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Invited Paper: Characterization of Few Mode Fibers and Devices

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Abstract

This paper reviews characterization techniques for few mode fibers and components. The focus is on interferometric techniques including spatial and spectral resolved imaging (S^2). Simple interferometric measurements are shown to be able to give quite comprehensive information including measurement of relative group delay. A new improved algorithm for analyzing S^2 data for the case of only two modes is presented. The effect of polarization and rotation of asymmetric modes is treated as well.

Keywords: Few Mode Fibers, Optical Fiber Measurements, Interference

1. Introduction

Few mode fibers are an interesting and currently a hot research topic with a wide range of applications, including communication, non-linear signal processing, sensing and imaging. The first interest in few mode fibers goes back to the very beginning of fiber optics where Stolen et al. [1] looked on inter modal four wave mixing, a topic which is still heavily investigated [2, 3]. Later, interest in use of higher order modes for compensation of the dispersion of transmission fibers [4, 5] appeared. Besides dispersion compensation for telecommunication use around 1550 nm, there has also been interest in use of a higher order modes for dispersion compensation at other wavelengths such as in the ytterbium gain band (1030-1060 nm) [6] and around 800 nm for

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all fiber delivery of pulses from Ti:Sapphire femtosecond lasers [7]. Another application, also within fiber lasers, of a higher order mode of a few mode fiber has been to obtain very large effective areas [8]. Also within fiber sensors there is an interest in use of few mode fibers [9]. However, with no doubt, for the last 10 years, the main interest in few mode fibers has been for application for space division mode multiplexing as a mean to overcome the capacity limit of single mode fibers [10, 11, 12] where now transmission on up to 45 modes has been demonstrated [13].

Proper characterization of few mode fibers and components are important. The ultimate characterization technique for a few mode fiber system might be the coherent swept wavelength interferometric method [14, 15], which measure the full transfer matrix including phase and polarization. However, this method requires a complicated coherent detection setup and furthermore the complex transfer matrix might not always be easy to interpret. In this paper, the focus is on somewhat simpler methods, which have been found useful by the authors. In Section 2 measurement of properties of individual fiber modes is treated, while the rest of the paper is devoted to interferometric methods. Section 3 reviews the basic theory of interference. In Section 4, it is shown, how a simple setup with a few mode fiber spliced between two single mode fibers provides information about both mode coupling and relative group delay. Section 5 is devoted to the S^2 (Spatial and Spectral imaging) method. A new improved algorithm for treatment of the measurement data for the case of two modes is presented. The influence of noise on the measurement results is, as far as we know, also treated for the first time. Finally, the influence of polarization and rotation of asymmetric modes is discussed. This important topic has so far not been treated much in the literature.

Besides the measurement techniques discussed in this paper, several other methods has been proposed and used. Especially worth mentioning, there has been a lot of work on Optical Time Domain Reflectometer (OTDR) based methods [16, 17, 18, 19].

2. Measurement of properties of individual fiber modes

For measurement of typical fiber properties such as attenuation, chromatic dispersion, and effective area of the individual modes of a few mode fiber, a setup as shown in Fig. 1 can be used. For good measurements, it is important that the mode converter delivers a mode with high mode purity.

Furthermore, the mode coupling in the coupling/splice from the mode converter to the few mode fiber under test should be low. Finally, the distributed mode coupling in the fiber should be low. This can be a problem for some fiber designs, such as graded index fibers, where the modes are grouped in mode groups. For example in a graded index fiber, which supports four LP modes (LP_{01} , LP_{11} , LP_{21} , and LP_{02}), LP_{21} (or strictly speaking LP_{21a} and LP_{21b}) and LP_{02} will have almost the same propagation constant and will therefore couple strongly during propagation. If for example a LP_{02} mode is launched into the fiber, a superposition of LP_{21a} , LP_{21b} , and LP_{02} will be detected at the end. Therefore measurement of properties such as dispersion and effective area is difficult for these modes.

Figure 1: Measurement of properties of individual fiber modes

For measurement on LP_{01} , the input single mode fiber is typically spliced directly to the few mode fiber and then followed by a suitable mode stripper for example two or more perpendicular bends as mentioned in Section 4. For the higher order modes, long period gratings [12], spatial light modulators [20], single phase plates [21], or multi plane phase control [22, 23] are typically used as the mode converter.

The specific property can then be measured using standard setups for single mode fibers with some precautions. Examples on measurement results on fibers supporting two and four LP modes can for example be found in [12, 24]. For attenuation measurements, using the cut back technique, it is important that the detector setup can fully detect the higher order mode. For Optical Time Domain Reflectometry (OTDR) no special precautions are needed. The back reflected signal is converted back to LP_{01} by the mode converter. Dispersion measurement setups typically require both single mode in and out, so a second mode converter needs to be added at the end to convert back to LP_{01} . For measurement of the effective area of a higher order mode, the conventional far field scan technique can be used, however, a generalized Hankel transform should be used when transforming to the near field for the asymmetric modes [23].

78 3. Basics of interference

79 The rest of the paper is devoted to methods based on interference. In
80 this section, the basic of interference is first reviewed.

81 The true modes of an optical fiber are the so called vector modes, which
82 are the solution to the wave equation for an optical fiber. However, in this
83 paper, the LP modes are used. The LP modes are the solutions to the wave
84 equation using the weakly guiding approximatio. For the fibers treated in
85 this paper, it is a good approximation. For the degenerate asymmetric LP
86 modes (LP_{lm} , where $l > 0$), the two spatial modes LP_{lma} and LP_{lmb} have
87 the exact same propagation constant. Therefore, in the analysis below, the
88 electrical field of a degenerate LP mode is simply represented by the sum
89 $E_{lma} + E_{lmb}$.

Consider two modes with the electrical field distributions E_1 and E_2 ,
transverse amplitude distribution $A_i(x, y)$, propagation constant β_i , and phase
distributions $\varphi_i(x, y)$ ($i = 1, 2$), propagating the distance z in a two mode
fiber then using complex representation:

$$\begin{aligned} E_1 &= A_1(x, y)e^{-i(\beta_1 z + \varphi_1(x, y))} \\ E_2 &= A_2(x, y)e^{-i(\beta_2 z + \varphi_2(x, y))}. \end{aligned} \quad (1)$$

90 In this paper the $e^{i\omega t}$ term is omitted and the fields are normalized so $A_i(x, y) =$
91 $\sqrt{I_i(x, y)}$, where $I_i(x, y)$ is the intensity distribution. When the two modes
92 are added together, the intensity $I = E \cdot E^*$ is:

$$\begin{aligned} I(x, y, z) &= I_1(x, y) + I_2(x, y) + \\ &2A_1(x, y)A_2(x, y) \cos(\Delta\beta z + \Delta\varphi(x, y)). \end{aligned} \quad (2)$$

93 Here I_1 and I_2 are the intensity distributions for mode 1 and 2, $\Delta\beta = \beta_2 -$
94 β_1 , and $\Delta\varphi(x, y) = \varphi_2(x, y) - \varphi_1(x, y)$. Further, using a first order Taylor
95 expansion:

$$\Delta\beta z \approx \Delta\beta_0 z - DGD \frac{2\pi c}{\lambda^2} \Delta\lambda, \quad (3)$$

96 where DGD is the differential group delay between mode 1 and 2 after prop-
97 agating the distance z , and c is the speed of light. From Eq. (3), it is found
98 that the intensity fluctuates versus wavelength λ with the period:

$$\lambda_B = \frac{\lambda^2}{DGD \cdot c}, \quad (4)$$

From Eq. (4), the range of DGD, which can be characterized by the interferometric techniques described in this paper, may be found. As the DGD is proportional to the fiber length, then the length, which can be characterized is likewise determined. The minimum DGD is determined by bandwidth of the spectral measurement. The maximum DGD is determined by the resolution of the spectral measurement.

It is noted that the total power found by integrating over (x,y) is stable.

$$P(z) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} I(x, y, z) dx dy = P_1(z) + P_2(z). \quad (5)$$

This is due to the orthogonality between the modes, which leads to:

$$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} A_1(x, y) A_2(x, y) \cos(\Delta\beta z + \Delta\varphi(x, y)) dx dy = 0 \quad (6)$$

That is, an oscillation in power is seen, only if a part of the field is detected. That could be in a splice to a single mode fiber as described in Section 4 or by looking at a small fraction of the field as in S² as described in Section 5.

For the general case of N modes Eq. (2) generalizes to

$$I(x, y, z) = \sum_{i=1}^N I_i(x, y) + \sum_{i=1}^{N-1} \sum_{j=i+1}^N 2A_i(x, y) A_j(x, y) \cos(\Delta\beta_{i,j} z + \Delta\varphi_{i,j}(x, y)), \quad (7)$$

where $\Delta\beta_{i,j} = \beta_i - \beta_j$, and $\Delta\varphi_{i,j}(x, y) = \varphi_i(x, y) - \varphi_j(x, y)$. That is, the intensity pattern contains beat frequencies corresponding to differential group delay between the individual modes as given by Eq. (4).

The above treatment is under the assumption that both modes have the same polarization. Further, by utilizing the vector version of Eq. (1) and (2), it can be shown that the above treatment is also applicable for random polarization as long as the birefringence of the few mode fiber is the same for all modes, which is a good approximation as long as the fiber is not too long. That is, the fiber length should be short enough to assure that $(\beta_x - \beta_y)z \ll \pi$.

120 4. Simple interferometric measurements

121 In this section a simple setup is considered, where a few meter of few mode
 122 fiber is spliced between two single mode pigtailed. The setup gives information
 123 about both power distribution between the modes and differential group
 124 delay. First, the case of a two mode fiber is treated and then, the analysis is
 125 expanded to more modes.

126 4.1. Two modes

Figure 2: Simple interferometric setup

127 The setup considered is shown in Fig. 2. A short length of two mode
 128 fiber (TMF) is spliced to single mode fiber (SMF) pigtailed and connected
 129 between a broadband light source and an optical spectrum analyzer (OSA).
 130 In the two splices there will be coupling to the two modes of the fiber. The
 131 coupling to the two modes in splice 1 is given by the overlap integral:

$$P_{i,in} = \frac{\left| \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E_i(x, y) E_{SMF,in}^*(x, y) dx dy \right|^2}{\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E_i(x, y) E_i^*(x, y) dx dy}, \quad (8)$$

132 where $P_{i,in}$ is the input power to the two mode fiber and $E_i(x, y)$ ($i=1,2$) are
 133 the two fields in the two mode fiber (Eq. (1)), and $E_{SMF,in}(x, y)$ is the field
 134 in the input SMF. Similar at the output, the power of the output SMF is
 135 found as:

$$P_{SMF,out} = \frac{\left| \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (E_1(x, y) + E_2(x, y)) E_{SMF}^*(x, y) dx dy \right|^2}{\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E_{SMF}(x, y) E_{SMF}^*(x, y) dx dy}, \quad (9)$$

136 The setup can be modelled as a Mach-Zehnder interferometer as shown
 137 in Fig. 3 and setting $E_1 = E_{01}$ and $E_2 = E_{11a} + E_{11b}$. By combining Eq. (2),
 138 (8), and (9) the power out of the output SMF is found as:

$$P_{SMF,out} = P_{1,out} + P_{2,out} + \sqrt{P_{1,out}} \sqrt{P_{2,out}} \cos(\Delta\beta z) \quad (10)$$

Figure 3: Model of setup in Fig. 2

139 where:

$$P_{i,out} = \frac{\left| \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E_i(x, y) E_{SMF}^*(x, y) dx dy \right|^2}{\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E_{SMF}(x, y) E_{SMF}^*(x, y) dx dy}, \quad (11)$$

140 and $i=1,2$. From Eq. (10), it can be found that the ratio between $P_{1,out}$
 141 and $P_{2,out}$ in dB (referred to as HOM^{dB}) is determined simply from the
 142 peak-peak variation in $P_{SMF,out}$ in dB (ΔP^{dB}) [25]:

$$HOM^{dB} = 10 \log_{10} \left(\frac{P_{2,out}}{P_{1,out}} \right) = 20 \log_{10} \left(\frac{10^{\frac{\Delta P^{dB}}{20}} - 1}{10^{\frac{\Delta P^{dB}}{20}} + 1} \right). \quad (12)$$

143 Finally:

$$HOM^{dB} = HOM_{Splice1}^{dB} + HOM_{Splice2}^{dB} + HOM_{MS}^{dB}, \quad (13)$$

144 where $HOM_{Splice1}^{dB}$ and $HOM_{Splice2}^{dB}$ are the coupling to mode 2 relative to
 145 mode 1 in splice 1 and 2, and HOM_{MS}^{dB} is the difference in loss between
 146 mode 2 and mode 1 in the mode stripper.

147 The described method has been used to optimize a bend based mode
 148 stripper for an OFS two mode step index fiber [26]. The mode stripper
 149 consist of two perpendicular bends of the same radius. Perpendicular bends
 150 are used to get the same bend loss for the two degenerate LP_{11} modes (LP_{11a}
 151 and LP_{11b}). Fig. 4 shows a picture of the used mode stripper where the bend
 152 diameter can be easily changed.

153 The measured spectra are shown in Fig. 5. The two splices were purposely
 154 made with an offset by misaligning the fibers 20 μm before the splice to get
 155 a better LP_{11} excitation (due to the surface tension the misalignment after
 156 the splice is considerable smaller, around 10 μm). Three measurements are
 157 shown: One without modes stripper, and two with perpendicular bends with
 158 a diameter of 18 and 13 mm respectively.

Figure 4: Mode stripper with perpendicular bends and configurable bend diameter

159 It is clearly seen how the addition of the mode stripper reduce the ampli-
 160 tude of oscillations. For the $\varnothing 18$ mm bends, it is observed that the LP_{11} is
 161 only effectively attenuated for wavelength above ~ 1550 nm. For the $\varnothing 13$ mm
 162 bends LP_{11} is effectively attenuated over the entire measurement wavelength
 163 range from 1470 to 1630 nm. Furthermore, a general loss increase is observed
 164 for the $\varnothing 13$ mm bends, as LP_{01} is also attenuated for this bend diameter.
 165 The peak-peak variation in power in the wavelength range 1550-1590 nm
 166 has been estimated for the three cases, ignoring the slow variations in the
 167 spectra. The slow variations are attributed to accidental mode coupling over
 168 short distances. The results and corresponding HOM^{dB} are summarized in
 169 Tab. 1.

170 From the results in Tab. 1 it is concluded that the $\varnothing 18$ mm bends give
 171 an LP_{11} suppression of 14 dB and the $\varnothing 13$ mm bends a suppression of 18
 172 dB. Furthermore, assuming the same mode coupling in the two splices, it is
 173 seen that the offset splices give around -14 dB coupling to LP_{11} .

174 The setup of Fig. 2 (without the mode stripper) can also be used to

Figure 5: Measured spectra with and without mode strippers (MS)

Table 1: Measured HOM content		
Configuration	ΔP^{dB}	HOM^{dB}
Two offset splices	0.8	-27
MS Ø 18 mm inserted	0.15	-41
MS Ø 13 mm inserted	0.10	-45

175 measure the difference in group delay and group velocity dispersion between
 176 the two modes as first proposed by Menashe et al. [27]. An example of such
 177 a measurement, on an OFS two mode graded index fiber [12], is shown in
 178 Fig. 6.

179 The measured transmission spectrum is shown in Fig. 6(a) together with
 180 fitting of the peaks. From the fits the wavelength of each peak λ_n is found
 181 for $n = 0$ to the number of peaks N . In Fig. 6(b) is then showed "phase
 182 measured" (PM) defined as:

$$PM(\lambda_n) = \frac{\lambda^2}{c} \pm PM(\lambda_{n-1}), \quad (14)$$

183 where $+$ is used for positive DGD and $-$ for negative DGD. In Fig. 6(b)
 184 is further shown a fourth order polynomial fit to the measured phase. By
 185 differentiating the polynomial with respect to wavelength, the DGD is found.
 186 By differentiating once more, the difference in dispersion between the two
 187 modes is found. This can be understood as follows. When going from one
 188 peak to the next $\Delta\beta z$ change by 2π , that is $PM(\lambda) = \Delta\beta z \frac{\lambda^2}{2\pi c}$. Further,
 189 $DGD = \frac{d\beta}{d\omega} z = \frac{\lambda^2}{2\pi c} \frac{d\beta}{d\lambda} z$.

(a)

(b)

Figure 6: (a) Measured spectra interference spectrum for 20 m of two mode fiber and fits to the peaks (b) Phase shift calculated from peak position, fourth order polynomial fit to measured phase, and calculated differential group delay

Figure 7: (a) Result scanning Fourier transform no mode filter (b) Calculated DGD

191 The simple setup of Fig. 2 is also very useful in the case of few mode
 192 fibers supporting more than two modes. In this case, analyzing the data with
 193 a sliding window Fourier transform can be very useful. This spectrogram
 194 approach has been proposed for data taken by S^2 by Jasapara and Yablon
 195 [28]. However, as proposed by Nicholson et al. [29] as well as Pedersen
 196 [30], similar information can be achieved from the simpler setup of Fig. 2.
 197 An example of results of a measurement on an OFS 4 mode step index
 198 fiber [26] is shown in Fig. 7. 5.2 meter of few mode fiber is used and the
 199 spectrum is taken from 1050 to 1250 nm. In this wavelength range the fiber

200 supports, 10 LP modes. The few mode fiber is spliced to the single mode
 201 fibers (Corning HI1060) with an offset. From Eq. (7), it is seen that the
 202 spectrum contains beatings corresponding to the difference in DGD between
 203 the individual modes. A Gaussian filter with a width of 5 nm is moved over
 204 the spectrum in 50 steps. For each step a Fourier transform is taken. The
 205 Gaussian filter is used instead of a simple step filter to avoid side lobes. The
 206 axis of the Fourier domain is re-scaled to DGD using Eq. (4) and the results
 207 is shown as a 3D plot in Fig. 7(a). Beating with the fundamental mode is
 208 assumed to dominate. In Fig. 7(b), the DGD relative to LP_{01} is shown for all
 209 guided higher order modes calculated from the refractive index profile with
 210 a scalar mode solver. When comparing Fig. 7(a) and 7(b) beating between
 211 LP_{01} and LP_{11} , LP_{02} , LP_{12} , and LP_{03} is clearly identified.

Figure 8: Result scanning Fourier transform with two perpendicular $\varnothing$ 12 mm bends as mode stripper

212 The effect of inserting a mode stripper consisting of two perpendicular
 213 bends with a diameter of 10 mm is illustrated in Fig. 8. Most of the beatings
 214 except the LP_{01} to LP_{11} beating is observed to be almost vanished. Still, the
 215 amplitude of the LP_{01} to LP_{11} beating is reduced.

216 5. S^2 measurements

217 5.1. Introduction

218 Spatial and Spectral resolved imaging (S^2) [31, 32] is a very useful tech-
 219 nique for characterization of few mode fibers and components. It has for
 220 example been used for measurement of DGD and distributed mode coupling
 221 in few mode fibers [12, 26], mode coupling in few mode splices [24], bend loss

222 measurement on few mode fibers [33], and cross talk in mode multiplexers
 223 [12, 34].

224 The standard implementation of S^2 [32] has the limitation that it requires
 225 one dominant mode. For characterizations of components such as mode con-
 226 verters, it can be important to be able to measure cases where one mode
 227 is not so dominant. C^2 [35] is another technique, which does not require
 228 a dominant mode. However, C^2 has the drawback that it requires tedious
 229 balancing of the lengths of the interference arms. Modifications to the S^2
 230 analyzing algorithm has been proposed by Otto et al. [36] and Sévigny et al.
 231 [37]. The Otto algorithm has, as far as we know, not been implemented
 232 for practical use, and the Sévigny methods are for cases with more than two
 233 modes. As pointed out by Otto, the case with two modes is the most diffi-
 234 cult. In this paper a modified version of the Otto method for the case with
 235 two modes will be demonstrated and verified by simulations and measure-
 236 ments. The new method results in relaxed conditions for a dominant mode.
 237 Furthermore, influence of noise is analyzed, which to our knowledge has not
 238 been done earlier.

239 An alternative technique to S^2 is the time of flight [38, 39] method, which
 240 for longer few mode fiber lengths is very useful. However, this technique is
 241 not treated in this paper.

242 5.2. Theory

Figure 9: S^2 setup. Light from a tunable laser is input to the device under test. The output from the two mode fiber is imaged on a camera.

243 The basic concept of the S^2 method is to measure the wavelength depen-
 244 dency of the near field distribution out of a few mode fiber ($I(x, y, \lambda)$ of Eq.
 245 (2), (7)) with a setup as in Fig.9. For each pixel of the near field, a Fourier
 246 transform is done. For measurement on a two mode fiber, when adding the
 247 amplitudes of the Fourier transforms, a result as shown in Fig. 10 is ob-
 248 tained in the ideal case. For a real measurement, the result might look like
 249 in Fig. 12. Two peaks are observed, one DC peak (DGD = 0) corresponding
 250 to the DC term in Eq. (2) and a peak centered at the DGD between the

251 two modes corresponding to the cosine term in Eq. (2). There are several
 252 possible reasons for the broadening of the peak seen in real measurements,
 253 such as variation of the DGD of the fiber with wavelength, and wavelength
 254 dependent mode coupling [32]. For asymmetric LP modes, the broadening
 255 could further be due to different DGD of the two degenerate modes due to
 256 a non-circular core. A small non-circularity can give quite large splitting in
 257 DGD [40]. In our measurements, the broadening of the peaks of the Fourier
 258 spectrum is further due to the wavelength dependent response of the camera
 259 used. As described in [32] and [36], the slowly varying DC term $I_{DC}(x, y, \lambda)$
 260 and the fast varying AC term $I_{AC}(x, y, \lambda)$ can be found by low pass filtering
 261 and band pass filtering respectively of the Fourier transform of $I(x, y, \lambda)$ and
 262 transforming back to the wavelength domain. In the traditional S^2 analysis
 263 of [32], which works for several modes, instead of finding $I_{AC}(x, y)$, one find
 264 $J_{AC}(x, y, \lambda)$ by band pass filtering of the function $J(x, y, \lambda)$ defined below.

$$J(x, y, \lambda) = \frac{I(x, y, \lambda) - I_{DC}(x, y, \lambda)}{2\sqrt{I_{DC}(x, y, \lambda)}}. \quad (15)$$

265 In case of a dominant mode 1, one gets according to the conventional method
 266 of [32]:

$$\begin{aligned}
 I_1(x, y) &\approx I_{DC}(x, y) = \frac{1}{\lambda_e - \lambda_s} \int_{\lambda_s}^{\lambda_e} I_{DC}(x, y, \lambda) d\lambda \\
 I_2(x, y) &\approx J_{AC}(x, y) = 2 \frac{1}{\lambda_e - \lambda_s} \int_{\lambda_s}^{\lambda_e} J_{AC}^2(x, y, \lambda) d\lambda,
 \end{aligned} \quad (16)$$

267 where λ_s , and λ_e , are the start and end measurement wavelengths.

268 The above analysis works well as long as the power of the dominant modes
 269 is 10 to 12 dB higher than the other modes. Here, for the case of two modes,
 270 a modified version of the method of Otto et al. [36] is proposed to relax the
 271 requirement for a dominant mode. Otto suggest for the case of only two
 272 modes to use the exact solution:

$$I_{1,2} = \frac{1}{2} \left(I_{DC}(x, y) \pm \sqrt{I_{DC}^2(x, y) - I_{AC}^2(x, y)} \right), \quad (17)$$

273 where $I_{AC}(x, y)$ is found like $I_{DC}(x, y)$ by integrating over the measurement
 274 wavelength range. However, we have found that the use of Eq. (17) do not
 275 give any improvement compared to the conventional analysis using Eq. (16).

276 Therefore, we instead suggest a further development of the method of [32],
 277 where for the case of only two modes, we use the exact solution:

$$I_{1,2} = \frac{1}{2} \left(I_{DC}(x, y) \pm \sqrt{I_{DC}^2(x, y) - 4J_{AC}(x, y)I_{DC}(x, y)} \right). \quad (18)$$

278 Finally, the relative content of I_2 is found as:

$$Rel_power = \frac{\iint I_2(x, y) dx dy}{\iint I_{DC}(x, y) dx dy}. \quad (19)$$

279 It is noted that a problem in using Eq. (18) (as well as Eq. (17)) is that
 280 whether addition or subtraction should be used for $I_{1,2}$ might depend of the
 281 position (x,y). For example, in the case of a dominant LP_{11} and a weaker
 282 LP_{01} , the LP_{01} might still have the highest intensity in the center. For
 283 simplicity, in this work, + for I_1 and - for I_2 will always be used. This
 284 will as seen lead to an error when the power in mode 2 is not much smaller
 285 than in mode 1. Still, as shown below, by using Eq. (18) relative powers up to
 286 to -5 dB can be detected for the weak mode compared to -10 dB for the
 287 conventional analysis using Eq. (16).

288 In real measurements, the measured intensity $I(x, y, \lambda)$ is overlaid with
 289 noise, which can lead to negative values. To avoid numerical problems a
 290 DC level is therefore added to $I(x, y, \lambda)$, and it is again subtracted from
 291 $I_{DC}(x, y)$. Due to the band pass filtering, the added DC level have no effect
 292 on $J_{AC}(x, y)$. Furthermore, to reduce influence from noise, negative values
 293 are set to zero after calculating $I_{1,2}$ with Eq. (18).

294 5.3. Simulation

295 To confirm the analyzing method, it has been tested on simulated mea-
 296 surement data. The electrical field of a LP_{01} and LP_{11} like modes, with
 297 intensity distribution as shown in insert 1 and 2 in Fig. 10 are added to-
 298 gether. The difference in propagation constants is calculated using Eq. (3)
 299 with a DGD of 4 ps. Furthermore, noise with Gaussian distributions is added
 300 for each pixel of the field. Insert 3 in Fig. 10, shows an example at a single
 301 wavelength of the total intensity distribution after addition of noise. Fig.
 302 10 shows an example of the sum of the Fourier transforms for each pixel as
 303 well as the position of the DC low pass filter and the AC band pass filter for
 304 finding $I_{DC}(x, y, \lambda)$ and $J_{AC}(x, y, \lambda)$. The power of LP_{11} relative to the total

power calculated using the the standard (Eq. (16)) and the improved (Eq. (18)) method as function of the actual (as set in the simulation) LP_{11} power are shown in Fig. 11. The case of no noise as well as the case of noise are also shown. A standard deviation of the noise equal to 0.015 times the maximum intensity for LP_{01} is used. This noise level is believed to be somewhat higher than the real noise in the actual setup described in the next section, judging from the minimum detection level of the setup.

It is observed, that the improved method allows for measurement of higher LP_{11} contents. If in the case on no noise, a 1 dB error is allowed, relative power content up to -10 dB can be measured with the standard method, while up to -5 dB with the improved method. It is also observed that noise not only limits the minimum of LP_{11} power, which can be detected but also the maximum. In the presence of noise, the improved method enhances both the minimum and maximum relative power, which can be detected.

Figure 10: Sum of Fourier amplitudes for relative LP_{11} power of -8 dB. Insert 1+2: intensity distributions for LP_{01} and LP_{11} used for simulation. Insert 3: Example of combined intensity distribution for relative LP_{11} power of -8 dB.

5.4. Measurements

The used measurement setup is shown in Fig. 9. The tunable laser source is an Agilent 81689A, which is tunable between 1524 and 1576 nm and the camera is a Scintacor CamIR. The camera has a standard silicon camera chip. To get sensitivity in the 1550 nm region, it is coated with a fluorescent coating. This results in a quadratic response to the input power. For the

Figure 11: Relation between actual relative LP_{11} power and found from S^2 analyze using standard and improved algorithm. Standard deviation of the noise equal to 0.015 times the maximum intensity for LP_{01} is used.

325 measurements, the dark current output from the camera is first measured,
 326 the dark current is then subtracted from the measurement and the square
 327 root is taken. It is verified by measurements that this lead to a linear relation
 328 between the input optical power to the camera and the measured value.

Figure 12: Sum of Fourier amplitudes for measurement on a photonic lantern with 3.5 m two mode output fiber.

329 For this investigation, a few meters of OFS two mode step index fiber
 330 was used and the device with mode coupling was either a photonic lantern
 331 [34] or a long period grating [41]. A typical sum of the Fourier transforms is

Figure 13: Difference between relative powers obtained using improved and standard method as function of result from standard method. Standard deviation of the noise equal to 0.004 times the maximum intensity for LP_{01} is used for simulation with noise.

shown in Fig. 12 together with the positions of the AC and DC Fourier filters. The difference in relative powers deduced from improved and standard method on measurements on photonic lanterns as well as long period gratings are shown in Fig. 13. Simulated result with and without noise are shown for comparison. A standard deviation of the noise equal to 0.004 times the maximum intensity for LP_{01} is used. This noise level is thought to be representative for the noise of the setup. Up to a relative power level of -12 dB a good agreement between measurements and simulations is observed. For relative power levels above -12 dB the improvement for the measured data is lower than expected from the simulation. If a 1 dB error is allowed then by combining the data from Fig. 11 and 13, it is found that the maximum detectable relative power is around -7 dB for the real measurements.

In conclusion, using the proposed improved analyzing method for S^2 data for the case of only two interfering modes, the maximum detectable relative power is improved from -10 dB with the current method to -5 dB in the ideal case, and -7 dB for real measurement. This makes S^2 even more useful for example for support of development of mode converters.

6. Polarization effects

The effect of polarization and rotation of asymmetric modes in few mode systems is a topic, which so far has not been treated much in the literature.

352 However, as shown in this chapter it is an important topic, which should not
 353 be neglected.

354 6.1. Mode rotation and polarization dependent loss

Figure 14: Measurement of effect of polarization

355 If the system in Fig. 14 is considered, the effect on changing the polar-
 356 ization in the input single mode (SM) fiber depends on whether coupling
 357 to non degenerate symmetric modes (LP_{0m}) or degenerate asymmetric mode
 358 (LP_{lm} , where $l > 0$) is considered. For symmetric modes, change of input
 359 polarization just transforms into a change of the polarization of the output
 360 mode. However, for asymmetric modes, the mode rotates. This is illustrated
 361 in the measurement of Fig. 15, where the measured near field out of a 2
 362 meter long two mode fiber is shown and the device with mode coupling is a
 363 long period grating for conversion from LP_{01} to LP_{11} . The near field pattern
 364 is shown for 4 randomly chosen positions of the polarization controller.

365 It is clearly observed, how the mode rotates, or strictly speaking, how
 366 the power change between LP_{11a} and LP_{11b} . This is explained as follows.
 367 All practical fibers have a slight non-circularity of the core, which leads to a
 368 difference in propagation constant for the two degenerate asymmetric modes
 369 (LP_{11a} and LP_{11b} in this case). This causes a rotation of the mode as it prop-
 370 agates along the fiber with one rotation per the beat length $\frac{\lambda}{n_{eff,11a} - n_{eff,11b}}$,
 371 where $n_{eff,11a}$ and $n_{eff,11b}$ are the effective index for the two modes. Further-
 372 more, as all practical fibers also have some birefringence, the effective indices
 373 further change with polarization and so the beat length. The change in the
 374 beat length leads to rotation at the end of the fiber. The above explanation
 375 is a little simplified. If one looks on the true vector modes then there is,
 376 even for a perfectly symmetric fiber, a difference in effective index for the
 377 two degenerate mode. However, a full analysis of this is outside the scope of
 378 this paper.

379 This rotation of asymmetric modes with polarization often leads to po-
 380 larization dependent loss (PDL). As an example, PDL has been measured

Figure 15: Measured mode rotation out of LPG when input polarization is changed

381 using the polarization scanning method [42]. In this method, the output
 382 power is monitored while the polarization controllers are varied randomly
 383 for at time period long enough to assure that the entire Poincare sphere is
 384 covered. Max and min output power is noted and the PDL is the difference.
 385 Using a thermal induced long period grating [41], the PDL directly out of
 386 the two mode fiber wherein the grating is written is measured to be 0.02 dB
 387 (which is around the detection limit of the setup). After adding a splice to
 388 a few meters of additional two mode fiber, the PDL increased to 0.18 dB.
 389 This is understandable as a small offset in the splice leads to different mode
 390 overlaps depending of the rotation of the LP_{11} mode.

391 Another example of PDL due to rotation of the LP_{11} mode is shown in
 392 Fig. 16. Here, PDL measurements on a mode selective photonic lantern
 393 are shown. A mode selective photonic lantern [43] can be described as a
 394 number of single mode fibers, which are adiabatically tapered down with a
 395 surrounding (cladding after down tapering) with lower refractive index than
 396 the silica cladding of the fibers. The modes of the single mode fibers then,
 397 due to the down tapering, spread out and the cladding forms a new core at
 398 the end of the taper. The end of the taper is then coupled to the few mode
 399 fiber. The number of fibers should correspond to the number of modes in the
 400 few mode fiber and different types of single mode fiber used for each mode

group of the fiber. If things are done right, it is then possible to multiplex
and demultiplex each mode group of the fiber.

(a)

(b)

Figure 16: Measured PDL versus wavelength for a photonic lantern. (a) PDL for demultiplexing of LP_{01} (b) PDL for demultiplexing of LP_{11}

In this example, a three fiber photonic lantern for coupling in and out of a two mode fiber, where air is used as the low index surrounding (cladding after down tapering) [34], is considered. The taper is made using one standard single mode fiber (SSMF) and two Corning HI1060 (HI). The SSMF is for coupling to the fundamental mode of the two mode fiber and the two HI for coupling to the LP_{11a} and LP_{11b} mode group. PDL versus wavelength using the polarization scanning method for demultiplexing of the LP_{01} mode is shown in Fig. 16(a). Due to the mode selectivity of the lantern, a low loss

411 (2.7 dB with a PDL of 0.1 dB) is observed for SSMF output fiber and a much
 412 higher loss for the sum of the HI outputs (11.8 dB with a PDL of 0.5 dB). Due
 413 to the mode degeneracy of LP_{11a} and LP_{11b} the sum of the two HI outputs
 414 are monitored to get the total power of LP_{11} . PDL versus wavelength for
 415 demultiplexing of the LP_{11} mode is further shown in Fig. 16(b). The mode
 416 selectivity is also observed here with a lower loss for the sum of the HI
 417 outputs (5.4 dB with a PDL of 2.4 dB) than for the SSMF output (17 dB
 418 with a PDL of 8.5 dB). As expected, the polarization dependency of the loss
 419 is observed to be much higher for demultiplexing of LP_{11} than for LP_{01} . For
 420 demultiplexing of LP_{01} it is observed that the polarization dependency for
 421 the SSMF output is very low and higher for the sum of the HI outputs. The
 422 later can be explained by that the HI fibers couple to modes in the taper,
 423 which are asymmetric and therefore rotates when the polarization is changed.
 424 That is, the overlap of the asymmetric lantern mode with the LP_{01} mode of
 425 the few mode fiber changes with polarization.

426 6.2. Polarization effects in interferometric measurements

427 The effect of input polarization of the simple set up of Fig. 2 has been
 428 investigated using 1.6 m of OFS two mode step index fiber. First, a tunable
 429 laser followed by a polarization controller has been used as the light source
 430 and a power meter on the output. The wavelength was stepped between 1525
 431 and 1575 nm in 0.5 nm steps. This was done for 10 fixed input polarizations.
 432 Furthermore, PDL was measured for each wavelength step using the polar-
 433 ization scanning method as described earlier. The result for two out of the
 434 ten fixed polarizations compared to the PDL measurement are shown in Fig,
 435 17(a). For visual clarity only the wavelength range from 1540 to 1560 nm
 436 is shown. A significant difference between the spectra for the two polariza-
 437 tions is observed. It is further noticed that the PDL measurements envelope
 438 the measurements for fixed polarization. This is true for all 10 fixed input
 439 polarizations measurements.

440 Further, 5 measurements was done, where an un-polarized ASE source is
 441 used as light source and an OSA used as detector. Between each measurement
 442 the fiber was moved. Results for three different arrangements of the fiber are
 443 shown in Fig. 17(b). A significant change in the spectrum is observed due
 444 to the fiber movement.

445 The found peak to peak variation in the spectrum and the correspond-
 446 ing calculated power ratio of the two mode HOM^{dB} is summarized for all
 447 measurements in Table 2.

(a)

(b)

Figure 17: Measured interference spectra for 1.6 m of two mode fiber. (a) Measured using tunable laser for two fixed polarizations and PDL (b) Measured using ASE source and OSA and three different arrangements of the fiber

448 The results are interpreted as follows. In splice 1 of Fig. 2, a fraction
 449 of the power is due to an offset in the splice coupled to LP_{11} . As described
 450 earlier, the mode rotates during propagation in the two mode fiber with the
 451 rate of rotation depending of the input polarization. Also moving the two
 452 mode fiber changes the rotation of the LP_{11} mode. In splice 2, a fraction of
 453 the LP_{11} power couples into the single mode fiber due to offset in the splice.
 454 This fraction depends on the rotational orientation of the LP_{11} relative to the
 455 offset. That is, the amount of LP_{11} power coupled in splice 2 ($HOM_{Splice2}^{dB}$)
 456 depends of the input polarization as well as the lay up of the two mode
 457 fiber. In a PDL measurement all rotations are sampled, that is maximum and
 458 minimum coupling in splice 2 are sampled. This is why the PDL measurement

Table 2: Measured peak peak variation and corresponding HOM content for different input conditions

Meas.	ΔP^{dB}	HOM^{dB}
pol 1	2.20	-18
pol 2	2.50	-17
pol 3	2.26	-18
pol 4	2.19	-18
pol 5	1.98	-19
pol 6	1.61	-21
pol 7	2.50	-17
pol 8	1.39	-22
pol 9	2.32	-18
pol 10	2.32	-18
PDL	2.64	-16
ASE 1	1.24	-23
ASE 2	2.43	-17
ASE 3	0.60	-29
ASE 4	1.19	-23
ASE 5	2.26	-18

459 envelope all of the fixed polarization measurements.

460 It is further observed that the un-polarized ASE source gives similar in-
461 terference patterns as the polarized laser. This confirms the statement in
462 Section 3 that random polarization of the two interfering modes gives same
463 results as when the two modes are polarized with the same polarization.

464 7. Conclusion

465 This paper has reviewed measurements on few mode fibers mostly fo-
466 cusing on interferometric methods. It has been shown that a simple mea-
467 surement of the transmission spectrum of a piece of few mode fiber spliced
468 between two single mode pigtails can be used to measure the group difference
469 between guided modes even in the case of several guided modes. It can also
470 be used for optimization of mode strippers.

471 A new algorithm to analyse the result of spatial and spectral resolved
472 imaging (S^2) for the case of two modes have been presented, which increase
473 the maximum detectable relative power compared to the present algorithm.

474 The effect of the rotation of asymmetric modes either due to fiber move-
475 ment or change of input polarization has been analyzed as well.

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479 9. References

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