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A Simple Coupled-Bloch-Mode Approach To Study Active Photonic Crystal Waveguides and Lasers

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Abstract—By applying a coupled-Bloch-mode approach, we have derived a simple expression for the transmission properties of photonic crystal (PhC) line-defect waveguides with a complex refractive index perturbation. We have provided physical insights on the coupling mechanism by analyzing the frequency dependence and relative strength of the coupling coefficients. We have shown the impact of the perturbation on the waveguide dispersion relation and how the gain-induced distributed feedback limits the maximum attainable slow-light enhancement of the gain itself. We have then applied our approach to analyze the threshold behaviour of various PhC laser cavities and proved the significant impact of coherent distributed feedback effects in these lasers. Importantly, our approach also reveals that a structure simply consisting of an active region with zero back reflections from the passive output waveguides can achieve lasing oscillation with reasonable threshold gain.

Index Terms—Photonic crystal (PhC), photonic crystal waveguides and lasers, coupled-mode theory, Bloch modes, photonic integrated circuits.

I. INTRODUCTION

PHOTONIC-crystal (PhC) waveguides are made by creating a line-defect in a PhC slab. The defect introduces guided modes in the photonic band-gap of the crystal and allows for an efficient propagation of optical signals. In particular, a major advantage of PhC line-defect waveguides (LDWGs), as compared to conventional waveguides, is the possibility to exploit slow-light (SL) propagation [1]. In the SL region of the waveguide dispersion relation, the group velocity is greatly reduced and ideally tends to zero at the band edge. As a consequence, in the SL region, a Bloch wave propagating within an active PhC waveguide experiences an effective gain per unit length which is greatly enhanced with respect to the modal gain coefficient of a conventional waveguide [2]. This gain-enhancement allows for the realization of shorter devices and makes PhC waveguides ideal candidates for high-density photonic integrated circuits. PhC lasers have attracted large interest of gain in this section was treated as a weak perturbation to the field expansion in a finite-length, active section; the presence of in-plane waveguides can achieve lasing oscillation with reasonable threshold gain.

heterostructure (BH) embedded in a PhC LDWG. Furthermore, differently from PhC lasers based on LN cavities [7], [8], in LEAP lasers light can be emitted into in-plane waveguides. The output waveguide can be either shifted with respect to the active region [9], [10] or placed in-line with it [11]. Therefore, LEAP lasers are promising sources for photonic integrated circuits [3].

Rigorous approaches to analyze PhC devices, such as FDTD [4], [12] or RCWA [13], [14], [15], are time- and memory-consuming and cannot always provide an intuitive understanding of the physical phenomena into play. The aim of this paper is to present an alternative and simple approach for analyzing active PhC LDWGs and lasers based on this type of waveguides. Our approach is based on coupled-mode theory (CMT) [16], [17], which has proved to be an effective tool to study and design lasers with periodic gain and/or refractive index perturbation, such as edge-emitting DFB lasers [18]. More recently, CMT has also been used to analyze both passive [19], [20] and active [21] PhC waveguides. By following a simple CMT approach, we show that a weak perturbation of the gain and/or refractive index of a PhC waveguide causes a strong coupling between the forward- and backward-propagating Bloch modes of the unperturbed waveguide, with coupling coefficients strongly increasing as we move towards the band edge. We start from the formulation already presented in [21], where the Bloch modes of a passive, reference waveguide were used as a basis for the field expansion in a finite-length, active section; the presence of gain in this section was treated as a weak perturbation to the passive structure, coupling the otherwise independent, counter-propagating Bloch modes. However, the system of coupled propagation equations was solved numerically in [21], thus not providing important insight on the coupling mechanism. In this work, we push further the formulation of [21] to generally take into account both a real and imaginary refractive index perturbation. Avoiding the numerical solution, we derive a
simple, closed-form expression for the unit cell transmission matrix of an active PhC waveguide. Therefore, consistently with the rigorous, non-perturbative approach of [22], we show that, in the presence of gain, the group index does not diverge at the band edge and that the maximum attainable SL gain enhancement is limited by the gain itself.

In this work, our coupled-Bloch-mode approach is then applied to analyze the threshold condition of various types of PhC lasers. As a first example, we analyze a laser cavity made up of different sections of both passive and active PhC LDWGs. This laser is conceptually similar to the one characterized in [11]. Consistently with [11], we show that three different operating regimes can be identified, which can be explained on the basis of the interplay between the distributed feedback in the active region and the passive mirrors. This reveals the great impact of coherent distributed feedback effects in these PhC lasers. As a second example, we analyze a typical PhC laser based on a LN cavity, such as that of [8]. In this case, the laser cavity is an active PhC LDWG bounded on either side by classical PhC mirrors. As a last example, we examine a structure simply consisting of an active section, with zero back reflections from the interfaces with the passive output waveguides. Interestingly, in this type of structure the distributed feedback, caused entirely by the gain perturbation in the active region, is enough to allow lasing with reasonable threshold gain.

The paper is organized as follows: in Section II, we present our model and discuss the peculiar characteristics of the self- and cross-coupling coefficients of an active PhC LDWG. In Section III, we present the numerical results applied to both active PhC waveguides and lasers. In Section IV, we finally draw the conclusions.

II. NUMERICAL MODEL

Fig. 1: Reference PhC waveguide supercell (a). Cross section of the reference waveguide supercell at either the input or output plane. The black lines represent QWs or QDs layers (b). Finite-length, perturbed section in red and reference waveguide in grey (c).

We consider an active, PhC LDWG of finite length bounded on either side by semi-infinite, passive PhC waveguides, as illustrated in Fig. 1(c). The passive sections have the same lattice constant and membrane thickness as the active section. In this context, for membrane we simply mean a thin layer of semiconductor, suspended in a low refractive index medium and periodically patterned with holes [23], as shown in cross-section in Fig. 1(b). The material gain $g$ in the active section may be provided by layers of quantum wells (QWs) or QDs embedded within the PhC membrane, shown as black lines in Fig. 1(b). For simplicity, the entire membrane is assumed to contain active material, as in the case of optically pumped waveguides [2] and lasers based on LN cavities [7], [8], which lack a structure for lateral carrier confinement. However, we note that the coupled-Bloch-mode approach is also applicable to structures with a buried active region, such as LEAP lasers. We consider the gain of the active section and the variation of its refractive index with respect to that of the passive sections as a weak perturbation. The forward- (+) and backward-propagating (-) Bloch modes of the reference waveguide in the frequency-domain are denoted by $E_{0, \pm}(r, \omega) = e^{\pm ikz_{0}(\omega)}$, where $k_{z}(x, y, z, \omega) = k_{z}(x, y, z + a, \omega)$, with $k_{z}$ being the Bloch wave number along the length of the waveguide ($z$) and $a$ the PhC lattice constant. In the active section, the complex refractive index variation compared to the reference waveguide is $\Delta n_{g} = \Delta n_{i} + i\Delta n_{r}$, where $\Delta n_{i}$ is the variation of refractive index and $\Delta n_{r}$ reflects the modal gain coefficient $g_{0}$, with $\Delta n_{i} = -(c/\omega)g_{0}/2$. Here, $g_{0}$ is given by $\Gamma_{\gamma}g$, where $\Gamma_{\gamma}$ is...
the optical confinement factor of the considered electric field guided mode along the y-direction and within the QWs or QDs layers. This polarization perturbation couples to each other in the active waveguide the otherwise independent Bloch modes of the reference waveguide. Therefore, in the limit of a weak perturbation, the electric field in the active section can be expanded in the basis of the Bloch modes of the reference waveguide, with slowly-varying amplitudes \( \psi_z(z, \omega) \) caused by the coupling. The electric field in the active section thus reads as \( E(r, \omega) = \psi_+(z, \omega)E_{0,+}(r, \omega) + \psi_-(z, \omega)E_{0,-}(r, \omega) \).

At a given \( \omega \), by neglecting nonlinear effects, two coupled differential equations in \( \psi_z \) are derived [21]:

\[
\begin{align*}
\partial_z \psi_+(z) & = i \kappa_{11}(z) \psi_+(z) + i \kappa_{12}(z) e^{2ik_\parallel z} \psi_-(z) \\
-\partial_z \psi_-(z) & = i \kappa_{21}(z) e^{2ik_\parallel z} \psi_+(z) + i \kappa_{11}(z) \psi_-(z)
\end{align*}
\]

The self- and cross-coupling coefficients are written as \( \kappa_{11,21,21}(z, \omega) \equiv \frac{\Delta n_z(\omega) + i \Delta n_\omega(\omega)(\omega/c)}{n_{r,0}(\omega)/n_{r}(\Gamma_{11,12,21}(z, \omega))} \), where \( n_{r,0} \) and \( n_{r} \) are the group index and the membrane material refractive index of the reference waveguide. \( \Gamma_{11,12,21}(z, \omega) \) are given by

\[
\begin{align*}
\Gamma_{11}(z, \omega) & = \frac{a \int_V \mathbf{e}_0(\mathbf{r}, \omega) \cdot \mathbf{e}_0(\mathbf{r}, \omega) |F(\mathbf{r})|^2 dV}{\int_V |\mathbf{e}_0(\mathbf{r}, \omega) \cdot \mathbf{e}_0(\mathbf{r}, \omega)|^2 dV} \\
\Gamma_{12}(z, \omega) & = \frac{a \int_V \mathbf{e}_0(\mathbf{r}, \omega) \cdot \mathbf{e}_0(\mathbf{r}, \omega) \cdot \mathbf{e}_0(\mathbf{r}, \omega) |F(\mathbf{r})|^2 dV}{\int_V |\mathbf{e}_0(\mathbf{r}, \omega) \cdot \mathbf{e}_0(\mathbf{r}, \omega)|^2 dV}
\end{align*}
\]

with \( \Gamma_{21} = \Gamma_{12}^{*} \). Here, \( V \) is the volume of the supercell (see Fig. 1(a)), \( S \) the transverse section at position \( z \) and \( n_0(\mathbf{r}) = n_s = n_{\text{holes}} \) in the membrane (holes) is the reference waveguide background refractive index. The spatial distribution of the perturbation is taken into account by \( F(\mathbf{r}) \), which is equal to 1 (0) in the membrane (holes) of the active section. \( \Gamma_{11} \) is the ratio between the electric field energy in the gain region of a supercell and that stored in the whole supercell, by assuming the modal gain \( g_0 \) to be uniform through the semiconductor slab. We note that the self-coupling coefficient is caused by the fact that the modes used as a basis for the electric field see, in the perturbed waveguide, an average complex refractive index profile which is different from that of the reference waveguide where the modes are defined. Due to the \( z \)-periodicity of \( \mathbf{e}_{0,\pm} \) and \( F(\mathbf{r}) \), \( \Gamma_{11} \) and \( \Gamma_{12} \) and, therefore, the coupling coefficients are periodic with \( z \). As an example, we consider a reference waveguide with refractive index \( n_s = 3.171 \); the other parameters are those of the PhC waveguide on which the lasers in [8] are based, with \( a = 438 \) nm, \( h = 250 \) nm. The membrane is assumed to be suspended in air, consistently, for instance, with [7], [8] or [9], [10], which investigate air-bridge structures. Band structure and TE-like Bloch modes of the reference waveguide are computed by the plane wave eigensolver MIT Photonic-Bands (MPB) [24] (Fig. 2). The blue lines represent a subset of the continuum of photonic crystal slab modes found by MPB [25]. These modes are confined to the slab along the y-direction, but are delocalized in the x- and z-direction. We apply the coupled-Bloch-mode approach to the fundamental guided mode (red, solid line), which is confined both along the x- and y-direction. The magnitude and phase of \( \Gamma_{11} \) and \( \Gamma_{12} \) over a unit cell are displayed in Fig. 3 at various \( k_z \) values. Given

![Fig. 3: Spatial dependence of \( \Gamma_{11} \) and \( \Gamma_{12} \) of the reference waveguide at various \( k_z \) values. The membrane refractive index is \( n_s = 3.171 \); the other parameters are those of the PhC LDWG on which the lasers in [8] are based. (3a) \( \Gamma_{11} \). (3b) Magnitude of \( \Gamma_{12} \). (3c) Phase of \( \Gamma_{12} \).](image)

![Fig. 4: Magnitude of the Fourier components of the self-coupling \( \kappa_{11}(z) \) (a) and cross-coupling coefficient \( \kappa_{12}(z) \) (b) at various \( k_z \) values.](image)
the spatial distribution of the perturbation (that is $F(\mathbf{r})$), they only depend on the reference waveguide geometry and Bloch modes and not on the magnitude of the perturbation. For uniformly pumped membranes, $\Gamma_{11}$ and $\Gamma_{12}$ are intrinsic parameters of the reference waveguide and they determine how strong the cross-coupling is with respect to self-coupling. The coupling coefficients are indeed proportional to $\Gamma_{11}$ and $\Gamma_{12}$ through $n_{\varepsilon,0}$ and the complex refractive index perturbation. Therefore, they can be expanded in a Fourier series as $k_{11,12}(z, \omega) = \sum_{q} k_{11,12,q}(\omega) \exp(i q 2 \pi z / a)$. Since the phase of $\Gamma_{12}$ is approximately linear with $z$ with a slope equal to $\pm 2 \pi / a$, $k_{12,q=1}$ is proportional to $|k_{12}(z)|$, which is comparable with $k_{11,q=0}$. This is illustrated in Fig. 4, showing the Fourier components of $k_{11}(z)$ and $k_{12}(z)$ in arbitrary units at various $k_z$ values. Interestingly, we find that the complex refractive index perturbation not only produces a self-coupling coefficient proportional to the average of the perturbation (i.e. the average of $\Gamma_{11}(z)$ in Fig. 3(a)), but also a strong cross-coupling, whose magnitude is close to that of the self-coupling. This strong cross-coupling is possible thanks to the linear phase of $\Gamma_{12}(z)$; if this linear phase component were not present, the cross-coupling would be negligible. By analyzing the contribution of the various field components ($\hat{x}$, $\hat{y}$ and $\hat{z}$ component) to $\Gamma_{12}(z)$, we have found that the linear phase variation is caused by a significant $\hat{z}$ component of the TE-like guided mode, which is typically negligible in standard waveguides. Here the non-negligible $\hat{z}$ component is due to the strong lateral ($x-$direction) confinement obtained by the PhC.

Having now more insight into the main properties of the coupling coefficients, we derive an analytical expression for the transmission matrix describing the field propagation over a unit cell. As illustrated in Fig. 4, all harmonics in the self-coupling coefficient, other than $q = 0$, are negligible; similarly, the harmonic with $q = 1 (= -1)$ is the dominant one in the cross-coupling coefficient $k_{12}$ ($k_{21}$). This is true over a wide frequency range and especially close to the band edge. Therefore, by only retaining the dominant harmonics, Eqs. (1) are turned into

$$
\begin{align*}
\frac{\partial}{\partial z} \psi_+ &= i k_{11,12}(z, \omega) \psi_+ + i k_{12,12}(z, \omega) e^{2i \delta_1(z)z} \psi_- \\
-\frac{\partial}{\partial z} \psi_- &= i k_{21,12}(z, \omega) e^{-2i \delta_1(z)z} \psi_+ + i k_{11,11}(z, \omega) \psi_-
\end{align*}
$$

where $\delta_1(z) = \pi/a - k_z(z)$ is the detuning from the band edge. By defining $b^\pm(z, \omega) = \psi_{\pm}(z, \omega) \exp\{\mp i \delta_1(z)z\}$, the system of Eqs. (3) is turned into a pair of partial differential equations with $z$-independent coefficients. This allows to analytically solve it over a unit cell as an initial value problem. That is, $b^\pm(z_0, a, \omega)$ are computed by assuming $b^\pm(z_0, \omega)$ to be known, with the input coordinate $z_0$ of the unit cell conveniently chosen to be zero. Therefore, the unit cell transmission matrix in terms of $b^\pm$ is obtained. However, we need the transmission matrix for $c^\pm(z, \omega)$, with $c^\pm(z, \omega) = \psi_{\pm}(z, \omega) \exp\{\pm i k_z(z)z\}$. Since $b^\pm(z, \omega) = c^\pm(z, \omega) \exp\{\mp i (\pi/a)z\}$, the unit cell transmission matrix in terms of $c^\pm$ is obtained through the change of variables $b^\pm(z_0, a, \omega) = c^\pm(z_0 + a, \omega) \exp\{\mp i \pi/a\}$ and $b^\pm(z_0, \omega) = c^\pm(z_0, \omega)$. Therefore, we can relate $c^\pm$ at the input (N = 1) and output (N) of the generic $N_{th}$ cell by

$$
\begin{bmatrix}
\psi_N^+ \\
\psi_N^-
\end{bmatrix} = T_a(\omega) \begin{bmatrix}
\psi_{N-1}^+ \\
\psi_{N-1}^-
\end{bmatrix}
$$

(4)

where $T_a$ is the unit cell transmission matrix in terms of $c^\pm$. The elements of $T_a$ are given by

$$
\begin{align*}
T_{a,11,a,22} &= -\cos(\gamma a) + i \left( \frac{\delta}{\gamma} \right) \sinh(\gamma a) \\
T_{a,12,a,21} &= \mp i \left( k_{12,q=1,21,q=-1} \gamma \right) \sinh(\gamma a)
\end{align*}
$$

(5)

with $\delta_1(\omega) = k_{12,q=0}(\omega) - \delta_1(\omega)$ and $\gamma(\omega) = \sqrt{k_{12,q=-1}(\omega) k_{12,q=1} - \delta_1^2(\omega)}$. This approach allows to directly relate the eigenvalues of $T_a$ to the coupling coefficients and the detuning. In fact, the eigenvalues $\lambda_{1,2}$ are readily obtained as $\lambda_{1,2} = \exp(\pm \text{Re} \{\gamma a\}) \exp(\pm \text{Im} \{\gamma a + \pi\})$. From here, we define $\pm \beta_{g,a} = \pm \text{Im} \{\gamma a + \pi\}$ as the effective gain per unit length, representing the gain experienced by the forward (+) and backward (−) Bloch modes of the active section while propagating in a unit cell. Similarly, $\pm \beta_{b,a} = \pm \text{Re} \{\gamma a\}$ is the phase shift per cell of these Bloch modes. From this phase shift, we can then calculate the group index of the Bloch modes of the active waveguide as $\gamma_{g,a}(\omega) = \lambda_{g,a}(\omega)/\beta_{g,a}(\omega)$. By applying Frobenius theorem [26], the transmission matrix of an active waveguide of $N$ unit cells is computed as

$$
T_a^N = \lambda_{g,a}^N M^{-1}
$$

(6)

where $\lambda$ and $\lambda_{g,a}$ are given by

$$
M = \begin{bmatrix}
u_{11} & \nu_{12} \\
u_{21} & \nu_{22}
\end{bmatrix}, \quad \lambda_{g,a} = \begin{bmatrix}\lambda_1 & 0 \\
0 & \lambda_2\end{bmatrix}
$$

(7)

with $\nu_{11} = [u_{11} \ u_{21}]^T$ and $\nu_{22} = [u_{12} \ u_{22}]^T$ being the eigenvectors of $T_a$ and $T$ denoting the transpose operator. As well as being numerically efficient, this approach for computing $T_a^N$ allows for a useful physical interpretation: $\lambda_{g,a}$ can be seen as the transmission matrix describing the propagation of the Bloch modes of the active section over the $N$ unit cells. $M^{-1}$ and $\lambda$ can be interpreted as the transmission matrices of the interfaces between the active section and, respectively, the left and right passive waveguides; they physically account for the mismatch between the Bloch modes of the active and passive sections. By applying the relationships between a transmission and a scattering matrix [27], the transmission matrix $T_a^N$ can be turned into the corresponding scattering matrix. The scattering parameters are given by

$$
\begin{align*}
S_{11} &= \frac{\lambda_{1,1,1,1}^N - \lambda_{2,2,1,1}^N}{(\lambda_1 - \lambda_2) T_{a,11} + (\lambda_{2,2,1,1}^N - \lambda_{1,1,1,1}^N)} \\
S_{12;21} &= \frac{(\lambda_2 - \lambda_1)}{(\lambda_1 - \lambda_2) T_{a,11} + (\lambda_{2,2,1,1}^N - \lambda_{1,1,1,1}^N)} \\
S_{22} &= \frac{(\lambda_1 - \lambda_2)}{(\lambda_1 - \lambda_2) T_{a,11} + (\lambda_{2,2,1,1}^N - \lambda_{1,1,1,1}^N)}
\end{align*}
$$

(8)

As outlined in [21], the coupled-Bloch-mode approach breaks down at large values of $g_0$. Specifically, the larger $n_{\varepsilon,0}$ becomes, the smaller is the value of the gain coefficient at
A. Line-defect active waveguide

This approach will be applied in section IIIA.

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III. SIMULATION RESULTS

A. Line-defect active waveguide

Fig. 5 shows the magnitude of the self- (\( \kappa_{11,q=0} \)) and cross-coupling coefficient (\( \kappa_{12,q=1} \)) for \( g_0 = 0 \) (blue) and \( g_0 = 50 \text{ cm}^{-1} \) (red). In both cases, \( \Delta n_s = -0.001 \).

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Fig. 6: Dispersion curve computed by MPB for \( n_s = 3.171 \) (black) and \( n_s = 3.170 \) (pink). Phase shift per cell of the Bloch modes of the perturbed waveguide with (light-blue, dotted) and without (light-blue, solid) cross-coupling; the reference waveguide has \( n_s = 3.171 \) and the real refractive index perturbation is \( \Delta n_s = -0.001 \), with \( g_0 = 0 \). The dark-blue, dotted curve is for \( \Delta n_s = -0.002 \), \( g_0 = 0 \).

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III. SIMULATION RESULTS

A. Line-defect active waveguide

Fig. 5 shows the magnitude of the self- (\( \kappa_{11,q=0} \)) and cross-coupling coefficient (\( \kappa_{12,q=1} \)) for \( g_0 = 0 \) (blue) and \( g_0 = 50 \text{ cm}^{-1} \) (red). In both cases, \( \Delta n_s = -0.001 \). This figure proves that the cross-coupling coefficient is always comparable to the self-coupling coefficient. This is consistent with results also shown in [20]. However, as compared to [20], we have clarified here the physical origin of this peculiar behaviour. We also observe that the coupling coefficients depend on the intensity of the perturbation, as expected, but also on frequency and they significantly increase as the frequency approaches the band edge as consequence of the SL effect. Therefore, Bloch modes at smaller frequency and/or with higher gain of the active section experience stronger distributed feedback as compared to higher frequency Bloch modes and/or lower active section gain.

In this section, we analyze how a generally complex refractive index perturbation impacts the dispersion relation of a PhC waveguide. To validate our model, we first study the case of a purely real refractive index perturbation, because the results obtained by our approach can be compared with MPB simulations. We consider a reference passive waveguide with \( n_s = 3.171 \) and \( g_0 = 0 \) and we report in black in Fig. 6 the corresponding dispersion relation calculated by MPB. We then consider a small, real refractive index perturbation \( \Delta n_s < 0 \) and we compare the dispersion relation calculated with our model (i.e. \( \beta_{\text{eff}}(\omega)a \)) with that calculated by MPB for a passive waveguide with refractive index \( n_s + \Delta n_s \). As \( n_s \) decreases,
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The dispersion curve shifts to higher frequencies, meaning that the Bloch modes become evanescent at lower frequencies. The formation of this stopband for Bloch modes is correctly reproduced by our approach: the light-blue, dotted curve in Fig. 6 is the dispersion relation calculated by our approach for $\Delta n = -0.001$ and it perfectly overlaps with that obtained by MBP for $n_s = 3.170$. If the contribution of the cross-coupling coefficients in Eqs. (3) is neglected, we find the dispersion relation shown by the light-blue, solid curve in Fig. 6: in this case, the Bloch modes of the perturbed waveguide are not evanescent in the stop-band and the dispersion relation disagrees with the MBP prediction. These results validate the correctness of our approach and emphasize the role of the cross-coupling terms. Furthermore, the possibility to analyze the propagation of the Bloch-modes in a portion of the stopband of the perturbed, passive waveguide will be exploited in the next section to study laser configurations similar to the LEAP lasers of [6], [28].

As a second example, we consider the impact of a gain perturbation, assuming now $\Delta n_s = -0.001$ and $g_0 > 0$. Fig. 7(a) shows how the dispersion relation of the Bloch modes of the active waveguide is modified by the gain perturbation and Fig. 7(b) reports the corresponding group index at various gain values. For $g_0 = 0$, the group index diverges at $f \simeq 189.388$ THz, because this frequency corresponds to the band edge of the perturbed guide. At lower frequencies, the Bloch modes are evanescent and the group index is zero, because evanescent waves do not carry any active power. As $g_0$ increases, the group index is gradually reduced in the passband, thus limiting the SL effect; on the contrary, it gradually increases in the stopband, where $\beta_{\text{eff}}$ is now different from zero. This is consistent with results reported in [22], which were obtained through a non-perturbative approach. The corresponding $\beta_{\text{eff}}$ versus frequency is shown in Fig. 8(a) at various values of $g_0$; we also report in Fig. 8(b) the gain enhancement factor $g_{\text{eff}}/g_0$ at $f \simeq 189.388$ THz obtained by our approach. Since cross-coupling is not negligible, the gain-induced distributed feedback becomes more and more important as the gain increases, thus causing the decrease of the group index; as a consequence, the gain enhancement factor, which is based on the SL effect, is reduced by increasing the pumping of the active region. This result is consistent with [22] and confirms that a fundamental limitation to the achievable SL enhancement of the gain is imposed by the gain itself.

**B. PhC Lasers**

In this section, we apply the coupled-Bloch-mode approach to model the threshold characteristics of various types of PhC laser cavities. The threshold condition is found by calculating the complex loop-gain (LG) of the cavity.

As a first example, we analyze the cavity shown in Fig. 9. The cavity is made up of three sections, perturbed in refractive index (passive mirrors) and gain (active section) with respect to a reference, passive waveguide with $n_s = 3.171$. The section to the left of the active region (broad-band mirror) is passive, with $\Delta n_a = -0.002$ and length fixed to $L = 30a$; the active section has $\Delta n_t = -0.001$, $g_0 > 0$ and $L = 10a$. The section to the right of the active region is also passive and acts as a buffer mirror, with $\Delta n_b = -0.001$ (Type A), $\Delta n_b = -0.0005$ (Type B) or $\Delta n_t = -0.002$ (Type C) and variable length $L_{\text{buffer}}$. The output waveguide coincides with the reference waveguide. The position of the reference plane to compute the complex loop-gain is denoted by $z_{\text{ref}}$.

![Fig. 9: Laser cavity made up of three sections, perturbed in refractive index (passive mirrors) and gain (active section) with respect to a reference, passive waveguide with $n_s = 3.171$. The section to the left of the active region (broad-band mirror) is passive, with $\Delta n_a = -0.002$ and length fixed to $L = 30a$; the active section has $\Delta n_t = -0.001$, $g_0 > 0$ and $L = 10a$. The section to the right of the active region is also passive and acts as a buffer mirror, with $\Delta n_b = -0.001$ (Type A), $\Delta n_b = -0.0005$ (Type B) or $\Delta n_t = -0.002$ (Type C) and variable length $L_{\text{buffer}}$. The output waveguide coincides with the reference waveguide. The position of the reference plane to compute the complex loop-gain is denoted by $z_{\text{ref}}$.](image-url)
The laser threshold behaviour is mainly dominated by the mirrors the reflectivity of both mirrors is high (because the lasing of the broad-band mirror. Therefore, at the lasing frequency, the buffer refractive index perturbation coincides with that band and buffer mirror reflection spectrum. In Type C (black), following) in the three cases, with respect to both the broad-band mirror in the three different operating conditions: Type A (red), Type B (blue) and Type C (black). In all three buffer mirror scattering matrices, i.e.,

\[
\begin{align*}
    r_{eq,R} &= S_{11,active} + r_{eq,2} \\
    r_{eq,2} &= \frac{S_{12,active} S_{11,buffer} S_{21,active}}{1 - S_{11,buffer} S_{22,active}}
\end{align*}
\]  

(9)

with \(S_{ij,buffer}\) and \(S_{ij,active}\) denoting the buffer and active section scattering parameters computed by Eqs. (8). The longitudinal resonant modes are those satisfying \(\Delta k = 2\pi \xi\); the threshold gain \(g_{0,th}\) is the smallest \(g_0\) value ensuring \(|\Delta k| = 1\) at the frequency of the longitudinal modes. Fig. 10 shows the reflectivity of the broad-band mirror (green) and that of the buffer mirror in the three different operating conditions: Type A (red), Type B (blue) and Type C (black). In all three cases, the buffer length is fixed to \(L_{buffer} = 15a\). The bullets denote the position of the lasing mode (calculated in the following) in the three cases, with respect to both the broad-band and buffer mirror reflection spectrum. In Type C (black), the buffer refractive index perturbation coincides with that of the broad-band mirror. Therefore, at the lasing frequency, the reflectivity of both mirrors is high (because the lasing mode lies in the stop-band of the buffer mirror) and the laser threshold behaviour is mainly dominated by the mirrors feedback. In Type A, the buffer refractive index perturbation is \(\Delta n_a = -0.0001\); the lasing frequency is slightly shifted with respect to Type C, but the buffer reflectivity is smaller. In Type B, the buffer refractive index perturbation is \(\Delta n_b = -0.0005\); as a result, the band edge of the buffer dispersion relation is in the stopband of the active section. The lasing frequency is practically the same as for type A, but the buffer reflectivity is even smaller. In Type A and B, we can expect the interplay of the feedback in both the active section and the mirrors to play a role in determining the laser threshold. Fig. 11(a) shows threshold gain (solid curve) and lasing frequency (dashed line) as a function of the buffer mirror length (a) for type A (red), B (blue) and C (black). Output coupling efficiency (b). The active section has \(\Delta n_a = -0.001\) and \(L_{active} = 10a\).
As a second example, we analyze the PhC lasers shown in Fig. 14(a). This configuration is similar to the optically pumped PhC laser of [8], where the cavity mirrors are classical PhC mirrors. Referring to Fig. 14(a), \( r_{m,R} \) is the reflectivity that the forward-propagating Bloch mode of the reference, passive waveguide (whose perturbation is accounted for, in the active section, by the coupled-Bloch-mode equations) undergoes when impinging on the right mirror; this mode is reflected back to the active section because it becomes evanescent within the mirror. A similar interpretation holds for \( r_{m,L} \). This reflectivity cannot be computed by our approach, because the classical PhC mirror cannot be viewed as a weak perturbation to a reference waveguide (due to the high refractive index contrast between slab and air holes). However, it has been computed in [30] by modelling the cavity as an effective Fabry-Perot (FP) resonator and by fitting the Q-factor with that obtained through a RCWA approach [14]. For the sake of simplicity and neglecting the impact of disorder, we assume a high, two different buffer lengths, \( |r_{eq,R}| \) is smaller in the second case, thus requiring a larger gain for achieving threshold. For \( L_{\text{buffer}} = 28a \), the lasing frequency is the same again, but the threshold gain practically coincides with that of the structure with \( L_{\text{buffer}} = 5a \). This is because the mode is now close to the top of the buffer reflection spectrum side-lobe, rather than to the zero (see Fig. 13); furthermore, as for \( L_{\text{buffer}} = 5a \), \( S_{\text{11,active}} \) and \( r_{eq,2} \) are nearly in-phase (see Fig. 12). For the case of \( L_{\text{buffer}} = 17a \), although \( |r_{eq,2}| \) is maximum, the threshold gain is not minimum, but rather close to its maximum value. The reason is that \( S_{\text{11,active}} \) and \( r_{eq,2} \) are now nearly out-of-phase. We note that \( S_{\text{11,active}} \) and \( r_{eq,2} \) are also nearly out-of-phase for the case \( L_{\text{buffer}} = 21a \); in this case, however, the magnitude of \( r_{eq,2} \) is too small to significantly affect \( r_{eq,R} \). As a result of this complex interaction between the active section distributed feedback and the buffer back reflection, the Type B configuration exhibits an optimum number of buffer cells minimizing the threshold gain. These examples prove the great impact of coherent distributed feedback effects in PhC cavities like that in Fig. 9, which is similar to a LEAP laser with an in-line coupled waveguide.

Fig. 14: Typical PhC laser based on a LN cavity (a). PhC laser cavity consisting of an active section with zero back reflections for Bloch modes at the interfaces with the passive output waveguides (b). In both cases, the reference waveguide has \( n_s = 3.171 \) and the active section \( \Delta n = -0.001 \). The position of the reference plane to compute the complex loop-gain is denoted by \( z_{\text{eff}} \).
As a result, frequency-independent reflectivity $r_{m,L} = r_{m,R} = 0.98$, which represents a reasonable approximation \[8\], \[30\], \[31\]. To compute the LG, we choose the reference plane at the interface between the active section and the left mirror (see Fig. 14(a)). As a result, $r_{eq,L}$ is equal to $r_{m,L}$ and $r_{eq,R}$ is obtained from Eq. (9) by replacing $S_{11_{\text{buffer}}}$ with $r_{m,R}$. The laser threshold behaviour is summarized in Fig. 15, showing the numerically computed resonant frequencies (a) and threshold gain (b); each colour corresponds to a different longitudinal resonant mode. In the existing literature, similar cavities have been studied in \[4\] through FDTD simulations. In \[4\], it has been shown that the resonant modes of a passive LN cavity correspond to the fulfillment of the condition

\[
0.5 - \frac{k_c \alpha}{2\pi} = m/2N \tag{10}
\]

with $N = L/a$ being the number of unit cells, $m$ the mode order and $k_c$ the dispersion relation of the passive PhC LDWG of the cavity. For this reason, we have evaluated the quantity $0.5 - \beta_{eff} a/2\pi$ at the numerically computed resonant frequencies and corresponding threshold gain of Fig. 15; it is shown as the solid curve in Fig. 16. We also note that the quantity $0.5 - \beta_{eff} a/2\pi$ practically coincides, at a given cavity length, with $m/2N$ (dashed curve in Fig. 16). Since the required threshold gain is low, the gain-induced distributed feedback is negligible. As a consequence, the gain does not impact on the position of the resonant modes. As the cavity length increases, the modes move towards the SL region along the dispersion relation of the umpumped waveguide. This effect is consistent with experimental \[8\] and numerical \[32\] trends and is independent of the perturbation-induced distributed feedback. In fact, by setting $\Delta n = 0$ and $k_{2,q=1} = k_{21,q=-1} = 0$ in Eqs. (5), Eq. 10 can be easily obtained. The threshold gain reported in Fig. 15(b) is compared with the expression [1/(SL active)] ln[1/(r_{m,L} r_{m,R})] (dotted curve in Fig. 15(b)) \[8\], with $S = n_g/n_e$ being the slow-down factor, evaluated at the resonant frequencies, and $L_{\text{active}}$ the cavity length; again, the group index is that of the umpumped waveguide. This expression resembles that of a standard FP laser, but the threshold gain is scaled down by the slow-down factor. Since the gain is low, $n_g$ is not reduced and the SL enhancement of the gain is not limited. On the basis of these considerations, we conclude that the laser with classical PhC mirrors modelled as high-reflectivity, frequency-independent reflectors behaves as a SL-enhanced FP laser for Bloch modes.

As a last example, we focus on the structure in Fig. 14(b), which consists of an active section with zero back reflections for Bloch modes at the interfaces with the passive output.
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waveguides. The practical implementation of this matching condition is outside the scope of this paper; various promising solutions are reported in the existing literature [33], [34]. We show for the first time, to the best of our knowledge, that this type of structure can indeed achieve lasing with reasonable threshold gain. The reference waveguide has \( n_r = 3.171 \), while the active section has \( \Delta n_s = -0.001 \) and \( g_0 > 0 \). Similarly to the second example, here a given \( \Delta n_s \) is only considered in order to shift away the active section dispersion relation from the critical point \( k_c = \pi/a \) and investigate the laser operation also at frequencies near the band edge (as explained in the end of section II). To compute the LG, we choose a reference plane within the active region, at the interface between any two unit cells; \( N_R \) unit cells are located on the right of the reference plane and \( N_L \) on the left, with \( N = N_L + N_R \) being the total number of unit cells in the whole active region. Therefore, \( r_{eq,R} \) is given by the \( S_{11} \) parameter of Eqs. (8) with \( N = N_R \); similarly, \( r_{eq,L} \) is computed as the \( S_{22} \) parameter of Eqs. (8) with \( N = N_L \). The threshold gain and corresponding resonant frequencies of this laser are shown in Fig. 17(b) and compared with those of the laser with classical PhC mirrors (Fig. 17(a)). The cavity length ranges from 10\( a \) to 20\( a \) and each colour corresponds to a different longitudinal resonant mode. As a first remark, we note that the threshold gain is considerably larger for the laser with zero back reflections; however, the threshold gain of the lasing mode (\( M_1 \)) turns out to be reasonable, being of the same order of magnitude as observed for the Type B configuration in Fig. 9. Secondly, the lasing mode frequency is essentially independent of the cavity length and it is located close to the band edge of the active section dispersion relation with \( g_0 = 0 \). On the contrary, the higher-order modes shift towards the SL region as the cavity length increases. This is somehow similar to what occurs in purely gain-coupled DFB lasers, whose lasing frequency is independent of the cavity length and exactly located at the Bragg frequency [18]. Finally, we note that the laser in Fig. 14(b) exhibits a much better spectral selectivity as compared to a laser with classical PhC mirrors. This is illustrated in Fig. 18, comparing the gain margin (defined as the difference between \( M_2 \) and \( M_1 \) threshold gain) for the lasers in Fig. 14 as a function of cavity length.

IV. Conclusions

By starting from a set of two coupled-Bloch-mode equations [21], we have derived a simple, closed-form expression for the unit cell transmission matrix of a PhC LDWG with a generally complex refractive index perturbation as compared to a reference waveguide. This allows for a simple and numerically efficient analysis of active PhC LDWGs and lasers based on this type of waveguide, such as LEAP lasers.

In particular, we have derived the expression of the coupling coefficients and explained that the magnitude of the cross-coupling is always comparable to that of self-coupling; this is due to the non-negligible longitudinal component of TE-like Bloch modes in PhC LDWGs. We have shown that our approach can correctly reproduce the formation of a stop-band for Bloch modes as a consequence of a purely real refractive index perturbation. We have further validated it by computing the group index and gain enhancement factor of an active PhC waveguide; consistently with the rigorous, non-perturbative approach of [22], we have shown that the maximum attainable SL gain enhancement is limited by the gain itself.

We have then applied our coupled-Bloch-mode approach to analyze the threshold condition of three types of PhC laser cavities. The first cavity is conceptually similar to that characterized in [11]. Depending on the buffer refractive index perturbation, we have identified, consistently with [11], three different operating regimes, thus proving the great impact of coherent distributed feedback effects in this type of PhC cavity. The second cavity is the one characterized in [8]. By neglecting the impact of fabrication disorder and modelling the classical PhC mirrors as standard reflectors with a high, frequency-independent reflectivity, we have shown that the gain-induced distributed feedback is negligible in this type of cavity, which simply behaves as a SL-enhanced FP laser for Bloch modes. As a last example, we have analyzed a structure consisting of an active section bounded on either side by passive waveguides, which are assumed to be matched for the reference waveguide Bloch modes. This means that this configuration is different from the typical LEAP laser implementation. Interestingly, we have shown that this cavity can lase with reasonable threshold gain, with lasing only sustained by the active region distributed feedback.

In conclusion, we have presented an effective approach that will be useful to provide insights on the characteristics of PhC lasers and might be also extended to study the laser dynamics of these structures.

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