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High-Order Harmonic Resonances in Traction Power Supplies: A Review Based on Railway Operational Data, Measurements and Experience

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Abstract—Harmonic resonances in traction power supply systems (TPSSs) have attracted great attention due to their potentially destructive impact on the safe and stable operation of railways. Based on operation data acquired from Chinese railways, this paper presents a comprehensive review of this issue considering both academic and engineering requirements. Analysing actual incidents, the general patterns and effects of TPSS resonances are derived. The relation between the TPSS and the locomotive is illustrated by circuit models. Relevant methods for modeling locomotives and TPSSs are discussed. Both advanced resonance analysis methods giving general influence factors and a simplified resonance analysis explaining resonance features in a practical manner are discussed. Multiple ground-based and on-board solutions for resonance elimination are presented. At last, pre-identifying resonances in actual systems is investigated for addressing the open topic of resonance prevention.

Index Terms—Electric railway; harmonic resonance; resonance elimination; traction power supply system; traction drive system.

I. INTRODUCTION

AILWAYS are key infrastructure of national economic importance, forming the backbone of integrated traffic and transport systems for many countries. With the advantages of fast and heavy loading capability, environmentally friendly, electric railways, especially high-speed ones, are becoming popular worldwide. However, their load, electric locomotives, are single-phase, non-linear, rectifying load consuming reactive power, generating harmonics and negative sequence current injected into traction power supply systems (TPSSs) and then into the public grid. Thus, the problem of power quality in the public grid and TPSS as well as their interaction with the locomotive operation is an old aged but often new issue for both railway and power grid companies all over the world [1]–[8].

Among the reported power quality issues in TPSSs (e.g. power imbalance [9], reactive power [10], low-frequency oscillation and harmonic amplification due to grid-converter instability [11]–[15], harmonic resonance [16], etc.), high-order/frequency harmonic resonances (or simply harmonic resonances) have received great attention by scholars and practitioners [16]–[18]. This problem stems from the use of power electronics in electric locomotive drives. With the development of high power semiconductor devices and associated control techniques, ac-drives have become the most popular choice in traction applications globally [19]–[24]. Ac drive electric locomotives, including electric multiple units (EMUs), use a number of interleaved four-quadrant converters (4QCs) as front-end rectifiers to either draw power from the supply network or inject power back into the supply network during regenerative braking [25]–[27]. Compared with diode and/or thyristor rectifiers of traditional dc electric locomotives [24], the 4QCs ensure unity power factor and a nearly sinusoidal current on the network side. However, the interleaved 4QCs, operating under phase-shifted pulse-width modulation (PS-PWM), still generate relatively high amount of sideband harmonics located around the multiples of the carrier frequency. If some of current harmonics injected into a TPSS coincide with the resonant frequency of this TPSS, a resonance can be excited [28], [29]. When a resonance occurs, amplified resonant voltages and currents appear in the TPSS, resulting in supply voltage distortion, interference with adjacent communication lines, erroneous operation of protective devices, even damaging some high-voltage (HV) devices [30]–[35]. There are two stipulations in the following analysis. Firstly, due to the nature of high-order harmonic resonance, where locomotives are the harmonic source while the TPSS provides the resonance path, the incidents are treated as a power quality issue and not as cases of TPSS instability [36]. Secondly, the high-order harmonic resonance incidents correspond to a parallel resonance which is the main concern in TPSS, while series resonances discussed in some literature, e.g. [37]–[39], are mainly related to background harmonics at lower frequencies seen from an ESS or upper grid.

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A. Previous Studies on TPSS Resonances

An early instance of high-order resonance was encountered in the 1969 test of the first commercial AT (auto-transformer) feeding in the Kagoshima line (Japan) [21]. The cause of the resonance was the large capacitance in the 65-km long supply network, eventually being resolved through the installation of passive filters (PFs) in section posts (SPs). Technical papers reporting TPSS resonances date back to the late 1970s [40], with indications that when two or more electric locomotives haul one train there will be certain risk of resonance which may cause overvoltage and even equipment failure [1]. A digital computer method and an analogue simulation were presented for precalculating the possibility of excessive overvoltage at the early stage of newly designed schemes, with measured resonant voltage from Taiwan railways provided for verification. A damping filter to suppress resonant overvoltages appearing in Zimbabwe national railways was presented in [2].

In the past decades, TPSS resonances in other countries and regions have been reported, e.g. Korea [17], China [41], Czech Republic [42] and Italy [43]. Most studies can be classified into two sets: i) those focusing on network-side modeling to describe the TPSS impedance-frequency characteristics, and ii) those referring to electric locomotive load characteristics to explain how harmonic currents are generated and injected into the overhead supply network.

1) Network-side studies: In early works, wave propagation theory along distributed lines was used to analyse the propagation of harmonic currents generated by electric locomotives. The analysis was based on a “two-transmission-line” simplified representation of the 15 kV, 16.7 Hz German TPSS [18]. A method for online identification of resonant frequencies [44] as well as further elimination approaches [45]–[49] were also proposed. However, German railways have special TPSS features such as network structure, voltage and frequency, so resonance related analysis cannot be directly applied to TPSSs in many other countries.

The impedance-frequency characteristics of the Italian 3 kV dc supply network was analysed in [50] and harmonics propagation in the network-locomotive system were investigated through analytical [51] and probabilistic [52] methods. The Korean high-speed railway (HSR) 2 × 25 kV AT supply network was modeled as an eight-port system, representing the entire system model for harmonic and resonance simulation studies [17]. Similar approaches for the Tehran-Karaj railway in Iran (including both 2 × 25 kV and 25 kV sections) and the Suvarnabhumi Airport rail link in Thailand (25 kV supply) were presented in [53] and [54], respectively.

2) Locomotive-side studies: The main cause of harmonics in TPSSs is the switching operation of locomotive line-side converters, so explaining harmonic injection from power electronics converters is necessary for TPSS resonance studies. Two different methods for harmonic analysis of the multiple interleaved single-phase, two-level PWM rectifiers [55], [56] were used to explain the generation and frequency characteristics of locomotive line current harmonics. The double Fourier series (DFS), a well-accepted tool for PWM waveform analysis, was adopted for harmonic analysis of two-level [56] and three-level [57], [58] multiple interleaved converters. The impact of dc-link voltage control in the low-order harmonics together with the high-order PWM harmonics was presented in [28], [59] and a probabilistic model of locomotive current harmonics based on operational data was developed in [60].

B. State-of-Art and Operation Experience of Chinese Railways

In China, railway electrification started in the 1960s, somewhat later compared to some European countries and Japan. Since entering the new century, electric railways, especially the high-speed ones, have experienced a very fast growing period. By the end of 2017, Chinese railways covered 127,000 km with 25,000 km of HSRs, accounting for 66.3 % of the total HSR lines globally. The electrification rate is above 65 % with more than 82 % of the transportation task accomplished by electric traction. All electrified lines adopt a standard single-phase, 25 kV1, 50 Hz ac supply, which is now the most common supply mode worldwide. Since 2007, the HXD series of ac electric locomotives and the CRH, CR series of EMUs, including a total of more than 20 models, have been successively rolled out. Though all of them used ac drives, i.e., employing 4QCs as line-side converters, the number of interleaver converters and the carrier frequency of each converter are discrepant for different models leading to diverse harmonic spectra.

In a way, due to the long electrified mileage in the common supply mode and the variety of electric locomotives in operation, Chinese railways provide rich operation experience especially incidents related to resonance studies. Since the first high-frequency harmonic resonance occurred back in 2007 in the Beijing-Harbin line, more than 15 other Chinese lines have experienced similar instances. All of the resonances occurred in the 3 kHz frequency span despite being caused by over 10 different models of locomotives and EMUs.

Regarding these resonances, first, a number of field tests have been conducted to identify the features of harmonic resonance, e.g., frequency range, over-voltage/current level [41], [62]–[64]. Then, referring to the measured data, line current harmonics of those locomotive models exciting resonances have been investigated by either DFS analysis [28], [65] or probabilistic modeling [66], [67]. On the other hand, harmonic propagation on the supply network has also been analysed (see [16], [29]–[31], [68]–[73], etc.). Harmonic elimination approaches have been explored from both a TPSS [31], [74] and locomotives perspective [75]–[77]. Recently, [12] and [13] summarised multiple power quality issues (i.e., low-frequency oscillation, harmonic resonance and harmonic instability) in railway electrification systems. Through supply network and electric locomotives modeling in frequency domain and related stability analysis, those issues were defined as a type of network-train instability. In [78], the issue of harmonic

\(^1\) including 2 × 25 kV mode [61], a special 25 kV supply mode, which provides higher supply capability adopted by HSRs and heavy-haul freight lines.
resonance of electric railways was independently reviewed from the aspects of identification, mechanism and elimination. The work focused on advanced modeling of the networktrain system and verification through simulations and scale-down experiments without strong links to operational data or practical cases of TPSS resonance.

C. Contribution and Organization

Among previous technical papers related to TPSS resonances, there is a lack of comprehensive analysis, linking real incidents with mechanisms that explain resonances and recommend elimination methods. Even if all such aspects can be found in different papers, prevention of resonances remains a key topic open for contributions. This paper contributes to the subject of TPSS resonance, seeing the supply network and locomotives as an interactive system, comprehensively discussing the phenomena, mechanism, elimination and pre-identification based on a large amount of operational data from Chinese railways.

The rest of the paper is organized as follows. Section II discusses two typical TPSS resonance cases providing details on their features and influence, then summarises the TPSS resonance incidents that occurred in Chinese railways over the past decade to conclude general patterns and effects. In Section III, an entire system model is first presented and simplified to illustrate the locomotive-network interaction. Then, modeling methods for both the locomotive side (line current harmonic characteristics) and the TPSS side (impedance-frequency characteristics) are discussed. At last, advanced resonance analysis methods and general influence factors are summarised; considering engineering implementations, a simplified resonance analysis is presented to explain the resonance features in a practical manner. Multiple ground-based and on-board resonance elimination approaches are discussed in Section IV. In Section V, the resonance pre-identification in real systems is investigated for addressing the open topic of resonance prevention. Finally, Section VI summarises this work.

II. RESONANCE PHENOMENA ON CHINESE RAILWAYS

A. Case I: 2007, Beijing-Harbin Railway Line

From July to August 2007, reconnection CRH2 EMUs were put into operation in succession on the Beijing-Harbin railway line. During that time, the two sections of the line supplied by the Jixian south electrical substation (ESS), Jixian south ESS to Yanjiao section post (SP) and Jixian south ESS to Hanjialin SP, frequently experienced high-frequency resonances. This led to equipment failure in the network. In Yanjiao SP, catenary network arresters exploded three times and a spark gap broke down once; in Hanjialin SP, dc power charging block and ac power surge protector burnt down; in Jixian south ESS, the reactive power compensation branch breaker tripped frequently leading to branch malfunction.

Generally, a reconnection EMU train has 16 cars consisting of two standard EMUs with 8 cars per each.

Outside the TPSS, low power appliances (e.g. air conditioners, televisions, low power leakage protectors, etc.) along the two sections were also damaged. Fig. 1 shows the burnt arresters of that incident.

Field tests indicated that CRH2 EMUs were the harmonic source exciting resonance in the 750-1150 Hz (17-23 p.u.) frequency range. Fig. 2(a) shows the rms of supply voltages at Jixian south ESS and Yanjiao SP, which reached 60 kV and 30 kV, respectively. Fig. 2(b) shows the ESS supply voltage waveform and spectrum. Measurements showed a peak value higher than 110 kV with 43.85% of 18th harmonic, 26.72% of 19th harmonic and about 10% of 16th, 17th and 20th harmonics. Meanwhile, the measured rms voltages of indoor power supply in ESS and SPs reached 260 V (nominal 220 V).

B. Case II: 2011, Wuhan-Guangzhou HSR Line

In the afternoon of 23rd January 2011, the section between Guangzhou south ESS and Lishui SP of the Wuhan-Guangzhou HSR Line was put into operation. Among previous technical papers related to TPSS resonances...
Guangzhou HSR\(^4\) experienced six tripping instances of the breakers due to harmonic resonance, leading to the destruction of four arresters in the catenary network.

Through extensive field tests, it was found that a resonance in the 2250-2750 Hz (45-55 p.u.) frequency range was excited by CRH380A EMUs. Fig. 3(a) shows the measured rms of supply voltages and load currents, (b) waveforms and spectrum of supply voltages. (273 and 274 are the labels of down and up links respectively. AT stands for the AT midpoint absorbing current.)

### Table I.

<table>
<thead>
<tr>
<th>Harmonic Order</th>
<th>Harmonic Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>51st</td>
<td>25.26%</td>
</tr>
<tr>
<td>49th</td>
<td>13.95%</td>
</tr>
<tr>
<td>53rd</td>
<td>18.32%</td>
</tr>
<tr>
<td>51st</td>
<td>37.31%</td>
</tr>
<tr>
<td>26th</td>
<td>14.32%</td>
</tr>
</tbody>
</table>

A resonance event may last between several seconds to minutes, with extreme cases resulting in sustained overvoltage of more than ten minutes. Supply voltage waveforms gradually change to distorted shapes, which have maintained for the duration of the event. In contrast to other general transients, TPSS resonances can be defined as quasi-steady-state phenomena.

iv) Location: Resonances have occurred in sections and stations of HSRs, conventional passenger lines and heavy-haul lines. There is limited correlation between a location and a resonance, but a common feature of the locations will be explained in Section III-D.

v) Vehicle model: Early TPSS resonances in China were mainly excited by CRH2 series\(^5\) EMUs running on HSRs. Later on, the line-side converter controls of those models were improved, significantly decreasing the resonance risk. After 2011, most resonances were excited by HX\(_D\) series locomotives running on conventional railways.

On the other hand, the impact of the TPSS resonances can be categorised into two classes:

i) Activation of protection relays: Resonances always trigger overvoltage protections either in the ESSs or on-board the trains, causing breakers to trip. Both ground and on-board breaker tripping actions result in trains stopping, service interruption and delays across the traffic network.

ii) Damages on electrical devices: Both ground and on-board HV devices, especially arresters, may be burnt by resonant overvoltages. In locations where the power grid is too weak to provide independent supply for ESSs and SPs, low power appliances, powered by the traction network through a step-down transformer, may be also damaged. For arresters, the resonant overvoltage with high frequency and high amplitude may increase the leakage current (from several to hundreds mA). When a resonance is sustained for a time interval longer than a few minutes, the arresters may be burnt by overheating of their value plates (usually made by zinc oxide, ZnO).

### III. MECHANISM OF TPSS RESONANCE

In order to study the resonance mechanism, it is important to identify the interaction between the supply network and the train. Mathematical model for each part of the system need to be formed in the frequency domain. Finally, the TPSS resonance can be defined through impedance-frequency characteristics.

\[^{4}\text{It is now called Beijing-Guangzhou HSR after the Zhengzhou-Wuhan and Beijing-Zhengzhou parts were completed in Sept. and Dec. 2012, respectively.}\]

\[^{5}\text{Other resonance incidents occurred out of China are not included in this table due to lack of detailed data. Interested readers can still find related information from [1], [2], [17], [21], [40], [42], etc.}\]
A. Network-Train Interaction System

The structure of a double-track 2 × 25 kV AT supply system, which is the most popular choice for HSR and heavy-haul lines worldwide [79]–[81], is presented in Fig. 4(a). A basic TPSS consists of an ESS, a feeding network section, an AT post (ATP) and an SP. The electric locomotives act as nonlinear loads injecting harmonics into the TPSS. In the ESS, a Vx consists of an ESS, a feeding network section, an AT post (ATP) and an SP. The electric locomotives act as nonlinear loads injecting harmonics into the TPSS. In the ESS, a Vx

<table>
<thead>
<tr>
<th>No.</th>
<th>Time</th>
<th>Location</th>
<th>Vehicle (Vx [p.u.])¹</th>
<th>f_c [p.u.]*</th>
<th>Main Impact</th>
<th>Damaged devices</th>
<th>Breaker tripping</th>
<th>Stopping or delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2007-7-8</td>
<td>Sections of Beijing-Harbin line supplied by Ixian south ESS</td>
<td>CRH2 (25)</td>
<td>15-23</td>
<td>Supply network arresters</td>
<td>Appliances in ESS and SP</td>
<td>At ESS</td>
<td>Yes</td>
</tr>
<tr>
<td>2</td>
<td>2008.8</td>
<td>Wuxing ESS-Yongle SP of Beijing-Tianjin HSR</td>
<td>CRH2 (25)</td>
<td>25-31</td>
<td>Contact wire arresters (leading power off)</td>
<td>At ESS</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>3</td>
<td>2008.12</td>
<td>Sections of Hefei-Nanjing HSR supplied by Longcheng ESS</td>
<td>CRH2 (25)</td>
<td>17</td>
<td>Supply network arresters</td>
<td>None</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2009.4-5</td>
<td>Sections of Hefei-Wuhan HSR supplied by Luan and Chang' anji ESSs</td>
<td>CRH2 (25)</td>
<td>17-21</td>
<td>Unknown</td>
<td>At ESS</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2011.1</td>
<td>Zhaozhuang-Bengbu over-zone feeding of Beijing-Shanghai HSR</td>
<td>CRH380AL (25)</td>
<td>45-55</td>
<td>Arresters on CRH380BL³</td>
<td>None</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>2011.1</td>
<td>Guangzhou south ESS-Lishui SP of Wuhan-Guangzhou HSR</td>
<td>CRH380A (25)</td>
<td>47-55</td>
<td>Supply network arresters</td>
<td>At ESS</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>2011.11</td>
<td>Shanhaiguan station</td>
<td>HX2³3B</td>
<td>23-29</td>
<td>RC branches of SS4†</td>
<td>On SS4†</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>2011.11</td>
<td>Sections of Harbin-Dalian line supplied by Changchun north ESS</td>
<td>HX2³3B</td>
<td>23-29</td>
<td>Arresters on CRH380BL³</td>
<td>None</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>2012</td>
<td>Qinjiazhuang ESS-Wuzhai Station of Ningwu-Kelan line</td>
<td>HX2³2 (16)</td>
<td>19-29</td>
<td>Arresters on HX2³2</td>
<td>At ESS</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>2013.11</td>
<td>Multi-sections of Beijing-Jilong line between Fuyang and Xiangtang</td>
<td>HX2³1B</td>
<td>53-55</td>
<td>Supply network arresters</td>
<td>Arresters on locomotives</td>
<td>HX2³1B</td>
<td>Yes</td>
</tr>
<tr>
<td>11</td>
<td>2013.11</td>
<td>Handan SP-locomotive depot of Beijing-Guangzhou line</td>
<td>HX2³2B</td>
<td>41-49</td>
<td>Arresters on locomotives</td>
<td>Appliances in ESS and SP</td>
<td>At ESS</td>
<td>Yes</td>
</tr>
<tr>
<td>12</td>
<td>2013.11</td>
<td>Wuliu station of Yangpingguan-Ankang line</td>
<td>HX2³21000</td>
<td>41-49</td>
<td>Unknown</td>
<td>None</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>2016.3-7</td>
<td>Shanghai EMU depot</td>
<td>CRH380D</td>
<td>63-75</td>
<td>None</td>
<td>On CRH380B³</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>2016.11</td>
<td>Sections of Beijing-Baotou line supplied by Kongjiyang ESS</td>
<td>Unknown³</td>
<td>37-51</td>
<td>Appliances in ESS and SP</td>
<td>None</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>2017.4</td>
<td>Sections of Watang-Rizhao line supplied by Liangshan north ESS</td>
<td>Unknown³</td>
<td>19-25</td>
<td>Leading abnormal noise of traction transformer of ESS</td>
<td>None</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>2018.5</td>
<td>Section of Qian’an-Caofedian line supplied by Tanghai ESS</td>
<td>HX2³1 (5)</td>
<td>17-21</td>
<td>Supply network arresters</td>
<td>Discharging gap of HX2³1</td>
<td>At SP, On HX2³1</td>
<td>Yes</td>
</tr>
</tbody>
</table>

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* The frequency of f_c represents the resonant frequency per-unit of 50 Hz.

† Multiple models of locomotives were operating under the same supply, but the exact model that initiated the resonance was not identified.

§ The frequency of f_c represents the carrier frequency of the 4QC modulation per-unit of 50 Hz. The f_c values of HX2³21000, HX2³1B, HX2³2B, HX2³3B and CRH380D are not available.
impedance of the locomotive, respectively:

\[ v_L = \frac{\sum_{i=1}^{n} V_{ab,i}}{n}, \]

(1)

\[ Z_L(j\omega) = Z_1(j\omega) + \frac{Z_2(j\omega)}{n}. \]

(2)

The equivalent multi-level PWM voltage \( v_L \) is the harmonic source in the system. Neglecting any background harmonics, the \( k \)-th harmonic \( v_{L,k} \) of \( v_L \) generated by the PWM process of 4QCs will give rise to a harmonic voltage \( v_{p,k} \) at the pantograph:

\[ v_{p,k} = \frac{Z_{in}(j\omega_1)}{Z_{in}(j\omega_1) + Z_{L}(j\omega_1)} v_{L,k}, \]

(3)

where \( \omega_1 \) stands for the fundamental angular frequency. According to the analysis above, the following can be remarked.

**Remark 1:** Both the equivalent impedances of the TPSS and the locomotive, \( Z_{in} \) and \( Z_L \), create a coupling impedance system containing inductive and capacitive elements. The frequencies corresponding to extreme values of this system’s impedance-frequency characteristics are the resonant points. The equivalent voltage \( v_L \) contains higher-order harmonics which may coincide with the resonant frequency stimulating very high harmonic voltage at the pantograph, \( v_{p,k} \), causing a series of detrimental effects, which is collectively referred to as TPSS resonance.

**Remark 2:** The equivalent impedance, \( Z_{in} \), of TPSS refers to the impedance-frequency characteristics of the ESS and the feeding network. The ATs installed in ATP and SP can be omitted from the resonance analysis since their leakage impedances have limited effect on the resonant frequency. Nevertheless, an SP defines the length of a TPSS that affects \( Z_{in} \) as well as the resonant frequency. Modeling the complex TPSS in frequency domain is an important aspect of resonance analysis which is significant to get a comprehensive understanding of the resonance mechanism.

**B. Modeling of Locomotive Harmonic Characteristics**

1) **Analytical Method:** PS-PWM is generally used for modulating all interleaved 4QCs in a locomotive. DFS can be used to describe one 4QC ac-side PWM voltage waveform in the frequency domain [83], then to express one 4QC ac-side current and entire line current of a train by synthetically considering the circuit parameters and phase shift angles. One can solve the DFS of a PWM waveform by means of 3-dimensional (3-D) [56]–[58] or 2-D [28] graphical method.

Taking a CRH2 series EMU as an example, the DFS expressions of 4QC ac-side voltage and current, \( v_{ab} \) and \( i_{N21} \), and the on-board transformer primary current, \( i_{N1} \) (mainly consisting of two 4QC ac-side currents, \( i_{N21} \) and \( i_{N22} \), for this kind of models) are presented in (4)-(6), respectively. In these equations, \( V_{dc} \) and \( M \) represent the converter dc-link voltage and the modulation index, \( J_{2n+1} \) is the Bessel function of first kind of order \( 2n+1 \), \( \omega_c \) and \( \omega_o \) stand for the angular frequencies of carrier and modulation waveforms respectively, \( \theta_c \) and \( \theta_o \) are the corresponding initial phase angles. Detailed derivation for (4)-(6) can be found in [28], where the DFS expressions, simulation results and measured data are compared. Through the DFS analysis, one can get:

i) Line-side current harmonics of a locomotive are generated by the modulation process used to shape the 4QC ac-side voltages.

ii) The spectrum of a 4QC line-side current is fairly broad, containing odd sideband harmonics around even multiples of the carrier frequency, i.e., \( 2n\omega_c + (2n+1)\omega_o \).

iii) For a particular frequency, the modulation index \( M \) associated with 4QC output power, is the only factor affecting the harmonic amplitude. It is seen as a coefficient of the independent variable of the Bessel function \( J_{2n+1}(2m\pi M) \), which is a bounded function and has a shape of a decaying sinusoidal waveform. \( M \) varies within a narrow margin for

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7Theoretically, there may be more than one resonant frequencies in a TPSS [18], [82]. Fortunately, only the lowest one should be considered for two reasons: this one is easily within the range of the PWM harmonics of lower sideband groups; at higher frequencies the system resistance is high enough to provide a high damping factor [1].
a PWM rectifier [55], therefore, line current harmonic content is relative stable for different operating conditions of a locomotive.

iv) Neither $\theta_c$ or $\theta_o$ affect harmonic amplitudes but influence harmonic phases. To properly set $\theta_c$ for every 4QCs of a locomotive, i.e., PS-PWM, will effectively eliminate the harmonics of the combined line current. Comparing (5) and (6), harmonics around odd multiples of $2m\omega_c$ are canceled since $\theta_c$ of $i_{N21}$ is shifted by $\frac{\pi}{2}$ to that of $i_{N21}$.

This analytical method is good for understanding the generation mechanism of such harmonics and their influence factors, but is mostly suitable for qualitative analysis. In practice, when quantitatively comparing the analytical harmonic solutions and operational data of locomotive line current, there are certain differences. One of the reasons for these differences is that the derivation process sees the supply voltage as an ideal harmonic amplification which will be more apparent when operation and power transmission on the supply network, e.g., voltage source, neglecting the interactions between locomotive operational data of locomotive line current, there are certain differences. One of the reasons for these differences is that

\[
v_{ab}(t) = V_{dc}M \cos(\omega_c t + \theta_o) + \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \frac{V_{dc}}{\pi m} J_{2m+1}(2m\pi M) \cos(n\pi) \cos \left[ 2m\omega_c t + (2n+1)\omega_c t + 2m\theta_c + (2n+1)\theta_o \right],
\]

\[
i_{N21}(t) = \pm \frac{\sqrt{(V_{dc}M)^2 + 2V_c^2}}{\omega_c L} \cos(\omega_c t) - \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \frac{V_{dc}}{\pi m[2m\omega_c t + (2n+1)\omega_o]} J_{2m+1}(2m\pi M) \cos(n\pi) \sin \left[ 2m\omega_c t + (2n+1)\omega_c t + 2m\theta_c + (2n+1)\theta_o \right]
\]

\[
i_{N1}(t) = \frac{i_{N21} + i_{N22}}{N_T} = \frac{1}{N_T} \left\{ \begin{array}{l}
\pm \frac{2 \sqrt{(V_{dc}M)^2 + 2V_c^2}}{\omega_c L} \cos(\omega_c t) \\
- \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \frac{V_{dc}}{\pi m[4m\omega_c + (2n+1)\omega_o]} J_{2m+1}(4m\pi M) \cos(n\pi) \sin \left[ 4m\omega_c t + (2n+1)\omega_c t + 4m\theta_c + (2n+1)\theta_o \right] \end{array} \right\}
\]

\[
i_{N22}(t) = \frac{1}{N_T} \left\{ \begin{array}{l}
\pm \frac{2 \sqrt{(V_{dc}M)^2 + 2V_c^2}}{\omega_c L} \cos(\omega_c t) \\
- \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \frac{V_{dc}}{\pi m[4m\omega_c + (2n+1)\omega_o]} J_{2m+1}(4m\pi M) \cos(n\pi) \sin \left[ 4m\omega_c t + (2n+1)\omega_c t + 4m\theta_c + (2n+1)\theta_o \right] \end{array} \right\}
\]

By observing (5) and (6), it is clear that the second step is to describe the stochastic volatility of the harmonic ratios. The measured data present, in fact, a certain statistical property, i.e., harmonic ratios located around the fitting curves. For each fitting curve, a probability density function, conventionally the well-known Gauss Formula, could be defined to present the distribution property.

iii) The last step is to describe the distribution of harmonic phase angles. Since the angles are affected by the locomotive operation state, i.e., accelerating, constant-speed, regenerative braking and coasting, it is preferable to identify the angles’ varying trends for each operation state. Typically, these trends cannot be easily described by simple expressions, so that look-up tables become an option. Fortunately, the angle distribution property along these trends is also subject to the Gauss formula [67].

For a specific locomotive type, through the three-step process above, line current harmonic characteristics will be formed as a probabilistic model which can be used to predict the locomotive line current for simulations at a design stage. Fig. 5 gives a comparison of CRH2 EMU line currents from simulation using probabilistic model and measurements. As the modeling is based on operational data with all the stochastic and nonlinear factors of the real system, the accuracy satisfies the requirements of related quantitative analysis. An obvious drawback of this method is that it considers the line current spectrum as a kind of external feature of the locomotive thus failing to explain how harmonics are generated.

3) Equivalent Locomotive Model to TPSS: When studying issues on the network-train system, e.g. the resonance, a simple locomotive model connected with the TPSS is usually required. Based on DFS solutions and the on-board transformer impedance, a Thévenin model can be formed to represent a locomotive (see Fig. 6(a) as well as Fig. 4(c) with relevant analysis). Alternatively, it can be readily transferred to a Norton model (see Fig. 6(b)). Treating a locomotive as a branch of a TPSS, according to the substitution principle, a simple and useful way is to represent the locomotive by the
branch current forming a current source mode (see Fig. 6(c)) that could be the line current of analytical solutions, simulation results, probabilistic model based predictions or measured samples.

C. Modeling of TPSS Impedance-Frequency Characteristics

An integrated TPSS impedance consists mainly of three parts: an external power source, an ESS and a section of supply network.

1) External Power Source: When discussing the high-order harmonic resonance in TPSS, the external power source is usually modeled as simple as possible. In practice, limited parameters on grid side are available so that an external power source can be simply modeled as a three-phase Thévenin [85] or Norton [86] equivalent circuit based on the short-circuit source can be simply modeled as a three-phase Thévenin [85] or Norton [86] equivalent circuit based on the short-circuit capacity and nominal voltage of the primary side of an ESS.

2) Electrical Substation: In the ESS, the main component is the traction transformer which should be expressed by its node admittance matrix. Despite various connections, a traction transformer can be generally described by:

\[
\begin{bmatrix}
I_P \\
I_S
\end{bmatrix} = \begin{bmatrix}
Y_{PP} & Y_{PS} \\
Y_{SP} & Y_{SS}
\end{bmatrix} \begin{bmatrix}
V_P \\
V_S
\end{bmatrix},
\]  
(7)

where \(I_P, V_P\) and \(I_S, V_S\) are the current and voltage vectors of the transformer primary and secondary sides. Method for forming transformer models with specific connections is referred to [87]. Node admittance matrices of all other elements (e.g. reactive power compensation branches, tuned filters) need to be formulated separately. In the end, an entire ESS model is established by superposing all matrices according to the ESS main connection.

3) Supply Network: Modeling the supply network is a key part of creating an appropriate TPSS impedance-frequency characteristics. Regardless of supply mode (e.g. direct supply (with negative feeder), AT supply, boost transformer (BT) supply [88]), and regardless of the track number (i.e. single or double), the backbone of a supply network are paralleled multiconductor transmission lines (MTL) forming a chain. A chain network model, which is given in Fig. 7(a), was initially introduced for modeling 2×25 kV AT supply network [89], then expanded to a generalised model for describing all types of traction supply networks [90], [91]. This model consists of both series and shunt elements. The shunt elements, i.e., locomotives, ATs or other cross links, split a network into several series segments with different lengths. Each segment is formed by homogeneous series elements, i.e., MTL.8 The chain network can be described by a node admittance equation of (8), where \(V_i\) and \(I_i\) (\(i = 1, 2, ..., N\)) are the voltage and current vectors at node \(i\). Assuming there are \(m\) paralleled conductors in the network, thus \(Z_i\) and \(Y_i\) (\(i = 1, 2, ..., N\)) of (8) are \(m \times m\) dimension impedance and admittance matrices. It is clear that the node admittance matrix of the network is a tridiagonal banded matrix which can be easily solved by Schechter LU decomposition. Fig. 7(b) illustrates the method for forming this node admittance: one can firstly formulate node admittance matrix of each series element in a \(2m \times 2m\) scale; then superpose all the series element matrices in sequence (two adjacent matrices have a \(2m \times 2m\) overlap); finally the network model can be completed by adding the \(m \times m\) node admittance matrices of shunt components into the overlaps in sequence.

The MTL of each series segment consisting of \(m\) conductors with distributed parameters can be modeled by an equivalent \(\pi\)-type circuit as shown in Fig. 7(c) which is described by:

\[
\begin{align*}
Z_L &= \sinh(\sqrt{ZY})/(ZY)^{-1}Z \\
Y_L/2 &= Y( Zy)^{-1/2} \tanh(\sqrt{ZY}/2) \\
\end{align*}
\]  
(9)

where \(Z\) and \(Y\) are the MTL parameter matrices formed by self- and mutual- impedances and admittances per unit length of the conductors, \(l\) stands for segment length. Generally, for a double-track electric railway \(m > 10\), which can be reduced by combining the equipotential conductors9 [92], [93]. To solve (9), both phase-modal transformation [94] and matrix series [95] can be adopted. Modeling of other series and shunt elements of the network, which are relatively easier, can be found in literature, e.g. [91], [96].

For harmonic studies, i.e. resonance, all basic parameters of aforementioned modeling are linked to frequency. In practice, only fundamental parameters of external grid and transformers are usually available, thus they are assumed linear for high-frequency analysis. For MTL, frequency-dependent parameters can be acquired analytically using the conductors’ material, shape and spatial position, e.g. through the Carson equation. The skin effect is non-negligible at/close to the resonant 8In BT systems the BTs should be seen as series elements as well.
9See the cross section of supply network in Fig. 4(a) where conductors with same colors have same potentials.
D. Resonance Mechanism Analysis

1) Advanced Analysis Methods: Time-domain simulation (TDS) may be the most efficient approach to test harmonic response in a TPSS and is commonly utilised, e.g. [17], [30], [42], [53], [69], etc.. However, in general simulation environments (Matlab/Simulink, PSCAD/EMTDC, etc.) the accuracy is limited for the following reasons: the distributed supply network can just be represented by lumped RLC cells with arbitrary defined small lengths; implementing frequency-dependent RLC parameters is not possible; the skin effect or other nonlinearities are also hard to be included.

More accurate analysis is usually carried out by means of frequency scan (FS) [82] and resonance mode analysis/assessment (RMA) [96]. The FS is based on the calculation acting on system node admittance, $Y_s$, which has a inverse, $Y_s^{-1} = Z_s$, with impedance nature. Its element $z_{k-j}$ is defined as driving impedance and scanned in frequency range of interest, yielding the information that how much voltage at node $i$ will be excited by 1 p.u. current injected into node $j$; the maxima and minima of the resulted impedance-frequency curve indicate parallel and series resonances, respectively.

The RMA is based on the eigen-analysis, which transfers the system into modal domain, i.e., $Y_s \rightarrow Y_{sm}$. The parallel resonances can be identified through modal analysis which results in sensitivity information of the components of a complex system. Details of the RMA are provided in [99], [100] and [96] for its application for TPSS resonances. The main advantages and disadvantages of FS and RMA as well as TDS are summarised in Table II.

A number of factors impacting the resonance in terms of frequency and magnitude are available regardless of the analysis methods leveraged. However, some, especially the quantitative ones, are quite dependent to specific cases studied with detailed parameters. From literature ( [17], [53], [69], [82], [96], etc.) there are some general findings for the primary parallel resonance concerned: i) the MTL of supply network and traction transformer in ESS dominate the TPSS resonance, other elements (e.g. external grid) have less prominent effect; ii) longer distance of the supply network corresponds to lower resonant frequencies; iii) resonant harmonics tend to be amplified more at the far-end of a supply section; iv) the resonance magnitude will decrease when increasing inductive

TABLE II
ADVANCED RESONANCE ANALYSIS METHODS

<table>
<thead>
<tr>
<th>Methods</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>TDS</td>
<td>Higher efficiency</td>
<td>Lower accuracy</td>
</tr>
<tr>
<td>FS</td>
<td>Great accuracy</td>
<td>Scan nodes one by one</td>
</tr>
<tr>
<td></td>
<td>Parallel &amp; series resonance</td>
<td>No sensitivity information</td>
</tr>
<tr>
<td>FS</td>
<td>Explicit physical meaning</td>
<td></td>
</tr>
<tr>
<td>RMA</td>
<td>Sensitivity information</td>
<td>No straight link between node number &amp; node location</td>
</tr>
<tr>
<td></td>
<td>Quantizes component impact</td>
<td>Complex process</td>
</tr>
</tbody>
</table>

With detailed parameters. From literature ( [17], [53], [69], [82], [96], etc.), there are some general findings for the primary parallel resonance concerned: i) the MTL of supply network and traction transformer in ESS dominate the TPSS resonance, other elements (e.g. external grid) have less prominent effect; ii) longer distance of the supply network corresponds to lower resonant frequencies; iii) resonant harmonics tend to be amplified more at the far-end of a supply section; iv) the resonance magnitude will decrease when increasing inductive
parameters; an opposite effect will be obtained by increasing capacitive parameters; v) the resonant frequency can be simply seen independent of the train location\(^ {11}\), however, the resonant amplification is more apparent when the train is located further away from the ESS.

2) Simplified Analysis: When addressing real resonant incidents, it is preferable to use simpler modeling and analysis methods. Therefore, a simplified resonance mechanism analysis is adopted here. A TPSS can be simplified as an equivalent model (Fig. 8), where \(L\) is the equivalent inductor of the ESS (mainly the transformer leakage impedance and the equivalent impedance of external power source), \(z\) and \(c\) stand for the series impedance and parallel capacitance of the equivalent single-phase circuit per unit length, \(i_h\) represents the locomotive (as current harmonic source), \(D\) and \(x\) are the distances from the ESS to the SP and the locomotive, respectively.

The equivalent impedance seen from the harmonic source to the ESS (left) is [1]:

\[
Z_1 = Z_c \cosh(\gamma x) + Z_e \sinh(\gamma x),
\]

where \(Z_e = \sqrt{z/c}\) and \(\gamma = \sqrt{j\omega c}\) are the characteristic impedance and propagation constant of the equivalent single-phase circuit, respectively. The equivalent impedance seen from the harmonic source to the SP (right) is:

\[
Z_r = \frac{Z_c}{\tanh[\gamma(D - x)]}.
\]

The integrated impedance of the TPSS seen from the harmonic source is:

\[
Z = Z_1/Z_r = \frac{Z_1 Z_r}{Z_1 + Z_r}
\]

\[
= Z_e \cosh[(\gamma D - x)] \frac{j\omega L \cosh(\gamma x) + Z_e \sinh(\gamma x)}{j\omega L \sinh(\gamma D) + Z_e \cosh(\gamma D)}.
\]

According to (13), it is easy to find \(Z = \infty\) at the resonant frequency, implying:

\[
j\omega L = \frac{Z_c}{\tanh(\gamma D)}.\]

Since \(\gamma D \ll 1\), \(\tanh(\gamma D) \approx \gamma D\), eq. (14) can be rewritten as:

\[
j\omega L \approx -\frac{Z_c}{\gamma D} = -\frac{1}{j\omega D} = -\frac{1}{j\omega C},
\]

where \(C = cD\) is the total parallel capacitance of the whole supply network. Therefore, from (15), an approximate resonant frequency equation is acquired:

\[
f \approx \frac{1}{2\pi\sqrt{LC}}.
\]

According to the analysis above, the following conclusions about TPSS resonance can be reached:

i) Inherently, the TPSS resonance can be seen as a parallel resonance between the distributed capacitance of the supply network and the equivalent inductance of the ESS, always stimulating overvoltages in the TPSS.

ii) The TPSS resonant frequency depends on its own distributed electrical parameters of the supply network, the impedance frequency characteristics of the transformer and external power source of the ESS. The location of the locomotive within a section does not affect the resonant frequency.

iii) Under the same conditions of external power source, transformer and conductors (supply network), the longer a supply section length is, the larger its parallel capacitance (\(C\)) leading to lower resonant frequencies. There is an approximately inversely proportional relationship between the resonant frequency of a supply section to the square root of its length.

These useful conclusions from the simplified analysis readily explain the resonance phenomena summarised in Table I of Section III: a parallel resonance always excites overvoltage damaging electrical devices or triggering the protective relay; for a specific TPSS section (see each case of Table I), the frequency range of harmonic amplification is fixed, however different sections have different resonant frequencies because of divergent ESSs and supply networks. Although the incidents took place in different zones, they share a common condition, long equivalent parallel supply networks leading to lower resonant frequencies which are prone to be excited by PWM harmonics of electric locomotives or EMUs.

IV. RESONANCE ELIMINATION

Since the resonance is related to both the TPSS and electric locomotive, one can eliminate it through ground-based and on-board approaches.

A. Ground-based Harmonic Suppression

1) Installing Passive Filters in ESSs/SPs: Passive filters (PFs), consisting of RLC elements without complex control, are structurally simple, easy to design and reliable. Typical PF topologies of power industry are analysed in [101] and their features are summarised in Table III.

In railway supplies, single-tuned filters (STFs) are usually installed in ESSs to filter out the low-order harmonics; the first-order high-pass filter (1-HPF), i.e. series RC, can be an option addressing higher-order harmonic issues (e.g. applied in...
Advantages and Disadvantages of Typical Passive Filters

<table>
<thead>
<tr>
<th>Topologies</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>STF</td>
<td>Strong attenuation</td>
<td>Frequency specific</td>
</tr>
<tr>
<td></td>
<td>Low cost</td>
<td>Introduce resonances</td>
</tr>
<tr>
<td>1-HPF</td>
<td>Simple structure with HP nature</td>
<td>Require large capacitor</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Large fundamental loss</td>
</tr>
<tr>
<td>2-HPF</td>
<td>Best HP performance</td>
<td>Large fundamental loss (but less than 1-HPF)</td>
</tr>
<tr>
<td>3-HPF</td>
<td>Good HP performance</td>
<td>Introduce resonances</td>
</tr>
<tr>
<td></td>
<td>low fundamental loss</td>
<td></td>
</tr>
<tr>
<td>CTF</td>
<td>Better HP performance</td>
<td>Sensitive to parameter variation</td>
</tr>
<tr>
<td></td>
<td>low fundamental loss</td>
<td></td>
</tr>
</tbody>
</table>

South Korea [102]); second-order HPFs (2-HPFs), also named as second order damping filters (SODFs), are an effective solution to TPSS resonances (applied in Japan [21], Zimbabwe [2] and China [74]); third-order HPFs (3rd HPFs) are not a common choice in this application; C-type filters (CTFs) with their promising features, have certain potential in TPSS for harmonic problems [31].

Fig. 9 (a) presents the main circuits of SODFs with their connection to 25 kV and 2×25 kV TPSSs. Fig. 9 (b) shows a set of SODF equipment which is installed in the Changchun north sub-feeder switching post (SFSP) to solve the resonances (case 8 of Table I). As a result, the supply voltage there has been filtered to a nearly sinusoidal waveform after the SODF operation, as shown in Fig. 9 (c). Up to now, 12 sets of SODFs have been installed and commissioned in ESSs and SPs in Chinese railways. Instances of harmonic resonance in these sections have been successfully eliminated.

Note that all PFs, regardless of topology, do not only filter out harmonics but move resonant frequencies. Despite their high cost and additional land requirements, installing them in ESSs or SPs is a simple, reliable and validated approach for addressing harmonic resonance issues.

2) Installing Active Power Filters in ESSs/SPs: Active power filters (APFs) have been widely used in the electric utility industry for power quality regulation owing to their distinct advantages of small size, controllable output (both frequency and amplitude) and compatible compensation (reactive power, voltage drop and imbalance) compared to traditional PFs [103]–[106]. Nevertheless, APFs are still not a common choice in TPSS applications. According to the technical literature (e.g. [107], [108]), a step-down transformer is usually designed to match the voltage level between the IGBTs and the traction supply network. As reported in [109], [110], a set of straight hanging APF based on cascaded H-bridge (CHB) is designed and installed in an ESS of Beijing-Shanghai HSR line. This is, up to now, the only APF in use in the Chinese railways.

Unfortunately, APFs usually compensate harmonics lower than the 11st. In a word, for the purpose of TPSS resonance elimination, current APF techniques are hard to simultaneously satisfy the requirements of high capacity (several MVA), high voltage (line to ground 27.5 kV) and real-time high-frequency precise control for high-order harmonic compensation (microsecond precision). However, with the development of wide band-gap semiconductor devices (SiC), using APFs to solve power quality issues of TPSSs is still a promising application.

3) Other Provisional Measures on the Ground: Other approaches include: i) switching the external power source of the ESS, ii) changing the supply network operation mode (e.g. changing over-zone supply), etc. These approaches, that efficiently change the TPSS impedance-frequency characteristics (resonant frequency) without the time or cost of construction of the two previous methods, can be provisional measures for resonance suppression.

B. On-Board Harmonic Suppression

1) Conventional PS-PWM: Generally, a locomotive or an EMU has a number of interleaved 4QCs operating under PS-PWM for getting a higher equivalent switching frequency and lower harmonic content. This is the most conventional on-board harmonic suppression approach [56], [58]. In practice, in June 2009, the PS-PWM scheme and controller parameters of the 4QCs of CRH2-200 EMUs were upgraded to improve the line current harmonic characteristics. A set of comparative spectra of CRH2-200 EMU line current measured before and after the upgrade is given in Fig. 10. Since then, few resonances stimulated by CRH2 series EMUs (See Table I).

2) Improved PWM methods: From the PWM perspective, a properly designed PWM scheme that does not generate harmonic components within the resonant frequency range is a straightforward solution. At this point, selective harmonic elimination PWM (SHE-PWM) with its distinct advantage of tight spectrum control [111] can be chosen as an alternative.
to traditional PS-PWM. In [112], SHE-PWM was initially introduced for two interleaved EMU 4QCs, but the resonance issue was not considered in that work. A SHE-PWM, named resonant harmonic elimination PWM (RHEPWM), was proposed in [75] for resonant harmonic elimination. Nevertheless, the RHEPWM provides a fixed SHE-PWM pattern for a specific resonant frequency, it fails to tackle the resonant frequency variation when the train travels across different supply sections (Section III-D).

The windowed SHE-PWM (WSHE-PWM) scheme for interleaved 4QCs of [76] provides two distinct windows of eliminated harmonics: a base window (BW) fixed in low frequency range plus a moving window (MW) within the potential resonant frequency range (see Fig. 11). In the BW, the fundamental component is controlled to specific modulation index, \( M \), and all the odd harmonics are eliminated. The MW addresses the resonance for a particular TPSS section and can move accordingly. The frequency range of potential resonances is split into several sub-regions, W1-W5; each of them is with a frequency width equal to the MW. When the resonant frequency is in a sub-region, all harmonics within the window are eliminated. Figs. 12(a) and (b) present a set of experimental performance of PS-PWM and WSHE-PWM with same equivalent switching frequency and equivalent resonant circuit applied. The differences of the equivalent pantograph voltage, \( v_p' \), on both waveform shapes and spectra show the resonant harmonic elimination effect of the WSHE-PWM. Interested readers are referred to [76] for details of WSHE-PWM and more papers, e.g. [113], [114], for methods about solving SHE-PWM problem of multiple interleaved and multi-level cases.

However, it should be noted that implementing a low frequency SHE-PWM scheme in a closed-loop system is not a trivial task. As a result, there are few successful cases of SHE-PWM used in traction 4QCs applications, despite its inherent suitability for the purpose of resonance elimination.

3) On-board filters: Some earlier models of electric locomotives, such as the German Class 120 locomotive, the Chinese SS4 locomotive and the CRH1 EMU [18], [115], were equipped with RLC based PFs connecting to either the primary or secondary windings of the on-board transformer. Thus, in most situations, harmonics can be suppressed from the generation-end. In contrast, an on-board PF offers a low damping path absorbing resonant harmonic currents from the overhead line in case of a section already being in resonance stimulated by other locomotives. In general, the PF capacity, which is designed for a single locomotive, is inadequate for a whole section. Such PFs, especially their resistors, can be easily burnt during extreme resonances. Considering the reliability, volume and cost, a number of new locomotive models have abandoned this filter design approach.

An alternative is an on-board APF which provides controllable frequency-domain characteristics compared to on-board PF and has lower capacity and voltage requirements compared to ground APF. German and Japanese researchers proposed similar solutions [48], [116]; adding an auxiliary winding to ground APF. German and Japanese researchers proposed similar solutions [48], [116]; adding an auxiliary winding to an on-board transformer and connecting an APF to the additional winding. This APF is used to compensate all the harmonic currents from the 4QCs connected on secondary windings. Mainly restricted by the switching frequency of the IGBTs fitted, on-board APFs cannot cover all potential

![Fig. 10. CRH2-200 EMU line current harmonic spectra before and after the PS-PWM scheme upgrade.](image1)

![Fig. 11. Concept of windowed SHE-PWM utilising W3, eliminating resonant harmonics between 2000 and 2500 Hz.](image2)

![Fig. 12. Experimental result on the resonant circuit: (a) PS-PWM with 250 Hz carrier frequency, (b) windowed SHE-PWM with W3, M = 0.71. CH1: \( v_p' \), 100 V/div, CH2: \( i_p' \), 20 A/div, CH3: \( v_{PWM} \), FFT for CH3, 20 V/div-y, 500 Hz/div-x.](image3)
TABLE IV
TPSS Resonance Elimination Approaches

<table>
<thead>
<tr>
<th>Approaches</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground based</td>
<td>Simple and reliable for resonance elimination</td>
<td>Large land requirement, High cost, Large power consumption</td>
</tr>
<tr>
<td>PFs</td>
<td>Smaller than PFs, Controllable output</td>
<td>Limited to low-order harmonics, Less reliable than PFs</td>
</tr>
<tr>
<td>APF</td>
<td>Other provisional measures</td>
<td>Fast implementation, Low cost, Limited elimination effect</td>
</tr>
<tr>
<td>SHE-PWM</td>
<td>PS-PWM</td>
<td>Easy to implement, Generate fixed side-band harmonics</td>
</tr>
<tr>
<td>PS-PWM</td>
<td>SHE-PWM</td>
<td>Controllable spectra, Difficult to implement</td>
</tr>
<tr>
<td>PFs &amp; APFs</td>
<td>PEs &amp; APFs</td>
<td>Smaller sizes, lower power and voltages than ground based counterparts, Need space in carriages, Train reliability</td>
</tr>
</tbody>
</table>

resonant frequency ranges. A hybrid solution with active and passive filters for locomotives, consisting of an APF, an RC branch and an SODF, can also be used [117]. Full-scale experimental results show that in the hybrid system, the higher-order harmonics are filtered through the SODF, the APF plays the role of lower-order harmonic compensator, and the RC branch absorbs sideband harmonics generated by the APF itself. The main advantages and disadvantages of the TPSS resonance elimination approaches discussed above are summarised in Table IV.

V. IDENTIFICATION OF RESONANCES IN REAL SYSTEMS

Generally, one treats TPSS resonances as sporadic issues taking elimination measures after a resonance incident occurs. Practically, it is hard to prevent TPSS resonances. On the one hand, there is a large number of electric locomotive models with diverse line-side current harmonic features. Moreover, details of modulation schemes and control strategies of the locomotive line-side converters (i.e., the 4QCs), are confidential to the manufacturers, but important for resonance elimination. Although harmonic features of locomotives could be found from their type-test reports, TPSS resonant frequencies are difficult to accurately calculate, predict or evaluate through a model-based analytical way.

A. Assessing TPSS resonant frequencies

The resonant frequencies of a TPSS depend on the impedance-frequency characteristics of both the supply network and the ESS (Section III-C and Section III-D). However, the extremely high diversity of sections in the railway network, in terms of lengths, structure, construction (i.e. bridges, tunnels, etc.) together with non-linearities, especially the skin effect in the conductors, creates a large number of permutations and introduces complexities in mathematical modeling as well as implementation in a time-domain simulation environment such as MATLAB/Simulink or PSCAD.

Furthermore, the harmonic impedance of the external power supply, which supplies and is also part of the ESS impedance, is typically not carefully considered when modeling the TPSS in the high-frequency harmonic range. In fact, network configuration and customer loads connected to the external power supply will affect the harmonic impedance and the resonant frequencies of a given TPSS. Reliable parameter estimation of impedances from model-based calculation or assessment is typically limited to 20 p.u. in frequency and does not cover the whole range of possible resonant frequencies [118], [119].

In short, lack of comprehensive data, the almost infinite number of permutations and non-ideal behaviour of the actual system pose severe difficulties in accurately calculating TPSS resonant frequencies.

B. Resonant frequency measurement

An alternative to model-based calculations for identifying the resonant frequencies of a TPSS is based on harmonic injection and measurement through a power electronics based harmonic generator (HG) [120]–[122].

The concept of TPSS resonant frequency measurement based on the HG is shown in Fig. 13(a). The HG is connected between the catenary and the rail, similar to a locomotive. A harmonic current \( i_g \) is then injected into the TPSS resulting in a harmonic voltage \( v_g \) as a response. Neglecting background harmonics, the equivalent input impedance of the TPSS at the test location can be calculated as follows:

\[
Z_{TPSS}(f_g) = |Z_{TPSS}| \theta_{TPSS} = \frac{v_g(f_g)}{i_g(f_g)}, \tag{17}
\]

where \( f_g \) is the objective harmonic frequency of generated current \( i_g \). Through a frequency sweep across the range of interest, the TPSS impedance-frequency characteristics can be acquired. The peak value of (17) corresponds to the resonant frequency of the TPSS under measurement.

How the HG generates a current with arbitrary spectrum becomes a power electronics issue. As shown in Fig. 13(b), the HG is based on a standard CHB converter connecting to the TPSS through a reactor, \( L \), and a step-down transformer. In simple terms, this is a very feasible plan to control harmonic up to thousands Hz\(^{12}\) at the voltage level of 27.5 kV. Through dedicated control and modulation process, the CHB ac-side composite voltage \( v_{CHB} \) can be fully controlled, injecting current \( i_g \) with harmonic controllable in both frequency and amplitude. In practice, inter-harmonics can be generated to enhance measurement precision and minimise error from background noise. Interested readers are referred to [121], [122] for details of the measuring method and the HG.

Utilising the proposed measuring method and HG, a first test was carried out in May 2018 on a newly designed Chinese HSR line. Configuration of the field test is given in Appendix A. Fig. 13(c) provides a set of impedance-frequency characteristics of a TPSS section measured at ESS and SP, respectively, illustrating the following:

\(^{12}\)In practical implementation, 5000 Hz is set for the developed HG as the upper limit to cover the potential resonant frequency range.
Primary resonance: A parallel resonant point is identified at 1325 Hz where $|Z_{TPSS}|$ shows a maximum. As expected, this frequency is independent of the test location. However, the maximum $|Z_{TPSS}|$ at this frequency seen from tail end (SP) is about twice that of seen from the head end (ESS). The phase response, $\theta_{TPSS}$, gives additional information: the system is almost purely inductive at low frequencies (<1000 Hz); the system turns to almost purely capacitive (>1500 Hz); after crossing through the resonant point where the system behavior is resistive; all these features can be readily explained by a LC parallel circuit like Fig. 8.

Series resonance: Although not the main concern, series resonant points are also identified. Zooming in $|Z_{TPSS}|$ at high frequencies (>2000 Hz), two minima at approximately 2325 Hz and 2825 Hz appear for ESS and SP test results respectively; $\theta_{TPSS}$ shows that at these frequencies the system is resistive, and crossing them turns the system from capacitive to inductive. It confirms that the further a test point to ESS, the higher series resonance is [69].

Testing error: At high frequencies, the measured $\theta_{TPSS}$ deviate from their expected trajectories for reasons such as the decreasing phase-angle precision of the current sensor and the higher electromagnetic noise introduced from the environment.

Knowledge of the TPSS impedance-frequency characteristics is a key requirement in providing methods to prevent resonances. These methods will be beneficial to the commissioning of newly designed railway lines and locomotives.

VI. DISCUSSION AND SUMMARY

This paper addressed the issue of TPSS resonance, discussing phenomena, mechanism as well as elimination and pre-identification. The provided analysis is based on a large amount of data, extensive measurements and operation experience from Chinese railways over the past decade. Considering both academic and engineering requirements, the work is expected to provide a comprehensive overview of this issue to both researchers and practitioners in the field.

In summary, the high-order harmonic resonance is a typical power quality issue linked to both the TPSSs (impedance frequency characteristics) and the electric locomotives (line current harmonic characteristics). The two parts constitute an interaction system; the former as the dominant resonant circuit that carries the energy flow and the later playing the role of harmonic source that generates high-frequency harmonics injected into the system. Occurrence of harmonic resonance depends on the coincidence of the two in frequency.

Although resonances in TPSSs have occurred for decades, they still represent an unsolved issue in railway supply. The basis to figure out this issue is to break the resonant conditions at either the TPSS or the locomotive side. Practical experience shows that simpler engineering approaches should be considered before resolving to advanced elimination methods. For future work, if reliable information on the locomotive current harmonics and TPSS resonant frequencies can be acquired, prevention at the design stage is a more efficient approach to elimination after an event.

APPENDIX A
FIELD TEST CONFIGURATION

Fig. 14 demonstrates the test sections and points of the line attached with a photo from the field testing. By switching the breakers in ESSs, SP and ATPs, a 51.5 km long line (from ESS2 at K487+011 to ESS1 at K538+511 including over-zone supply) was set to 11 operation modes. As a result, the TPSS impedance frequency characteristics under all these modes were obtained. The results in Fig. 13(c) are of section 1 (from ESS1 at K538+511 to SP at K512+261) operating under all parallel 2×25 kV AT supply mode (This is the normal operation mode for Chinese HSRs where the supply network of up and down tracks are connected in parallel at ESS, ATP and SP to provide higher current-carrying capability and more redundancy.).
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