Highly–sensitive phase and frequency noise measurement technique using Bayesian filtering

Zibar, Darko; Chin, Hou-Man; Tong, Yeyu; Jain, Nitin; Guo, Joel; Chang, Lin; Gehring, Tobias; Bowers, John E.; Andersen, Ulrik Lund

Published in:
IEEE Photonics Technology Letters

Publication date:
2019

Document Version
Peer reviewed version

Link back to DTU Orbit

Citation (APA):
Highly–sensitive phase and frequency noise measurement technique using Bayesian filtering

Darko Zibar, Hou–Man Chin, Yeyu Tong, Nitin Jain, Joel Guo, Lin Chang, Tobias Gehring, John E. Bowers and Ulrik L. Andersen

Abstract—Spectral purity of laser sources is typically investigated using phase or frequency noise measurements, which require extraction of the optical phase. This is a challenging task if the signal–to–noise–ratio (SNR) of the spectral line or the linewidth–to–noise–ratio (LNR) are not sufficiently high. In this paper, we present a statistically optimal method for optical phase noise measurement that relies on coherent detection and Bayesian filtering. The proposed method offers a record sensitivity, as the optical phase is measured at a signal power of -75 dBm (SNR of -11 dB in 1.1 GHz receiver bandwidth). Practically, this means that the phase noise measurements are, up to a high–degree, not limited by the measurement noise floor. This allows measurements down to -200 dB rad²/Hz and up to 10 GHz, which is useful when measuring the Schawlow–Townes (quantum noise limited) laser linewidth. Finally, the estimated optical phase is highly accurate allowing for quantum limited signal demodulation. The method thus holds the potential to become a reference measurement tool.

Index Terms—phase noise, lasers, frequency combs, Bayesian filtering, machine learning

I. INTRODUCTION

Optical phase and frequency noise characterization of laser sources and frequency combs relies on accurate optical phase measurements [1]. What complicates this measurement is the presence of measurement noise which may originate from the shot noise and thermal noise due to the photodetection process, analogue–to–digital converters, mixers, local oscillator synthesizers and amplification. The measurement noise defines an upper bound on the available SNR, defined as the ratio between the power of the carrier and the integrated measurement noise power in the bandwidth of interest. Another important, but highly overlooked parameter, is the LNR. The LNR is especially important when performing phase noise measurements of lasers with ultra–narrow linewidth (<1 Hz).

For a given SNR and LNR, we seek to find an optimum method that results in the most accurate optical phase noise measurement. The statistically optimum, and thereby, theoretically, the most accurate optical phase noise measurement, is obtained by defining and solving the minimum mean square error (MMSE) problem for the optical phase [2]. The MMSE is defined as: \( \phi_{\text{MMSE}} = \arg\min_{\phi} E[(\phi - \phi_{\text{true}})^2] \), where \( \phi_{\text{MMSE}} \) is the phase \( \phi \) that is the solution to the MMSE problem and \( E[\cdot] \) is the expectation operator.

In this paper, we demonstrate both numerically and experimentally that optical phase and frequency noise measurement methods which solve the MMSE problem significantly outperform – in the achievable measurement range and accuracy – conventional techniques relying on delay interferometer and direct demodulation, respectively [3], [4]. The reason for this is that the methods reported in [3], [4], are not a solution to the MMSE problem and thereby do not employ any filtering of the measurement noise. Another widely used approach, not compared to in this paper, is based on cross–correlation [5]. The phase noise measurement based on cross–correlation does employ long–term averaging to reduce the impact of measurement noise. However, if the measurement noise contains non–white noise, the averaging may introduce bias. Finally, the cross–correlation does not provide the evolution of the optical phase noise as a function of time which can be useful in evaluating stability of laser sources in terms of Allan variance.

It should be noted that the phase and frequency noise measurement techniques reported in [3], [4], [5] do provide accurate phase and frequency noise measurements if the optical power is sufficiently high and the frequency range of interest is not too large (up to \( \sim 10 \) MHz). However, the conventional methods do not scale well for low optical signal powers and when measuring frequencies exceeding \( \sim 10 \) MHz (SNR decreases for increasing measurement bandwidth). Unfortunately, there is no reported work on the accuracy of phase noise measurement techniques presented in [3], [4], [5] as a function of SNR or LNR.

The field of Bayesian filtering, which is an important part of the machine learning toolkit, and digital coherent detection, which recently has had a large impact in the field of optical communications [6], offer a novel approach for optical phase noise measurements. The digital coherent receiver combines a balanced receiver architecture capable of retrieving the in–phase or quadrature components of the beat signal which are quantized by high speed analogue–to–digital–converters (ADC). The acquired samples can then be stored and processed offline. The phase noise is then estimated using advanced signal processing methods such as state–space based Bayesian filtering in combination with various machine learning techniques. The obtained phase noise corresponds to the solution of the MMSE problem and results in the optimal phase noise measurement.

The disadvantage of the techniques based on Bayesian filter-
ing is that they rely on coherent detection and thereby require a stable local oscillator (LO) laser with significantly lower phase noise compared to the laser under test. An approach to solve this issue would be to use an identical laser for the LO and for the laser under test.

In this paper, we present a state-space based Bayesian filtering framework for phase and frequency noise characterization. Machine learning techniques such as Monte Carlo Markov Chain, (MCMC) and Metropolis–Hasting algorithm are employed to learn the parameters, (variance of the process and measurement noise), of the state-space model [2].

II. BAYESIAN FILTERING FRAMEWORK

Given the sampled photocurrent, $y_k$, the Bayesian filtering framework requires a state–space model that consists of: 1) a measurement equation, which describes the relation between $y_k$ and the time-varying optical phase, $\phi_k$, and, 2) state equation, which is an approximation to the evolution of the optical phase difference between the laser under test (LUT) and the LO: $\phi_k = \phi_k^{\text{LUT}} - \phi_k^{\text{LO}}$. The proposed state-space framework is as following:

\begin{align*}
\phi_k &= \phi_{k-1} + v_{k-1} \\
y_k &= A \sin(\Delta \omega T_s k + D(T_s k)^2 + \phi_k) + n_k
\end{align*}

where $y_k$ are the discrete–time samples after the ADC, $k$ is an integer representing time, $T_s = 1/T_\text{ADC}$ is the sampling frequency of the ADC and $A = 2R \sqrt{P_s P_{\text{LO}}}$. $R$ is the responsivity of the photodiodes, $P_s$ and $P_{\text{LO}}$ are the powers of the LUT and the LO. $n_k$ is the measurement noise contribution, corresponding to the balanced receiver shot noise and the electronics’ thermal noise. It is assumed that $n_k$ has Gaussian distribution with zero mean and variance $\sigma^2_n$, $v_k$ is a Langevin noise source with variance $\sigma^2_L$. The frequency difference between the laser under test and the LO is expressed as $\Delta \omega$ and $D$ is a linear laser frequency drift expressed in Hz\(^2/\text{s}\). Since $D$ is included in the measurement equation, the phase noise estimation is not affected by it.

The state-space representation framework requires that $\phi_k$ is described as a first-order Markov process. To obtain a first-order Markov model of $\phi_k$, one approach is to consider physics-based equations [6]. However, it may not be always feasible to obtain the physics-based model or the physical parameters entering those model equations. In this paper, it will be shown that a relatively simple state equation, Eq. (1), is a good approximation for describing the evolution of phase noise. Even though Eq. (1) is an approximation to the true phase noise evolution, the measurements $y_k$, Eq. (2), correct for this model mismatch through Kalman filtering equations as they carry information about the optical phase.

Once the state–space framework has been formulated, Bayesian filtering can be employed for optical phase extraction. The framework of Bayesian filtering provides a recursive algorithm that computes a statistically optimum joint estimation of phases $\phi_k$, given $y_k$ for $k = 1, ..., L$ where $L$ is the total number of samples. This is referred to as dynamic parameter estimation as the phase $\phi_k$ is evolving with time. The optimum estimates will be the mean values of the phase evolution $\phi_k$ denoted by $\overline{\phi_k}$.

In terms of optical phase noise measurements, the biggest advantage of Bayesian filtering is in filtering out the measurement noise. This is because given the noisy measurements $y_k$, the Bayesian filter estimates phases, $\phi_k$, that minimizes the MMSE criterion. Bayesian filtering is an iterative process and it computes $\overline{\phi_k} = E[\phi_k | y_{1:k}]$ which implies averaging across the entire measurement rather than measuring the distribution describing phi at time $k$. Indeed Eq. (1) specifies the quantity that we want to estimate from $y_k$. This will result in a phase which is closer to the true phase and thereby significantly less affected by the measurement noise compared to the direct demodulation Eq. (3). One can consider Bayesian filtering as a matched and adaptive filter around the carrier.

In this paper, the Bayesian filtering framework is implemented using the unscented and the extended Kalman filter, (UKF and EKF). The UKF is a special case of Gaussian filtering and is highly accurate when the system under consideration is nonlinear which is the case if the laser has significant amount of phase noise. The UKF filter employs matrix inversion and is thereby more computationally complex compared to EKF.

The state–space model expressed by Eq. (1) and (2) also includes unknown static parameters $\Theta = [\sigma^2_n, \sigma^2_L, D]$ that need to be jointly estimated together with the dynamic parameters. The objective is to perform joint learning of the dynamic and the static parameters of the state-space model in a statistically optimal way. Joint static and dynamic parameter learning is achieved by embedding the MCMC approach implemented as a Metropolis-Hastings (MH) algorithm within the UKF or EKF framework [2]. First, the static parameters are learned by employing the MCMC algorithm which has a prediction error provided by the UKF or EKF as a cost function. Once the static parameters are learned, we proceed with UKF or EKF. Detailed explanation of the entire procedure can be found in [2], Chapter 12. There are several other methods for learning the static parameters of the state–space model as explained in [2], Chapter 12. Comparison of different methods in terms of speed and computational complexity would be useful and could potentially be addressed in a future work.
First, we investigate the performance of the Bayesian filtering framework for phase noise characterization using numerical simulations. The advantage of numerical simulations is that the reference phase is available. The phase noise of the LUT and the LO is modelled as a Wiener process, i.e. \( \phi_{k}^{\text{LUT/LO}} = \phi_{k-1}^{\text{LUT/LO}} + \nu_{k}^{\text{LUT/LO}} \), where the variance is expressed as \( \sigma_{k}^{2} = 2\pi \Delta \nu_{k}^{\text{LUT/LO}} T_{k} \). For the numerical simulations, the linewidth of the LUT, \( \Delta \nu_{\text{LUT}} \), and the LO, \( \Delta \nu_{\text{LO}} \), are set to 5 mHz, respectively. The performance of the direct demodulation method, (Eq. 3), for three different SNR levels is illustrated in Fig. 2. In general, the accuracy of Bayesian filtering decreases as a function of SNR [2].

The measured phase noise spectra are shown in Fig. 2. Fig. 2 illustrates that there is fairly good agreement between the EKF and OEwaves instrument up to \( \sim 2 \) MHz. The conventional method is in general not able to provide correct phase noise measurement. The phase noise spectrum obtained by the OEwaves instrument stops at \(-140 \) dB rad\(^2\)/Hz which is the instrument measurement noise floor. The EKF phase noise measurement is however, not limited by the measurement noise floor in the same way and it therefore continues down to \(-200 \) dB rad\(^2\)/Hz. The EKF based phase noise measurement thereby provides a more accurate measurement of the Schawlow-Townes linewidth (quantum noise laser limited linewidth) compared to the OEwaves instrument and the conventional phase noise measurement based on direct demodulation.

Next, we investigate the sensitivity of the proposed Bayesian filtering. We vary the LUT power to the coherent receiver down to \(-74.9 \) dBm. For the LUT and the LO we use narrow linewidth (\(<0.1 \) kHz) single frequency low–noise distributed feedback (DFB) fiber lasers from NKT Photonics. The LO power is \(7 \) dBm. For the signal detection, we use a balanced receiver followed by the A/D converter. The balanced receiver is shot noise limited up to \(2.1 \) GHz and has \(3 \) dB bandwidth of \(1.1 \) GHz [8]. The sampling rate of the A/D converter is \(10 \) GSa/s.

Fig. 3 shows the frequency noise spectra obtained using the UKF and the conventional method, (Eq. 3), for three different LUT power levels. The frequency noise spectra according to the LUT datasheet, (denoted Specs.) is shown as a dashed line in the figure. This spectrum has been measured by a very stable self–heterodyne method and therefore does not include filtering of the measurement noise. The frequency noise spectra are obtained from the phase noise spectra by multiplication with \(f^2\), where \(f\) is the frequency.

For the optical signal power of \(-56.8 \) dBm (2.1 nW), corresponding to the SNR of 11 dB, the FN curves based on the UKF and conventional approach overlap up to approximately 500 kHz. Beyond this frequency, the conventional approach for FN measurement becomes unreliable. For signal powers lower than \(-56.8 \) dBm, Fig. 3 indicates that the approach based on the UKF method significantly outperforms the conventional method. In fact, the UKF method provides fairly accurate and consistent FN spectra estimation down to a signal power of \(-74.9 \) dBm (SNR = -11 dB).

It should however be noticed that as the LUT power is decreased, FN spectra start rising in the frequency range from \(1 \) kHz–\(10 \) kHz. The reason is that as the SNR is decreased, the filtering of the measurement noise becomes more challenging. The elaboration: since we have noisy measurements, \( \nu_{k} \), the proposed method can never provide the exact replica of the laser phase noise but an approximation to it. Therefore, there will always be a mismatch between the true and the estimated phase noise. For the UKF, the error will increase as the optical signal power or SNR is decreased as shown in Fig. 3. In general, the accuracy of Bayesian filtering decreases as a function of SNR [2].

For the simulation results, the error between the estimated and the true phase noise can be computed as a function of
sensitive phase noise measurements, demonstrating the record
sensitivity in terms of power level and measurement range. The framework has been demonstrated to operate in a wide–
range of SNRs (-11 dB in 1.1 GHz bandwidth), wide frequency
range (10 GHz) and be able to measure down to -200 dB
rad$^2$/Hz. The presented framework could potentially also be
useful in many applications that rely on accurate optical phase
tracking from low-power optical signals such as: continuous-
variable quantum key distribution (CV–QKD), state-estimation
for cavity opto–mechanics, low-power optical communication
and quantum metrology.

ACKNOWLEDGMENT
This work is supported by the European Research Council
through the ERC-CoG FRECOM project (grant agreement
no. 771878), the EU project CiViQ (grant agreement no.
820466), the Innovation Fund Denmark, Quantum Innovation
Center (Qubiz) and the Center for Macroscopic Quantum
States (bigQ, DNRF142).

REFERENCES
A. Joshi, S. Datta, C. Alexandre, M. Lours, P. Tremblin, G. Santarelli,
R. Holzwarth, and Y. L. Coq, “Photonic microwave signals with
and A. Peters, “Accurate frequency noise measurement of free-running
P. Signoret, “Time-dependent laser linewidth: beat-note digital acquisition
[Online]. Available: https://www.osapublishing.org/abstract.cfm?URI=oe-
24-24-27961
Coq, “Phase noise characterization of sub-hertz linewidth lasers via digital
Techniques in Optical Communication,” Journal of Lightwave Technol-
Morton, and J. E. Bowers, “High-power sub-kHz linewidth lasers fully
S. Pirandola, and U. L. Andersen, “8 GBit/s real-time quantum random
number generator with non-1id samples,” pp. 1–12, 2018. [Online]. Available:
http://arxiv.org/abs/1812.05377