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Distributed Optimal Voltage Control for VSC-HVDC Connected Large-Scale Wind Farm Cluster Based on Analytical Target Cascading Method

Sheng Huang, Qiuwei Wu*, Jin Zhao, Wu Liao

Abstract—This paper proposes an analytical target cascading (ATC) based distributed optimal voltage control (DOVC) scheme for the voltage-source-converter high-voltage-direct-current (VSC-HVDC) connected large-scale wind farm cluster (WFC). The aims are to minimize the voltage fluctuations of the point of connection (POC), collector buses, and wind turbine (WT) terminal buses inside the WFC, and to improve the reliability and to guarantee the optimality of the primal optimization problem. A WFC with 5 wind farms and each wind farm consisting of 20 WTs is used to validate the proposed DOVC scheme.

Index Terms—analytical target cascading (ATC), distributed, voltage control, wind farm cluster (WFC).

I. INTRODUCTION

Due to its excellent continuity and wind energy capture capability, offshore wind power has attracted increasing attention. With the rapid development of offshore wind farms (WFs), the cost effective collection and transmission technology for large-scale offshore wind power plants are becoming increasingly relevant [1], [2]. From a cost-benefit point of view, voltage-source-converter high-voltage-direct-current (VSC-HVDC) is preferred as a suitable transmission technology for large-scale offshore wind power plants [3], [4].

With the increasing penetration of wind power in power systems, large-scale offshore wind farm clusters (WFCs) with many sub-wind farms (WFs) are being connected to grids through the VSC-HVDC transmission system, creating challenges for system operators. Modern WFCs are required not only to provide ancillary services to comply with grid code requirements [5], [6], but also to enhance the operation performance of the WFC to adapt to the stochastic and intermittent nature of wind energy. In general, the wind turbines (WTs) inside the WFC are connected through long medium voltage (MV) feeders. The active power change has a significant impact on the voltage variation due to the low X/R ratio of the MV feeder [7]. The WTs located at the end of feeders have a risk of being tripped because their terminal voltages may exceed their threshold [8]. Therefore, the voltage control of a large-scale WF/WFC is an important task to guarantee secure operation while considering the economical operation of the WF/WFC [9], which has motivated a number of studies. In [10], a variable droop gain control scheme was proposed to mitigate voltage fluctuations at the point of connection (POC), which fully utilizes the voltage regulation capability of each WT converter. An MPC based centralized optimal voltage control scheme was proposed in [11], [12] for VSC-HVDC connected offshore WFs. The aims are to keep the voltage within a feasible range while taking the economical operation of the WFs into consideration. Ref. [13] proposed an approach for pilot-bus selection that considers both the response of the generators and the wind power fluctuation to maintain the system at the desired voltage profile. In [14], [15], centralized optimal reactive power dispatch strategies were proposed to minimize the total electrical losses of the WF, including not only the losses in cables and WT transformers but also the losses inside the wind energy generation systems. For distributed/hierarchical voltage control, a distributed control scheme based on the consensus protocol method was proposed in [16]. The aim is to improve the voltage profile and eliminate the centralized communication. In [17], an ADMM based hierarchical voltage control scheme was proposed. The central unit only needs to solve an optimization problem without constraints, while the constraints are solved with a simple augmented Lagrangian consensus protocol method.
lower-level control scheme in a hierarchical manner.

According to the above analysis, voltage control of WFs/WFCs can be classified into decentralized control, centralized control, and distributed/hierarchical control. Decentralized control mainly regulates voltages by using a droop controller or a proportional integral (PI) controller. Although the decentralized control can eliminate the requirement of the central unit, the optimal solution cannot be obtained. With the WFC growing both in number and size, considering the stochastic and intermittent nature of wind energy, centralized control might fail to obtain the optimal solution due to the high computation burden of the central unit. For the existing distributed/hierarchical control schemes, hierarchical control still requires a central unit to coordinate all the WTs inside the WFC. Distributed control mainly uses a two-tier control framework. From the WFC point of view, the solution is suboptimal.

To address the above issues, an analytical target cascading (ATC) method-based distributed optimal voltage control (DOVC) scheme is proposed for the VSC-HVDC connected large-scale WFC. The ATC method is an augmented Lagrangian relaxation based decomposition method [19], [20], which provides certain flexibilities in the coordination of problems and the selection of penalty functions [21], [22]. The aims of the proposed method are to minimize the voltage fluctuations of the POC, collector buses, and WT terminal buses while regulating the bus voltages close to the rated voltage, and smooth the reactive power outputs of the WTs inside the WFC. With the ATC method, the large-scale optimization problem is decomposed to several optimization subproblems and solved in the WFCVSC and sub-WF controllers in a distributed manner. Each sub-WF controllers only needs to coordinate the reactive power outputs of their corresponding WTs inside the sub-WF with local constraints and local measurements. The main contributions are summarized as follows:

1. A DOVC scheme is designed for the large-scale VSC-HVDC connected WFC. The large-scale strongly coupled sensitivity-based voltage optimization problem is decomposed. With the DOVC scheme, the requirement of a central controller is eliminated.

2. The WFCVSC and sub-WF controllers are operated in a distributed manner which considers $N-1$ principle to improve the reliability of the WFC. Compared to the conventional centralized optimal control, the optimization task is distributed to several controllers to reduce the computation burden of the WFC central controller.

3. The exchanged information only includes the shared control variables with their neighboring controllers, implying better protection of the information privacy.

The rest of the paper is organized as follows. Section II provides an overview of the DOVC scheme. Centralized optimal voltage control of the WFC is presented in Section III. The ATC-based distributed solution method for the VSC-HVDC connected WFC is described in Section IV. Finally, simulation results are presented and discussed in Section V, followed by the conclusions.

II. CONTROL ARCHITECTURE

A. Configuration of WFC

Fig. 1 shows the configuration of a VSC-HVDC connected large-scale offshore WFC. The WFC consists of several sub-WFs. Each sub-WF is connected to a 33 kV/155 kV transformer. The high voltage (HV) side of each transformer is connected to the POC through a 20 km HV transmission cable. The power generated by the WFC is transmitted to the onshore AC grids through a VSC-HVDC system. Each sub-WF consists of two feeders. Several WTs are connected by a feeder and displaced with a distance of 4 km.

B. Control Concept

Fig. 2 shows the control structure. The WFCVSC is equipped with a WFCVSC controller and each sub-WF is equipped with a sub-WF controller. The WFCVSC
controller generates an optimal voltage reference for the controlled bus while minimizing the voltage fluctuations of the POC and regulating the voltage of POC close to the rated value. The sub-WF controllers generate the optimal reactive power references for their corresponding WTs. The aims are to minimize the voltage fluctuations of the collector buses and WT terminal buses while regulating the bus voltages close to the rated value and smooth the reactive power outputs of the WTs inside the sub-WF. The WFCVSC controller and sub-WF controllers are operated in a distributed manner. The updated shared control variable vector is exchanged between the neighboring controllers, which are described in Section IV in detail. The optimization subproblem is solved in each sub-controller with local constraints and local measurements while guaranteeing the optimality of the primal optimization problem. A ring communication topology is used in the WFC, which considers the $N - 1$ principle.

III. VOLTAGE CONTROL OF LARGE-SCALE WFC

A. WFCVSC and WT model

The WFCVSC is used to regulate the voltage magnitude and frequency of the WFC AC grid while transmitting power from the AC side to the DC side. Based on [12], [17], the control diagram of the WFCVSC is shown in Fig. 3.

![Fig. 3. Control diagram of the WFCVSC.](image)

In Fig. 3, $V_c$ is the controlled AC bus voltage, $V_s$ is the terminal voltage of the WFCVSC, which can be seen as the slack bus of the WFC, and the phase reactor is represented by $R_c + L_c$. The operation of the WFCVSC is achieved by using a standard cascaded control structure, which includes an inner current control loop and an outer voltage control loop. Since the $d$-axis of the reference frame is oriented along the voltage-oriented vector of the WFC AC grid, $V_c \approx u_{ed}$, and $u_{eq} = 0$. Since the time constant of the outer voltage loop is much larger than that of the inner current loop, the inner current loop can be modeled as a first-order lag function by selecting suitable parameters of the PI controller. Thus, the WFCVSC control diagram is shown in Fig. 4.

![Fig. 4. Control diagram of the WFCVSC.](image)

Define $V_c^{ref}$ and $V_c$ as the voltage reference and measurement of the controlled AC bus, respectively, $K_p$ and $K_i$ as the proportional and integral gains of the PI controller of the outer voltage control loop, respectively, $i_{sd}$ as the $d$-axis current of the WFCVSC, $V_c^{ref}$ as the integral of the error between $V_c^{ref}$ and $V_c$, $C_f$ as the capacitance, $T_1$ as the time constant of the inner current loop, and $\Delta$ stands for the incremental operator. Thus, the incremental state-space model of the WFCVSC in the matrix form can be represented as,

$$
\Delta \dot{x}_v = A_v \Delta x_v + B_v \Delta u_v
$$

where

$$
\Delta x_v = \begin{bmatrix} \Delta V_c, \Delta V_c^{ref}, \Delta i_{sd} \end{bmatrix}^T, \Delta u_t = V_c^{ref}
$$

$$
A_v = \begin{bmatrix} 0 & -1/\tau_1 & 0 \\ 1 & 0 & -1/\tau_1 \\ K_p/T_1 & 0 & K_i/T_1 \end{bmatrix}, B_v = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}
$$

With power electronic converters, the WTs can regulate active power and reactive power independently. The WTs can provide reactive power support for the power system and regulate the bus voltages inside the WFC while operating in the maximal power point tracking (MPPT) mode. According to [23], the dynamic response of the WT converter can be modeled as a first-order transfer function by considering perfect control decoupling between the active and reactive power. Then, considering the time delay of the communication system and dynamic reactive power response of the WT, the dynamic behavior of WT reactive power output can be obtained as,

$$
\Delta Q_{wt} = 1/(1 + sT_Q) \Delta Q_{ref}^{wt}
$$

where $Q_{wt}$ is the reactive power output of the WT, $Q_{ref}^{wt}$ is the reactive power reference of the WT, and $T_Q$ is the time constant of the reactive power dynamic behavior of the WT. Then, we can obtain,

$$
\Delta Q_{wt} = -\frac{1}{T_Q} \Delta Q_{wt} + \frac{1}{T_Q} \Delta Q_{ref}^{wt}
$$

B. Objective Function

In the conventional optimal voltage control problem, the voltage sensitivity coefficients can be obtained from the updated Jacobian matrix, which has to be rebuilt and inverted for every change in the operational conditions of the network. Moreover, this method cannot be used to calculate the sensitivity coefficients with respect to the slack bus voltage. To avoid the above issues and improve the computational efficiency, the analytical voltage sensitivity calculation method in [24] is used to formulate the objective function. Once the voltage sensitivity with respect to active and reactive power injections of the
WTs and slack bus voltage are obtained, the first objective function can be formulated. The aims of the first objective function are to minimize the voltage fluctuations of the POC, collector buses, and WT terminal buses while regulating the bus voltages close to the rated voltage. Moreover, the power losses of the WFC network can also be reduced efficiently by regulating the voltage profile across the WFC network [25]. Then, the first objective function can be expressed as,

$$\text{Obj}_1 = \min \left[ \| V_{POC} - V_{\text{rated}} \|^2_{Q_c} + \sum_{i=1}^{\left| N_{ct} \right|} \| V_{ch,i} - V_{\text{rated}} \|^2_{Q_x} + \sum_{i=1}^{\left| N_{ct} \right|} \| V_{wt,i} - V_{\text{rated}} \|^2_{Q_w} \right]$$

where $N_C$ is the set of the collector buses, $N_{ct}$ is the set of the WT terminal buses, $Q_c$, $Q_x$ and $Q_w$ are the weighting factors for minimizing the fluctuations of the POC, collector buses, and WT terminal buses, respectively, $V_{POC}$ is the voltage of the POC, $V_{ch,i}$ is the voltage of the $i$th collector bus, $V_{wt,i}$ is the voltage of the $i$th WT terminal bus, and $V_{\text{rated}}$ is the rated voltage of the WFC. In each control period, $V_{POC}$, $V_{ch,i}$, and $V_{wt,i}$ can be described by,

$$V_{POC} = V_{POC}^0 + \frac{\partial V_{POC}}{\partial V_c} \Delta V_c + \frac{\partial V_{POC}}{\partial Q_w} \Delta Q_w$$

$$V_{ch,i} = V_{ch}^0 + \frac{\partial V_{ch,i}}{\partial V_c} \Delta V_c + \frac{\partial V_{ch,i}}{\partial Q_w} \Delta Q_w, \quad \forall i \in N_C$$

$$V_{wt,i} = V_{wt}^0 + \frac{\partial V_{wt,i}}{\partial V_c} \Delta V_c + \frac{\partial V_{wt,i}}{\partial Q_w} \Delta Q_w, \quad \forall i \in N_{ct}$$

with

$$\frac{\partial V_{POC}}{\partial Q_w} = \left[ \frac{\partial V_{POC}}{\partial Q_{w1}}, \frac{\partial V_{POC}}{\partial Q_{w2}}, \ldots, \frac{\partial V_{POC}}{\partial Q_{w|N_{ct}|}} \right]$$

$$\frac{\partial V_{ch,i}}{\partial Q_w} = \left[ \frac{\partial V_{ch,i}}{\partial Q_{w1}}, \frac{\partial V_{ch,i}}{\partial Q_{w2}}, \ldots, \frac{\partial V_{ch,i}}{\partial Q_{w|N_{ct}|}} \right]$$

$$\frac{\partial V_{wt,i}}{\partial Q_w} = \left[ \frac{\partial V_{wt,i}}{\partial Q_{w1}}, \frac{\partial V_{wt,i}}{\partial Q_{w2}}, \ldots, \frac{\partial V_{wt,i}}{\partial Q_{w|N_{ct}|}} \right]$$

$$Q_w = \left[ Q_{w1}, Q_{w2}, Q_{w3}, \ldots, Q_{w|N_{ct}|} \right]^T.$$

In order to facilitate the long-term stable operation of the generators and converters inside the WFC, the aim of the second control objective is to smooth the reactive power outputs of the WTs, which can be expressed as,

$$\text{Obj}_2 = \min \left[ \sum_{i=1}^{\left| N_{ct} \right|} \| Q_{wt,i} - Q_{w0,i} \|^2_{Q_R} \right]$$

where $Q_{w0,i}$ is the initial reactive power output of the $i$th WT in a control period, and $Q_R$ is the weighting factor. Combining (4) and (8), the total objective function of the WFC can be obtained as,

$$F_{WFC} = \text{Obj}_1 + \text{Obj}_2$$

where $Q_{w0,i}$ and $Q_{w0,i}$ are the minimal and maximal available reactive power of the $i$th WT, respectively. For the WFCVSC, the controlled voltage should be kept within a feasible range,

$$V_{c\min} \leq V_c \leq V_{c\max}.$$  (11)

where $V_{c\min}$ and $V_{c\max}$ are the minimal and maximal voltages of the controlled bus, respectively.

### IV. DISTRIBUTED SOLUTION METHOD BASED ON ATC

In the large-scale WFC voltage control, each bus voltage is affected by the reactive power injections from all WTs inside the WFC. In the conventional centralized-based voltage control methods, the WFC central controller must gather all the required measurements from all WTs and buses, as well as the whole voltage sensitivity coefficients to obtain the optimal solution for the WFC. With the development of large-scale wind power projects, the centralized optimization algorithms might not be suitable for the large systems. Therefore, an ATC based distributed solution method is proposed in this section to solve the large-scale strongly coupled optimization problem separately and restore the optimality and adaptability through an iterative message-passing algorithm.

#### A. Optimization Problem of the WFCVSC Controller

The large-scale optimization problem can be decomposed into several optimization subproblems and distributed to the WFCVSC controller and several sub-WF controllers. Since the WFCVSC controller is close to the POC, the optimization subproblem of the WFCVSC is to minimize the voltage fluctuations of the POC by optimizing the controlled bus voltage reference and reactive power output of the sub-WFs. For the WFCVSC controller, it is assumed that each sub-WF is equivalent to a current source, injecting active and reactive power to the collector bus. Thus, the equivalent configuration of the WFC for the WFCVSC controller is shown in Fig. 5.

![Fig. 5. Equivalent configuration of the WFC for the WFCVSC controller.](image)
\[ \Delta x_{wv} = \left[ \Delta x_v, \Delta Q_{wv,1}, \Delta Q_{wv,2}, \cdots, \Delta Q_{wv,|N_i|} \right]^T, \]
\[ \Delta u_{wv} = \left[ \Delta V_r, \Delta Q_{wv,1}^{ref(v)}, \Delta Q_{wv,2}^{ref(v)}, \cdots, \Delta Q_{wv,|N_i|}^{ref(v)} \right]^T, \]
\[ \Delta y_{wv} = \left[ \Delta V_s, \Delta Q_{wv,1}, \Delta Q_{wv,2}, \cdots, \Delta Q_{wv,|N_i|} \right]^T, \]
\[ A_{wv} = \begin{bmatrix} A_v & A_{wv} \end{bmatrix}, B_{wv} = \begin{bmatrix} B_v & B_{wv} \end{bmatrix}, \]
\[ C_{wv} = \begin{bmatrix} C_v & C_{cb,1} & \cdots & C_{cb,|N_i|} \\ 1 & \cdots & \cdots & 1_{|N_i|} \end{bmatrix}, \]

with
\[ A_{wv}^v = \text{diag} \left[ -\frac{1}{T_Q,1}, -\frac{1}{T_Q,2}, \cdots, -\frac{1}{T_Q,|N_i|} \right], \]
\[ B_{wv}^v = \text{diag} \left[ \frac{1}{T_Q,1}, \frac{1}{T_Q,2}, \cdots, \frac{1}{T_Q,|N_i|} \right], \]
\[ C_v = \left( \frac{\partial V_c}{\partial V_s} \right)^{-1} \left[ -1, 0, 0 \right], \]
\[ C_{cb,i} = \left( \frac{\partial V_c}{\partial Q_{cb,i}} \right)^{-1} \left( \frac{\partial V_c}{\partial Q_{cb,i}} \right), \forall i \in N_C, \]

where \( V_c^{ref(v)} \) is the controlled bus voltage reference optimized in the WFCVSC controller, \( Q_{wv}^{ref(v)} \) is the total reactive power reference of the \( i \)th sub-WF optimized in the WFCVSC controller, and \( \frac{\partial V_c}{\partial Q_{cb,i}} \) is the controlled bus voltage sensitivity with respect to the reactive power injection from the \( i \)th sub-WF collector bus. According to (12) and (13), the objective function of the WFCVSC is,
\[ f_{voc} = \min \left\| V_{Poc} - V_{\text{rated}} \right\|_{Q_v}^2 \]
\[ \text{s.t.} \quad (14) \]
\[ A_{wv} = \begin{bmatrix} A_v & A_{wv} \end{bmatrix}, B_{wv} = \begin{bmatrix} B_v & B_{wv} \end{bmatrix}, C_{wv} = \begin{bmatrix} C_v & C_{cb,1} & \cdots & C_{cb,|N_i|} \\ 1 & \cdots & \cdots & 1_{|N_i|} \end{bmatrix}, \]
\[ \Delta x_i = A_i \Delta x_i + B_i \Delta u_i \]
\[ \Delta y_i = C_i \Delta x_i \]

where
\[ \Delta x_i = \left[ \Delta x_v, \Delta Q_{wv,1}, \cdots, \Delta Q_{wv,|N_i|} \right]^T, \]
\[ \Delta u_i = \left[ \Delta V_r, \Delta Q_{wv,1}^{ref(v)}, \cdots, \Delta Q_{wv,|N_i|}^{ref(v)} \right]^T, \]
\[ \Delta y_i = \left[ \Delta V_s, \Delta Q_{wv,1}, \cdots, \Delta Q_{wv,|N_i|} \right]^T, \]
\[ A_i = \begin{bmatrix} A_v & A_{wv} \end{bmatrix}, B_i = \begin{bmatrix} B_v & B_{wv} \end{bmatrix}, C_i = \begin{bmatrix} C_v & C_{cb,1} & \cdots & C_{cb,|N_i|} \end{bmatrix}, \]
\[ C_1 = \begin{bmatrix} I_{3 \times 3}^1, \cdots, I_{3 \times 3}^{N_i} \end{bmatrix}, C_2 = \begin{bmatrix} I_{2 \times 2}^\alpha \end{bmatrix}, \]
\[ C_3 = \begin{bmatrix} I_{3 \times 3}^\alpha, \alpha = |N_i| \right\|_{Q_v}^2 \]

\[ A_{wv} = \text{diag} \left[ -\frac{1}{T_Q,1}, -\frac{1}{T_Q,2}, \cdots, -\frac{1}{T_Q,|N_i|} \right], \]
\[ B_{wv} = \text{diag} \left[ \frac{1}{T_Q,1}, \frac{1}{T_Q,2}, \cdots, \frac{1}{T_Q,|N_i|} \right], \]
\[ C_{wv} = \begin{bmatrix} C_v & C_{cb,1} & \cdots & C_{cb,|N_i|} \\ 1 & \cdots & \cdots & 1_{|N_i|} \end{bmatrix}, \]

\[ \Delta x_i = \left[ \Delta x_v, \Delta Q_{wv,1}, \cdots, \Delta Q_{wv,|N_i|} \right]^T, \]
\[ \Delta u_i = \left[ \Delta V_r, \Delta Q_{wv,1}^{ref(v)}, \cdots, \Delta Q_{wv,|N_i|}^{ref(v)} \right]^T, \]
\[ A_i = \begin{bmatrix} A_v & A_{wv} \end{bmatrix}, B_i = \begin{bmatrix} B_v & B_{wv} \end{bmatrix}, C_i = \begin{bmatrix} C_v & C_{cb,1} & \cdots & C_{cb,|N_i|} \end{bmatrix}, \]
\[ C_1 = \begin{bmatrix} I_{3 \times 3}^1, \cdots, I_{3 \times 3}^{N_i} \end{bmatrix}, C_2 = \begin{bmatrix} I_{2 \times 2}^\alpha \end{bmatrix}, \]
\[ C_3 = \begin{bmatrix} I_{3 \times 3}^\alpha, \alpha = |N_i| \right\|_{Q_v}^2 \]

where \( V_c^{ref(v)} \) is the controlled bus voltage reference optimized in the \( j \)th sub-WF controller, \( Q_{wv,j}^{ref(v)} \) is the total reactive power reference of the \( j \)th sub-WF optimized in the \( j \)th sub-WF controller, and \( Q_{wv,i}^{ref(v)} \) is the reactive power reference of the

---

**B. Optimization Problem of the Sub-WF Controller**

The aims of each sub-WF controller are to minimize the voltage fluctuations of the corresponding collector bus and WT terminal buses and smooth the reactive power outputs of the WTIs inside the sub-WF. The equivalent configuration of the WFC for the \( i \)th sub-WF controller is shown in Fig. 6.
is the shared variable vector optimized in the \( i \)th sub-WF controller. The objective function of the \( i \)th sub-WF controller is,

\[
 f_i = \min \| V_{ch,i} - V_{rated} \|^2_{Q_{c,i}} + \sum_{i=1}^{\left| N_{wt,i} \right|} \| V_{wt,i} - V_{rated} \|^2_{Q_{w,wt,i}} \\
 + \sum_{i=1}^{\left| N_{wt,i} \right|} \| Q_{wt,i} - Q^0_{wt,i} \|^2_{Q_w} \tag{17}
\]

s.t. \( Q_{wt,i}^\min \leq Q_{wt,i} \leq Q_{wt,i}^\max, \forall i \in N^{(i)}_{wt} \).

The line constraints also can be considered into the optimization sub-problem. Define \( A^{(i)}_{L} \) as the set of cables of the \( i \)th sub-WF and \( A^{(i)}_{B} \) as the set of buses of the \( i \)th sub-WF. Then we can obtain,

\[
 V_{wt} = V_{wt}^0 + S\Delta y_i \tag{18}
\]

where \( V_{wt} = [V_1, V_2, V_3, V_4, \ldots V_{A^{(i)}_L}]^T, V_{wt}^0 = [V_{1,wt}, V_{2,wt}, V_{3,wt}, V_{4,wt}, \ldots V_{A^{(i)}_B}]^T, \) and \( S \) is the voltage sensitivity coefficient matrix related to \( \Delta y_i \). Then, we can get,

\[
 I_{wt} = G_{wt,i}V_{wt} \tag{19}
\]

where \( I_{wt} = [I_1, I_2, I_3, I_4, \ldots I_{A^{(i)}_B}]^T, \) and \( G_{wt,i} \) is the real part of the admittance matrix of the \( i \)th sub-WF. Thus, the line constraints are obtained as,

\[
 I_{i,\min} \leq I_i \leq I_{i,\max}, \forall i \in A^{(i)}_L. \tag{20}
\]

where \( I_{i,\min} \) and \( I_{i,\max} \) are the minimal and maximal current limits of the \( i \)th cable inside the sub-WF, respectively.

C. ATC-Based Solution Method

According to Sections III. A and B, the sum of the objective function of the sub-controllers of the WFC controller is equal to the primal objective function of the WFC controller,

\[
 f_{vsc} + \sum_{i=1}^{\left| N_{i} \right|} f_i = F_{WFC} \tag{21}
\]

Each sub-controller has local control variables and shared control variables with other sub-controllers. Based on the ATC method, the sub-controllers solve the optimization subproblems with their corresponding constraints in parallel while exchanging the value of the shared variables with their immediate neighbors. Once the value of the shared variables converges, the optimal solution of the primal optimization problem is obtained. Denote \( \mathcal{V} \) as the set of the WFCVSC, and the optimization subproblem of the WFCVSC can be formulated in an augmented Lagrangian form,

\[
 \min f_{vsc} + \sum_{j \in \mathcal{N}_{i}} \tau_{v,j}(\alpha_i \xi - \xi_j) + \| \beta_{v,j} \circ ((\xi - \xi_j)) \|^2_2 \tag{22}
\]

where \( \xi_v = [\Delta V^{\text{ref}(v)}, \Delta Q^{\text{ref}(v)}_{wt,1}, \Delta Q^{\text{ref}(v)}_{wt,2}, \ldots, \Delta Q^{\text{ref}(v)}_{wt,|\mathcal{N}_{w}|}]^T \) is the shared variable vector optimized in the WFCVSC controller, \( \xi_j = [\Delta V^{\text{ref}(j)}, \Delta Q^{\text{ref}(j)}_{wt,1}, \Delta Q^{\text{ref}(j)}_{wt,2}, \ldots, \Delta Q^{\text{ref}(j)}_{wt,|\mathcal{N}_{w}|}]^T \) is the shared variable vector optimized in the \( j \)th controller, and \( \tau_{v,j} \) is the adjacancy coefficient. If there exists a directed communication path from the \( i \)th controller to the \( j \)th controller, \( \tau_{ij} = 1 \), otherwise \( \tau_{ij} = 0 \). \( \alpha_{ij} \) and \( \beta_{ij} \) are multipliers associated with the linear and quadratic terms, respectively, which are updated during the iterative solving process, and the operator \( \circ \) is the Hadamard product. Similarly, the optimization subproblem of the \( i \)th sub-WF controller can be formulated in an augmented Lagrangian form,

\[
 \min f_i + \sum_{j \in \mathcal{N}_{i} \cup \mathcal{V}} \tau_{i,j}(\alpha_i \xi - \xi_j) + \| \beta_{i,j} \circ ((\xi - \xi_j)) \|^2_2 \tag{23}
\]

To be noticed, the shared variable \( \Delta Q^{\text{ref}(i)}_{wt,i} \) in \( \xi_i \) is expressed as,

\[
 \Delta Q^{\text{ref}(i)}_{wt,i} = \sum_{i=1}^{\left| N_{wt,i} \right|} \Delta Q_{wt,i} \tag{24}
\]

With the ATC method, each sub-controller solves the local optimization problem with local measurements in a parallel manner. Taking the \( i \)th sub-WF as an example, the solution procedures are illustrated as follows.

Step 1: Choose initial values for \( \alpha_{ij} \) and \( \beta_{ij} \), and set the initial values of the iteration number \( k \), \( \xi_{i-1} \) and \( \xi_{i+1} \) to zero.

Step 2: Set \( k = k + 1 \). Update the local control variables and shared control variables by using (22) with \( \xi^{[k]}_{i-1} \) and \( \xi^{[k]}_{i+1} \) from the \( k \)th iteration,

\[
 \Delta u^{[k+1]}_i = \arg \min_{\Delta u_{i}} \sum_{j \in \mathcal{N}_{i} \cup \mathcal{V}} \tau_{i,j}(\alpha^{[k]}_{i,j} \xi_i - \xi_j) \\\n + \| \beta^{[k]}_{i,j} \circ ((\xi_i - \xi_j)) \|^2_2 \tag{25}
\]

with the constraints,

\[
 Q_{wt,i}^{\min} \leq Q_{wt,i} \leq Q_{wt,i}^{\max}, \forall i \in N^{(i)}_{wt} \tag{26}
\]

Step 3: Check the following necessary-consistency and sufficient conditions,

The necessary-consistency condition is,

\[
 \xi^{[k+1]}_i - \xi^{[k]}_i \leq \varepsilon_1 \tag{27}
\]

The sufficient condition is,

\[
 \frac{|f_i(\Delta u^{[k+1]}_i) - f_i(\Delta u^{[k]}_i)|}{f_i(\Delta u^{[k]}_i)} \leq \varepsilon_2 \tag{28}
\]

where \( \varepsilon_1 \) and \( \varepsilon_2 \) are coefficients. If the conditions are satisfied, set a variable as \( \eta^{[k]}_{i+1} = 1 \), otherwise, \( \eta^{[k]}_{i+1} = 0 \). Check the condition,

\[
 \frac{\xi^{[k]}_{i+1} + \xi^{[k]}_{i} + \eta^{[k]}_{i+1}}{3} = 1 \tag{29}
\]

where \( \eta^{[k]}_{i-1} \) and \( \eta^{[k]}_{i+1} \) are sent from neighboring controllers with \( \xi^{[k]}_{i-1} \) and \( \xi^{[k]}_{i+1} \) at the same time. If the condition is satisfied, set \( \eta^{[k+1]}_{i+1} = 1 \), otherwise, \( \eta^{[k+1]}_{i+1} = 0 \). Go to step 4.
The collector bus voltage of WF1 is shown in Fig. 8. The voltage with the PI control scheme fluctuates between 1.001 p.u. and 1.003 p.u.. From 0 to 250 s, the voltage with the TT-OVC scheme fluctuates between 0.9996 p.u. to 1.0008 p.u.. From 250 to 400 s, the voltage fluctuates between 0.999 p.u. and 1 p.u. The voltage performances with the DOVC scheme are the same as those with the COVC scheme and are much better than those with the PI control scheme and TT-OVC scheme. The voltage fluctuates between 0.9998 p.u. and 1.0002 p.u. during the whole control period. Fig. 9 shows the voltage performances of the POC. The voltage of the POC is the only control objective of the PI control scheme. The POC voltage with the PI control scheme is controlled around 1 p.u., and the voltage of the POC with the remaining control schemes is controlled below 1 p.u.. The voltage fluctuations with the PI control scheme are larger than those with the other control schemes.

V. CASE STUDY

A. Test System

In this section, the WFC illustrated in Fig. 1 is used for the case study. The WFC consists of 5 sub-WFs and each sub-WF consists of 20 × 5 MW WTs. The total simulation time is set as 600 s. The control action is carried out every 2 s. Each sub-WF is operated in the MPPT mode under different wind conditions. To examine the performance of the proposed control method, the simulation results are compared with those based on a centralized optimal voltage control scheme (COVC), the two-tier optimal voltage control (TT-OVC) scheme in [18], and a conventional PI control scheme. In the PI control scheme, a PI controller is used to generate a total reactive power reference to maintain the voltage of the POC close to the rated value. The reactive power references of the WTs are dispatched to the WTs based on a proportional distribution control scheme. The test system parameters of the WFC are shown in Table I.

B. Control Performance

Fig. 7 shows the total available active power of the WFC. From 0 to 250 s, the available active power fluctuates around 360 MW. From 250 to 400 s, the available active power increases to approximately 460 MW gradually. From 400 to 600 s, the available active power decreases to approximately 380 MW.

---

**TABLE I**

<table>
<thead>
<tr>
<th>TEST SYSTEM PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>33kV Cable</td>
</tr>
<tr>
<td>155kV Cable</td>
</tr>
<tr>
<td>0.9kV/33kV Transformer</td>
</tr>
<tr>
<td>33kV/155kV Transformer</td>
</tr>
<tr>
<td>155kV/380kV Transformer</td>
</tr>
<tr>
<td>WFC rated capacity</td>
</tr>
<tr>
<td>Wind farm rated capacity</td>
</tr>
<tr>
<td>WT rated capacity</td>
</tr>
</tbody>
</table>

Fig. 10 shows the voltage performance of WT20, which is located at the end of the feeder of sub-WF1. The voltage performances with the TT-OVC scheme are much better than those with the PI control scheme. The voltage performances with the DOVC and COVC schemes are better than those with the TT-OVC scheme. The DOVC and COVC schemes can effectively control the voltage of the WT20 terminal bus within 1.002-1.005 p.u.. The zoomed part shows that the performances with the DOVC scheme are the same as those with the COVC scheme. The DOVC scheme guarantees the optimality of the primal centralized optimization problem. Fig.
Fig. 8. Collector bus voltage of WF1.

Fig. 9. POC voltage of the WFC.

Fig. 10. Terminal bus voltage of WT20.

Fig. 11. Overall voltage control performance.

Fig. 12. Reactive power output of WT20.

Fig. 13. Active power loss of the WFC.

The reactive power output of WT20 is shown in Fig. 12. The reactive power outputs of the four control schemes are different. The reactive power output with the PI control scheme fluctuates from -0.6 to -0.4 MVar. The reactive power output with the TT-OVC scheme fluctuates from -1.4 to -1 MVar. The reactive power outputs with the DOVC and COVC schemes are the same, which fluctuates from -1.4 to -1.2 MVar. The power losses of the WFC are shown in Fig. 13. The performances with the DOVC and COVC schemes are slightly better than those with the TT-OVC scheme and PI control scheme.

Fig. 14. Convergence performance.

The reactive power outputs with the DOVC and COVC schemes are the same, which fluctuates from -1.4 to -1.2 MVar. The power losses of the WFC are shown in Fig. 13. The performances with the DOVC and COVC schemes are slightly better than those with the TT-OVC scheme and PI control scheme.

The reactive power output of WT20 is shown in Fig. 12. The reactive power outputs of the four control schemes are different. The reactive power output with the PI control scheme fluctuates from -0.6 to -0.4 MVar. The reactive power output with the TT-OVC scheme fluctuates from -1.4 to -1 MVar. The reactive power outputs with the DOVC and COVC schemes are the same, which fluctuates from -1.4 to -1.2 MVar. The power losses of the WFC are shown in Fig. 13. The performances with the DOVC and COVC schemes are slightly better than those with the TT-OVC scheme and PI control scheme.

The convergence performance with the ATC method is shown in Fig. 14. The active power references of the sub-WF3 and sub-WF5 optimized in the each sub-controllers are selected as the representative shared control variables to illustrate the performance of the convergence. The shared variables converge and remain constant at the same value after approximately 35 iterations, implying excellent convergence performance.

Fig. 15 shows the current of the representative cable. The current of the cable between WT12 and WT13 are selected as the representative cable. The current of the cable with the PI control scheme reaches to approximately 0.02 p.u., which is much higher than those with the DOVC, COVC, and TT-OVC control schemes.
VI. CONCLUSION

In this paper, a DOVC scheme based on the ATC is proposed for the VSC-HVDC connected large-scale WFC. A voltage-sensitivity-based optimization problem is formulated to minimize the voltage fluctuations of the POC, collector buses, and WT terminal buses while regulating the bus voltages close to their rated voltage and smooth the reactive power outputs of the WTs inside the WFC. A distributed solution method based on the ATC is proposed to solve the optimization problem in a parallel manner, which improves the reliability and scalability while protecting the information privacy of the WFC. As verified by case studies, the DOVC scheme can efficiently reduce the fluctuations of the POC, collector buses, and WT terminal buses and smooth the reactive power outputs of the WTs inside the WFC. The ATC-based distributed method achieves the same performance as a centralized control method and guarantees the optimality of the primal optimization problem. Due to its advantages with regard to scalability, flexibility, reliability and information privacy, the DOVC scheme is preferred for voltage control of the large-scale VSC-HVDC connected WFC.

REFERENCES

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