Modelling of Compliant-type Gas Bearings: A Numerical Recipe

von Osmanski, Sebastian; Larsen, Jon S.; Santos, Ilmar

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Modelling of Compliant-type Gas Bearings: A Numerical Recipe

Sebastian von Osmanski¹, Jon S. Larsen², Ilmar F. Santos¹

¹ Dept. of Mechanical Engineering, Technical University of Denmark, 2800 Kgs. Lyngby, Denmark
² GEA Process Engineering A/S, 2860 Søborg, Denmark
E-mail addresses: asvosm@mek.dtu.dk (S. von Osmanski), jonsteffen.larsen@gea.com (J.S. Larsen), and ifs@mek.dtu.dk (I.F. Santos)

Abstract

Despite its merits, the Gas Foil Bearing (GFB) suffers from several inherent limitations which could likely be overcome using active radial injection. This hence represents a natural next step, even though several issues relating to the foil structure modelling remain unresolved. A prerequisite for the development of any model-based feedback control scheme is a model capturing the effects of gas injection on the system dynamics. The currently presented work on a generic numerical recipe for GFB simulation is intended as a step towards such a model.

The aim of the present work is to consolidate the existing state of the art knowledge on GFB modelling into a generic framework that can act as an efficient platform for further research. By creating a structure where the same pieces of code can be applied for simulation of a wide range of rotors supported by any number of rigid or compliant type gas bearings — with or without injection — the available experimental results can be utilized optimally for validation. Furthermore, the programming intensive optimizations necessary to obtain tolerable solution times can be more easily reused.

The framework consists of three domains treating the rotor, the foil structure and the fluid film respectively, along with clear-cut domain interfaces based on linear mappings. In the fluid domain, the film is modelled using the Modified Reynolds Equation discretised using finite volume, leaving the injection flow to an auxiliary model. Both the rotor and the compliant structure are represented in generic state-space formats facilitating various different models for both domains. The global system is solved both statically and in time using efficient general purpose routines exploiting e.g. analytical Jacobian matrices and sparse direct linear solvers.

Nomenclature

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>BC</td>
<td>Boundary Condition</td>
</tr>
<tr>
<td>CDS</td>
<td>Central Differencing Scheme</td>
</tr>
<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
</tr>
<tr>
<td>CSR</td>
<td>Compressed Sparse Row</td>
</tr>
<tr>
<td>CV</td>
<td>Control Volume</td>
</tr>
<tr>
<td>DAE</td>
<td>Differential/Algebraic Equation</td>
</tr>
<tr>
<td>DOF</td>
<td>Degree of Freedom</td>
</tr>
<tr>
<td>EOM</td>
<td>Equation of Motion</td>
</tr>
<tr>
<td>FD</td>
<td>Finite Difference</td>
</tr>
<tr>
<td>FE</td>
<td>Finite Element</td>
</tr>
<tr>
<td>FV</td>
<td>Finite Volume</td>
</tr>
<tr>
<td>GFB</td>
<td>Gas Foil Bearing</td>
</tr>
<tr>
<td>IC</td>
<td>Initial Condition</td>
</tr>
<tr>
<td>IVP</td>
<td>Initial Value Problem</td>
</tr>
<tr>
<td>LUDS</td>
<td>Linear Upwind Differencing Scheme</td>
</tr>
<tr>
<td>MRE</td>
<td>Modified Reynolds Equation</td>
</tr>
<tr>
<td>ODE</td>
<td>Ordinary Differential Equation</td>
</tr>
<tr>
<td>SEFM</td>
<td>Simple Elastic Foundation Model</td>
</tr>
<tr>
<td>UDS</td>
<td>Upwind Differencing Scheme (1st order)</td>
</tr>
<tr>
<td>(')</td>
<td>Time derivative, $d^2/dt^2$</td>
</tr>
<tr>
<td>(&quot;)</td>
<td>Time derivative, $d/d\tau$</td>
</tr>
</tbody>
</table>

Abbreviations

- A_b: Bearing surface area $A_b = [0, L] \times [0, 2\pi]$
- A_i: In-film area of the i-th CV
- L, \(\tilde{L}\): Bearing length, \(\tilde{L} = L/R\)
- M: Molar mass of gas
- M_{inj,i}: Integrated injection flux for the i-th CV
- R: Journal radius
- \(R_u\): Universal gas constant
- S_i: In-film circumference of the i-th CV
- S_\tau: Non-dimensional group, $S_\tau = \frac{6R^2m_{inj}}{C^2p_a}$
- T_{iso}: Isothermal gas temperature
- U: Unbalance, mass \times eccentricity
- a_j: FV stencil coefficients constituting A_{FV}
- f\_tol: Global force tolerance
- h, \(\tilde{h}\): Film height, \(\tilde{h} = h/C\)
- l\_tol: Global length tolerance
- l_f: Foil domain length scale
- l_r: Rotor domain length scale
- m_r: Rotor domain mass scale

Non-dimensional quantity

- (')
- (")
- \(\nabla\cdot\): Divergence
- \(\nabla\): Gradient
for GFB simulation is intended as a step towards such a model. The currently presented work on a generic numerical recipe bases of experimental and theoretical gas bearing results from the literature to be utilised optimally for validation.

This could potentially overcome some of the passive GFB’s inherent issues and work on hybrid GFBs have been presented [21, 22].

A prerequisite for the development of any model-based feedback control scheme would be a model capturing the non-linear behaviour of the compressible gas film and its interaction with the compliant structure. To Aims of the present work is to generate a fast and versatile simulation tool for radial GFBs. This should be capable of modelling an arbitrary number of rigid or compliant type gas bearings supporting a variety of rigid or flexible rotors. The tool should rely on fast and readily available general purpose solvers to provide simultaneous steady-state solutions as well as time integrations while reusing as much code as possible. This allows the available base of experimental and theoretical gas bearing results from the literature to be utilised optimally for validation purposes.

A prerequisite for the development of any model-based feedback control scheme would be a model capturing the effects of gas injection on the system dynamics. The currently presented work on a generic numerical recipe for GFB simulation is intended as a step towards such a model.

\[ m_{inj} \] Injection flux field
\[ n_c \] Number of CVs (per bearing)
\[ n_{dof} \] Number of rotor domain DOFs
\[ n_{ub} \] Number of unbalance masses
\[ n_{GB} \] Number of GFBs
\[ p, \hat{p} \] Film pressure, \( \hat{p} = p/p_a \)
\[ p_{tol} \] Global (derived) pressure tolerance
\[ p_a \] Ambient pressure, fluid pressure scaling
\[ p_f \] Foil domain pressure scale
\[ p_i \] Pressure at \( i \)-th CV centre
\[ \rho_f \] FV equation residual for \( i \)-th CV
\[ t \] Physical time
\[ x, y, z \] Fluid film coordinates, \( y_g \) is curvilinear
\[ x_c, y_c, z_c \] Rotor coordinates
\[ \Omega, \dot{\Omega} \] Angular velocity \( \dot{\Omega} = \Omega/\omega_t \)
\[ \Xi \] FV stencil, i.e. a set of CV indices
\[ \alpha \] Bearing index, \( \alpha = 1, \ldots, n_b \)
\[ \mu \] Dynamic gas viscosity
\[ \omega_r \] Characteristic frequency
\[ \phi \] Phase of unbalance
\[ \rho_g \] Density of gas
\[ \tau \] Dimensionless time, \( \tau = \omega_t t \)
\[ \theta \] Circumferential angle
\[ \tilde{\theta} \] Circumferential coordinate, \( \tilde{\theta} = y_g/R \)
\[ f_{ab} \] Rotor unbalance force vector
\[ f_{fa,a} \] Foil structure friction forces in bearing \( \alpha \)
\[ f_{fb} \] Bearing force vector
\[ f_{fw,a} \] Foil structure static loads in bearing \( \alpha \)
\[ f_{p,a} \] Foil structure pressure loads in bearing \( \alpha \)
\[ f_{w} \] Rotor static force vector
\[ h_{c,c} \] Initial film heights at CV centres
\[ h_{c,e} \] Initial film heights at CV edges
\[ h_{c,cve} \] Compliant film heights at CV centres
\[ h_{c,eve} \] Compliant film heights at CV edges
\[ h_{r,c} \] Rigid film heights at CV centres
\[ h_{r, eve} \] Rigid film heights at CV edges
\[ n \] Outward pointing unit normal vector
\[ p_a, \hat{p}_\alpha \] Bearing \( \alpha \) CV centre pressures, \( p_a = p_a \hat{p}_\alpha \)
\[ r_{scale} \] Nondimensionalization scalings for \( r_r \)
\[ r_{tol} \] Dimensional system residual tolerances
\[ r_r \] Rotor residual vector
\[ r_{FV,a} \] FV residual vector for bearing \( \alpha \)
\[ r_{FV,\alpha} \] FV residual vector in bearing \( \alpha \)
\[ u_g, \hat{u}_g \] Gas in-film velocity vector, \( u_g = \{ u, u_z \}^T \)
\[ \chi_r, \hat{\chi}_r \] Rotor DOF vector
\[ \chi_{f,\alpha}, \hat{\chi}_{f,\alpha} \] Foil structure DOF vector for bearing \( \alpha \)
\[ z_r \] Rotor state vector
\[ z_{f,\alpha} \] Foil structure state vector for bearing \( \alpha \)
\[ A_f \] Foil structure state-space inertia
\[ A_r \] Rotor state-space inertia
\[ A_{FV} \] FV coefficient matrix, Coutte & Poiseuille
\[ B_f \] Foil structure state-space stiffness/damping
\[ B_r \] Rotor state-space stiffness/damping
\[ B_{FV} \] FV coefficient matrix, squeeze term
\[ C_r \] Rotor state-space gyroscopic matrix
\[ C_{FV} \] FV coefficient matrix, local expansion term
\[ D_r, D_f \] Rotor damping matrix
\[ G_r, G_{f} \] Rotor gyroscopic matrix
\[ H_{c,c} \] Mapping from \( \chi_{f,\alpha} \) to \( h_{c,cve} \)
\[ H_{c,e} \] Mapping from \( \chi_{f,\alpha} \) to \( h_{c,eve} \)
\[ H_{c,v} \] Mapping from \( \chi_{f,\alpha} \) to \( h_{c,v} \)
\[ H_{f,v} \] Mapping from \( \chi_{f,\alpha} \) to \( h_{f,v} \)
\[ H_{f,w} \] Mapping from \( \chi_{f,\alpha} \) to \( h_{f,w} \)
\[ H_{f,\alpha} \] Mapping from \( \chi_{f,\alpha} \) to \( h_{f,\alpha} \)
\[ H_{f,\alpha} \] Mapping from \( p_a \) to \( T_f \hat{p}_{b,\alpha} \)
\[ I \] Identity matrix
\[ K_f, \hat{K}_f \] Foil structure stiffness matrix
\[ K_r, \hat{K}_r \] Rotor stiffness matrix
\[ M_f, \hat{M}_f \] Foil structure mass matrix
\[ M_r, \hat{M}_r \] Rotor mass matrix
\[ T_f \] Foil structure DOF to state-space mapping
\[ T_r \] Rotor DOF to state-space mapping

1 Introduction

Rotor–bearing systems supported by Gas Foil Bearings (GFBs) often display complex dynamics originating from the non-linear behaviour of the compressible gas film and its interaction with the compliant structure. To understand and predict such a behaviour, several researchers have contributed improvements to the available GFB modelling tools over the recent years. These include the introduction of the simultaneous solution approach [2] and work on various foil structure models [11, 19, 15]. Even though several issues remain unresolved for conventional GFBs, especially regarding the foil structure damping, a natural next step concerns active radial injection. This could potentially overcome some of the passive GFB’s inherent issues and work on hybrid GFBs have been presented [21, 22].

The aim of the present work is to generate a fast and versatile simulation tool for radial GFBs. This should be capable of modelling an arbitrary number of rigid or compliant type gas bearings supporting a variety of rigid or flexible rotors. The tool should rely on fast and readily available general purpose solvers to provide simultaneous steady-state solutions as well as time integrations while reusing as much code as possible. This allows the available base of experimental and theoretical gas bearing results from the literature to be utilised optimally for validation purposes.

A prerequisite for the development of any model-based feedback control scheme would be a model capturing the effects of gas injection on the system dynamics. The currently presented work on a generic numerical recipe for GFB simulation is intended as a step towards such a model.
to the rotor and hence has been found from the usual integration of the projected fluid film pressures as stemming from all GFBs in the system and hence has obvious.

force values. For a more complex rotor, based on e.g. FE discretization with beam elements, a good choice is less

scales depends on the specific rotor and the underlying mathematical model, but these can be tuned to enhance

spectively. The static forces are kept in \( \tilde{x}_r \) and \( \tilde{y}_r \), while \( \tilde{z}_r \) gives a non-dimensional angular velocity \( \tilde{\omega} = \Omega/\omega_r \) which is shared between the domains. It will often be sensible to define \( \omega_r \) simply as the angular velocity implying \( \tilde{\omega} = 1 \), but other choices could be advantageous for some simulations [12, 2]. The choice of rotor length and mass scales depends on the specific rotor and the underlying mathematical model, but these can be tuned to enhance the numerical properties of the rotor matrices. A point mass rotor supported by a single GFB will have \( \tilde{D}_r, \tilde{G}_r \) and \( \tilde{K}_r \) equal to zero, hence the scales could be chosen to provide \( \tilde{M}_r = I \) while maintaining sensible numerical force values. For a more complex rotor, based on e.g. FE discretization with beam elements, a good choice is less obvious.

The force vector \( \tilde{f}_b \) contains the vertical \( x_r \) and horizontal \( y_r \) components of the fluid film forces on the rotor stemming from all GFBs in the system and hence has \( 2n_b \) non-zero elements. For each bearing, the forces are found from the usual integration of the projected fluid film pressures as

\[
\begin{pmatrix}
\int_{A_{\text{b}}} (p - p_{\text{s}}) \left( \cos \theta \right) R dA \\
\int_{A_{\text{b}}} (p - p_{\text{s}}) \left( \sin \theta \right) R dA
\end{pmatrix}
\]

where \( A_{\text{b}} = [0, L] \times [0, 2\pi] \).

The force vector \( \tilde{f}_{\text{ab}} \) contains the components of the unbalance forces due to each unbalance mass \( U \) attached to the rotor and hence has \( 2n_{\text{ab}} \) non-zero elements. For one particular unbalance mass attached to a node of the

2 Rotor Domain

The model is build to support any type of rotor representable in a general state-space form using the rotor Degree of Freedom (DOF) vector \( \tilde{x}_r \) as

\[
\begin{pmatrix}
0 & \tilde{M}_r \\
I & 0
\end{pmatrix}
\begin{pmatrix}
\tilde{x}_r \\
\dot{\tilde{x}}_r
\end{pmatrix}
+ \begin{pmatrix}
\tilde{K}_r & \tilde{D}_r \\
0 & -\tilde{G}_r
\end{pmatrix}
\begin{pmatrix}
\tilde{x}_r \\
\dot{\tilde{x}}_r
\end{pmatrix}
+ \tilde{\Omega} \begin{pmatrix}
0 & -\tilde{G}_r \\
0 & 0
\end{pmatrix}
\begin{pmatrix}
\tilde{x}_r \\
\dot{\tilde{x}}_r
\end{pmatrix}
= \begin{pmatrix}
1 \\
0
\end{pmatrix}
(\tilde{f}_{\text{rv}} + \tilde{f}_{b} + \tilde{f}_{\text{ab}}) = \tilde{r}_r,
\]
rotor, the unbalance forces are found using the usual projections as

\[
\begin{bmatrix}
\hat{f}_{ub,x} \\
\hat{f}_{ub,y}
\end{bmatrix} = \begin{bmatrix}
\cos(\Omega_t \tau + \phi) \\
\sin(\Omega_t \tau + \phi)
\end{bmatrix} U \tilde{\Omega}^2,
\]

where the non-dimensional unbalance mass is given as \( U = U/(m_r l_r) \).

3 Foil Domain

Various different foil structure models are available in the literature of which the majority are based on variations over the Simple Elastic Foundation Model (SEFM). The available models include purely algebraic equation systems (stiffness only), first order systems (including stiffness and damping) as well as full second order systems including also mass and possibly friction. In the current implementation, a format capable of containing all these variations is desirable. Therefore the foil structural model in a bearing \( \alpha \) is represented using the non-dimensional foil DOF vector \( \tilde{x}_{f, \alpha} \) in the general state-space residual format as

\[
\begin{bmatrix}
0 & M_f \\
1 & 0
\end{bmatrix} \begin{bmatrix}
\dot{x}_{f, \alpha} \\
x_{f, \alpha}
\end{bmatrix} + \begin{bmatrix}
K_f & C_f \\
0 & -I
\end{bmatrix} \begin{bmatrix}
\dot{x}_{f, \alpha} \\
x_{f, \alpha}
\end{bmatrix} + \begin{bmatrix}
1 & 0
\end{bmatrix} (\hat{f}_{fw, \alpha} + \hat{f}_{p, \alpha} + \hat{f}_{f, \alpha}) = r_{f, \alpha},
\]

which is suitable for the most general class of foil models. The non-dimensional foil structure mass, damping and stiffness matrices are given as \( M_f, D_f, K_f \) respectively, while the static loads are represented in \( \hat{f}_{fw, \alpha} \), the fluid film pressure loads are contained in \( \hat{f}_{p, \alpha} \) and the friction forces (if any) are held by \( \hat{f}_{f, \alpha} \). For a non-zero mass matrix, this represents a system of Ordinary Differential Equations (ODEs), but for many models, such as the widely applied case of an inertialess SEFM incorporating a structural loss factor, eq. (4) turns into a system of Differential/Algebraic Equations (DAEs). If friction forces are not included, the DAE structure could still be used directly as discussed in [20], but this is unnecessarily complicated. Instead, the implementation also supports a decayed version of eq. (4) given simply as

\[
\begin{bmatrix}
\tilde{C}_f & \tilde{K}_f \\
A_f & B_f
\end{bmatrix} \begin{bmatrix}
\dot{x}_{f, \alpha} \\
x_{f, \alpha}
\end{bmatrix} + \begin{bmatrix}
1 & 0
\end{bmatrix} (\hat{f}_{fw, \alpha} + \hat{f}_{p, \alpha}) = r_{f, \alpha},
\]

representing a system of first order ODEs for non-zero damping matrices or an algebraic equation system in the case of zero damping. As for the rotor domain equations, all quantities in the foil domain are non-dimensionalized. This is achieved using the characteristic frequency \( \omega_c \), a foil domain length scale \( l_f \) and a foil domain pressure scale \( p_f \). For an SEFM-like model, the obvious choices for these would be the clearance and ambient pressure, but for more complex models other scales might be advantageous.

4 Fluid Film Domain

To model the gas film separating the journal from the rigid or compliant surfaces of the bearing, we will apply the Modified Reynolds Equation (MRE) under the assumption of isothermal, isoviscous conditions. The compressibility is introduced by assuming ideal gas behaviour providing the relation for the gas density \( \rho_g \)

\[
\rho_g = \frac{M_g}{R_u T_{iso}} p,
\]

where \( M_g, R_u \) and \( T_{iso} \) are the (average) molar mass of the gas, the universal gas constant and the isothermal gas temperature, respectively. A natural extension would be to drop the assumption of isothermal conditions and simultaneously solve the energy equation. This has been done for rigid gas bearings, a least using non-simultaneous approaches [16], but for foil bearings the thermal BCs are less clearly defined. The maximum temperature differences in the circumferential and axial directions were found experimentally by Zywica et al.
[23] for a specific GFB to be approximately 35 °C and 2 °C, respectively. This would imply density variations on the order of ±5%, which is not considered sufficient to justify the inclusion of the energy equation. If this was nevertheless to be attempted, experimental validation of the resulting temperature field would be necessary to adjust the applied thermal BCs.

4.1 The Modified Reynolds Equation in Terms of Flows

To formulate a discretization scheme for the MRE, it is convenient to consider the underlying velocity profiles. Expressing the journal surface velocity as \(u_0 (x_s = 0) = \Omega R\) and assuming the pressure and viscosity to be constant across the film thickness \(h\), the velocity profiles can be derived from the Navier–Stokes equations using an order of magnitude analysis followed by an integration across the film thickness [7] as

\[
\mathbf{u}_g = \left\{ u_0 \ u_z \right\}^T = \left\{ -x_g \frac{h-x_g}{2\mu} \frac{\partial p}{\partial x_g} + \Omega R \frac{h-x_g}{h} \ -x_g \frac{h-x_g}{2\mu} \frac{\partial p}{\partial x_g} \right\}^T,
\]

giving the in–film fluid velocities in m/s as functions of the pressure gradients and the journal surface velocity. Note that the axial velocity of the journal is neglected leaving only the Poiseuille component in this direction.

The MRE can be obtained by inserting the velocity profiles into the continuity equation augmented with a source term to represent injection and integrating it across the film thickness as

\[
\int_0^h \left( \frac{\partial \rho g}{\partial t} + \nabla \cdot (\rho g \mathbf{u}_g) \right) \, dx_g = m_{inj},
\]

where \(\rho g\) is the fluid density and \(m_{inj} = m_{inj} (y_g, z_g)\) represents an injection source term field in kg/(m³ s). The inflow due to injection could be obtained using analytical expressions, possibly including empirical correction factors, as suggested by various authors [1, 14, 17, 21] or by an external Computational Fluid Dynamics (CFD) routine, but this is outside the present scope.

Replacing the fluid density by its average across the film thickness and applying Leibniz’ rule, eq. (8) produces the MRE on the form

\[
\frac{\partial}{\partial t} \left( \frac{\rho_g h^3}{12 \mu} \frac{\partial p}{\partial y_g} \right) - \frac{\partial}{\partial z_g} \left( \frac{\rho_g h^3}{12 \mu} \frac{\partial p}{\partial z_g} \right) + \frac{\omega R}{2} \frac{\partial \rho_g h}{\partial y_g} + \frac{\partial (\rho_g h)}{\partial t} = m_{inj},
\]

which can alternatively be expressed in terms of the flow vector \(\mathbf{q}_g\) with units kg/(m³ s) [3] as

\[
\nabla \cdot \mathbf{q}_g = -\frac{\partial (\rho_g h)}{\partial t} + m_{inj}
\]

where

\[
\mathbf{q}_g = \left\{ q_y \ q_z \right\}^T = -\frac{\rho_g h^3}{12 \mu} \nabla p + \frac{\omega R}{2} \rho_g h \left\{ 0 \ 1 \right\}^T.
\]

Equation (10) can be non-dimensionalised using the clearance \(C\) and radius \(R\) as through-film and in-film length scales respectively, the ambient pressure \(p_a\) as the pressure scale and a frequency \(\omega_R\) as time scale. Assuming isothermal conditions in the fluid film, the viscosity can be assumed constant and the gas density can be expressed using the ideal gas law. This gives the version of the MRE implemented in the model

\[
\nabla \cdot \tilde{\mathbf{q}}_g = -2S_T \frac{\partial (\tilde{\rho} h)}{\partial T} + \tilde{m}_{inj},
\]

where

\[
S_T = \frac{6R^2 \mu \omega_T}{C^2 p_a}, \quad \tilde{\mathbf{q}}_g = \left\{ \tilde{q}_y \ \tilde{q}_z \right\}^T = \left( -\tilde{\rho} h^3 \nabla \tilde{p} + S_T \tilde{\Omega} \tilde{p} h \left\{ 0 \ 1 \right\}^T \right) \quad \text{and} \quad \tilde{m}_{inj} = \frac{12R^2 \mu R_a T_{iso}}{p_a^3 C^3 M_g} m_{inj}.
\]

Notice that before eq. (11) is non-dimensionalised, insertion of the ideal gas law and multiplication by \(R_a T_{iso}/M_g\) changes its units into J/(m³ s), such that the conserved quantity is rather energy than mass. Subsequently, the equation is divided by the viscosity \(\mu\) resulting in the rather awkward unit J Pa s/(m³ s) or N²/m³.
Scheme (UDS) relying on a single upstream cell is used. The current work is limited to structured grids where upwind procedure to stabilize the numerical solution. The approach presented in [1] is intended for unstructured approximation. It is, however, argued by Arghir et al. [1] that these “convected pressures” should be treated using an fourth-order CDS using three or four cell centres, respectively.

4.2 Finite Volume Discretization

Having the MRE represented in terms of flows as given in eq. (11), theFinite Volume (FV) method can be applied and implemented following an intuitive approach. Equation (11) represents the differential form of the conservation equation which can be integrated over a Control Volume (CV) $i$ with in-film area $A_i$ and circumference $S_i$. Applying the divergence theorem, this can be written as

$$
\int_{S_i} \tilde{q}_e \cdot n \, dS = -2S_i \int_{A_i} \frac{\partial}{\partial \tau} (\tilde{p}\tilde{h}) \, dA + \int_{A_i} \tilde{m}_{inj} \, dA,
$$

(13)

where the three terms represent the flux across the CV boundary to the neighbouring cells, the accumulation within the CV and the injection influx, respectively. Limiting the implementation to rectilinear non-uniform grids, i.e. grids of quadrilateral CVs, the integral over the CV circumference in eq. (13) can be split into four components as illustrated in fig. 2b. Representing the flow $\tilde{q}_e$ at each face by its midface value and the unsteady term $\frac{\partial}{\partial \tau} (\tilde{p}\tilde{h})$ by its cell centre value (both being second order approximations), the FV residual equation for the $i$-th CV can be written as

$$
\Delta \tilde{z}_i (\tilde{q}_{\theta,\tau}\mid_e - \tilde{q}_{\theta,\tau}\mid_n) + \Delta \tilde{h}_i (\tilde{q}_{\overline{z}}|_e - \tilde{q}_{\overline{z}}|_n) + 2S_i \Delta \tilde{z}_i \Delta \tilde{h}_i \left( \tilde{h}_i \frac{\partial \tilde{h}_i}{\partial \tau} + \tilde{h}_i \frac{\partial \tilde{q}_e}{\partial \tau} \right) - \tilde{M}_{inj,i} = r_{FV,i},
$$

(14)

where $\Delta \tilde{h}_i$, $\Delta \tilde{z}_i$ are the CV dimensions, $\tilde{h}_i$, $\tilde{q}_e$ are the CV centre pressure and film height and $\tilde{q}_{\theta,\tau}$, $\tilde{q}_{\overline{z}}$ are the flow components evaluated at the midpoint of the respective CV faces as indicated in fig. 2a. For now, the integrated value of the injection flux over the cell area is denoted simply as $\tilde{M}_{inj,i}$ since its treatment depends on the applied injection model.

4.3 Cell Face Fluxes

To evaluate eq. (14), the components of the flow must be known at the CV faces. These are given from eq. (12) and hence depend on the face pressure, face pressure gradient and face film height. The gradient at the midpoint of each face is reconstructed using linear interpolation between the centres of the two CVs sharing the face following a Central Differencing Scheme (CDS). This approximation is of second order on a uniform mesh, but formally decays to first order for a non-uniform mesh. For reasonable grid expansion rates it will, however, still produce convergence characteristics close to that of a second order scheme [5]. Referring to fig. 2a, the gradient at the eastern face is thus reconstructed using the CV centre values $i$ and $E$. If formal second order accuracy was requested, one could apply the "Quadratic Upstream Interpolation for Convective Kinematics" scheme or a fourth-order CDS using three or four cell centres, respectively.

The face pressure values could likewise be reconstructed using CDS to provide a second order accurate approximation. It is, however, argued by Arghir et al. [1] that these "convected pressures" should be treated using an upwind procedure to stabilize the numerical solution. The approach presented in [1] is intended for unstructured grids where the implementation of higher order upwind schemes is complicated, thus the Upwind Differencing Scheme (UDS) relying on a single upstream cell is used. The current work is limited to structured grids where
it is straightforward to include an additional upwind cell in the stencil to produce the Linear Upwind Differencing Scheme (LUDS). This provides second order accuracy for the face pressure, but has the potential to produce unbounded solutions when used without limiters.

Considering the structure of the resulting equation system, the combination of CDS for face pressure gradients and UDS for face pressure values gives a computational molecule for each cell with five contributions: the cell itself and its four immediate neighbours \( E, N, W \) and \( S \). Using LUDS for the face pressures, four additional contributions from the next set of neighbours \( EE, NN, WW \) and \( SS \) should furthermore be included.

Prior to applying any of the upwind schemes, the flow direction at the given face must be evaluated. Noting that the pressure is always positive, the sign of the flow vector components can be judged from eq. (12) as

\[
\text{sign} (\tilde{q}_a) = \text{sign} \left( -\hat{h} \frac{\partial \tilde{p}}{\partial \theta} + S \hat{\Omega} \hat{f} \right) \quad \text{and} \quad \text{sign} (\tilde{q}_c) = \text{sign} \left( -\hat{h} \frac{\partial \tilde{p}}{\partial z} \right),
\]

where the CDS approximations for the pressure gradients are used while the film height at the face is supplied by the rotor and foil models. Knowing the sign of the flow, the face pressure itself can be reconstructed using cell centres in the upwind direction only.

At the outer boundaries where ambient pressure is prescribed (see fig. 2a), corresponding to a Dirichlet type Boundary Condition (BC), only the face gradient needs reconstruction. This is achieved using a one-sided FD scheme relying on two CV centres into the mesh. If a symmetry condition is imposed at the bearing mid plane, corresponding to a Neumann type BC, the flux at these faces should simply be set to zero. Both types of BCs are used if the LUDS is requested to reconstruct face pressures too close to a boundary for two upstream CV centres to be available.

Lastly, the model implements a cyclic condition which is physically meaningful for rigid and single-pad foil bearings. In this case, the cyclic boundary face fluxes are reconstructed exactly as for the internal faces using the corresponding CVs from the opposite end of the mesh.

### 4.4 Finite Volume Residual Equation

Using the FD schemes described in the preceding section to evaluate the fluxes across each CV face and plugging these into eq. (14), one obtains a residual equation on the form

\[
\sum_{\xi \in \Xi} a_j \tilde{p}_j + 2S \Delta z \Delta \hat{h} \left( \tilde{h}_i \tilde{p}_i + \Delta \tilde{h}_i \tilde{p}_i \right) - M_{\text{inj},i} = r_{\text{FV},i},
\]

where \( \Xi \) is the set of stencil members making up the computational molecule, e.g. \( \Xi = \{ i, E, N, W, S \} \) for the case of a CDS+UDS combination. The stencil coefficients \( a_j \) depend on the present pressure values across the stencil, the current film heights at the cell faces and the dimensions of the cell. In a steady-state simulation, the cell centre temporal pressure derivative \( \dot{\tilde{h}}_i \) is zero, while it is directly available in a time integration. The film heights at the cell centre and faces as well as the temporal derivative at the cell centre \( \hat{\tilde{h}}_i \) are given from the rotor and foil domains.

For a bearing \( \alpha \) discretized using \( n_{cv} \) CVs, the pressure state vector holding all CV centre pressures can be written

\[
p_{\alpha} = \left( \tilde{p}_1, \ldots, \tilde{p}_{n_{cv}} \right)^T,
\]

using which the FV residual vector can be assembled for the entire bearing as

\[
A_{\text{FV}} \left( p_{\alpha}, x_r, x_f, \alpha \right) p_{\alpha} + B_{\text{FV}} \left( x_r, x_f, \alpha \right) p_{\alpha} + C_{\text{FV}} \left( x_r, x_f, \alpha \right) p_{\alpha} + f_{\text{FV}} = r_{\text{FV}, \alpha},
\]

where \( A_{\text{FV}} \) holds the FV coefficients originating from the Couette and Poiseuille terms of the MRE, while the coefficients in \( B_{\text{FV}} \) and \( C_{\text{FV}} \) represent the squeeze and local expansion terms respectively. The injection contribution is represented by \( f_{\text{FV}} \) which will generally depend on the bearing pressure and possibly variables from the rotor and fluid domains.
5 Domain Interfaces

To evaluate the FV residuals from eq. (16), the film height $\tilde{h}$ is required at all CV edges and centres. This contains three contributions: (a) a constant describing the initial geometry such as clearance and pad inlet slopes; (b) a rigid contribution stemming from the movement of the rotor; and (c) the compliant contribution due to deformation of the foil structure (if simulating a compliant bearing).

Recalling the rigid contributions to be given merely as trigonometric projections of the eccentricity components, written usually like $e_r \cos \theta + e_y \sin \theta$, this can be represented as a constant linear mapping applied to the rotor DOF vector $\vec{x}_r$. Collecting the rigid film heights from all CV centres and edges across all bearings in the vectors $h_{r,cvc}$ and $h_{r,cve}$ respectively, these can be calculated as

$$ h_{r,cvc} = H_{cvc,r} \tilde{x}_r \quad \text{and} \quad h_{r,cve} = H_{cve,r} \tilde{x}_r, $$

where the mapping matrices can be assembled offline knowing: (a) the angular positions of the CV centres/edges; (b) the indices of the rotor DOFs representing the eccentricity components in each bearing; and (c) the length scaling between the two domains. The row dimensions correspond to the combined number of CV centres/edges across all bearings while the column dimensions equal the number of rotor DOFs $n_{rdof}$, but only $2n_b$ columns will contain non-zero elements.

Analogously to the rigid contributions in eq. (19), the compliant film heights at all CV centres and edges can be represented as constant linear mappings from the foil DOF vectors $\vec{x}_{f,ar}$. This can be written as the linear combinations

$$ h_{c,cvc} = H_{cvc,f} \left[ \hat{s}^T_{f,1} \cdots \hat{s}^T_{f,n_b} \right]^T \quad \text{and} \quad h_{c,cve} = H_{cve,f} \left[ \hat{s}^T_{f,1} \cdots \hat{s}^T_{f,n_b} \right]^T $$

but the content of the mapping matrices will be dependent on the applied foil model. The currently implemented SEFM places one foil node for each circumferential CV centre position. In this case, $H_{cvc,f}$ maps each foil DOF directly to all CV centres sharing the same circumferential position and applies a length scaling. The mapping to CV edges $H_{cve,f}$, on the contrary, will furthermore contain FD coefficients interpolating from the foil DOFs locations to the CV edges. Having prebuild the mapping matrices and the constant contribution vectors $h_{r,cvc}$ and $h_{r,cve}$, the film heights can hence be found at runtime simply as

$$ h_{cvc} = h_{c,cvc} + h_{r,cvc} \tilde{x}_r + H_{cvc,f} \left[ \hat{s}^T_{f,1} \cdots \hat{s}^T_{f,n_b} \right]^T \quad \text{and} \quad h_{cve} = h_{c,cve} + h_{r,cve} \tilde{x}_r + H_{cve,f} \left[ \hat{s}^T_{f,1} \cdots \hat{s}^T_{f,n_b} \right]^T. $$

To evaluate the rotor residual from eq. (1), the bearing forces must be obtained by integration of the pressure as given in eq. (2). For a single bearing, this is evaluated numerically by summing the projected force contributions from all CVs as

$$ \begin{bmatrix} \tilde{f}_{b,x} \\ \tilde{f}_{b,y} \end{bmatrix} = \frac{p_a R^2}{l_i m_r \omega^2} \sum_{i} \Delta \tilde{p}_i \Delta \tilde{z}_i \left( \cos \theta_{cvc,i} \right) - 1 \begin{bmatrix} \cos \theta_{cvc,i} \\ \sin \theta_{cvc,i} \end{bmatrix}, $$

where $\tilde{p}_i$ and $\theta_{cvc,i}$ are the $i$-th CV centre pressure and circumferential position respectively, while $\frac{p_a R^2}{l_i m_r \omega^2}$ is the inter-domain force scaling. Notice that eq. (22) effectively applies the midpoint rule within each CV, but a higher order numerical integration scheme could likewise have been applied. This would require the trigonometric functions to be sampled and the pressure value to be reconstructed using FD at a number of locations within each CV, thus making the force contribution from each CV dependent also on the neighbouring cells. Regardless of the integration scheme applied within each CV, the scaling, integration and summation can be represented using a constant linear mapping

$$ T \tilde{f}_b = H_{f_b} \left[ p_{1}^T - 1, \ldots, p_{n_b}^T - 1 \right]^T, $$

calculating the state-space sized bearing force vector as a single matrix–vector product. $H_{f_b}$ has row dimension equal to the rotor state-space, i.e. $2n_{rdof}$, and column dimension matching the combined number of CVs across all
bearings, i.e. \( n_b n_c \). Generally, each bearing will give rise to two bearing force components, meaning that \( \mathbf{H}_{b,p} \) will contain \( 2n_b n_c \) non-zero elements. For GFBs it is common to apply the Gumbel condition such that sub-ambient pressures are discarded during the integration. In this case, \( \mathbf{H}_{b,p} \) becomes non-constant as the columns corresponding to CVs with sub-ambient pressures should be zeroed. In practice, this is achieved by prebuilding the full non-Gumbel \( \mathbf{H}_{b,p} \) and continuously restoring a copy of this before zeroing out the sub-ambient columns.

Lastly, the pressure forces on the foil structure should be integrated to calculate \( \mathbf{f}_{p,a} \) in eq. (4) or eq. (5). For each bearing \( \alpha \), this can be achieved analogously to the bearing force integration in eq. (23) using a constant linear mapping as

\[
\mathbf{T}_f \mathbf{f}_{p,a} = \mathbf{H}_{f,p} \mathbf{p}_a,
\]

where the exact content of \( \mathbf{H}_{f,p} \) depends on the foil model. For the currently implemented SEFM, the mapping will simply calculate the average pressure at each circumferential position multiplied by the surface area ascribed to each foil DOF and the inter-domain force scaling \( \frac{\rho_{a} R^2}{\rho_{f/j}} \). The row dimension should match the number of foil states while the column dimension is \( n_c \), but the number of non-zero elements depends on the foil model.

The mapping matrices introduced in this section usually have sparse structures and are used mostly for calculating matrix–vector products. They are hence build and stored in Compressed Sparse Row (CSR) format to lower the storage requirements and provide efficient matrix–vector products.

### 6 System Assembly

For a rotor–bearing system comprising \( n_b \) GFBs, the coupled state-space and residual vectors are defined as

\[
\mathbf{z} = \begin{bmatrix}
\mathbf{p}_1^T & \cdots & \mathbf{p}_{n_b}^T & \mathbf{z}_{f,1}^T & \cdots & \mathbf{z}_{f,n_b}^T & \mathbf{x}_r^T & \mathbf{x}_\tau^T
\end{bmatrix}^T
\quad \text{and} \quad
\mathbf{r} = \begin{bmatrix}
\mathbf{r}_{\text{FV,1}}^T & \cdots & \mathbf{r}_{\text{FV,n_b}}^T & \mathbf{r}_{f,1}^T & \cdots & \mathbf{r}_{f,n_b}^T & \mathbf{r}_r^T
\end{bmatrix}^T
\]

using which the global non-linear system of equations can be written on fully implicit form like

\[
\mathbf{f}_G (\tau, \mathbf{z}, \dot{\mathbf{z}}) + \mathbf{f}_{G,t} (\tau, \mathbf{z}, \dot{\mathbf{z}}) + \mathbf{f}_{G,s} (\mathbf{z}) + \mathbf{f}_{G,0} = \mathbf{0},
\]

with a global residual function \( \mathbf{f}_G (\tau, \mathbf{z}, \dot{\mathbf{z}}) \) which can be partitioned into its transient, steady and constant components \( \mathbf{f}_{G,t} (\tau, \mathbf{z}, \dot{\mathbf{z}}) \), \( \mathbf{f}_{G,s} (\mathbf{z}) \) and \( \mathbf{f}_{G,0} \) respectively. The main advantage of this partitioning is to clearly separate out all time dependent terms such that \( \mathbf{f}_{G,t} (\tau, \mathbf{z}, \dot{\mathbf{z}}) = \mathbf{0} \) at steady-state, thus allowing the exact same implementation of \( \mathbf{f}_{G,s} (\mathbf{z}) \) and \( \mathbf{f}_{G,0} \) to be used for steady-state solutions as well as in time integrations.

It is common to treat IVPs for ODE systems on explicit first order form where a system function evaluates the temporal derivative of the state vector directly. In the current model, the fluid film FV equations could have been formulated explicitly in terms of the alternative variable \( \psi = \tilde{\rho} \tilde{h} \), if this had been substituted into eq. (11) as demonstrated by Bonello and Pham [2]. For many cases, explicit forms of the rotor and foil equations could likewise have been obtained by inverting \( \mathbf{A}_r \) and \( \mathbf{A}_f \) in eqs. (1), (4) and (5). In the GFB models by Larsen and Santos [10] and Gu et al. [6], the global systems are formulated on linearly implicit form where the product of a "mass" matrix and the state vector derivative is provided by the system function. In [10], this matrix is constant such that the system can be solved on explicit form by evaluating and inverting it, while a dedicated solver for the linearly implicit form is used in [6] where the "mass" matrix is state dependent. In the present work, the fully implicit form is maintained for several reasons. It allows the implementation to span the largest possibly variety of foil structure models, including those resulting in a DAE system. Any explicit matrix inversions can be avoided and the use of only physically meaningful variables simplifies the implementation of BCs as well as the interpretation of errors and intermediate results. Furthermore, it is an advantage when it comes to friction models that the variable \( \mathbf{z} \) is available when evaluating the residual. The disadvantages of using the fully implicit form are mainly the added complexity of defining consistent Initial Conditions (ICs) and a smaller selection of available time integrators.

Assembling the contributions from the three domains, the global equation system can be written on the fully
case where matrices. This is the case for steady-state solutions, but particularly when integrating in time. For the steady-state most injection models would be state dependent and hence should be relocated to the non-constant parts.

The lower right block is constant and contains the rotor stiffness, damping and gyroscopic contributions governing
due to pressure changes across all bearings. As discussed in relation to eq. (23), this block is pressure dependent
foil domains are only coupled through the fluid film. The lower left block governs the change in rotor residuals

\[
\begin{bmatrix}
B_{\text{FV}} (\dot{x}_r, \dot{\dot{x}}_{f,1}) & p_1 + C_{\text{FV}} (\dot{x}_r, \dot{\dot{x}}_{f,1}) & \dot{p}_1 \\
& \vdots \\
B_{\text{FV}} (\dot{x}_r, \dot{\dot{x}}_{f,n_b}) & p_{n_b} + C_{\text{FV}} (\dot{x}_r, \dot{\dot{x}}_{f,n_b}) & \dot{p}_{n_b} \\
A_f z_{f,1} + T_f \tilde{f}_p (p_1, z_{f,1}) & \cdots & \cdots \\
& \vdots \\
A_f z_{f,n_b} + T_f \tilde{f}_p (p_{n_b}, z_{f,n_b}) & \cdots & \cdots \\
A_r z_r + T_r \tilde{f}_{ab} (\tau) & \cdots & \cdots
\end{bmatrix}
\begin{bmatrix}
f_{G,t} (\tau, z, \dot{z})
\end{bmatrix}
\]

\[
\begin{bmatrix}
A_{\text{FV}} (p_1, \dot{x}_r, \dot{\dot{x}}_{f,1}) & p_1 \\
& \vdots \\
A_{\text{FV}} (p_{n_b}, \dot{x}_r, \dot{\dot{x}}_{f,n_b}) & p_{n_b} \\
B_f z_{f,1} + T_f \tilde{f}_p (p_1) & \cdots & \cdots \\
& \vdots \\
B_f z_{f,n_b} + T_f \tilde{f}_p (p_{n_b}) & \cdots & \cdots \\
(B_r + \tilde{\Omega} C_r) z_r + T_r \tilde{f}_b (p_1, \ldots, p_{n_b}) & \cdots & \cdots
\end{bmatrix}
\begin{bmatrix}
f_{G,s} (z)
\end{bmatrix}
\]

\[
f_{G,0} = \begin{bmatrix}
f_{\text{FV},1} \\
\vdots \\
f_{\text{FV},n_b} \\
T_f \tilde{f}_{fw,1} \\
\vdots \\
T_f \tilde{f}_{fwn,n_b} \\
T_r \tilde{f}_{aw}
\end{bmatrix} = \mathbf{r}
\]

where the upper rows represent the non-linear FV equations governing the fluid film while the midmost rows can
either be left out (to model a rigid bearing) or represent a linear/non-linear foil structure. The lowermost rows hold
the rotor model. Notice that the injection contributions, if any, are currently located in the constant part \(f_{G,0}\), while
most injection models would be state dependent and hence should be relocated to the non-constant parts.

Depending on the system size, ranging potentially from a few hundred to tens of thousands of states, and the
system stiffness, it is advantageous or downright necessary to formulate analytical expressions for the Jacobian
matrices. This is the case for steady-state solutions, but particularly when integrating in time. For the steady-state
case where \(f_{G,t} (\tau, z, \dot{z}) = 0\), the Jacobian with respect to the state vector \(z\) is given as

\[
J_{z,s} = \frac{\partial f_{G,s}}{\partial z} = \begin{bmatrix}
\frac{\partial \lambda_{fV} p_1}{\partial p_1} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \frac{\partial \lambda_{fV} p_{n_b}}{\partial p_{n_b}} \\
\vdots & \ddots & \vdots \\
0 & \cdots & 0 \\
0 & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
H_{f_p,p} \\
B_f \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\frac{\partial \lambda_{fV} p_1}{\partial \dot{x}_r} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \frac{\partial \lambda_{fV} p_{n_b}}{\partial \dot{x}_{f,n_b}} \\
\vdots & \ddots & \vdots \\
0 & \cdots & 0 \\
0 & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
H_{f_p,p} \\
B_f \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
0 \\
B_r + \tilde{\Omega} C_r \\
0 \\
0
\end{bmatrix}
\]

The upper left and midmost blocks reflect the changes in FV residuals within each bearing as the pressure and foil
deflection (governing the compliant film height) vary within that same bearing. The upper right block gives the FV
residual changes due to the rotor position (governing the rigid film height). These blocks are all state-dependent,
i.e. non-constant. The blocks in the midmost rows are constant and represent the changes in foil residuals within
each bearing due to changes in pressure and foil states within that same bearing. If a friction model was included,
this would likely cause additional non-constant contribution to the left and midmost of these blocks. Notice that
the foil residuals are not directly affected by the rotor states, which is physically meaningful since the rotor and
foil domains are only coupled through the fluid film. The lower left block governs the change in rotor residuals
due to pressure changes across all bearings. As discussed in relation to eq. (23), this block is pressure dependent
if the Gumbel condition is applied, but constant if the full bearing surface is included in the pressure integration.
The lower right block is constant and contains the rotor stiffness, damping and gyroscopic contributions governing
the change in rotor residuals due to changes in rotor states. Again, the nature of the foil–rotor coupling is evident,
as the rotor residuals are not directly affected by the foil states.

For the transient case, the Jacobian with respect to both the state vector and to its temporal derivative are
necessary. The Jacobian with respect to \(z\) is now the sum of \(J_{z,s} = \partial f_{G,s}/\partial z\) from eq. (28) and an additional
contribution $\partial f_{G_1}/\partial z$ given as

$$
\frac{\partial f_{G_1}}{\partial z} = \begin{bmatrix}
B_{FV} & \cdots & 0 & \frac{\partial C_{FV} p_1}{\partial x_1} & \cdots & 0 & \frac{\partial C_{FV} p_1}{\partial x_r} & 0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & \cdots & B_{FV} & 0 & \cdots & \frac{\partial C_{FV} p_{nb_1}}{\partial x_1} & \frac{\partial C_{FV} p_{nb_1}}{\partial x_r} & 0 \\
0 & \cdots & 0 & 0 & \cdots & \frac{\partial C_{FV} p_{nb_2}}{\partial x_1} & \frac{\partial C_{FV} p_{nb_2}}{\partial x_r} & 0 \\
\end{bmatrix},
$$

(29)

which represents the additional pressure and film height dependencies originating from the local expansion and squeeze terms of the MRE. If a friction model was included, this would possibly give rise to additional contributions in this matrix. Since $\partial f_{G_2}/\partial z = 0$ by definition, the Jacobian with respect to the temporal derivative of the state vector $z$ is given as

$$
J_z = \frac{\partial f_{G_1}}{\partial z} = \begin{bmatrix}
C_{FV} & \cdots & 0 & \frac{\partial B_{FV} p_1}{\partial x_1} & \cdots & 0 & \frac{\partial B_{FV} p_1}{\partial x_r} & 0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & \cdots & C_{FV} & 0 & \cdots & \frac{\partial B_{FV} p_{nb_1}}{\partial x_1} & \frac{\partial B_{FV} p_{nb_1}}{\partial x_r} & 0 \\
0 & \cdots & 0 & 0 & \cdots & \frac{\partial B_{FV} p_{nb_2}}{\partial x_1} & \frac{\partial B_{FV} p_{nb_2}}{\partial x_r} & 0 \\
\end{bmatrix},
$$

(30)

where the FV residual derivatives in the uppermost blocks are, once again, stemming from the MRE’s local expansion and squeeze terms. $A_f$ represents either foil inertia or damping depending on whether eq. (4) or eq. (5) is employed and $A_r$ holds the rotor inertia. Notice that the content of eq. (30) is decisive to the overall system structure. When $J_z$ has full rank, the model is a system of ODEs, while a singular $J_z$ implies a DAE system.

7 Steady-state and Transient Solution

The entire model is implemented in C where eq. (27) is solved in steady-state using the general purpose non-linear algebraic solver "KINSOL" and integrated in time using the ODE/DAE IVP solver "IDA". Both are readily available and part of the "SUNDIALS: SUite of Nonlinear and DIfferential/ALgebraic Equation Solvers" developed by the Lawrence Livermore National Laboratory [8]. In both cases, a Modified Newton iteration is employed to solve the non-linear equation systems using the Jacobian matrices given in eqs. (28) to (30). Like the mapping matrices, the Jacobians are constructed in the CSR format and the linear equation systems emerging from the Newton iterations are solved using sparse LU factorization provided by the KLU sparse solver library [4].

7.1 Ill-conditioning and Tolerances

Equation (27) presents a multi-domain problem, hence the numerical quantities involved will have varying orders of magnitude. To some extent, this can be alleviated using the scaling parameters within each domain, but the specification of meaningful solver tolerances still requires each equation to be assessed separately. Rotordynamics codes are often organized in a segregated fashion using an inner routine to solve for the pressure distribution (and related fields, such as temperature) while an independent outer routine searches for the rotor static equilibrium or steps the dynamic rotor equations forward in time. In these constructions, separate equation systems are solved for each domain and the solver tolerances should hence also be set separately. This can usually be achieved using scalar values as the variables/residuals within each domain are homogeneous. In the present model, all equations are solved simultaneously by a single solver, meaning that a vector of absolute tolerances must be supplied.

This is achieved by supplying global absolute force and length tolerances $f_{tol}$ [N] and $l_{tol}$ [m] which are then used to construct equivalent tolerances for each state and residual quantity involved. As an example, the fluid film state variables physically represent pressure with unit Pa. Assuming the absolute tolerances to be specified in the vicinity of 1 N and 1 m respectively, the corresponding relative tolerances can be added and multiplied by the
derived base value for the relevant quantity as

\[ p_{\text{tol}} = \frac{1 \text{ N}}{(1 \text{ m})^3} \left( \frac{f_{\text{tol}}}{1 \text{ N}} + \frac{2l_{\text{tol}}}{1 \text{ m}} \right), \]  

which for realistic values \( f_{\text{tol}} = 1 \times 10^{-3} \text{ N} \) and \( l_{\text{tol}} = 1 \times 10^{-8} \text{ m} \) is completely dominated by the force tolerance and gives \( p_{\text{tol}} = 1.00002 \times 10^{-3} \text{ Pa} \). One could argue that other base values than 1 N and 1 m should be used, but for now this is utilized as a systematic way of producing tolerances across the equation system. This approach is beneficial since the tolerances on some quantities, such as the FV residual given in J Pa s/(m² s) = N²/m³, are challenging to define based on physical intuition.

Collecting the system residual scaling factors implied by the non-dimensionalizations in the vector \( r_{\text{scale}} \) and the corresponding absolute tolerances derived as shown above in the vector \( r_{\text{tol}} \), the convergence criterion used for steady-state solutions can be written as

\[ ||\text{diag}(r_{\text{scale}})\text{diag}(r_{\text{tol}})^{-1}r||_{\infty} < 1, \]  

where \( || ||_{\infty} \) denotes the infinity norm. When integrating in time using the IDA solver, the tolerances are set on the local truncation error as estimated by the integration routine within each time step [9]. This is done in a way similar to the well-known Matlab integrators using a vector of absolute tolerances and a scalar relative tolerance. The first is here calculated as \( \text{diag}(r_{\text{scale}})^{-1}r_{\text{tol}} \) consistently to eq. (32), while the latter is set to around \( 1 \times 10^{-6} \).

8 Results

The current implementation has been validated against two previously presented codes treating rigid and compliant gas bearings respectively.

In the case of a rigid bearing, the midmost rows representing the foil structure in eqs. (25) and (27), along with the corresponding Jacobian rows and columns in eqs. (28) to (30), are simply left out. The same is the case for the compliant film height contribution in eq. (21). In this configuration, the code has been used to simulate the rotor–bearing system presented in [18]. This comprises a flexible rotor weighing approximately 4 kg modelled using FE which is supported by a ball bearing and a single rigid gas bearing. Placing an unbalance of 10 g mm at the disc, this system has been integrated in time to obtain the forced steady-state orbit at 5 kRPM. The same has been simulated using the code presented in [13] utilising an FD discretization of the fluid film, a reduced order rotor and an explicit time integration scheme. The resulting orbits are very similar, as it can be seen from the comparison in fig. 3a.

To validate the full model including a foil structure, the test rig presented in [10] has been simulated. This is composed of a rigid shaft with a mass of 21.1 kg supported by two identical three-pad GFBs. The SEFM is employed to model the foil structure under the assumption of an axially uniform displacement field. For the present code, this is implemented by locating foil nodes at each circumferential CV position and to incorporate the calculation of the average pressure at each of these locations in the pressure–foil mapping matrix from eq. (24).

Simulating in time at 15 kRPM with unbalances of 40 g mm and –2.5 g mm at the bearing locations, the forced steady-state orbits measured in the two bearings are obtained as shown in fig. 3b. These are compared to results from the code presented in [10], where a FE discretized fluid film is used, and the orbits are seen to agree well for both bearings.

9 Conclusions & Future Aspects

In the paper, a recipe for a simultaneously formulated model for simulation of rigid as well as compliant type gas bearings supporting a wide variety of rotor configurations has been treated. This has included the mathematical formulation as well as selected aspects of the numerical implementation.

Two main features of the presented model are the full coupling between the domains and the residual form of the system. For steady-state solutions, the presented formulation allows a single general purpose non-linear algebraic solver to be used while given full system knowledge through the Jacobian. This is not possible in the more common segregated solution approaches involving (at least) two independent solvers. For time integrations, the same formulation allows all state variables to be solved simultaneously in time using a general purpose IVP solver. The residual, or fully implicit, form of the equation system has numerical advantages as no explicit matrix
FIGURE 3: Time integrated unbalance driven steady-state orbits compared to benchmark codes: (a): Flexible rotor supported by a rigid gas bearing as described in [18] simulated as given in [13] at 5 kRPM with a 10 g mm unbalance at the disc. (b): Rigid rotor supported by two identical GFBs ("A" and "B") using the geometry and code described in [10] at 15 kRPM with 40 g mm and −2.5 g mm unbalances at the bearings.

Inversions are necessary, but it is mainly chosen to allow a wider variety of foil models. Unfortunately, a limited number of IVP solvers are available for this form, and calculation of consistent ICs is required, something which is not generally necessary for explicitly formulated systems.

In the future, several extensions to the model should be made. One is the inclusion of reduced order rotors, e.g. through modal truncation, which will primarily require the left eigenvectors or similar to be included in the $T_r$ matrix of eq. (1). The resulting reduction in overall system size would usually be limited, as even the full rotor would most often require much fewer states than the fluid film, but removing the high frequency modes of a full FE rotor model would allow the IVP solver to take much larger time steps. Another interesting extension would be to include a bump foil model with frictional dissipation as described in [19], possibly coupled to a top foil as described in [15].

The original and main motivation for developing the presented model has been to prepare an efficient platform for research into GFBs with active radial injection. For rigid bearings, several analytical expressions are available in the literature, but the additional compliance–injection interaction in GFBs complicates the identification of the empirical correction factors, or "discharge coefficients", most often involved. This could possibly be circumvented by simultaneously solving a miniature CFD model for the injection zones, but this has not yet been proven feasible.

REFERENCES


