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Experimental and numerical analysis of a reciprocating piston expander with variable valve timing for small-scale organic Rankine cycle power systems

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Abstract

This paper presents the modeling and experimental results of a reciprocating piston expander for organic Rankine cycle applications using variable timing admission valve, enabling the adjustment of the expansion ratio in real time while the expander is running. An organic Rankine cycle experimental test rig with the n-pentane as the working fluid and single-cylinder reciprocating piston expander was developed. Experiments were conducted for evaporation temperature ranging from 125 °C to 150 °C, and condensation temperature ranging from 20 °C to 40 °C. The performance of the reciprocating piston expander was investigated in terms of the torque of expander, pressure inside the cylinder, isentropic efficiency of the expander, and net power produced by the expander. Based on the experimental data, a dynamic model of the system was formulated in object-oriented language, Modelica. The model was experimental validated and used to predict the performance of the expander. Special attention was paid to the robust modelling of the valve actuation to avoid computational inefficiencies caused by singularities of state variables or their derivatives. Results indicate that the expander produces up to 2.5 kW of electricity from a low-temperature heat source while operating at pressure ratios ranging from 10 to 16.5 with an isentropic efficiency of approximately 70%. The relative differences between the model and the measurements of the isentropic efficiency and power output of the expander per revolution was $\pm 10\%$ and $\pm 30\%$ respectively. The variable valve timing approach play a major role for the efficient operation of a reciprocating machine and enable it to adjust the expansion ratio under the varying load without deteriorating the expander performance.

Keywords: Organic Rankine cycle, Reciprocating piston expander, Variable valve timing, Dynamic modelling, Pentane, Waste heat recovery

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1. Introduction

Low-grade thermal energy cannot be converted efficiently to electric power by conventional power generation methods, and a large amount of low temperature heat remains unused or discarded as waste heat into the environment. The low-grade thermal energy resources include renewable energy (geothermal, solar thermal, and ocean thermal energy) and waste heat from industries (e.g. cement, steel, textile, paper, and chemical industries), thermal power plants, and the transportation sector. In this context, research on how to convert these low-grade temperature heat sources into electric power is of great importance. The organic Rankine cycle (ORC) technology provides a way to convert low temperature heat into electricity by evaporating a working fluid with a lower boiling point than water and extracting work from a subsequent expansion process. The ORC system has advantages over conventional thermodynamic power cycles for low-grade waste heat to power conversion, and has been widely used in recovering different low-grade heat sources and waste heat [1]. There has been an increasing interest in the area of ORC technology, and progressively adopted as premier technology for low temperature and waste heat to power conversion [2].

The expansion machine in an ORC system is a critical component that affects the investment cost and overall performance of the ORC system. The selection of the expansion machine of the ORC system depends on a number of factors, such as the operating conditions, working fluid, space and weight restrictions, cost, availability, and the size of the system [3]. Expanders can be classified into two groups: volumetric expanders (displacement expanders) and turbo-expanders. The use of turbo-expanders for small-scale ORC systems (less than 50 kW) presents challenges with respect to their high rotational speed, relatively low expansion ratio, and potentially high cost [4]. For small to medium-scale ORC systems, the volumetric expanders can be considered a good choice due to their low cost, low rotational speed, and ability to operate in a two-phase region. The most common types of volumetric expanders are scroll expanders [5], vane expanders [6], screw expanders [7], and piston expanders [8].

The geometry of the volumetric expanders is characterized by the internal built-in volume ratio and the internal built-in pressure ratio. The internal built-in pressure ratio is defined as the ratio between the expander supply pressure and the pressure at the end of the expansion process. The pressure at the end of the expansion process may be lower or higher than the pressure at the exhaust of the expander. The operating conditions corresponding to the equal internal built-in pressure ratio and external pressure ratio represents the best isentropic efficiency point of the expander [9]. Scroll expanders have a relatively low internal built-in volume ratio, ranging from 1.5 to 3, and pressure ratio ranging from 2 to 15 [10].

Due to the risk of thermal expansion and increase in the internal leakage losses, the operating temperature of the scroll expander is limited to 250 °C [11]. The screw expander and vane expander has similar range of the pressure ratio as scroll expander, ranging from 2 to 15. The reciprocating piston expanders outperform these volumetric expanders due to their broad range of operating conditions. The reciprocating piston expanders have built-in volume ratio ranging from 6 to 14, pressure ratio ranging from 2 to 30, operating temperature and pressure up to 600 °C and 9 MPa respectively [12]. Despite these unique performance characteristics, the application of the reciprocating piston expanders in ORC technology is scarce due to the complexity of the valve control mechanism of these expanders.

In the open literature, three types of the piston expanders have been studied: the rolling piston expander [13], free piston expander [14], and reciprocating piston expander [15]. Apart from the piston being the common part of the expander, the working principles and performance of these machines differ significantly from each other. Free-piston expanders were used in the marine and stationary power plants in the mid-20th century, and attempts were made to use this principle in automotive applications [16]. Previous works on free piston expanders mainly focused on the development and testing of small-scale expanders (lower than 1 kW) and tested at relatively low operating conditions in terms of inlet temperature and pressure [17]. Rolling piston expanders had been used in the CO₂ transcritical cycle to recover the energy from the expansion process, replacing the expansion valve with a rolling piston expander [18]. The rolling piston has higher frictional losses and leakage losses than the reciprocating piston expanders, resulting in lower isentropic efficiencies [19].

The first research work on the reciprocating piston expanders was reported in early 1970 for waste heat recovery application from car engines using steam Rankine cycle [20,21]. Syniuta et al. [22] conducted experimental study of reciprocating piston expanders with steam at temperature of 540 °C and a pressure of 69 bar and the maximum power out was 104 kW. At Australian National University, a reciprocating piston expander was developed for the steam Rankine cycle for solar thermal applications [23,24]. The 2.6 liters three-cylinder lubricated expander has the built-in volume ratio between 10 and 15. The inlet conditions of the expander were 400 °C and a pressure between 15 and 60 bar. The power output of the expander was reported between 15 and 35 kW [24]. Bouvier et al. [15] presented an experimental study of an oil-free piston expander for a micro combined heat and power (CHP) system based on the steam Rankine cycle. The experiments were performed for a range of inlet temperatures (60 °C to 340 °C) and pressures (20 bar to 34 bar), and the maximum power output and the isentropic efficiency of the expander were 2.4 kW and 40%, respectively.

Latz et al. [25] conducted a performance assessment of a reciprocating piston expander used in a steam Rankine cycle for heat recovery from a heavy-duty diesel engine. The expander had a compression ratio of 21 together with a steam outlet timing that caused high re-compression. Simulations indicated that reducing the expander's compression ratio from 21 to 13 would allow 30% lower steam supply pressures without adversely affecting the expander's power output. The system achieved a thermal efficiency of 10%. Bianchi et al. [26] presented experimental characterization of a micro-scale ORC system which employs a piston expander prototype, made of three cylinders arranged radially around the drive shaft. The results show that the gross electric power output varied between 250W and 1150 W, depending on the expander speed and on the number of electric loads activated.

Olivier et al. [27] performed an experimental comparison of a piston, screw, scroll, and roots expander. The volumetric expanders delivering 2 kW to 4 kW were tested on two similar small-scale ORC units with the working fluid R245fa. The experimental result showed that the highest isentropic efficiency achieved by the piston expander was 57%. Ancona et al. [28] developed and reported the preliminary experimental result of a 3-piston reciprocating expander for an ORC based micro-CHP unit. The experiments were conducted with pressure ratio of 1.3 to 1.7, and maximum power output of the expander was 0.5 kW. The inlet and exhaust port has constant opening and closing interval. Zhang et al. [29] investigated the working performance, flow loss mechanism and its internal flow structure of the single-valve reciprocating expander in the simulation environment of Fluent software. The results suggests that the largest pressure drop occur at the air intake process due to the higher pressure difference. Maria et al. [30] conducted off-design optimization of organic Rankine cycle (ORC) engines with piston expanders for medium-scale combined heat and power applications. The optimization tool predict the performance of the CHP system under varying heat-source conditions considering the time-varying operational characteristics of the individual components. The results shows that the power output of the ORC system, isentropic efficiency of the expander, and heat exchanger effectiveness is improved by 17% , 16%, and 15% respectively.

The valves used for admission and exhaust of the working fluid are a potential source of losses and have to be designed and controlled carefully. Gao et al. [31] evaluated the performance of a steam Rankine cycle system with a reciprocating piston expander. Their results indicated an increase in the engine power output by 12% due to the steam Rankine cycle system, when the diesel engine operated at 80 kW/2590 rpm. Authors concluded that the intake and exhaust valve could be phased in optimum opening and closing timing for large power output and efficiency. Antonelli et al. [32] investigated the valve timing effect on the performance of the wankel expander for the working fluids R600a and R152a.

The results show that the speed of the expander and opening degree of the inlet and exhaust valves affect the isentropic efficiency of the expander significantly. Badami and Mura [33] conducted preliminary design and controlling strategies of a small-scale steam Rankine cycle with reciprocating expander. Authors provided the guidelines for a rapid sizing of the expander and investigated three control strategies of the expander namely; variable speed at constant cut-off, variable cut-off at constant speed, and constant speed at constant cut-off. The variable cut-off strategy provide higher efficiency but leads to the complex design and development of the expander. The variable speed leads to lower efficiency, and has lower design complexity comparing with variable cut-off design.

The previous studies conclude that the opening and closing of the inlet and exhaust port has significant effect the performance of the reciprocating expander. The application of reciprocating piston expander for ORC applications is challenging task due to difficulty in valve control mechanism. However, none of the previous studies presented the experimental results of inlet and exhaust valve control for the optimum performance of expander at different operating conditions. In addition, predicting the expander performance plays a critical role for the design of a suitable reciprocating expander. To this end, dynamic model can significantly reduce the cost and time to predict and evaluate the performance and control of the expander. None of the previous papers on reciprocating expanders presented a dynamic model to predict the performance of the expander. The primary objective of the paper is to contribute to the knowledge of reciprocating piston expander and to promote the practical application and commercialization of reciprocating piston expander for ORC or micro-combined heat and power systems. The novel contributions of the paper are the following:

- Development and experimental investigation of an ORC system with a reciprocating piston expander employing variable admission valve timing. The variable admission valve timing allow the expander to adjust the expansion ratio while the expander is running.
- Dynamic modelling of the reciprocating piston expander to predict the performance at the different operating conditions. The dynamic model will not only serve as a tool to evaluate the performance of the expander, but will also serve as framework for further studies concerning optimal valve control of reciprocating expanders.

The paper is divided in seven sections. The research background, literature review, and state of the art are described in section 1. Details of the experimental set-up, operational conditions, and valve control are presented in section 2. The overview of the research methodology, expander model, and dynamic model of the ORC system are discussed in section 3. The section 4 of the paper presents the data

acquisition system and uncertainty analysis. The results of the study are provided in section 5, and a detailed discussion is provided in section 6. Finally, the conclusions are outlined in section 7.

2. Experimental set-up

The initial design, modelling, and testing of a proof-of-concept of a double-acting reciprocating expander were reported in our previous work [34]. Afterwards, a more production-oriented second version of the machine was designed and developed for present study. The new design is more conventional and features two ‘ordinary’ single-acting pistons that only produce work during the half of one crankshaft revolution each. In addition, the cylinders can be connected by a valve allowing for either two-stage operation in series or as a parallel configuration to increased power output. Figure 1 shows both versions of the reciprocating piston expander.

Figure 1: Reciprocating piston expander: a) proof-of-concept design, b) second expander prototype (used in the present study).

This reciprocating expander is the first machine with rotating variable admission valves. The data presented here were recorded during operation with only one of the two cylinders from the machine shown in Figure 1b. The reciprocating machine is described on a per-component basis following the structure of the physical device.

The most relevant parts of the reciprocating expander are shown in Figure 2. A bearing holds the crankshaft, which is connected to the crank arm with length l_{cr} . A connecting rod with length l_{ro} links the piston pin to the crank arm. Since the piston pin is not necessarily situated right above the crankshaft, there is a piston pin offset that is denoted by δ_{pi} . The piston itself has a bore of d_{pi} and its movement is limited to the vertical direction by a cylindrical wall of a mass m_{wa} , and a temperature T_{wa} . All mechanical parts shown in Figure 2 were modelled as ideally stiff and with their mass distributed evenly along their main axis.

Figure 2: Control volume and geometry of the reciprocating machine.

The fluid-related indices “in”, “cyl” and “out” used in Figure 2 refer to the thermodynamic states in the supply pipe, expansion chamber, and exhaust line, respectively. Mass can enter and leave the control volume through three ports: (a) inlet valve, (b) outlet valve, and (c) the leakage gap of hydraulic diameter δ_{wa} between the piston and cylinder wall.

Energy exchange across the boundaries of the control volume takes place by means of heat transfer to and from the walls at temperature T_{wa} and by the amount of work extracted or added via the piston. Since the piston is located directly above the crankshaft, δ_{pi} is 0 m in the design presented here. The clearance volume, which is left in the expansion chamber when the piston reaches the top dead center (TDC), is 36 cm³. All other geometric details are listed in Table 1. Here, the masses of the bearings at each end of the connecting rod were added to the piston and the crank arm, respectively. Since the latter was a lumped variable accounting for the rotating masses below the cylinder, it is an estimated value. The dagger symbol † indicates that the mass could not be measured directly.

Table 1: Properties of the different mechanical parts.

Component	Value of the geometrical parameter	
Piston	$d_{pi} = 46 \text{ mm}$	$m_{pi} = 400 \text{ g}$
Conrod	$l_{ro} = 163 \text{ mm}$	$m_{ro} = 423 \text{ g}$
Crank arm	$l_{cr} = 55 \text{ mm}$	$m_{cr} = 750 \text{ g}^{\dagger}$
Wall	$\delta_{wa} = 0 \text{ mm}^{\dagger}$	$m_{wa} = 25 \text{ kg}^{\dagger}$
Inlet	$d_{in} = 25.6 \text{ mm}$	$l_{in} = 0 \text{ mm}^{\dagger}$
Outlet	$d_{out} = 22 \text{ mm}$	$l_{out} = 0 \text{ mm}^{\dagger}$

193 The crank arm weight was found from a simplified geometric sketch assuming an approximate
 194 density of 7600 kgm^{-3} . Using an average density of 2700 kgm^{-3} , the same method was applied to find the
 195 mass of the total jacket mass attributed to one cylinder. While the crank arm is made from stainless steel,
 196 the expander jacket is made of aluminum; hence the different density figures. Both values were rounded
 197 to account for the inaccuracy of the weight estimation. Setting δ_{wa} to 0 mm removed leakage flow from
 198 the calculations. The lengths of 0 mm for l_{in} and l_{out} were used since the valves were located in the
 199 cylinder head.

200 The simplified process diagram in [Figure3](#) main components that were used during the
 201 experiments. Besides the expander, the experimental rig included an air-driven piston pump to pressurize
 202 the working fluid from state 1 to state 2 before it enters the evaporator. After passing the expander in
 203 state from 3 to 4, the fluid enters the water-cooled condenser and leaves it in state 1.

Figure 3: A simplified sketch of the investigated power cycle.

Both heat exchangers are brazed plate type devices. An oil loop with 0.4 m³ of heat transfer fluid and five 12 kW electrical heaters supplies the heat flow rate $\dot{Q}_{oil}$, to the evaporator, while cooling the heat transfer medium from the state a to state b. The cooling water loop removes the heat flow $\dot{Q}_{water}$ from the condenser by heating the water flow $\dot{m}$ water from the state I to state II.

2.1 Operating conditions

The operating conditions are categorized based on the opening timing of the exhaust valve. The operating conditions S refer to the exhaust valve opening period of 75° and result in high pressure ratios. The operating conditions L refer to the exhaust valve opening period of 115° and result in a low pressure ratio. For each S and L case, experiments were performed at two different evaporation temperatures, $T_{ev} = 150$ °C and $T_{ev} = 125$ °C, and two different condensation temperatures, $T_{co} = 20$ °C and $T_{co} = 40$ °C, resulting in eight operating conditions:

S1: $T_{ev} = 150$ °C and $T_{co} = 40$ °C

S2: $T_{ev} = 150$ °C and $T_{co} = 20$ °C

S3: $T_{ev} = 125$ °C and $T_{co} = 40$ °C

S4: $T_{ev} = 125$ °C and $T_{co} = 20$ °C

L1: $T_{ev} = 150$ °C and $T_{co} = 40$ °C

L2: $T_{ev} = 150$ °C and $T_{co} = 20$ °C

L3: $T_{ev} = 125$ °C and $T_{co} = 40$ °C

L4: $T_{ev} = 125$ °C and $T_{co} = 20$ °C

However, for the operating conditions with the lowest pressure ratio (S3 and L3), the expander could not be operated reliably. The power output was very low, and the machine did not reach steady state due to issues with the evaporator temperature. Therefore, the operating conditions with the lowest pressure ratio (S3 and L3) were excluded, and results of the remaining six operating conditions (S1, S2, S4, L1, L2, L4) are presented in the paper. These operating conditions sufficiently cover the variations in evaporation temperature, condensation temperature, and pressure ratio.

2.2 Valve control

Experiments were carried out with two different camshaft designs for the exhaust valve and variable timing for the injection system. The first cam design “S” had a short exhaust opening period of 75° . The valve operation during one crankshaft revolution of the S4 test series with a short exhaust valve-opening period is shown in Figure 4.

Figure 4: Valve operation during one crankshaft revolution of the S4 test series with a short exhaust valve-opening period.

The solid line in Figure 4 shows the operation of the exhaust valve, which was the same for all “S” cases, and the dotted line shows the specific inlet port control for the case S4. The early exhaust closing angle of -97° before the TDC leads to a high recompression ratio. Hence, more mass remains in the chamber leading to a higher cylinder pressure at the beginning of the admission of a new batch of working fluid. At high pressure ratios, compressing a large amount of gas before injection reduces throttling losses in the early injection phase, caused by a large pressure difference between the supply line and the expansion chamber. However, at low pressure ratios, a high recompression ratio can lead to a cylinder pressure higher than the evaporator pressure introducing new losses to the system. Therefore, a second, less aggressive, exhaust valve timing scheme “L” with a longer opening period of 115° was implemented to allow for testing of lower pressure ratios and hence lower evaporation temperatures.

The valve operation during one crankshaft revolution of the L1 test series with a long exhaust valve-opening period is shown in Figure 5.

Figure 5: Valve operation during one crankshaft revolution of the L1 test series with a long exhaust valve-opening period.

Comparing Figure 4 to Figure 5, one can see the impact of the different exhaust valve timings. Both systems opened shortly after the BDC, but the longer opening in Figure 5 led to a period with a fully opened exhaust line, while the exhaust in Figure 4 started to close again shortly after it reached the fully opened position. The valve timing, temperatures, and pressures at the outlet of the evaporator and condenser for each of the operating points are listed in Table 2. The inlet system of the expander was equipped with a variable valve timing. The timing for the admission valve was found experimentally by minimizing the length of the admission period ($\theta_{in,open} - \theta_{in,close}$). Starting with a long opening interval, the cut-off angle $\theta_{in,close}$ and the inlet opening angle $\theta_{in,open}$ were carefully moved towards the TDC until the final pressure after expansion approached the average pressure during the exhaust stroke.

Based on the measured crankshaft position and the valve control setpoint, two high accuracy servo drives controlled a rotary admission valve, presented in Figure 6. Using two rotary valves like the one shown in Figure 6 in series, fluid admission can only occur if the openings in both rotors are inside the supply pipe flow path. The rotational speed of both rotors is directly linked to the crankshaft, but the operator is able to adapt the phase shift between the two. This phase shift can be used to control the overlap of opening of the two individual rotary valves, which effectively controls the onset of the fluid admission and the admission cutoff. The operation of the inlet valve of the expander has been patented [36] and operating of the valve is briefly explained in the patent.

Figure 6: Rotary admission valve of the tested expander.

The angles documented in Table 2 represent the shortest possible admission period that was found to run reliably. No stable valve configurations with an acceptable performance were found for the missing cases S3 and L3, which were supposed to operate between 125 °C and 40 °C.

Table 2: Operating points for the experiments in terms of valve timing, temperatures, and pressures at the outlet of evaporator and condenser.

Case	S1	S2	S4	L1	L2	L4
$\theta_{in,open}$ (°)	-11	-10	-10	-2	-5	-19
$\theta_{in,close}$ (°)	7	15	25	16	27	10
T_{ev} (°C)	153	145	125	155	157	126
p_{ev} (bar)	15.4	14.2	10.1	14.7	15.3	9.8
T_{co} (°C)	19.4	37.5	19.5	20.0	39.2	20.4
p_{co} (bar)	0.9	1.4	0.9	0.9	1.6	0.9

The other values in the Table 2 are the exact evaporator and condenser outlet temperatures T_{ev} , T_{co} and pressures p_{ev} , p_{co} for all six cases. These values can be used to calculate the degree of superheating $\Delta T_{ev,sup}$ and the degree of subcooling $\Delta T_{co,sub}$ at the heat exchanger outlets. For the presented data, $\Delta T_{ev,sup}$ varies between 0 K and 10 K for S4 and L1, while $\Delta T_{co,sub}$ varies between 8 K and 13 K for L1, S1, and S2.

3. Piston expander modelling

The model of the reciprocating piston expander is developed in Dymola, a commercial Modelica simulation environment. Modelica is non-proprietary object-oriented, declarative, and equation-based language for the modelling and simulation of the complex physical systems. Dymola uses the Modelica modelling language to define models with open access to the Modelica language. It provide freedom to create one's own libraries and models or extend any model from the existing Modelica libraries or commercially available libraries to accelerate development times, reduce maintenance efforts and improve the level of reuse across projects.

The model of the expander is based on the free Modelica library, Modelica.Fluid, which provide interfaces and basic components for the device-oriented modeling of one-dimensional thermo-fluid flow in networks containing vessels, pipes, fluid machines, valves and fittings. The reciprocating expander model is extended from the Modelica library, Modelica.Fluid.Machines.SweptVolume, developed by Franke et al. [37]. The geometrical calculations, equations to determine the heat flow rate, and three possible paths for mass transport are added in the base model of the expander. The next sections 3.1, 3.2, and 3.3 explain the numerical model of the system. The section 3.4 explain the development of the system model in the Dymola simulation environment.

3.1 Conservation equations

The primary governing equations of the expander model are based on the law of conservation of energy and mass. Based on meaningful initial guesses for internal energy U_0 , and confined mass m_0 , energy and mass balances were calculated for the expansion chamber. The law of conservation of the energy across the cylinder volume has the form

$$\frac{dE}{dt} = \dot{U} = \left(H + \frac{C^2}{2} + gz \right)_{in} - \left(H + \frac{C^2}{2} + gz \right)_{out} + \dot{W} + \dot{Q} \quad (1)$$

$$0 = \dot{H}_{in}^* + \dot{H}_{out}^* + \dot{W} + \dot{Q} - \dot{U} \quad (2)$$

The $\dot{U}$ is the rate of change in internal energy in the control volume, and $\dot{H}^*$ denotes the enthalpy flows across the system boundaries with a positive sign for flows entering the control volume. These flows were defined by temperature and static pressure and could include a dynamic part based on the fluid velocity in the ports, in case detailed knowledge about the flow losses in the valves is available. The mass balance of the expander is given by

$$\frac{dm}{dt} = \sum m_{in} - \sum m_{out} \quad (3)$$

3.2 Mass flow rate

The absolute values for the mass and the internal energy define the state of the working fluid as given by Eq. 4 and Eq. 5

$$m = V\rho \quad (4)$$

$$U = \int \dot{U} dt \quad (5)$$

Eventually, the total mass balance equation was expressed in terms of mass of working fluid entering and leaving the control volume and leakage mass:

$$\dot{m}_{cyl} = \dot{m}_{in} + \dot{m}_{out} + \dot{m}_{leak} \quad (6)$$

The mass balance is coupled with a vectorized component mass balance to provide the possibility of using mixtures as working fluids by means of a composition vector $\mathbf{X}$.

$$\dot{m}_{cyl}\mathbf{X}_{cyl} = \dot{m}_{in}\mathbf{X}_{in} + \dot{m}_{out}\mathbf{X}_{out} + \dot{m}_{leak}\mathbf{X}_{leak} \quad (7)$$

The thermophysical properties of the working fluid are obtained from an equation of state (EOS) by Span and Wagner [38] via CoolProp [39] with the balance equations. Internal power was a function of volume and pressure only, and hence defined by

$$\dot{W}_{cyl} = -\dot{V}_{cyl}p_{cyl} \quad (8)$$

3.3 Heat transfer model

Due to the temperature difference between the working fluid and walls of the cylinder, the heat flows from working fluid towards the cylinder. The heat transfer rate to and from the cylinder wall $\dot{Q}_{wa}$ was calculated according to Adair et al. [40] and is given by

$$\dot{Q}_{wa} = \alpha_{fluid}A_{cyl,wa}(T_{fluid} - T_{wa}) \quad (9)$$

This empirical correlation for convective heat transfer coefficient α_{fluid} [40] is based on Reynolds number Re and Prandtl number Pr and is given by

$$\alpha_{fluid} = \left(\frac{\lambda}{\Gamma}\right) 0.053 Re^{0.8} Pr^{0.6} \quad (10)$$

$$Re = \frac{\rho \Lambda \Gamma}{\mu} \quad (11)$$

331 The computation of Re requires the characteristic velocity Λ [40] and the characteristic length Γ
 332 [40] of the expander, given by

$$\Lambda = \frac{1}{2} d_e \Omega_e \quad (12)$$

$$\Gamma = \frac{1}{2} d_e \quad (13)$$

333 In order to account for a high degree of turbulence after admission of the working fluid, the
 334 equivalent swirl velocity $\Omega_e = f(\Omega, \theta)$ is assumed to decrease from working fluid admission to exhaust
 335 port employing an equivalent diameter, given by

$$d_e = \frac{6V}{A_s} \quad (14)$$

336 3.4 Valve model

337 The valves make up a crucial part of the system, since pressure drops during admission and
 338 exhaust have a major influence on the operation. The system studied here was equipped with a rotary
 339 admission valve and a poppet-type exhaust system. Admission was controlled electrically and could
 340 therefore operate with different opening and cut-off timings. However, being triggered by the crankshaft
 341 rotation, the process of opening and closing was fixed in terms of crank angle degrees. The nominal
 342 opening of the valves is given by

$$a_0 = \max[0, g(\theta, \theta_{open}) - g(\theta, \theta_{close})] \quad (15)$$

343 For a given crankshaft angle, the nominal opening a_0 of both valve types could be calculated using
 344 a smooth transition for the opening and the closing angles, θ_{open} and θ_{close} , respectively. The closing and
 345 opening angles of both valves are estimated by Eq. 16:

$$g(\theta, \theta_{act}) = \begin{cases} 0 & \text{if } -\pi/2 \geq \phi \\ \psi & \text{if } -\pi/2 < |\phi| \\ 1 & \text{if } -\pi/2 \leq \phi \end{cases} \quad (16)$$

$$\psi = 1 + \frac{1}{4} [\cos(\phi)^2 + 2] \cdot \sin \phi - \frac{1}{2} \quad (17)$$

$$\phi = \frac{\pi(\theta - \theta_{act})}{\Delta\theta} \quad (18)$$

This implementation provides a continuous transition up to the second order and originates from smoothing functions presented by Richter [41]. Two different transition angles θ_{act} were used to determine the opening θ_{open} and closing of the valve θ_{close} with the transition durations $\Delta\theta_{open}$ and $\Delta\theta_{close}$ of 55° and 100° for all cases. Based on the nominal opening a_0 , the actual opening a was calculated by a characteristic function. This function translated the opening signal a_0 to a factor a that could be multiplied with the maximum flow area to obtain the instantaneous flow area. The Figure 7 shows the opening characteristics of the inlet and outlet valves of the reciprocating expander.

Figure 7: Opening characteristics of the inlet and outlet valve.

Figure 7 illustrates the large differences between the rotary valve used for admission control and the exhaust valve of the traditional poppet type. The former was modelled using Eq. 16 while replacing θ with a_0 and setting $a_{0,act}$ to 0.5 and $\Delta a_{0,act}$ to 0.5. The second curve in Figure 7 exhibits a typical poppet valve behavior that sometimes is referred to as the quick opening characteristic. Based on the actual design, the valve opening is given by

$$a = 1 - (b - 0)(1 - a_0) + (b - 1)(1 - a_0)^c \quad (19)$$

The parameter $b = 0.5$ and the exponent $c = 5$, which approximate the behavior of the actual design. The valve implementation in Modelica followed the standard EN 60534 [42].

Hence, the mass flow rate was calculated from a nominal throat area A_t defined by the hydraulic inlet and outlet diameters in Table 1, and the upstream density ρ_1 and pressure p_1 . The suction flow rate of the working fluid is given by

$$\dot{m} = aY A_t \sqrt{\Delta p_{1,t} \rho_1} \quad (20)$$

The Eq. 20 employs a flow expansion factor Y and a modified throat pressure difference to account for pseudo choking conditions, given by

$$\Delta p_{1,t} = x p_1 \quad (21)$$

The pressure differential ratio factor x was obtained from the total pressure difference, given by

$$x = \frac{\Delta p_{1,2}}{p_1} \quad \text{for} \quad x \leq 0.5 a_o \quad (21)$$

Finally, the flow expansion factor Y is given by

$$Y = 1 - 2x/3 \quad (22)$$

An orifice flow equation was used to approximate the pressure drop in the pipes between the heat exchangers and the expander. As suggested by Franke et al. [37]

$$\Delta p_{1,2} = 0.5 \zeta p_1 w^2 \quad (23)$$

The pressure loss factor ζ depends on the pipe length l , the pipe diameter d , and the pipe wall friction f_{pipe} .

$$\zeta = f_{pipe} \frac{l}{d} \quad (24)$$

The pipe friction factor f_{pipe} was estimated using the Moody diagram [43]. For the operating conditions under consideration, the value of the parameter ζ vary between 0.5 to 4.5. Finally, the pressure drop in the supply and exhaust pipe can be estimated from the homogeneous fluids velocity w and pressure loss factor ζ .

Unlike the volumetric efficiency concept in compressors, the positive displacement expanders are characterized by the filling factor [15]. The filling factor is defined as the ratio between the actual mass admitted to the expander (measured from experiment) to the theoretical mass. The filling factor of the reciprocating piston expander is estimated by

$$FF = \frac{\dot{m}_{in}}{(N/60)\rho_{in}V_{in}} \quad (25)$$

3.5 Modelica modelling

The modelled system consists of the mechanical components attached to the expander and the internal mechanism displayed in [Figure 2](#). The internal components of the slider-crank mechanism were regarded as ideally stiff with all their mass concentrated in the center of gravity. The base model of the cylindrical and box-shaped parts are extended from the Modelica Library, Modelica.Mechanics.Multibody [44]. The components were connected by joints computed by revolute objects, and a one-dimensional slider was implemented as an instance of the prismatic model. The piston position couples the mechanical system to the volume of the expansion chamber, and the revolute component for the main bearing connects to the crankshaft assembly using the connectors from the Modelica Library, Modelica.Mechanics.Rotational. The modelled system assembly and Modelica model of the system are shown in [Figure 8a](#) and [Figure 8b](#), respectively.

In addition to the parts described in [Table 1](#), external mechanical components were also included. The whole crankshaft assembly is shown in [Figure 8a](#), which also contains the heat exchangers and valves. The crankshaft itself had a mass of 1 kg, and an idealized cylindrical shape with a radius of 2 cm and a length of 20 cm. The flywheel was directly connected to the crankshaft, and the model concentrated almost all-rotational inertia in a bulk value of 120 gm². The total rotational inertia was approximately four times the inertia of the installed flywheel, which is equal to 30.6 gm². A stiff rotational spring described by a constant of 8 kNrad⁻¹ accounts for the flexibility of the shaft connectors and the deformation of the shaft itself. At the end of the model assembly, an additional rotational inertia of 12 gm² represents the last part of the shaft, including the torque transducer and the hydraulic brake used to dissipate the generated energy. In the experimental installation, the torque was measured directly at the load making the dynamic behavior of the shaft visible in the torque measurements. Mechanical losses were lumped in an artificial quadratic friction term that dissipated energy by imposing a torque M_f that acts against the direction of rotation according to the equation

$$M_f = M_0 \left(\frac{\omega}{\omega_0} \right)^2 \quad (26)$$

The value of the nominal friction M_0 is 2.5 Nm at the nominal rotational speed ω_0 of 1000 rpm.

Figure 8: Modelled system assembly: a) schematic, b) model in Dymola.

4. Measurement and data acquisition

A high speed data acquisition system running at 10 kHz was used to record cylinder pressure, torque, and crankshaft position. The cylinder pressure was obtained from a strain gauge sensor with a full-scale value of 35 bar and a maximum combined uncertainty of $\pm 0.25\%$ thereof. In addition, the torque sensor was based on strain gauges and had a nominal torque of 200 Nm and a maximum deviation of $\pm 0.3\%$ of the rated value. This sensor recorded the torque applied to a hydraulic brake mounted at the end of the crankshaft. Before that brake, the shaft position was measured with an incremental magnetic pulse generator allowing the calculation and storage of the average rotational frequency of the crankshaft. The rotational speed of the shaft was controlled by manually setting the resistance of the hydraulic brake.

A slower sampling rate of 1 Hz was used to store temperature and pressure data in the pipes connecting the expander to the evaporator and condenser based on an averaged sensor signal. The 25 bar version of a piezoresistive pressure transducer recorded the corresponding pressures with an uncertainty of $\pm 0.5\%$. Temperatures at the evaporator outlet, condenser outlet, and expander outlet were measured using thermocouples with an uncertainty of ± 0.5 K. Based on the measured crankshaft position and the valve control set points, two high accuracy servo drives control the rotary admission valve presented in [Figure 8b](#). Using separate drives to control the opening and the closing of the admission line makes it possible to close the inlet before the maximum opening degree is reached. This explains the relative admission valve opening of less than unity occurring in [Figure 4](#). On the cooling water side, two energy meters from accuracy class 2 of EN1434 [45] were employed to measure incompressible volume flow rate of water $\dot{V}_{water}$ with a minimum accuracy of $\pm 5\%$ of the permanent flow rate values of $16 \text{ m}^3\text{h}^{-1}$ and $3 \text{ m}^3\text{h}^{-1}$, respectively. Inlet and outlet temperatures of the secondary side of the condenser were measured with two PT100 class A resistance thermometers with an uncertainty of approximately 0.25 K in the cooling water temperature range.

Combining the measured cooling water volume flow and temperatures on the secondary side of the condenser with the working fluid temperature and pressure measurements, the heat uptake of the cooling water could be used to calculate the average working fluid mass flow rate, $\dot{m}_{fluid}$. The incompressible volume flow rate of water $\dot{V}_{water}$ at a constant density ρ_{water} was multiplied with the constant specific heat capacity $C_{p,water}$ and the temperature difference ΔT_{water} to obtain the heat flow rate in the condenser:

$$\dot{Q}_{water} = \dot{V}_{water} \rho_{water} C_{p,water} \Delta T_{water} \quad (27)$$

Using the energy balance on the primary side, the mass flow rate of the working fluid was calculated as

$$\dot{m}_{fluid} = \frac{\dot{Q}_{water}}{(h_{out} - h_{co})} \quad (28)$$

where $\dot{Q}_{water}$ is the heat transfer rate, h_{out} is the expander outlet enthalpy obtained from the measured temperatures and pressures, and h_{co} is the condenser outlet enthalpy. Multiplying this value with the specific work obtainable from an isentropic expansion from evaporation pressure p_{ev} to condensing pressure p_{co} yields the reference power $\dot{W}_{is}$.

In order to find the isentropic efficiency η_{is} , the actual shaft power $\dot{W}$ was calculated from the measured torque M and the rotational speed ω and divided by the isentropic work $\dot{W}_{is}$. The isentropic efficiency was calculated in the same way for the experimental data and in the simulations. Both formulations were based on an average value of the rotational speed.

The uncertainties were processed with Engineering Equation Solver (EES) [46] resulting in slightly different values for the different operating conditions. The measurement-related absolute error ε of a derived quantity Z from the partial derivatives with respect to all involved measurements X_i and their respective uncertainties ε_{Xi} was calculated as follows:

$$\varepsilon_Z = \sqrt{\sum_i \left(\frac{\partial Z}{\partial X_i} \right)^2 (\varepsilon_{Xi})^2} \quad (29)$$

The partial derivatives needed for this procedure were computed numerically for each operating point. Since EES does not include uncertainties for the fluid property functions, those had to be included manually adding more elements to the calculation of the final uncertainty ξ_Z . Previous works regarding the employed equation of state [25,33] suggest a maximum acceptable uncertainty of $\pm 1\%$ for the caloric properties of gaseous n-pentane and $\pm 0.2\%$ for those of liquid water. These values were combined with Eq. 29 using the equation

$$\frac{\xi_Z}{Z} = \sqrt{\left(\frac{\varepsilon_Z}{Z} \right)^2 + (0.01)^2 j + (0.02)^2 k} \quad (30)$$

where j and k are the total number of fluid property calls for n-pentane and water required to compute the dependent variable Z . Applying the above to the subsequent determination of $\dot{Q}_{water}$, $\dot{m}_{fluid}$, and η_{is} , the corresponding (j, k) pairs of (0, 2), (2, 2) and (5, 2) were obtained. For better understanding, the detail calculations of the uncertainty analysis are provided in the Appendix A.

5. Results

A comparison between results of the dynamic model and the experimental results for the torque and pressure of the expander for one complete revolution are shown in [Figure 9](#) and [Figure 10](#). Both figures show scatter plots of measured data from twenty subsequent revolutions at steady state operation for each of the operating points S1 through L4. Recorded at a sampling frequency of 10 kHz, each series contains more than 35000 samples. The solid line over the scattered points represents a single revolution of the modelled operation at the same evaporator and condenser conditions.

Figure 9: Measured torque data from twenty revolutions and simulation results for one revolution versus crankshaft angle at six operating points.

Running in steady state, the modelled data does not change from revolution to revolution, and there is no need to show more than one revolution. Figure 9 depicts the torque measured at the hydraulic load as a function of crankshaft angle. The abscissa values of -180° and 180° denote the bottom dead center (BDC), while the top dead center (TDC) is located at 0° since there is no piston pin offset. The exhaust and admission periods are marked by the area enclosed by the grey lines that are labelled with “out” and “in”, respectively. Starting from the BDC on the left hand side of Figure 9, the torque decreases while the fluid leaves the expansion chamber. After the exhaust valve is closed, more energy is extracted from the rotating masses to recompress the fluid left in the expansion chamber. New working fluid is injected close to the TDC, and the fluid performs work on the piston making the torque curves rise sharply. The oscillations observed in the torque measurements are related to the mechanical properties of the crankshaft. Very small deformations in the crankshaft make it act as a rotary spring introducing an additional force. The sharp increase in torque is interpreted as an excitation of the crankshaft oscillations.

Figure 10: Measured pressure data from 20 revolutions and simulation results for one revolution versus cylinder volume at six operating points.

The pressure-volume graphs in Figure 10 display the pressure inside the cylinder with respect to the volume of the expansion chamber. The evaporator pressure is indicated by a line marked with p_{ev} , while the condenser pressure is annotated with p_{co} . The piston moved between BDC and TDC causing a change in the trapped volume from 770 cm^3 to 36 cm^3 . Hence, the presented design could operate with a theoretical maximum expansion ratio of more than twenty in the low mass flow rate limit.

Following the lower branch of the pressure curve from right to left, the exhaust process takes place until the valve closes and the remaining gas is compressed as the piston travels towards the TDC. The cylinder pressure peaks around the minimum clearance volume, and the process continues with fluid intake and expansion along the high pressure branch of the cycle until it reaches the BDC again. The experimental campaign produced similar results for the isentropic expansion efficiency regardless of the operating conditions. Only run L4 with little pre-compression and a low expansion ratio yielded an isentropic efficiency of below 55%.

The expander performance and operating conditions and their associated uncertainties are listed in Table 3. All other operating conditions from Table 2 resulted in an isentropic efficiency around 70%, as listed in Table 3.

Table 3: Additional dependent figures obtained from the experimental data including their uncertainties.

Operating conditions	T (K)	P (bar)	τ (Nm)	$\dot{Q}_{water}$ (kW)	$\dot{m}_{fluid}$ (g s ⁻¹)	FF (%)	η_{is} (%)	$\dot{W}$ (kW)
S1	± 0.5	± 0.25	± 0.3	11.4 ± 0.58	22.3 ± 1.18	88.1 ± 4.0	72.5 ± 5.4	1.8 ± 0.07
S2	± 0.5	± 0.25	± 0.3	5.6 ± 0.27	11.7 ± 0.58	79.5 ± 3.1	74.0 ± 4.9	0.8 ± 0.05
S4	± 0.5	± 0.25	± 0.3	7.7 ± 0.39	16.6 ± 0.88	94.2 ± 3.8	70.5 ± 5.7	1.1 ± 0.06
L1	± 0.5	± 0.25	± 0.3	15.2 ± 0.77	30.2 ± 1.58	77.4 ± 4.3	70.8 ± 5.3	2.4 ± 0.06
L2	± 0.5	± 0.25	± 0.3	18.6 ± 0.89	38.8 ± 1.95	90.2 ± 3.7	68.0 ± 4.3	2.4 ± 0.07
L4	± 0.5	± 0.25	± 0.3	10.8 ± 0.55	22.6 ± 1.20	83.7 ± 3.4	53.0 ± 4.3	1.1 ± 0.06

The uncertainties regarding the determination of the heat transfer rate $\dot{Q}_{water}$ and the mass flow rate $\dot{m}_{fluid}$ are dominated by the accuracy of the water volume flow meter that delivers the input to Eq. 27. For both quantities, the flow rate measurement errors account for almost 62% of the total error while the errors originating from the water temperature measurements contribute with about 19% of the total error for both the inlet and the outlet temperature measurement. The calculated uncertainty for the isentropic efficiency is governed by the condenser pressure measurement errors that contributes over 43% of the final uncertainty, while the water flow error and the shaft work error are responsible for slightly above 21% of the final uncertainty each. The water temperature measurement errors effect the accuracy of the isentropic efficiency with about 6.6% and all other quantities contribute less than 1% of the final efficiency error.

A comparison between the values predicted by the model and the experimental values for the isentropic efficiency and expander power output are shown in [Figure 11](#) and [Figure 12](#), respectively. The error bars show the total uncertainty $\xi_{\eta_{is}}$ from the measurements and the employed equations of state according to Eq. 29 and Eq. 30. No errors are shown for the simulated data sets. Instead, the vertical bars illustrate the change in isentropic efficiency that could be observed for calculations using the modified pipe flow loss coefficients described in Section 3.4. All points are within the $\pm 10\%$ band except for L4, which approaches the $\pm 20\%$ margin.

Figure 11: Calculated and measured isentropic expander efficiencies; the filled areas indicate a difference of $\pm 10\%$ and $\pm 20\%$, respectively.

Figure 12: Calculated versus measured expander output; the filled areas indicate a difference of $\pm 15\%$ and $\pm 30\%$.

Dividing the power output in the last column of [Table 3](#) by the average rotational speed yields the work extracted per revolution. [Figure 12](#) compares the work per revolution from the measurements and from the simulations in a similar fashion as [Figure 11](#) compares the efficiencies. The runs with a short valve opening “S” deliver between 55 Jrev^{-1} and 100 Jrev^{-1} , while the simulations predict between 60 Jrev^{-1} and 75 Jrev^{-1} . Allowing more mass to enter the cylinder increases the output to approximately 150 Jrev^{-1} for L1 and L2, while L4 produces less work per revolution than S4.

6. Discussion

6.1 Experimental campaign

In section 2.1 above, measurements of six different operating points were presented. The plots in [Figure 9](#) show that all measurements were taken at steady state conditions. The reoccurring torque oscillation in the measurements in [Figure 9](#) is of an amplitude of approximately 30% of the maximum torque for all cases. This behavior is attributed to the periodical application of force to the crankshaft and the accompanying deformations. The three upper series in [Figure 9](#) exhibit a minimum torque below 0 Nm shortly before the TDC. At those points of the revolutions, work is performed on the trapped gas in the cylinder. This caused a measurable deceleration of the shaft extracting energy from the flywheel and the hydraulic equipment, reversing the sign of the torque for a short period in cases S1, S2 and S4. The size of the flywheel limits the pre-compression, since a small flywheel could cause unacceptably large variations in the rotational speed.

Comparing S2 in [Table 2](#) and [Figure 10](#), it can be seen that the peak pressure of almost 16 bar is higher than the evaporator pressure of 14.2 bar. This over-compression of the remaining gas before injection not only consumes energy from the flywheel, but also increases flow losses, since a part of the working fluid charge has to leave the cylinder in the direction of the evaporator and enter it again before the actual injection process could start, which also reduces the average mass flow rate per cycle. In fact, S2 produces the lowest measured power of all test cases. However, the recompression pressure is not high enough to cause a large efficiency penalty. In addition, case S4 suffers from too much recompression, but the lower pressure levels reduce the forces on the piston and thereby limit the impact on the measured torque. The large pressure ratios and high mass flow rates produced the most power. From S1 to L1, power output increases by a third from 1.8 kW to 2.4 kW. The lower isentropic efficiency in case L1 is related to the throttling losses caused by large pressure differences occurring in the early admission phase due to an incomplete pre-compression in the expansion chamber.

The measured expander isentropic efficiency of test L4 is worse than in all other cases; see [Table 3](#). This indicates that shifting the admission window forward by approximately 15° has a negative impact on the efficiency. An early opening of the admission valve avoids large pressure differences at the TDC, but at the same time, it requires larger amounts of fluids to travel back and forth through the admission valve. In addition to the early opening, the PV-diagram of L4 in [Figure 10](#) also exhibits a peculiarity in the exhaust phase. In all plotted revolutions, there is an increase in pressure in the low pressure branch at around 540 cm³.

This suggests that there has been either a temporary increase in flow resistance in the exhaust system or a leakage from the inlet valve. High frequency condenser pressure measurements could be used to determine the cause of this “bump”. A blocked exhaust would lead to a decreased pressure in the pipe towards the condenser, and a shortcut to the evaporator would cause a pressure peak to occur in that pipe. However, the low sampling frequency used for the pressure transducers did not allow for this kind of analysis. It has to be mentioned that L4 is the case with the earliest admission of working fluid and a long opening of the exhaust valve, which could explain why a shortcut from the evaporator to the condenser only appears in this case. Recording the inlet pressure with a higher resolution is expected to be beneficial as well. Pressure variations are expected to occur in the evaporator and the admission system due to the relatively large swept volume of the piston pump. The pump operates at a lower frequency than the fluid injection valves, and one stroke of the feed pump supplies working fluid for several crankshaft revolutions. The actual injection pressure is thus likely to be different from the average supply line pressure.

6.2 Modelling

[Figure 9](#) shows measurements and calculated data points for the instantaneous crankshaft torque. The spring constant of 8 kNmrad^{-1} used to approximate the crankshaft stiffness captures parts of the dynamic behavior. The model does not reproduce higher order phenomena, but the fluctuations in torque occur with a magnitude similar to that of the measurements. The results indicate that the model is not suited for a mechanical analysis of the whole assembly. However, additional spring and damper components could be used to obtain a better match with the experimental data. For cases L1 and L2, the torque is over-predicted. The torque is related to the chamber pressure and the mechanical assembly connecting the piston and crankshaft as well as the dynamics of the connected parts. Since the pressures are not over-predicted in the same manner as the torque, it is expected that the higher torque is caused by a positive interference (amplification) of the shaft oscillation and the fluid admission.

The measured and calculated pressure traces in the cylinder in [Figure 10](#) do not match equally well at all operating conditions. The modelled exhaust of working fluid occurring at decreasing chamber volume at low pressures takes place at a lower pressure than what was observed in the experiments. The flow resistance in the pipes connecting the expander and the condenser might be underestimated in the model. Despite the lower modelled back pressure in the exhaust pipes, the chamber pressure peaks at the same or higher values as the piston reaches the TDC on the left hand side of the PV-diagrams. Additionally, the pressure also decreases faster in cases with a short admission period, S1 and S2. Such effects can be attributed to a delayed valve operation in the model.

The larger the pressure drop over the exhaust valve, the earlier the recompression in the cylinder starts, since the fluid cannot leave the cylinder fast and a larger pressure difference is needed to force the fluid out of the expansion chamber. A high pressure difference in the early admission phase keeps the fluid from leaving the chamber, leading to an overshoot at the end of the recompression. This is also a possible explanation for the notable over-compression in case L4, which has the earliest admission opening, -19° . The fluid pushed out of the cylinder at the TDC could increase the pressure in the supply line, which would result in more flow into the cylinder in the late admission phase, effectively “widening” the PV-curve in the cases S1 and S2 in [Figure 10](#).

Using the average mass flow rate per cycle, the model calculates a lower efficiency than the one derived from the laboratory tests; see [Figure 11](#). Both flow losses and thermal losses occur in the model. As discussed above, the flow resistance in the partially opened valves might be too high, which also decreases the overall efficiency. Another possibility is that the heat transfer rate is lower than predicted by Eq. 9. At high pressures and temperatures, heat is transferred from the working fluid to the wall. At the end of the expansion, the heat flow is reversed causing an additional entropy generation.

The implemented equations have been published as an integrated part of the ThermoCycle library [48], which is available at <https://github.com/thermocycle/Thermocycle-library> and a parameterized version of the model is provided as a digital attachment to this paper. Using the open standard libraries of the Modelica language, others may create new components that seamlessly integrate with the presented code to refine certain aspects of the model to meet the user’s requirements in terms of both accuracy and computational performance.

7. Conclusions

The experimental and modelling results of a reciprocating piston expander for organic Rankine cycle applications using a novel rotating variable admission valve system are presented. The novel rotating variable admission valve system enable the expander to adjust the expansion ratio in real time under different operating conditions. Experiments were conducted for evaporation temperature ranging from 125 °C to 150 °C, and condensation temperature ranging from 20 °C to 40 °C. Based on the experimental data, a dynamic model of the system was formulated in object-oriented language, Modelica. Special attention was paid to the robust modelling of the valve actuation to avoid computational inefficiencies caused by singularities of state variables or their derivatives. The performance of the reciprocating piston expander was investigated in terms of the torque of expander, pressure inside the cylinder, isentropic efficiency of the expander, and net power produced by the expander. The core results of the study are listed below

- Experimental data showed that variable injection valve timing can be used to operate a reciprocating machine at an almost constant isentropic efficiency of 70% for pressure ratios of approximately 16.5 (S1, L1), 9.7(S2, L2), and 11 (S4, L4).
- The torque of the expander and pressure inside the cylinder predicted by the model have reasonable agreement with the experimental values of the torque and pressure. The relative difference between the model predicted values and experimental measurements are within $\pm 30\%$.
- The valve characteristics play a major role for the efficient operation of a reciprocating machine. Switching quickly between fully closed and fully opened is essential to minimize throttling losses during injection and exhaust.
- A comparison between the results of the dynamic model and the experimental results suggests that the dynamic model is able to predict the expander efficiency within $\pm 10\%$ without extended parameter tuning, and produced work is predicted by the dynamic model within $\pm 30\%$.

The optimum control of the admission and exhaust valve in terms of variable admission angle and cut-off angle can further improve the performance of the expander and being considered for future study by the authors. Besides the valve control, parametric studies and multi-objective optimizations, such as [49], could potentially improve the performance and design of the reciprocating piston expander.

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691 Nomenclature

692 Parameters

A	Area, m ²	T	Temperature, K
A_s	Surface Area, m ²	U	Internal energy, J
a	Valve opening	u	Specific internal energy, J kg ⁻¹
b	First valve coefficient	$\dot{U}$	Change in internal energy, W
c	Second valve coefficient	V	Volume, m ³
d	Diameter, m	v	Specific volume, m ³ kg ⁻¹
FF	Filling factor, -	$\dot{V}$	Volume change rate, m ³ s ⁻¹
g	Helper function for valve calculations	w	Flow velocity, m s ⁻¹
h	Specific enthalpy, J kg ⁻¹	$\dot{W}$	Power, W
$\dot{H}$	Enthalpy flow rate, W	X	Composition vector
l	Length, m	X	Arbitrary quantity
$\dot{m}$	Mass flow rate, kg s ⁻¹	x	Pressure differential ratio
p	Pressure, bar	Y	Flow expansion factor
$\dot{Q}$	Heat flow rate, W	Z	Arbitrary quantity
s	Specific entropy, J kg ⁻¹ K ⁻¹		

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694 Greek symbols

Δ	Difference	ω	Rotational frequency, s ⁻¹ , Hz
δ	Hydraulic diameter or distance, m	φ	Relative phase angle
ε	Absolute error	ψ	Smoothing function
Γ	Characteristic length, m	ρ	Specific density, kg m ⁻³
Λ	Characteristic velocity, m s ⁻¹	θ	Crankshaft angle, °
λ	Thermal conductivity, W m ⁻¹ K ⁻¹	ξ	Total absolute error
μ	Dynamic viscosity, Pa s	ζ	Pressure loss factor
Ω	Swirl velocity, m s ⁻¹	η	Efficiency
α	Heat transfer coefficient, W m ⁻² K ⁻¹		

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696 Subscripts

0	Nominal value	<i>ev</i>	Evaporator	<i>sub</i>	subcooling
1	Upstream	<i>fluid</i>	Working fluid	<i>sup</i>	superheating
<i>Act</i>	Valve actuation	<i>in</i>	Inlet	<i>t</i>	Throat
<i>close</i>	Valve closing	<i>is</i>	Isentropic	<i>water</i>	Cooling water
<i>co</i>	Condenser	<i>oil</i>	Heat transfer oil	<i>wa</i>	Wall
<i>cr</i>	Crank arm	<i>leak</i>	Leakage	<i>out</i>	Outlet
<i>cyl</i>	Cylinder	<i>open</i>	Valve opening	<i>pi</i>	Piston
<i>e</i>	Equivalent				

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