle$ and $|\beta\rangle$ is [55]

$$\langle \alpha|\beta \rangle = e^{-\frac{1}{2}(|\alpha|^2 + |\beta|^2) + \alpha\beta^*},$$

which implies they are never orthogonal. It follows that

$$\alpha\beta^* = |\alpha||\beta|e^{-i\theta} = \frac{1}{2} (|\alpha|^2 + |\beta|^2) + \ln|\alpha\beta|,$$

$$\beta\alpha^* = |\alpha||\beta|e^{i\theta} = \frac{1}{2} (|\alpha|^2 + |\beta|^2) + \ln|\alpha\beta|.$$  

Hence, we obtain

$$2|\alpha||\beta| \cos(\theta) = |\alpha|^2 + |\beta|^2 + |\alpha|(|\beta|)\beta^* = |\alpha|^2 + |\beta|^2 + \ln F(\rho_\alpha, \rho_\beta),$$

where $F(\rho_\alpha, \rho_\beta)$ is the Uhlmann’s fidelity [47], with $\rho_\alpha \equiv |\alpha\rangle \langle \alpha|$ and $\rho_\beta \equiv |\beta\rangle \langle \beta|$. Written in terms of the fidelity, the coherent absorption coefficient (9) reads

$$A_{\text{coh}}^C = 1 - \left[|\alpha|^2 + |\beta|^2 + |\alpha|(|\beta|)\beta^* / |\alpha|^2 + |\beta|^2 \right] \ln F(\rho_\alpha, \rho_\beta),$$

which holds for arbitrary lossy beam splitters and for any pair of coherent input states.

Now we turn to the condition of coherent perfect absorption for the specific type of lossy beam splitters with $t = 1/2$ and $r = -1/2$. These values for transmission and reflection amplitudes give rise to maximum incoherent absorption of 1/2 for a thin film (a beam splitter) in a homogeneous background, meaning that for a single incident coherent state, such beam splitters absorb half the light, since $|\alpha|^2 + |\beta|^2 = 1/2$. For the two incident coherent states, we end up with the coefficient for coherent absorption:

$$A_{\text{coh}}^C = 1 + \frac{\ln F(\rho_\alpha, \rho_\beta)}{|\alpha|^2 + |\beta|^2}.$$  

Here, the natural logarithm is well defined, since the fidelity of two arbitrary coherent states always is greater than zero due to their non-orthogonality property (10). As a check, we find back $A_{\text{coh}}^C = 1/2$ for a single incident coherent state (take $\alpha \neq \beta = 0$). CPA according to Eq. (14) occurs when $F(\rho_\alpha, \rho_\beta) = 1$, i.e., when the incoming coherent states are indistinguishable. This is satisfied if and only if the coherent states have the same phases and amplitudes. These are the same well-known requirements of CPA as in classical optics: the expression Eq. (9) for the coherent absorption reduces to that obtained through classical scattering amplitudes for $t = 1/2$ and $r = -1/2$ [56]. Indeed, these values are required for CPA to occur in two-port lossy systems [3,21,56] and will be used subsequently in our analysis of CPA. Thus, with the “quasi-classical” coherent states, we recover the classical condition for CPA, but this time explained in terms of quantum mechanical indistinguishability. Incidentally, while counterpropagating normally incident waves are commonly used in CPA literature,
one can have a more general setup, as given in Fig. 1 (see, e.g., Ref. [7]. Decisive for CPA are the values \( t = -r = 1/2 \), not the angle of incidence.

By discussing CPA in quantum rather than classical optics, we replaced interference by quantum interference. The latter is described by transition probabilities between quantum states, which in the case of pure states, as we have here, are given by the fidelity [57]. It is intriguing that by Eq. (13) or Eq. (14), fidelities can be measured in terms of absorption. Thus, practically, one may distinguish two quantum states through a dissipative process, e.g., by using an absorbing metamaterial with known properties.

3. COHERENT ABSORPTION OF SQUEEZED COHERENT STATES

Prepared by the theory and results for coherent states in Section 2, we will now investigate the effects of squeezing on the coherent absorption of light. Mathematically, a squeezed coherent state \(|\alpha, \zeta\rangle\) is obtained by the action of a squeeze operator on a coherent state \(|\zeta\rangle\), e.g., for mode 1,

\[
|\alpha, \zeta_1\rangle_1 = \hat{S}_1(\zeta_1)|\alpha\rangle_1 = \hat{S}_1(\zeta_1)\hat{D}_1(\alpha_1)|\rangle.
\]

(15)

Here, the squeeze operator is defined as

\[
\hat{S}_1(\zeta_1) \equiv e^{i\zeta_1}\hat{a}_1^{\dagger}e^{-i\zeta_1}\hat{a}_1
\]

(16)
in terms of the mode creation and annihilation operators \(\hat{a}_1^{\dagger}\) and \(\hat{a}_1\). The degree of squeezing is determined by the complex coefficient \(\zeta_1 = \zeta_1^*\) exp \(i\phi_1\). Here, \(\zeta_1\) is called the squeezing parameter, while the angle \(\phi_1\) quantifies the amount of rotation of the field quadratures in the corresponding quantum phase space.

Let us now assume two squeezed coherent states \(|\alpha, \zeta_1\rangle_1\) and \(|\beta, \zeta_2\rangle_2\) as the input states of a general lossy beam splitter. The input states are then characterized by two sets of complex parameters: the coherence parameters \(\alpha = |\alpha|\exp(\theta_1)\) and \(\beta = |\beta|\exp(\theta_2)\), and the squeezing parameters \(\zeta_1^1 = \zeta_1\) exp \(i\phi_1\) and \(\zeta_2^2 = \zeta_2\) exp \(i\phi_2\), with \(\zeta_1, \zeta_2\) real-valued. The total input state has the form \(|\psi\rangle_{in} = |\alpha, \zeta_1\rangle_1 \otimes |\beta, \zeta_2\rangle_2 \otimes |\rangle_{BS}\).

A. Perfect Coherent Absorption of Coherence

The expected values of the input operators with respect to the state \(|\psi\rangle_{in}\) read

\[
\langle \hat{a}_1 \rangle_1 = |\alpha|\exp(\theta_1)\cosh(\zeta_1) - |\alpha|\exp(\theta_2)\sinh(\zeta_1),
\]

(17a)

\[
\langle \hat{a}_2 \rangle_2 = |\beta|\exp(\theta_2)\cosh(\zeta_2) - |\beta|\exp(\theta_2)\sinh(\zeta_2).
\]

(17b)

For the input coherence (4), i.e., the coherent content within squeezed states introduced by the Glauber’s degree of freedom, it then follows that

\[
C_{in} = \gamma_1^2|\alpha|^2 + \gamma_2^2|\beta|^2,
\]

(18)

where \(\gamma_1^2 = \cosh(2\zeta_1) - \cosh(\eta_1)\sinh(\eta_1)\), \(\gamma_2^2 = \cosh(2\zeta_2) - \cosh(\eta_2)\sinh(\eta_2)\), with \(\eta_1 \equiv 2\theta_1 - \phi_1\) and \(\eta_2 \equiv 2\theta_2 - \phi_2\). Similarly, using the input–output relations (1), we obtain the output coherence

\[
C_{out} = (|t|^2 + |r|^2)C_{in} + \Gamma(tr^* + rt^*),
\]

(19)

where

\[
\Gamma = \langle \hat{a}_1 \rangle_1|\hat{a}_2\rangle_2^* + \langle \hat{a}_2 \rangle_2|\hat{a}_1\rangle_1^* =
2|\alpha|/\beta|\exp(\theta_1)\cosh(\zeta_1)\cos h(\zeta_2)
- \cos(\theta_1 + \theta_2 - \phi_2)\cosh(\zeta_1)\sinh(\zeta_2)
- \cos(\theta_1 + \theta_2 - \phi_1)\sinh(\zeta_1)\cosh(\zeta_2)
+ \cos(\theta - \phi)\sinh(\eta_1)\sinh(\eta_2),
\]

(20)

and where \(\theta = \theta_1 - \theta_2\) and \(\phi = \phi_2 - \phi_1\). Therefore, by Eq. (5), the fraction of the coherence that gets lost is

\[
\mathcal{A}_c = 1 - |t|^2 + |r|^2 + \frac{\Gamma(tr^* + rt^*)}{r_1^2|\alpha|^2 + r_2^2|\beta|^2}.
\]

(21)

In the following we will analyze whether it is possible that all coherence gets lost at the absorbing beam splitter, i.e., whether perfect absorption of coherent photons (\(\mathcal{A}_c = 1\)) can be achieved with two incoming squeezed states. We will study the corresponding coherent absorption of intensities in Section 2.B.

We study how squeezing affects the coherent absorption of quantum coherence by exploring Eq. (21) for several parameter regimes. Let us first assume two squeezed beams with unit coherent amplitudes, \(|\alpha| = |\beta| = 1\), with coherent phase angles \(\theta_1 = \theta_2 = \delta\) and also equal squeezing angles \(\phi_1 = \phi_2 = \phi\), such that \(\delta \neq \phi\). So we take many parameters to be equal, but we do allow the squeezing amplitudes \(\zeta_1\) and \(\zeta_2\) to be different. It follows that

\[
\mathcal{A}_c = 1 - \left(|t|^2 + |r|^2 + \frac{\Omega_1}{\Omega_2}\right),
\]

(22)

where

\[
\Omega_1 = 2[\cosh h(\zeta_1) - \cos(\zeta_1)\cosh h(\zeta_2)](|r|^2 + |t|^2),
\]

\[
\Omega_2 = \cosh h(2\zeta_1) + \cosh h(2\zeta_2),
\]

\[
- \cos(\zeta_1)\sinh h(2\zeta_2) + \sinh h(2\zeta_2),
\]

and we defined \(\zeta_1 = \zeta_1\) and \(\zeta_2 \equiv 2\delta - \phi\). We specify again \(r = 1/2\) and \(r = 1/2\) and then find the condition for complete absorption of coherence to be \(\Omega_1/\Omega_2 = 1/2\), which is satisfied only if \(\zeta_1 = \zeta_2\). Therefore, for squeezed coherent states, equal coherent amplitudes and phases are not sufficient criteria for perfect absorption of coherence. Equal degree of squeezing is an additional requirement. For coherent states, this is trivially satisfied (\(\zeta_1 = \zeta_2 = 0\)). For squeezed states, the additional requirement is nontrivial, generalizing the requirement of indistinguishability that we identified for coherent input states in Section 2.C.

Further non-trivial effects of quantum squeezing on the absorption of quantum coherence can be revealed by considering the special case of two input states with equal coherent amplitudes (\(\alpha = \beta\), a nonvanishing coherent phase difference \(\theta_1 = \theta\) and \(\theta_2 = 0\)), equal squeezing amplitudes \(\zeta_1 = \zeta_2\), and vanishing squeezing phases \(\phi_1 = \phi_2 = 0\). As before, the absorbing beam splitter is characterized by \(r = 1/2\) and \(r = -1/2\). We obtain

\[
\mathcal{A}_c = \frac{1}{2} + \frac{\cos(\theta)}{1 + e^{2\zeta_1}\cosh h(2\zeta_1) - \cos(2\theta)\sinh h(2\zeta_1)}.
\]

(23)

Thus, for all phase differences \(\theta\), in the limit \(\zeta \to +\infty\) (amplitude squeezing), the coefficient of absorption of quantum coherence converges to the maximum incoherent absorption of \(\mathcal{A} = 1/2\). With squeezing, up to \(\zeta \approx 3.5\) say, (almost) perfect absorption of quantum coherence is recovered for \(\theta\) equal to a multiple of \(2\pi\). In the opposite limit \(\zeta \to -\infty\) (phase squeezing), the coefficient of absorption...
becomes $A_{sq}^{C} = 1/2 + (\cos(\theta)/[1 + (1/2)(1 + \cos(2\theta))]$, i.e., we have a similar behavior as in the case of bare coherent states. An illustration of these analytical results is depicted in Fig. 2.

While Eq. (23) was found for different coherent phases but equal squeezing, as a final example to illustrate loss of coherence in the presence of squeezing, we revert the situation and take the coherence phases and amplitudes to be equal, but different squeezing amplitudes (and again $\delta = \phi = 0$ and the same beam splitter with $t = 1/2$ and $r = -1/2$). This gives

$$A_{sq}^{C} = 1 - \frac{1}{2} \left( 1 - \frac{2r e^{-\xi^2}}{e^{-\xi^2_1} + e^{-\xi^2_2}} \right).$$

(24)

This formula is depicted in Fig. 3 and implies that in the absence of phases, squeezing always works against absorption, provided that $\xi_1 \neq \xi_2$. Indeed, the perfect absorption of coherent photons occurs only if $\xi_1 = \xi_2$. Furthermore, for equal squeezing, we regain the symmetry such that in the limits $\xi \to \pm \infty$, the absorption saturates at its classical maximum of 0.5.

### B. Coherent Absorption of Intensity

We showed that, for two coherent incident states, the fraction of quantum coherence lost is equal to that of intensity, i.e., $A_{coh}^{C} = A_{coh}$ or $\Delta C = \Delta I = 0$. In the case of two squeezed incident states, the loss of intensity reads

$$\Delta I = \mathcal{I}_{in} \left( 1 - (|t|^2 + |r|^2) - \frac{\Gamma(t^*r + r^*t)}{\mathcal{I}_{in}} \right),$$

(25)

where $\Gamma$ is given in Eq. (20). Hence, the coefficient of absorption of intensity becomes

$$A_{sq}^{C} = 1 - (|t|^2 + |r|^2) - \frac{\Gamma(t^*r + r^*t)}{\mathcal{I}_{in}} = A - \frac{\Gamma(t^*r + r^*t)}{\mathcal{I}_{in}},$$

(26)

which can be rewritten as

$$\mathcal{I}_{in} A_{sq}^{C} = \mathcal{I}_{in} A - \Gamma(t^*r + r^*t).$$

(27)

Similarly, from Eq. (21), we have

$$C_{in} A_{sq}^{C} = C_{in} A - \Gamma(t^*r + r^*t).$$

(28)

Equations (27) and (28) can be combined with $C_{in} A_{sq}^{C} = \Delta C$ and $\mathcal{I}_{in} A_{sq}^{C} = \Delta I$ to give

$$\Delta I - \Delta C = (\mathcal{I}_{in} - C_{in}) A.$$  

(29)

Important about this identity is its generality: it relates coherence losses and intensity losses for arbitrary lossy beam splitters and arbitrary squeezed coherent input states. It is the generalization of the simple identity $\Delta I = \Delta C$ that we obtained for coherent states. If $A_{sq}^{C} \neq A$, then Eq. (29) implies that one has equal quantum absorption coefficients $A_{sq}^{C} = A_{sq}^{C}$ if and only if the total input intensity is equal to the total input coherence, i.e., $C_{in} = \mathcal{I}_{in}$. But if the latter are not equal, and the coherent part of the squeezed state is completely absorbed ($\Delta C = C_{in}$), then it follows from Eq. (29) that $C_{out} = (\mathcal{I}_{in} - C_{in})(1 - A) \neq 0$; in other words, a quantum state with finite intensity survives the coherent absorption process, as the noise parts of the squeezed states are incoherent with respect to each other and thus cannot interfere and create a CPA-like effect. We will analyze this output state in Section 4.

For a fair comparison of coherence and intensity absorption, we now consider the same special cases that we already investigated in our analysis of the coefficient of absorption of quantum coherence. First, we choose $\theta_1 = \theta$, $\xi_1 = \xi_2 = \xi$, $\theta_2 = \phi_1 = \phi_2 = 0$, and $t = 1/2$ and $r = -1/2$ with equal coherent amplitudes. We obtain

![Fig. 2. Coefficient of coherent absorption $A_{sq}^{C}(\xi, \theta)$ of Eq. (23) upon variation of parameters of the squeezed coherent input states, for a beam splitter with $t = 1/2$ and $r = -1/2$. We vary the coherence angle $\theta = \theta_1$ and the squeezing parameter $\xi = \xi_1 = \xi_2$, while keeping $\theta_2 = \phi_1 = \phi_2 = 0$ fixed. Figure is valid for arbitrary equal coherence amplitudes $\alpha = \beta$.](image1)

![Fig. 3. Variations in the coefficient of coherent absorption (a) $A_{sq}^{C}(\xi_1, \xi_2)$ of Eq. (24) and (b) $A_{sq}^{C}(\xi_1 = \xi, \xi_2 = 0)$ of Eq. (31a). All other parameters are as explained in the main text.](image2)
which should be compared with Eq. (23), which gives $A_{\theta,0}^C$ for the same input states. In the limit $\xi \to +\infty$, we have $A_{\theta,0}^C = 1/2$ as before. Thus, in this limit, all the quantum contributions due to quantum coherence and squeezing are lost, and the corresponding absorption coefficients reduce to that of maximum incoherent one, i.e., $A_{\theta,0}^C = A_{\theta,0} = A$. In the opposite limit of $\xi \to -\infty$, we obtain $A_{\theta,0}^C = (1/2) + 2 \cos(\theta)/[3 + \cos(2\theta) + (1/|\alpha|^2)] < 1$ for all $\alpha \in \mathbb{C}$, thus characteristically different from absorption of quantum coherence but complying with our general relation Eq. (29). Figure 4 illustrates that perfect absorption of intensity is possible if and only if there is no squeezing. The discrepancy between absorption of coherence in Fig. 2 and of intensity in Fig. 4 is evident. The quantum coefficient $A_{\theta,0}^C$ saturates to its classical incoherent value of $1/2$ faster than $A_{\theta,0}^C$ for $\xi > 0$. In the opposite regime where $\xi < 0$, we have coherent oscillations similar to the case of absorption of quantum coherence, though always in the interval $A_{\theta,0}^C \in [0, 1]$.

A crucial difference between the coefficients $A_{\theta,0}^C$ and $A_{\theta,0}^I$ is that the latter depends explicitly on the mean number of photons in the initial states. This leads to the breaking of the parity symmetry such that $A_{\theta,0}^C(\xi, \theta = 0, \delta = 0) \neq A_{\theta,0}^C(-\xi, \theta = 0, \delta = 0)$. To clarify this, let $\xi_2 = 0$ and $\xi_1 = \xi$, so that the first beam is prepared in a squeezed state, while the second one is in a coherent state with equal amplitudes $|\alpha|$. By setting all optical phases to zero, the coefficient of absorption of coherent degree of freedom of squeezed states Eq. (24) is found to be

$$A_{\theta,0}^C = \frac{1}{2} + \frac{e^{-\xi}}{1 + e^{-2\xi}} = \frac{1 + \cosh \xi}{2 \cosh \xi},$$

while the intensity absorption coefficient becomes

$$A_{\theta,0}^I = \frac{1}{2} + \frac{e^{-\xi}}{1 + e^{-2\xi} + \sinh(2\xi)/|\alpha|^2}.$$  

Hence, in this regime, the coefficient $A_{\theta,0}^C$ is an even function and thus totally symmetric in the squeezing parameter $\xi$ [Fig. 3(b)]. The symmetry-breaking term in the expression for $A_{\theta,0}^C$ is now easily recognized as the ratio $R = \sinh^2(\xi)/|\alpha|^2$ of the mean photon contributions of independent squeezing and coherent degrees of freedoms to the total intensity of the squeezed coherent state. The maximally asymmetric and symmetric regimes are then identified by $R \gg 1$ and $R \ll 1$, respectively. In Fig. 5, we plot the coefficient of coherent absorption of intensity $A_{\theta,0}^C(\xi_1 = \xi, \xi_2 = 0)$ of Eq. (31b), as a function of the squeezing parameter $\xi$, for a set of coherent amplitudes $|\alpha|^2 \in \{10^{-3}, 10^0, 10^3, 10^6\}$. The figure illustrates clearly that intensity absorption in general is not symmetric in $\xi$. To restore the symmetry in the considered regime, the minimum number of coherent photons is found to be of order $10^6$.

4. CONTINUOUS-VARIABLE QUANTUM STATE PREPARATION WITH CPA

We would like to know the quantum states of light produced at the output of a beam splitter that exhibits CPA. For equal coherent states $|\alpha\rangle_1$ and $|\alpha\rangle_2$ as input, we found in Section 2B that there is zero intensity in the output, meaning that the output state for the two optical output modes has to be the vacuum state. The remarkable robustness of this way of producing the vacuum as output is that the same output state is produced whatever the coherence amplitude $|\alpha|$ of the input state.

Quantum state preparation becomes even more interesting for squeezed coherent input states, which we write in terms of squeezing and displacement operators as

$$\psi_{in} = |\alpha, \xi_1\rangle \otimes |\beta, \xi_2\rangle \otimes ||_{BS},$$

$$= \hat{S}_2(\xi_1) \hat{D}_2(\beta) \hat{S}_1(\xi_1) \hat{D}_1(\alpha) ||_{BS},$$

$$= \hat{S}_1(\xi_1) \hat{S}_2(\xi_2) \hat{D}_1(\alpha) \hat{D}_2(\beta) ||_{BS},$$

$$= e^{i(\xi_1^2 - \xi_2^2 + \xi_1 \xi_2)} \hat{D}_1(\alpha) \hat{D}_2(\beta) ||_{BS}.  \quad (32)$$

where in the third and fourth equalities we used that $\hat{a}_1$ and $\hat{a}_2$ commute. In the previous section, we saw that for equal input amplitudes and squeezing, and for $t = -r = 1/2$, all coherence can be coherently absorbed, but some output intensity will always remain. We will now use the input state (32), specify $\beta = \alpha$ and $\xi_2 = \xi_1 = \xi$, and then determine the output state for this specific case.

In Eq. (1), output operators were defined in terms of input operators. We need to invert this, writing the input operators in terms of the output operators. Thereby we can obtain the
sought output state by writing the input state in terms of the output operators. We will use the known quantum optical input–output theory for absorbing beam splitters [48–51], in particular Ref. [51], and identify what is special about quantum state transformation by absorbing beam splitters that exhibit CPA. Following Ref. [51], we write the input–output operator relations Eq. (1) in matrix notation as

$$\hat{b} = T \hat{a} + A \hat{g}. \quad (33)$$

Here, $T$ is the 2×2 transmission matrix. The Langevin noise of the absorbing beam splitter is accounted for by linear combinations of bosonic device input operators $\hat{g}_1$ and $\hat{g}_2$ that together form the vector $\hat{g}$. The corresponding linear coefficients form the 2×2 absorption matrix $A$. Besides optical output operators $\hat{b}_{1,2}$, there are device output operators $\hat{b}_{1,2}$. Also the latter pair can be written as a linear combination of all four input operators. The 4×4 matrix that relates all four output operators in terms of the four input operators is restricted by the requirement that output operators satisfy standard bosonic commutation relations and are canonically independent. This restricts $A$ once $T$ is given, for example.

The formalism of Ref. [51] simplifies particularly for the CPA beam splitter because $T$ is a real symmetric matrix in this special case, and the absorption matrix $A$ is then also easily found:

$$T_{\text{cpa}} = \begin{pmatrix} t & r \\ r & t \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = \frac{1}{2}(1 - \sigma_x),$$

$$A_{\text{cpa}} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \frac{1}{2}(1 + \sigma_x). \quad (34)$$

in terms of both the 2×2 unit matrix $I$ and the Pauli matrix $\sigma_x$. This simplifies the input–output relation Eq. (33), and the corresponding relation for the device output operators becomes $\hat{b} = -A_{\text{cpa}} \hat{a} + T_{\text{cpa}} \hat{g}$, which combined with Eq. (33) provides the full 4×4 input–output operator matrix relation.

For the CPA beam splitter, by matrix inversion, the inverse relationship of Eq. (33) becomes

$$\hat{a} = T_{\text{cpa}}^{-1} \hat{b} - A_{\text{cpa}}^{-1} \hat{b}. \quad (35)$$

Now we can use these relations to write $\hat{a}_1$ and $\hat{a}_2$ in the input state (32) in terms of the four output operators $\hat{b}_{1,2}$ and $\hat{b}_{1,2}$, and it is easy to show that $\hat{a}_1^2 + \hat{a}_2^2 = (\hat{b}_1 - \hat{b}_2)^2 + (\hat{b}_1 + \hat{b}_2)^2)/2$. We thereby obtain as one of our main results the output state $|\psi\rangle_{\text{out}} = e^{i\delta c} (\hat{b}_1 - \hat{b}_2) |\psi\rangle_{\text{in}} \otimes e^{i\delta c} (\hat{b}_1 + \hat{b}_2) |\alpha, \alpha\rangle_{\text{BS}}, \quad (36)$

which is remarkable for several reasons. First, just like the input state, it is a direct-product state of optical output states and beam splitter device states. In other words, the optical output state is not entangled with the absorbing beam splitter that was used to produce it. Tracing out the beam splitter’s internal degrees of freedom therefore leaves the output state of light in a pure state, rather than the usual mixed state that requires a density-matrix description. This remarkable outcome is the main reason that the CPA beam splitter, despite being lossy, can become a useful component in continuous-variable quantum state engineering.

The second remarkable property of the state is the perfect coherent absorption: all coherence of the input state $|\psi\rangle_{\text{in}} = |\alpha, \zeta_1\rangle \otimes |\alpha, \zeta_2\rangle \otimes |\psi\rangle_{\text{BS}}$ resided in the optical channels and ends up in the material modes of the beam splitter. There are no coherent photons, in Glauber’s sense, in the optical output state, i.e., it does not depend on the coherence amplitude $\alpha$ at all. This explains that we found $A_{\text{sq}}^C = 1$ in Section 3A.

As a special case and check of our results, for vanishing squeezing, we indeed find standard CPA behavior: for the two-mode coherent input state $|\psi\rangle_{\text{in}} = |\alpha\rangle_1 \otimes |\alpha\rangle_2 \otimes |\psi\rangle_{\text{BS}}$, we find from Eq. (36) the corresponding output state $|\psi\rangle_{\text{out}} = |\psi\rangle_{\text{BS}}$. This, indeed, is a direct product of the optical vacuum state and coherent states for the device modes of the beam splitter. So for coherent states, the coherent absorption is indeed perfect; no photons leave the CPA beam splitter and $A_{\text{sq}}^C = 1$.

Returning to the general case of squeezed coherent input, the optical output state $e^{i\delta c} (\hat{b}_1 - \hat{b}_2)^i |\psi\rangle_{\text{in}} \otimes e^{i\delta c} (\hat{b}_1 + \hat{b}_2)^i |\alpha, \alpha\rangle_{\text{BS}}$ in Eq. (36) is a one- and two-mode combined squeezed vacuum state. The one-mode squeezing corresponds to quadratic operators in the exponent such as $\hat{b}_1^i$, and the two-mode squeezing to the products of different operators such as $\hat{b}_1 \hat{b}_2$. Being generalizations of the generic squeezed vacuum, the optical output states can be used for the implementations of quantum teleportation [26], quantum metrology [30], quantum dense coding [32], quantum dialogues [33], and electromagnetically induced transparency protocols [38].

Squeezed vacuum states have non-vanishing intensities, and since a beam splitter under CPA conditions emits squeezed vacuum states of light, coherent perfect absorption of intensity is not possible, and $A_{\text{sq}}^C < 1$ for non-vanishing squeezing. This optical output state is independent of the coherence amplitude $\alpha$ of the incident squeezed coherent states. This quantum property explains why the coherent absorption coefficient $A_{\text{sq}}^C$ in Eq. (31b) became dependent on the input intensity via $|\alpha|^2$, while such a nonlinear dependence is absent for $A_{\text{sq}}^R$ in Eq. (31a).

These one- and two-mode combined squeezed vacuum states have been studied before, albeit in a different setting [43]. The exact form that emerges here was first proposed by Abdalla [44,45] and later studied by Yeoman and Barnett [46]. In the latter contribution, it was identified that these states could be generated by superposing two identical (equally squeezed) single-mode squeezed vacuum states via a 50/50 ideal beam splitter. It was found that $|\alpha|^2 = |\alpha|^2$ should hold to produce such states on a beam splitter, a condition that also the lossy CPA beam splitter satisfies.

So while the ideal beam splitter requires squeezed vacuum input states, the CPA beam splitter can take any pair of identical squeezed coherent states to distill [58] a two-mode entangled squeezed vacuum state out of it. Moreover, squeezing takes place via an absorption process resulting in beam-splitter internal modes that end up in one- and two-mode combined squeezed coherent states. We find that half the squeezing is absorbed into internal modes of the beam splitter. This leaves the other 50% of the squeezing for the optical output modes, in accordance with a spectral analysis performed for the special case of incoming squeezed vacuum states [20]. The distillation of squeezed vacuum states that we propose is not possible with non-absorbing beam splitters. Thus, our results generalize the previous ones and propose a new engineering procedure to produce pure quantum states via perfect absorption of quantum coherence.

5. CONCLUSION

In conclusion, we investigated the coherent absorption of light when two squeezed coherent beams are superposed on an absorbing beam
splitter. We first reconsidered the generic case of two incoming bare coherent states and distinguished two types of absorption, namely, of quantum coherence introduced to the system by the complex amplitude $\alpha$ and of intensity. We showed that the corresponding absorption coefficients are identical for the case of bare coherent state inputs and can be written in terms of quantum fidelity, suggesting a general condition of indistinguishability of input states for CPA to occur that holds for squeezed coherent states as well.

In the case of squeezed coherent beams, the coherent degree of freedom is completely absorbed, provided that the CPA conditions hold. By Eq. (29), we provided a general argument that the input intensity will not be fully absorbed in the presence of quantum squeezing. More specifically, we revealed that an entangled squeezed vacuum state is produced at the output, leaving the absorber in a one- and two-mode combined squeezed coherent state. In some cases, both states might be reused as quantum resources [59,60].

We propose to test and use the lossy CPA gate as a new tool for quantum state preparation. Since quite remarkably the CPA gate produces a direct-product state of an optical output state and an internal beam-splitter state [see Eq. (36)], it does not suffer from the usual disadvantage of lossy optical components in becoming entangled with optical fields, producing mixed reduced quantum states for the light fields. Instead, the optical output states of the CPA gate are pure quantum states. Yet the action of the gate crucially depends on the CPA beam splitter being lossy: all coherence is absorbed.

It is interesting to compare the CPA gate with the usual practical implementation of “phase-space displacement” by which a squeezed vacuum state and a strong coherent state are mixed on a low-reflectivity non-lossy beam splitter [61], resulting in a squeezed coherent output state. Our CPA gate does more or less the reverse, separating squeezing from coherence, but the crucial difference is that it does so for arbitrary (but equal) input coherence amplitudes. This arbitrariness constitutes a potentially useful robustness of this gate. In particular, the CPA gate would work in a small-signal regime where saturation effects in absorption can safely be neglected.

Our proposal of the CPA quantum gate is part of an interesting wider trend to engineer quantum dissipation and to use it for quantum state preparation and other quantum operations (see, e.g., Refs. [62–69]). Also for our CPA gate for continuous-variable quantum state preparation, loss is a resource to obtain new functionality: the CPA gate prepares its pure quantum states both despite being lossy and because it is lossy.

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