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Analysis and Comparison of Push-Pull Class-E Inverters with Magnetic Integration for Megahertz Wireless Power Transfer

Xiaosheng Huang, Yipeng Kong, Ziwei Ouyang, Senior Member, IEEE, Wei Chen, and Shuyi Lin

Abstract—This paper presents the circuit design and magnetic integration of push-pull class-E inverters for wireless power transfer (WPT) up to megahertz. The design criterion for achieving ZVS of a class-E inverter with coupled windings is derived mathematically. The approaches of magnetic integration for push-pull class-E inverters are analyzed and compared. Then, a new magnetic structure with hybrid magnetic materials is proposed to build the integrated inductors with either coupled windings or uncoupled windings. A 3 MHz WPT system is built to verify the analysis. The detailed comparison of the class-E inverters with magnetic integration is presented in terms of switch voltage, efficiency, harmonic currents and thermal distribution. In the optimized design example, the switches keep ZVS over the entire load range without using any closed-loop control. The system efficiency reaches 87.1% at 350 W output power.

Index Terms—wireless power transfer, class-E, integrated magnetic, inverter, rectifier

NOMENCLATURE

$\nu_S, \nu_{S1}, \nu_{S2}$ Switch voltages.

$\omega$ Angular frequency.

$R$ Equivalent load resistance of inverters.

$i_R$ Output current of the inverter.

$I_R$ Amplitude of $i_R$.

$\varphi$ Initial phase of $i_R$.

$Q_L = \omega L_0/R$, loaded quality factor.

$L_{in}$ Inductance of winding $L_{in1}$ and $L_{in2}$.

$L_c$ Mutual inductance of the coupled windings.

$L_f$ Leakage inductance of the coupled windings.

$k_{in}$ Coupling coefficient of the coupled windings.

$i_{in1}, i_{in2}$ Winding current.

$I_{DC}$ DC component, i.e., DC bias of $i_{in1}, i_{in2}$.

$i_{AC}$ AC component of $i_{in1}, i_{in2}$.

$C_f$ capacitance of $C_{in}, C_{in}$.

$L_e = 2L_f$, equivalent resonant inductance for the coupled windings.

$V_{in}$ Input DC voltage.

$q_e = 1/\omega^2 L_c C_f$, the normalized resonance frequency of $L_c$ and $C_f$.

$p_e = I_R \omega L_e/V_{in}$, the normalized output current.

$p_{re} = R/\omega L_e$, the $R$ normalized by the reactance of $L_e$.

$\nu_{SN}$ = $\nu_S/V_{in}$ the normalized switch voltage.

$I_{SN}$ Initial value of $i_{in1}$ at $\omega t = \pi$.

$q_{SN} = I_{SN} \omega L_e/V_{in}$, the normalize $I_{SN}$.

$q = 1/\omega \sqrt{L_f C_f} = \sqrt{2q_e}$, the normalized resonance frequency of $L_f$ and $C_f$.

$p = I_R \omega L_f/V_{in} = 1/2\pi$, the normalized output current.

$p_r = R/\omega L_f = 2p_{re}$, the $R$ normalized by the reactance of $L_f$.

$L_{decp}$ = $L_{in}$, resonant inductance for the uncoupled windings.

$L_{cpl}$ = $L_f$, resonant inductance for the coupled windings.

I. INTRODUCTION

Wireless power transfer (WPT) based on magnetic coupling is increasingly applied in portable devices, medical implants, electric vehicles and so on [1]–[5]. In most common situations, increasing the frequency is conducive to reduce the size and extend the transfer distance, but it is quite challenging to design a high-efficiency inverter for the WPT up to megahertz. The efficiency deteriorates because of the substantial increase in the losses of switches and magnetic components.

A class-E inverter with finite DC-feed inductance is often used for the WPT of which the load varies in a wide range. The switch can naturally achieve zero-voltage-switching (ZVS) without using closed-loop control, but this expected feature is essentially based on using much lower input inductance [6]–[8]. It results in a large AC current through the input inductor and the DC source. The current causes not only additional losses but also conducted electromagnetic interference (EMI).

Interleaving two class-E inverters is effective for suppressing the odd-order harmonics through the DC source. The combined topology is known as push-pull class-E inverter [9], [10], differential-mode class-E inverter [11]. The even-order harmonics remaining can also be reduced by coupling the input inductors of the interleaved inverters. Hence, the ripple current through the DC source can be suppressed significantly to reduce EMI noise. This approach was introduced in [12], but the design criterion to achieve ZVS was still unclear.

In the push-pull class-E inverter with finite DC-feed inductance, the amplitude of the AC current through the input

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inductors is kept relatively large to maintain ZVS when the load varies. For loss reduction, a possible approach is reducing the amplitude by an adjustable input voltage, but it requires an additional DC-DC converter [13]. The other approach is reducing the ESR (equivalent series resistance) of the inductors by using a larger size of the magnetic elements. Consequently, individual inductors with air-core or low-permeability cores are usually applied in the class-E inverters. The inductor size is relatively big and difficult to be reduced.

The design methodologies of the inductors of class-E inverters are basically the same as conventional power converters. [14] proposes a detailed design method of choke inductors for parallel-capacitor class-E inverters, but it cannot be used for the inductor with a large AC current. [15] introduces a design method to reduce the volume of inductors with significant AC current. However, by now, most existing literatures about class-E inverters focus on the circuit design and closed-loop control, but not the magnetic design [6], [8], [16]–[21].

Magnetic integration methods, particularly coupling multiple inductors, have been proved highly effective for improving performance and reducing the size of power converters [22]–[24]. When the two inductors of a push-pull class-E inverter are integrated, the circuit board area occupied by the inductors can be reduced directly. Moreover, the class-E inverter presents improved harmonic suppression by coupling the inductor windings. However, so far, few studies have discussed the magnetic integration of class-E inverter.

This paper presents the circuit design and magnetic integration of push-pull class-E inverters for the WPT up to megahertz. The circuit analysis, design and comparison of the class-E inverters with either coupled or uncoupled windings are first presented in section II. It shows that the even-order harmonic currents through the integrated inductor are suppressed by coupling the windings. Then, a design criterion to satisfy ZVS of the push-pull class-E inverter with coupled windings is proposed. In section III, the approaches of magnetic integration are analyzed and compared. A new magnetic structure, which consists of hybrid materials with different permeability, is proposed to reduce the number of gaps and simplify the magnetic design. The structure is implemented for the two kinds of integrated inductors with either coupled or uncoupled windings. In section IV, a 3 MHz WPT system is built, and the topologies of the class-E inverters are also applied for rectification. A detailed comparison (regarding efficiency, harmonic current and temperature rise) for the different integrated inductors is presented.

II. CIRCUIT DESIGN

As in Fig.1, the windings of the integrated inductor can be either coupled or uncoupled, thus the circuit design is different for the cases. If the windings are uncoupled, i.e., $k_{in} = 0$, the circuit design is the same as a single-switch class-E inverter and has been well studied in existing literature [6], [8]. Note that the current amplitude of the inductor windings is kept large to satisfy ZVS when the load resistance varies from an optimal value to infinity. Since the switches are driven by complementary signals, the inverter operates in differential mode. Hence, the fundamental and odd-order harmonic currents circulate in the inverter, while the even-order ones can pass through the DC source.

For the other case, i.e., $k_{in} \neq 0$, the ripple current of DC source can be suppressed by coupling the windings. This topology has been proposed in [12], but its design criteria to achieve ZVS are still unknown. Therefore, in this section, the analysis focus on the inverter with coupled windings.

A. Switch Voltage for Coupled Windings

For the topology with coupled windings, the designed inductance for different order harmonics can be separated. The inductance for even-order harmonic currents is equal to the self-inductance of the windings. On the other hand, the inductance for fundamental and odd-order harmonic currents is equal to the leakage inductance. Therefore, it is an essential principle that increasing the self-inductance for ripple suppression and keeping a particular leakage inductance to satisfy ZVS. Hence, the ideal integrated inductor with coupling windings acts like a transformer with particular leakage inductance. Since the switching behavior is changed by the coupled windings, it requires a new design criterion to achieve ZVS as well as conventional class-E inverters. To simplify the analysis below, some assumptions are made as follows.

1) The switches have zero on-resistance and infinite off-resistance. The switch $S_1$ is on for $0 < \omega t \leq \pi$ and off for $\pi < \omega t \leq 2\pi$. In reverse, $S_2$ is off for $0 < \omega t \leq \pi$ and on for $\pi < \omega t \leq 2\pi$. Hence, the switch voltages satisfy

$$v_s(\omega t) = v_{s1}(\omega t) = v_{s2}(\omega t + \pi)$$

2) The equivalent series resistance of passive components is ignored.

3) $Q_L$ is high enough to make the output current $i_R$ a purely sinusoidal wave expressed as

$$i_R(\omega t) = I_R \cdot \sin(\omega t + \phi)$$

The $L_o$ and $C_o$ resonate at the operation frequency, i.e., $\omega = 1/\sqrt{L_oC_o}$.

4) The mutual inductance and leakage inductance, i.e.,

$$L_c = k_{in}L_{in}$$

Fig. 1. Push-pull class-E inverter with an integrated inductor. The windings ($L_{in1}$ and $L_{in2}$) can be either coupled or uncoupled.
The even-order harmonic currents can be eliminated theoretically. The winding currents are coupled and the self-inductance is large enough. Since the circuit units are symmetrical, the winding currents satisfy

\[ i_{\text{in1}}(\omega t) = I_{\text{DC}} + i_{L_f}(\omega t) \]

\[ i_{\text{in2}}(\omega t) = I_{\text{DC}} - i_{L_f}(\omega t) \]

(5)

(6)

Since the circuit units are symmetrical, the winding currents satisfy

\[ i_{L_f}(\omega t) = -i_{L_f}(\omega t + \pi), \]

(7)

which implies that there is no common-mode current if the windings are coupled and the self-inductance is large enough. The even-order harmonic currents can be eliminated theoretically.

Fig. 2a illustrates the equivalent circuit of the inverter when \( S_1 \) is off for \( \pi < \omega t \leq 2\pi \). The coupling coefficient \( k_c = 1 \). The value of \( k_d \) depends on the magnetic structure, and \( L_f = (1 + k_d)L_d \).

\[ L_f = (1 - k_{in})L_{in} \]

(4)

satisfy \( L_c \gg L_f \). Thus, the winding current can be decomposed by a direct current and an alternating current as

\[ i_{\text{in1}}(\omega t) = I_{\text{DC}} + i_{L_f}(\omega t) \]

\[ i_{\text{in2}}(\omega t) = I_{\text{DC}} - i_{L_f}(\omega t) \]

(5)

(6)

From (7) and (8), the charging current of \( C_f1 \) for \( \pi < \omega t \leq 2\pi \) satisfies

\[ \omega C_f \frac{dv_{S1}(\omega t)}{d\omega t} = \frac{-1}{\omega L_c} \int_{\pi}^{\omega t} v_S(\omega t_1) d\omega t_1 + i_{L_\pi} + i_R(\omega t) \]

(9)

which can be represented by a second-order differential form expressed as

\[ \frac{1}{q_e^2} \frac{d^2v_{S1}(\omega t)}{d\omega t^2} + v_{S1}(\omega t) - p_e \cdot \cos(\omega t + \varphi) = 0 \]

(10)

The general solution of equation (10) is

\[ v_{S1}(\omega t) = \xi_1 \cos(\omega t + \varphi) - \xi_2 \sin(\omega t + \varphi) \]

(11)

where \( \xi_1 \) and \( \xi_2 \) are the coefficients related to the circuit parameters of the inverter. Calculating the derivative of (11) as

\[ \frac{dv_{S1}(\omega t)}{d\omega t} = \xi_2 \cos(\omega t + \varphi) - \xi_1 \sin(\omega t + \varphi) + \xi_1 \sin(\omega t + \varphi) \]

(12)

From equation (9) and (12) at \( \omega t = \pi \), the initial conditions can be expressed by

\[ v_{S1}(\pi) = 0 \]

\[ \frac{dv_{S1}(\omega t)}{d\omega t} \bigg|_{\omega t=\pi} = q_e^2 p_{\pi \varphi} - q_e^2 p_e \sin(\varphi) \]

(13)

(14)

Solving equation (11) based on the initial conditions and resulting in the expressions of \( \xi_1 \) and \( \xi_2 \) given by

\[ \xi_1 = -\frac{\cos(\pi q_e) \cos(\varphi)}{q_e} + \frac{1}{q_e} \sin(\pi q_e) \sin(\varphi) \]

\[ -q_e \sin(\pi q_e) \cdot [p_{\pi \varphi} - p_e \sin(\varphi)] \]

\[ \xi_2 = -\frac{\sin(\pi q_e) \cos(\varphi) - \frac{1}{q_e} \cos(\pi q_e) \sin(\varphi)}{q_e} \]

\[ + q_e \cos(\pi q_e) \cdot [p_{\pi \varphi} - p_e \sin(\varphi)] \]

To get the switch voltage for a particular value of \( q_e \), it requires three constraint expressions for calculating the unknown state variables \( p_{\pi \varphi}, p_e, \varphi \). In this paper, the constraint conditions include a) Volt-second balance of windings; b) Energy conservation; c) Ohm’s law for the fundamental voltage and current. The equations are deduced as follows.

1) According to the volt-second balance principle, the off-state switch voltage \( v_{S1} \), defined in assumption 1) for \( \pi < \omega t \leq 2\pi \) satisfies

\[ 2\pi = \int_{\pi}^{2\pi} v_{S1}(\omega t_1) d\omega t_1. \]

(16)

Substituting (11) into (16), resulting in

\[ p_{\pi \varphi} = \frac{2}{\omega} \cdot \frac{\pi + \xi_2 \sin(\varphi)}{1 - \cos(\pi q_e)} + \frac{\xi_2 \cos(\varphi) \sin(\pi q_e)}{q_e (1 - \cos(\pi q_e))} \]

\[ \left( p_e - \frac{\xi_2}{q_e} \right) \sin(\varphi) \]

(17)

2) As the input voltage is constant, the input energy \( g_{in} \) per period is

\[ g_{in} = \frac{2\pi}{\omega} \cdot \frac{V_{in} I_{\text{DC}}}{2} = \frac{2}{\omega} \cdot \frac{V_{in} \cdot i_{\text{in1}}(\pi) + i_{\text{in1}}(2\pi)}{2} \]

(18)

From (5), (6), (7) and (12), getting

\[ \beta_{\text{1DC}} = \frac{2I_{\text{DC}}}{V_{in} \omega C_f} \]

\[ = q_e^2 p_{\pi \varphi} - q_e^2 p_e \sin(\varphi) + \frac{dv_{S1}(\omega t)}{d\omega t} \bigg|_{\omega t=2\pi} \]

(19)
The energy dissipation per period $q_{out}$ consists of the energy consumption of $R$ and the loss of $C_f$.

\[
\frac{q_{out}}{V_{in}^2C_f} = \pi q_e^2 p_e^2 + \frac{1}{2} v_{Sn}(2\pi)^2
\]

Combining (18), (19) and (20), the equation for energy conservation is

\[
\pi \cdot \beta_{1DC} = \pi q_e^2 p_e^2 + \frac{1}{2} v_{Sn}(2\pi)^2
\]

3) According to the symmetry shown in (5), (6) and (7). The input current $i_{in}$ can be normalized as

\[
\beta_{in}(\omega t) = \frac{i_{in}(\omega t)}{V_{in}\omega C_f}
\]

\[
= \begin{cases} \beta_{1DC} - \beta_{in}(\omega t + \pi) & 0 < \omega t < \pi \\ \frac{d\beta_{Sn}(\omega t)}{d\omega t} - q_e^2 p_e \sin(\omega t + \varphi) & \pi < \omega t < 2\pi \end{cases}
\]

From (9), the normalized switch current $i_S(\omega t)$ for $0 < \omega t < \pi$ is

\[
\beta_S(\omega t) = \frac{i_S(\omega t)}{V_{in}\omega C_f} = \beta_{in}(\omega t) - q_e^2 p_e \sin(\omega t + \varphi)
\]

At the switching-on instant without ZVS condition, $C_f$ is discharged through $S_1$. According to charge conservation, the transient integral can be theoretically given by

\[
\int_0^{0+} i_S(\omega t_1) d\omega t_1 = V_{in}\omega C_f + v_{Sn}(2\pi).
\]

The fundamental current flowing through the switch can be decomposed into two quadrature components. Since the inductance $L_c$ only affects the common-mode currents, the third constraint condition can be written as

\[
\int_0^\pi \beta_S(\omega t_1) \cos(\omega t_1 + \varphi) d\omega t_1 + v_{Sn}(2\pi) \cos(\varphi)
\]

\[
\int_0^\pi \beta_S(\omega t_1) \sin(\omega t_1 + \varphi) d\omega t_1 + v_{Sn}(2\pi) \sin(\varphi)
\]

\[
= \frac{R}{2} \left( \omega C_f - \frac{1}{\omega L_f} \right) = \left( \frac{1}{q_e^2} - 1 \right) p_r
\]

As a result, the three state-variables ($p_e$, $p_{re}$, $\varphi$) can be numerically solved out from the constraint equations with particular circuit parameters $q_e$ and $p_{re}$.

**B. Parameters for Satisfying ZVS and ZVDS Simultaneously**

The optimum condition of a class-E inverter is normally defined as satisfying ZVS and zero-voltage-derivative-switching (ZVDS) simultaneously. As the switching voltage and current are zero, the inverter can achieve minimum switching losses in practical systems. The optimum conditions can be expressed as

\[
v_{Sn}(2\pi) = 0
\]

\[
\frac{dv_{Sn}(\omega t)}{d\omega t} \bigg|_{\omega t=2\pi} = 0
\]

Solving the equation set formed by (16), (21), (25), (26) and (27), the optimum $q$ and $p_r$ for the inverter with coupled windings are

\[
q_{optm1} = 1.659, \quad p_{optm1} = 1.197.
\]

In the definition $q = 1/\omega \sqrt{L_f C_f}$ and $p_r = R/\omega L_f$ for the inverter with coupled windings, $L_f$ equals to the inductance for the differential-mode current (i.e., fundamental and odd-order harmonic currents).

In contrast, for the inverter with uncoupled windings, the definitions should be changed to $q_{trd} = 1/\omega \sqrt{L_{in} C_f}$ and $p_{trd} = R/\omega L_{in}$, where the inductance for the differential-mode current is $L_{in}$.

The optimum $q_{trd}$ and $p_{trd}$ can be calculated as a class-E inverter with finite DC-feed inductance. According to the calculation in [7]. The optimum circuit parameters are

\[
q_{optm2} = 1.412, \quad p_{optm2} = 1.364.
\]

Note that, $L_{in}$ is limited by the design criterion of the class-E inverter with finite DC-feed inductance. Therefore, using the uncoupled windings will not directly improve the suppression of odd-order harmonic currents. Contrarily, it may reduce the suppression since $q_{optm2}$ is slightly lower than $q_{optm1}$. As in Fig.3, there is no distinct difference between the switch voltages for the coupled and uncoupled windings, while the winding current amplitude is observably reduced by using the coupled windings.

**C. Parameters for Satisfying Only ZVS**

The selection of circuit parameters can be more flexible by relaxing the switching conditions to achieving only ZVS. Tab.1 is solved by removing (27) from the equation set and presetting $q$ values. The waveforms for different cases are illustrated in Fig.4. It shows that the peak switch voltage decreases with a lower $q$ value, but the winding current amplitude at $q = 1.45$
TABLE I
PARAMETER SETS FOR SATISFYING ZVS

<table>
<thead>
<tr>
<th>Cases</th>
<th>$q$</th>
<th>$p_r$</th>
<th>$p$</th>
<th>$\varphi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1.45</td>
<td>2.341</td>
<td>0.675</td>
<td>0.114</td>
</tr>
<tr>
<td>b</td>
<td>1.5</td>
<td>1.624</td>
<td>0.983</td>
<td>0.179</td>
</tr>
<tr>
<td>c</td>
<td>1.659</td>
<td>1.197</td>
<td>1.381</td>
<td>0.316</td>
</tr>
<tr>
<td>d</td>
<td>1.8</td>
<td>1.151</td>
<td>1.491</td>
<td>0.415</td>
</tr>
<tr>
<td>e</td>
<td>1.95</td>
<td>1.191</td>
<td>1.515</td>
<td>0.514</td>
</tr>
</tbody>
</table>

$L_f = \frac{R}{p_r \cdot \omega}$  \hspace{1cm} (31)

$C_f = \frac{1}{L_f \cdot \omega^2}$  \hspace{1cm} (32)

where $P_{out}$ is the output power of the inverter, $V_{in}$ is the DC input voltage. The designed $R$ is based on the rated output, thus the ZVS can be satisfied when the load resistance varies from its rated value to infinity.

III. MAGNETIC DESIGN FOR INTEGRATED INDUCTORS

Magnetic integration is commonly implemented by combining multiple inductors of a converter. The inductor windings of the inverter after integration can be either coupled or uncoupled. These different cases result in a significant change in the circuit design. As in Fig.1, the parallel capacitance $C_f$ resonates with the self-inductance $L_{in}$ if the windings are uncoupled. In contrast, if the windings are coupled, the $C_f$ resonates with the leakage inductance $L_f$. In this paper, both two kinds of integrated inductors are built and compared. We define the resonance inductance as $L_{decpl}$ and $L_{cpl}$ to highlight the difference between the cases.

A. Integrated inductor with uncoupled windings

Designing the integrated inductor with uncoupled windings is basically the same as that for a class-E inverter with finite DC-feed inductance. The winding inductance $L_{decpl}$ can be expressed as

$L_{decpl} = L_{in} = N^2 \frac{\mu_0 A_e}{l_e}$  \hspace{1cm} (33)

$l_e = \frac{l_{core}}{\mu_r} + l_{gap}$  \hspace{1cm} (34)

where $N$ is the number of turns, $\mu_0 = 4\pi \cdot 10^{-7}$ H/m, $A_e$ is the effective cross-sectional area of the magnetic core, $l_{core}$ is the effective magnetic path length of the core, $\mu_r$ is the relative permeability of the core, $l_{gap}$ is the gap length, $l_e$ is the effective magnetic path length of $L_{decpl}$.

Fig.6 illustrates the magnetic integration for the uncoupled windings. The windings share the central column after integration. The even-symmetric common-mode fluxes are unchanged in the central column. In contrast, the odd-symmetric differential-mode ones are canceled. Therefore, the core loss can be slightly reduced by suppressing the flux through the shared column.

For size reduction, the integrated method can also reduce $A_e$ of the central column based on the same losses of individual inductors.

B. Integrated inductor with coupled windings

As mentioned in subsection II.A, the integrated inductor with coupled windings acts like a transformer with designed leakage-inductance $L_f$ and mutual-inductance $L_c$. The inductance for the even-order harmonic currents is $L_f + L_c$, while the inductance for the fundamental and odd-order harmonic currents is $L_f$. Hence, the values of $L_c$ and $L_f$ can be designed separately.

even increases to about 2 times that at $q = 1.659$. In contrast, when $q > 1.8$, the amplitude tend to be unchanged.

Fig.5 illustrates the switch voltages with different $q$ and $R$. The switch satisfies ZVS when $R > R_{rated}$. Note that switching with a conductive body-diode of MOSFET is regarded as ZVS. The peak switch voltage has a larger range of variation when $R$ varies with a higher $q$ value. On the other hand, the switch voltages at $q = 1.45$ tend to be load-independent. Therefore, the value of $q$ affects the winding currents and switch voltages significantly.

For an actual inverter, the circuit parameters can be calculated by

$R = \frac{p_r^2 \cdot p_e^2 \cdot V_{in}^2}{2P_{out}}$  \hspace{1cm} (30)
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common-mode flux through the central column is canceled by combining the individual inductors. Fig. 7a is an “E-E” type magnetic structure for building an integrated inductor with two coupled windings. Three gaps are set avoid core saturation and adjusting the inductance. The common-mode flux, which is excited by the DC bias (defined in equation (5)) and the even-order harmonic currents, flows through the gaps of side-columns and bypasses the central gap. In contrast, the differential-mode flux, which is excited by the fundamental and odd-order harmonic currents, flows through all the three gaps. The suppression of even-order harmonic currents can be improved by reducing \( L_{gapd} \), but the DC bias of inverter may cause saturation of the magnetic cores. Therefore, the gaps should be precisely set to suppress the harmonic currents and prevent a significant increase in inductor loss.

As illustrated in Fig.7b, a new magnetic structure formed by hybrid materials is proposed to reduce the number of gaps and simplify the magnetic design. The low-permeability material is introduced to avoid core saturation without using gaps on the side-columns, while the high-permeability material benefits size reduction and makes sure \( L_c \) is large enough to suppress the common-mode currents. Therefore, the gap \( c \) is removed from the new structure, and the central gap is adjusted to get a preferred \( L_f \). The inductance for common-mode and differential-mode harmonic currents can be respectively expressed as

\[
L_c = \frac{N^2 \mu_0 A_e}{l_{ec}} \quad (35)
\]

\[
l_{ec} \approx \frac{l_{coreh}}{\mu_r} + \frac{l_{coreh}}{\mu_l} \quad (36)
\]

and

\[
L_{cdl} = L_f = \frac{N^2 \mu_0 A_e}{l_{ef}} \quad (37)
\]

\[
l_{ef} = \frac{l_{coreh}}{\mu_r} + \frac{l_{corel}}{\mu_l} + l_{gapd} \quad (38)
\]

where \( l_{gapc} \) and \( l_{gapd} \) are the length of \( gap_c \) and \( gap_d \), respectively. \( l_{ef} \) and \( l_{ec} \) are the effective magnetic path length of \( L_c \) and \( L_f \). \( l_{coreh} \) and \( l_{corel} \) are the effective magnetic path length of the high-permeability core and the low-permeability core. \( A_e \) is the effective cross-sectional area. Note that, the defined variables are based on one “E-I” type core.

Since \( l_{ef} \gg l_{ec} \), the value of \( L_o \) is much higher than \( L_f \). Hence, the integrated inductor with coupled windings can achieve the suppression of even-order harmonic currents without using a filter.

Besides, since the magnetic cores are arranged next to each other, the differential-mode fluxes through the central column are also canceled. However, due to the higher field density bias caused by DC bias, the total core loss is still higher than that of the integrated inductor with uncoupled windings.

C. Qualitative analysis for core losses

Steinmetz empirical equation and its improved forms are usually applied to calculate the core-losses of magnetic components [14], [25]–[27]. The estimated core loss \( P_{core} \) under sinusoidal flux density is given by

\[
P_{core} = \gamma f \alpha B_m^{\beta} \cdot V_{core} \quad (39)
\]

where \( \gamma, \alpha, \beta \) are the coefficients extracted from the material data sheets of magnetic cores, \( V_{core} \) is the core volume, \( B_m \) is the amplitude of excitation magnetic density. The prime part of \( B_m \) caused by the fundamental harmonic of switch voltage can be approximately calculated by

\[
B_{m1} = \frac{p \cdot p_r \cdot 2V_{in}}{NA_e \cdot \omega} \quad (40)
\]

where \( N \) is the turns of each winding, \( A_e \) is the area of the core section. According to Fourier series of the switch voltage in Fig.3a, the amplitude of the second-order harmonic voltage is about 0.54 times the fundamental harmonic voltage. Hence,
the flux density is non-sinusoidal, the material data sheets and Steinmetz equation cannot be used for calculating the core losses of integrated inductors.

Due to the non-sine excitation, estimating or measuring the losses of the integrated inductor is difficult to get accurate results. A more accurate method needs to be investigated in future works. To verify the effect of magnetic integration, a more feasible method is measuring and comparing the losses for different magnetic designs. As in Fig.8, two individual inductors (0.67 μH) using "E-E" type cores (EQ25/5.6/18, PC200 material) are applied in a class-E inverter. The open-load loss is about 11.6W (at 3MHz). Moving and pressing the two inductors together directly when measuring the open-load loss. The measurement is repeated continuously, and it shows that the open-load loss always instantly drops by about 0.2W when the inductors are pressed together. There is no distinct change when measuring the system efficiency since the loss reduction is too small.

Generally, the optimization of a gaped inductor is a multi-objective problem, which relates to winding turns, core material, fringe-effect around gaps and so on. The losses of the integrated inductor are highly dependent on a particular design. Since there is no distinct difference in system features when the individual inductors are integrated together without coupling the windings, the integrated inductor with uncoupled windings can also represent the individual inductors. In this paper, therefore, we use the proposed magnetic structure to build the two kinds of integrated inductors for a fair comparison based on the same conditions. As illustrated in Fig.9, there are two high-permeability cores (EQ25/5.6/18) of PC200 material placed on a low-permeability core (4.1mm thickness plate) of 4F1 material. The detailed comparison is presented below.

IV. MEGAHertz WPT PROTOTYPE WITH CLASS-E INVERTERS AND RECTIFIERS

Several inverters and rectifiers with different integrated inductors are built to verify the analysis above. As illustrated in Fig.10, the WPT system consists of three parts, i.e., the class-E inverter, the magnetic resonant tank, and the class-E rectifier. Since class-E inverters can also operate for rectification, we use the same topology for rectifiers but replace the MOSFETs by diodes. The design criteria for inverters is also applied for the rectifiers.

A. Magnetic Resonant Tank of WPT system

The magnetic resonant tank consists of three resonant loops of which parameters meet

$$\frac{1}{\omega^2} = L_1 C_1 = L_2 C_{2eq} = L_3 C_3.$$  

where \(C_{2eq} = C_1 C_2/(C_1 + C_2)\). The first resonant loop acts as an output filter of the class-E inverter, and the third loop acts as an input filter of the class-E rectifier. Therefore, the topology is well simplified. For a better comparison, we use the same parameters for the inverter and rectifier. Since they are symmetrical, the voltage gain and the load resistance of the system can be approximately given by

$$\frac{V_{out}}{V_{in}} \approx \frac{k_{23} \sqrt{L_2}}{k_{12} \sqrt{L_1}}$$  

$$R_L = \frac{V_{out}^2}{P_{out}}$$

where \(k_{23}\) are the coupling coefficient between \(L_2\) and \(L_3\), \(k_{12} = \sqrt{L_1/(L_1 + L_2)}\) is defined as the equivalent coupling coefficient. As in Fig.11, the WPT system is designed to achieve a DC-DC converter (48 V to 48 V) with rated output power about 300 W. The value of \(k_{23}\), which is about 0.3 in this paper, is limited by the transfer distance and the coil diameters. Hence, the values of \(L_1\), \(L_2\) and \(L_3\) are adjusted to realize the expected voltage gain as in Tab.II.

B. Experiment Setup for Inverters and Rectifiers

1) Circuit Design and Component Selection: The parameters of the class-E inverter and rectifier with uncoupled windings are calculated based on the optimal \(q_{trd}\) and \(p_{trd}\) as in equation (29). The circuit design is the same as a class-E inverter with finite DC-feed inductance [7].

The circuit parameters of inverters and rectifiers with coupled windings are calculated by (30), (31) and (32). The variables in these equations are obtained by solving the equation set discussed in subsection II.C.

Note that, the parameters are calculated based on the rated power of 360 W, and realize an actual output power of about 300 W of the receiver considering the system losses. The operation frequency is 3 MHz according to the optimal frequency range described in material data sheets. The applied values of \(C_I\) are lower than the calculated values due to the nonlinear parasitic capacitance of switches (about 130 pF for MOSFETs, 30 pF for diodes).

Fig. 8. Individual inductors for verifying loss reduction of the central column after magnetic integration. The output terminal is opened directly.

Fig. 9. Magnetic structure used for integrated inductors. For a fair comparison, we use the same cores for both kinds of integrated inductor with coupled windings and uncoupled windings.

Fig. 10, the WPT system consists of three parts, i.e., the class-E inverter, the magnetic resonant tank, and the class-E rectifier.
For the class-E inverters, since the duty cycle is fixed at 50\%, the requirement for rise time and fall time of switches is relaxed compared to PWM mode converters of which duty cycle can substantially shift from 50\%. The selection of the switches of class-E inverters is related to various specifications, such as the gate charge, on-resistance, withstand voltage, rise time and fall time. The driving losses and frequency are limited by the gate charge. The power dissipation of the MOSFET package is highly dependent on the on-resistance, rise time and fall time. In addition, the parallel capacitor of the MOSFET package is highly dependent on the on-resistance, rise time and fall time. The driving losses and frequency are limited by the gate charge. The power dissipation of the MOSFET package is highly dependent on the on-resistance, rise time and fall time. In this paper, the IXF36N20X3 MOSFET, which has a relatively low parasitic capacitance and switching time, is selected for the class-E inverters operating at 3 MHz.

2) Integrated Inductors: The integrated inductors which were built in section III.C are applied in the prototype. For a class-E inverter with coupled windings, the resonance inductance $L_{cpl}$ is half of the measured value between terminal $b$ and $c$ of the integrated inductor illustrated in Fig.2b. Likewise, for a class-E inverter with uncoupled windings, the resonance inductance $L_{decpl}$ can be measured directly. Hence, there are two values of $L_{cpl}$ or four values of $L_{decpl}$ for each group of one inverter and one rectifier as in Tab.II.

Besides, an optimized integrated inductor with coupled
Fig. 12. Measure voltages of inverters and rectifiers. Left: drain-source voltages of MOSFETs of inverters. Right: reverse voltages of diodes of rectifiers.

windings is also implemented for the comparison of magnetic losses with different core size. The inductance of the optimized inductor is kept the same as that with $q = 1.659$ in Tab.II, while the high-permeability cores (PC200, EQ25/5.6/18) are replaced by four pieces of smaller cores (PC200, ER23/5/13). The $A_e$ is increased from 93.51 mm$^2 \times 2$ to 50.3 mm$^2 \times 4$, i.e., by 7.6%.

C. Measurement

1) Switch Voltage: Fig.12 illustrates the measured voltages over the output power range from zero to 300 W. At the output power of 300 W, the input power of the inverters is about 350 W, which is closed to the rated value. Due to the nonlinear parasitic capacitance of MOSFETs and diodes, the measured peak voltages are higher than the theoretical values above.

Fig. 13. Measured efficiency and output voltage with varying output power.

Fig. 14. Efficiency and output voltage of the prototype with the optimized integrated inductor ($q = 1.659$).
As shown in the left sub figures, the ZVS is maintained over the entire load range without using closed-loop control. Comparing Fig.12a and Fig.12c, when the values of \( q \) equal the optimum ones in (28) and (29), the voltage waveforms are similar despite whether the windings are coupled or uncoupled. In contrast, the voltage waveforms have lower peak values when \( q = 1.53 \). Therefore, a lower value of \( q \) helps reduce the required maximum \( V_{DS} \) of the MOSFETs selected.

For the rectifiers, according to the basic circuit analysis of the magnetic resonant tank, the input and output currents, i.e., \( i_{inv} \) and \( i_{rinv} \), have the same initial phase \( \phi_r \). When the system operates at the rated load, the phase of \( i_{rinv} \) shifted from the switching phase of \( D_3 \) meets \( \phi_r = \phi_3 + \pi - \phi \) for rectification. Thus, the switching phases of the class-E rectifiers satisfy

\[
\phi_3 = 2\phi, \quad \phi_4 = \pi + 2\phi, \quad (44)
\]

where \( \phi_3 \) and \( \phi_4 \) are the initial switching phases of \( D_3 \) and \( D_4 \) shifted from the driving phase of \( S_1 \). The waveforms of rectifiers and inverters are supposed to be symmetrical when MOSFETs are applied for rectification. However, since the diodes are used for the rectifiers in the prototype, the duty cycle is changed adaptively when the load varies.

As in Fig.12c, due to the slight difference in the inductance of the uncoupled windings, the diode voltages show large unbalance at 50 W and 100 W. The unbalance in the voltages results in an efficiency drop. In contrast, as in Fig.12a and Fig.12b, the diode voltages are balanced over the entire load range. Therefore, the issue of unbalanced voltages no longer exists when the windings are coupled.

The experiment results show that the class-E rectifiers present similar voltage waveforms no matter the windings are coupled or uncoupled. However, it is worth noting that the analytic models of the class-E rectifier with coupled windings are still unclear because the analysis of the inverter cannot be applied for the rectifier when both of diodes are in off-state.

2) Efficiency and Output Voltage: Fig.13 shows the system efficiency and output voltage over the output power range. The driving losses (about 0.7 W) are not considered. The output voltage of the rectifier with uncoupled windings varies shapely when lightly loaded, while that with coupled windings are more stable. The peak efficiency of the system with uncoupled windings (86.8% at 300 W) is slightly higher than that with coupled windings (86.3% at 340 W) by about 0.5%. When the output power is higher than 300 W, the ZVS tends to be lost, and the efficiency will decrease, eventually. As in Fig.14, the reduced efficiency can be made up by enlarging the volume of the cores of the integrated inductor with coupled windings. Considering the existing magnetic cores are not optimized for the integrated inductor with coupled windings, there are chances to improve the efficiency in the future works further.

Fig.15 illustrates the steady-state thermal images of the circuit boards. Cooling fans are applied but without heatsink. The inverters and rectifiers have similar thermal characteristics. The temperature rise of the integrated inductor with coupled windings is apparently higher than that with uncoupled windings. In contrast, the diode temperature of the rectifiers with coupled windings is lower. As a result, coupling the windings will increase the core losses while decrease the switch losses at the same conditions.

When the operation frequency is up to megahertz, a small difference in time-delay between the current probe and the voltage probe will result in a non-ignorable phase shift, thus...
the integral of instantaneous power will be inaccurate. Consequently, measuring the inverter/rectifier efficiency or breaking down the total losses of the circuit broad is quite difficult.

To estimate the efficiency of the class-E inverters and rectifiers, we assume that the inversion and rectification have the same efficiency, i.e., $\eta_{\text{inv}} = \eta_{\text{rec}}$ because they use the similar topology, integrated inductor and component package, except the switch types (MOSFETs for inverters, diodes for rectifiers). The thermal images also show that the inverters and rectifiers have similar temperature rise at 300 W. In addition, the ESRs of the resonant loops of the magnetic resonant tank are almost constant since the core losses of the coupling coils are negligible. The total loss of magnetic resonant tank can be calculated according to the measured loop currents and ESRs (at 3MHz).

The total loss of the magnetic resonant tank $P_{\text{mag}}$ can be expressed by

$$P_{\text{mag}} = P_{\text{in}} \cdot \eta_{\text{inv}} - P_{\text{out}}/\eta_{\text{rec}}$$

(45)

where $P_{\text{in}}$ and $P_{\text{out}}$ are the input and output power of the WPT system. $\eta_{\text{inv}}, \eta_{\text{rec}}$ is the efficiency of inverter and rectifier respectively.

Tab. III shows the estimated losses of the resonant loops and the efficiency of the class-E inverter/rectifier at the output power of 300W. Assuming $\eta_{\text{inv}} = \eta_{\text{rec}}$, resulting in

$$\eta_{\text{inv}} = \frac{P_{\text{mag}}}{\sqrt{P_{\text{mag}}^2 + 4P_{\text{in}}P_{\text{out}}}}$$

(46)

$$P_{\text{mag}} = P_{\text{loop1}} + P_{\text{loop2}} + P_{\text{loop3}}$$

(47)

where $P_{\text{loop}}$ is the loss of the resonant loop of the magnetic resonant tank. It shows that the efficiency of the inverter/rectifier with uncoupled windings is increased by about 0.6% for the same magnetic cores. While the estimated $\eta_{\text{inv}}, \eta_{\text{rec}}$ increases to 95.2% for the optimized inductor with coupled windings.

3) Harmonic Currents: Fig.16 shows the measured winding currents, i.e., $i_{1n1}$. By coupling the windings of the integrated inductor, the winding current amplitudes are much lower. As shown in Fig.16b, the amplitude of the second-order harmonic current is distinctly reduced from 1.89 A to 0.58 A at 300W. Fig.17 is the FFT results of the input currents, i.e., $i_{in}$, it shows that the second-order harmonic (6 MHz) of the input current is also far lower than before. In this example, the amplitude (0.8A) is about 27% of that (2.99A) with uncoupled windings. In additions, when the windings are uncoupled, the fundamental harmonic ripple is not canceled completely due to the slight difference in the winding inductance. Note that the measured results, which are not illustrated, for the rectifiers are similar to that of the inverters.

<table>
<thead>
<tr>
<th>Coupled windings ($q = 1.659$)</th>
<th>$P_{\text{in}}$</th>
<th>$P_{\text{out}}$</th>
<th>$P_{\text{loop1}}$ (406.87mΩ)</th>
<th>$P_{\text{loop2}}$ (1.032Ω)</th>
<th>$P_{\text{loop3}}$ (553.79mΩ)</th>
<th>$\eta_{\text{inv}}, \eta_{\text{rec}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coupled windings ($q = 1.412$)</td>
<td>346.3 W</td>
<td>299.7 W</td>
<td>3.42 W</td>
<td>4.81 W</td>
<td>4.27 W</td>
<td>95.1%</td>
</tr>
<tr>
<td>Coupled windings (optimized) ($q = 1.659$)</td>
<td>402.6 W</td>
<td>350.6 W</td>
<td>3.72 W</td>
<td>4.73 W</td>
<td>4.65 W</td>
<td>95.2%</td>
</tr>
</tbody>
</table>

Fig. 16. Measured winding currents ($i_{in1}$).

Fig. 17. Measured ripples of input currents ($i_{in}$).
Therefore, although the efficiency is not improved by coupling the windings of the integrated inductors, it can naturally balance the winding currents of the class-E inverters and rectifiers, and significantly suppress the harmonic currents without using EMI filters.

V. CONCLUSION

This paper focuses on the circuit design and magnetic integration of push-pull class-E inverters for the wireless power transfer up to megahertz. The design criterion is proposed to achieve ZVS of class-E inverters with coupled windings. In addition, the approach of magnetic integration is implemented. The new magnetic structure with hybrid magnetic materials is proposed to build the integrated inductors of the class-E inverters with either coupled windings or uncoupled windings. The following conclusions can be made:

1) By using the design criterion proposed, the class-E inverters with coupled windings maintain ZVS when the load resistance varies from the rated value to infinity without using any closed-loop control.

2) The integrated inductor with uncoupled windings can reduce the core loss slightly. In contrast, the integrated inductors with coupled windings can suppress the harmonic currents without using filters. In the experiment, the amplitude of the second-order harmonic current through the DC source is reduced to 27%.

3) The core losses of the class-E inverter with coupled windings are higher than that with uncoupled windings. In the experiment, the system efficiency decreases by 0.5%.

4) The winding currents of the rectifier with coupled windings are self-balanced, thus it provides a more stable output voltage.

5) The proposed magnetic structure with hybrid materials reduces the number of the gaps of the integrated inductor and simplifies the magnetic design of class-E inverters with coupled windings.

REFERENCES


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