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Generating Maximal Entanglement between Spectrally Distinct Solid-State Emitters

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We show how to create maximal entanglement between spectrally distinct solid-state emitters embedded in a waveguide interferometer. By revealing the rich underlying structure of multiphoton scattering in emitters, we show that a two-photon input state can generate deterministic maximal entanglement even for emitters with significantly different transition energies and linewidths. The optimal frequency of the input is determined by two competing processes: which-path erasure and interaction strength. We find that smaller spectral overlap can be overcome with higher photon numbers, and quasimonochromatic photons are optimal for entanglement generation. Our work provides a new methodology for solid-state entanglement generation, where the requirement for perfectly matched emitters can be relaxed in favor of optical state optimization.

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Quantum technologies promise dramatic improvements in computing and communication by utilizing quantum entanglement between qubits [1]. Although many promising quantum technology architectures have emerged over the past two decades, none are free from the practical challenges presented by high-fidelity quantum control and scalability. For example, superconducting circuit implementations enjoy excellent coherence properties but operate slowly [2], while trapped ion qubits can be prepared with almost unit fidelity but are difficult to scale [3]. Solid-state architectures, such as optically coupled spin systems, compete on speed and scalability. They include semiconductor quantum dots and nitrogen-vacancy centers. Large optical nonlinearities in solid-state systems are now very common [4–6], and solid-state emitters are readily integrated into complex photonic structures, further enhancing the light-matter interaction [7]. However, there are many challenges still to overcome. For example, charge noise and phonon scattering have limited the size of the optical nonlinearities observed thus far [4].

Another major drawback to solid-state emitters is that the central energies and lifetimes of their transitions are highly dependent on the fabrication process, and vary significantly both across and within samples [8]. Known methods for entangling solid-state qubits require emitters with identical energies to facilitate path-erasure techniques [9,10]. This adds a practically insurmountable overhead to the process of matching multiple solid-state qubits for creating large entangled states [11]. Stark shifting and strain tuning the emitter transitions has been employed to tune solid-state emitters onto resonance [12–14], but this requires a substantial technical overhead, and arbitrary emitters in a sample cannot in general be tuned onto resonance. Here, we propose a process for generating entanglement that is robust against spectral variations in the emitters’ transition energies and linewidths. We show that photons, linear optics, and photon counting suffice to create deterministic entanglement between imperfectly matched emitters, revealing a rich underlying structure of multiphoton scattering off two nonidentical emitters. While many challenges remain, this work removes a major obstacle to a scalable solid-state quantum technology architecture.

Our setup is shown in Fig. 1. Two solid-state emitters each have an $L$-type level structure, with two stable low-lying spin states ($|\uparrow\rangle_i$, $|\downarrow\rangle_i$), and a dipole transition that
couples a spin state to an excited state \( |e\rangle \). The transition energy for emitter \( \alpha = 1, 2 \) is \( E_\alpha \), and the polarization is circular due to selection rules. The emitters are initially prepared in the product state \( (|\uparrow\rangle + |\downarrow\rangle)(|\uparrow\rangle + |\downarrow\rangle)/2 \) and embedded in a waveguide Mach-Zehnder interferometer at \( c \) points, where perfect correlation between propagation direction and circular polarization occurs [15]. Consequently, the emitters scatter circularly polarized light only in the forward direction, as was demonstrated recently using semiconductor quantum dots under an applied magnetic field [16–19]. For a lossless waveguide, the emitter will impart a \( \pi \)-phase shift to each photon that is on resonance with the transition [20–23]. The input to the interferometer is a two-mode Fock state \( |n,m\rangle \), and the detectors \( D_1 \) and \( D_2 \) produce classical signatures \( (p,q) \) indicating the presence of \( p \) and \( q \) photons, respectively.

Assuming the emitters are identical, a single monochromatic resonant photon injected into either one of the input arms of the interferometer will scatter from one of the emitters, and after the final beam splitter the light-matter interaction direction and circular polarization occurs [15]. The input to the interferometer is a two-mode Fock state \( |n,m\rangle \), and the detectors \( D_1 \) and \( D_2 \) produce classical signatures \( (p,q) \) indicating the presence of \( p \) and \( q \) photons, respectively.

In practice, both the linewidths and transition energies vary significantly between solid-state emitters, and it was generally assumed that this prohibits the creation of perfect entanglement using linear optics and photodetection. In this case, the input photon can no longer be resonant with both emitters simultaneously. With \( \hbar \omega \) the single-photon energy, \( \Gamma_\alpha \) the unidirectional emission rate of emitter \( \alpha = 1, 2 \) into the waveguide, and \( \gamma_\alpha \) the corresponding coupling to nonguided modes, the scattering process is described by the transmission coefficient [25]:

\[
t_{\alpha}(\omega) = \frac{\hbar \omega - E_\alpha - i\hbar(\Gamma_\alpha - \gamma_\alpha)/2}{\hbar \omega - E_\alpha + i\hbar(\Gamma_\alpha + \gamma_\alpha)/2}.
\]

We characterize the emitter loss by \( \beta_\alpha \equiv \Gamma_\alpha/(\Gamma_\alpha + \gamma_\alpha) \). For nonzero emitter detuning \( \delta \equiv E_2 - E_1 \), \( t_{\alpha}(\omega) \) ceases to be a \( \pi \)-phase shift, and for \( \beta_\alpha < 1 \), \( t_{\alpha}(\omega) \) is no longer a pure phase shift. The setup then does not create maximally entangled states deterministically anymore. Nevertheless, we will now demonstrate how tailoring the optical input state \( |n,m\rangle \) into the Mach-Zehnder interferometer leads to deterministic maximal entanglement between two spectrally distinct emitters.

In general, a detector signature \( (p,q) \) indicates that the two emitters are in a mixed entangled state. We use the concurrence \( C(\rho) \) for a two-qubit state \( \rho \) to quantify this entanglement [26]. Each signature \( (p,q) \) occurs with probability \( \Pr(p,q) \) and results in an emitter state \( \rho_{(p,q)} \), leading to a concurrence \( C(\rho_{(p,q)}) \). We define the average concurrence as

\[
C_{av} = \sum_{(p,q)} \Pr(p,q)C(\rho_{(p,q)}).
\]

This is an appropriate figure of merit, since it provides a lower bound for the amount of entanglement expected from a given experiment without postselection. The entanglement in the two-qubit state can be increased by discarding measurement outcomes corresponding to below-average concurrences. This comes at the expense of the rate of entanglement generation.

The amount of entanglement that can be generated between the two spectrally distinct emitters with a single probe photon is shown in Fig. 2. The single-photon protocol is analyzed using linear optics transformations.
while a multiphoton input requires taking into account the nonlinear nature of the interaction [22] [see Supplemental Material (SM) for details [27]]. As expected, for spectrally distinct emitters the average concurrence does not reach its maximal value [Fig. 2(b)]. The amount of entanglement is determined by two competing processes. On the one hand, which-path information for the probe photon must be erased, while at the same time the phase shift induced by the photon scattering event must be maximized. Tuning closer to either emitter increases the relative phase shift but also imparts a degree of path information onto the probe, as the light-matter interaction is now stronger for one of the emitters. For emitters with finite detuning and linewidth it is not obvious which photon energy maximizes the average concurrence. Three emitter linewidths are shown in Fig. 2(a), and Fig. 2(b) shows the corresponding $C_{av}$. The linewidths correspond to emitters with 1, 0.66, and 0.33 ns lifetimes, typical of semiconductor QDs benefiting from modest Purcell enhancements [28]. Increasing the linewidth of the emitters leads to a larger spectral overlap, thereby erasing some of the which-path information and increasing $C_{av}$. Figure 2(c) shows the optimal frequency of the input photon that maximizes $C_{av}$. For narrow linewidths it is preferable to tune the photon energy away from the mean emitter energy $(\hbar \omega - E_1 = 0.5 \mu eV$ for $\delta = 1.0 \mu eV)$, and towards resonance with one of the emitters. Though this reduces the concurrence in the state heralded by a click at detector $D_2$, it does increase the probability of a successful scattering event.

One may expect that a photon with a wide frequency bandwidth that overlaps with both emitters will improve the entanglement generation. Figure 2(d) shows the average concurrence for a single probe photon with Lorentzian, Gaussian, and square spectral profiles, centered at $\hbar \omega = E_1 + \delta/2$, as a function of the photon bandwidth. We find that increasing the bandwidth of the input photon only degrades the average concurrence, and a narrow band probe is always preferable. We attribute this to the reduced temporal extent of the photon at larger bandwidths, which increases the probability of exciting the emitter, and thus the fraction of light emitted incoherently through spontaneous emission. This reduction is particularly noticeable for a Lorentzian wave packet, where a close spectral match with the emitter increases the excitation probability. We conclude that for given emitter detuning and linewidths, the maximum $C_{av}$ of the single-photon case is limited by the competing requirements of maximizing the induced phase shift and path erasure.

Next, we consider whether two photons can increase the average concurrence. Consider an input state of two identical monochromatic photons $|n, m\rangle = |1, 1\rangle$ entering the interferometer. They will evolve into a two-photon NOON state $(|2, 0\rangle - |0, 2\rangle)/\sqrt{2}$ via Hong-Ou-Mandel interference on the first beam splitter [29,30] and interact

![FIG. 3. Two-photon (i.e., |1, 1\rangle) entanglement generation for a pair of detuned L-type emitters with equal linewidth $\Gamma$ and energies $E_1$ and $E_2 = E_1 + \delta$, where $\delta = 1.0 \mu eV$. (a) Average concurrence versus monochromatic two-photon energy. The emitters have equal linewidth of 0.66 $\mu eV$ (blue line), 1.0 $\mu eV$ (red line), and 2.0 $\mu e V$ (purple line), (b) $C_{av}$ for Lorentzian (solid line), Gaussian (dotted line), and square (dashed line) single-photon envelopes as a function of FWHM pulse width $\sigma$. Here, $\hbar \omega = (E_1 + E_2)/2$ and $\Gamma = 1.0 \mu e V$. The vertical line indicates the linewidth of the emitters. (c) Average concurrence as a function of $\beta = \beta_1 = \beta_2$ for a monochromatic two-photon pulse (solid line) and monochromatic single-photon pulse (dashed line); both emitters have linewidths of 1.0 $\mu e V$. (d) Average concurrence as a function of normalized emitter detuning $\delta/\Gamma$ for two monochromatic photons (red lines) and a single monochromatic photon (blue lines). Solid lines represent $\beta = 1$ and dashed lines represent $\beta = 0.9$. The optimal photon frequencies for various coupling and detuning ratios are discussed in the Supplemental Material [27].](image-url)
The emitter and therefore achieves the required \( \pi \)-phase shift. We consider more general emitter detuning and linewidth examples in Fig. 4 and in the Supplemental Material [27].

We study again the effect of broadband probe photons on the entanglement generation process. In this case the phase shift is no longer additive in photon number, since the interaction becomes nonlinear. When two photons are present, the emitter may be excited, which opens a pathway for stimulated emission, such that the coherence of the wave packet is maintained. Competition between spontaneous and stimulated emission processes leads to the two-photon case, each photon imparts a \( \pi/2 \) phase shift to the emitter and therefore achieves the required \( \pi \)-phase shift. We consider more general emitter detuning and linewidth examples in Fig. 4 and in the Supplemental Material [27].

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In Figs. 4(a) and 4(b) we show the maximum \( C_{av} \) (optimized over photon frequency \( \omega_{opt} \)) for the one- and two-photon input states as a function of the emitter detuning and the emitter linewidth ratio. The single-photon case outperforms the two-photon case if the emitters are spectrally collocated, or if one of the emitters significant overlaps the other, however narrow the linewidth. Crucially, however, by exploiting the multiphoton additivity of the phase shift, a two-photon process can efficiently generate entanglement for any finite detuning without requiring arbitrarily small emitter lifetimes. The converse of this is also true: for any combination of linewidths \( \Gamma_1 \) and \( \Gamma_2 \) there exists a nonzero emitter detuning which creates deterministic maximal entanglement given the optimal two-photon input state. In practice, this means a much greater freedom in matching solid-state emitters for entanglement generation in a Mach-Zehnder interferometer than previously thought.

We extended the entanglement generation process to monochromatic \( [n,m] \) Fock states into the interferometer. In Figs. 4(c) and 4(d) we show the maximum \( C_{av} \) as a function of the emitter detuning and the emitter linewidth ratio for input states \([2,1]\) and \([2,2]\), respectively (for more examples, see the SM [27]). There is a marked improvement in the entanglement generation over the single- and two-photon processes, with larger areas of parameter space achieving a near unity \( C_{av} \). Remarkably, this indicates that a wide range of imperfections in the fabrication of two identical emitters can be overcome by optical state optimization. Note that the \([2,1]\) case inherits features from both the \([1,1]\) and \([1,0]\) processes. It therefore performs well for both spectrally collocated emitters and those with finite detuning. A similar compound structure is visible in Fig. 4(d), where an input state \([2,2]\) shows a double two-photon structure compared to the \([1,1]\) input in Fig. 4(b). A clear trend emerges, where larger spectral emitter detuning can be overcome by higher number input states \([n,m]\) (see SM [27]).

There are several practical challenges turning this entanglement generation process into a useful quantum
technology. First, a reduced coupling of the emitter to the waveguide mode will reduce the phase shift imparted on the photons, and therefore lower the average concurrence. Second, the photons may be scattered out of the waveguide mode or be lost in the detection process. However, the use of fast, high-efficiency photon number resolving detectors will mitigate this problem, and such detectors are actively developed [31,32]. Third, the photons must be created in tunable identical quasi-monochromatic modes. There are a number of ways this can be achieved over a wide frequency range. Spontaneous parametric down-conversion (SPDC) is inherently tunable [33,34] and frequency filtering will create the optimal quasi-monochromatic pulses as well as remove unwanted frequency entanglement. The resulting photon generation rate reduction can be mitigated using multiplexing, which has been demonstrated for both SPDC photon sources [35] and tunable quantum dot sources [36]. Alternatively, tuning of single-photon pulses is possible via frequency conversion [37]. Finally, dephasing will have an impact on the entanglement generation process. The dominant dephasing mechanism for spin-doped solid-state emitters is nuclear spin interactions [38]. While this naturally leads to a random precession of the spin ground state, there are a number of strategies based on dynamical decoupling that may be used to suppress its impact [39–41]. In addition, solid-state emitters are subject to charge fluctuations and phonon interactions. The former leads to spectral wandering occurring on a microsecond timescale, which may be overcome by operating the process on shorter timescales [42]. Phonon scattering leads to sidebands [43] that can be removed through frequency filtering or by placing the emitter in an optical cavity [44,45].

In conclusion, we presented a robust entanglement generation mechanism between two solid-state qubits embedded in a Mach-Zehnder interferometer. Entangling techniques that use solid-state emitters are well known to place very stringent requirements on the spectral identity of the emitters [9]. Our approach overcomes these restrictions by showing how to tailor multiphoton input states, mitigating a long-considered weakness of solid-state emitters. We found that maximal deterministic entanglement between increasingly distinct emitters is possible using higher photon number input states, revealing a rich structure in multiphoton scattering from two emitters with different energies and linewidths. Our work provides a new methodology for solid-state entanglement generation, where the requirement for perfectly matched emitters can be relaxed in favor of optical state optimization.

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[27] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.123.023603 for a description of the “average concurrence” figure of merit, detailed analyses of single, two and N-photon transport through the Mach-Zehnder interferometer and plots showing the optimum optical state energy to employ over a range of the emitter parameter space.