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MATH FOR ENGINEERING STUDENTS

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ABSTRACT

At the Technical University of Denmark all the Bachelor of Engineering programmes have the same courses in Mathematics given by the Department of Mathematics. In this paper, two exceptions to this are discussed: A successful course and one cancelled after a pilot run. At the Arctic Engineering programme math is combined with physics in a course, where the students learn math in a CDIO way by translating a physical model for an engineering case into a solvable mathematical model. Most students think they get a good understanding of math and the way the math works behind the physics. In 2014 a math course modelled on the Arctic Engineering course, but with the Math Department giving the math classes, was developed for the Civil Engineering programme. However, the math teacher did not use the cases for motivation but required the students to do traditional math exercises. The students were unhappy with the course, even though the failure rate was lower and the average grade higher than for the standard Math course. The good experience from the Arctic Engineering programme shows that it is possible to use engineering cases as motivation for math without losing focus on the math theory. But math teachers must be convinced that engineering students have better ways to learn math than with the traditional theoretical approach.

KEYWORDS

Mathematics, interdisciplinary courses, course design, Standards: 2, 7, 8

INTRODUCTION

How do you teach mathematics to engineering students?

At the Technical University of Denmark (DTU) the Department of Mathematics gives the math courses. All the Bachelor of Engineering (BEng) programmes have the same math courses with a few exceptions.

This paper first analyses the challenges in teaching math to engineering students and then discusses two alternative math courses – one successful for 10 years and one cancelled after a pilot run.

THE CHALLENGE TO TEACH ENGINEERING MATH

Teaching math to engineering students is a challenge – both to the teacher and to the students. The teacher faces students that do not seem to be overly interested in math, and the students often face teachers that do not know the students' use of math in other courses and future jobs. Math classes to engineering students often become more a question of survival than of engaged learning.

The question is how to teach math, so it is useful in engineering and thereby motivating for the student, and at the same time gives the student a general understanding of math without reducing math to a tool to solve an engineering problem.

What is the role of math in modern engineering?

In the beginning engineering was not based on science but only on practice. The old Romans build their aqueducts, some of which are still standing, without the mathematical basis used for modern bridge design. With the scientific rebirth in Western Europe after the Enlightenment engineering got its science foundation, and during the last century engineering education became increasingly theoretic. As a reaction to this, several initiatives have been initiated to bring back practice to engineering education (ALE, CDIO). This trend, however, has not to any extent fundamentally changed the way engineering students are taught math.

Actually, there is a lot of focus on teaching math to engineering students – there are written many papers on teaching engineering math (e.g. Croft & Ward 2001). This shows the need for developing math teaching – unfortunately it also shows how the target is missed. Most papers on math for engineers are written by mathematicians, who discuss how to optimize teaching in a math for math way. New IT tools have been introduced, and math classes have been supplemented by projects and active learning (Ferreira, Nicola & Figueiredo 2011), but this is not enough to motivate students, if they do not see a purpose for learning the math.

Math is an abstract science with an epistemology based on coherence – you seek a set of axioms that are not inconsistent. The epistemology of engineering is to seek something that works, so you can meet specified goals – it does not really matter, if the model used is self-consistent as long as you get useful results. Most engineering disciplines have a mathematical basis, but for practical engineering math is rarely very visible. In modern engineering math has become a behind the scene activity carried out by machines. Only for very specialized engineering math is a primary tool (see e.g. Journal of Engineering Mathematics), but most engineering students do not end up in these jobs – certainly not BEng graduates.

What the engineering student needs is to be able to transform the physical world into a strong mathematical model, put this into a computer, and assess the validity of the output.

Now back to the original question: How to teach math to engineering students?

There are basically three approaches to teaching math in an engineering programme: Stand-alone math courses, integrated math in engineering courses, or integrated engineering applications in math courses.

The standard way to teach math to engineering students is to leave it to the Math Department. The Math Department is staffed with mathematicians, who think math is beautiful in its own right and get very disappointed, when their engineering students do not share this fascination. And they often have large classes with students from different programmes, making it difficult to motivate and engage the student.

An alternative to this is to let an engineering teacher give pure math classes. However, over time the task of giving the math is often delegated to specific engineering teachers, who end up as math teachers.

The two other methods may seem like much of the same, but there is a significant difference. In one you get rid of the mathematicians from the Math Department and integrate the math into an engineering context, which in principle is the optimal solution giving high motivation (Brandsford et al. 2000 p. 60). In reality, it is not so simple. First, many engineering programmes do not have suitable engineering courses requiring the right level of math. Second, it is difficult for an engineering teacher to focus sufficiently on the theoretical side of math, when the real interest is in the engineering application. Third, learning math based on applications in only one field results in the math being context bound (Brandsford et al. 2000 p. 62); you need examples from different applications in order to generalize.

In the last approach, the math teachers still teach the math, but teachers from the engineering departments participate. Math is the primary learning objective – the engineering part is primarily for motivation; learning the engineering topic is an added benefit. A variation of this is that a qualified engineering teacher also teaches the math. Both variations will be discussed here. First the approach with an engineering teacher giving the entire course, and then the approach with a mathematician giving the math.

MATH AT THE BEng PROGRAMMES AT DTU

The BEng programmes focus on the practical aspect of engineering in order to make the graduates ready for a job, in contrast to the more theoretical Bachelor of Science in Engineering programmes, which are the first step to a Master degree.

Each BEng programme had their own math courses taught by an engineering teacher until 1995, when a restructuring resulting in all BEng programmes should have the same two 5 ECTS math courses given by the Math Department. An exception to this was the programme in Arctic Engineering as described in the next section.

Following a new reorganisation the plan was that the two math courses should be replaced by small math modules, from which the different programmes should select 10 ECTS relevant math topics. In reality, all programmes ended up including a common 5 ECTS math course in their first semester: BasicMath. An exception to this was an experiment at the programme in Civil Engineering as described later.

The students have many difficulties with the BasicMath course. Many students fail the course and the average grade is low. The results for the first six times the course was given are shown in the first part of Table 1.

Most students start in the fall right after high school. The students starting in the spring are older students with varying backgrounds, and they have generally more difficulties with math than the younger students directly from high school.

Table 1: Passing rates and average grades on a scale from 0 to 12 with 02 to pass.

		Passing rate	Average grade
BasicMath	Spring 2017	66	3.7
	Fall 2016	72	4.8
	Spring 2016	62	3.7
	Fall 2015	80	5.6
	Spring 2015	80	5.3
	Fall 2014	81	5.9
	Average all	**74**	**4.8**
	Average Spring	**69**	**4.2**
Math in Physics	2017	63	4.6
	2016	77	5.0
	2015	70	5.2
	2014	55	2.5
	2013	73	5.1
	2012	80	5.0
	Average	**69**	**4.6**
Math for CivEng	**Spring 2017**	**79**	**5.8**

THE BEng PROGRAMME IN ARCTIC ENGINEERING

The BEng programme in Arctic Engineering is special, since a majority of the students are Greenlandic, and the first three semesters take place in Greenland. Due to this the programme for logistics reasons has a compressed math course, with classes given every day. In 2007 the curriculum for the first three semesters was according to the CDIO principles reorganized into large interdisciplinary courses (Christensen 2008, Ingeman-Nielsen & Christensen 2011), and a new combined math and physics course was introduced: Math in Physics.

The Math in Physics course

Before the reorganization the Math Department gave the math, and the math teacher had actually designed some very good examples from engineering: ballistics, vibrations of a moving car due to bumps in the road, and oscillation of a high rise building due to an earthquake. But the examples were used in the classical math class approach: First the necessary math was given, and the physical model was elaborated and transferred into a mathematical model by the teacher – then it was up to the student to solve the resulting differential equations.

In the new course given by an engineering teacher, this is turned somewhat upside down. The students are given a realistic (although often somewhat tinkered) case – it could very well be about ballistics (all in Greenland goes hunting) or about the suspension of a car (and even better a snowmobile) – and have to create the physical model and translate this into a mathematical model themselves, before they can solve the equations.

Teaching in the course is planned with an inductive need-based approach (Kurki-Suonio & Hakola 2007). A typical teaching sequence starts in the middle of a lesson, when new concepts are introduced, and a new assignment is given. The students then study the theory and learn the new math by applying it in problem solving. In the start of the next lesson the theory from the previous lesson is summed up, and the solution to the assignment is discussed. The sequence is finished, after a new is started, when handed in assignments are quickly returned with the teacher's comments.

Right from the start complex assignments based on realistic cases are used, since standard textbook drills with given results do not crater understanding and creativity (Cropley 2001 p. 160). The challenge is that real-world problems very quickly get much more complicated than you want for the first course in math. So the complex assignments have to be very structured and solved in a spiral way. The same case may be used for several sequences – becoming more and more realistic and requiring more and more mathematical concepts to handle.

The exam must satisfy two requirements. From an engineering point of view you want to test the students' ability to solve problems – in math you want to test their conceptual understanding. The ability to solve problems is assessed with a 1½ day assignment. To assess the students' conceptual understanding a 1 hour closed-book multiple-choice like test is given.

Evaluation and discussion

The exam results for the last six years are given in the middle section of Table 1. The results are comparable to those for the BasicMath course, but in reality, they are better than could be expected, since the Greenlandic students for cultural reasons come with a weaker background and score in average approximately 2 grade points lower than their Danish colleagues.

All courses at DTU are evaluated by a standard student evaluation. The following quotes are from the evaluation in 2011, where additional evaluation was done.
Good: *The assignments were fun, even though they were difficult. It was good that [the assignments] were structured step by step. The feedback on each assignment helped me understand the course topics.*
Not so good – suggested changes: *More assignments, but smaller and easier. Fewer assignments so we can work more in depth and get a better understanding.*
The teacher had to agree with these complaints. It is easy to get carried away, when you want a case to get as realistic as possible. Since then there has been more focus on also giving the weak students some good experiences. And fewer assignments are to be handed in.

In 2011 the students were also asked to do a self-evaluation of their learning in the course. A few states that they have learned math like differentiation and integration, but many writes about the understanding they have obtained of physics and the relation between math and physics: *I have obtained a better understanding of differential equations concerning acceleration, velocity, and position. To transform physics to math language. A lot about the formulas behind the formulas we have used e.g. in statics.*

The students should indicate the most important they have learned and the most difficult in the course.

Most important learning: *To understand the laws of physics via math. To set up Newton's 2nd law in Maple and transfer this to a differential equation.*
Most difficult: *To think in math-physics terms. To understand and get into this way of thinking.*
The practice-based approach used in this course should help bridge the gap between engineering and abstract thinking, but it probably also highlights the difference in the beginning.

Most students find the learning is good in this course. However, not all students are completely happy with the course. The two most frequent complaints are "too much work" and "the realistic cases are too complex and not easy to solve". Most students have the idea that a good assignment is one, they can solve, not realizing that they learn little by solving problems they already know how to solve. A little frustration is good for learning, as long as it does not kill the motivation. Most students appreciated the structured approach with increased difficulty, and they liked to work with the same case for several days, giving them time to get a good understanding of the problem.

An encouraging aspect of the student evaluation was the positive comments about feedback – one of the most common complains in student evaluations is insufficient feedback. The comments about learning the math behind the physics and the better understanding of dynamics are also encouraging – and should be seen in relation to the severe difficulties many students have with the conceptual understanding of fundamental physics. And even though the students may feel that they mostly learn physics, in reality, what they learn is mostly math, and how to use math to model the real world.

THE BEng PROGRAMME IN CIVIL ENGINEERING

After some changes to the BEng programme in Civil Engineering the standard BasicMath course was as an experiment in spring 2017 replaced by a specially developed course on Math and Physics: Math for CivEng.

The Math for CivEng course

The math teachers did not want to participate in this, but the dean required that the Math Department should be responsible for developing and giving this course. It was designed by two math teachers and an engineering teacher, and the compromise was a course structured like the Arctic Engineering course, but with only four teaching cycles called Mini Physics Projects, even though the content was mostly math. Each cycle consisting of a) modelling a physics problem into a math model with the engineering teacher, b) presentation of the math needed to solve the problem with a math teacher, c) the students solving the problem with the help of teaching assistants, and d) discussing the physical meaning of the mathematical solution with the engineering teacher.

This seemed reasonable. The cases were derived from those used in Arctic Engineering and designed and formulated so the students had to use the math given in the math classes. However, in the implementation the math teacher used purely standard x-y examples without reference to the case and gave the students traditional math homework in addition to the work on the Mini Projects. And the math teacher insisted that the final part of the projects were not given to the students before they had done the pure math assignments. This meant that the students had very little time to work on the projects.

This structure made the students very frustrated. They loved the high-school way the math was given but hated the more complex physics problems, which they felt were too difficult with the limited time and teacher assistance available.

The exam consisted in assessments of the Mini Physics Projects and the math homework, and a written math exam – except for the Mini Physics Project this was the same model as for the BasicMath course.

Evaluation and Discussion

69 students participated in the Math for CivEng course. The students had a slightly better grade average than for the standard BasicMath course as shown in the last line of Table 1.

The planned throughout evaluation of this experimental course was not carried out, since it was clear that the structure and implementation had to be changed.

The answers to the three most relevant statements in the student course evaluation are compared to the answers for the BasicMath course in Figure 1.
The statement Q1.1 is: I think I learn a lot in this course
The statement Q1.5 is: I think the teacher/teachers create a good connection between the different teaching activities
The statement Q1.8 is: All in all I think it is a good course

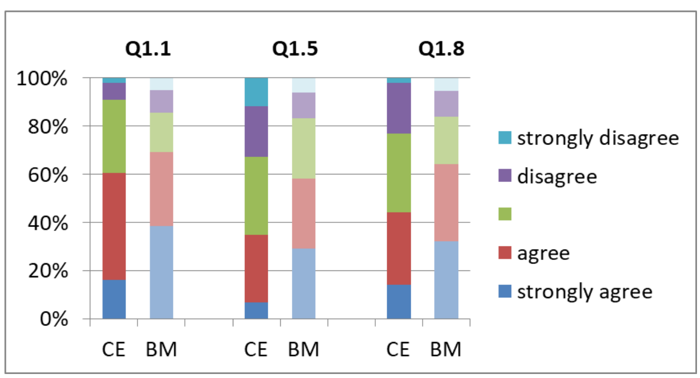

Figure 1: Answers to statements Q1.1, Q1.5 and Q1.8 in the student course evaluation.
CE stands for the Math for CivEng course.
BM stands for the BasicMath course average from fall 2014 to spring 2017.

The evaluation for the Math for CivEng course is bad, as it should be, but not as bad as it could be expected to be due to all the problems with the course. It is interesting to observe that relatively fewer students than for the BasicMath course think they have not learned anything and strongly dislike the course. A few students even think that there was a good connection between the math and physics part of the Math for CivEng course.

The following is a few of the students' comments.
The physics part has been a bit confusing, particularly because there was so long time between each lesson…
After the physics part (phase 1) you get 4-6 questions, which generally is not that difficult to do. But as soon as the math part is over, 10 physics questions (phase 2) are added to the assignment, which makes the assignment very unclear, considering you only have 2 days to finish the assignment.

There were very few positive comments:
I think the [connection] between physics and math is good, since you realize, where you use the math you learn in reality.
The physics teacher got bad evaluation:
I have difficulties to understand the course, since the teacher shows very little interest in teaching us.
Whereas the math teacher got very positive evaluation:
The teacher makes good use of the blackboard, which makes it easier to follow the lecture and take notes. At the same time, the tempo is adequate, and you feel you that you achieve a lot each time.
The students were told at the start of the course that they would meet very different teaching styles – traditional high school teaching in math and university style teaching in physics. But with the skewed planning and very little teaching assistant time allocated to the casework, the students could not appreciate the independent way of studying.

The basic problem was of course that the Math Department did not want to participate in this experiment, so they did not accept the principles for the course and align the math part accordingly. But then the engineering teacher was not prepared for the fear that the math teachers had for not giving the math in the traditional way. The engineering teacher was under pressure to make this course ready for implementation and in hindsight accepted too many compromises. However, it might have worked, if the original intentions had been implemented.

An acting dean decided that the experimental course should not continue after the pilot run, and the programme should return to use the BasicMath course until it would be possible to design and implement a better course.

CONCLUSION

As the experience with the Math in Physics course at the Arctic Engineering programme shows, it is possible to do a useful integration between math and applications, so the engineering aspect of mathematical modelling is enhanced and at the same time focus is kept on the math theory. However, it is not easy to implement. The assumed optimal way, where math teachers work with engineering teachers, was a failure due to a reluctant participation from a Math Department.

To succeed math teachers must be convinced that engineering students have better ways to learn math than the classical deductive way. And something has to be done, since far too many students, as the statistics for the BasicMath course show, have difficulties in getting a good experience with math.

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