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Optimization of realistic loudspeaker models with respect to basic response characteristics

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Abstract

A numerical model for optimization of loudspeakers mounted in infinite baffles is presented in this paper. The optimization is carried out by using objective functions based on basic response characteristics for loudspeakers such as the on axis frequency response. In-order to create a realistic model of the speaker we include the excitation of the speaker from the motor system. The interaction between acoustic medium and structure is modelled with the finite element method. As most loudspeaker drivers are nearly symmetric, the presented model is a 2D axisymmetric finite element model. The exterior domain is modelled with perfectly matched layers, which ensure free-field radiation conditions. Optimized designs for a selection of objective functions are presented and discussed.

Keywords: Optimization, FEM, Loudspeaker, Materials

1 INTRODUCTION

Since its discovery three decades ago [2], structural topology optimization has increased in popularity [3] and is now widely used as a design tool to for improving a wide range of engineering structures such as airplane fuselages [1], waveguide filters [10] and periodic microstructures [8]. The usage of optimization techniques has over the years expanded to many different scientific disciplines, including acoustic-structure problems [9].

Topology optimization has previously been used when performing structural-acoustic optimization. One of the pitfalls of using structural topology optimization in conjunction with acoustics, is the physical interpretation of structural elements with intermediate densities, e.g. how to interpret an element with 40 % air and 60 % solid. Several methods have been proposed to combat this issue, [13] introduces the concept of a mixed finite element formulation, [6] uses artificial mechanical and acoustic parameters in the non-structural and non-acoustic domains together with self coupling elements and [11] proposes level set based topology optimization.

If we consider the design of loudspeakers from different brands the choice of shape and materials are converging towards similar solutions. This paper will look at the choice of materials in loudspeakers and how these choices can improve the performance. To do this we use advanced numerical optimization techniques in conjunction with a 2D axisymmetric finite element model. This paper is about distributed parameter optimization. Here the mechanical structure is not changeable, and only the material of which it is made of is optimized. This of course limits the design freedom in the sense that new geometries are not created, instead we explore the opportunities of existing mechanical structures.

2 THEORY

Finite element analysis (FEA) is used to calculate the acoustic wave propagation from a vibrating mechanical structure. The model consists of a structural mechanics domain where the governing equation are the dynamic equation of motion and an acoustic domain described with the Helmholtz equation. These domains are discretized into elements with quadratic shape functions. The two domains are coupled together in the interface between the mechanical structure and the acoustic domain, for details see e.g. [5]. Perfectly Matches Layers

(PMLs) are used to mimic exterior acoustic conditions, the details of this method can be found in [4]. The following matrix vector equation is used to calculate mechanical displacements and acoustic pressure

$$\left(\begin{bmatrix}\mathbf{K} & -\mathbf{S}^T \\ \mathbf{0} & \mathbf{K}_F(\omega)\end{bmatrix} - \omega^2 \begin{bmatrix}\mathbf{M} & \mathbf{0} \\ \rho_F \mathbf{S} & \mathbf{M}_F(\omega)\end{bmatrix}\right) \begin{Bmatrix}\mathbf{u}_s \\ \mathbf{p}\end{Bmatrix} = \begin{Bmatrix}\mathbf{f}_s(\omega) \\ \mathbf{0}\end{Bmatrix}, \tag{1}$$

where $\mathbf{K}$ is structural stiffness matrix, $\mathbf{S}$ is the coupling matrix, ω is the harmonic excitation frequency, $\mathbf{M}$ is the structural mass matrix, $\mathbf{u}_s$ is the structural nodal displacements $\mathbf{f}_s(\omega)$ is the externally applied harmonic excitation of the mechanical structure, $\mathbf{K}_F(\omega)$ is the acoustic stiffness matrix that is frequency dependent in the PML region, ρ_f is the density of air, $\mathbf{M}_F(\omega)$ is the acoustic mass matrix which is frequency dependent in the PML region and $\mathbf{p}$ is the nodal pressure in the acoustic domain.

3 DISTRIBUTED PARAMETER OPTIMIZATION

This paper proposes an optimization scheme in which the material distribution within the structure is optimized. The optimization procedure relies on being able to change the element stiffness and density in each individual structural element, which is controlled by the design variable α^e that can take on values between 0 and 1. The following linear interpolation is used in each structural element

$$\rho^e = \rho_{min} + \alpha^e \left(\rho_{max} - \rho_{min}\right) \tag{2}$$

$$E^e = E_{min} + \alpha^e \left(E_{max} - E_{min}\right), \tag{3}$$

where E^e is the Young's modulus in an element, ρ^e is the element density, ρ_{min} is a lower bound for the element density, ρ_{max} is the upper bound, E_{min} is the lower bound of the element Young's modulus and E_{max} is the upper bound. Young's modulus and density are chosen as the parameters to be changed during the optimization because they are very much linked to the vibration pattern of the mechanical structure. The parameters needs change with some co-dependency (here a linear dependency) such that the achieved material configuration is kept realistic and to avoid trivial solutions.

The system of equations in (1) can be written in a compact form

$$\left(\tilde{\mathbf{K}} - \omega^2 \tilde{\mathbf{M}}\right) \tilde{\mathbf{u}} = \tilde{\mathbf{S}} \tilde{\mathbf{u}} = \tilde{\mathbf{f}}, \tag{4}$$

where $\tilde{(\cdot)}$ indicates that the matrix or vector is written with compact notation.

The full sensitivity analysis is not carried out in this paper, the reader is referred to [7, 6] for a detailed explanation of the derivation of the adjoint sensitivities.

3.1 The Optimization Problem

Figure 1 shows the domain to be optimized, Ω, which resembles a piston-like structure mounted in an infinite baffle. The figure also shows half of the acoustic domain Ω_F, the acoustic domain is also present below the piston, however it is not included in this figure and finally, Ω_A, which is the region with PMLs.

In Figure 1 the region marked by the red rectangle is the region of interest for the optimization. In this region it is desired to enhance the performance of the speaker, consequently, the magnitude of the pressure needs to be increased. This can be cast as a optimization problem

$$\begin{aligned} \max_{\alpha} \quad & \Phi_0 = |\tilde{\mathbf{u}}|^2 , \\ \text{s.t.} \quad & \tilde{\mathbf{S}}\tilde{\mathbf{u}} - \tilde{\mathbf{f}} = 0 , \\ & 0 \leq \alpha_e \leq 1 , \quad e = 1, \ldots, n , \end{aligned} \tag{5}$$

which can be solve using the Method of Moving Asymptotes [12]. It is only the nodal degrees of freedom (DOF) inside the boxed region that should be included in the objective function. Consequently, the diagonal

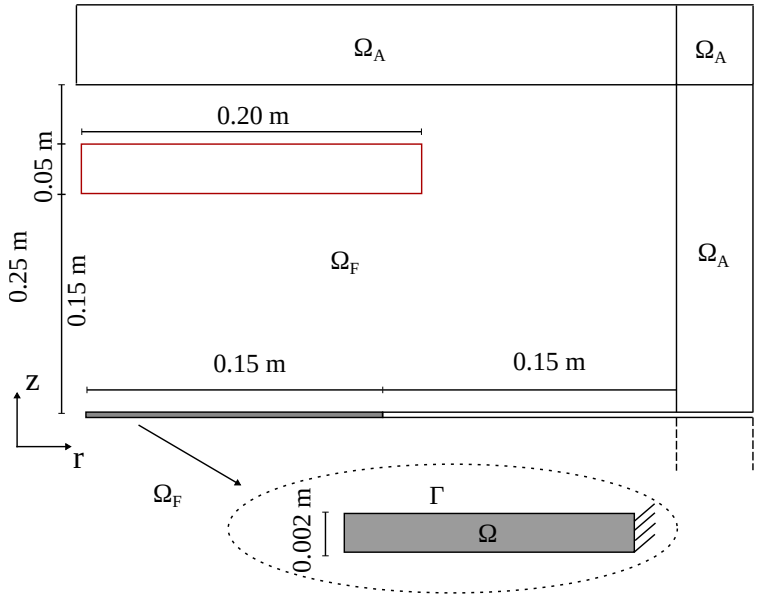

Figure 1. Piston-like-structure mounted in an infinite baffle.

matrix $\mathbf{L}$ is constructed, the matrix contains ones in the diagonal corresponding to the DOF in the boxed region in Figure 1.

$$\Phi_0 = |\tilde{\mathbf{u}}|^2 = |\tilde{\mathbf{u}}|^T \mathbf{L} |\tilde{\mathbf{u}}|. \tag{6}$$

The adjoint equation for this particular optimization problem becomes [7]

$$\tilde{\mathbf{S}}\boldsymbol{\lambda} = -2\mathbf{L}^T \tilde{\mathbf{u}}, \tag{7}$$

the equation should be solved for $\boldsymbol{\lambda}$, which is the adjoint variable that can be used to compute the sensitivities by inserting into:

$$\Phi' = \mathrm{Re}\left(\boldsymbol{\lambda}^T \frac{\partial \tilde{\mathbf{S}}}{\partial \alpha^e} \tilde{\mathbf{u}}\right), \tag{8}$$

where the term $\frac{\partial \tilde{\mathbf{S}}}{\partial \alpha^e}$ is the derivative of the system matrix with respect to the design variable α^e. Evaluating the term yields

$$\frac{\partial \tilde{\mathbf{S}}}{\partial \alpha^e} = \frac{\partial \tilde{\mathbf{K}}}{\partial \alpha^e} - \omega^2 \frac{\partial \tilde{\mathbf{M}}}{\partial \alpha^e}, \tag{9}$$

in which the derivative of the global mass and stiffness matrix is

$$\frac{\partial \tilde{\mathbf{K}}}{\partial \alpha^e} = \begin{bmatrix} \frac{\partial \mathbf{K}}{\partial \alpha^e} & -\frac{\partial \mathbf{S}^T}{\partial \alpha^e} \\ \mathbf{0} & \frac{\partial \mathbf{K}_F}{\partial \alpha^e} \end{bmatrix} = \begin{bmatrix} \frac{\partial E^e}{\partial \alpha^e}\mathbf{K} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \quad , \quad \frac{\partial \tilde{\mathbf{M}}}{\partial \alpha^e} = \begin{bmatrix} \frac{\partial \mathbf{M}}{\partial \alpha^e} & \mathbf{0} \\ \rho_F \frac{\partial \mathbf{S}}{\partial \alpha^e} & \frac{\partial \mathbf{M}_F}{\partial \alpha^e} \end{bmatrix} = \begin{bmatrix} \frac{\partial \rho^e}{\partial \alpha^e}\mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \tag{10}$$

where E^e and ρ^e are determined from (3) and (2), the derivative of the element interpolation functions with respect to the design variable is

$$\frac{\partial E^e}{\partial \alpha^e} = E_{max} - E_{min} \quad , \quad \frac{\partial \rho^e}{\partial \alpha^e} = \rho_{max} - \rho_{min} \tag{11}$$

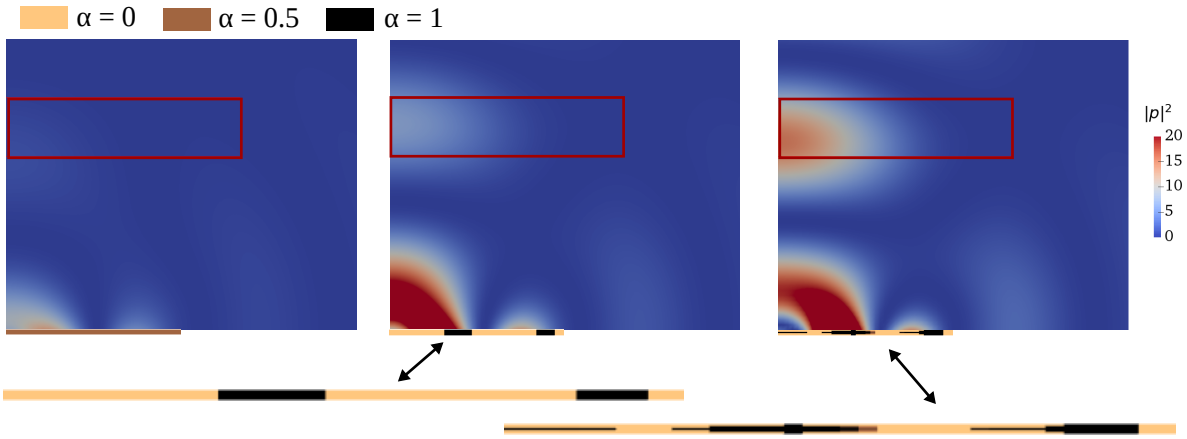

Figure 2. 1500Hz excitation frequency, $|p|^2$ evaluated for the starting guess (left), optimized design with 1 element in the vertical direction (middle) and optimized design with 5 elements in the vertical direction (right).

4 NUMERICAL RESULTS AND DISCUSSION

The optimization problem (5) is solved for the structure presented in Figure 1 at three different excitation frequencies. This structure corresponds to a loudspeaker unit of 12 inches, which is a large bass unit that is used to produce low frequency content. The results in this section is obtained for 1500Hz, 2500Hz and 3500 Hz, these frequencies are well above the operating range of a 12 inch unit. Furthermore the loudspeaker unit is flat and with no dust cap, consequently, the structure does not benefit from geometrical stiffness. These factors makes the optimization problem harder to solve, but solving it might give an indication whether we could extend the operating range of larger units by utilizing optimization techniques.

This section presents the optimized results and these are compared to the original structure with homogeneous material distribution. The vibration pattern of the optimized- and original structures are presented, furthermore the objective functions as function of iteration history are shown. Two optimization results are presented for each excitation frequency, the first result only has one element in the vertical direction and 36 elements in the radial direction. The second result has a higher degree of design freedom, these design consists of 5 elements in the vertical direction and 36 elements ind the radial direction.

Table 1 shows the parameters used to determine the Young's modulus and density in each element.

Table 1. Lower- and upper bound for element stiffness and density

E_{min}:	35 MPa	E_{max}:	105 MPa
ρ_{min}:	1350 kg/m^3	ρ_{max}:	4050 kg/m^3

Figure 2 shows $|p|^2$ for three different structures, all of whom are excited by a tip force of 2N at 1500 Hz. The leftmost structure has a uniform material distribution with $\alpha^e = 0.5$ in all elements. This is the initial design that also serves as a starting guess for the optimization. From the plots of the pressure magnitude, it can be observed that the optimized designs vastly improves the magnitude of the pressure in the desired region. Especially the rightmost design achieves a very good design, which is further validated by looking at the objective function in Figure 8.

The vibration pattern of the structures in Figure 3 shows that the rightmost structure can create larger structural displacements compared to the structure in the middle, it is noted that the overall displacement pattern is similar for the two optimized structures. The structural displacements have been scaled with a factor of 5000 for better visualization.

Figure 3. 1500Hz excitation frequency, vibration pattern for the the starting guess (left), optimized design with 1 element in the vertical direction (middle) and optimized design with 5 elements in the vertical direction (right).

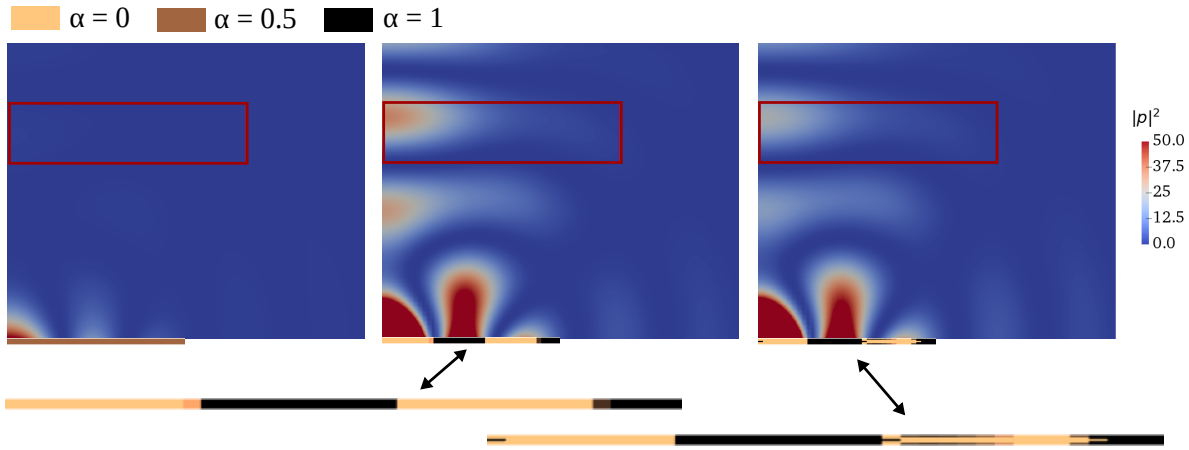

Figure 4. 2500Hz excitation frequency, $|p|^2$ evaluated for the starting guess (left), optimized design with 1 element in the vertical direction (middle) and optimized design with 5 elements in the vertical direction (right).

In Figure 4 the structure is excited with 2N at 2500 Hz. In this figure it is observed that optimized structure in the middle performs slightly better than the rightmost structure. This is surprising since the rightmost structure has larger design freedom and therefore should perform better when optimized, as it was the case in Figure 2. From Figure 8 one can see that the structure containing 5 elements exhibits a strange convergence pattern during the optimization, which might be due to an unfeasible local minimum. The vibration pattern of the structure can be seen in Figure 3.

The structure in Figure 6 is excited at 3500 Hz with a tip force of 2N. It can be seen in the figure that the pressure magnitude is indeed increased, this is however not as significant as the previously shown result. This is supported by Figure 8 that shows that the pressure magnitude in the rectangular area only is increased by a factor of 8. Figure 7 shows the vibration pattern of homogeneous- and optimized structures.

Figure 5. 2500Hz excitation frequency, vibration pattern for the the starting guess (left), optimized design with 1 element in the vertical direction (middle) and optimized design with 5 elements in the vertical direction (right).

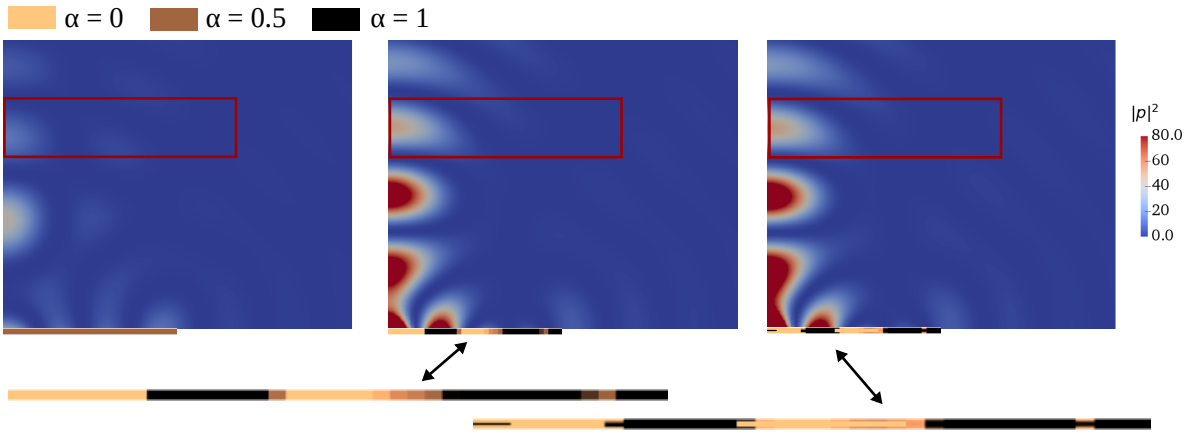

Figure 6. 3500Hz excitation frequency, $|p|^2$ evaluated for the starting guess (left), optimized design with 1 element in the vertical direction (middle) and optimized design with 5 elements in the vertical direction (right).

Figure 7. 3500Hz excitation frequency, vibration pattern for the the starting guess (left), optimized design with 1 element in the vertical direction (middle) and optimized design with 5 elements in the vertical direction (right).

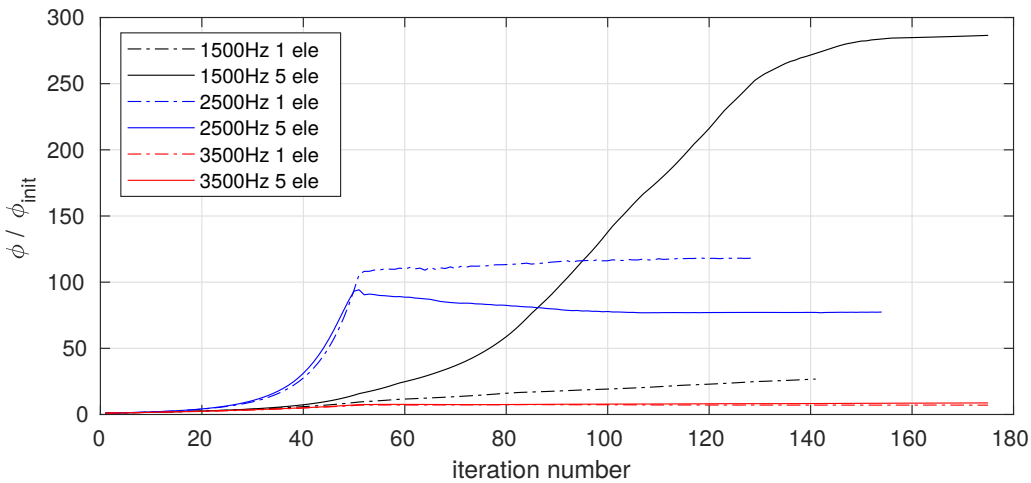

Figure 8. Objective functions evaluated for the optimized designs as function of iteration history.

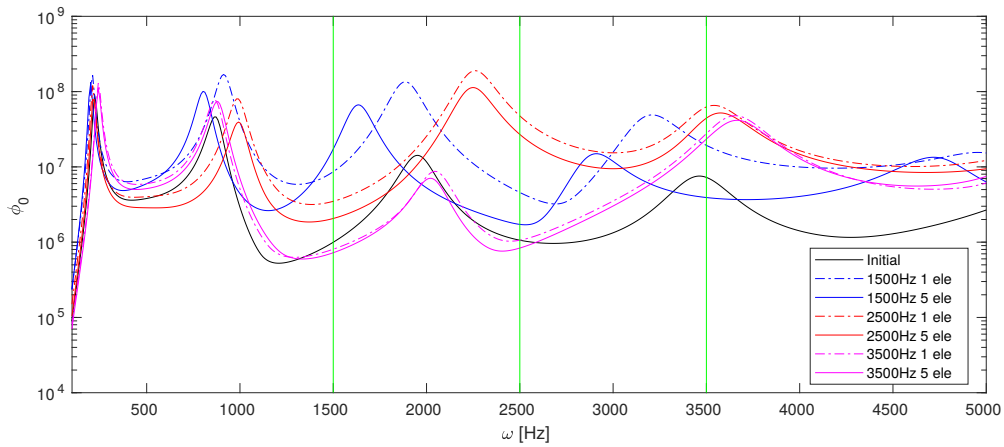

Figure 9. Evaluation of the objective function as a function of frequency for the initial homogeneous solution and for the optimized designs.

Figure 9 shows how the optimized designs performs in the frequency range from 100 Hz to 5000 Hz. The green lines indicate the target frequencies, the blue lines are the performance of the design optimized for 1500 Hz, the red curve is 2500 Hz and the magenta curve is 3500 Hz and the black curve is the initial homogeneous design. The optimizer tries to shift the resonance frequency such that it aligns with the target frequency for the optimization. This is especially pronounced for the blue and red curve where the third resonance frequency of the initial structure is shifted down in frequency for the blue curve and shifted up for the red curve. The first resonance frequency of the initial structure is hardly affected by the optimization whereas the second resonance frequency is affected to some extend. The optimized structure for 2500 Hz actually has a higher objective function value at 3500 Hz than the structure which was optimized for that frequency. This shows the existence of local minimums that can affect the end result of the optimization.

The optimized designs takes advantage of the rather simple objective function by increasing the magnitude of the pressure close to the center axis where it is easiest. This gives a rather uneven pressure response, due to the fact that the pressure on axis is greatly increased relative to the pressure off-axis. If one wants the increase in pressure to be more evenly distributed a more complex objective function should be used; in which the optimization favors the off-axis response over the on-axis response. Another option could be to minimize the standard deviation of the pressure magnitude for the optimized designs presented in this paper.

The designs presented in this paper is optimized for one specific frequency. It was expected that the performance at other frequencies would be sacrificed to achieve optimal performance at the target frequency. This has however not been the case and this could be due to the fact that the optimized structures has regions with lower structural stiffness and thereby larger structural displacements compared to the initial design.

5 CONCLUSION

A 2D axisymmetric numerical framework has been developed and extended to include distributed parameter optimization of mechanical structures coupled to acoustic domains. The optimization is based on well known gradient based methods. The advantage of the method is that it does not require any interpolation scheme, because the interface between fluid and solid is well defined throughout the optimization. The developed method is tested with three simple examples in which the initial design is improved. The design obtained at 1500 Hz shows major improvements when compared to the starting point, the design with 5 vertical elements improves with a factor of 280 when compared to the initial design. This comes to show that large improvements of

existing designs can be obtained by optimizing the material within the structure.

The method can readily be extended to consider more sophisticated objective functions, which i.e. could be used to achieve an even pressure response at higher frequencies. The method should be extended, such that the optimization includes multiple frequencies and thereby making the obtained solutions valid in a broader range.

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