Data-Driven Security-Constrained OPF

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Abstract—In this paper we unify electricity market operations with power system security considerations. Using data-driven techniques, we address both small signal stability and steady-state security, derive tractable decision rules in the form of line flow limits, and incorporate the resulting constraints in market clearing algorithms. Our goal is to minimize redispatching actions, and instead allow the market to determine the most cost-efficient dispatch while considering all security constraints. To maintain tractability of our approach we perform our security assessment offline, examining large datasets, both from measurements and simulations, in order to determine stable and unstable operating regions. With the help of decision trees, we transform this information to linear decision rules for line flow constraints. We propose conditional line transfer limits, which can accurately capture security considerations, while being less conservative than current approaches. Our approach can be scalable for large systems, accounts explicitly for power system security, and enables the electricity market to identify a cost-efficient dispatch avoiding redispatching actions. We demonstrate the performance of our method in a case study.

I. INTRODUCTION

Market-clearing processes and security assessment are separated in most electricity markets [1]. Market operators determine the economic dispatch in an auction based on Optimal Power Flow algorithms (OPFs) considering only a few physical limitations, such as generation and line flow limits. Transmission System Operators (TSOs), in charge of the secure and reliable system operation, determine actual physical bounds and carry out necessary redispatching measures. In order to optimally establish secure system operation and avoid expensive redispatching actions, market-clearing processes need to account for security considerations. To this end, algorithms are required, which capture the security requirements imposed by TSOs while at the same time being scalable and rapidly deployable in market-clearing auctions. We propose conditional line transfer limits as preventive security measures, which are optimally chosen in market-clearing auctions.

Security requirements include the N-1 criterion and stability margins. The N-1 criterion guarantees safe system operation even after the outage of any single component, while the stability margin ensures that oscillations caused by small disturbances are sufficiently damped. Security-Constrained OPF algorithms (SC-OPFs), i.e. OPFs which explicitly account for security criteria, have primarily focused on the N-1 criterion. Constraints, which for example ensure small-signal stability, are based on differential algebraic equations and cannot be included in the OPF as linear or non-linear constraints in a straightforward way. The authors in [1] and [2] use eigenvalue-sensitivities with respect to OPF decision variables to maintain tractability of small-signal stability constraints. However, both papers conclude that the required computational effort is a major challenge and the methods perform poorly when applied to larger systems. Additionally, compared to this paper, the method developed in [1] determines only the optimal redispatching action of the TSO and does not incorporate small-signal stability constraints in market-clearing auctions. An online preventive control strategy accounting for contingencies and transient stability was developed in [3]. The security considerations are translated to generator capacity limits of the most critical generators, which are included as hard constraints in the OPF. This approach, however, does not allow the market participants to bid all their available capacity and imposes very conservative bounds, which can prevent the deployment of cheaper generation capacity. In [4] a security boundary-constrained DC-OPF ensuring voltage and small-signal stability is proposed, where hyperplanes are used to linearly approximate the security boundary. Thus, operating points outside the convex space constructed by the hyperplanes are not considered in the OPF.

The contribution of this paper is a data-driven SC-OPF, which (a) incorporates the N-1 criterion, (b) ensures small-signal stability and compliance with stability margins, (c) avoids the complex reformulation of small-signal stability constraints inside the OPF by incorporating decision rules, (d) is applicable to large systems, (e) allows for a fast online solution as all computation related to the small-signal model is done offline and (f) encapsulates all security considerations for the OPF in conditional transfer limits of the lines. Traditionally, transfer limits, which are calculated by TSOs and provided to market operators, tend to be very robust and can lead to an inefficient use of the network. In this paper, we derive conditional transfer limits in the form of decision rules, which can be easily incorporated in OPF algorithms using Mixed Integer Linear Programming (MILP) and allow the market to optimally decide on power flows which will always guarantee power system security. Furthermore, this SC-OPF is not limited to market operation but it can also be used to minimize losses in the system or for other objectives. For this purpose, a large database of operating points is created using a small-signal model of the system. The operating
points are evaluated for small-signal stability and classified according to their compliance with required stability margins and the N-1 security criterion. Decision tree learning tools are then applied to derive decision rules suitable for implementation in OPF problems, thereby reducing the complexity of the SC-OPF. This allows for a more detailed modeling of the system given that all computations with respect to the small-signal model are done offline and not within the OPF. Less constraints are used compared to conventional SC-OPFs, which makes the method suitable for large systems.

The remainder of this paper is organized as follows: Section II introduces small-signal stability. An overview of the methodology to derive the SC-OPF is given in section III. The small-signal model and database creation are described in section IV and V, followed by a summary of the methodology for feature selection and knowledge extraction in section VI. Section VII describes the implementation of small-signal stability constraints in an OPF problem, while the application of the proposed method is demonstrated in section VIII. Finally, section IX concludes the paper.

II. SMALL-SIGNAL STABILITY

Small-signal stability assessment belongs to standard practice for almost all power system operators. In [5], a joint Cigre and IEEE Task Force defines small-signal rotor angle stability as the ability of a power system to maintain synchronism under small disturbances. They conclude that small-signal (or small disturbance) stability depends on the initial operating point and that nowadays instability is usually a result of insufficient damping of oscillations.

Small-signal instability is associated with one or more complex eigenvalues, whose real parts become positive [1]. Usually, TSOs determine a certain stability margin in terms of minimum damping ratio which should not be violated at any time to ensure a secure operation of the system.

Although power system stabilizers (PSS) and other damping controllers have been proposed and shown to be effective in improving small-signal stability, the authors of [6] argue that it is usually not possible to design a damping controller for all possible operating conditions, which makes additional remedial measures necessary. Thus, an OPF considering small-signal stability constraints may be an efficient way to avoid such operating conditions and therefore small-signal stability problems.

III. METHOD

The methodology to derive the data-driven SC-OPF proposed in this paper is shown as a flowchart in Fig. 1.

The first part consists of the data generation block. If available, data from phasor measurement units (PMUs) could be used. Further, if SCADA data is available a state estimator could be used to map the data into a power system model to perform the required simulations. However, due to the unavailability of such data we used data obtained by simulation of small-signal models in this paper. The derivation of such models is described in section IV, while the consideration of N-1 security and the challenges of the data generation are discussed in the following subsections. The result of this first part of the methodology is a large database of operating points which are classified for the base case as well as for credible contingencies. This is described in section V. The second part of the methodology uses this database to derive rules, which can later be incorporated in the OPF formulation. The second part includes feature selection and knowledge extraction and is described in section VI. The third part consists of implementing such rules in the OPF algorithm using MILP (section VII). Finally, the SC-OPF is tested in a case study in section VIII.

IV. MODELING

In order to create a database of possible operating points for which we can perform a small-signal stability assessment, we first need to derive a small-signal model.

A sixth order synchronous machine model [7] with an Automatic Voltage Regulator (AVR) Type I (3 states) is used in this study. With an additional state for the bus voltage delay this leads to a state-space model of 10m states, with \( N_G \) representing the number of generators in the grid. The model is based on the following assumptions without loss of generality:

- Stator and network transients are neglected.
• Constant mechanical torque is assumed as a result of neglecting turbine governor dynamics.
• The damping torque is assumed to be linear.

For a detailed description of the derivation of a multi-machine model, the interested reader is referred to [7], [8]. The relevant equations are briefly summarized in Appendix A. The small-signal models are derived symbolically until equation (A2), due to the fact that the symbolic inversion of the Jacobian given in (35) is computationally extremely demanding, if not impossible (for larger systems). The symbolic derivation enables us to initialize the model for every possible combination of load and generation using Matpower 6.0 [9].

The small-signal model is adjusted to account for the N-1 criterion by explicitly incorporating the outage of a single component. Thus, the multi-machine model of the base case, i.e. without any contingency, is adapted for every credible contingency resulting in \( \Gamma + 1 \) different small signal models. Variable \( \Gamma \) represents the number of considered contingencies.

V. DATABASE GENERATION

The required database contains the necessary information on possible operating points of the system including their stability and stability margins. This set of credible operating points contains all possible combinations of generator dispatches corresponding to typical load patterns. Thus, in general, depending on the number of generators in the grid, \( N_G \), their maximum capacity, \( P_{i \text{max}} \), the number of load patterns, \( \Lambda \) and the minimum generation bid size, \( \alpha \), the number of operating points in this set is given by:

\[
\Psi = \Lambda \cdot \prod_{i=1}^{N_G-1} \left( \frac{P_{i \text{max}}}{\alpha} + 1 \right).
\]

Obviously, this number corresponds to only one approach. The study of how to define and determine the whole space of operating points is equally important and needs to be addressed. We will discuss this in more detail in subsection V-C. Thus, (1) provides only a first impression of the size of the required database and which variables influence the database size specifically.

In case that PMU and SCADA data is available for all credible operating points, the necessary database for the base case already exists. Noise reduction and data correction methods might be necessary and the data needs to be analyzed with respect to the stability margin of the operating points. However, it is very likely that there does not exist data for all credible contingencies, thus additional simulations will be necessary. Hence, a state estimator could be used to map the SCADA data into a power system model to perform additional simulations.

A. Base Case

If PMU / SCADA data is not (sufficiently) available for the base case, i.e. considering the whole grid without contingencies, (additional) simulations are necessary. The base case model is initialized for every (missing) operating point using ac power flow in Matpower 6.0 [9]. Each operating point is examined for small signal stability considering the chosen stability margin and classified as "fulfilling the stability margin" or "not fulfilling the stability margin". The results are saved in a database together with the necessary information of the operating point. The necessary information depends on the chosen mapping of the operating points between the ac and dc power flow and is described in more detail in section VI-A.

B. N-1 Security

In order to evaluate the N-1 security of the system, \( \Gamma \) different small signal models adjusted for all considered contingencies are also initialized for all \( \Psi \) operating points. Here, however, the choice of the slack bus in the ac power flow used for initializing the various contingency models may differ between the contingencies considered. Ideally the distributed slack buses should be chosen according to the location of the primary reserves, which balance the power mismatch in case of the specific contingency based on the experience of the TSO. The operating points are classified according to the outcome of the small signal analysis of the different contingency models.

The database contains all necessary information of the operating points and the results of the small signal analyses for the base case and all considered contingencies.

C. Challenges

The data generation is one major challenge of this approach. In particular, two partially contradicting goals exist:

• keeping the database as small as possible to minimize computational demand
• determining the stability boundary as precise as possible

Large grids with many generators lead to a very large database if all possible operating points were evaluated as seen in (1). However, since the goal is to determine line flow limits it is not really necessary to analyze all possible combinations of generation patterns as indicated in Fig. 2. The figure shows the

![Fig. 2. Scatter plot of two angle differences, which correspond to line flow limits, for all operating points. Operating points fulfilling the stability margin (3%) are marked in orange, those not fulfilling the stability margin in blue.](image-url)
important, in particular at the boundary, to find the step size to power changes. Nonetheless, Fig. 3 indicates that it is
distance between the different simulated operating points is
operating points should be smaller than the minimum bid size
area around the stability boundaries the step size between the
efficient line flow limits without increasing risk. Thus, in the
as accurately as possible, in order to provide the most cost-
the existing knowledge of experienced engineers at TSOs.
the database can be reduced. This can be achieved by e.g. using
and neglecting operating points far away from the boundary
area of operating points fulfilling the stability margin (orange)
and the area of operating points not fulfilling the stability
marginal (blue). Thus, by focusing on the stability boundary
and the area of operating points not fulfilling the stability
area of operating points fulfilling the stability margin (orange)
and unstable (red) operating points is minimized.

Fig. 3. Step size (d) of simulation needs to be chosen in a way that the
area of stable operating points (purple) is maximized while the area of
uncertain/potentially stable operating points (grey) which lies between stable
(blue) and unstable (red) operating points is minimized.

VI. FEATURE SELECTION AND KNOWLEDGE EXTRACTION

The creation of the database is followed by the feature
selection, which determines the level of conservatism of the
line flow limits. The knowledge extraction constitutes the final
step in the offline derivation of the decision rules.

A. Feature Selection

In order to be able to substitute simulation data with PMU
data in the future, only easily accessible variables should be
considered as inputs. The goal is to incorporate all security
considerations into conditional transfer limits, which provide
the boundaries for the market and keep the complexity of the
SC-OPF comparably low.

The transfer limits for the market-clearing are derived from
the data obtained by using ac power flow as initialization for
the small signal models. Since market operations are usually
based on linearized dc power flow equations, the main issue
here is to achieve a good mapping between the ac power flow
operating points and the dc power flow points. This mapping
is needed due to the fact that for the same generation pattern
the line flows in an ac power flow can differ (significantly)
from the ones in a dc power flow depending on the voltages
and angles in the system. Thus, a better mapping of operating
points leads to more accurate and less conservative line flow
limits.

We focused on three different approaches to determine the
active power transfer limits. Eq. (2) shows the active power
transfer over a line \( n - m \) based on the full AC power flow
equations.

\[
F_{nm} = |V_n||V_m| \left( G_{nm} \cos(\theta_n - \theta_m) + B_{nm}\sin(\theta_n - \theta_m) \right),
\]

where \( F_{nm} \) and \( V_i \) denote the active power flow between
nodes \( n \) and \( m \) and the voltages at node \( i \), respectively. \( G_{nm} \) and \( B_{nm} \)
correspond to the conductance and susceptance of the line.

1) Standard DC Approximation: The standard dc approxi-
mation is based on the following assumptions:
- the resistance of the lines is very small compared to the
  reactance resulting in: \( G = 0 \) and \( B_{nm} = \frac{1}{x_{nm}} \)
- the angle difference between adjacent nodes is small
  allowing for the sine to be replaced by its argument
- voltage magnitudes \( |V_n| \) are equal to 1 p.u..

The line flows are then given by:

\[
F_{nm} = \frac{\theta_n - \theta_m}{x_{nm}},
\]

2) Less conservative approximation: More accurate line
flows can be computed if nodal voltage magnitudes and the
sine of the angle difference are accounted for:

\[
F_{nm} = \frac{|V_n||V_m| \sin(\theta_n - \theta_m)}{x_{nm}}.
\]

3) Exact Mapping of DC and AC operating points: The
most precise mapping of the dc and ac operating points has
been achieved by incorporating the mapping into the data
generation. We derived the line losses of the ac power flow
of each generation pattern and included them as additional loads
in a subsequent dc power flow. The losses are added to the
nodes in the following way:

\[
P_{Loss,n} = \frac{1}{2} \sum_{m \in I_n} (F_{nm} + F_{mn}),
\]

where \( F_{nm} \) and \( F_{mn} \) represent the line flows obtained from
the ac power flow analysis and \( I_n \) denotes the set of nodes
connected to node \( n \). The dc power flow is thus augmented
by the loss approximation of the corresponding ac power flow and the obtained line flows of the dc power flow represent an exact mapping of operating points.

B. Knowledge Extraction

This paper proposes classification trees as knowledge extraction/classification methods. A classification tree is a subset of decision trees (DT) where the decision outcome, called "target variable", can take a finite set of values. A complete introduction to DT theory for power systems can be found in [11]. Within the generated databases containing all operating points, we distinguish between those "fulfilling the stability margin" and those "not fulfilling the stability margin". TSOs require stability margins to ensure sufficient damping of oscillations. The classification tree is used to reveal patterns of the interconnection between observation attributes and target variables with the goal of using these patterns as decision rules to predict the decision outcome of new observations. These attributes of observations used as decision variables are called "predictors". A simple example of a DT is shown in Fig. 4. It contains nodes, branches and terminal decisions. At each node a test of a predictor splits the data set into subsets according to the test result. The branches connecting this node with the nodes one level below represent the possible test outcomes. The leaf nodes with terminal decisions constitute the final classification of the input observation. The use of a DT instead of other data mining or machine learning techniques provides a better understanding of the entire system’s small-signal stability and what influence different parts of the system have. Moreover, we can define conditional transfer limits by using an appropriate feature selection and associating all full DT paths from the root to the leaves with binary decision variables. The optimal dispatch is then chosen by allowing only one path to be activated.

Note that before training the DT, the database should be analyzed with respect to the potential problem of skewed classes, i.e. it needs to be examined whether one class ("fulfilling the stability margin" / "not fulfilling the stability margin") is significantly over-represented compared to the other. If this is the case, three options exist: (a) limiting the number of the over-represented class by neglecting the appropriate amount of samples, (b) over-sampling the minority class or (c) raising the cost of misclassifying the minority class. While option (a) might leave out exactly the most important information close to the stability boundary, (b) degrades the learning speed due to duplicate instances. Option (c) allows to introduce risk-aversion, which might lead to more conservative and less cost-efficient transfer limits. This however, can be improved through smaller step sizes during the analysis described in V-C.

The DT is trained using a subset of the whole database, called training set A. The derived rules are tested using another subset B of the database with $A \cup B = \emptyset$.

It is worth mentioning that although DTs achieve pretty high accuracies, there will always be misclassifications, which in theory could cause unstable solutions of the proposed data-driven SC-OPF. However, actions have been taken to minimize, if not completely avoid this threat. First, by increasing the cost for misclassifications of unstable cases we reduce the misclassifications in this direction. This, however, might also cause less cost-efficient line flow limits. Further, by choosing a certain stability margin and differentiating between "fulfilling the stability margin" and "not fulfilling the stability margin", instead of "stable" and "unstable", we can reduce the risk even further, since misclassifications do not necessarily lead to instability, but just to lower stability margins than required - at least in all scenarios tested in this paper. This is discussed in more detail in section VIII-B.

Finally, in order to avoid over-fitting, the obtained trees need to be pruned appropriately. Different pruning techniques will be investigated in future work.

VII. OPF Implementation

As usual in market applications, the network is modeled using a linear DC approximation.

A. Standard DC Optimal Power Flow (DC-OPF)

The objective function to be minimized represents the total generation cost (6). The optimization problem is formulated as follows:

$$\min_{\mathbf{P}_G} \sum_{i=1}^{N_G} c_{G,i} P_{G,i},$$

$$\text{s.t.} \sum_{i=1}^{N_G} P_{G,i} - \sum_{n=1}^{N_B} P_{D,n} = 0,$$

$$-P_{L}^{\max} \leq P_{L} - \sum_{n=1}^{N_B} P_{D,n} \leq P_{L}^{\max},$$

$$0 \leq P_{G} \leq P_{G}^{\max},$$

where symbols in bold denote matrices or vectors. $N_G$ and $N_B$ refers to the number of generators and buses, respectively. Equality constraint (7) ensures balance between total generation and total demand. Demand $\mathbf{P}_D$ is assumed to be inelastic.

![Fig. 4. A simple example of a decision tree](image-url)
and thus, not a decision variable. Power Transfer Distribution Factors (PTDF) are used to translate net injections into line flows, which are constrained to their line flow limits $F_{L,p}^{\text{max}}$ in Eq. (8). Eq. (9) constrains the generator outputs below capacity limits.

### B. Security Constrained OPF (SC-OPF)

The SC-OPF ensuring N-1 security and small signal stability extends the standard DC-OPF by including additional constraints representing conditional line transfer limits, derived from the decision tree in Section VI-B. The DT is incorporated into the optimization using Mixed Integer Linear Programming (MILP). Each full path $p$ from the root to a leaf node is represented by a set of minimum and maximum line transfer limits $F_{L,p}^{\text{min}}$ and $F_{L,p}^{\text{max}}$ respectively. Eq. (8) is replaced by:

$$PTDF \cdot (P_G - P_D) \leq F_{L,p}^{\text{max}} y_p + F_{L,p}^{\text{max}} (1 - y_p) \quad \forall p \in P \quad (10)$$

$$PTDF \cdot (P_G - P_D) \geq F_{L,p}^{\text{min}} y_p - F_{L,p}^{\text{min}} (1 - y_p) \quad \forall p \in P, \quad (11)$$

where $P$ is the set of full paths representing the decision tree. Note that the conditional line transfer limits do not limit the flows symmetrically as in Eq. (8), i.e. the lower bound does not necessarily correspond to the negative upper bound. Each path is associated with a binary variable $y_p$, which, if chosen, ensures the activation of the corresponding conditional line flow limits or otherwise, leaves the usual line flow limits unchanged. The number of binary variables corresponds to the number of leaf nodes in the DT. The choice of line limits to one path only is constrained through $\sum_p y_p = 1$.

### C. Matching of SC-OPF and Database

In contrast to the operating points saved in the database, losses are not inherently considered in the standard DC-OPF or SC-OPF described above. Thus, the operating points resulting from the OPFs need to be matched with the ones saved in the database by incorporating losses. Due to the fact that the optimal power flow is not known prior to delivering the line flow limits and the line flows and losses vary depending on the generation pattern, an initial best guess of the optimal operating point is crucial to achieve a good matching. By searching the database for the cheapest operating point (assuming that costs are known a priori), which fulfills the stability margin and meets the load level, we found a "best guess" operating point, whose losses were incorporated in the standard DC-OPF and the SC-OPF according to (5). The losses of the "best guess" OP are determined through an ac power flow analysis. The DC-OPF and SC-OPF are thus augmented by a loss approximation and yield more realistic results.

### VIII. CASE STUDY

The proposed method ensuring small signal stability and N-1 security is applied to the IEEE 14 bus test system shown in Fig. 5. All parameters are given in the appendix.

The small signal models were derived using Mathematica, the initialization and small signal analysis were carried out using Matlab and Matpower 6.0 [13]. All DC-OPF simulations were carried out in Python using the Gurobi Optimizer [14].

A minimum damping ratio of 3% is chosen as stability margin. We chose to require the same minimum damping ratio also for the steady states obtained after an outage of a single element during the N-1 security analysis. However, this could also be chosen differently by e.g. allowing lower damping ratios after certain faults.

The system has been simulated for several load levels (±20%) and various step sizes between the generation patterns. Generator $g_1$ has been chosen as the slack bus, also in all N-1 scenarios, due to the small size of the other generators. Since the IEEE 14 Bus system is not N-1 stable, a few contingencies and reactive power limits could not be considered for the N-1 security assessment. All line faults have been considered except for a fault on the line connecting bus 1 and bus 2. All bus faults have been analyzed, except for a fault at bus 1 and bus 2. These contingencies could not be considered because either they lead to instability / violation of the AVR limits for any operating point of the considered load patterns (bus 1) or the stable operating points are not stable for the remaining contingencies. Hence, this would leave us with an empty set of feasible operating points and therefore we had to neglect these contingencies.

First, we will compare the results of the novel data driven SC-OPF, when using different features for mapping the operating points of ac and dc power flows. Then, we will investigate the impact of various step sizes between the generation patterns on the results of the data-driven SC-OPF, i.e. different accuracies and database sizes. Finally, we will discuss additional benefits of data-driven approaches.

### A. Comparison between Standard DC-OPF and Data-driven SC-OPF using different Mapping Approaches

The comparison between the data-driven SC-OPFs using different mapping approaches is done using a 2.5 MW step size during the data generation. Table I lists the results of both
the standard DC-OPF and the SC-OPF using different features of the database, in particular the three different mapping approaches described in VI-A. Additionally, the database best guess operating point is listed, which is used for determining the losses to be incorporated in the OPFs.

Table I shows that the standard DC-OPF fails to ensure N-1 security, since for this generation pattern the most critical pair of eigenvalues of the most critical fault in the N-1 security assessment (Fault at Bus 5) indicates a negative damping ratio, i.e. instability. The most critical eigenvalues of those faults in the N-1 security assessment violating the minimum required stability margin of 3% are shown in Fig. 6. It shows that all SC-OPFs ensure N-1 security of the system by a change of the generation pattern resulting in the most critical eigenvalues of the same faults being moved into the left half-plane, i.e. in the safe area with a minimum damping ratio larger than the required stability margin.

The figure visualizes also that the operating points obtained by the SC-OPFs using different mapping approaches vary by their minimum damping ratio, which reflects the more conservative line flow limits in case of both dc approximations compared with the exact mapping approach. These more conservative line flow limits lead consequently also to higher cost of security, as indicated in Tab. I.

Finally, Tab. I indicates also the importance of the matching of operating points, i.e. the importance of the best guess of power flow. While in the 1st run column of the SC-OPFs the losses are implemented according to the data based best guess, the 2nd run uses the result of the 1st run as initial best guess. In this case, the loss distribution and therefore the line flows are more accurate, i.e. more comparable to dc power flows obtained in the data generation. Thus, the matching of operating points is improved which leads to better results in all cases.

### B. Comparison of different Step Sizes between the analyzed Generation Patterns

Table II shows various aspects and results for the database generation of a SC-OPF comparing different step sizes used during data generation.

Obviously, following the simple approach with constant step size between each analyzed generation pattern the database size grows as indicated in (1). However, as discussed in V-C there exist improvement potential on which we will focus in future work. Further, the required simulation time is related to the database size. It naturally increases with the number of operating points. The database was generated using 20 cores in parallel on the high performance computing farm at DTU.

The tree accuracy is also related to the available database size. In general, it should be easier to differentiate between the case fulfilling the stability margin and not fulfilling the stability margin if there is a higher step size in between the analyzed generation patterns. This is reflected in Tab. II by the decreasing number of false prediction with risk with increasing step size. False predictions with risk are those predictions which falsely predict operating points not fulfilling the stability margin as "fulfilling the stability margin". The results show that the chosen risk averse approach, discussed in section VI-B, which increases the costs of this kind of false predictions, works fine since the vast majority of false predictions is made in the other direction. This is indicated by the small percentage of these kind of false predictions within the total number of false predictions.

However, on the other hand there exists also less data for training which may decrease the tree accuracy as indicated in Tab. II. This also leads to potentially shorter trees resembled by a smaller number of binary variables and a faster execution time, shown in Tab. III.

Table II supports also the discussion in section V-C about the importance of the step size, in particular close to the boundary. This is reflected by the significantly increasing number of risky false predictions with increasing step size when the DTs are tested with the database of the other step sizes. Here, the DTs of 2.5 MW and 5 MW are tested on the 1 MW database while the 1 MW DT is tested on the 2.5 MW database. It is shown that a wider step size in between the generation patterns leads to potentially more and unsafer risky false predictions, as indicated by the increasing number of false predictions with risk and the decreasing minimum damping ratio of those false predictions.

However, the tree accuracy should not be confused with the cost efficiency of the line flow limits resembled by the generation costs. All SC-OPFs with the different step sizes lead to points fulfilling the required stability margin. Here, we used two runs using the 1st run as initialization for the 2nd run as described in the previous subsection.

### TABLE I

<table>
<thead>
<tr>
<th>Dispatch (MW)</th>
<th>DC-OPF (w. losses)</th>
<th>Best Guess (Data Based)</th>
<th>SC-OPF (clas. DC)</th>
<th>SC-OPF (less con. DC)</th>
<th>SC-OPF (exact)</th>
</tr>
</thead>
<tbody>
<tr>
<td>g1</td>
<td>275.509</td>
<td>183.73</td>
<td>165.962</td>
<td>165.262</td>
<td>176.535</td>
</tr>
<tr>
<td>g2</td>
<td>0</td>
<td>55</td>
<td>60</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>g3</td>
<td>0</td>
<td>27.5</td>
<td>26.603</td>
<td>26.671</td>
<td>22.51</td>
</tr>
<tr>
<td>g4</td>
<td>0</td>
<td>2.5</td>
<td>16.17</td>
<td>15.687</td>
<td>9.689</td>
</tr>
<tr>
<td>g5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total Cost (€)</td>
<td>2755.5</td>
<td>2981.1</td>
<td>3091.63</td>
<td>3077.21</td>
<td>3022.56</td>
</tr>
<tr>
<td>Min. damping ratio (%)</td>
<td>-20.33%</td>
<td>4.07%</td>
<td>9.73%</td>
<td>9.59%</td>
<td>6.59%</td>
</tr>
<tr>
<td>Cost of Security (€)</td>
<td>-</td>
<td>225.59</td>
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<td>321.7</td>
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<td></td>
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<td></td>
<td>264.39</td>
<td>204.97</td>
</tr>
</tbody>
</table>

The results of standard DC-OPF and data-driven SC-OPF using different mapping approaches are listed in Table I. It shows that the standard DC-OPF fails to ensure N-1 security, since for this generation pattern the most critical pair of eigenvalues of the most critical fault in the N-1 security assessment (Fault at Bus 5) indicates a negative damping ratio, i.e. instability. The most critical eigenvalues of those faults in the N-1 security assessment violating the minimum required stability margin of 3% are shown in Fig. 6. It shows that all SC-OPFs ensure N-1 security of the system by a change of the generation pattern resulting in the most critical eigenvalues of the same faults being moved into the left half-plane, i.e. in the safe area with a minimum damping ratio larger than the required stability margin.

The figure visualizes also that the operating points obtained by the SC-OPFs using different mapping approaches vary by their minimum damping ratio, which reflects the more conservative line flow limits in case of both dc approximations compared with the exact mapping approach. These more conservative line flow limits lead consequently also to higher cost of security, as indicated in Tab. I.

Finally, Tab. I indicates also the importance of the matching of operating points, i.e. the importance of the best guess of power flow. While in the 1st run column of the SC-OPFs the losses are implemented according to the data based best guess, the 2nd run uses the result of the 1st run as initial best guess. In this case, the loss distribution and therefore the line flows are more accurate, i.e. more comparable to dc power flows obtained in the data generation. Thus, the matching of operating points is improved which leads to better results in all cases.
Table III shows that in fact the SC-OPF using the widest step size leads to the cheapest operating point. This is due to the wider uncertain area in between the analyzed operating points resulting in more relaxed line flow limits. However, the difference is very small (max. 6.2 €/h). It is important to emphasize that, although the widest step size achieves the best performance in this particular scenario, smaller step sizes are safer as discussed before and indicated in Tab. II.

**C. Additional Benefit of the Data-driven Approach**

Besides the possibility to deliver cost-efficient line flow limits the data-driven approach provides the additional benefit of serving as an easy accessible source of knowledge for the TSO which could be useful during normal operation but in particular during training of new employees.

First, similar as visualized in Fig. 6, the database enables the user to get an overview which faults are the most critical for any operation point. Unlike for the specific generation pattern provided by the DC-OPF this may only be a single fault.

Second, the use of a DT instead of other knowledge extraction methods provides the user with additional comprehensible knowledge about safe and unsafe line flows in the system. By just visualizing the tree as e.g. in Fig. 4 the user is able to...
access this valuable comprehensive information, which would not be possible by most other knowledge extraction methods, which work mostly like a black-box from the outside.

IX. CONCLUSION

A novel data-driven SC-OPF method ensuring small-signal stability has been proposed. It uses a small-signal model for data generation, and classification trees for the knowledge extraction. Conditional transfer limits in the form of decision rules are derived from the decision tree and implemented in the OPF problem by means of Mixed Integer Linear Programming. We investigated the impact of different features for the mapping of operating points from the ac to the dc space and emphasized specific aspects of this method, e.g. the importance of matching the operating points. Additionally, we discussed the challenges of this method, such as the problems arising when using a wide step size (in particular at the stability boundary). Finally, we emphasized the additional benefits of the data-driven approach as it serves as a source of knowledge of the system. The case study indicates a good performance.

ACKNOWLEDGMENT

This work is supported by the EU project Best Paths under the 7th Framework Programme, Grant Agreement No. 612748 and by the ForskEL-projekt 12264 Best Paths for DK.

APPENDIX

A. Small-Signal Model

1) Notation:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{ik}$</td>
<td>Angle of the $ik$-th entry of the network bus admittance matrix.</td>
</tr>
<tr>
<td>$\delta_i$</td>
<td>Rotor angle of generator $i$.</td>
</tr>
<tr>
<td>$\omega_i$</td>
<td>Rotor speed of generator $i$.</td>
</tr>
<tr>
<td>$\omega_s$</td>
<td>Synchronous rotor speed.</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Base synchronous frequency.</td>
</tr>
<tr>
<td>$\psi_{d,i}$</td>
<td>Magnetic flux of generator $i$.</td>
</tr>
<tr>
<td>$D_i$</td>
<td>Additional damping Torque of generator $i$.</td>
</tr>
<tr>
<td>$e_{d,i}$</td>
<td>Transient voltage of generator $i$ in d-/q-axis.</td>
</tr>
<tr>
<td>$r_{d,q}$</td>
<td>Field voltage of generator $i$.</td>
</tr>
<tr>
<td>$H_i$</td>
<td>Inertia constant of generator $i$.</td>
</tr>
<tr>
<td>$i_{d,q}$</td>
<td>Current of generator $i$ in d-/q-axis.</td>
</tr>
<tr>
<td>$K_{A_i}$</td>
<td>Amplifier gain of generator $i$.</td>
</tr>
<tr>
<td>$K_{E_i}$</td>
<td>Self-excited constant used for of generator $i$.</td>
</tr>
<tr>
<td>$K_{F_i}$</td>
<td>Stabilizer gain used for of generator $i$.</td>
</tr>
<tr>
<td>$m$</td>
<td>Number of machines.</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of buses.</td>
</tr>
<tr>
<td>$r_{F_i}$</td>
<td>Out of the stabilizer of generator $i$.</td>
</tr>
<tr>
<td>$r_{a,i}$</td>
<td>Armature resistance of generator $i$.</td>
</tr>
<tr>
<td>$S_{E_i}(E_{f_{di}})$</td>
<td>Saturation function of generator $i$.</td>
</tr>
<tr>
<td>$T_{A_i}$</td>
<td>Time constant of the voltage regulator of gen. $i$.</td>
</tr>
<tr>
<td>$T_{d0,q0}$</td>
<td>Transient time const. of d-/q-axis of generator $i$.</td>
</tr>
<tr>
<td>$T_{d0,q0}''$</td>
<td>Sub-transient time const. of d-/q-axis of gen. $i$.</td>
</tr>
<tr>
<td>$T_{E_i}$</td>
<td>Exciter time constant of generator $i$.</td>
</tr>
<tr>
<td>$T_{F_i}$</td>
<td>Time constant of the stabilizer of generator $i$.</td>
</tr>
<tr>
<td>$T_{M_i}$</td>
<td>Mechanical torque of generator $i$.</td>
</tr>
<tr>
<td>$v_{ref,i}$</td>
<td>Reference voltage of generator $i$.</td>
</tr>
<tr>
<td>$v_{R_i}$</td>
<td>Output of the voltage regulator of generator $i$.</td>
</tr>
<tr>
<td>$v_{R,max}$</td>
<td>Limit of the voltage regulator of generator $i$.</td>
</tr>
<tr>
<td>$x_{d,q_i}$</td>
<td>Transient reactance of generator $i$ in d-/q-axis.</td>
</tr>
<tr>
<td>$x_{l_i}$</td>
<td>Leakage reactance of generator $i$ in d-/q-axis.</td>
</tr>
<tr>
<td>$x_{d,q_i}''$</td>
<td>Synchronous reactance of gen. $i$ in d-/q-axis.</td>
</tr>
<tr>
<td>$Y_{ik}$</td>
<td>Sub-transient reactance of gen. $i$ in d-/q-axis.</td>
</tr>
<tr>
<td>$\gamma_{d,i}$</td>
<td>Magnetic flux of generator $i$.</td>
</tr>
<tr>
<td>$\gamma_{q,i}$</td>
<td>Magnitude of the output of the stabilizer of generator $i$.</td>
</tr>
<tr>
<td>$\gamma_{r,i}$</td>
<td>Base synchronous frequency.</td>
</tr>
<tr>
<td>$\gamma_{s,i}''$</td>
<td>Exciter time constant of generator $i$.</td>
</tr>
<tr>
<td>$\gamma_{s,i}''$</td>
<td>Field voltage of generator $i$.</td>
</tr>
<tr>
<td>$\gamma_{s,i}''$</td>
<td>Magnetizing function of generator $i$.</td>
</tr>
<tr>
<td>$\gamma_{s,i}''$</td>
<td>Sub-transient reactance of gen. $i$ in d-/q-axis.</td>
</tr>
<tr>
<td>$\gamma_{s,i}''$</td>
<td>Sub-transient reactance of gen. $i$ in d-/q-axis.</td>
</tr>
</tbody>
</table>

2) Model: The multi-machine model can be described with 10m differential algebraic equations (12)-(23) and 6m algebraic equations: 2m equations defining stator fluxes (24)-(25), 2m stator equations (26)-(27), and 2m network equations (28)-(29). For a more detailed description the interested reader is referred to [8].

$$\frac{d\delta_i}{dt} = \Omega(\omega_i - \omega_s)$$ (12)
$$\frac{d\omega_i}{dt} = \frac{1}{2H_i} (T_{M_i} - \psi_{d,i}q_{i,q} + \psi_{q,i}i_{d,i} - D_i(\omega_i - \omega_s))$$ (13)
$$\frac{de_{d,i}}{dt} = (-e_{d,i}' - (x_{d,i} - x_{d,i}')i_{d,i} - \gamma_{d,i}i_{q,i} + \gamma_{d,i}e_{q,i}' + e_{d,i}')/T_{d0,i}$$ (14)
$$\frac{de_{q,i}}{dt} = (-e_{q,i}' - (x_{q,i} - x_{q,i}')i_{q,i} - \gamma_{q,i}i_{d,i} + \gamma_{q,i}e_{d,i}')/T_{d0,i}$$ (15)
$$\frac{d\psi_{d,i}''}{dt} = (-\psi_{d,i}' + e_{d,i}' - (x_{d,i}' - x_{d,i}')i_{d,i})/T_{d0,i}$$ (16)
$$\frac{d\psi_{q,i}''}{dt} = (-\psi_{q,i}' + e_{q,i}' - (x_{q,i}' - x_{q,i}')i_{q,i})/T_{d0,i}$$ (17)
$$\frac{dv_{h,i}}{dt} = \frac{v_{h,i} - v_{t,i}}{T_{r,i}}$$ (18)
$$\frac{dv_{R_i}}{dt} = \frac{K_{A_i}(v_{ref,i} - v_{h,i} - r_{f,i} - \frac{K_{F_i} e_{f_{di}}}{T_{r,i}}) - v_{R_i}}{T_{A_i}}$$ (19)
$$\frac{dr_{F_i}}{dt} = \frac{r_{F_i} - \frac{K_{F_i} e_{f_{di}}}{T_{F_i}}}{(T_{F_i}) e_{f_{di}}}$$ (20)
$$\frac{de_{f_{di}}}{dt} = \frac{K_{E_i} + S_{E_i}(E_{f_{di}})}{T_{E_i}} e_{f_{di}} + \frac{v_{R_i}}{T_{E_i}}$$ (21)

$$0 = \psi_{d,i} + x_{d,i}'i_{d,i} - \gamma_{d,i}i_{q,i}' + x_{q,i}'i_{q,i} - (1 - \gamma_{d,i})\psi_{d,i}''$$ (22)
$$0 = \psi_{q,i} + x_{q,i}'i_{q,i} + \gamma_{q,i}i_{d,i}' - (1 - \gamma_{q,i})\psi_{q,i}''$$ (23)
$$0 = r_{a,i} + i_{d,i} + \psi_{q,i} + v_{d,i}$$ (24)
$$0 = r_{a,i} + i_{q,i} - \psi_{d,i} + v_{q,i}$$ (25)

for $i = 1, 2, 3, \ldots, m$.

and

$$0 = P_{G_i} + P_{L_i}(V_i) - \sum_{k=1}^{n} V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik})$$ (26)
$$0 = Q_{G_i} + Q_{L_i}(V_i) - \sum_{k=1}^{n} V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik})$$ (27)

for $i = 1, 2, 3, \ldots, m, m + 1, \ldots, n$.

with

$$P_{G_i} = I_d i_i \sin(\delta_i - \theta_i) + I_q i_i \cos(\delta_i - \theta_i)$$ (28)
$$Q_{G_i} = I_d i_i \cos(\delta_i - \theta_i) - I_q i_i \sin(\delta_i - \theta_i)$$ (29)
for $i = 1, 2, 3, \ldots, m$. The equations can be combined, linearized and rewritten in matrix notation as:

$$
\begin{bmatrix}
\Delta \dot{X}
\end{bmatrix} =
\begin{bmatrix}
A & B_1 & B_2
\end{bmatrix}
\begin{bmatrix}
\Delta X
\end{bmatrix} +
\begin{bmatrix}
E_1
\end{bmatrix}
\Delta U
$$

with $X = [X_1^T, \ldots, X_m^T]^T$ and $X_i = [\xi_i, \omega_i, e_{di}, e_{qi}, \psi_{di}, \psi_{qi}]^T$ including the machine state variables and $\Delta Z_1 = [v_1, v_{h,1}, \ldots, v_{h,m}]^T$ including the voltage angle of the slack bus as well as the voltage magnitudes of the generator buses. $\Delta Z_2 = [\theta_2, \ldots, \theta_n, v_{h,m+1}, \ldots, v_{h,n}]^T$ includes the voltage angles of the remaining buses and the voltage magnitudes of the load buses. The system matrix $A_{sys}$ with dimension $10m \times 10m$ fulfilling the equation

$$
\Delta \dot{X} = A_{sys} \Delta X + E \Delta U
$$

can be obtained as:

$$
A_{sys} = A - [B_1 \, B_2][J_{AE}]^{-1}
\begin{bmatrix}
C_1
\end{bmatrix}
\begin{bmatrix}
C_2
\end{bmatrix}
$$

with

$$
[J_{AE}] =
\begin{bmatrix}
D_{11} & D_{12}
\end{bmatrix}
\begin{bmatrix}
D_{21} & D_{22}
\end{bmatrix}
\begin{bmatrix}
\end{bmatrix}

$$

representing the network algebraic Jacobian.

### B. Parameters

#### Table IV

<table>
<thead>
<tr>
<th>Parameters</th>
<th>g1</th>
<th>g2</th>
<th>g3</th>
<th>g4</th>
<th>g5</th>
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<tbody>
<tr>
<td>MVA</td>
<td>615</td>
<td>60</td>
<td>60</td>
<td>25</td>
<td>25</td>
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<td>$r_s$ (pu)</td>
<td>0.0031</td>
<td>0.0031</td>
<td>0.0031</td>
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<td>$H$ (s)</td>
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<td>5.06</td>
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<td>2</td>
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<td>2</td>
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<td>2</td>
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<tr>
<td>$x_d$ (pu)</td>
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<td>1.05</td>
<td>1.05</td>
<td>1.25</td>
<td>1.25</td>
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<tr>
<td>$x_q$ (pu)</td>
<td>0.2995</td>
<td>0.1850</td>
<td>0.1850</td>
<td>0.232</td>
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<td>0.98</td>
<td>1.22</td>
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<td>$x_{dq}$ (pu)</td>
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<td>0.36</td>
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<tr>
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<td>$x_{pq}$ (pu)</td>
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<td>0</td>
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<td>0.134</td>
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<tr>
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<td>6.1</td>
<td>4.75</td>
<td>4.75</td>
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<tr>
<td>$T_{s0}$ (s)</td>
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<td>0.04</td>
<td>0.06</td>
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<tr>
<td>$T_{s1}$ (s)</td>
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<td>0.3</td>
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<tr>
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<td>0.099</td>
<td>0.099</td>
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<tr>
<td>$K_0$</td>
<td>120</td>
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<td>20</td>
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<td>$V_{max}$ (pu)</td>
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<td>$K_v$ (pu)</td>
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<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$T_v$ (s)</td>
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<td>$B_e$ (1/pu)</td>
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<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
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</tr>
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</table>

#### Economic data

| Cmin (€/MWh) | 10 | 12.5 | 15 | 17.5 | 20 |

---

## References


