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In memoriam Leonid V. Keldysh

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Leonid Keldysh – one of the most influential theoretical physicists of the 20th century – passed away in November 2016. Keldysh is best known for the diagrammatic formulation of real-time (nonequilibrium) Green functions theory and for the theory of strong field ionization of atoms. Both theories profoundly changed large areas of theoretical physics and stimulated important experiments. Both these discoveries emerged almost simultaneously – like Einstein, also Keldysh had his *annus mirabilis* – the year 1964. But the list of his theoretical developments is much broader and is briefly reviewed here.

1 Introduction On November 11 2016 Leonid Veniaminovich Keldysh passed away in Moscow. Keldysh was a Russian theoretical physicist who had a tremendous influence on many fields of physics. In this article we briefly describe Keldysh's most important contributions and discuss how they influenced and continue to influence modern physics. In particular, we concentrate on his work on nonequilibrium many-particle physics that is related to his discovery of nonequilibrium Green functions theory, see Sec. 3. At the same time it is of high interest to recall many of his other activities, see Sec. 4, including his contributions to strong field ionization, Sec. 4.1, and exciton physics, cf. Sec. 4.2.

Keldysh's heritage includes 77 scientific publications [1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,64,65,66,67,68,69,70,71,72,73,74,75,76,77], that reflect the broad range of topics, Keldysh was interested in and how this interest evolved over time. His two most influential papers on real time Green Functions [12] and strong field ionization [13] were published in 1964 in the Zhurnal Teoreticheskoi i Eksperimentalnoi Fiziki[1] and collected about 3300 and 6000 citations, respectively[2]. In fact, these two papers were written almost at the same time. They were submitted by Keldysh to the journal on April 23 and May 23 1964, respectively, making the year 1964 Keldysh's *annus mirabilis*. All articles of L.V. Keldysh are listed in chronological order in the reference section at the end of this paper.

Also, the activity of Keldysh in support of Russian science, in general, and the Academy of Sciences, in particular, is documented in 7 articles published between 1992 and 1999, [78,79,80,81,82,83,84]. They are interesting historical documents in their own but also show that Keldysh was speaking up publicly when he thought this is necessary, often together with other leading Russian colleagues. Some information on the often difficult political environment is contained in the biographical notes in Sec. 2. Moreover, Keldysh published a remarkable number of 61 short notes in honor of leading Russian physicists – a special tradition in Soviet and Russian science. These articles include 42 birthday congratulations and 29 obituaries.

Finally, we also include some remarks on Keldysh's students and Keldysh's work as a mentor, in Sec. 5.

[1] abbreviated ZhETF, the English translation is being published as JETP or Soviet Physics JETP)
[2] according to Google Schloar, December 2018

Figure 1 Leonid V. Keldysh (1931-2016), photograph from around 2010, provided by M. Sadovskii.

2 Biographical notes [85] Leonid Keldysh was born in Moscow on 7 April 1931 in a family of scientists. His mother, Lyudmila Vsevolodovna Keldysh, was a leading Soviet mathematician, her brother, an applied mathematician, Mstislav Vsevolodovich Keldysh was one of the leaders of the Soviet space program, later becoming the President of the USSR Academy of Sciences. Leonids's step father was Petr Sergeevich Novikov, a full member of the Academy and also a leading mathematician, while Leonid's younger step brother, Sergei Petrovich Novikov, also becoming a mathematician and Academy member, was later awarded the Fields medal. But Leonid's choice was theoretical physics.

In 1948 Keldysh enrolled in the Physics Department of Moscow State Lomonosov University (MGU), where he graduated in 1954 (attending also courses at the Department of mechanics and mathematics for an extra year). After this he started to work at the Theoretical Physics Department of the P.N. Lebedev Physics Institute (LPI) of the Academy of Sciences, which remained his work place until the end of his life. His scientific supervisor at LPI was Vitaly Lazarevich Ginzburg, and the Theoretical Department at that time was headed by Igor Evgenievich Tamm (both later becoming Nobel prize winners). However, since these early years, Leonid was essentially a self–made man in science.

In his early works (1957–1958) Keldysh developed a consistent theory of phonon-assisted tunneling in semiconductors which was immediately recognized by the semiconductor community. His most famous work of this period was devoted to the calculation of the electric field-induced shift of the absorption edge in semiconductors, what is now called the Franz–Keldysh effect [86,3], see Sec. 4.3.1. In the early 1960s he proposed to use spatial modulation of the lattice to create an artificial band structure [7]. This idea was later realized in semiconductor superlattices. He also developed an original theory of core levels in semiconductors [9]. One of his most famous works of this period was the 1964 theory of tunnel and (multi-)photon ionization of atoms by intense electromagnetic waves [13] that became the starting point for the entire field of intense laser–matter interaction, including atoms, ions, molecules, plasmas and solids, cf. Sec. 4.1. This field has recently been reviewed in Ref. [87], where it is concluded that the success behind the theory is that it precisely fulfills the criterion "making things as simple as possible, but not simpler". This feature is characteristic of many of Keldysh's other influential papers.

Leonid Keldysh started working in science during a period when quantum field theory methods were popular and successfully applied in condensed matter physics. Here he made his most famous contribution with his 1964 work on a general diagram technique for nonequlibrium processes [12]. Introducing Green functions with time–ordering along what is today known as the (Schwinger)-Keldysh time contour, he was able to construct the standard Feynman diagrams for these Green functions at finite temperatures and for general nonequilibrium states, see Sec. 3.

Figure 2 Leonid V. Keldysh around 1962, photograph provided by M. Sadovskii.

Strangely enough, even at that time, ten years after starting his work, he had not yet been awarded any higher scientific degree. However, when he finally submitted the Candidate of Science (PhD) thesis, in 1965, he was immediately awarded the degree of Doctor of Science (similar to habilitation in Germany). In 1968 he was elected corresponding member of the USSR Academy of Sciences, becoming a full member in 1976.

Since 1964 Keldysh's interests moved to semiconductors. In his work with Yu.V. Kopaev [15] he introduced the new concept of an excitonic insulator and to laser excited nonequilibrium exciton systems, exciton superfluidity [20] and their ioinization into an electron–hole quantum liquid of electron–hole droplets, for details see Sec. 4.2.

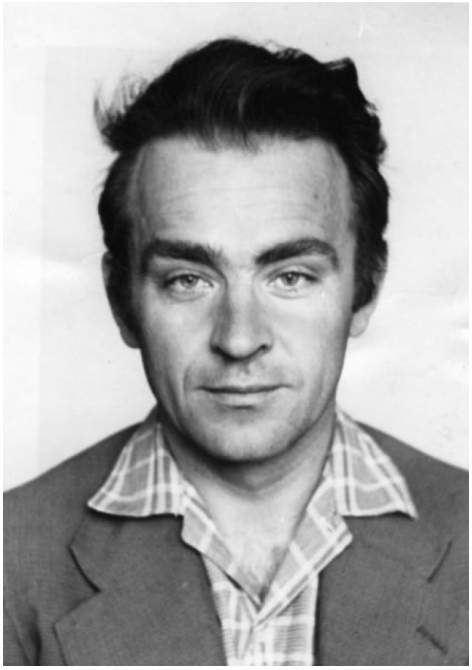

Figure 3 Leonid V. Keldysh around 1968, photograph provided by M. Sadovskii.

Since 1965 Keldysh was a professor at MGU heading the chair of quantum radiophysics (1978-2001). He had many PhD students, a number of which later became famous theoreticians, professors and members of the Russian Academy of Sciences, see Sec. 5. He was member of editorial boards of the leading Russian physics journals and served as Editor in Chief of Physics Uspekhi, from 2009 to 2016. Keldysh was awarded numerous prizes, including the Lenin prize (1974), the Hewlett–Packard Prize (1975), the Alexander von Humboldt Prize (1994), the Rusnanoprize (2009), the Eugene Feinberg Memorial Medal (2011), the Pomeranchuk Prize (2014) and the Grand Lomonosov Gold Medal of the Russian Academy of Sciences (2015). He was elected foreign member of the

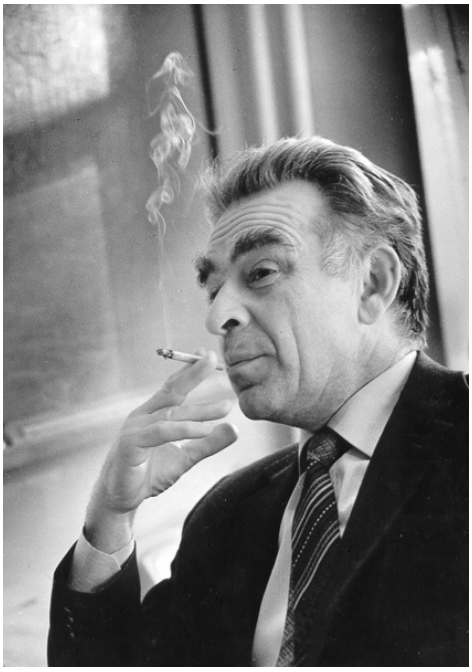

Figure 4 Leonid V. Keldysh around 1989, photograph provided by M. Sadovskii.

US National Academy of Sciences (1995) and became a Fellow of the American Physical Society in 1996.

In the late 1980s Keldysh had to perform various administrative duties, which he actually did not like at all, but considered impossible to reject during this difficult period for Russian Science. These included the head of the Theoretical Physics Department and the director of the Lebedev Institute (1989–1994) and also the position of a Secretary of the General Physics Department of the Russian Academy of Sciences (1991–1996). During this period he lived in his own way, never conforming to external circumstances. He always was a highly independent person, and it was impossible to persuade him to take a decision he did not agree with. He was among the leading RAS members who strictly rejected the Government–proposed reform of the Academy in 2013, becoming a member of the influential "Club of July 1" within the RAS, opposing this reform.

3 Nonequilibrium (Real-time or Keldysh) Green Functions (NEGF) Judging by its impact on a huge number of fields Keldysh's Real-time Green functions theory is, without question, his most important discovery, and we address it in some more detail.

3.1 The story of NEGF Quantum many-body systems have been described by many different approaches including wave function methods, reduced density operators (quantum BBGKY-hierarchy) of Bogolyubov, Krikwood and others, e.g. [88] as well as Green functions and Feynman diagrams by Schwinger, Dyson and Feynman.

Figure 5 Leonid V. Keldysh around 2006, photograph by Nikolay Gippius.

Following the results for the ground state soon the extension to thermodynamic equilibrium was developed in the 1950s by Matsubara, Kubo as well as Abrikosov, Gorkov, Dzyaloshinski in the U.S.S.R. which led to the concept of imaginary-time Green functions. The idea to rewrite the canonical density operator in thermodynamic equilibrium as a quantum-mechanical evolution operator, but in imaginary time, was then quite popular in a number of fields, including Feynman's path integral concept, so that step was rather natural.

However, the extension of the technique from thermodynamic equilibrium to *arbitrary nonequilibrium situations* is a huge step that is far less straightforward, and it took more time to develop. These developments occured almost independently in the U.S. and in the U.S.S.R. The works in the U.S. were mostly due to Martin and Schwinger who derived the generalization of the BBGKY-hierarchy to the case of many-time Green functions [89], and Baym and Kadanoff who derived and analyzed the generalization of the Boltzmann equation that includes memory effects [90]. These developments were reviewed in detail by Paul Martin and Gordon Baym in their lectures at the first Nonequilibrium Green Functions conference in Rostock, Germany, in 1999, cf. Refs. [91,92]. The Russian developments in the field of Nonequilibrium Green functions are due solely to Leonid Keldysh and were published in his seminal paper [12] where he introduced the "round trip time contour" – a small but ingeneous mathematical trick – that allowed him to rigorously extend Feynman's diagram technique to nonequilibrium. The Russian developments in thermodynamic and Nonequilibrium Green functions were reviewed by Alex Abrikosov [93] and Leonid Keldysh [66], respectively. The latter article is reprinted as a supplement [94] to this paper.

3.2 The PNGF conferences and Leonid Keldysh

Interestingly, after writing his paper introducing NEGF in 1964, Keldysh did not actively continue these developments (the same was true for Baym and Kadanoff). So it must have been a surprise for them that they were in 1999 invited to a conference entitled "Kadanoff-Baym equations–Progress and Perspectives for Many-Body Theory", 35 years after the original developments. In fact, in the 1970s and 1980s NEGF were used only by a few groups world wide but the activities increased significantly in the 1990s when NEGF methods were used in semiconductor optics and various groups learned to directly solve the Keldysh-Kadanoff-Baym equations (KBE) on modern computers, following the pioneering work of Danielewicz [95] on nuclear collisions. Not surprisingly, many theorists[3] expected that these equations would lead to breakthroughs in many fields which indeed turned out to be the case, see Sec. 3.3.

At the same time, the lengthy title of the conference in 1999 reflects some confusion in the community about the different contributions of the American and Russian founders of the theory and about the priorities. Even Baym was under the impression that Keldysh's work of 1964 was a follow up to their book [90], as he pointed out in his conference talk in 1999 and in the proceedings [92]. However, this was an incorrect assumption. Not surprisingly, Keldysh – who could not participate in the 1999 conference – was very upset when he became aware of Baym's article. He then took the opportunity to attend the second conference, "Progress in Nonequilibrium Functions (PNGF) II" in Dresden in 2002 and, in his lecture, to "straighten" things out. For everybody who uses NEGF today or will do so in the years to come, this turned out to be a very lucky case, because Keldysh summarized in some detail and in his honest style how his ideas emerged and who influenced him. We are lucky that he published his recollections in the conference book [101], and his article is reprinted as a supplement to this paper. A photo showing Leonid Keldysh at the PNGF II together with, among others, Alex Abrikosov, Pawel Danielewicz and Paul Martin is shown in Fig. 6.

The success story of nonequilibrium Green functions and the tremendous impact of Keldysh's paper [12] is clearly reflected in the next meetings of the conference series and their proceedings [96,102,103,104] culminating in the present issue of the proceedings of the 2018 conference.

3.3 Current research fields based on NEGF

During the last three decades NEGF have seen a dramatic increase in attention. This is mostly due to the increase in computing capabilities that have made direct solutions of the Keldysh-Kadanoff-Baym equations possible. Applications have been developed for a large number of fields where many-body effects, correlations and non-quasiparticle behavior are of relevance. This includes transport in metals [105], semiconductor optics and transport [100], nanostructures [106], atoms and

[3] Here we should mention, in particular, W. Schäfer [96,97], D. Kremp [98,99], H. Haug [100] and K. Henneberger in Germany and their schools.

Figure 6 Part of the participants of the Conference "Progress in Nonequilibrium Green Functions II", Dresden, August 2002. Front row from left: Alexei Abrikosov, Leonid Keldysh, Jörn Knoll, Pawel Danielewicz, Hendrik van Hees, Paul C. Martin. Part of the conference photo, from Ref. [101]. Second and third row, among others: Paul Gartner, Egidius Anisimovas, Vladimir Filinov, Robert van Leeuwen, Alexey Filinov, Roland Zimmermann, Antti-Pekka Jauho and Rolf Binder. Photograph by M. Bonitz

molecules [107,108], plasma physics [109,110], nuclear matter [111,112], cosmology [113,114], transport properties of strongly correlated cold fermionic atoms [115,116], among others.

4 Other research topics of L.V. Keldysh

4.1 Strong Field Ionization Cited more than 5500 times, Keldysh's paper [13] presented the first quantum theory of the ionization of an atom by an intense laser field. The paper introduced optical tunneling, multi-photon ionization, and above-threshold ionization, experimentally observed about 15 years later, e.g. [117]. Keldysh presented the first nonlinear quantum-mechanical calculation of the ionization probability of an atom in a strong electromagnetic field. Starting from the time-dependent bound state wave function (we follow the notation of Ref. [87]) $\Psi_0(t) = \psi_0(\mathbf{r})e^{iI_p t/\hbar}$ he computes the transition probability amplitude of the electron into a time-dependent continuum state in the presence of the field (i.e. a Volkov state [118]),

$$M(\mathbf{p}) = -\frac{i}{\hbar}\int_{-\infty}^{\infty} dt\, \langle \Psi_{\mathbf{p}}(t)|V_{\rm int}(t)|\Psi_0(t)\rangle, \qquad (1)$$

where I_p denotes the ionization potential, $\mathbf{p}$ the canonical momentum, and $V_{\rm int}$ the interaction potential of the electron with the field. Note that the explicit forms of $\Psi_{\mathbf{p}}$ and $V_{\rm int}$ depend on the chosen gauge, so the analysis requires some care. Indeed, many suggested modifications or improvements of Keldysh's work led to gauge-dependent results giving rise to debates in the community, for details see [87]. The momentum distribution of the photoelectrons is then $dW(\mathbf{p}) = |M(\mathbf{p})^2|d^3p$, and the total ionization probability is the momentum integral of dW. Using the dipole approximation for the field and neglecting Coulomb interaction and relativistic effects on $\Psi_{\mathbf{p}}$ Keldysh was able to obtain closed expressions for the ionization probability. The result contains an important dimensionless parameter – the "Keldysh parameter",

$$\gamma = \sqrt{2mI_p}\,\frac{\omega}{eE_0}\,, \qquad (2)$$

which determines the boundary between multiphoton and tunneling regimes. Here ω and E_0 are, respectively, the frequency and amplitude of the exciting electric field. The Keldysh parameter describes the ratio of the characteristic momentum of the electron in the bound state, $\sqrt{2mI_p}$,

to the momentum the electron gains from the field, eE_0/ω. For $\gamma < 1$ ($\gamma > 1$), ionization is dominated by the tunnel (multiphoton) mechanism. In case of a monochromatic field and $\gamma > 1$, multiphoton absorption is possible if the atom absorbs at least

$$N_{\min} = \frac{I_p + U_p}{\hbar\omega} + 1$$

photons which includes the average kinetic energy of the free electron in the field ("ponderomotive potential"), $U_p = (eE_0)^2/(4m\omega^2)$. If the photon number exceeds $N_{\min}$, ionization will lead to distinct peaks in the photoelectron energy spectrum – which has been called "above threshold ionization" – and has been accurately verified experimentally. With the dramatic progress in laser technology and the availability of coherent radiation sources from the infrared range to x-rays, these effects have achieved fundamental importance in countless fields.

Keldysh's theory triggered a tremendous wave of further improvements of the theory that include Coulomb interaction, relativistic effects or the field-induced modification of the bound states (Stark effect). The analysis of ionization processes was extended to more complex atoms, molecules and semiconductors, and similar approaches were developed for relativistic effects such as pair creation (Schwinger mechanism). For additional information and references, the reader is referred to the review [87].

4.2 Excitons and electron-hole systems Keldysh made important contributions to semiconductor physics. He was early on interested in the many–exciton problem in semiconductors. In his work with Yu.V. Kopaev [15] he introduced the new concept of an excitonic insulator. Actually this was a new mechanism of a metal–insulator transition. In later works by Keldysh and his collaborators it was shown conclusively, that there are no superfluidity properties in this model [27], as was initially suspected by some authors, and he moved to the study nonequilibrium systems of excitons, appearing under intense laser pumping of semiconductors, where superfluidity of excitons was shown to be possible [20][4]. However at that time (1968) Keldysh realized, that in most semiconductors (with multiple bands) the nonequilibrium system of many excitons actually transforms into an electron–hole quantum liquid (where excitons are ionized), forming electron–hole droplets. Interestingly enough, this idea was expressed only in his summary talk at the Moscow International Conference on Semiconductors [119] and was not published anywhere for a rather long time. However, it immediately stimulated experimental studies, and electron–hole droplets were soon discovered, leading to many further experimental and theoretical works on this new state of matter. Essentially, he supervised these works around the Soviet Union, continuing to introduce new concepts, such as the phonon "wind" in the system of electron–hole droplets [35]. An overview of the field of electron-hole droplets can be found in the review [120]. Electron-hole droplet formation was also verified in *ab initio* quantum Monte Carlo simulations [121]. The problem of limited life time of electron-hole pairs in optically excited semiconductors can be overcome with indirect excitons predicted by Lozovik and co-workers [122] which have interesting superfluidity properties [123].

4.3 Further research results Even though Keldysh is mainly famous for real-time Green functions, strong-field ionization and the theory of excitons, he has made important contributions to many other fields.

4.3.1 Franz-Keldysh effect It was a natural question to ask whether the Franz-Keldysh effect (the shift of the absorption edge due to an applied static electric field) could be extended to a situation where the absorbing sample was placed in a time-dependent field. Indeed, early theoretical work addressed some aspects of this situation[124, 125]. Experimentally, however, sufficiently strong time-dependent fields were not available until the first free-electron lasers started operation. A detailed study was published in Ref. [126], where the excitonic absorption of a quantum well system was studied as a function of the frequency of the impinging strong THz field emanating from the Santa Barbara free-electron laser. The theory developed for this situation agreed very well with the observations. The theory combines three concepts in whose development the pioneering ideas by Keldysh were crucial: strong field effects in semiconductors, excitonic dynamics, and nonequilibrium Green's functions. It is remarkable that all three ingredients originate from the same author.

4.3.2 Transport in mesoscopic systems The scattering theory of transport, developed by Landauer and Büttiker [127,128], which expresses the conductance of a mesoscopic sample in terms of its transmission properties, is – despite of its huge success and importance – only valid for systems where electron-electron or electron-phonon interactions can be ignored. The Keldysh diagram technique, which allows for a systematic treatment of interactions, is particularly well-suited for deriving extensions of the Landauer-Büttiker formalism. The Keldysh technique, as applied to transport physics, was introduced in the Western literature in an important series of papers by Caroli, Combescot, and co-workers [129,130,131,132]. These papers were mainly concerned with tunneling through a single barrier (including interactions with localized states and phonons in the barrier), but a real breakthrough occurred in 1992, when Meir and Wingreen showed [133] that the calculation of the conductance through a quantum dot with arbitrary interactions could be formulated in a similar manner. Literally thousands of papers have examined transport in situations where interactions are important.

[4] Here also a less known paper of 1972 on the coherent state of excitons should be mentioned, that was recently reprinted in Ref. [76] with comments by M. Sadovski. There Keldysh derived what is now called the Gross-Pitaevkii equation for coherent excitonic state in an external electromagnetic field.

One example is the tunneling of electrons between a tip and a metal through a single adsorbed molecule (or atom) in a scanning tunneling microscope. The Keldysh technique provides an elegant way to describe the inelastic stationary electron tunneling with emission and absorption of vibrational excitations of the molecule, interactions with the phonon baths in the substrate and tip, as well as the overheating of the molecule and its resulting motion – hopping or rotation [134, 135, 136, 137, 138]. Keldysh's theory provides the theoretical basis for inelastic tunneling electron spectroscopy, single molecule chemistry and motors, for details see, e.g., the text book [139].

The approach can be generalized to time-dependent situations [140], or situations where the partitioning of the system into separate leads and a central region must be re-examined [141]. The next level of abstraction can be achieved by formulating the nonequilibrium theory in a field-theory language. This powerful formulation has found a very large number of applications, which are reviewed, e.g. in a recent advanced text-book [142]. The field-theory formulation honors Keldysh by employing many technical terms that commemorate their inventor, e.g., Keldysh rotation, or Keldysh action.

4.3.3 The Rytova-Keldysh potential In 1979 Keldysh considered the Coulomb interaction in thin semiconductor and semimetal films, and proposed a form for the interaction potential between charged particles in such systems [41]. (Work along similar lines was reported earlier by Rytova [143]). A central theme in condensed matter physics in our millenium is concerned with two-dimensional materials, such as graphene, or transition metal dichalcogenides. The Rytova-Keldysh potential forms an important ingredient in the physics of these materials. Recent developments are reviewed, e.g. in [144] where many references to related work can be found.

4.3.4 Stochastic methods applied to the Keldysh contour The idea of treating quantum many-body systems out of equilibrium on the Keldysh time contour has been extended to various other methods. A stochastic sampling method of Feynman diagrams was developed by Werner *et al.* and is known as diagrammatic Monte Carlo, see [145] and references therein. Diagrammatic Monte Carlo extends earlier equilibrium simulations such as the continuous-time quantum Monte Carlo method for fermions [146] to arbitrary nonequilibrium situations. While it formally can treat strongly coupled systems and is successfully used in condensed matter systems and for cold atoms, it suffers from the dynamic fermion sign problem that strongly limits the simulation duration.

5 The Keldysh school Actually Keldysh's scientific interests were much broader than one could judge from his list of publications. This is, in part, reflected in the broad range of topics his PhD students worked on, see Sec. 5. One of us (MS) recalls "I first met him in 1969 when I was a third year student of the Ural State University and attended his lectures on exciton condensation and electron–hole droplets at the famous winter school on theoretical physics "Kourovka" near Sverdlovsk (now Ekaterinburg). In 1971 I became his PhD student at the Lebedev Institute in Moscow and, to my surprise, he proposed to me a PhD topic related to the construction of the theory of "liquid semiconductors"—a research field developed previously in the experimental works of the Ioffe–Regel group in Leningrad and still lacking serious theoretical foundation. This reflected Keldysh's interest in the general theory of electrons in disordered systems, being only developed at that time in the classical works of Neville Mott, Ilya Lifshits and Philip Anderson. In the following years we tried (in fact more or less in vain!) to construct such a theory. Our main idea was to produce a theoretical model of the pseudogap—a concept introduced by Mott on qualitative grounds to explain electronic properties of amorphous and liquid semiconductors. Here we were successful and formulated an exactly solvable model of the pseudogap, based on the summation of a complete series of Feynman diagrams for a simplified 1D model. Actually Keldysh declined co–autorship, so these results appeared under my name only, forming the ground for my future work in many years to follow, leading eventually to the studies of the pseudogap problem in high-T_C superconductors. This model was, in fact, a generalization of a similar diagram summation in Keldysh's studies of doped semiconductors, which appeared only in his dissertation (1965) and was later used or rediscovered by others. These are only few of many examples of his unpublished results. Most of them he was writing in large notebooks at his home, which some of his students were lucky enough to see."

The list of Keldysh's PhD students includes Yu. V. Kopaev, D.I. Khomski, R.R. Guseinov, V.S. Babichenko, B.A. Volkov, M.V. Sadovskii, A.P. Silin, V.E. Bisti, A.V. Vinogradov, S.G. Tikhodeev, E.A. Andryushin, T.A. Onishchenko, N.S. Maslova, and P.I. Arseev, and their year of graduation and research topics are presented in table 1. Many of them became successful scientists themselves. Kopaev, Khomskii, Volkov, Sadovskii, Tikhodeev, Ivanov, Arseev, Maslova, and Gippius later did their habilitation. Gippius, Khomskii, Sadovskii, Tikhodeev, and Vinogradov became professors. Arseev became a corresponding member and Kopaev and Sadovskii full members of the Russian Academy of Sciences.

6 Conclusions There have been a number of obituaries for Keldysh in the U.S. [85] and in Russia [147] that have covered various sides of Keldysh's scientific work and personality. The 2017 special issue of Physics Uspekhi (issue 11, volume 60) covers in detail Keldysh's scientific work. There is no need to reproduce this material here. Instead, we have taken the particular angle of view on Keldysh that concentrates on his contributions to nonequilibrium many-body physics, in general, and nonequilibrium Green functions, in particular. Keldysh's single pa-

Table 1 List of Keldysh's PhD (above the line) and master students (below) in chronological order, their year of graduation and their scientific topics. See also the list of references at the end of the paper.

Name	Graduation	Research topics
Yu. V. Kopaev	1965	Semimetal-dielectric phase transitions
D. I. Khomskii	1969	Systems with strong electronic correlations
R. R. Guseinov	1971	Electron-phonon interaction in systems with excitonic instabilities
M. V. Sadovskii	1974	Liquid semiconductors, Pseudogap, Disorder and Fluctuation effects on the 1D Peierls transition
A. P. Silin	1975	Condensation of excitons in semiconductors
B. A. Volkov	1976	Electronic properties of semiconductors with structural instabilities
A. V. Vinogradov	1976	Electronic mechanisms of light absorption of dielectrics in transparency range
E. A. Andryushin	1977	Electron-hole liquid in layered semiconductors
V. S. Babichenko	1977	Electron-hole liquid in strongly anisotropic semiconductors and semimetals
T. A. Onishchenko	1977	Electron-hole liquid in strong magnetic field
V. E. Bisti	1978	Exciton interactions in semiconductors
S. G. Tikhodeev	1980	Interaction of electron-hole liquid in semiconductors with deformations. Nonequilibrium diagram technique for relaxation processes
A. L. Ivanov	1983	Intensive electromagnetic wave in a direct-gap semiconductor
I. M. Sokolov	1984	Localization in the Anderson model with correlated site energies, percolation theory
P. I. Arseev	1986	Electrodynamics of rough surfaces of metals and semiconductors
N. S. Maslova	1987	Resonant interaction of light with a system of nonlinear oscillators. Non-equilibrium transport through correlated systems
N. A. Gippius	1988	Quantum reflection of an exciton from the surface of an electron-hole droplet. Interaction of electromagnetic radiation with semiconductors
S. S. Fanchenko	1975	Generalized diagram technique of non-equilibrium processes. The problem of arbitrary initial conditions

per on the subject [12] has dramatically changed the whole field providing us and future generations with a strict mathematical basis and an extremely powerful and fully general tool – nonequilbrium diagram technique. This has allowed the NEGF approach to being introduced in an enormously broad range of areas of physics and quantum chemistry, not just as a tool for recovering familiar kinetic equations or deriving improved approximations but, more and more, as a practical tool for quantitative analysis of time-dependent processes on all time scales. Judging by the impressive increase in the number of publications on NEGF, Keldysh's work has provided a tremendously fertile ground for theory developments in many decades to come.

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