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A Bi-objective efficiency-fairness model for scheduling slots at congested airports

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ABSTRACT

Slot scheduling has emerged as an issue of major concern at congested airports. A commonly used metric for assessing airport slot scheduling efficiency is total schedule displacement. In addition to schedule efficiency, fairness has also been identified as another important slot scheduling criterion. However, the literature currently does not provide sufficient modelling capabilities for investigating the single airport slot scheduling efficiency-fairness trade-off. The objective of this paper is to develop and solve a novel airport slot scheduling model which considers simultaneously slot scheduling efficiency and fairness objectives. We introduce a fairness metric for slot scheduling and formulate the airport slot scheduling problem as a bi-objective optimization model which considers fairness and efficiency simultaneously. We use an integrated solution framework that combines the ε-constraint method and a row generation algorithm to solve the proposed bi-objective model. We investigate the slot scheduling efficiency-fairness trade-off under the following two slot scheduling regimes: (i) a regime that considers historical slot usage rights (hierarchical), and (ii) a regime that does not consider historical slot usage rights (non-hierarchical). The proposed model generates the efficient frontiers describing the slot scheduling efficiency-fairness trade-off for both slot scheduling regimes. The results of the model are presented at the aggregate (airport-wide) and disaggregate (individual) airline level providing information that enhances the transparency of slot scheduling decisions. The efficient frontiers produced by the proposed model can facilitate discussions among different stakeholders in making slot scheduling decisions.

1. Introduction

The rapid growth of air transport demand coupled with inadequate provision of airport capacity has led to a serious imbalance between demand for airport services and supply of the required airport resources. As a result, 180 airports worldwide are schedule coordinated (Level 3 airports), while 123 airports are schedule facilitated (Level 2 airports) (IATA, 2017a). According to the IATA World Scheduling Guidelines (WSG) an airport is coordinated when demand exceeds its capacity, and its infrastructure cannot be expanded in the short term to meet demand; while an airport is considered as schedule facilitated when 'there is potential for congestion at some periods of the day, week or scheduling period, which is amenable to resolution by voluntary co-operation between airlines… ' (IATA, 2017b).

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Slots are used to express the capacity of schedule coordinated airports. A commonly used metric for assessing airport scheduling efficiency is the schedule delay or schedule displacement which is defined as the difference between the requested and allocated slot times (Koesters, 2007; Zografos et al., 2012). In congested airports, slots represent a scarce resource that should be allocated to competing users. The literature suggests (Marsh and Schilling, 1994; Kumar and Kleinberg, 2000; Bertsimas et al., 2011a, 2011b) that in addition to efficiency a major issue associated with the allocation of scarce resources is fairness. It is also worth noting that the IATA Worldwide Scheduling Guidelines (IATA, 2017b) require that the slot scheduling process should be non-discriminatory and transparent. Furthermore, IATA and Airports Council International (ACI) have jointly recognized the need to optimize the use of scarce airport capacity at congested airports, and they have agreed to develop a more efficient and effective slot allocation process which must ensure 'transparency, certainty, consistency, fairness, and non-discrimination' (ICAO, 2016). Therefore, in addition to schedule efficiency, fairness of slot scheduling represents another airport slot scheduling objective (Zografos et al., 2016). Clearly, there is a trade-off between slot allocation efficiency expressed by some form of schedule displacement and fairness. However, the existing literature does not provide sufficient modelling capabilities for examining the above stated trade-off for scheduling series[1] of airport slots for an entire scheduling season[2]. Therefore, it is important to develop decision support capabilities that will allow decision makers to explore the trade-off between slot scheduling efficiency and fairness, in making slot scheduling decisions.

The objective of this paper is to develop and solve a novel strategic single airport bi-objective slot-scheduling model which considers simultaneously slot scheduling efficiency and fairness objectives and generates and presents to all stake holders information supporting the transparency of the decision making process. It is worth emphasizing that this paper deals with the strategic slot scheduling problem at a single airport. This means that the proposed model considers decisions made twice per year for congested airports (once for the winter and once for the summer scheduling season).

Another issue associated with slot scheduling decisions relates to the consideration of historical slot usage rights. For instance, the current IATA Worldwide Scheduling Guidelines (IATA, 2017b) suggest that slot requests of airlines with historical usage rights (Grandfather Rights) should be given higher priority in making slot scheduling decisions over new entrants and other aircraft operators. However, the existing literature has not investigated adequately the effect of this rule on the slot scheduling efficiency-fairness trade-off for the scheduling of slots at a single airport.

Furthermore, as stated earlier, IATA and ACI have jointly recognized the need that slot scheduling decisions should be consistent and transparent (ICAO, 2016). Consistency in decision making requires the development of methods/models that can produce the same outcomes using the same set of inputs, i.e. decision objectives, constraints, priorities, input data, for all scheduling circumstances and stakeholders involved. Transparency calls for providing to all stakeholders sufficient information regarding the: (i) implications of a proposed solution on their slot requests, and (ii) means of comparing how fairly they have been treated in relation to their competitors. All existing mathematical optimization slot scheduling models, by their nature, provide the capability to produce consistent results. However, up to date, limited research effort has been devoted to model explicitly fairness. Furthermore, the results of existing models have been presented and discussed at an aggregate level, e.g. what is the impact of the slot scheduling outcome for all users (airport level), and do not provide information regarding the impact of slot scheduling decisions at the individual airline level. This paper addresses the above identified research gaps and presents a modelling approach for examining the slot scheduling efficiency-fairness trade-off. The results of the proposed model are presented at the aggregate (airport-wide) and disaggregate (individual airline level) providing information that enhances the decision making process.

The remainder of this paper is organized as follows: Section 2 discusses previous related work and establishes the contributions of this paper, Section 3 presents the proposed model, while Section 4 describes the solution approach. Section 5 presents the results of the application of the proposed model. Finally, Section 6 summarizes the research conclusions and provides directions for future research.

2. Literature review

Airport slot allocation and scheduling is a research area that has attracted considerable attention. Three major research streams can be identified in the slot allocation and scheduling literature, namely: administrative, market-based, and hybrid (i.e. approaches combining administrative and market-driven instruments). Fig. 1 presents the classification of the existing literature and identifies different types of methods that have been used in order to manage the airport demand capacity imbalance. For a more comprehensive discussion on the classification and use of slot allocation methods, the reader is referred to Madas and Zografos (2008), and Zografos et al. (2016).

In this work, we are dealing with slot scheduling in the context of pure administrative procedures. Specifically, we focus our attention on slot scheduling decisions that are based on the IATA Worldwide Slot Scheduling Guidelines. Therefore, we focus our review on this part of the literature. The primary focus and contribution of this paper is on the development of a novel bi-objective model for strategic slot scheduling at a single airport. The distinctive feature of the proposed model is its ability to consider simultaneously efficiency (total displacement) and fairness objectives. In this section we review relevant slot scheduling literature referring to each one of the two objectives involved in our formulation to demonstrate the research gap associated with the lack of formulations considering simultaneously, both efficiency and fairness objectives.

[1] A series of slots exists when 'at least 5 slots are requested for the same time on the same day-of-the-week during the same scheduling season' (IATA, 2017b).

[2] According to the IATA World Scheduling Guidelines there are two scheduling seasons. The Summer season starts the last Sunday of March, while the Winter season starts the last Sunday in October.

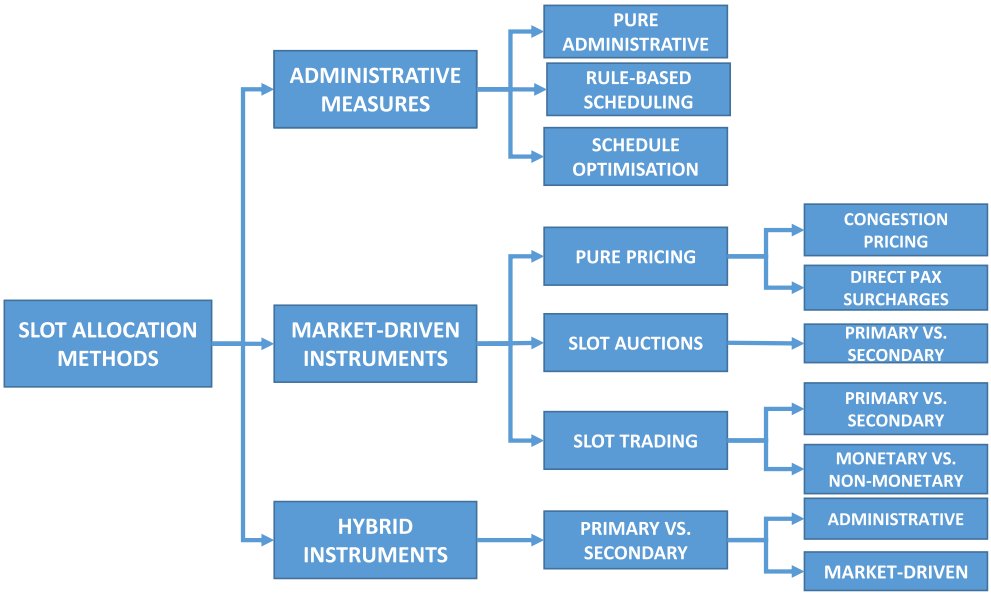

Fig. 1. Classification of the existing literature.

Koesters (2007) proposed a heuristic procedure to estimate single airport schedule displacement (delay), for various levels of declared capacity, demand, and slot utilization. Zografos et al. (2012) introduced an integer programming model for the deterministic single-airport slot scheduling problem. The airport declared capacity was considered as input for the slot scheduling problem. The application of the proposed model to a real world case resulted to a reduction of the slot scheduling delay. Sensitivity analysis performed demonstrated that a small increase in declared capacity may result in substantial (non-linear) improvements in schedule displacement (slot delay). Jacquillat and Odoni (2015) presented an integrated scheduling and operations approach to airport congestion mitigation. The proposed formulation involves a Dynamic Programming model for optimizing airport capacity utilization and an Integer Programming slot scheduling model. The proposed scheduling model is a bi-objective model that considers the minimization of the maximum displacement and the minimization of the total displacement. Zografos et al. (2017) introduced two bi-objective models which consider, besides the minimization of total schedule displacement, the minimization of the maximum schedule displacement, or the minimization of the violated slots (i.e. slots that are displaced beyond a maximum acceptable threshold value). Ribeiro et al. (2018) proposed an optimization model for allocating airport slots under IATA Guidelines. The proposed model extends the optimization model proposed by Zografos et al. (2012), by incorporating additional slot scheduling requirements stemming from the IATA World Scheduling Guidelines, and the bi-objective formulation presented in Zografos et al (2017) into a tri-objective formulation by introducing as a third objective the minimization of the rejected slot requests.

Fairness has been introduced as a criterion for slot allocation in the context of congestion pricing (Andreatta and Lulli, 2009), and the design of monetary compensation mechanisms for allocating slots following market-based approaches (Castelli et al., 2012). The fairness criterion used in this study (Castelli et al., 2012) requires that each airline should be financially penalized proportionally to its contribution to the infeasibility of the ideal solution. This means that if an airline asks to use slots during the periods when demand exceeds capacity, then it should be asked to compensate those that will not have the opportunity to use the airport during this congested period. Pellegrini et al. (2017) introduced the notion of fairness in slot scheduling decisions at the network level by considering the maximum cost of missed allocation, and the maximum total displacement costs of all airlines across all airports. Jacquillat and Vaze (2018), proposed a model for incorporating inter-airline equity in airport scheduling interventions. The proposed model minimizes lexicographically the disutility of airlines starting with the airlines having the largest disutility (minimax criterion). It is worth noting that both models mentioned above, i.e. Pellegrini et al. (2017) and Jacquillat and Vaze (2018), formulated the single day slot scheduling problem. Therefore these models did not capture the requirement regarding the scheduling of series of slots for the entire scheduling season. In contrast with the case of single airport slot scheduling decisions, fairness has been considered explicitly as a decision making criterion in the context of Air Traffic Management (ATM) decisions. Specifically, fairness has been considered in modelling the Ground Delay Problem (Manley and Sherry, 2010; Glover and Ball, 2013), the airspace planning problem (Sherali et al., 2002), and in the context of Air Traffic Flow Management Problem (Vossen et al., 2003; Pourtaklo and Ball, 2009; Glover and Ball, 2013; Liang et al., 2013). Manley and Sherry (2010) analyzed the performance and equity of the Ground Delay Programs (GDP). They postulated that from an equity point of view the delay that should be allocated to an airline should be proportional to the number of flights it has. Glover and Ball (2013) also studied the trade-off between equity and efficiency for Ground Delay Program (GDP) planning. The measure of equality used in this study was the deviation of the flight from its Ratio by Schedule (RBS) allocation. Vossen et al. (2003) introduced a framework for considering equity in air traffic flow management. In this study, equity was modelled as a deviation from the ideal allocation, which is based on

the unconstrained Ratio by Schedule (RBS) approach. Pourtaklo and Ball (2009) investigated the equitable allocation of enroute airspace resources and used a proportionality rule to determine the fair share of slots that should be allocated to a given airline. The proportionality rule used postulates that the number of slots allocated to an airline should be proportional to the requests made. Following the determination of the number of slots they allocated flights to slots by taking into account the priorities of each airline. Sherali et al. (2002) developed an airspace planning model for selecting flight plans. The proposed model considers air traffic controller workload, flight safety, and equity. The equity metric in this study is expressed through a utility function that expresses the effectiveness of a selected flight plan. This utility function is then restricted to arrange between an upper and a lower acceptable value.

The literature review revealed that currently there are not models considering simultaneously efficiency and fairness objectives for the scheduling of series of slots at a single airport for the entire season. It is worth noting that the consideration of fairness, as an additional criterion, in studies dealing with the allocation of scarce resources has been widely recognized in the literature (Marsh and Schilling, 1994; Kumar and Kleinberg, 2000; Bertsimas et al., 2011a, 2011b). The consideration of fairness in the resource allocation process contributes to the acceptability of the resulting resource allocation outcome especially when the resources are allocated to users with competing interests. Another aspect of slot scheduling decisions that has not been adequately addressed in the literature (Zografos et al., 2016) relates to the investigation of the effect of slot scheduling priorities on slot scheduling efficiency and fairness. The IATA World Scheduling Guidelines (IATA, 2017b) identify three different types of airline requests: (1) historic, i.e. requests of incumbent airlines, (2) new entrant[3], and (3) other, i.e. all remaining requests. According to the IATA WSG, slots are allocated based on the historic precedence principle. This means that historic requests should be satisfied first, followed by new entrant, and other requests. Pellegrini et al. (2017) studied the effect of the relaxation of the historic precedence rule on the number of missed allocations and on total displacement when allocating slots at the network level. Zografos et al. (2017) studied the effect of the relaxation of the historic rule on total and maximum displacement, and on total and maximum acceptable displacement at single airport level. In this paper, we are complementing the existing literature by analyzing the effect of simultaneous consideration of all slots on the efficiency-fairness trade-off.

Existing slot scheduling models present aggregate results regarding the performance of the slot scheduling process for an airport without presenting results regarding the implications of the slot scheduling decisions to individual airlines and flights. Knowledge of the impact of slot scheduling decisions at a disaggregate level (i.e. airline) is particularly useful since this information can ensure transparency of the decision making process. Therefore, it is useful to develop models that will provide the capability to investigate the effect of the use of slot scheduling priorities on the efficiency-fairness trade-off, while at the same time they ensure consistency and transparency of decisions.

In a nutshell, this paper contributes to the slot scheduling literature by: (i) introducing a new bi-objective slot scheduling model that considers simultaneously efficiency and fairness objectives and schedules series of slots at a single airport for the entire season, (ii) examining the effect of alternative slot scheduling regimes on scheduling efficiency and fairness for series of slots (iii) introducing a solution approach that can generate the entire efficient frontier of the slot scheduling fairness-efficiency trade-off, and (iv) presenting the implications of the slot scheduling decisions at a disaggregate level. The proposed model was implemented using data from a congested (coordinated airport).

3. Modelling slot scheduling efficiency and fairness

The following notation is used for the mathematical formulation of the proposed bi-objective slot scheduling model.

Sets

T	set of coordination slot time intervals
M	set of movements requested by all the airlines
P	set of movement pairs $(m^p_{arr}, m^p_{dep}) \in P$, where m^p_{arr} is the arrival movement and m^p_{arr} is the corresponding departure movement
M_a	set of movements requested by airline a
C	set of airport capacity constraints
T_c	set of slots associated with capacity constraint c
A	set of airlines

Parameters

t_m	requested slot time for movement m
ω^p	turnaround time for movement pair p
u_c	declared capacity associated with capacity constraint c
ε	parameter used for the ε-constraint method
Δ	step size for updating the value of ε
n	number of coordination time intervals

[3] "An airline's slot request should have new entrant status provided that the request, if accepted, would result in the airline holding, in total, fewer than five slots at that airport on that day." (IATA, 2017b).

Decision variables

x_m^t	$x_m^t = 1$, if movement m is allocated to slot t; otherwise, $x_m^t = 0$

Functions

ρ_a	fairness metric for airline a
f_m	displacement allocated to movement m
d_a	total displacement allocated to airline a
D	total displacement of all airlines

3.1. Displacement metric

Schedule displacement is defined (Koesters, 2007; Zografos et al., 2012) as the difference between the allocated and requested slot time. Mathematically the displacement of a movement m is expressed by Eq. (1).

$$f_m = \sum_{t \in T} |t - t_m| \, x_m^t, \quad \forall \, m \in M \tag{1}$$

Based on above equation, the total schedule displacement associated with all slot requests of an airline (a) is expressed by Eq. (2), while the total displacement of all the slots requested by all airlines is computed using Eq. (3).

$$d_a = \sum_{m \in M_a} f_m, \quad \forall \, a \in A, \text{ and} \tag{2}$$

$$D = \sum_{a \in A} \sum_{m \in M_a} f_m \tag{3}$$

The above stated metrics are defined without considering priorities for the satisfaction of slot requests. In the case that the slot requests are prioritized according to historical slot usage rights, i.e. historic, new entrant, and other, then M_a in Eqs. (2) and (3) is replaced by the slot requests corresponding to flights (movements) of different priority, i.e. historic, new entrant, and other.

3.2. The fairness metric

The concept of allocating delays (schedule displacement) proportionally to requests made has been proposed in the literature in the context of the Ground Delay Programme (Manley and Sherry, 2010). In this paper, we are adopting this concept to the slot scheduling decision making context and we postulate that the total displacement that should be allocated to an airline should be proportional to the slot requests that an airline has made. Eq. (4) provides the mathematical expression of the proposed fairness metric.

$$\rho_a = \frac{\frac{d_a}{D}}{\frac{|M_a|}{\sum_{a \in A} |M_a|}}, \quad \forall \, a \in A \tag{4}$$

The denominator in the above equation is the proportion of slots requested by airline (a), while the numerator is the proportion of the displacement allocated to airline (a). Based on the above fairness indicator, the following definitions are used throughout this paper.

(1) *Fairly treated airline:* An airline (a) is a fairly treated airline, if $\rho_a = 1.0$;
(2) *Favored airline:* An airline (a) is a favored airline, if $\rho_a < 1.0$;
(3) *Disfavoured airline:* An airline (a) is a disfavoured airline, if $\rho_a > 1.0$.

Combining Eq. (4) and the definition of fairly treated airlines (see case 1 above), we can state the following proposition.

Proposition. *At a congested airport, if all airlines are fairly treated, then the average displacement of each airline should be equal.*

Proof. Eq. (4) can be rearranged as:

$$\rho_a = \frac{\frac{d_a}{D}}{\frac{|M_a|}{\sum_{a \in A} |M_a|}} = \frac{\frac{d_a}{|M_a|}}{\frac{D}{\sum_{a \in A} |M_a|}}, \quad \forall \, a \in A \tag{5}$$

In the preceding equation, the numerator is the average displacement of airline (a), while the denominator is the average displacement of all airlines. It is worth reminding that the preceding definitions are established on the condition that the total displacement is positive (i.e. $D > 0$); otherwise, the denominator in Eq. (5) is not well defined. Considering that the slot allocation

problem is non-trivial for congested coordinated airports (i.e. that is in congested airports always there will be a schedule displacement), this condition is always satisfied. If airline (a) is a fairly treated airline, then the denominator equals the numerator, i.e. $\frac{\frac{d_a}{|M_a|}}{\frac{D}{\sum_{a\in A}|M_a|}} = 1$, implying that the average displacement of airline (a) equals the average displacement of all airlines. Thus, if all airlines are fairly treated, the average displacement of each airline should be equal to the average displacement of all airlines. Stated otherwise, the average displacements of all airlines are identical. □

Eq. (4) expresses the fairness under the assumption that all slot requests are of the same type. When different types of slots are considered, the fairness is calculated within each type of requested slots. We are using the term 'types of slot requests' to reflect slot allocation priorities based on the established historical slot usage rights. According to the IATA Worldwide Slot Scheduling Guidelines (IATA, 2017b), the requests of airlines with historical precedence slot rights should be satisfied first, followed by the satisfaction of new entrant, and other requests.

Based on the proposed fairness metric all airlines are fairly treated when $\rho_a = 1.0, \forall\, a \in A$. However, due to capacity and turnaround time constraints, in reality, perfect slot scheduling fairness for all airlines cannot be achieved. An alternative way of looking at the fairness issue is to compare the displacement of a given airline with the displacement of other airlines (relative as opposed to absolute fairness). In the next section, we propose a fairness objective that minimizes the maximum distance between the fairness metric of an airline and the average fairness metric of all airlines. This objective ensures that the worst case of un-fairness differs as little as possible from the average fairness.

3.3. The proposed bi-objective model

Based on the metrics defined above we propose the following mathematical model which considers simultaneous efficiency and fairness.

$$\min_{\mathbf{x}=[x_m^t]} Z(x) = \left\{ \overbrace{\sum_{a\in A}\sum_{m\in M_a} f_m}^{\text{Efficiency objective}},\ \max\left\{ \left| \rho_a - \overbrace{\frac{\sum_{a'\in A}\rho_{a'}}{|A|}}^{\text{Fairness objective}} \right|,\ a\in A \right\} \right\} \tag{6}$$

Subject to:

$$\sum_{m\in M}\sum_{t\in T_c} x_m^t \leqslant u_c, \quad \forall\, c\in C \tag{7}$$

$$\sum_{t\in[0,k)} x_{m_{dep}^p}^t + \sum_{t\in[k-\omega^p,n)} x_{m_{arr}^p}^t \leqslant 1, \quad \forall\, (m_{arr}^p, m_{dep}^p)\in P,\ k\in[\omega^p, n) \tag{8}$$

$$\sum_{t\in T} x_m^t = 1, \quad \forall\, m\in M \tag{9}$$

$$x_m^t \in \{0, 1\} \tag{10}$$

Eq. (6) of the proposed model seeks to minimize the total schedule displacement (efficiency) and the maximum deviation of fairness from the average fairness. Eq. (7) requires that the total number of movements considering series of slots for a given time interval cannot exceed the available capacity of the corresponding time interval. Eq. (8) expresses the aircraft turnaround time constraint requiring that the time interval between the arrival and departure slot be greater than or equal to the minimum aircraft turn-around time. n denotes the number of coordination time intervals, where each time interval is usually set at 5 min in practice. In short, this equation means that for any $k \geqslant \omega^p$, if the departure has been allocated to any interval $[0, k)$, it is not possible to allocate the corresponding arrival after interval $k - \omega^p$. Eq. (9) stipulates that every movement must be allocated to one and only one-time interval. It is important to stress here that the proposed bi-objective formulation considers the allocation of a series of slots (over the entire scheduling season). A series of slots exists when 'at least 5 slots are requested for the same time, e.g. 10:00 am, on the same day-of-the-week, e.g. Monday, (IATA, 2017b). For instance, a request for departing from an airport at 10:00 am for at least five Mondays during the same scheduling season defines a request for a series of slots. When a series of slots is considered the slots should be allocated at the same time for all requested days in the scheduling season. It is worth noting that IATA World Scheduling Guidelines (IATA, 2017b) state that if this is not possible, then the requests that belong to a series should be allocated approximately the same time. In our formulation, we require that all slots that belong to a series are allocated exactly at the same time they are requested. The turnaround, capacity, and unique scheduling constraints are modelled in a similar fashion to the model presented in Zografos et al. (2012).

4. Solving the bi-objective slot scheduling model

The ε-constraint method (Marler and Arora, 2004; Ehrgott, 2006) is adopted to solve the bi-objective formulation presented in Section 3. In order to use the ε-constraint method the fairness objective is transformed into the following ε-constraint:

$$\left|\rho_a - \frac{\sum_{a' \in A} \rho_{a'}}{|A|}\right| \leqslant \varepsilon, \quad \forall\, a \in A \tag{11}$$

The left-hand side of constraint (11) represents the allowable deviation of the fairness associated with the schedule of airline (a) from the average (overall airlines that have requested the same type of slots) fairness. Constraint (11) is nonlinear, as the denominator of ρ_a contains binary decision variable x_m^t. This constraint is linearized by multiplying by the total displacement expression (i.e. $\sum_{m \in M} \sum_{t \in T} |t - t_m|\, x_m^t$) on both sides of Eq. (11). This operation is valid when the total displacement is non-zero. This assumption holds for all congested airports (otherwise the bi-objective problem is trivial). The linear form of constraint (11) is given by Eq. (12).

$$\left|\frac{\sum_{m \in M_a} \sum_{t \in T} |t - t_m|\, x_m^t}{\frac{|M_a|}{\sum_{a \in A} |M_a|}} - \frac{\sum_{a \in A} \frac{\sum_{m \in M_a} \sum_{t \in T} |t - t_m|\, x_m^t}{\frac{|M_a|}{\sum_{a \in A} |M_a|}}}{|A|}\right| \leqslant \varepsilon \cdot \sum_{m \in M} \sum_{t \in T} |t - t_m|\, x_m^t, \quad \forall\, a \in A. \tag{12}$$

Thereby, the original bi-objective model is reformulated as follows:

$$\min_{\mathbf{x} = [x_m^t]} Z(\mathbf{x}, \varepsilon) = \sum_{a \in A} \sum_{m \in M_\alpha} f_m \tag{13}$$

Subject to:

Eqs. (7)–(10) and (12).

The objective function minimizes the total displacement under different values of the parameter (ε). The value of (ε) indeed represents the maximum allowable difference between the fairness value of a given airline and the average fairness value of all airlines.

4.1. Consideration of historical slot usage rights: hierarchical vs simultaneous slot scheduling

For the model discussed so far, we define two alternative cases which reflect two different slot scheduling policies. The hierarchical case (h) takes into consideration the current IATA Worldwide Scheduling Guidelines (IATA, 2017b), which give higher priority to the satisfaction of slot requests of airlines that have historical slot usage rights as compared to requests made by new entrant and other airline operators. The hierarchical satisfaction of slot requests allows us to decompose the slot scheduling problem into three interrelated problems (one problem for each level of the hierarchy, i.e. historic, new entrant, and other), which are solved sequentially. The sequential solution of the three problems implies that the solution to the first problem of the hierarchy acts as a constraint for defining the feasible region for the solution to the second problem. Similarly, the solution to the second problem acts as a constraint for defining the feasible region for the solution to the third problem. The non-hierarchical case (nh) satisfies all slot requests with the same priority. In this case, the problem is solved by considering all slot requests simultaneously.

4.2. An integrated ε-constraint and row generation approach for solving the bi-objective model

In order to solve the slot scheduling problem, we first select the case that we would like to consider, i.e. hierarchical vs non-hierarchical (see Fig. 2). The next step of the proposed solution framework is to define the initial value of (ε) that will be used in the right-hand side of constraints (12). The selection of (ε) should be made carefully. If the value of (ε) is too high, the fairness constraint would not be bounded, and the results would be identical to those resulting from a model that does not consider fairness. If the value of (ε) is too small, the problem may not have a feasible solution. The initial (ε) value is determined via the following procedure: Initially, the single objective model (3), (7), (8)–(10) that minimizes the total displacement is solved. The results obtained from the solution of the single object model are substituted into Eq. (4) to calculate the fairness metric for each airline. Subsequently, we are substituting the calculated fairness metric into Eq. (6) to calculate the value of the fairness objective. The calculated value of the fairness objective is set as the initial (ε) value. The proposed approach ensures that the solution to the single objective problem is a feasible solution to the bi-objective problem and the initial value of (ε) is tight. To update (ε), we iteratively reduce the value of (ε) by a predefined step size Δ and obtain (ε^I) (the (ε) value adopted in iteration I). For each (ε^I), the ε-constraint model is solved, and the objective values obtained are stored for determining the efficient frontier. The algorithm terminates when no feasible solution can be found for an (ε^I).

To improve the computational speed for solving the resulting slot scheduling model, we are implementing a row generation procedure (Fig. 3) that iteratively solves the model expressed by (13), (7)–(10) and (12). The first step of the row generation procedure (Zografos et al., 2012), is to build a basic integer programming (IP) model using the efficiency objective and constraints (9) and (10). After solving the basic model (i.e. the model that includes only definitional constraints), the satisfaction of constraints is checked. If any capacity, turnaround, or fairness constraint is violated, the violated constraint is added to the model, and the augmented model is solved. The solution algorithm terminates when all constraints are satisfied.

The algorithm was coded in C++ and compiled using Microsoft Visual Studio 2013. The ε-constraint model was solved by CPLEX 12.62 and the optimality gap is set to be 0.1. All tests were run on a desktop with 32 GB RAM and Intel(R) Xeon(R) CPU E5264 V3@ 2.6 GHz.

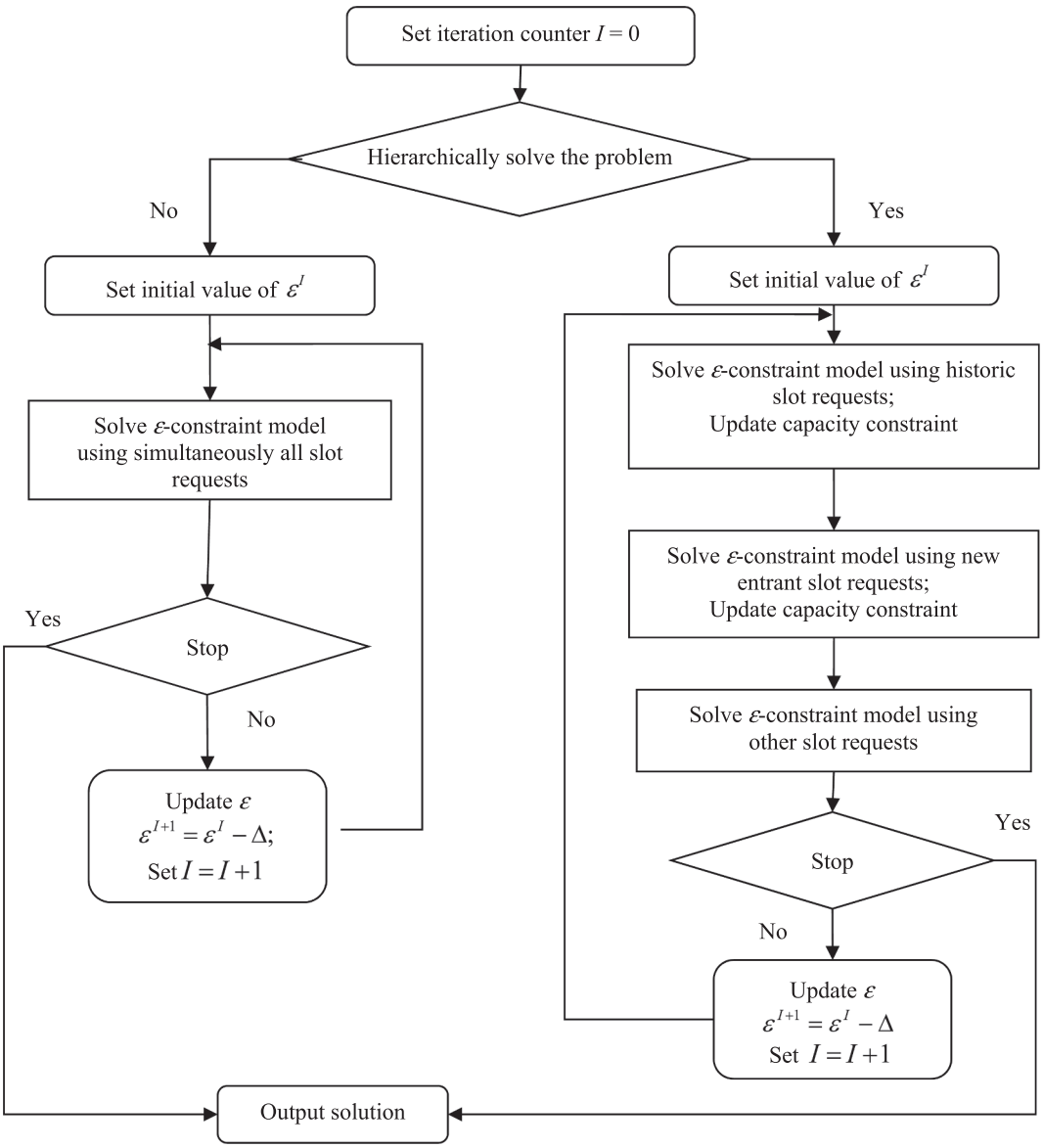

Fig. 2. Flowchart of the overall solution algorithm.

The graph presented in Fig. 4, compares the total computation time between the hierarchical (h) and non-hierarchical (nh) slot scheduling policies for different values of ε. The notation b/d/f/h and b/d/f/nh in Fig. 4 signifies the bi-objective (b), displacement (d), fairness (f) model solved hierarchically (h) and non-hierarchically (nh) respectively. Fig. 4 suggests that for both the non-hierarchical (blue[4] line) and the hierarchical (red line) the computational time increases significantly as the value of ε decreases (i.e. the problem becomes more constraint). For instance, in our experiments (reported in Section 5), the computational time increased from 729 s to 3 h when analyzing the simultaneous (non-hierarchical) slot scheduling policy, while in the case of the hierarchical policy runs, the computational time ranged between 142 s and 11 h. Please note that large values of ε, e.g. $\varepsilon = 3$ mean that the fairness objective (expressed as constraint (12) in the proposed bi-objective model) is not binding and practically the solution of this model corresponds to the solution of the single objective model that optimizes only the total displacement. Small values of ε, e.g. $\varepsilon = 1$, correspond to much tighter fairness constraints and to a much more constraint feasible region for the bi-objective problem, which is translated to more extensive computational time due to the difficulty associated with the identification of feasible solutions. Fig. 4 also suggests that the computational requirements of the hierarchical model are much more intensive as compared to the computational requirements of the non-hierarchical model when the fairness constraint becomes very tight ($\varepsilon = 1$).

[4] For interpretation of color in Fig. 4, the reader is referred to the web version of this article.

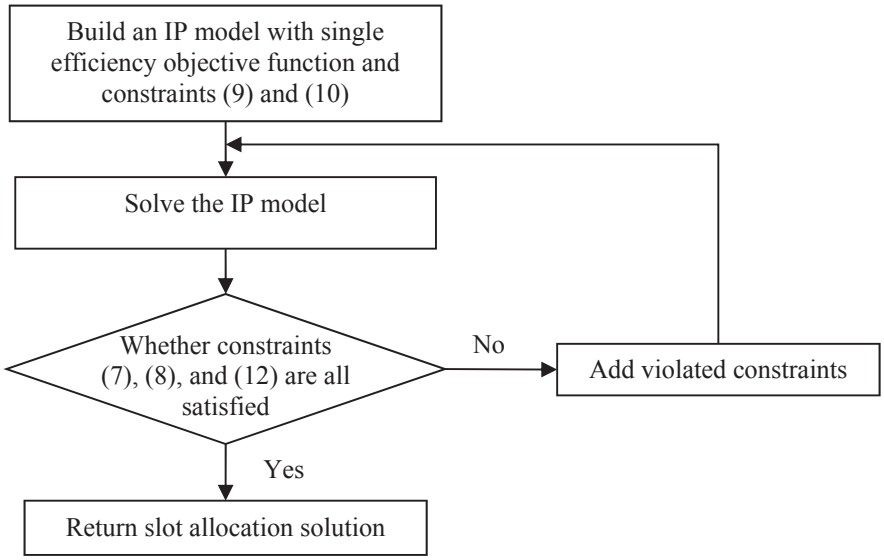

Fig. 3. Flowchart of the row generation procedure.

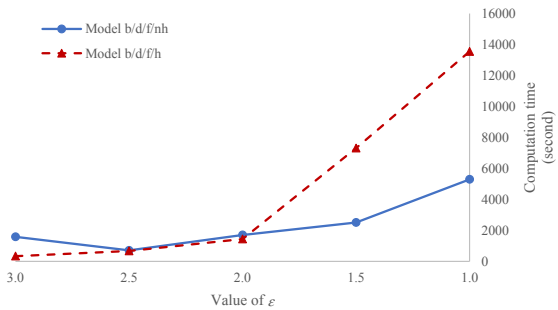

Fig. 4. The effect of the value of ε on computational performance.

Table 1
Declared capacity values for the airport under consideration.

Movement type	Duration (min)	Capacity (No. of movements)
Total movement	15	3
Total movement	60	10
Arrivals	60	4
Departures	60	6

As illustrated in Fig. 4, the computational times for developing the efficient frontiers of the instances of the bi-objective model reported in Section 5 are rather high. However, these computational times are not prohibitive for the decision making problem at hand, as the slot scheduling decisions under consideration are of strategic nature and are made twice per year (once per scheduling season). However, when the proposed model will be used to allocate slots at large airports, the computational time increases significantly. Therefore, future research efforts should address this issue through the development of efficient heuristics aiming to improve the computational efficiency.

5. Model application and results

The model presented in Section 3 was applied to data resembling real world conditions of a Coordinated Airport. The inputs required by the proposed model include the airport declared capacity, the slot request data, and the turnaround time. The airport declared capacity data for different time intervals are provided in Table 1.

Each slot request includes the code of the airline requesting the slot, the calendar day(s) for which the slot is requested, the requested arrival and departure slot times, the allocated arrival and departure slot times, and aircraft type. The number of allocated

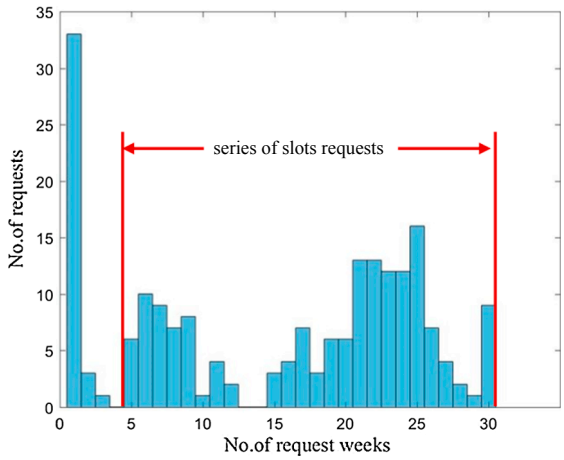

Fig. 5. Distribution of the frequency of historic requests by number of requested weeks.

slot requests for the historic, new entrant, and other categories were 202, 23 and 214 respectively. In our analyses, we are considering all series of slot requests for the summer scheduling season. In our modelling and solution approach the request of a series of slots is denoted, in Eqs. (6)–(10), by a single variable x_m^t representing the entire series. Through this process it is guaranteed that the request of a series of slots is allocated at the same time of the requested day. Fig. 5, illustrates the distribution of historic slot requests according to the number of weeks they have been requested and identifies the requests that have been made for the same day and time for more than five weeks (series of slots) for the scheduling period under consideration. For instance one can observe that 9 requests were made for the same day of the week and time of day for 6 weeks, or that around 35 requests were made for less than 5 weeks and therefore they do not belong to a series of slots.

5.1. Investigating the slot scheduling efficiency-fairness trade-off

In this section, we discuss the results of the proposed bi-objective efficiency-fairness model when it is solved hierarchically (b/d/f/h). The efficient frontiers for each slot priority type are shown in Fig. 6. For each graph shown in Fig. 6, the two end-points of the efficient frontiers, upper left-hand side and lower right-hand side points, correspond to the optimum value (minimum) of total displacement and the optimum value of the fairness objective respectively. In the brackets the first number represents the total displacement (efficiency), while the second number represents the value of the fairness objective. In fact, the points corresponding to the optimum solution of the single objective problem that minimizes total displacement (upper leftmost point) in the three graphs coincide with the solution of the single objective model that minimizes the total displacement.

For all three types of slot requests, we observe that any improvement (reduction) of the displacement (efficiency) objective results in deterioration of the fairness objective. Please recall that the value of (ε) represents the maximum allowable difference between the fairness values of a given airline from the average fairness value of all airlines. Therefore, higher values of the fairness objective represent lower levels of fairness. For all three types of slots, sacrifices in fairness yield gains in terms of total displacement (e.g. moving from the rightmost point of each graph to the next point to its left). However, the fairness-efficiency trade-off is not the same for all three slot categories. In comparing the three graphs of Fig. 6, it can be inferred that the range of the fairness values varies significantly among the three categories of slot requests. Specifically, the historic requests present a smaller range of fairness values while the values of fairness of the historic slot requests are smaller (please recall that smaller values of fairness represent a fairer scheduling of slots). Therefore, one can infer that it is much easier to achieve a fairer scheduling among the historic slot requests as compared to the scheduling of new entrant and other requests. Using the graphs of Fig. 6 we can calculate, for each type of slot requests, the price of fairness $P(\varepsilon)$ at a given fairness level (ε) according to Eq. (14):

$$P(\varepsilon) = \frac{D(\varepsilon) - D^*}{D^*} \tag{14}$$

where:

$D(\varepsilon)$ = value of displacement at fairness level (ε)
D^* = optimum displacement value

The price of fairness defined by Eq. (14) derives from the definition of fairness introduced in Bertsimas et al. (2011a, 2011b). The price of fairness represents the additional displacement, as a percent of the optimum displacement, encountered in order to achieve a given level of fairness. For instance, if we would like to calculate the price of achieving the optimum fairness for all types of slots ($\varepsilon = 0.7$, Fig. 6), the price of fairness for the historic requests will be $(144 - 122)/122 = 0.18$, while for the new entrant and other requests will be respectively $(176 - 80)/80 = 1.20$ and $(1799 - 1057)/1057 = 0.70$. These results suggest that it is much less

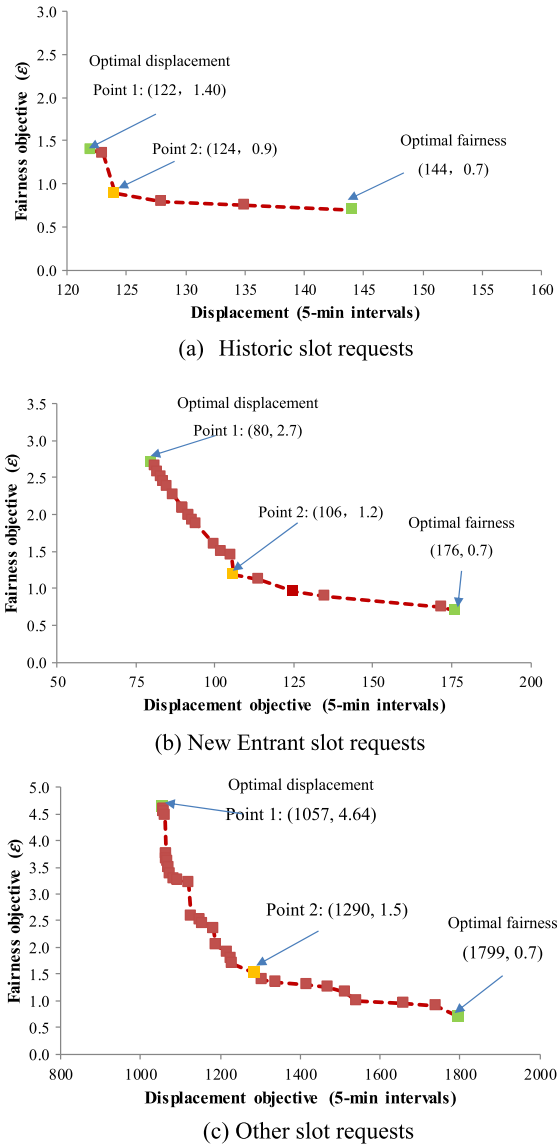

Fig. 6. Efficient frontiers produced by the proposed bi-objective model for the hierarchical slot allocation policy.

expensive to improve fairness among the historic requests, i.e. we need to increase the optimum displacement by 18% in order to achieve the best possible fairness for all historic requests, as compared to the new entrant and other requests for which the corresponding increase is 120% and 70% respectively.

The results of the bi-objective model, shown in Fig. 6, provide useful support for understanding the efficiency-fairness trade-offs involved in the slot allocation process. The efficient frontiers produced by the proposed model can facilitate discussions among different stakeholders, e.g. airlines, slot coordinators, and airports, in order to reach a commonly agreed solution. Most importantly, the efficient frontier produced by the proposed model can be used to avoid dominated (inferior) slot scheduling outcomes. Furthermore, the information provided by the price of fairness enhances the transparency of the decision making process as all stakeholders are informed about the impact (in terms of additional displacement cost) that slot scheduling decisions, that incorporate different fairness levels, may have on them.

Although the aggregate view of the results (i.e. the consideration of total displacement and fairness values) provides a good indication regarding the overall slot scheduling performance they do not shed light on the effect of the different schedules (resulting to different points of the efficient frontier) on each airline and type of slot request. Therefore, it is useful to look into the slot scheduling results at a disaggregate (individual airline, and type of request) level.

In what follows we are presenting an example of the information that can be derived from the disaggregate view of the results of our model. The major underlying assumption of the fairness objective proposed in this paper is that, for each slot category, the total displacement allocated to the slot requests of a given airline should be proportional to the slot requests that the airline has placed.

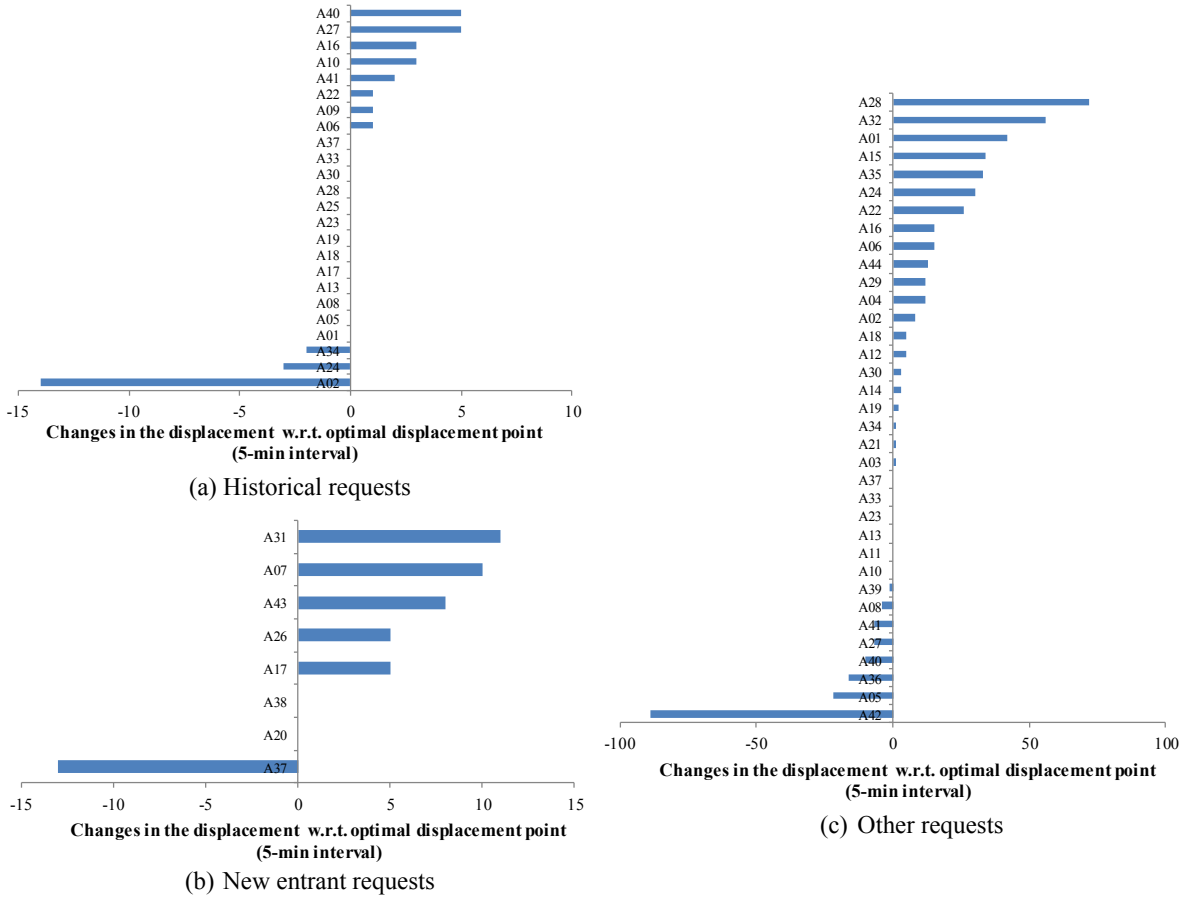

Fig. 7. Changes in displacement of knee point with respect to the optimal displacement point of model b/d/f/h for all types of slot requests.

The disaggregate analysis results are presented in Fig. 7. This figure presents the value of displacement that each airline is gaining, (reduction of displacement), or losing (increase in displacement), when one compares schedules corresponding to alternative points of the efficiency-fairness curve. In the example depicted in Fig. 7 we are comparing, for all types of slot requests, the differences in displacement resulting between the airport schedules corresponding to point 1 and point 2 of the efficiency-fairness trade-off curve depicted in Fig. 6. Point 1 represents the optimum displacement schedule, while point 2 represents the schedule resulting when the point indicating the knee-of-the curve (i.e the point beyond which there are diminishing fairness returns) is considered. The changes in displacement for each airline and type of slot request have been calculated by subtracting the displacements corresponding to point 1 (optimum displacement) from the displacements corresponding to point 2 (knee-of-the curve). Positive values of the change in displacements, for each type of slot request, indicate that the corresponding airlines experience larger schedule displacement while negative values indicate that the corresponding airlines experience smaller schedule displacement. Thus, for the case under consideration we observe that for the historic slot requests (Fig. 7a) seven airlines will experience larger schedule displacement; when a schedule with better fairness performance will be selected (point 2) as compared to the optimum displacement schedule (point 1), while three airlines will experience smaller displacement. Apparently the airline with the largest gain in this scenario is airline A02 which experiences the largest reduction in total displacement.

In fact, various comparisons can be carried out using different values of ε. Accordingly, the winner or loser status of each airline could change. The information provided by Fig. 7 is very useful when slot scheduling decisions have to be made since it clearly presents to each stakeholder (airline) the gains or sacrifices it has to make when a decision on an acceptable fairness level has to be made. Therefore, the provision of disaggregate information enhances transparency of slot scheduling decisions.

5.2. Effect of the simultaneous consideration of slot requests on the efficiency-fairness trade-off

The results discussed so far, are based on the basic assumption that the allocation of slots is based on a regime that considers historical slot usage rights in scheduling slots (hierarchical slot scheduling). According to this regime historic slot requests are scheduled first, followed by the allocation of the new entrant, and other slot requests (IATA, 2017b). An alternative way of modelling and solving the slot scheduling problem is to consider all slot requests simultaneously.

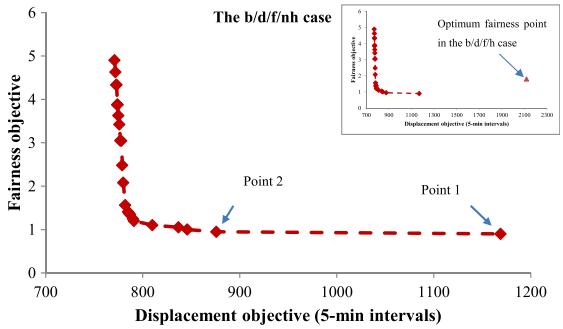

Fig. 8. Efficient frontier for the b/d/f/nh case.

Table 2
Comparing the optimum fairness points between the b/d/f/h case and the b/d/f/nh case.

	The b/d/f/h case		The b/d/f/nh case (Point 1 in Fig. 9)	
	Displacement objective	Fairness Objective	Displacement objective	Fairness objective
Historic	144		579	
New Entrant	176		61	
Other	1799		529	
Total	2119	1.80	1169	0.90

The trade-off between total displacement and fairness for the case that considers simultaneously all slot requests (b/d/f/nh) is shown in Fig. 8. The graph presented in Fig. 8, was generated by solving the proposed bi-objective model (see Section 4) by considering all slot requests simultaneously for different ε values. Point 1 (see Fig. 8) represents the solution with the optimum fairness ($\varepsilon = 0.9$) and maximum total displacement, while the leftmost point on the displacement-fairness curve represents the optimum displacement and minimum fairness point. Please recall that higher values of the fairness objective represent lower levels of fairness. The price of fairness for point 1 is (1169–771)/771 = 0.52, while the price of fairness for point 2 (see Fig. 8) is (876–771)/771 = 0.14. These findings suggest that moving from the optimum displacement point to point 2, fairness can be improved substantially with small displacement (efficiency) sacrifice. It is also worth noting, that moving from point 2 to point 1 significant sacrifice of the displacement objective is required to achieve small fairness gain.

The comparison of the optimum fairness points between the bi-objective hierarchical and non-hierarchical models is summarised in Table 2. The displacement values for each request type of the hierarchical case have been obtained from the optimum fairness points in Fig. 6. The total displacement for the b/d/f/h case is calculated as the sum of the displacements of all three request types. The total fairness value is calculated by calculating the fairness metric (ρ_a) for each airline and substituting the calculated fairness metric values into the fairness objective in Eq. (6). The fairness metric for each airline is calculated according to Eq. (4). The total displacement value used in this equation is $D = 2119$ (see Table 2). For the non-hierarchical case, the overall fairness is improved by 50%, while the total displacement is also improved by almost 45%. Furthermore it is clearly shown in Table 2, that the overall displacement reduction is due to the dramatic reduction of the displacement of the new entrant and other categories. It is also worth pointing out that the resulting improvements are at the expense of the significantly higher displacement of the historic slot requests.

Fig. 9 presents the disaggregate analysis of the results of the comparison between the non-hierarchical case (b/d/f/nh) and hierarchical (b/d/f/h) of the bi-objective slot scheduling problem. In Fig. 9, we have plotted the value of the displacement that each airline is gaining (reduction of displacement) or losing (increase in displacement) as compared to the hierarchical case. The comparisons presented in Fig. 9 are based on the results of displacement when a comparable level of fairness can be achieved between the hierarchical and non-hierarchical case ($\varepsilon = 1.80$). Various comparisons in terms of the winning and losing status of the identified airlines can be performed for different levels of fairness sought. For the case considered in the example presented in Fig. 9, we observe that the majority of the historical requests encounter significantly larger displacements, while the new entrant and other requests encounter significant gains.

6. Conclusions

In this paper, we have developed and solved a bi-objective slot scheduling model which allows the investigation of the trade-off between efficiency and fairness. The fairness objective used is based on a proportionality metric that postulates that the total schedule displacement of an airline should be proportional to the number of slots it has requested. A solution framework integrating the ε-constraint method with row generation was used to solve the proposed bi-objective model. Two slot scheduling regimes were investigated, namely, hierarchical and non-hierarchical. The results from the application of the proposed model, for the specific case analysed, suggest that: (i) for the hierarchical slot scheduling the price of fairness for new entrant and other requests is higher than that of the historic requests, (ii) the non-hierarchical scheduling improves both total displacement and fairness. However, it should be

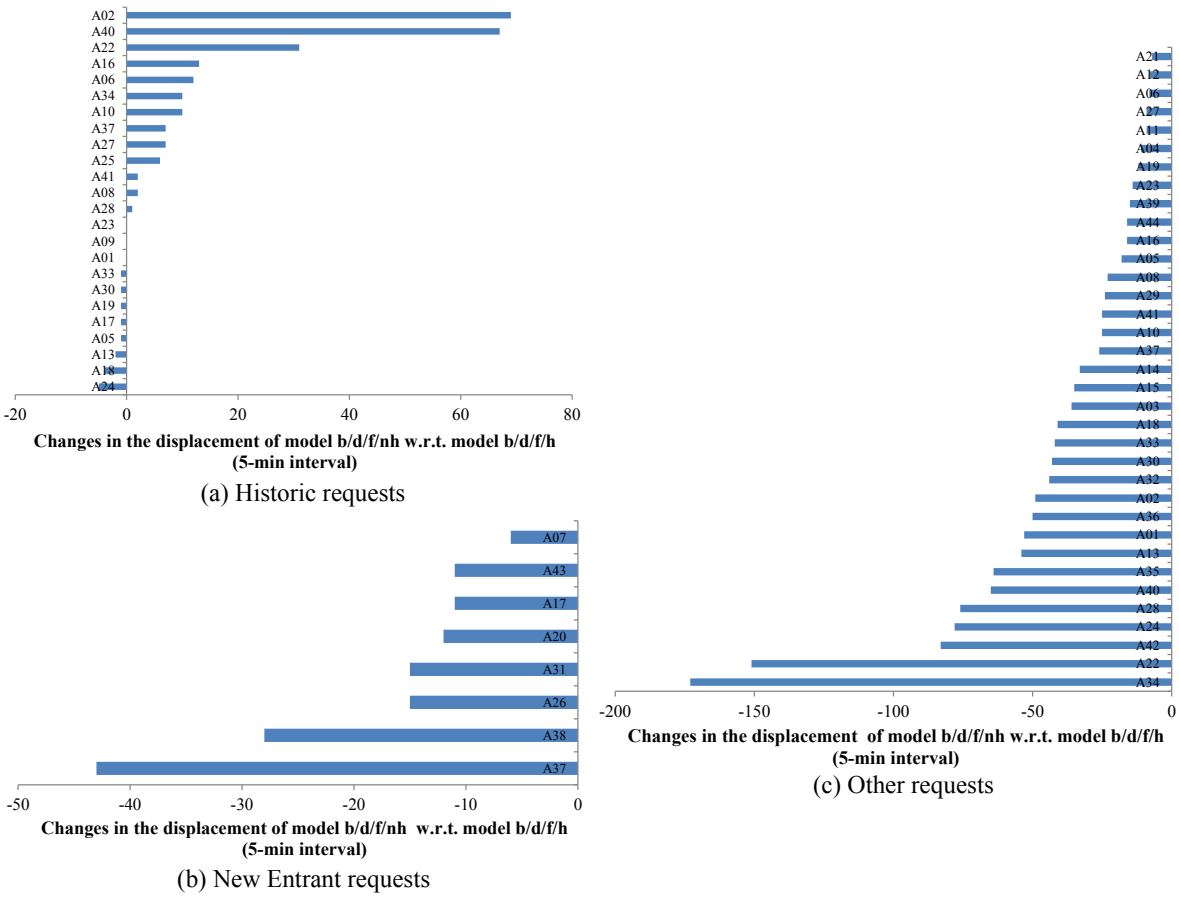

Fig. 9. Changes in displacement of model b/d/f/nh with respect to model b/d/f/h for all types of slot requests.

stressed that the total displacement improvement is on the expense of the displacement increase of the historic slot requests. The disaggregate presentation of the results provides information regarding the gains/losses of each stakeholder and enhances the transparency of the slot scheduling decisions.

The efficient frontiers resulting from the solution of the bi-objective model shed light in the efficiency-fairness trade-off for all types of slots, i.e historic, new entrant, and other. The generated efficient frontiers inform decision makers as to how much they should sacrifice (increase) in total displacement in order to achieve a desirable level of fairness. The fairness metric used discourages the artificial increase of slot requests, since it allocates displacement proportionally to the number of the requested slots. This means that more total displacement will be allocated to stakeholders that are placing more slot requests.

The experience derived by the application of the models presented in this paper suggests that the slot scheduling problem by its nature is multi-object and multi-stakeholder. Therefore, for the same demand profile of slot requests and the same level of airport declared capacity one can produce a wide array of alternative non-inferior schedules depending on the preference structure of different stakeholders. Furthermore, in order to achieve a commonly acceptable solution it is necessary to be able to explore a wide range of alternatives and to effectively communicate these results to the decision makers in order to understand the impact of the different choices offered by the efficiency-fairness trade-off curve on the displacement of the requests of each airline. In line with the decisions making characteristics identified above, the proposed bi-objective model may be included in a slot scheduling Decision Support System (DSS).

In the course of our research, we have identified a number of issues that provide a fertile ground for future research. For instance, the definition of the fairness metric makes the implicit assumption that the economic costs associated with the displacement of each flight is the same for all flights of the same airline, and that it does not differ among airlines. However it should be recognized, that depending on their business model, network structure, etc., different airlines may encounter completely different costs as a consequence of the displacement of their flights. This issue becomes more perplexed when considering that the economic value and importance of the displacement of each flight is very sensitive proprietary information which is not disclosed by airlines. Therefore, it is important to investigate how these differences in displacement costs among the flights of different airlines can be taken into account in formulating future slot allocation models.

Research work underway is focusing on: (i) the development of efficient exact and heuristic algorithms that can be used to decrease the computational time needed to solve the single airport bi-objective slot allocation problem, (ii) the development of a

Decision Support System for airport slot allocation, (iii) the comparison of alternative fairness measures proposed in the literature, (iv) the development of a slot allocation mechanism which will use both efficiency and fairness criteria

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