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***Abstract*—Spectral purity of laser sources is typically investigated using phase or frequency noise measurements, which require extraction of the optical phase. This is a challenging task if the signal–to–noise–ratio (SNR) of the spectral line or the linewidth–to–noise–ratio (LNR) are not sufficiently high. In this paper, we present a statistically optimal method for optical phase noise measurement that relies on coherent detection and Bayesian filtering. The proposed method offers a record sensitivity, as the optical phase is measured at a signal power of -75 dBm (SNR of -11 dB in 1.1 GHz receiver bandwidth). Practically, this means that the phase noise measurements are, up to a high–degree, not limited by the measurement noise floor. This allows measurements down to -200 dB rad^2/Hz and up to 10 GHz, which is useful when measuring the Schawlow–Townes (quantum noise limited) laser linewidth. Finally, the estimated optical phase is highly accurate allowing for quantum limited signal demodulation. The method thus holds the potential to become a reference measurement tool.**

***Index Terms*—phase noise, lasers, frequency combs, Bayesian filtering, machine learning**

I. Introduction

OPTICAL phase and frequency noise characterization of laser sources and frequency combs relies on accurate optical phase measurements [1]. What complicates this measurement is the presence of measurement noise which may originate from the shot noise and thermal noise due to the photodetection process, analogue-to-digital converters, mixers, local oscillator synthesizers and amplification. The measurement noise defines an upper bound on the available SNR, defined as the ratio between the power of the carrier and the integrated measurement noise power in the bandwidth of interest. Another important, but highly overlooked parameter, is the LNR. The LNR is especially important when performing phase noise measurements of lasers with ultra–narrow linewidth ($<$1 Hz).

For a given SNR and LNR, we seek to find an optimum method that results in the most accurate optical phase noise measurement. The statistically optimum, and thereby, theoretically, the most accurate optical phase noise measurement, is obtained by defining and solving the minimum mean square error (MMSE) problem for the optical phase [2]. The MMSE is defined as: $\phi^{MMSE} = \text{argmin}_{\phi} E[(\phi - \phi^{true})^2]$, where ϕ^{MMSE} is the phase ϕ that is the solution to the MMSE problem and $E[\cdot]$ is the expectation operator.

In this paper, we demonstrate both numerically and experimentally that optical phase and frequency noise measurement methods which solve the MMSE problem significantly outperform – in the achievable measurement range and accuracy – conventional techniques relying on delayinterferometer and direct demodulation, respectively [3], [4]. The reason for this is that the methods reported in [3], [4], are not a solution to the MMSE problem and thereby do not employ any filtering of the measurement noise. Another widely used approach, not compared to in this paper, is based on cross-correlation [5]. The phase noise measurement based on cross-correlation does employ long–term averaging to reduce the impact of measurement noise. However, if the measurement noise contains non–white noise, the averaging may introduce bias. Finally, the cross-correlation does not provide the evolution of the optical phase noise as a function of time which can be useful in evaluating stability of laser sources in terms of Allan variance.

It should be noted that the phase and frequency noise measurement techniques reported in [3], [4], [5] do provide accurate phase and frequency noise measurements if the optical power is sufficiently high and the frequency range of interest is not too large (up to $\sim$10 MHz). However, the conventional methods do not scale well for low optical signal powers and when measuring frequencies exceeding $\sim$10 MHz (SNR decreases for increasing measurement bandwidth). Unfortunately, there is no reported work on the accuracy of phase noise measurement techniques presented in [3], [4], [5] as a function of SNR or LNR.

The field of Bayesian filtering, which is an important part of the machine learning toolkit, and digital coherent detection, which recently has had a large impact in the field of optical communications [6], offer a novel approach for optical phase noise measurements. The digital coherent receiver combines a balanced receiver architecture capable of retrieving the in–phase or quadrature components of the beat signal which are quantized by high speed analogue–to–digital–converters (ADC). The acquired samples can then be stored and processed offline. The phase noise is then estimated using advanced signal processing methods such as state–space based Bayesian filtering in combination with various machine learning techniques. The obtained phase noise corresponds to the solution of the MMSE problem and results in the optimal phase noise measurement.

The disadvantage of the techniques based on Bayesian filter-

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ing is that they rely on coherent detection and thereby require a stable local oscillator (LO) laser with significantly lower phase noise compared to the laser under test. An approach to solve this issue would be to use an identical laser for the LO and for the laser under test.

In this paper, we present a state-space based Bayesian filtering framework for phase and frequency noise characterization. Machine learning techniques such as Monte Carlo Markov Chain, (MCMC) and Metropolis–Hasting algorithm are employed to learn the parameters, (variance of the process and measurement noise), of the state-space model [2].

II. Bayesian filtering framework

Given the sampled photocurrent, y_k, the Bayesian filtering framework requires a state–space model that consists of: 1) a measurement equation, which describes the relation between y_k and the time-varying optical phase, ϕ_k and, 2) state equation, which is an approximation to the evolution of the optical phase difference between the laser under test (LUT) and the LO: $\phi_k = \phi_k^{LUT} - \phi_k^{LO}$. The proposed state-space framework is as following:

$$\phi_k = \phi_{k-1} + v_{k-1} \quad (1)$$

$$y_k = A\sin(\Delta\omega T_s k + D(T_s k)^2 + \phi_k) + n_k \quad (2)$$

where y_k are the discrete–time samples after the ADC, k is an integer representing time, $F_s = 1/T_s$ is the sampling frequency of the ADC and $A = 2R\sqrt{P_s P_{LO}}$. R is the responsivity of the photodiodes, P_s and P_{LO} are the powers of the LUT and the LO. n_k is the measurement noise contribution, corresponding to the balanced receiver shot noise and the electronics' thermal noise. It is assumed that n_k has Gaussian distribution with zero mean and variance σ_n^2. v_k is a Langevin noise source with variance σ_{Ln}^2. The frequency difference between the laser under test and the LO is expressed as $\Delta\omega$ and D is a linear laser frequency drift expressed in Hz^2/s. Since D is included in the measurement equation, the phase noise estimation is not affected by it.

The state-space representation framework requires that ϕ_k is described as a first-order Markov process. To obtain a first-order Markov model of ϕ_k, one approach is to consider physics-based equations [6]. However, it may not be always feasible to obtain the physics-based model or the physical parameters entering those model equations. In this paper, it will be shown that a relatively simple state equation, Eq. (1), is a good approximation for describing the evolution of phase noise. Even though Eq. (1) is an approximation to the true phase noise evolution, the measurements y_k, Eq. (2), correct for this model mismatch through Kalman filtering equations as they carry information about the optical phase.

Once the state–space framework has been formulated, Bayesian filtering can be employed for optical phase extraction. The framework of Bayesian filtering provides a recursive algorithm that computes a statistically optimum joint estimation of phases ϕ_k, given y_k for $k = 1, ..., L$ where L is the total number of samples. This is referred to as dynamic parameter estimation as the phase ϕ_k is evolving with time. The optimum

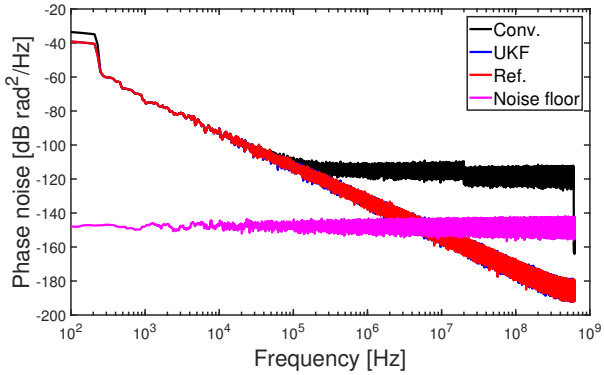

Fig. 1. *Simulations*: Phase noise spectra obtained using the conventional and the UKF based phase estimation methods.

estimates will be the mean values of the phase evolution ϕ_k denoted by $\overline{\phi}_k$.

In terms of optical phase noise measurements, the biggest advantage of Bayesian filtering is in filtering out the measurement noise. This is because given the noisy measurements y_k, the Bayesian filter estimates phases, ϕ_k, that minimizes the MMSE criterion. Bayesian filtering is an iterative process and it computes $\overline{\phi}_k = E[\phi_k | y_{1:k}]$ which implies averaging across the entire measurement rather than measuring the distribution describing phi at time k. Indeed Eq. (1) specifies the quantity that we want to estimate from y_k. This will result in a phase which is closer to the true phase and thereby significantly less affected by the measurement noise compared to the direct demodulation Eq. (3). One can consider Bayesian filtering as a matched and adaptive filter around the carrier.

In this paper, the Bayesian filtering framework is implemented using the unscented and the extended Kalman filter, (UKF and EKF). The UKF is a special case of Gaussian filtering and is highly accurate when the system under consideration is nonlinear which is the case if the laser has significant amount of phase noise. The UKF filter employs matrix inversion and is thereby more computationally complex compared to EKF.

The state–space model expressed by Eq. (1) and (2) also includes unknown static parameters $\mathbf{\Theta} = [\sigma_n^2, \sigma_{Ln}^2, D]$ that need to be jointly estimated together with the dynamic parameters. The objective is to perform joint learning of the dynamic and the static parameters of the state-space model in a statistically optimal way. Joint static and dynamic parameter learning is achieved by embedding the MCMC approach implemented as a Metropolis-Hastings (MH) algorithm within the UKF or EKF framework [2]. First, the static parameters are learned by employing the MCMC algorithm which has a prediction error provided by the UKF or EKF as a cost function. Once the static parameters are learned, we proceed with UKF or EKF. Detailed explanation of the entire procedure can be found in [2], Chapter 12. There are several other methods for learning the static parameters of the state–space model as explained in [2], Chapter 12. Comparison of different methods in terms of speed and computational complexity would be useful and could potentially be addressed in a future work.

III. Numerical results

First, we investigate the performance of the Bayesian filtering framework for phase noise characterization using numerical simulations. The advantage of numerical simulations is that the reference phase is available. The phase noise of the LUT and the LO is modelled as a Wiener process, i.e. $\phi_k^{LUT/LO} = \phi_{k-1}^{LUT/LO} + v_{k-1}^{LUT/LO}$, where the variance is expressed as $\sigma^2_{LUT/LO} = 2\pi\Delta\nu_{LUT/LO}T_s$. For the numerical simulations, the linewidth of the LUT, ($\Delta\nu_{LUT}$) and the LO, ($\Delta\nu_{LO}$), are set to 5 mHz, respectively. The powers of both LO and LUT are set at +15 dBm. High powers are deliberately chosen to illustrate the challenge of performing phase noise measurements of sub–Hz linewidth lasers even though the SNR and LNR are very high. The bandwidth under consideration is 1.25 GHz and the corresponding SNR and LNR is 80 dB and 77 dB, respectively. The frequency difference between the LUT and the LO is 20 MHz. For comparison reasons, we also employ a conventional method for phase noise estimation based on direct demodulation [3]:

$$\begin{aligned}\phi_k^{conv} &= \arg[(y_k + j\mathcal{H}\{y_k\})e^{-j\Delta\omega T_s k}] = \arg[I + jQ] \\ &= \tan^{-1}(Q/I) \end{aligned} \quad (3)$$

Here $\mathcal{H}\{\cdot\}$ denotes the Hilbert transformation, used for obtaining the quadrature component of the optical field. In general, the performance of the direct demodulation method, (Eq. 3), and delay–intereferometer for phase noise measurement is very similar [3]. Fig. 1, shows the phase noise spectra obtained using the conventional and the UKF method together with the reference phase noise spectrum and the noise floor. Fig. 1, illustrates that the phase noise spectrum obtained using the UKF method completely agrees with the reference for the entire frequency range. The conventional method is only accurate up to 100 kHz whereafter the phase noise spectrum deviates from the reference and flattens out. This illustrates challenges in measuring ultra–narrow linewidth lasers, using conventional methods, even in the presence of high SNR and LNR. Even though the reference phase noise spectra is below the noise floor for frequencies beyond 10 MHz, the UKF method achieves accurate phase noise extraction. The reason why a full match is obtained between the reference and the UKF estimated phase noise spectra is because we know an exact expression for the phase noise generation process. This information is then used in the state–space model, i.e. Eq. (1).

IV. Experimental results

In our experimental investigations, we compare three different phase noise measurement techniques: 1) the conventional approach given by Eq. (3) based on direct demodulation [3], [4], 2) a self-homodye delay-interferometer, implemented in the OEwaves OE4000 laser linewidth and phase noise system and 3) the Bayesian filtering framework, implementing the UKF or EKF. The LUT is a fully integrated ultra–low noise laser with an output power of -2.3 dBm[7]. The LO is a Koshin Kogaku tunable laser with output power of 2 dBm. The real–time sampling scope employed for the measurement has sampling rate of 50 GSa/s and an analogue bandwidth of 70 GHz (Tektronix DPO70000SX ATI).

The measured phase noise spectra are shown in Fig. 2. Fig. 2 illustrates that there is fairly good agreement between the EKF and OEwave instrument up to $\sim$2 MHz. The conventional method is in general not able to provide correct phase noise measurement. The phase noise spectrum obtained by the OEwaves instrument stops at -140 dB rad^2/Hz which is the instrument measurement noise floor. The EKF phase noise measurement is however, not limited by the measurement noise floor in the same way and it therefore continues down to -200 dB rad^2/Hz. The EKF based phase noise measurement thereby provides a more accurate measurement of the Schawlow-Townes linewidth (quantum noise laser limited linewidth) compared to the OEwaves instrument and the conventional phase noise measurement based on direct demodulation.

Next, we investigate the sensitivity of the proposed Bayesian filtering. We vary the LUT power to the coherent receiver down to -74.9 dBm. For the LUT and the LO we use narrow linewidth ($\sim$0.1 kHz) single frequency low–noise distributed feedback (DFB) fiber lasers from NKT Photonics. The LO power is 7 dBm. For the signal detection, we use a balanced receiver followed by the A/D converter. The balanced receiver is shot noise limited up to 2.1 GHz and has 3 dB bandwidth of 1.1 GHz [8]. The sampling rate of the A/D converter is 10 GSa/s.

Fig. 3 shows the frequency noise spectra obtained using the UKF and the conventional method, Eq. (3), for three different LUT power levels. The frequency noise spectra according to the LUT datasheet, (denoted Specs.) is shown as a dashed line in the figure. This spectrum has been measured by a very stable self–heterodyne method and therefore does not include filtering of the measurement noise. The frequency noise spectra are obtained from the phase noise spectra by multiplication with f^2, where f is the frequency.

For the optical signal power of -56.8 dBm (2.1 nW), corresponding to the SNR of 11 dB, the FN curves based on the UKF and conventional approach overlap up to approximately 500 kHz. Beyond this frequency, the conventional approach for FN measurement becomes unreliable. For signal powers lower than -56.8 dBm, Fig. 3 indicates that the approach based on the UKF method significantly outperforms the conventional method. In fact, the UKF method provides fairly accurate and consistent FN spectra estimation down to a signal power of -74.9 dBm (SNR = -11 dB).

It should however be noticed that as the LUT power is decreased, FN spectra start rising in the frequency range from 1 kHz–10 kHz. The reason is that as the SNR is decreased, the filtering of the measurement noise becomes more challenging. The elaborate: since we have noisy measurements, y_k, the proposed method can never provide the exact replica of the laser phase noise but an approximation to it. Therefore, there will always be a mismatch between the true and the estimated phase noise. For the UKF, the error will increase as the optical signal power or SNR is decreased as shown in Fig. 3. In general, the accuracy of Bayesian filtering decreases as a function of SNR [2].

For the simulation results, the error between the estimated and the true phase noise can be computed as a function of

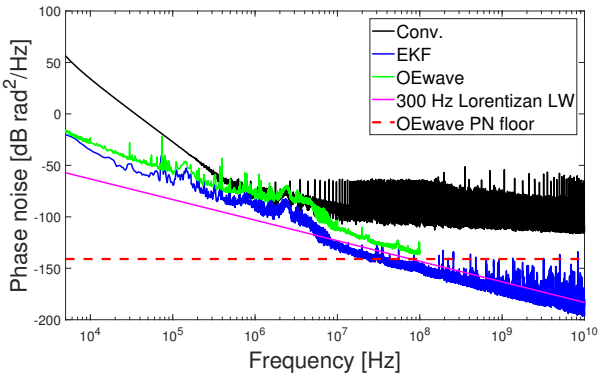

Fig. 2. *Experiment:* Phase noise spectra obtained using different phase noise measurement techniques. PN: phase noise.

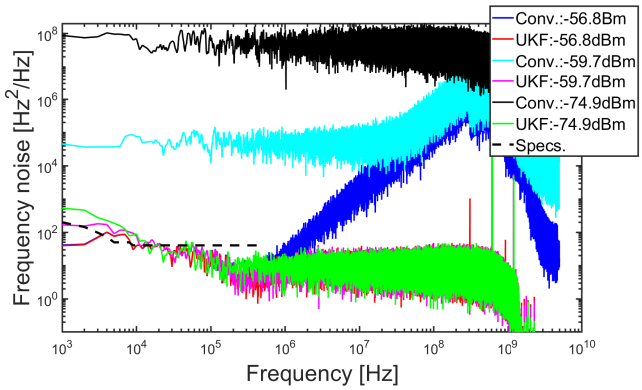

Fig. 3. *Experiment:* Frequency noise spectra for varying LUT power obtained. Legend entry 'Specs' denotes the measurement provided by the laser manufacturer.

increasing measurement noise level. However, for simulation results we use the same model for the generation of the phase noise and for the estimation. Therefore, the results may look too optimistic and may not be informative for experimental data where we do not know phase noise generation model. An approach to check the accuracy of the estimated phase noise is to modulate, y_k with binary data, ($\pm$ 1),and then use previously estimated phase noise to perform data demodulation. If the estimated phase is accurate we should be able to recover the data. Fig. 4 shows the evaluated bit error rate (BER), calculated from 10^7 bits, as a function of the LUT signal power, plotted together with the quantum-noise limited BER curve assuming ideal heterodyne detection and ideal phase noise compensation. It is observed that the experimental BER obtained using the UKF based phase estimation has negligible penalty to the quantum-limited curve. This demonstrates fairly accurate phase estimation down to -74.9 dBm. It should although be noted that the deviation of the UKF curve from the theoretical achievable quantum limit curve increases for decreasing input power which is in accordance with Fig. 3.

V. Conclusion and perspectives

The novelty of the paper is on the experimental demonstration of the Bayesian filtering framework for performing ultra-sensitive phase noise measurements, demonstrating the record

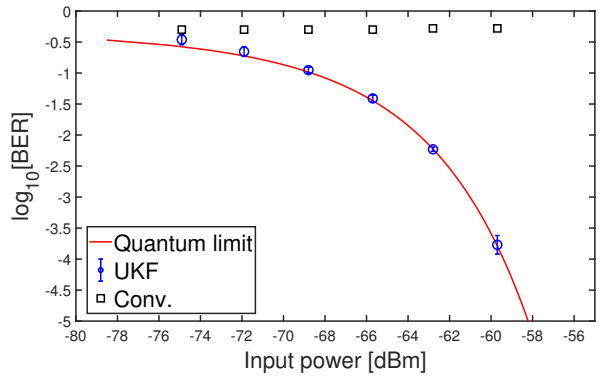

Fig. 4. Bit error rate (BER) as a function of input signal power

sensitivity in terms of power level and measurement range. The framework has been demonstrated to operate in a wide–range of SNRs (-11 dB in 1.1 GHz bandwidth), wide frequency range (10 GHz) and be able to measure down to -200 dB rad^2/Hz. The presented framework could potentially also be useful in many applications that rely on accurate optical phase tracking from low-power optical signals such as: continuous-variable quantum key distribution (CV–QKD), state-estimation for cavity opto–mechanics, low-power optical communication and quantum metrology.

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References

[1] X. Xie, R. Bouchand, D. Nicolodi, M. Giunta, W. Hansel, M. Lezius, A. Joshi, S. Datta, C. Alexandre, M. Lours, P. Tremblin, G. Santarelli, R. Holzwarth, and Y. L. Coq, "Photonic microwave signals with zeptosecond-level absolute timing noise," *Nat. Photonics.*, vol. 11, pp. 44–48, 2017.

[2] S. Sarkka, *Bayesian Filtering and Smoothing*. New York, NY, USA: Cambridge University Press, 2013.

[3] M. Schiemangk, S. Spießberger, A. Wicht, G. Erbert, G. Tränkle, and A. Peters, "Accurate frequency noise measurement of free-running lasers," *Applied Optics*, vol. 53, no. 30, p. 7138, 2014. [Online]. Available: https://www.osapublishing.org/abstract.cfm?URI=ao-53-30-7138

[4] N. Von Bandel, M. Myara, M. Sellahi, T. Souici, R. Dardaillon, and P. Signoret, "Time-dependent laser linewidth: beat-note digital acquisition and numerical analysis," *Optics Express*, vol. 24, no. 24, p. 27961, 2016. [Online]. Available: https://www.osapublishing.org/abstract.cfm?URI=oe-24-24-27961

[5] X. Xie, R. Bouchand, D. Nicolodi, M. Lours, C. Alexandre, and Y. L. Coq, "Phase noise characterization of sub-hertz linewidth lasers via digital cross correlation," *Opt. Lett.*, vol. 42, no. 7, pp. 1217–1220, Apr 2017.

[6] D. Zibar, M. Piels, R. Jones, and C. G. Schaeffer, "Machine Learning Techniques in Optical Communication," *Journal of Lightwave Technology*, vol. 34, no. 6, pp. 1442–1452, 2016.

[7] D. Huang, M. A. Tran, J. Guo, J. Peters, T. Komljenovic, A. Malik, P. A. Morton, and J. E. Bowers, "High-power sub-khz linewidth lasers fully integrated on silicon," *Optica*, vol. 6, no. 6, pp. 745–752, Jun 2019.

[8] T. Gehring, C. Lupo, A. Kordts, D. S. Nikolic, N. Jain, T. B. Pedersen, S. Pirandola, and U. L. Andersen, "8 GBit/s real-time quantum random number generator with non-iid samples," pp. 1–12, 2018. [Online]. Available: http://arxiv.org/abs/1812.05377