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Full-scale CFD simulation of tsunamis. Part 1: Model validation and run-up

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Abstract

This paper presents numerical simulations of the propagation, shoaling and run-up of full-scale tsunami waves. The simulations are performed with a model solving Reynolds-averaged Navier-Stokes equations with $k-\omega$ turbulence closure, and is one of very few studies involving CFD simulations at full tsunami scale, involving full resolution of small scale dispersive effects as well as wave breaking. It is demonstrated that a combination of previous analytical and empirical expressions for run-up heights and inundation speeds match those simulated well. This indicates that these are reasonable first approximation even in cases where the underlying assumption of linearity of the incoming tsunami is violated, as well as in cases where breaking occurs, though they generally slightly underestimate both run-up height and inundation speed. It is shown that the run-up of tsunamis can manifest in different ways depending on the initial wave shape and slope of the coast, and three qualitative run-up types previously identified in the literature are described in detail. It is further shown that the shorter waves of an undular bore, which appear during lengthy propagation in shallow water, can either maintain their shape the entire distance to shore, or break far offshore creating a breaking bore. It is demonstrated the front of the breaking bore will not appear as steep for N-waves as single waves because these need to re-wet the drawn-down region before reaching the original shoreline. Finally, the importance of shorter waves riding on the front of the tsunami is discussed. It is shown that they have little impact on the run-up height and inundation speed, but are important in terms of local flow velocities. The results presented here are Part 1 of a larger study, where Part 2 involves details of the tsunami-induced boundary layer dynamics, bed shear stresses

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and implication for sediment transport.

Keywords: Tsunamis, Computational Fluid Dynamics, Run-up, inundation, RANS, Turbulence modelling

1. Introduction

1 When tsunamis hit a coast they are potentially catastrophic as seen in
2 the two recent major tsunami incidents, the Boxing day tsunami in 2004
3 in the Indian Ocean, and the Tohoku tsunami in Japan in 2011 where ap-
4 proximately 230,000 and 20,000 people were killed, respectively (Suppasri
5 et al., 2012). These tsunamis also caused severe damage to buildings and
6 structures, and in many places houses and bridges were washed away. With
7 the destructive force of the tsunamis, run-up and inundation have naturally
8 received considerable attention in the past.

9 Experimentally, however, the run-up of tsunami waves is difficult to
10 study due to the scales involved, even at model scale with a large scal-
11 ing factor. Jiang et al. (2015) boldly stated that: "both the length and
12 time scales of an actual tsunami wave cannot be down-scaled in wave flume
13 experiments according to the Froude similarity law." A similar point was
14 made by Chen et al. (2012). As a result many have used solitary or other
15 short wave forms, which might be interesting in themselves (and useful e.g.
16 for model validation), but their resemblance to geophysical tsunamis is at
17 best questionable (Madsen et al., 2008).

18 Despite the challenges mentioned above a limited number of studies have
19 been made where reasonable scaling was achieved, using large scale facili-
20 ties. Matsuyama et al. (2007) did a properly scaled tsunami experiment
21 in their 200 m long wave flume. Their waves were sinusoidal and ran up
22 different bathymetries. While propagating, the waves developed into undu-
23 lar bores which shoaled, and the individual short waves ultimately broke.
24 They did not focus on run-up height, but rather on wave transformation,
25 wave breaking and velocities during breaking. To our knowledge this is the
26 first experiment of its sort where undular bores showed up in an experi-
27 mental study involving the run-up of tsunamis. Recently Schimmels et al.
28 (2016) and Sriram et al. (2016) generated properly scaled tsunami waves
29 with a piston wave-maker. The waves propagated on a flat bed, before run-
30 ning up a $1/6$ slope. Here the run-up heights were not measured, but they
31 demonstrated the splitting of the wave front into an undular bore, similar to
32 Matsuyama et al. (2007), and they also showed the reflected wave. Others
33 have achieved reasonable scaling using other generation methods, see e.g.

34 the pump generated tsunami experiments by Rossetto et al. (2011), Gose-
35 berg et al. (2013), Goseberg (2013), Drähne et al. (2016) and Larsen et al.
36 (2018a).

37 The run-up of tsunamis has also been studied numerically, often using
38 either non-linear shallow water (NLSW) or Boussinesq models. For good
39 reviews of NLSW models applied on tsunamis, the reader is referred to
40 Synolakis and Bernard (2006) and Levin and Nosov (2016). Despite not
41 being able to handle dispersion NLSW models have, as stated by Madsen
42 et al. (2016), had reasonable success in simulating geophysical tsunamis.
43 Over long propagation distances however, a dispersive tail might evolve,
44 and when the undular bores form in shallow water dispersion likewise be-
45 comes important. (For a more in depth discussion on the role of dispersion
46 for tsunami waves please see Grue et al. (2008), Kim and Lynett (2011),
47 Glimsdal et al. (2013), Løvholt et al. (2012), Grilli et al. (2013).) Dispersion
48 can obviously be handled by Boussinesq type models (see e.g Lynett and
49 Liu (2002), Lynett and Liu (2005) and Fuhrman and Madsen (2009)), but
50 such models are limited to a single-valued free surface and still require an
51 empirical breaking model.

52 Computational Fluid Dynamics (CFD) models, on the contrary, can in
53 principal handle both non-linearity and dispersion, and in contrast to NLSW
54 and Boussinesq models, actually resolve breaking and bottom boundary
55 layer processes directly. CFD studies of tsunamis are few, however, due
56 to their computational expense. Horrillo et al. (2006) simulated the propa-
57 gation of the Indian Ocean tsunami using both a NLSW model, a Boussi-
58 nesq model and a Reynolds-averaged Navier-Stokes (RANS) model and con-
59 cluded that over long propagation distances dispersion became important,
60 which only the Boussinesq and CFD model could describe. Horrillo et al.
61 (2006) did not go into details regarding the simulated run-up with the CFD
62 model, however. Biscarini (2010) first validated their CFD model against
63 an experimental landslide-generated tsunami, and was subsequently able
64 to accurately predict the run-up height of the 1958 Lituya Bay (Alaska)
65 landslide-generated tsunami. Montagna et al. (2011) validated their CFD
66 model (Flow 3D) with an experimental landslide generated tsunami and
67 concluded that the model was able to accurately reproduce surface eleva-
68 tions near the generation as well as in inundated areas, but did not proceed
69 to simulate full-scale tsunami events. Tomita and Takahashi (2014) used
70 the STOC-IC model, developed by Tomita et al. (2006), and were able to
71 accurately reproduce an experiment where undular bores showed up at the
72 tsunami wave front. The STOC-IC model, however, is not a standard CFD
73 model, as the surface elevation is solved for using a depth-averaged con-

74 tinuity equation. This mean that this type of model cannot handle the
 75 overturning of the breaking waves as well as the mixture of air and water
 76 that occur e.g. in the front of a breaking bore. Qu et al. (2017) compared
 77 run-up heights of properly scaled tsunamis with those obtained using a soli-
 78 tary wave. Similar to Madsen et al. (2008) they concluded, that there were
 79 great differences between solitary waves and properly scaled tsunami waves.
 80 Finally, Aniel-Quiroga et al. (2018a) used a hybrid approach, coupling a
 81 NLSW model for the initial propagation with a RANS model for the run-
 82 up, to simulate the run-up of tsunamis for a wide range of topographies and
 83 initial tsunami waves. Based on these they created an interpolation model
 84 which could output the run-up height for different input topographies and
 85 wave shapes. They showed that this approach compared reasonably well
 86 with past measured tsunami run-up heights. For a more in-depth review of
 87 tsunami literature focusing on propagation and run-up please see the PhD
 88 thesis by Larsen (2018).

89 Despite the large number of studies on tsunami run-up, the process is
 90 still not fully understood in detail. In this work we will use CFD to study
 91 and clarify how tsunamis run-up in different scenarios, both in terms of run-
 92 up height, inundation speed as well as a more qualitative description of the
 93 run-up sequence. With the latter we intend to focus on the various ways
 94 tsunamis can appear at the coast. Often in experiments a breaking bore is
 95 assumed either generated through dam breaks (see e.g. Yeh, 1991; Derschum
 96 et al., 2018; Stolle et al., 2018) or as a result of breaking elongated solitary
 97 waves (see e.g. Goseberg et al., 2016; Nistor et al., 2017), but a tsunami can
 98 run-up in other ways as well. We wish to characterise these scenarios and
 99 describe when they appear. Finally, as the CFD model naturally handles
 100 dispersion and resolves the breaking process directly, this study also aims to
 101 investigate the quantitative importance of shorter dispersive waves, some-
 102 times appearing at the tsunami wave front, on both the run-up, inundation
 103 speed and local flow velocities. The generated knowledge can potentially
 104 be important for hazard assessment by describing run-up heights and in-
 105 undation speeds for different scenarios, as well as design of experimental
 106 campaigns (as it can help identify more appropriate run-up scenarios).

107 The simulations presented in the present paper are also used in Larsen
 108 and Fuhrman (2018a) (hereafter referred to as Part 2). Here details of the
 109 tsunami-induced boundary layer dynamics, bed shear stresses and implica-
 110 tions for sediment transport are discussed.

111 The remainder of the study is organized as follows: Model description
 112 and boundary conditions are given in Section 2. In Section 3 the model
 113 is validated first for the propagation of long non-linear waves over large

distances by comparing with results from Sriram et al. (2016), and then for its performance on surface elevations and bed shear stresses in the surf and swash zone by comparing to measurements of Sumer et al. (2011). In Section 4 the case selection and model setup is described. Section 5 shows the model results for run-up heights and inundation speed of two archetype tsunami signals using different slopes. Section 6 entails a detailed description of different run-up types, and in Section 7 a discussion of when the different run-up types appear is performed. Finally, in section 8 overall conclusions are drawn.

2. Model description

The simulations are performed using the two-phase volume-of-fluid method (VOF) flow model **waves2FOAM** developed by Jacobsen et al. (2012). Here the Reynolds-averaged Navier-Stokes (RANS) (1) and continuity (2) equations are solved:

$$\frac{\partial \rho u_i}{\partial t} + u_j \frac{\partial \rho u_i}{\partial x_j} = -\frac{\partial p^*}{\partial x_i} - g_j x_j \frac{\partial \rho}{\partial x_i} + \frac{\partial}{\partial x_j} (2\mu S_{ji} + \tau_{ij}), \quad (1)$$

$$\frac{\partial u_i}{\partial x_i} = 0. \quad (2)$$

Here u_i are the ensemble averaged components of the velocities, x_i are the Cartesian coordinates, $\mu = \rho\nu$ is the dynamic molecular viscosity, ν is the kinematic viscosity, ρ is the density, p^* is the pressure in excess of hydrostatic, t is time, S_{ij} is the mean strain rate tensor given by

$$S_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \quad (3)$$

and τ_{ij} is the Reynolds stress tensor, which is expressed according to the Boussinesq approximation

$$\frac{\tau_{ij}}{\rho} = -\overline{u'_i u'_j} = 2\nu_T S_{ij} - \frac{2}{3} k \delta_{ij}. \quad (4)$$

Here the overbar signifies time (ensemble) averaging, ν_T is the eddy viscosity, δ_{ij} is the Kronecker delta, and

$$k = \frac{1}{2} \overline{u'_i u'_i} \quad (5)$$

136 is the turbulent kinetic energy density. In the above a prime superscript
137 denotes turbulent (fluctuating) velocity components.

138 To close the system the k - ω model of Larsen and Fuhrman (2018b) is
139 used, which is an extension of the Wilcox (2006) model. This model solves
140 transport equations for k

$$\frac{\partial \rho k}{\partial t} + u_j \frac{\partial \rho k}{\partial x_j} = \rho P_k - \rho P_b - \rho \beta^* k \omega + \frac{\partial}{\partial x_j} \left[\left(\mu + \rho \sigma^* \frac{k}{\omega} \right) \frac{\partial k}{\partial x_j} \right] \quad (6)$$

141 and the specific dissipation rate ω :

$$\begin{aligned} \frac{\partial \rho \omega}{\partial t} + u_j \frac{\partial \rho \omega}{\partial x_j} = & \rho P_\omega - \rho \beta \omega^2 + \rho \frac{\sigma_d}{\omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j} + \\ & \frac{\partial}{\partial x_j} \left[\left(\mu + \rho \sigma \frac{k}{\omega} \right) \frac{\partial \omega}{\partial x_j} \right]. \end{aligned} \quad (7)$$

142 The shear production term for k is

$$P_k = \tau_{ij} \frac{\partial u_i}{\partial x_j} = p_0 \nu_T, \quad p_0 = 2 S_{ij} S_{ij}. \quad (8)$$

143 Similarly, the buoyancy production for k is formulated as

$$P_b = -\frac{g_i}{\rho} \overline{\rho' u_i'} = p_b \nu_T, \quad p_b = \alpha_b^* N^2, \quad N^2 = \frac{g_i}{\rho} \frac{\partial \rho}{\partial x_i}, \quad (9)$$

144 where $(g_1, g_2, g_3) = (0, 0, -g)$ is gravitational acceleration and N^2 is the
145 square of the Brunt-Vaisala frequency. The production of ω is likewise taken
146 as

$$P_\omega = \alpha \frac{\omega}{k} \frac{\tilde{\omega}}{\tilde{\omega}} P_k = \alpha \frac{\omega}{\tilde{\omega}} p_0. \quad (10)$$

147 In this model the eddy viscosity is defined as

$$\nu_T = \frac{k}{\tilde{\omega}} \quad (11)$$

148 with

$$\tilde{\omega} = \max \left[\omega, \lambda_1 \sqrt{\frac{p_0 - p_b}{\beta^*}} \right], \quad (12)$$

$$\tilde{\omega} = \max \left[\tilde{\tilde{\omega}}, \lambda_2 \frac{\beta}{\beta^* \alpha} \frac{p_0}{p_\Omega} \omega \right]. \quad (13)$$

149 The formulation of the Larsen and Fuhrman (2018b) k - ω model above was
 150 designed to maintain stability in potential flow regions, while defaulting to a
 151 standard closure in sheared regions. Mayer and Madsen (2000) showed that
 152 two-equation closure models could be unstable in the potential flow region
 153 beneath surface waves, resulting in exponential growth of the turbulent ki-
 154 netic energy and eddy viscosity, leading to wide-spread over production of
 155 turbulence in the literature (see e.g the recent state-of-the art applications
 156 of Brown et al. (2016)). As shown by Larsen and Fuhrman (2018b), the
 157 inclusion of the two stress limited versions of ω (i.e. $\tilde{\omega}$ and $\tilde{\tilde{\omega}}$) arranged as
 158 above, formally solves this wide-spread and long-standing problem.

159 The standard closure coefficients utilized are those of Wilcox (2006):
 160 $\alpha = 0.52$, $\beta = 0.0708$ (constant for 2D flows), $\beta^* = 0.09$, $\sigma = 0.5$, $\sigma^* = 0.6$,
 161 $\sigma_{do} = 0.125$, with

$$\sigma_d = H \left(\frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j} \right) \sigma_{do}, \quad (14)$$

162 where $H(\cdot)$ is the Heaviside step function, which takes a value of unity if the
 163 argument is positive and zero otherwise. Additionally, we adopt the value
 164 $\alpha_b^* = 1.36$, which was derived by Larsen and Fuhrman (2018b), as well as
 165 the stress limiting coefficients $\lambda_1 = 0.2$ and $\lambda_2 = 0.05$.

166 A scalar field γ is used to track the two fluids, where $\gamma = 0$ represents
 167 pure air and $\gamma = 1$ pure water, with any intermediate value representing a
 168 mixture. The distribution of γ is governed by the advection equation

$$\frac{\partial \gamma}{\partial t} + u_j \frac{\partial u_i \gamma}{\partial x_j} + u_j^r \frac{\partial u_i^r \gamma (1 - \gamma)}{\partial u_j^r} = 0 \quad (15)$$

169 where u_j^r is a relative velocity used to compress the interface. The method
 170 is developed by OpenCFD, and it is documented in Berberovic et al. (2009).
 171 Any fluid property Φ in the flow is assumed to be given by

$$\Phi = \gamma \Phi_{water} + (1 - \gamma) \Phi_{air} \quad (16)$$

172 The boundary conditions are as follows: The bottom will have a no-
 173 slip condition imposed such that velocities are zero. For the turbulence
 174 quantities at the bottom a generalized wall function approach is used. The
 175 friction velocity is determined from the tangential velocity at the nearest

176 cell center, based on the profile of Cebeci and Chang (1978):

$$\frac{u}{U_f} = 2 \int_0^{y_c^+} \frac{dy^+}{1 + [1 + 4\kappa^2(y^+ + \Delta y_{cc}^+)^2 C]^{1/2}}, \quad (17)$$

$$C = [1 - \exp(-(y^+ + \Delta y_{cc}^+)/25)]^2, \quad (18)$$

$$\Delta y_{cc}^+ = 0.9 \left[\sqrt{k_s^+} - k_s^+ \exp\left(-\frac{k_s^+}{6}\right) \right], \quad (19)$$

177 who generalized the van Driest (1956) profile to incorporate potential rough-
 178 ness effects, with $y_c = \Delta y/2$ being the normal distance from the wall to the
 179 nearest cell center, where Δy is the thickness of the near wall cell, $k_s = 2.5d$
 180 is Nikuradse's equivalent sand roughness and $y_c^+ = y_c U_f / \nu$. The boundary
 181 conditions for k and ω are then as described by Fuhrman et al. (2014)

$$\frac{k}{U_f^2} = \min \left\{ A y_c^{+2}, \frac{1}{\sqrt{\beta^*}} \right\}, \quad (20)$$

$$\frac{\omega \nu}{U_f^2} = \max \left\{ \frac{B}{y_c^{+2}}, \frac{1}{\sqrt{\beta^*} \kappa y_c^+} \right\}. \quad (21)$$

182 The first arguments in these functions ensure that these variables follow their
 183 proper scaling $k \sim y^2$ and $\omega \sim 1/y^2$ for near wall cells within the viscous sub-
 184 layer (see e.g. Wilcox (2006)). The values $A = 1/(\delta^{+2} \sqrt{\beta^*}) = 0.02466$ and
 185 $B = \delta^+ / (\sqrt{\beta^*} \kappa) = 96.885$ are utilized, which ensure a continuous transition
 186 to the (fully-turbulent) second arguments at $y_c^+ = \delta^+$, where $\delta^+ = 11.626$ is
 187 taken as the viscous sub-layer thickness (in dimensionless wall coordinates).
 188 In the first cells nearest the bed, the eddy viscosity is not calculated from
 189 (11), but is instead calculated from

$$U_f^2 = \frac{\tau_b}{\rho} = (\nu + \nu_T) \frac{dU}{dz}. \quad (22)$$

190 The wall functions described above have been used previously to a great
 191 extent, see e.g. Baykal et al. (2015), Larsen et al. (2016), Bayraktar et al.
 192 (2016) and Larsen and Fuhrman (2018b).

193 The tsunami waves in this study, will be specified either as initial con-
 194 ditions or generated at the inlet. Specifically these will be represented by a
 195 general N-wave form (the summation of a positive and negative single wave)
 196 inspired by Madsen and Schäffer (2010), but with x variation now included.
 197 The free-surface is given by

$$\eta(x, t) = A_1 \operatorname{sech} \left[\Omega_1 \left((t - t_1) - \frac{x - x_0}{\sqrt{gh}} \right) \right]^2 - A_2 \operatorname{sech} \left[\Omega_2 \left((t - t_2) - \frac{x - x_0}{\sqrt{gh}} \right) \right]^2 \quad (23)$$

where A_1 and A_2 are the amplitudes of the two single waves, the effective frequencies Ω_1 and Ω_2 define the time span of the two single waves according to $\Omega_i = 2\pi/T_i$ where T_i can be interpreted as the effective period of the corresponding single wave, defined as the time separating where the surface elevations drops below 0.7% of the amplitude. The variable x_0 defines the center of the wave and t_1 and t_2 can be used to phase shift the two single waves. This signal can be reduced to a single (direction) wave by setting $A_2=0$. Please note that this formulation is different than the solitary wave as the period and wave height can be chosen independently. The solitary wave can be viewed as a special case of the single wave with $\Omega_1 = \sqrt{3gA_1/(4h)}$. The horizontal velocity is then given based on linear shallow water theory

$$u(x, t) = \sqrt{\frac{g}{h}} \eta(x, t) \quad (24)$$

The vertical velocity is derived from the local continuity equation, and the pressure is calculated as hydrostatic.

3. Model validation

Before simulating the actual tsunami-scale events it is important to validate that the model can, in fact, handle the necessary and relevant physics. Therefore it will be shown that the model is capable of handling the propagation and wave deformation of long waves over long distances. It will likewise be shown that the model can handle potential breaking of short waves as well as run-up in situations where breaking occurs.

3.1. Long wave propagation over long distances

In this section the model results are compared to one of the experiments of Sriram et al. (2016). Sriram et al. (2016) investigated how long waves with different shapes and amplitudes propagated on a flat bed and subsequently ran up a steep slope of $S=1/6$. The model will be compared to the test named "2/3" by Sriram et al. (2016). The incoming wave was a single wave (hence $A_2 = 0$), with a period $T = 30$ s, and an amplitude $A_1 =$

Figure 1: Layout of the computational domain for the Sriram et al. (2016) experiment. Note that this sketch is not to scale.

0.12 m at a water depth $h = 1$ m. This wave signal was used as an inlet condition with the inlet of the flume positioned at $x = 0$ m and the toe of the slope at $x = 251.5$ m. Figure 1 shows a sketch of the domain used in the simulations. The mesh in the majority of the domain has a height of $\Delta y = 0.01$ m and $\Delta x = 0.02$ m. This means that 12 cells are used to discretize the amplitude. This was shown by Larsen et al. (2018b) to be a reasonable resolution for propagating waves. All forthcoming simulations will have at least this number of cells per wave amplitude and keep the same aspect ratio, $\Delta x/\Delta y$. Near the bed the cells were gradually refined in the vertical, with near bed cells having $\Delta y = 6 \cdot 10^{-4}$ m. In this and all future simulations the time step was adjusted such that a maximum Courant number $Co = |u_i|\Delta t/\Delta x_i = 0.15$ is maintained at all times. This is lower than used in most CFD studies, but a low Courant number was shown by Larsen et al. (2018b) to be necessary to accurately simulate free surface waves with the `interFOAM` solver.

Figure 2 shows the modelled and experimental surface elevations at two

Figure 2: Comparison between modelled (—) and experimental (o) from Sriram et al. (2016) surface elevations at (a) $x=60$ m and (b) $x=225$ m. Included in the figures is also the initial time-shifted single wave (---) to illustrate how the wave deforms while propagating.

different locations in the flume. Included for comparison as a dashed line is also the initial analytical single wave expression (see again (23)), with the peak shifted to match the experiments. It can be seen that at $x = 60$ m (Figure 2a) the wave has deformed slightly, now having a steeper wave front, compared to the single wave expression. At $x = 225$ m (Figure 2b) the wave is starting to split and two clear peaks are visible. The third peak in Figure 2b (at approximately $t=117$ s) is the reflected wave. In general the comparison between the CFD model and the experiment is good. The root mean squared errors (RMSE) between the experiments and the model are $\text{RMSE}=0.005$ m and $\text{RMSE}=0.006$ m (i.e. approximately half the thickness of a cell) at $x = 60$ m and at $x = 225$ m respectively. The model captures both the initial deformation of the wave (Figure 2a) and the subsequent split and reflection (Figure 2b). This shows that the model, with the present mesh resolution, is capable of handling both dispersion and non-linearity. That the model is able to capture the reflection accurately also suggests that it is capable of handling the run-up and run-down of a non-breaking long single wave accurately.

3.2. Surface elevations and bed shear stresses in the surf and swash zone

Having shown that the model can handle propagation, deformation and reflection of long waves, we will now validate that the model can handle the more complex surf and swash zone. The model will therefore be compared

Figure 3: Layout of the computational domain and positions of the measurement sections for the Sumer et al. (2011) experiment. Note that this sketch is not to scale.

262 with experimental results involving a plunging solitary wave by Sumer et al.
 263 (2011). Here a solitary wave with amplitude $A_1 = 0.071$ m was generated on
 264 a water depth $h = 0.4$ m. As mentioned, the solitary wave can be viewed as a
 265 special case of the single wave with $\Omega_1 = \sqrt{3gA_1/(4h)}$. In the experiments,
 266 the wave propagated on a flat bed which was 8.01 m long, before continuing
 267 up a slope of $S \approx 1/14$. Surface elevations were measured with wave gauges
 268 at nine different cross-shore positions, and bed shear stresses were measured
 269 with hot film at eight different cross shore positions. The toe of the slope
 270 is positioned at $x = 0$ m. The layout of the computational domain, as
 271 well as the measurement positions (with the exception of section 0, which is
 272 positioned at the toe of the slope) are shown in Figure 3. The majority of the
 273 domain is discretized into cells having a size of $\Delta y = 0.005$ m and $\Delta x = 0.01$
 274 m, which corresponds to approximately 14 cells per wave amplitude, and an
 275 aspect ratio of $\Delta x/\Delta y = 2$. This is a very similar resolution as in the
 276 previous validation case. The approximate same number of cells per wave
 277 height has also shown to work well for breaking waves in Larsen and Fuhrman
 278 (2018b). For the bottom 0.05 m the grid is gradually refined in the vertical

279 over 21 cells with near bed cells having $\Delta y = 6 \cdot 10^{-4}$ m (this resolution
 280 gives 8 cells for the bottom 0.01 m). A solitary wave is initialized in the
 281 domain with the crest of the wave positioned at $x = -4.5$ m.

282 Figure 4 shows a sequence of the computed breaking and run-up of the
 283 solitary wave. In Figure 4a,b, a clear plunger is seen, with a small pocket of
 284 air trapped beneath the plunging wave (Figure 4b). In Figure 4c the wave is
 285 at the maximum run-up height. In Figure 4d a hydraulic jump is forming as
 286 the wave is drawing down. This hydraulic jump travels seaward as shown in
 287 Figure 4e, and a complicated air-water mixture is seen just seaward of the
 288 hydraulic jump. Finally, in Figure 4f a trailing wave, involving a mixture of
 289 air and water can be seen. Qualitatively, the sequence presented here is very
 290 similar to sketch in Figure 2 of Sumer et al. (2011), describing the sequence
 291 of breaking, run-up and draw-down.

292 Figure 5 shows the comparison between the experimental and modelled
 293 surface elevations at five different cross shore positions. It can be seen that
 294 the model is capable of capturing the surface elevation at the toe (Figure 5a),
 295 in the shoaling region (Figure 5b-c), in the surf zone (Figure 5d) and in the
 296 swash zone (Figure 5e). The largest discrepancy is found at section 1 during
 297 the run-down (Figure 5b at $t=8-12$ s). The reason for this discrepancy is
 298 probably that the hydraulic jump forms just onshore of the measurement
 299 position at this time and move seaward, as seen in Figure 4d,e. The hydraulic
 300 jump in the experiments occurred at this position (see Figure 2 in Sumer
 301 et al. (2011)), but it seems that there are discrepancies in the measured and
 302 computed surface elevations. These may be due to the model not resolving
 303 the complex hydraulic jump well enough, but it may also be due to difficulties
 304 in clearly defining a free-surface position in this complex region. In this
 305 situation there is a complex mixture of air and water as illustrated in the
 306 sketch in figure 2 from Sumer et al. (2011) as well as Figure 4b. In the
 307 experiments the free-surface was determined using resistive wave gauges.
 308 These are calibrated in pure water, and therefore it is questionable exactly
 309 which position is given in this situation with air mixture. In the model the
 310 surface elevations is found from the $\gamma = 0.5$ iso-line but in reality it span a
 311 minimum of two cells.

Figure 4: Sequence of the run-up of the solitary wave. The wave can be seen plunging in (a),(b), at full run-up height in (c), the initialization of the hydraulic jump in (d), the hydraulic jump moving seawards in (e) and the trailing wave in (f). Note that the axis in (c) is different from the other subplots.

Figure 5: Comparison between modelled (-) and experimental (o) from Sumer et al. (2011) surface elevations at (a) section 0, (b) section 1, (c) section 3, (d) section 5) and (e) section 8.

312 The run-up height in the experiments was estimated as 18 cm whereas
 313 in the simulation it was 21 cm, and is thus slightly overestimated. This
 314 is perhaps a bit surprising taking into account the good surface elevation
 315 comparison in the swash-zone (Figure 5e). The explanation for this is prob-

ably due to the near bed cells having somewhat large aspect ratios. In a VOF model, if γ is larger than zero, there will be a flux over the cell faces, irrespective of the actual location of the water within the cell. Large aspect ratios may therefore cause the tip of the wave to "smear out." Additionally, spurious velocities in the air phase may have added to the overestimation. The presence of the spurious velocities can be explained by the large density ratio between air and water, meaning that just a small erroneous transfer of momentum across the interface from the heavy to the light fluid will cause a large acceleration of the light fluid. Such spurious velocities have been documented in many places see e.g. Deshpande et al. (2012) and Wemmenhove et al. (2015). In Larsen et al. (2018b), it was shown that these only affect the wave very near the interface, but at the tip of the swash tongue, there are only one or two cells containing water, suggesting that it might have an effect here. In relation to tsunamis, this effect can be considered small, however, as the tsunamis are much longer, and the tip of swash tongue (with only a few cells of water) will only represent a very small fraction of the full tsunami wave.

The comparison between the bed shear stresses at five different cross shore positions is additionally shown in Figure 6. It can be seen that at all five measurement positions the bed shear stress during run-up is well captured (the first peak). It is slightly underestimated in section 8 (Figure 6e). Here, however, it is expected that the flow consist of a mixture of air and water, and it is therefore questionable exactly what shear stress was measured by the hot film in the experiments, which was calibrated for pure water. Furthermore, the model in this position also contained a mixture of air and water. For the comparison in Figure 6 (full line) a constant density of $\rho = 1000 \text{ kg/m}^3$, was used, but using the actual density of the near bed cell (shown as the dotted line), the non-dimensional shear stress would have been slightly higher than was measured.

During the draw-down larger differences are seen. In sections 1 and 2 (Figure 6a,b) the negative bed shear stress occurs slightly later in the model than in the experiments. These sections are, however, placed right around the position where the hydraulic jump forms (see Figure 3). Here slight differences between modelled and experimental surface elevations were seen (Figure 5b). The poorer comparison at this position is therefore apparently more due to the inability of the model to capture details within the hydraulic jump. At sections 5 and 6 the bed shear stress of the draw-down is also well captured by the model (Figure 6c,d). Finally in section 8 the bed shear stress during the draw-down is underestimated (Figure 6e). We ascribe this underestimation to the presence of spurious velocities in the air, which are in

Figure 6: Comparison between modelled using $\rho = 1000 \text{ kg/m}^3$ (-), using ρ_{cell} (·) and experimental (o) from Sumer et al. (2011) bed shear stresses at (a) section 1, (b) section 2, (c) section 5, (d) section 6 and (e) section 8. The dotted line in (e) is calculated using the actual density of the near-bed cell.

the opposite direction of the flow. In general the comparison of the bed shear stress is acceptable. There is some uncertainty regarding the performance right around the air/water/bed interface, but in relation to tsunamis this

uncertainty can be considered small, as a given cross shore position will either be fully in air or in water for the vast majority of a tsunami event.

4. Case selection and model setup

In this study we will now consider the simple canonical case of 2D tsunami waves propagating over an initial constant depth region, before running up a constant slope. As an inspiration for the present study we use the well-known Mercator yacht signal, taken during the Indian Ocean tsunami in 2004, which similarly has inspired other tsunami studies see e.g. Williams and Fuhrman (2016) and Larsen et al. (2017). The leading wave was estimated by Madsen and Fuhrman (2008) to be approximately sinusoidal and to have a wave height of 5 m and a period of $T=13$ min at a water depth of $h = 14$ m. This shape, wave height and period is obviously just one realization of a tsunami signal. As an example of the variability of tsunami signals please see Kawai et al. (2013). For the present simulations we will use various generalized N-waves, but all with a crest-to-trough wave height of $H = 5$ m and a period of $T=13$ min at a water depth of $h = 14$ m, similar to the measurements from the Mercator yacht.

Unless stated otherwise the waves will be initialized on a flat bed, which is one wave length long, such that the flat part of the domain houses the entire wave. The waves will then propagate and deform before reaching a constant slope region, where the slope will be systematically varied. A general setup of the computational domain can be seen in Figure 7. Note that $x = 0$ now defines the initial shoreline.

A zero gradient boundary condition was applied on the seaward boundary allowing water to flow out and thereby we do expect the seaward boundary to have any effect on the findings presented.

It is important to stress that the wave reaching the slope will no longer be precisely given by equation (23), as the wave, similar to the validation case in Figure 2, deforms while propagating. Furthermore, as will be shown later, the presence of the toe also effects surface elevation time series at the toe of the slope such that the surface elevations at the toe will be slightly different for differing slopes, even with the same initial wave form.

Table 1 gives a summary of the considered cases. In the first half of the table $A_2=0$ and therefore these waves are single waves, with only a positive displacement. The second half of the table contains N-waves with a leading depression. The periods and phase shifts of the N-waves are chosen to yield the same wave length and crest-to-trough wave height as the single

Figure 7: Layout of the computational domain, with origo positioned at the original shoreline.

396 wave cases. Only a leading depression wave was chosen, and not a leading
 397 elevation N-wave, as this would run-up similar to the single wave.

398 A_0 is the wave amplitude at the toe of the slope and ξ is the surf similarity
 399 parameter, which, as described by Madsen and Fuhrman (2008) and Madsen
 400 and Schäffer (2010) can be expressed as

$$\xi \equiv \frac{S}{\sqrt{H_0/L_\infty}} = \sqrt{\pi} \left(\frac{A_0}{h_0} \right)^{-1/2} \left(\frac{\Omega^2 h_0}{g S^2} \right)^{-1/2}, \quad (25)$$

401 where H_0 is the wave height at the toe, $h_0=14$ m is the depth at the toe,
 402 $L_\infty = gT^2/(2\pi)$ and $\Omega = 2\pi/T$ is the effective angular frequency. Also
 403 included in the table is the modelled run-up height R_{up}^* as well as the max-
 404 imum inundation speed V_{in}^* . Here V_{in} is calculated as

$$V_{in} = \frac{\partial x_{shore}}{\partial t}, \quad (26)$$

405 where x_{shore} is the horizontal position of the shoreline taken as the most
 406 landward position where $\gamma > 0.5$ at a distance 2 cm above the bed. At
 407 first the threshold was used on the bottom cells, but this resulted in the
 408 shoreline barely retreating during draw-down as a very thin layer of water
 409 remained for a long time. During run-up the two approaches yielded very
 410 similar results. V_{in}^* is subsequently found as the maximum inundation speed
 411 averaged over a minute. This particular time for averaging was chosen as
 412 we later discuss the chance of outrunning the tsunamis, and a minute seem

Table 1: Labels, slopes, initial wave parameters and computed run-up heights and inundation speeds of the simulated cases. The cases in bold represent the cases to be discussed in detail in the following sections.

Name	S	A_1 [m]	A_2 [m]	T_1 [s]	T_2 [s]	t_1 [s]	t_2 [s]	x_0 [m]	A_0 [m]	ξ	R_{up}^* [m]	V_{in}^* [m/s]
S005	1/5	5	0	780	0	0	0	-4640.5	10.8	41.9	11.3	0.81
S015	1/15	5	0	780	0	0	0	-4780.5	10.1	14.4	14.5	3.01
S030	1/30	5	0	780	0	0	0	-4990.5	8.03	8.11	16.3	6.13
S075	1/75	5	0	780	0	0	0	-5620.5	5.49	3.92	15.4	11.4
S100	1/100	5	0	780	0	0	0	-5970.5	5.34	2.98	14.4	13.2
S200	1/200	5	0	780	0	0	0	-7370.5	5.13	1.52	11.0	14.4
S500	1/500	5	0	780	0	0	0	-11571	5.02	0.62	6.18	13.8
S030L	1/30	5	0	780	0	0	0	-9561	8.56	7.86	16.8	8.33
N005	1/5	2.52	2.52	520	520	130	-130	-4640.5	5.07	61.2	8.75	1.13
N015	1/15	2.52	2.52	520	520	130	-130	-4780.5	4.73	21.1	11.6	4.03
N030	1/30	2.52	2.52	520	520	130	-130	-4990.5	4.18	11.2	12.0	8.26
N075	1/75	2.52	2.52	520	520	130	-130	-5620.5	2.66	5.63	11.3	11.4
N100	1/100	2.52	2.52	520	520	130	-130	-5970.5	2.41	4.43	9.64	11.1
N200	1/200	2.52	2.52	520	520	130	-130	-7370.5	2.51	2.17	4.89	10.3

appropriate in this regard. Using the entire time until maximum inundation did not properly represent how the tsunamis would be experienced, as the initial inundation in some cases were slow followed by a significantly more rapid inundation speed. Similarly, averaging over a significantly shorter time did not seem representative of the scenarios as local inundation speeds could be very high when only using a few seconds.

Figure 8 shows a time signal of the initial single wave and N-wave used in the simulations. Here it can clearly be seen that the two initial signals represent quite different tsunami realizations despite having the same crest-to-trough height and period. The roughness for the present cases is given by $k_s = 2.5d$ with $d = 4 \cdot 10^{-4}$ m. For all cases the domain is discretized into cells $\Delta y = 0.25$ m and $\Delta x = 0.5$ m and from 5 m above the bed and lower the grid is gradually refined (using 51 cell) in the vertical with near-bed cells having $\Delta y = 0.0014$ m corresponding to $\delta y/k_s = 1.4$. The bottom 0.5 m is discretized into 21 cells. The simulations to be presented in what follows each took between two weeks and two months to complete when simulated in parallel on 12 modern processors (Intel Xeon Processor E5-2680 v2).

5. Model results: Run-up heights and inundation speeds

This section will describe and discuss the modelled run-up heights and inundation speeds. Before continuing, the effect of the slope on the surface

Figure 8: Time signal of the initial waves.

433 elevation signal at the toe of the slope will be discussed, as this affects the
 434 analysis of the run-up.

435 As previously stated, the slope itself actually has an impact on the sur-
 436 face elevation at the toe of the slope, such that tsunamis with identical
 437 initial wave fields, running up different slopes, will have different surface el-
 438 evation signals at the toe of the slope. To illustrate this Figure 9 shows the
 439 surface elevations as a function of time for both single waves and N-waves
 440 for three different slopes. Most noticeably both cases with $S=1/5$ result in
 441 much larger wave heights than the initial wave signal (see Figure 8) and
 442 are substantially higher than the other cases. This can be explained by the
 443 tsunami being reflected even before the crest of the tsunami arrives at the
 444 slope. The reflected part of the tsunami is then super-positioned on top of
 445 the incoming wave, resulting in higher surface elevations. This is essentially
 446 a transient standing wave, similar to the periodic form shown in Madsen
 447 and Fuhrman (2008). For the cases with $S=1/30$ the peak of the surface
 448 elevations comes at a later time. This is again due to reflection and the
 449 reflected wave interacting with the incoming wave. For the single wave with
 450 $S=1/100$, the surface elevation signal shows two distinct peaks, one for the
 451 incoming and one for the reflected wave. However, following the crest of the
 452 incoming wave, the surface elevation never returns to the still water level
 453 before the arrival of the reflected wave. For the single wave, this only occurs
 454 for the very mild slope of $S=1/500$ (not included in the figure). For the
 455 N-waves, the slope of $S=1/100$ is mild enough that the reflected and the

456 incoming wave can be clearly distinguished.

Figure 9: Development of the surface elevation at the toe of the slope for (a) single waves and (b) N-waves.

457 The surface elevations presented here demonstrate the difficulty in iden-
 458 tifying incoming and reflected tsunami waves from a surface elevation signal
 459 alone. This can pose a challenge when trying to reproduce a tsunami event,
 460 using a measured surface elevation signal as an inlet condition in model
 461 simulations.

462 In Madsen and Schäffer (2010) expressions for the run-up height and
 463 inundation speed were given as a function of the surf-similarity parameter
 464 ξ for both single waves and N-waves propagating on a constant depth be-

Table 2: Values for χ_{elev} and χ_{break} for single waves and N-waves, based on the analysis Madsen and Schäffer (2010).

Wave type	χ_{elev}	χ_{break}	χ_{velo}
single wave	3.043	0.1561	2.516
N-wave	4.397	0.0373	8.450
Sinusoidal wave	$2\sqrt{\pi} \approx 3.545$	$(2\sqrt{\pi})^{-1} \approx 0.282$	$2\sqrt{\pi} \approx 3.545$

fore running up a constant slope, very similar to this study. Their analysis assumes a linear wave at the toe of the slope and although this is not entirely the case in the present study, we will nevertheless compare the run-up heights and inundation speeds from the simulations to the analytical results from Madsen and Schäffer (2010). Their analysis builds on the hodograph transform by Carrier and Greenspan (1958). They then expressed their solution in terms of both inverse Fourier transformations and convolution integrals to arrive at a solution for the time varying run-up. They provided a breaking criterion which corresponds to the theory breaking down, which happens when the Jacobian vanishes. This corresponds to the time and position where the wave front becomes vertical.

By this analysis the maximum run-up is then given by

$$R_{up}^* = \min(R_{up}, R_{up}^{limit}) \quad (27)$$

where

$$\frac{R_{up}}{A_0} = \chi_{elev} \pi^{1/4} \left(\frac{A_0}{h_0} \right)^{-1/4} \xi^{-1/2}, \quad (28)$$

is the expression for tsunamis not expected to break, and

$$\frac{R_{up}^{limit}}{A_0} = \frac{\chi_{elev} \chi_{break}}{\pi} \xi^2. \quad (29)$$

is the expression for tsunamis where breaking could be expected to occur.

Similarly the maximum inundation speed can be found as

$$V_{in}^* = \min(V_{in}, V_{in}^{limit}) \quad (30)$$

where

$$\frac{V_{in}}{\sqrt{gA_0}} = \chi_{velo} \pi^{3/4} \left(\frac{A_0}{h_0} \right) \xi^{-3/2} \quad (31)$$

482 and

$$\frac{V_{in}^{limit}}{\sqrt{gA_0}} = \frac{\chi_{velo}\chi_{break}}{\sqrt{\pi}}\xi \quad (32)$$

483 In the above χ_{elev} , χ_{break} and χ_{velo} comes from the analytical analysis of
 484 Madsen and Schäffer (2010). They are different for sinusoidal, single- and
 485 N-waves and the values used are presented in Table 2. Note that the values
 486 in Table 2 for both sinusoidal and single waves are identical to those pre-
 487 sented in the text in Madsen and Schäffer (2010), but that the values for
 488 N-waves are different. These have been re-calculated following the approach
 489 described in Madsen and Schäffer (2010), as the N-waves in the present
 490 study use a different phase shift and did not have $\Omega = \Omega_1 = \Omega_2$ as simply
 491 assumed in Madsen and Schäffer (2010).

492 Figure 10 compares the simulated run-up heights with those obtained
 493 with equation (27), for the single waves (included in the figure are also
 494 selected labels for those cases described in detail in the next section). As
 495 previously shown the amplitudes of the incoming waves are not identical with
 496 the different slopes. Therefore the analytical curve has been created using
 497 the initial wave amplitude $A_0 = 5$ m, whereas the simulated results are based
 498 on the actual A_0 at the toe (indicated in Table 1). This is the most legitimate
 499 comparison possible since the Madsen and Schäffer (2010) incident wave
 500 contained both the incoming and reflected wave. The analytical solutions
 501 of Madsen and Schäffer (2010) are actually a family of curves, and for the
 502 cases with A_0 higher than 5 m, the analytical curve would be positioned
 503 slightly, but not significantly, lower than that shown in Figure 10.

504 The analytical curve has two distinct regions. Starting from small ξ and
 505 increasing the expected run-up heights increase rapidly with ξ . This part of
 506 the curve (full line) represents the line given by (29), which is the breakdown
 507 limit of the theory. To the right of the peak, the curve follows a line given
 508 by (28) which corresponds to no breaking and hence full reflection at the
 509 shore. Included in the figure is also a linear function given by

$$\frac{R_{up}^{Hunt}}{A_0} = \xi, \quad (33)$$

510 which was suggested by Hunt (1959) to be a better estimate for breaking
 511 waves, see also Madsen and Fuhrman (2008) who used this for irregular
 512 waves. An alternative formulation for the run-up is therefore

$$R_{up}^* = \min(R_{up}, R_{up}^{Hunt}) \quad (34)$$

513 which is depicted as a dashed line (This was also used by Madsen and
514 Fuhrman (2008)).

515 It can be seen that, even though the assumption of linearity at the toe is
516 violated the analytical solution and the simulated results agree reasonably
517 well in the region where (28) is dominating. Similar to observations by Park
518 et al. (2015), the run-up heights are underestimated with the analytical
519 expressions in the region where (29) is dominating. Here the expression of
520 Hunt (1959) seems to give more accurate predictions. This can be explained
521 by the breakdown of NLSW theory not directly corresponding to breaking.
522 Furthermore, even after NLSW theory predicts a vertical surface elevation
523 (approximately interpreted as the tsunamis breaking), the tsunami waves
524 continue to run-up.

Figure 10: Comparison between modelled (*), analytical (27)(-) and (34)(- -) run-up of the single wave as a function of surf-similarity parameter.

525 Figure 11 compares the simulated run-up heights with those obtained
526 utilizing equation (27) for the N-waves. Similar to before the analytical
527 curve is made using the initial wave amplitude $A_0 = 2.5$ m and the modelled
528 results have used the actual A_0 at the toe.

Figure 11: Comparison between modelled (*), analytical (27) (-) and (34) (- -) run-up of the N-wave as a function of surf-similarity parameter.

Here, in general, a good agreement between the modelled run-up heights and the analytical run-up heights is again achieved. Similar to before, the modelled run-up heights are slightly above those obtained with the analytical approach in the region dominated by (29) and the expression of Hunt (1959) (33), depicted as a dashed line seems more appropriate.

Figure 12 compares the simulated maximum inundation speed with the that obtained using equation (30). Included as dashed line is also an alternative expression given by

$$V_{in}^* = \min(V_{in}, V_{in}^{Break}) \quad (35)$$

where V_{in}^{Break} has been approximated by

$$V_{in}^{Hunt} = \frac{\Omega R_{up}^{Hunt}}{S} \quad (36)$$

The above formulation is a way to estimate the inundation speed based on the predicted run-up height in situations where breaking would be expected to occur and follows the relation between run-up height slope and frequency for sinusoidal waves (see Madsen and Fuhrman (2008)). Similar to the predicted run-up height it can be seen that the analytical expression by Madsen and Schäffer (2010) gives a reasonable approximation for the inundation speed in cases where waves are not predicted to break (right part of the solid

Figure 12: Comparison between modelled (*), analytical (30) (-) and (35) (- -) maximum inundation speed for the single wave as a function of surf-similarity parameter.

line). In cases where waves are predicted to break the prediction based on the run-up height from Hunt (1959), (36) shown as a dashed line seem to more appropriate, and as whole the combined expression in (35) seems to give a reasonable approximation, although this expression in general slightly underestimates the inundation speed compared to the model.

Finally, Figure 13 compares the simulated maximum inundation speed with that obtained using (30) and (35). Similar to the results for the single waves, the combined expression (35) gives a reasonable prediction, although it also slightly underestimates the inundation speed compared to the model.

Overall, based on the decent match with the more advanced CFD simulations, we would argue that the combined expressions from Madsen and Schäffer (2010) and Hunt (1959), expressed in (34) and (35), performs relatively well in predicting the run-up heights and inundation speeds across a wide range of cases, and can hence be used as a quite reasonable first assessment of the run-up height and inundation speed, respectively. As they are based on NLSW equations, however, the detailed information of the actual process is limited. This information can, in contrast, be gathered by the advanced CFD model, as will be done in the following sections.

Figure 13: Comparison between modelled (*), analytical (30) (-) and (35) (- -) maximum inundation speed for the N-wave as a function of surf-similarity parameter.

6. Qualitative run-up description

The various tsunami signal and slope combinations can yield a wide array of different run-up types. According to Sriram et al. (2016), Shuto (1985) states that the observed tsunamis hitting the coast can broadly be placed into three categories; (1) rapidly rising tide, (2) breaking bore or "wall of water" and (3) undular bore. These three run-up types have all been experienced in the present simulations although we will recategorize these as:

- 1) Surging tsunami
- 2) Breaking bore
- 3) Undular bore

This change is done as we do not approve of the phrase "tide" when describing a tsunami and second that we do like the phrase "wall of water" as this is more of a popular description. We would also like to add that some cases appeared initially as undular bores but turned into breaking bores before hitting the coast, and this we will define as a fourth (intermediate) category:

- 4) Undular bore evolving into a breaking bore

In what follows, based on the highly resolved CFD results, we will now describe and study in detail the characteristics of the run-up process for each of these categories. Additionally, we will discuss different run-up features

Table 3: Sections, run-up types, case names and computed maximum inundation speed of the cases used for the detailed discussion.

Section	Run-up type	Case	V_{in}^* [m/s]
6.1	Surging tsunami	S005	0.81
6.2	Breaking-bore	S100	13.2
6.3	Undular bore turning into a breaking bore	S500	13.8
6.4	Undular bore	S030L	8.33
6.5	Breaking bore	N100	11.1

between single- and N-waves. The cases chosen to illustrate these different run-up categories are listed in Table 3.

6.1. Surging tsunami

We will first consider the surging tsunami run-up type. As a representative of this type we choose case S005, which is a single wave running up a steep $1/5$ slope (see again Table 1 for a full description). Figure 14 shows results of surface elevations and velocities from case S005 at several different instants. In Figure 14a the initial wave can be seen. In Figure 14b the wave has steepened slightly and has started to inundate the coast, and in Figure 14c the wave is at its maximum run-up. Finally, in Figure 14d the reflected wave can be seen with a steep wave front which ultimately evolves into an undular bore as shown in Figure 14e. From this representation it is very clear that the steep slope of $1/5$ is experienced almost as a vertical wall by the long tsunami, which is fully reflected.

It is noteworthy that there is a phase shift between the maximum velocity and the maximum surface elevation, as can be seen in Figure 14c. This shift can more clearly be seen from a time perspective and therefore Figure 15a shows a time series of the surface elevations and velocities at $x = 0$, corresponding to the original shoreline. In Figure 15b the temporal evolution of the horizontal position of the shoreline is shown. From Figure 15a the phase-shift between the maximum free stream velocity and the maximum surface elevation is completely clear. This phase shift is very similar to what has been observed beneath the 2010 Chilean tsunami at Monterey Bay by Lacy et al. (2012), and is a result the steep slope acting as a vertical wall, essentially creating a standing wave. This explanation, we believe, provides an alternative and more likely (and simpler) explanation for the phase shift, than that given by Lacy et al. (2012). They explained the phase shift through a near-bed momentum balance, where the bed friction, they argued, resulted in the 90° phase lag between η and u . The analogy to a

Figure 14: Surface elevation and velocities of S005 at different times. Note that this figure is not to scale.

standing wave explains this observation in a much simpler manner. It is worth noting that this is also reflected in the new edition of the ASCE 7-16

Figure 15: Results for case S005 showing (a) Velocities and surface elevations at $x = 0$, (b) the horizontal position of the shoreline.

standard on tsunami loads and effects (ASCE, 2016). Here a tsunami load case 2 is defined, where a flow depth of $2/3$ of the maximum flow depth with maximum flow velocity will be used for tsunami design. This is similar to prescribing a phase shift between the maximum surface elevation and maximum velocity.

While inundating the tsunami wave front is almost horizontal as can also be seen in Figure 14f-g, which shows zoomed in positions of the wave fronts. At the same time, the increase in water depth at the original shoreline is not very rapid, as illustrated in Figure 15a. Over the initial approximately 3 min = 180 s the depth at the shore goes from 10 cm to 1 m. From here the increase in water depth is faster and after an additional 1.5 min (to $t = 270$ s in Figure 15a) the depth increases from approximately 1 m to 10 m. Although the increase in water depth is large, the duration is still the order of minutes rather than seconds. Finally, Figure 15b illustrates that the inundation speed is indeed very low. The fastest inundation speed (averaging over a minute) is reached during $t = 194$ – 254 s, where it reaches approximately 0.8 m/s (2.92 km/h=1.81 mi/h). This present tsunami, could thus easily be outrun. The above observations imply, that the present tsunami will not be seen as a steep wave, but rather "tide-like" as classified by Shuto (1985)

(it is still obviously important to stress that this is not a tide). A real life example of this run-up type can be found in the youtube video "TD Interesting Topics Tube (2017)" at the time 3:50–5:05 and also at 10:25–12:40. These two sequences show the Tohoku tsunami running into the harbour of Miyako with a seemingly constant, but gradual, increase in water depth. The tsunami enters the harbour next to a seawall, which is finally over-topped in the second video sequence.

As previously mentioned, an undular bore forms at the front of the reflected wave (Figure 14e). This actually also happened in simulated cases S015, S030, N005, N015 and N030. That the undular bore develops in front of the reflected wave is also interesting, and can perhaps help explain why undular bores have been observed at many places in real tsunami events, as reflected tsunami waves will have travelled a long distance in relatively shallow water. The run-up behaviour of undular bores will be discussed later in this section.

6.2. *Breaking bore*

To represent the scenario of tsunamis running up as a breaking bore, we will now show results of case S100, which is a single wave running up a 1/100 slope (see again Table 1). Figure 16 shows surface elevations and velocities from case S100 at different instants. In Figure 16a the initial wave is shown. Figure 16b shows the shoaled wave with a steep wave front, and significantly higher velocities, as would be expected. The wave front has actually split at this time, and an undular bore is starting to form, as can be seen in the zoom-in of the wave front in Figure 16e. The short wave riding on top of the main tsunami wave cannot maintain its shape for very long, however, as this wave shoals and steepens. After only four seconds, the shorter wave riding on the front of the tsunami breaks, as seen in Figure 16c, and in more detail in the zoom in of the front in Figure 16f. From here the tsunami runs up the beach like a breaking bore, before it reaches its maximum run-up shown in Figure 16d.

According to the breaking criterion of Madsen and Schäffer (2010) this tsunami would be predicted to break (Figure 10), though it is right near the theoretical threshold. It is important to note, however, that in this case, it was not the main tsunami wave that broke, but rather a short wave riding on the wave front; that the main tsunami front develops into a breaking bore is still generally consistent with this characterization. In all the simulated cases appearing as breaking bores, the wave front initially split into one or more short waves which broke, and then turned the entire wave into a breaking bore. Behind the breaking wave front (Figure 16c,f), the surface

Figure 16: Surface elevation and velocities of S100 at different times. Note that this figure is not to scale.

674 elevations actually continue to increase, indicating that only a very small
 675 fraction of the entire wave is affected by breaking.

676 In contrast to the previous case (S005) this case did not result in a stand-
 677 ing wave, and as a result the free surface elevations and the velocities are in
 678 phase. This can be seen more clearly in Figure 17a where the velocities and
 679 free-surface elevations at $x = 0$ m are shown. In Figure 17b the temporal
 680 development of the shoreline position is likewise shown. From the represen-
 681 tation in Figure 17a it can be seen why the popular term "wall of water"
 682 suggested by Shuto (1985) could sometimes be used to describe certain run-
 683 up scenarios. Similar to before, the depth goes from 10 cm to 1.3 m in the
 684 first approximately 3 min (180 s), but hereafter the depth increases rapidly
 685 to 4 m in a matter of 4 s. Additionally the inundation speed is also much
 686 faster, as can be seen from Figure 17b. The fastest inundation speed (av-
 687 eraging over a minute) during $t = 344\text{--}414$ s corresponds to approximately
 688 13 m/s (47 km/h=29 mi/h). This is in stark contrast to the low inundation
 689 speed seen in case S005, and this tsunami could not be outrun. It is worth
 690 noting that the shoreline fluctuates a little during draw-down. These fluc-
 691 tuations are caused by small variations in the surface elevation in the very
 692 thin layer of water directly just above the bed. These small variations in
 693 surface elevations can cause the shoreline to jump as it is determined by the
 694 γ value of cells 2 cm above the bed.

695 Breaking bores have been observed in many instances, and have also
 696 often been used as tsunami representations in experiments. Real examples
 697 of breaking bores can e.g. be found in videos of the Tohoku tsunami in
 698 TD Interesting Topics Tube (2015) at times 3:00–3:15, 4:40–4:48 and 7:50–
 699 8:50. Images of the first situation was also used by Nistor et al. (2017) to
 700 justify the choice of wave used in the study. S100 demonstrates the potential
 701 devastation that can arise when tsunamis are near the peak of (34) in Figure
 702 10, i.e. where $R_{up} = R_{up}^{Hunt}$ corresponding to

$$\xi = \chi_{elev}^{-2/3} \pi^{-1/6} \left(\frac{A_0}{h} \right)^{1/6}. \quad (37)$$

703 This represents a worst case scenario where the tsunami front steepens sig-
 704 nificantly, but does not dissipate much energy before reaching the shoreline
 705 as the breaking starts very close to the shore or breaking do not occur.

706 6.3. Undular bore evolving into a breaking bore

707 As could be seen from the previous case, the tsunami started evolving
 708 towards an undular bore, but the resulting short waves quickly broke before
 709 turning into a full undular bore. There were cases where several shorter
 710 waves were riding on the wave front, thus forming a more proper undular

Figure 17: Results for case S100 showing (a) Velocities and surface elevations at $x = 0$, (b) the horizontal position of the shoreline.

bore. Similar to before, the individual waves broke before reaching the shore, and as a result turned into a breaking bore. Although these tsunamis essentially run-up as breaking bores, we will illustrate this run-up process in more detail using case S500, which is a single wave running up a 1/500 slope (see again Table 1 as well as Figure 10 for it's position on the run-up curve).

Figure 18 shows results of surface elevations and velocities at different instants from case S500. Figure 18a shows the initial wave. During shoaling an undular bore forms on the tsunami wave front as seen in Figure 18b. Figure 18e shows a zoomed in portion of the wave front, and here the shorter waves riding on the front are clearly visible. These waves, though "short", are still rather large, with wave lengths of approximately 100 m and wave heights of approximately 10 m, very similar to what have been observed many places, see e.g. Madsen and Fuhrman (2008) or the videos of the Tohoku tsunami in TD Interesting Topics Tube (2015) from at time 2:10–2:25.

As the tsunami propagates, the waves riding on the wave front shoal and steepen until they at some point break. This is illustrated in Figure 18c as well as the zoom-in Figure 18f where the tsunami is seen after propagating

Figure 18: Surface elevation and velocities of S500 at different times. Note that this figure is not to scale.

730 approximately 2 km further than in Figure 18b. Figure 18f clearly shows
 731 the that the front waves have broken. Traces of the undular bore can still
 732 be seen, but when this tsunami finally hits the coast (Figure 18d) there are
 733 no undulations at the wave front which is purely a breaking bore. This is
 734 also illustrated in Figure 19a where the time series of the surface elevations

Figure 19: Results for case S500 showing (a) Velocities and surface elevations at $x = 0$, (b) the horizontal position of the shoreline.

and velocities are shown at the original shoreline ($x = 0$ m). Here it can be seen that for the first 12 min (720 s), only small increases in water depth are seen, but then in 6 s the depth suddenly increases to approximately 2 m, before gradually increasing to a maximum of 4 m. From the figure it can also be seen that the milder slope, compared to case S100 (Figure 17), causes the duration over which the shore is inundated to be substantially longer. The total inundation is approximately 3 km in this case (Figure 18d), which is around twice the inundation distance from case S100. From the temporal development of the shoreline, shown in Figure 19b, it can be gathered that the fastest inundation speed (averaged over a minute) occurs from $t = 792$ – 852 s, and results in an inundation speed of approximately 13.8 m/s (49.7 km/h=30.9 mi/h), very similar to the previous case (S100) running up as a breaking bore. Obviously, this tsunami could also not be outrun.

6.4. Undular bore

As none of the cases with the initial set-up, with the flat part of the domain being one wave length long, resulted in an undular bore reaching all the way to the coast, an additional simulation has been made (S030L). This

simulation entailed the single wave running onto a slope of $S=1/30$, but with the flat part of the domain being two wave lengths long instead of one, and with the initial tsunami positioned an entire wave length away from the shore. This was done as the formation of undular bores is dependent on the propagation distance in relatively shallow water, as highlighted by Madsen et al. (2008), among others.

Figure 20 shows surface elevations and velocities from case S030L at different times. In Figure 20a the initial wave can be seen, positioned now further away from the coast. While propagating an undular bore evolves at the front of the tsunami as seen in Figure 20b. The undular bore keeps growing with more and more individual waves forming as shown in Figure 20c and in the zoom in Figure 20e. While shoaling these waves, similar to the previous case S500, steepen and break, as shown in Figure 20d,f. However, due to the steeper slope ($1/30$ relative to $1/500$, see Figure 20 vs. Figure 18) the individual waves do not lose all their energy while breaking and survive all the way to the shore as seen in Figure 20f.

The shorter waves riding in front of the tsunami have been given much importance in the past, and the appearance of these is probably one of the reasons why solitary waves have often been used to study tsunamis in the past, despite the previous mentioned scaling issues. To discuss the importance of these shorter waves relative to the bulk tsunami, we will now compare directly case S030 with S030L. These cases have the same initial wave form and are running up the same slope, but have propagated different distances before reaching the slope. Figure 21 shows surface elevations at two different times for S030 and S030L, with the time indicated by $t^* = t - t_p$, where t_p is the time where the maximum inundation distance is reached. At the first instant, the wave is approaching the shore. In Figure 21a the wave has steepened slightly compared to the initial single wave, whereas the undular bore is clearly visible in Figure 21b. At the second instant shown the tsunamis are at their respective maximum run-up. Despite the difference in the approaching wave, it can be seen that the final run-up heights of the two cases are very similar.

Another interesting comparison is the inundation speed of the two cases. Figure 22 shows the temporal evolution of the shoreline for cases S030 and S030L. From this figure it can clearly be seen that the initial inundation speed of case S030L is significantly larger (approximately 10 m/s versus 1 m/s), but overall the inundation speed and general process in both cases are very similar. For case S030 the largest inundation speed over a minute is 6.1 m/s ($22.1 \text{ km/h} = 13.7 \text{ mi/h}$) and for case S030L it is only slightly larger, namely 8.3 m/s ($29.9 \text{ km/h} = 18.6 \text{ mi/h}$), and is due to initial inundation

Figure 20: Surface elevation and velocities of S30L at different times. Note that this figure is not to scale.

of the very steep undular bore front. It is worth noting, that despite not breaking, the inundation speed of case S030 is substantially larger than

Figure 21: Comparison of run-up of single waves on a $S=1/30$ slope at different times.

795 that of case S005, but still also significantly lower than cases running up as
 796 breaking bores.

797 The similar run-up height and inundation speed of these two cases in-
 798 dicates that the bulk tsunami is much more important than the relatively
 799 shorter waves riding on top of it. This point was also previously hypothe-
 800 sized by Madsen et al. (2008) and Madsen and Schäffer (2010), but is more
 801 clearly and directly demonstrated and confirmed in the present fully-resolved
 802 CFD simulations. Photographs of the tsunami events show undular bores
 803 on the tsunami front without (very clearly) showing the main and much
 804 longer tsunami wave. This is natural due to the long wave length, but can
 805 result in too much importance given to the shorter waves. To illustrate this,
 806 Figure 23 shows the undular bore seen from the side (Figure 23a) and from a
 807 perspective view (Figure 23b). This illustration is very similar to one shown
 808 in Madsen et al. (2008), but it raises an important point which needs to
 809 be emphasized. When seen from the side, it can clearly be seen that front
 810 waves are riding on a much longer main wave. However, when looking from

Figure 22: Development in shoreline position for cases S030 and S030L.

above, only the shorter waves are visible, which is similar to photographs and videos of real tsunami events (see e.g. Madsen et al. (2008)).

While it has been shown that the undular bore forming in front of a tsunami does not matter much for the run-up distance and inundation speed and process, it does however matter in terms of local flow velocities. These will be important when assessing forces on structures or estimating sediment transport. To illustrate this, Figure 24 shows the velocities and surface elevation of the front of the tsunami for cases S030 and S030L with the front of the tsunami having just inundated past the original shoreline ($x = 0$). It can be seen that the maximum flow velocity in case S030 is right at the tip of the inundating tsunami with velocities reaching approximately 10 m/s. The maximum flow velocities in case S030L, on the other hand, occur at the tip of the breaking short waves, with velocities here reaching up to 19 m/s. These differences in velocities may be important for the impact forces on structures, which would be expected to scale with the square of the flow velocities (see e.g. Chock, 2016). This indicates, that if attempting to model tsunami forces on structures, choosing a model which can handle dispersion and wave breaking is likely important.

6.5. *N-waves*

Thus far, all example run-up types have been illustrated using single waves. The same run-up types as just described (except for the undular bore, which required a longer domain to fully evolve) have also been experienced

Figure 23: A (a) side looking (a) and (b) perspective view of the undular bore from case S30L

833 with the leading depression N-waves. There are, however, also some distinct
834 differences between the run-up of single waves and leading depression N-
835 waves. These will be discussed in what follows, by considering specifically
836 case N100 (see again Table 1 as well as Figure 11 for this case's position on
837 the run-up curve).

838 Figure 25 shows surface elevations and velocities at different instants
839 for case N100. In contrast to the previous cases, the initial wave shown in
840 Figure 25a now has a leading depression. The leading depression results in

Figure 24: Surface elevations and velocities of (a) case S030 and (b) case S030L. Note that this figure is not to scale.

an initially retreating shoreline, as seen in Figure 25b. Leading depression tsunamis with retreating shorelines have been observed in many places; one of the most memorable situations is probably the tourist standing on a dry beach in Thailand with the front of the Indian Ocean tsunami approaching, see Figure 1 of Foster (2014).

Figure 25b also reveals an additional feature which is distinctively different than the previously simulated single waves, namely that the split of the front wave is not happening at the crest of the tsunami, but rather in between the trough and the crest. This can more clearly be seen in Figure 25e. A similar observation was also made in the experimental study of Matsuyama et al. (2007), where it was shown that the individual waves in some cases evolved in front of the crest of the wave. Here the waves were leading elevation sinusoidal-like waves, and thus the split occurring in front of the crest in the simulation is not due to the leading depression. The split rather occurs where the local surface elevation steepness is largest. In these simulations this was the case between the trough and crest, whereas in the experiments, due to the imperfect initial sinusoidal shape, it happened in front of the wave crest.

Similar to case S100 the individual wave seen in Figure 25b,e cannot sustain its shape during shoaling, and breaks before reaching the slope. This wave therefore runs up as a breaking bore as shown in Figure 25c,f. As can be seen in Figure 25f, the front of the bore is only approximately 1 m high and fluctuations in the surface elevations are clearly visible as could be expected of breaking bores running up (see e.g. pictures from O'Donoghue et al., 2010; Johnson et al., 2016). At the same time the increase in water depth is not nearly as rapid as in cases S100 and S500 (Figures 17 and 19). This can be seen in Figure 26a where the time series of the surface elevations and velocities at $x = 0$ m are shown for case N100. Here it can clearly be seen that the depth increases much more gradually than cases S100 and S500, going from 0–3 m in slightly over 1 min. The reason for this could be the leading depression, which causes the shoreline to retreat as seen in Figure 26b. The breaking tsunami front will therefore re-wet the drawn-down region prior to reaching the original shoreline at $x = 0$. The appearance of this tsunami wave front is more similar to the appearance of cases S100 and S500 further inland.

The inundation speed of the present case is ultimately very similar to the single wave cases running up as a breaking bore. The maximum inundation speed (averaging over a minute) occurs at $t = 500$ – 560 s with a value of 11 m/s (39.6 km/h = 24.6 mi/h), indicating again that it would be impossible to outrun this tsunami run-up type.

7. Expected occurrence of the run-up types

In the previous subsections the run-up types have been described in detail. We will now discuss in which situations these run-up types can be expected to show up.

The first identified category, was the surging tsunami. This run-up type was only experienced in cases where the slope was very steep. In fact, it was only experienced in cases where the run-up from (28), from the analysis of Madsen and Schäffer (2010) was less than (33), corresponding to the right solid line in Figures 10 and 11. In that sense the surf-similarity parameter can be used to assess whether a surging tsunami is possible. However, as shown in case S030L, if the tsunami has propagated sufficiently far before reaching the toe of slope, an undular bore can develop. In this case the wave at the toe of slope was no longer at all described by (23). Furthermore, near the intersection of the two curves, it is not certain that a clear distinction between breaking and non-breaking cases can be made. We therefore believe it is reasonable to assume, that if the tsunami reaching the slope is linear (as

Figure 25: Surface elevation and velocities of N100 at different times. Note that this figure is not to scale.

897 was a pre-requisite for the Madsen and Schäffer (2010) analysis), then cases
 898 far to the right of the intersection between (28) and (33), will be surging.

Figure 26: Results for case N100 showing (a) Velocities and surface elevations at $x = 0$, (b) the horizontal position of the shoreline.

899 This can be expressed in terms of the surf-similarity parameter by

$$\xi \gg \chi_{elev}^{-2/3} \pi^{-1/6} \left(\frac{A_0}{h} \right)^{1/6} \quad (38)$$

900 or alternatively in terms of a non-dimensional tsunami-similarity parameter

$$\Upsilon = \frac{h_0^{1/6} \sqrt{g} S}{\Omega A_0^{2/3}} \gg (\chi_{elev} \pi)^{2/3} \approx 5 \quad (39)$$

901 where the sinusoidal value for $\chi_{elev} = 2\sqrt{\pi}$ has been invoked for the numer-
 902 ical approximation. Alternatively χ_{elev} can be found in Table 2, but since
 903 the actual wave shape cannot be known beforehand, the sinusoidal value
 904 for χ_{elev} would seem to be a reasonable approximation, as this also lies be-
 905 tween the values for the single wave and the N-wave. For all forthcoming
 906 calculations the sinusoidal value for χ_{elev} will be used.

907 The second category, the breaking bore, appeared in situations near the
 908 intersection of (28) and (33) or where $\xi < \chi_{elev}^{-2/3} \pi^{-1/6} \left(\frac{A_0}{h} \right)^{1/6}$, alternatively
 909 expressed as $\Upsilon < (\chi_{elev} \pi)^{2/3} \approx 5$. The latter corresponds to cases where
 910 wave breaking is predicted to be present, corresponding to the left dashed
 911 line in Figures 10 and 11. However, this will not alone make the tsunami

run-up as a breaking bore. In situations where the slope is very mild, an undular bore can form on the front of the tsunami.

The undular bores turning into a breaking bore was only experienced in situations where $\xi \ll \chi_{elev}^{-2/3} \pi^{-1/6} \left(\frac{A_0}{h}\right)^{1/6}$ or $\Upsilon \ll (\chi_{elev} \pi)^{2/3} \approx 5$. For the undular bore to properly form, the slope had to be very mild, otherwise resulting in immediate breaking of the short waves riding on the front of the tsunami, due to shoaling effects.

The final category, an undular bore, has only been experienced in one case, and required the tsunami to propagate a long distance in shallow water, and subsequently run-up a steep slope. In reality tsunamis will not propagate on a flat bed before going up a constant slope region. Often it might propagate on a very mild slope before going up a steeper slope. As this is similar to the scenario of S030L, this could explain why undular bores have been observed in so many places with real geophysical tsunamis. With this kind of bed profile an undular bore developed on a very mild slope will be able to reach all the way to the shore due to the subsequent steeper slope.

The different tsunami categories exemplify the variety of ways a tsunami can appear. In research tsunamis have often been treated as breaking bores, but the classification and descriptions shown here can be used as an inspiration for alternative scenarios in experiments. An attempt to capture the undular bore followed by the much longer main tsunami wave was presented very recently in the study by Aniel-Quiroga et al. (2018b), where breakwater stability due to a tsunami attack was investigated experimentally. Here the tsunami was represented first by a series of solitary waves followed by a continuous inflow of water, to represent the main wave. As mentioned, bores have been used as tsunami representations, and the results shown in Figure 16 could be used as a full-scale inspiration. Here the breaking bore lasted approximately 500 s, and the maximum bore velocity was close to 10 m/s. Assuming a scaling factor of 100, this would (by Froude scaling) result in a model scale bore duration of approximately 50 seconds, and a model scale bore velocity of 1 m/s. The surging tsunami has, to our knowledge, only been used in Goseberg (2013). As illustrated here, however, if considering a very steep beach, this is the most likely run-up type and representing the tsunami as a bore (though conservative) would not represent a likely tsunami scenario. Knowing the most likely tsunami scenario can be important for evacuation purposes. That breaking bores are used extensively in the literature is still understandable and justifiable, however, as these will represent a destructive scenario, which designers naturally needs to take into account.

951 8. Conclusion

952 In this paper the results of 14 full scale tsunami simulations have been
953 presented. The simulations have been performed with a RANS models,
954 using the new stabilized k - ω model of Larsen and Fuhrman (2018b), which
955 avoids exponential growth of the turbulent kinetic energy in a potential
956 flow region beneath surface waves, which is characteristic of two-equation
957 closures. The model has been first validated by comparing with surface
958 elevations from a properly scaled tsunami experiment by Sriram et al. (2016).
959 The model was shown to be capable of predicting the initial deformation
960 and steepening of the wave as well as the subsequent splitting of the wave
961 front, run-up and reflection. The model was further validated for surface
962 elevations and bed shear stresses in the surf and swash zone by comparing
963 to the plunging solitary wave experiments of Sumer et al. (2011). In general
964 the model predicted both the surface elevations and the bed shear stresses
965 well, although some differences were seen around the hydraulic jump during
966 run-down, as well as for the bed shear stresses at the air/water/bed interface.

967 Using single waves and N-waves as generic tsunami wave forms, simula-
968 tions were performed of full-scale tsunamis propagating on a flat bed before
969 running up a constant slope. The modelled run-up heights and inundation
970 speeds for both the single waves and the N-waves were shown to compare rea-
971 sonably well with the analytical expression by Madsen and Schäffer (2010)
972 in cases where breaking would not be expected to occur. In cases where
973 breaking could be expected to be present, the analytical solution underes-
974 timated the run-up height and inundation speed, and the expression from
975 Hunt (1959) for the run-up height, as well as a newly proposed expression for
976 the inundation speed, gave better estimations. The overall good compari-
977 son between analytical expression and the modelled run-up heights indicates
978 that the combined non-breaking expressions of Madsen and Schäffer (2010)
979 and the empirical breaking expressions from Hunt (1959) and (35) are suit-
980 able as first estimates of the run-up height and inundation speeds even in
981 situations where the wave is non-linear at the slope, and in situations where
982 waves break.

983 It has been demonstrated that, despite identical initial waves, the time
984 series of surface elevations at the toe of the slope can be quite different
985 for varying slopes due to the long tsunamis waves being reflected. It has
986 been emphasized that this can pose a challenge in identifying incoming and
987 reflected wave signals, even for what appears to be the leading wave.

988 Three previously defined run-up types were identified in the simulations,
989 and their characteristics were described. Additionally a fourth type was

identified and described. It was shown that the tsunami can appear as: (1) surging, (2) a breaking bore, (3) an "undular bore evolving into a breaking bore" and finally (4) an undular bore. In the surging scenarios the surface elevations are essentially horizontal and the inundation speeds are generally low. Furthermore, these case are characterized by full reflection from the beach, resulting in a standing wave, causing a phase shift between the maximum surface elevation and flow velocity. The surging tsunami would primarily occur in situations where the slope was sufficiently steep to be felt more or less as a vertical wall. If the tsunami is linear at the toe of the slope, the simulated results indicate that cases clearly predicted to be non-breaking by the analytical method of Madsen and Schäffer (2010) would be surging.

On milder slopes, the tsunami steepened significantly, and the wave front split into shorter waves. In most situations these waves broke almost immediately, turning the whole tsunami into a breaking bore. In some situations more and more short waves developed, turning the front of the tsunami into an undular bore which sometimes broke before reaching the shore, and in a single case survived all the way to the shore.

It has been shown that the shorter waves of the undular bores could appear at the crest of the tsunami, as well as of the front of the crest depending on the local surface elevation steepness. It was further shown that these shorter waves had little impact on the run-up height and inundation speed, confirming previous hypothesis made by Madsen et al. (2008). They were, however, shown to be important in relation to local flow velocities, which have been demonstrated to be significantly higher when shorter waves were present.

It is hoped that the present paper will shed light on, in general, how tsunamis run-up, and from this understanding contribute to improved hazard assessment.

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