Fundamental Properties of Mie Resonances in Water Cylinders – TM and TE Case Studies

Jacobsen, Rasmus Elkjær; Arslanagic, Samel; Laurynenka, Andrei

Published in:
Proceedings of URSI EM Theory Symposium

Publication date:
2019

Document Version
Peer reviewed version

Link back to DTU Orbit

Citation (APA):
**Fundamental Properties of Mie Resonances in Water Cylinders – TM and TE Case Studies**

Rasmus E. Jacobsen* (1), Samel Arslanagić (2), and Andrei V. Lavrinenko (3)

(1), (3) Department of Photonics Engineering, Technical University of Denmark, Kongens Lyngby, 2800, Denmark, e-mail: (1) raja@fotonik.dtu.dk, (3) alav@fotonik.dtu.dk,

(2) Department of Electrical Engineering, Technical University of Denmark, Kongens Lyngby, 2800, Denmark, e-mail: sar@elektro.dtu.dk

**Abstract**

All-dielectric metamaterials have recently attracted great attention in the artificial material design. They consist of high permittivity inclusions which enable resonances in sub-wavelength structures. In contrast to optics, several high permittivity materials exist in the microwave range. Among these we find one of the most abundant materials on earth: water. In its liquid state, it offers great tunable dynamic properties that can be used in material design. To this end, we presently examine analytically the so-called Mie resonances in water cylinders. Particular attention is devoted to the ability of such cylinders to support electric and magnetic dipole modes, and how these behave with temperature and frequency. Subsequently, we demonstrate that directive forward and backward patterns can be achieved by specific water cylinders with balanced electric and magnetic dipole responses. The results of this work may be used directly in or as a guideline for metamaterial design as well as for simple, cheap and rather directive antennas.

1. Introduction

The possibility to obtain a variety of permittivity and permeability responses using low-loss high permittivity materials, have made all-dielectric metamaterials (MMs) a favorable choice in artificial material design [1-2]. They enable sub-wavelength Mie resonances in high permittivity inclusions employed in a low index matrix. Recently, water has been proposed as an alternative material for inclusions of all-dielectric MMs and metasurfaces (MSs) at microwave frequencies. With its abundance, bio-friendliness, inexpensiveness and temperature dependent high permittivity, water offers many advantageous properties [3-7]. It was shown that the response of water-based MMs can be tuned/reconfigured thermally by a simple temperature control of the material [3, 9], chemically [10] and mechanically by rotating partially filled containers or simply by reshaping its volume [3, 11]. In [10], the cylindrical shape made it easy to contaminate the water. Besides the high permittivity, water is quite lossy in a wide frequency range rendering it of great use for broadband MS absorbers [12-13]. A MS composed of ‘rod-like’ water-elements in a foam material was shown to have high reflection and transmission modulation capabilities through a simple mechanical rotation of the surface [7]. The modulation was improved significantly by stacking several MSs in a multilayer array.

Typically, 2-D and 3-D arrays of water elements are studied numerically. In this work, we investigate Mie resonances in isolated water cylinders illuminated by plane waves of different polarizations (both TM and TE cases will be examined). The canonical problem is treated analytically using the cylindrical wave expansion formulation, which enables us to study fundamental properties and various derived effects of water cylinders. The temperature and frequency variation of the induced dipole resonances is evaluated, and throughout the paper, we discuss the potential of such water cylinders to support both isolated (for MM design) and balanced (for directive antenna design) dipole resonances. We mainly focus on the low frequencies where the losses in water have minimal influence on the resonances, and we show that the two polarizations induce completely different electric dipoles, while their magnetic dipoles have identical magnitudes, but different orientations. The magnetic dipole resonances can be improved in both intensity and isolation by increasing the temperature. Moreover, we show that Kerker’s conditions can be satisfied for only the TE polarization at non-resonant frequencies, which results in mainly forward/backward scattering. Throughout the work, the time-factor exp (jωt), with ω being the angular frequency and t being the time, is assumed and suppressed.

2. Configuration and analytical expressions

We consider a linearly polarized plane wave with the electric (magnetic) field \( \mathbf{E}_0^x \) (\( \mathbf{H}_0^x \)) incident on a water cylinder of radius \( r_c \) situated in a free-space medium, see Figure 1(a). Both TM\( ^x \) and TE\( ^x \) polarization states of the incident wave are investigated. A Cartesian coordinate system \((x, y, z)\), and an associated cylindrical coordinate system \((r, \phi, z)\), is placed with their \(z\)-axes coincident with the cylinder axis. The cylinder gives rise to the scattered \((\mathbf{E}_s^x, \mathbf{H}_s^x)\) and internal \((\mathbf{E}_0^x, \mathbf{H}_0^x)\) fields which are easily found through the application of boundary conditions at the cylinder surface \( r = r_c \). Their amplitude coefficients are given by [14, Sec. 8.4].
The current density inside the water is

\[ \mathbf{j_n} = \frac{J_n(k_0 r_n)J_n'(k_0 r_n)I_n(k_0 r_n) - \alpha_p J_n'(k_0 r_n)J_n(k_0 r_n)}{\alpha_p J_n'(k_0 r_n)H_n^2(k_0 r_n) - J_n(k_0 r_n)H_n^2(k_0 r_n)} \]  

and

\[ \mathbf{j_b} = \frac{J_n(k_0 r_b)H_n^2(k_0 r_b) - J_n(k_0 r_b)H_n^2(k_0 r_b)}{\alpha_p J_n'(k_0 r_n)H_n^2(k_0 r_n) - J_n(k_0 r_n)H_n^2(k_0 r_n)} \]  

respectively, where \( P = \text{TM} \) or TE giving \( \alpha_{TM} = \sqrt{\varepsilon_{r,w}} \), or \( \alpha_{TE} = \frac{1}{\sqrt{\varepsilon_{r,w}}} \). \( k_n \) and \( k_b \) are the wavenumbers in water and free-space, respectively, while \( J_n \) and \( H_n^2 \) are the \( n \)th order Bessel function of the first kind and Hankel function of the second kind, respectively. The prime denotes the derivative with respect to the entire argument.

With the fields at hand, absorption \( Q^p_{\text{abs}} \), extinction \( Q^p_{\text{ext}} \), and scattering \( Q^p_{\text{scat}} \) efficiencies are determined; these are used to describe the response of the water cylinders. In mathematical terms, we have [14, Sec. 3.4]

\[ Q^p_{\text{abs}} = Q^p_{\text{ext}} - Q^p_{\text{scat}} \]  

where

\[ Q^p_{\text{scat}} = \frac{2}{k_0 r_c} \left( |a_0|^2 + 2 \sum_{n=1}^{\infty} |a_n|^2 \right) \]  

\[ Q^p_{\text{ext}} = \frac{2}{k_0 r_c} \Re \left( a_0 + 2 \sum_{n=1}^{\infty} a_n \right) \]  

The induced electric \( \mathbf{p}_p \) [C] and magnetic \( \mathbf{m}_p \) [A-m] dipole moments per unit length can be determined as [15]

\[ \mathbf{p}_p = \frac{1}{j \omega} \int \mathbf{j}_p ds' \]  

\[ \mathbf{m}_p = \frac{1}{2} \int \mathbf{r}' \times \mathbf{j}_p \]  

where \( \mathbf{j}_p = j \omega \mathbf{E}_0 (\varepsilon_{r,w} - 1) \mathbf{E}_p \) being the polarization current density inside the water cylinder, \( S \) the cross section of the cylinder, \( ds' \) the differential surface element and \( \mathbf{r}' \) the position vector. Performing the integrals, one gets for the two polarization cases the following

\[ \mathbf{p}^\text{TM} = \bar{\mathbf{e}}^\text{TM} E_0^2 r^2 \varepsilon_0 (\varepsilon_{r,w} - 1) E_0 b_0^\text{TM} f_1(k_w r_c) k_w r_c \]  

\[ \mathbf{m}^\text{TM} = \mathbf{k}^\text{TM} \times \bar{\mathbf{e}}^\text{TM} E_0^3 r^3 \varepsilon_0 (\varepsilon_{r,w} - 1) E_0 b_0^\text{TM} f_2(k_w r_c) k_w r_c \]  

In the temperature range from 0 to 100 °C and under normal pressure, water is a liquid. The temperature dependent dielectric permittivity is described by the Debye formula [8]

\[ \varepsilon_{r,w}(\omega, T) = \varepsilon_{r,w}(T) - j \varepsilon_{r,w}(T) \frac{\varepsilon_J(T) - \varepsilon_{\infty}(T)}{1 - j \omega \tau(T)} \]  

with \( T \) [°C] being the temperature of water. \( \varepsilon_{\infty}(T) \) and \( \varepsilon_J(T) \) are the optical and static permittivities, respectively, while \( \tau(T) \) is the rotational relaxation time. The real \( (\varepsilon'_{r,w}) \) and imaginary \( (\varepsilon''_{r,w}) \) parts of (11) are shown in Figure 1(b).

3. Resonant water cylinder

We investigate resonances in a water cylinder with a radius of 10 mm illuminated by z- (TM²) and y-polarized (TE²) incident plane waves of amplitudes \( E_0 \) [V/m]; both propagate in the +x-direction. At a temperature of 20 °C, the absorption and scattering efficiency spectra are shown in Figure 2(a). For validation, the scattering efficiencies calculated in COMSOL Multiphysics is included. For TM² incidence, the resonance induced at 0.38 GHz is the electric dipole. The magnetic dipole resonance is present for both polarizations at 1.26 GHz and, as shown in Figure 2(b), they have identical magnitudes. Consequently, it follows that \( b_0^\text{TM} = \sqrt{\varepsilon_{r,w} b_1^\text{TM}} \) in (8) and (10), and that the magnitude of the induced magnetic dipole is polarization insensitive. This can be used, e.g., separate the magnetic and electric dipole contributions to the efficiencies for TM² polarization. For TE² at 2.02 GHz, there is an electric dipole resonance of a very low magnitude; this is partly due to the presence of other resonances, but also due to water losses.

There is an interesting point around 2 GHz for TM², where the scattering efficiency has a minimum, while there is a peak in absorption. The minimum is caused by the second harmonic (peak) of the \( n = 0 \) mode, where \( \mathbf{p}_{\text{TM}} \) has a minimum. The increase in absorption comes from the electric quadrupole resonance (\( n = 2 \)).

The total electric field normalized by the incident field is shown in Figure 2(c) and (d) in the \( xy \)-plane at 1.26 GHz (magnetic dipole) for TM² and TE², respectively. The field profiles are very different even though they exhibit magnetic dipoles of equal magnitudes, which is due to the difference in their orientations and the presence of other resonances for TM² as shown in Figure 3. This is also why the scattering and absorption efficiencies are different. The scattering efficiency is highest for \( \mathbf{p}_{\text{TM}} \), while the
due to the change in water’s permittivity: at low
causing a lower scattering efficiency, while opposite
intensity of

\[ T \] to the second (first resonance (internal
dipoles

\[ \text{Isolation} \] it

losses are too high, the induced dipoles will be affected by
high internal field intensities

0

more isolated making it more suitable for
absorption is

frequencies with
coefficients at the magnetic and electric dipole resonance

Figure 3. Isolation of resonances shown by internal
coefficients at the magnetic and electric dipole resonance
frequencies with \( T = 20 \) °C in (a) TM² and (b) TE².

Isolation of dipole resonances plays an important role in
MM design in particular. The isolation of the induced
dipoles is shown in Figure 3 by the magnitude of the
internal coefficients for their first and second modes. The
first resonance (\( n = 0 \)) is clearly better isolated compared to the second (\( n = 1 \)) for both polarizations.

The temperature variation is investigated for the scattering
efficiency as well as for \( p_{TM} \) and \( m_{TM} \) (remember \( m_{TM} \) and \( m_{TE} \) are identical except for their orientations). The
intensity of \( p_{TM} \) decreases with increasing temperature
causing a lower scattering efficiency, while opposite
behavior is observed for the other induced dipoles. This is
due to the change in water’s permittivity: at low

absorption is most profound for \( m_{TM} \). However, \( m_{TE} \) is
more isolated making it more suitable for e.g., MMs. Absorbed power in a dielectric is given by

\[ P_{\text{abs}} = 0.5 \varepsilon_0 c \varepsilon''_w \int \left| E_{\text{abs}}' \right|^2 dv \; \] ; thus the high absorption is related to
high internal field intensities and small losses. If the
losses are too high, the induced dipoles will be affected by
it. This absorbed energy results in the heating of water.

The dipole moments shown in Figure 2(b) and 3(c)-(d) are normalized as

\[ \frac{m}{(\varepsilon_r^2 \varepsilon_0 E_0)} \] and \[ \frac{p}{(\varepsilon_r^2 \varepsilon_0 E_0)} \], where \( c \) is the speed of
light, since the difference in their units is [m/s]. In the

case of \( m'/p' = c \) (where \( p' = \hat{e}_x p' \) and \( m = \hat{k}_x \times \hat{e}_y m' \)), the scattering will be maximized in the forward
direction, while if \( m'/p' = -c \), it will be in the backward
direction. The frequencies, where these conditions are met,
are marked by the black circles in Figure 2(b) for TE². In Figure 5, the normalized differential scattering width [14, Sec. 3.4] is shown at these frequencies as well as at the
magnetic dipole resonance frequency. The orientations of the
dipoles are drawn for each frequency. Such effects are interesting for MMs, MSs and directive antennas designs, since only forward (backward) scattering corresponds to
zero reflection (transmission). TM² polarization does not exhibit any cases with the required conditions due to the
strong electric dipole. For enhanced scattering in one
direction, one could consider tilting the cylinder to enhance
the electric dipole moment. A different shape exhibiting
greater induced dipole moments (or higher order modes)
could also be pursued.

Several ways to maximize scattering in one direction have been proposed, e.g., one considers pairing dipoles and
quadrupoles [16]. We next show that it is possible to attain
directed scattering by satisfying the so-called Kerker’s
conditions in the examined water cylinders. The dipole
moments shown in Figure 2(b) and 3(c)-(d) are normalized as

\[ \frac{m}{(\varepsilon_r^2 \varepsilon_0 E_0)} \] and \[ \frac{p}{(\varepsilon_r^2 \varepsilon_0 E_0)} \], where \( c \) is the speed of
light, since the difference in their units is [m/s]. In the

case of \( m'/p' = c \) (where \( p' = \hat{e}_x p' \) and \( m = \hat{k}_x \times \hat{e}_y m' \)), the scattering will be maximized in the forward
direction, while if \( m'/p' = -c \), it will be in the backward
direction. The frequencies, where these conditions are met,
are marked by the black circles in Figure 2(b) for TE². In Figure 5, the normalized differential scattering width [14, Sec. 3.4] is shown at these frequencies as well as at the
magnetic dipole resonance frequency. The orientations of the
dipoles are drawn for each frequency. Such effects are interesting for MMs, MSs and directive antennas designs, since only forward (backward) scattering corresponds to
zero reflection (transmission). TM² polarization does not exhibit any cases with the required conditions due to the
strong electric dipole. For enhanced scattering in one
direction, one could consider tilting the cylinder to enhance
the electric dipole moment. A different shape exhibiting
greater induced dipole moments (or higher order modes)
could also be pursued.

Figure 4. The temperature and frequency variation of the
scattering efficiency for (a) TM² and (b) TE² cases, as
well as the dipole moments (c) \( p_{TM} \) and (d) \( m_{TE} \). In all
cases, the cylinder radius is set to \( r_c = 10 \) mm.

frequencies \( \varepsilon''_w \) decreases only slightly with increasing

\[ \text{tems} \] temperature, while \( \varepsilon''_w \) decreases significantly causing the
blue-shift. At even higher frequencies, an increase in
temperature will have a more profound effect on the
efficiencies. In general, the dipole resonance frequencies
are blue-shifted by approximately 25 % with an increase of
temperature from 0 to 100 °C. The losses in water must be
minimized in order to achieve pronounced and isolated
resonances if water inclusions are to be used in MMs and
MSs. Generally, this is achieved at low frequencies and/or
high temperatures.
4. Conclusions

The electromagnetic response of a water cylinder resonating around 1 GHz was investigated for both TM and TE polarization states of the incident wave. The analytical expressions for the scattering and absorption efficiencies as well as the magnetic and electric dipole moments were outlined in Section 2. The two polarization states result in two different electric dipoles, while their magnetic dipole moments are identical except for their orientations. The TM case lead to a profound electric dipole at very low frequencies, while for the TE case, the magnetic dipole was the most pronounced resonance. Both of these were the only well-isolated resonance, while all other modes were influenced by other resonances. Increasing the temperature resulted in more profound and isolated magnetic resonances, which are desirable features for prospective MM and MS designs. For TM incidence, the temperature increase had a negative influence on the electric dipole resonance at very low frequencies. At last, we showed that balanced magnetic and electric dipole moments, (Kerker’s conditions), were achievable for the TE incidence, while the electric dipole was too intense in the TM case. These conditions resulted in primarily forward or backward scattering. These effects are of interest in various areas of artificial materials and electrically small and directive antennas.

5. References


