Full-scale CFD simulation of tsunamis. Part 2: Boundary layers and bed shear stresses

Larsen, Bjarke Eltard; Fuhrman, David R.

Published in:
Coastal Engineering

Link to article, DOI:
[10.1016/j.coastaleng.2019.04.011](https://doi.org/10.1016/j.coastaleng.2019.04.011)

Publication date:
2019

Document Version
Peer reviewed version

[Link back to DTU Orbit](#)

Citation (APA):
Larsen, B. E., & Fuhrman, D. R. (2019). Full-scale CFD simulation of tsunamis. Part 2: Boundary layers and bed shear stresses. *Coastal Engineering*, 151, 42-57. <https://doi.org/10.1016/j.coastaleng.2019.04.011>

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Full-scale CFD simulation of tsunamis. Part 2: Boundary layers and bed shear stresses

Bjarke Eltard Larsen^{a,*}, David R. Fuhrman^a

^a*Technical University of Denmark, Department of Mechanical Engineering, Section of Fluid Mechanics, Coastal and Maritime Engineering, DK-2800 Kgs. Lyngby, Denmark*

Abstract

This paper presents results from numerical simulations of the propagation and run-up of full scale tsunamis, using a Reynolds-Averaged Navier-Stokes model, with emphasis on the resulting boundary layers and bed shear stresses. Spatial distributions of the Shields and Rouse parameters during run-up and draw-down show that for the tsunamis considered, with a sediment grain size corresponding to medium sand, considerable sediment transport (Shields parameter $O(10)$ - $O(100)$) can be expected during run-up and that the sediment transport can be expected to be dominated by suspended load. The results likewise show that the expected sediment transport during draw-down will similarly be considerable and dominated by suspension. The tsunami-induced boundary layers are monitored in time, and the observed boundary layer thickness ranges from spanning only a small fraction of the water depth to spanning the entire depth. The velocity profiles beneath the tsunamis are shown to have good correspondence with a logarithmic profile within the boundary layer. Similarly, the bed shear stresses beneath the tsunamis are investigated and a new and simple engineering model is developed for predicting the temporal variation of the bed shear stress based only on a free-stream velocity signal. The new engineering model is shown give better predictions for the bed shear stress than a standard Manning approach, and is likewise shown (in the Appendix) to produce reasonable predictions for shorter (wind scale) waves. It is finally also shown how the temporal evolution of the boundary layer thickness can be predicted based on the free-stream velocity signal alone. The results presented here are Part 2 of a larger study, where Part 1 presents the model validation and detailed descriptions of the run-up process.

*Corresponding author

Email address: bjelt@mek.dtu.dk (Bjarke Eltard Larsen)

Keywords: Tsunamis, Computational Fluid Dynamics, Boundary layers, Shear stress, Turbulence modelling

1. Introduction

1 The potentially fatal impact of large tsunamis is well known. The two
2 recent major tsunami events, the Boxing Day tsunami in 2004 in the Indian
3 Ocean, and the Tohoku tsunami near Japan in 2011 were responsible for
4 approximately 230,000 and 20,000 fatalities, respectively (Suppasri et al.,
5 2012). Therefore, it is natural that most tsunami research in the past has
6 rightly focused on predicting run-up height and inundation distance, as in-
7 creased knowledge in these areas can potentially save human lives.

8 In addition to fatalities, tsunamis can also potentially have a long term
9 effect on entire coastal regions. Beaches can get eroded, scour can occur
10 around coastal structures, large sediment deposits can enter harbours, and
11 sediment deposited inland can hinder recovery work. Following the Tohoku
12 tsunami, it was reported that 90% of the beach had disappeared in the
13 Takata Matsubara area (Japan), and it was estimated that the total volume
14 of morphological change over an approximately 2000 m stretch was $1.9 \times$
15 10^6 m^3 (Kato et al., 2012; Yamashita et al., 2016). Before-and-after satel-
16 lite images of the beaches of Banda Aceh, Indonesia from the 2004 Indian
17 Ocean tsunami likewise show large scale erosion, with the coast locally re-
18 treating up to 200 m (Paris et al., 2009; Borrero et al., 2006). For other field
19 studies of the morphological impacts of tsunamis see e.g. Udo et al. (2016)
20 and Kuriyama et al. (2014). The above-mentioned field studies all provide
21 some useful insight on tsunami induced sediment transport and morphology.
22 They do not, however, give insight into the detailed transport patterns that
23 emerge, which is needed if prediction of tsunami-induced sediment transport
24 should be improved.

25 Despite the potentially large effect on beaches and coastal profiles the
26 tsunami-induced sediment transport and morphology has, apart from the
27 previous mentioned surveys, received relatively little attention compared to
28 e.g. run-up height and inundation distance. The few studies that have
29 treated this have primarily been numerical using the non-linear shallow wa-
30 ter (NLSW) equation models. These have either been two-dimensional hor-
31 izontal (2DH) where the flow is assumed uniform across the depth (e.g. Li
32 et al. (2012), Kuriyama et al. (2014), Ontowirjo et al. (2013), Sugawara et al.
33 (2014b)) or three-dimensional where the vertical direction is resolved using
34 a number of σ -layers. Often these have been reduced to two-dimensional

vertical models (2DV) (e.g Apotsos et al., 2011a,b,c). For a more detailed review on numerical modelling of tsunami induced sediment transport using NLSW models see Sugawara et al. (2014a). Such models are reasonably good at handling tsunami propagation and inundation. They do not describe dispersion, however, and will therefore not be able to capture the splitting of steep tsunami wave fronts into an undular bore, nor do they describe wave breaking accurately. Furthermore, in 2DH models the flow is uniform across the depth, and hence the bed shear stress and resulting sediment transport is typically based on either a Manning coefficient, a Colebrook-White type friction formula or an assumed logarithmic velocity profile. Due to the very limited number of studies on tsunami-induced boundary layers it is not known how well these approximations work in such instances, however.

Studies relating to tsunami-induced boundary layers are seemingly limited to the field measurement of Lacy et al. (2012), as well as the numerical studies of Williams and Fuhrman (2016) and Tanaka et al. (2016). Lacy et al. (2012) showed that the tsunami-induced boundary layer from the 2010 Chilean Tsunami, measured off the coast of California, were expectedly larger than typical wind wave boundary layers, though still only spanning a small portion of the entire depth. In a one dimensional vertical (1DV) Reynolds-Averaged Navier-Stokes (RANS) model Williams and Fuhrman (2016) simulated a series of tsunami-scale boundary layers, emphasizing that they are both current-like due to their long duration, but also wave-like, in the sense that they are unsteady and that the boundary layer may not span the entire water depth, similar to observations made by Lacy et al. (2012). Williams and Fuhrman (2016) likewise proposed formulas for predicting tsunami boundary layer thickness and friction factors for various tsunami signals and various bed roughnesses. Their study was limited to offshore regions (depths ≥ 10 m) to prevent the boundary layer being depth-limited, and the driving tsunami signal was treated as a time varying current. Treating the tsunami induced boundary layer as a time varying current has also been utilized in studies of tsunami induced scour around monopiles by Larsen et al. (2017) and Larsen et al. (2018). In an attempt to combat the uncertainties of the NLSW models regarding the bed-shear stress Tanaka et al. (2016) simulated tsunami run-up with a NLSW model coupled with a RANS model, using a $k-\omega$ turbulence model for describing the boundary layer. They showed that this gave different shear stress approximations compared to those that would be predicted using a Manning coefficient. It is unclear, however, exactly how they coupled the two models. Despite this, it is an interesting attempt to remedy some of the uncertainties regarding the bed shear stresses predicted by NLSW models.

75 Finally, tsunami-induced sediment transport has also been considered
76 in a branch of the paleo-tsunami research community trying to predict flow
77 characteristics of historic tsunamis based on sediment deposits, using what is
78 normally referred to as inverse models. Many different inverse models exist
79 see e.g. Moore et al. (2007), Soulsby et al. (2007) and Jaffe and Gelfenbuam
80 (2007). These models all rely on numerous assumptions, and amongst the
81 most common are: (1) sediments travel in suspension and (2) are not re-
82 suspended during draw-down. In an attempt to contribute to the paleo
83 tsunami research, Johnson et al. (2016) studied deposits left by a breaking
84 bore and concluded that an existing advection-settling model was able to
85 predict flow depths within a factor two and velocities within a factor 1.5,
86 thus showing a reasonable performance of one of the paleo tsunami inverse
87 models. Yoshii et al. (2017, 2018) studied experimentally tsunami deposits
88 in their 205 m long flume for a range of topographies. They concluded
89 that the majority of the deposits came from the onshore flow. The return
90 flow created seaward fining (sediment tending to have smaller grain sizes),
91 something contrary to many paleo tsunami inverse models, a point also made
92 by Paris et al. (2012). For a more complete overview of inverse models see
93 Sugawara et al. (2014a), and a review on tsunami-seabed interaction in
94 general can be found in Larsen (2018).

95 With the use of computational fluid dynamics (CFD) the present work
96 aims at both qualitatively and quantitatively describing tsunami-induced
97 boundary layers and shear stresses as well as the implications for sediment
98 transport. The knowledge on boundary layers and bed shear stresses can
99 be useful for implementation in "simpler" potential flow models (i.e. NLSW
100 type or Boussinesq type), possibly improving their sediment transport pre-
101 dictive capabilities. The results can likewise be useful for the previously de-
102 scribed inverse models, as it will shed light on the validity (or lack thereof)
103 of some of the common assumptions made in these models.

104 The results presented in this work build on the same simulations pre-
105 sented in Larsen and Fuhrman (2018a) (hereafter referred to as Part 1). In
106 Part 1 the numerical model was described, validated (for free surface eleva-
107 tions, run-up, wave breaking and bed shear stresses) and used to investigate
108 in detail the run-up processes of full-scale tsunamis.

109 The remainder of the paper is organized as follows: In Section 2 the
110 model is briefly described and the set-up of the model is provided. In Section
111 3 the model results for (1) Shields and Rouse parameters and (2) boundary
112 layer dynamics are presented, and a new method for predicting instantane-
113 ous tsunami-induced boundary layer thickness and bed shear stress is
114 proposed. In Section 4 overall conclusions are drawn. In Appendix A the

115 performance of the proposed method is demonstrated for short waves.

116 2. Model description and setup

117 For a complete description of the CFD model employed in the present
 118 work the reader is referred to Part 1. For the sake of efficiency, only an ab-
 119 breviated description is provided here. The CFD simulations are performed
 120 in the two-phase volume of fluid method (VOF) flow model **waves2FOAM**
 121 developed by Jacobsen et al. (2012). Here, the Reynolds-averaged Navier-
 122 Stokes (RANS) equations are solved. To close the system the k - ω model
 123 of Larsen and Fuhrman (2018b) is used, who extended the Wilcox (2006)
 124 model such that it is formally stable (i.e. does not un-physically produce
 125 turbulence) in the potential flow region beneath surface waves. For the
 126 turbulence quantities at the bottom the generalized wall function approach
 127 presented in Fuhrman et al. (2014) is used. These wall functions have also
 128 been used by e.g. Baykal et al. (2015), Larsen et al. (2016), Bayraktar et al.
 129 (2016) and Larsen and Fuhrman (2018b).

130 The tsunami waves in this study, will be specified as initial conditions and
 131 are the same as considered in Part 1. As an inspiration for the tsunamis the
 132 well-known Mercator yacht signal, taken during the Indian Ocean tsunami
 133 in 2004 was used. The leading wave of this signal was estimated by Madsen
 134 and Fuhrman (2008) to be approximately sinusoidal to have a wave height of
 135 5 m and a period of $T=13$ min at a water depth of $h = 14$ m. Specifically the
 136 tsunamis will be represented by a general N-wave form (the summation of a
 137 positive and negative single wave) inspired by Madsen and Schäffer (2010),
 138 but with x -variation now included. The free-surface is given by

$$\begin{aligned} \eta(x, t) = & A_1 \text{sech} \left[\Omega_1 \left((t - t_1) - \frac{x - x_0}{\sqrt{gh}} \right) \right]^2 \\ & - A_2 \text{sech} \left[\Omega_2 \left((t - t_2) - \frac{x - x_0}{\sqrt{gh}} \right) \right]^2 \end{aligned} \quad (1)$$

139 where A_1 and A_2 are the amplitudes of the two single waves and g is the
 140 gravitational acceleration. The effective frequencies Ω_1 and Ω_2 defines the
 141 time span of the two single waves according to $\Omega_i = 2\pi/T_i$ where T_i can be
 142 interpreted as the effective period of the corresponding single wave, defined
 143 as the time where the surface elevations drops below 0.7% of the amplitude.
 144 In the above x_0 defines the center of the wave and t_1 and t_2 can be used
 145 to phase shift the two single waves. This signal can be reduced to a "single
 146 wave" by setting $A_2=0$. The initial horizontal velocity is then given by linear

Figure 1: Layout of the computational domain, with origo positioned at the original shoreline. This sketch is a repeat from Part 1.

147 shallow water theory

$$u(x, t) = \sqrt{\frac{g}{h}} \eta(x, t). \quad (2)$$

148 The vertical velocity is derived from the local continuity equation, and the
149 pressure is calculated as hydrostatic.

150 In the simulations we consider the simple canonical case of 2D tsunami
151 waves propagating over an initial constant depth region, before running up
152 a constant slope. The waves will be initialized on a flat bed, which is one
153 wave length long, such that the flat part of the domain houses the entire
154 wave. The waves will then propagate and deform before reaching a constant
155 slope region, where the slope will be systematically varied. A general setup
156 of the computational domain can be seen in Figure 1.

157 Table 1 gives a summary of the cases to be considered in detail in this
158 study, corresponding to a sub-set of those in Part 1. In the first two rows
159 of the table $A_2=0$ and therefore these waves are single waves, with only a
160 positive displacement. The third row contains an N-wave with a leading
161 depression. The periods and phase shifts of the N-wave are chosen to yield
162 the same wave length and crest-to-trough wave height as the single wave
163 cases. In Part 1 a total of 14 cases were presented, but here we only go
164 into great detail with three of these, for the sake of brevity. These three are
165 specifically chosen to represent different run-up scenarios as detailed in Part
166 1. The grain size of the bed is taken as $d = 0.4$ mm (medium sand) such that
167 the roughness for the present cases is given by $k_s = 2.5d = 0.001$ m. For all
168 cases the domain is discretized into cells $\Delta y = 0.25$ m and $\Delta x = 0.5$ m and

Case	S	$A_1[m]$	$A_2[m]$	$T_1[s]$	$T_2[s]$	$t_1[s]$	$t_2[s]$	$x_0[m]$
S030	1/30	5	0	780	0	0	0	-4990.5
S200	1/200	5	0	780	0	0	0	-7370.5
N100	1/100	2.52	2.52	520	520	130	-130	-5970.5

Table 1: Cases, slopes and initial wave parameters for the simulated cases considered in the present work.

from 5 m above the bed and lower the grid is gradually refined (using 51 cell) in the vertical with near-bed cells having $\Delta y = 0.0014$ m corresponding to $\Delta y/k_s = 1.4$. The bottom 0.5 m is discretized into 21 cells.

3. Model results

In this section the modelled results will be presented. The first part of this section will focus on the spatial distribution of the Shields and Rouse parameters during run-up and draw-down of the three characteristic tsunami events in Table 1. The next part will focus on the tsunami-induced boundary layers, describing both the temporal development in boundary layer thickness and the shape of the velocity profiles. Finally, based on the results from the detailed boundary layer analysis a new engineering approach for estimating the friction velocity (and hence the bed shear stress) will be developed.

In this section some mentioning of the run-up characteristics of the tsunami will be made, but for a full discussion on these, please see Part 1.

3.1. Shields and Rouse parameters

In the present study to save computational time, only the hydrodynamics were solved for (i.e. velocities, surface elevations, turbulence and bed shear stresses). However, some conclusions regarding sediment transport can be made by looking at the Shields and Rouse parameters. The Shields parameter is a non-dimensional number proportional to the ratio of driving and stabilizing forces on sediment grains and is defined (Fredsøe and Deigaard, 1992) as

$$\theta = \frac{U_f |U_f|}{g(s-1)d} \quad (3)$$

where $s = 2.65$ is the assumed relative sediment density and U_f is the friction velocity. The Rouse parameter can be interpreted as proportional

195 to the ratio between settling velocity and near-bed turbulent fluctuations,
 196 and is conventionally defined (Fredsøe and Deigaard, 1992)

$$Z = \frac{w_s}{\kappa|U_f|} \quad (4)$$

197 where $\kappa = 0.4$ is the von Kármán's constant and w_s is the fall velocity. On
 198 this basis, analogous to (3), we define

$$Z^{-2} = \frac{\kappa^2 U_f |U_f|}{w_s^2} = \frac{\kappa^2 g(s-1)d}{w_s^2} \theta \quad (5)$$

199 such that (for a given grain size d and relative density, s) θ and Z^{-2} differ by
 200 only a constant and can thus be represented with a single plot. Throughout
 201 the present work $w_s = 0.0527$ m/s is taken as the settling velocity (calculated
 202 as described in Fredsøe and Deigaard, 1992). According to Fredsøe and
 203 Deigaard (1992) and Sumer and Fredsøe (2002) $Z < 2$ – 2.5 ($|Z^{-2}| > 0.18$ –
 204 0.25) is typically necessary for sediment to go into suspension, and sediment
 205 will be distributed more uniformly across the boundary layer as Z is lowered
 206 (or $|Z^{-2}|$ increased).

207 It is important to stress that the result for θ and Z^{-2} presented herein,
 208 are based on the single grain size considered, again corresponding to medium
 209 sand. Other grain sizes would naturally result in different values for θ and
 210 Z^{-2} , and as a result conclusions regarding sediment transport must be in-
 211 terpreted in this context. This will be further elaborated in the end of this
 212 section.

213 As a first case we will consider case S200 (a single wave running up a
 214 slope of $S=1/200$), which runs up as a breaking bore. Figure 2 shows θ and
 215 Z^{-2} as a function of both space and time, and to complement the analysis
 216 Figure 3 shows surface elevations and velocities at three different instants,
 217 which are marked by the horizontal dashed lines in Figure 2. The times
 218 correspond to the tsunami shoaling (Figure 3a), inundating (Figure 3b) and
 219 receding (Figure 3c). During shoaling it can be seen that the steep tsunami
 220 wave front has split, and that a short wave is riding on the front (Figure
 221 3a). The short wave was not able to maintain its shape, however, and it
 222 broke before reaching the shore (this was shown in Part 1 to be typical of
 223 tsunamis running up as breaking bores). For more details regarding the
 224 characteristics of tsunamis running up as breaking bores please see Part
 225 1. The very steep tsunami wave front in Figure 3a can be seen to give a
 226 very sudden increase in the velocity. This increase in velocity, of course,
 227 causes θ and Z^{-2} to increase, as shown in Figure 2 (close to the lower

Figure 2: Shields parameter, θ and inverse of the Rouse parameter squared Z^{-2} as a function of x and t for case S200. The horizontal dashed lines corresponds to the instants shown in Figure 3. The maximum and minimum θ for this case are 258 and -122, respectively.

dashed line), and the wave front can easily be identified by the very large gradient in either θ or Z^{-2} with respect to time. Behind the wave front θ decreases. This is due to the decreasing velocity, but also, as will be shown in the forthcoming sections, due to the boundary layer increasing behind the tsunami front. This reduces the velocity gradients and in turn the bed shear stresses. During shoaling θ is already high and $Z^{-2} > 0.25$ for most of the wave front meaning that sediment can be expected to be transported largely in suspension. It is important to stress that the boundary layer may or may not span the entire water depth, as will be discussed in the following section. Thus even in cases with very high Z^{-2} , the suspended sediment may not necessarily span across the entire depth. It should be noted that the tsunami is much longer (approximately 9 km) than shown in Figure 3a, and further behind the tsunami front $Z^{-2} < 0.25$ meaning that bed load transport would likely be the most important transport mechanism.

While inundating, the velocities increase further as seen in Figure 3b.

Figure 3: Surface elevations and velocities of case S200 at different times.

243 This results in significantly higher θ as seen in Figure 2 (near the dashed
 244 line in the middle). Near the front θ reaches $O(100)$ (though the colorscale
 245 has been limited to $-10 < \theta < 10$ to make the figure more readable). On the
 246 other hand, Z^{-2} reaches $O(10)$, which indicates that the suspended sediment
 247 will be present to a large degree throughout the boundary layer. Behind the
 248 breaking tsunami front Z^{-2} is decreasing meaning that suspended sediment
 249 will likely be more and more confined to the lower part of the tsunami-
 250 induced boundary layer. From Figure 2 it can also be seen that during
 251 inundation the horizontal span with large Shields parameters widens compared
 252 to the shoaling situation. The reason for this is that the depth is
 253 lower (causing larger shear), but also that the boundary layer here (as will
 254 be shown in the Section 3.2) spans the entire depth. Therefore there will
 255 not be large differences in the near bed velocity gradients at the tip of the
 256 tsunami front and some distance behind it. It is worth noting that during
 257 inundation larger regions with significantly lower θ and Z^{-2} are present
 258 around $x \approx 1500$ m and $t \approx 600$ s. These regions occur due to undulations
 259 in the surface elevation in the front of the breaking bore. This appears to
 260 be similar to the process described in Lu et al. (2018), where new bores

261 developed at the bore front in a model scale experiment. In the vertical the
 262 difference in surface elevation of these undulations is not well resolved (only
 263 by approximately 5 cells), but this is still enough cells that we believe it to
 264 be a physical phenomenon.

265 While receding (Figure 3c and near the upper dashed line in Figure 2) it
 266 can be seen that the magnitudes of the velocities and resulting θ values are
 267 somewhat lower than those near the breaking tsunami front. It can however
 268 be seen that high-magnitude θ values are present over a much longer span
 269 (>2 km as can also be seen from Figure 2). This can be explained by
 270 the water depth being much lower compared to the height of the tsunami
 271 front, and as a result vertical velocity gradients and shear stresses become
 272 larger. This results in a much broader region with high Z^{-2} , meaning that
 273 the sediment transport during the draw-down would again also seemingly
 274 be characterized to a high degree by suspended sediment load. It is worth
 275 noting the whitish lines/streaks which appear during recession ($x \approx 0 -$
 276 1000 m and $t \approx 700 - 1200$ s). These appear to be related mixture of
 277 air and water in the vicinity of the bed. This mixture locally enhances
 278 turbulent production and cause variations in the velocities. These mixtures
 279 were generated during the breaking and have a difficulties escaping due to
 280 the simulations being two-dimensional. The mixture regions are only $O(\text{cm})$
 281 in the vertical (a few cells thin), but often span several meters (the cells are
 282 0.5 m long). This is due to the large aspect ratios of the bottom boundary
 283 layer cells. The whitish while receding is therefore believed to be a numerical
 284 artefact caused by the two dimensional simulation and large aspect ratios.
 285 Though unfortunate, we do not believe that these streaks significantly affect the
 286 overall run-up and draw-down process and thereby the conclusions drawn
 287 in the current paper.

288 The θ values and Z^{-2} values will also naturally depend on how the
 289 tsunami runs-up. To demonstrate other characteristic scenarios, we will
 290 now show results for two additional cases, namely case S030 and case N100
 291 (Table 1). These were chosen to also demonstrate the results in cases where
 292 the tsunami is not breaking (case S030, categorized as surging, see Part 1)
 293 or has a leading depression (case N100) which runs up as a breaking bore
 294 similar to case S200 (see Part 1 for the a detailed description of the run-up
 295 of breaking bores).

296 Figure 4 shows the Shields, θ and inverse Rouse parameter squared Z^{-2}
 297 as a function of both time and space for case S030, which is a single wave
 298 running up a slope of $S = 1/30$, and Figure 5 shows surface elevations
 299 and velocities at three different instants. The times were again selected to
 300 show the tsunami during shoaling (Figure 5a), inundation (Figure 5b) and

draw-down (Figure 5c). While shoaling the tsunami wave front is not

Figure 4: Shields parameter, θ and inverse of the Rouse parameter squared, Z^{-2} as a function of x and t for case S030. The horizontal dashed lines corresponds to the instants shown in Figure 5. The maximum and minimum θ for this case are 123 and -12, respectively.

very steep, as seen in Figure 5a. This results in lower velocities, and as a result also lower θ values compared to case S200 (Figure 4 compared to Figure 2 near the lower dashed lines). As a result $Z^{-2} < 1$ is generally the case while shoaling. As $Z^{-2} > 0.252$, this still means that sediment will likely be transported in suspension, but due to the lower values relative to those from case S200 the sediment would not be expected to be as uniformly distributed across the boundary layer, and would likely be more confined to the bottom part of the boundary layer. During inundation (Figure 5b and 4 near the middle dashed line) it can be seen that the tsunami wave front has accelerated significantly and the velocities and resulting θ values here are similar in size to the front of case S200 during inundation (Figure 3b and 2 near the middle dashed line), as are the Z^{-2} values, which are larger than 0.25 all the way to the toe of the slope. Similar to case S200, it can be seen that while receding (Figure 5c and near the top dashed lines in Figure 4) θ

Figure 5: Surface elevations and velocities of case S030 at different times.

316 does not reach such large extreme values as near the front of the tsunami.
 317 Values exceeding unity are still observed all the way to the toe of the slope,
 318 again indicating significant sediment transport also during draw-down.

319 Finally, in Figure 6 θ and Z^{-2} are shown as a function of space and
 320 time for case N100, which is a leading depression N-wave running up a
 321 slope of $S = 1/100$, and Figure 7 shows surface elevations and velocities at
 322 three different instants (during shoaling Figure 7a, inundation Figure 7 and
 323 draw-down Figure 7c). The leading depression can easily be seen from the
 324 shoaling position in Figure 6a. Here regions with lower surface elevations
 325 and negative velocities are clearly visible in front of the tsunami wave front.
 326 This can likewise be seen in Figure 6 where the shoreline has moved offshore
 327 (negative x). Similar to case S200, the front of the tsunami has split during
 328 shoaling, and a few shorter waves can be seen riding on the main tsunami
 329 wave front. Also similar to case S200, these shorter waves cannot sustain
 330 their shape during shoaling, and break some distance from the shore. The
 331 resulting Shields parameter near the crest of the shoaling tsunami is high
 332 (Figure 6 near the bottom dashed line), but not as high as in case S200
 333 (Figure 2 near the bottom dashed line), where a significantly larger wave

Figure 6: Shields parameter, θ and inverse of the Rouse parameter squared, Z^{-2} as a function of x and t for case N100. The horizontal dashed lines corresponds to the instants shown in Figure 7. The maximum and minimum θ for this case are 185 and -29, respectively.

front was present. It can also be seen that the leading depression, causing the shoreline to retreat, has the potential to move significant amounts of sediment as the magnitude of θ is generally larger than one.

During inundation (Figure 7b and near the middle dashed line in Figure 6) and draw-down (Figure 7c and near the top dashed line in Figure 6) the image is very similar to the two previous cases. Here, the front of the inundating tsunami has very high θ and Z^{-2} , indicating significant sediment transport as well as significant amount of sediment transported in suspension. During draw-down θ and Z^{-2} are again generally high indicating likely significant suspended sediment transport during draw-down. Similar to case S200 some whitish lines/streaks appear in θ and Z^{-2} . This is again explained by undulations in front of the wave during run-up as well as a mixture of air and water in the vicinity of the bed during recession. Additionally, in this case the scar-like features seen during recession around $x \approx 400 - 600$ m and $t \approx 700 - 900$ s are believed to come from the wa-

Figure 7: Surface elevations and velocities of case N100 at different times.

349 ter depth not decreasing monotonically towards the shore. Shorter waves
 350 are generated due to the complex run-up and when these shorter waves run
 351 down it causes variations in both velocities and bed shear stresses.

352 Tsunamis of the size considered in the present work, clearly have the
 353 potential to move a considerable amount of sediment. The largest magnitude
 354 θ are seen at the tsunami front while inundating, indicating highest sediment
 355 transport rates here. Generally the Z^{-2} values in the considered cases were
 356 so high that the sediment transport would be expected to be dominated by
 357 suspension. At the front of the inundating tsunami Z^{-2} where even higher,
 358 such that suspended sediment could be expected to be present throughout
 359 the boundary layer and potentially mixed further via any breaking waves.

360 The results presented here can help assess the assumptions commonly
 361 used by inverse paleo tsunami models. In the models by Moore et al. (2007),
 362 Soulsby et al. (2007) and Jaffe and Gelfenbuam (2007) it was assumed that
 363 sediment was transported only during run-up, and not re-suspended during
 364 draw-down. As just shown, during the draw-down phase, suspended sedi-
 365 ment transport can be expected to be present to a large degree. It should
 366 be stated, however, that the present tsunami cases can all be considered to

367 be rather large events, and smaller tsunamis would naturally have less sedi-
 368 ment transport potential. It should also be stated that the current setup has
 369 a constant slope whereas the field will have significantly more complex to-
 370 pographies. Jaffe and Gelfenbuam (2007) argues that the tsunami will have
 371 a tendency to draw-down in topographic low areas and thereby not carry
 372 much of the deposited sediment. The Shields parameters during draw-down
 373 are large, but generally slightly lower than those present near the tsunami
 374 wave front. If the sediment was comprised of widely varying grain sizes, this
 375 would mean that the largest grains, which could be carried by the tsunami
 376 wave front will not necessarily move during draw-down, and the assump-
 377 tion that sediment will not be re-suspended during draw-down may, in some
 378 cases, be more reasonable for the largest grains in a sample. This scenario
 379 would mean that deposits will not tend to have smaller grain sizes mov-
 380 ing inland (deposits fining inland), as assumed by many inverse models, as
 381 the finer sediments would be re-suspended and be drawn seaward, while the
 382 coarser sediments would remain. This is similar, in fact, to the experimental
 383 finding by Yoshii et al. (2017).

384 As mentioned previously the conclusions drawn here, are based on one
 385 assumed grain type. Assuming other grain sizes would mostly affect when
 386 and where sediment is expected to travel in suspension and how uniformly
 387 it would be distributed within the boundary layer. As Z^{-2} in three cases
 388 are significantly above the threshold for when sediment go into suspension
 389 during large parts of both the run-up and draw down, it would require
 390 significantly larger grains (probably gravel type) to alter this conclusion,
 391 however. In contrast, assuming finer grains would mean that sediment could
 392 be expected to travel in suspension and be more uniformly distributed for
 393 even larger parts of the tsunami events.

394 3.2. Boundary layers

395 In this section we will focus on the tsunami induced-boundary layers that
 396 develop during the CFD simulations. For brevity, we will not go into detail
 397 on all three cases. Rather, we will discuss in detail the tsunami-induced
 398 boundary layer of case S200 which can be considered as a typical example of
 399 the simulated cases. Result of boundary layer thickness and friction velocity
 400 of the two remaining cases (S030 and N100) will be shown to supplement
 401 the analysis.

Figure 8: (a) free-stream velocity signal where (:) and (o) marks the separation between the phases and times for the velocity profiles shown in (b)-(e) and (b)-(e) Modelled velocity profiles (-), modelled boundary layer thickness (*), predicted from (8) boundary layer thickness (- -) for case S200 at the toe of the slope ($x = -2800$ m).

Figure 8 shows the free-stream velocity signal (Figure 8a) as well as the velocity profiles (Figure 8b-e) at different times for case S200 at the toe of the slope ($x = -2800$ m). Included in the figure as asterisks are also the modelled boundary layer thickness δ , which has been estimated as the lowest vertical position where

$$\frac{y - y_0}{|u|} \frac{\partial |u|}{\partial y} < 0.03. \quad (6)$$

Here y_0 is the bed elevation. If the above is not fulfilled at any position the boundary layer thickness is simply taken as the water depth. This is somewhat different from the standard way of estimating boundary layer thickness, which is sometimes taken as the vertical position where $u < 0.99u_0$, u_0 being free stream velocity. From the modelled results, however, it was difficult to estimate u_0 , as the crest of the steep waves sometimes had higher velocities than the near bed flow. Overcoming this difficulty therefore led to the above formulation, and as can be seen from Figure 8, the modelled boundary layer thicknesses defined using (6) corresponds well with visual expectations. The threshold of 0.03 was chosen as the lowest value not causing large frequent oscillations in the calculated boundary layer thickness. A lower value would have the calculated boundary layer thickness suddenly jumping to water depth as the threshold was not met. For the remainder of this study u_0 is defined as the velocity at $y = y_0 + \delta$.

Williams and Fuhrman (2016) proposed that the boundary layer thickness beneath the peak flow in a tsunami could be predicted by

$$\frac{\delta}{a} = 0.05 \left(\frac{a}{k_s} \right)^{-0.11} \quad \text{or} \quad \delta = 0.05a^{0.89}k_s^{0.11} \quad (7)$$

where $a = U_m/\Omega$ is the amplitude of the orbital motion and U_m is the maximum free-stream velocity. As we would like to predict the time varying boundary layer thickness, we propose to extend this formula such that

$$\delta(t) = \min \left(0.05a(t)^{0.89}k_s^{0.11}, h \right) \quad (8)$$

The first argument is similar to (7) and the second argument ensures that the predicted boundary layer thickness will not exceed the local water depth. In (8) $a = U_m/\Omega$ has been replaced with

$$a(t) = \int_{t_0}^t |u_0(t)| dt \quad (9)$$

429 where t_0 is the time of the previous zero crossing of the free stream velocity.
 430 $a(t)$ is then a measure of the distance travelled by a free-stream particle
 431 following flow reversal. This definition of $a(t)$ will for the standard sinusoidal
 432 wave case give the same boundary layer thickness at the peak velocity as
 433 the formulation by Williams and Fuhrman (2016). The predicted value for
 434 the (now time varying) boundary layer thickness using (8) is included in the
 435 figure as a dashed line.

436 The flow during first acceleration phase is shown in Figure 8b, the subse-
 437 quent deceleration phase in Figure 8c, the second acceleration phase, which
 438 is the tsunami drawing down in Figure 8d, and finally the second decelera-
 439 tion phase in Figure 8e.

440 During the first acceleration phase (Figure 8b) and the first deceleration
 441 phase (Figure 8c) it can be seen that the boundary layer continually grows.
 442 It starts as only a small fraction of the water depth, but grows nearly to the
 443 free surface by the end of the first deceleration phase. This supports the
 444 results by Williams and Fuhrman (2016) which highlighted that the tsunami-
 445 induced boundary layers may be both wave-like and current-like. During
 446 the draw-down (Figure 8d-e) it can be seen that the boundary layer grows
 447 both during the acceleration and deceleration phases. It can also be seen
 448 that the predicted boundary layer thickness from (8) agrees well during the
 449 acceleration phase, but slightly under-predicts the boundary layer thickness
 450 during the deceleration phase. In Figure 8d a small oscillation can be seen
 451 in dashed line towards the end of the acceleration phase. This caused by a
 452 very small decrease in the free-stream velocity between $t = 1120 - 1150$ s
 453 velocity as seen in Figure 8 which comes from very small differences in the
 454 free-surface.

Figure 9: (a) free-stream velocity signal where (:) and (o) marks the separation between the phases and times for the velocity profiles shown in (b)-(e) and (b)-(e) Modelled velocity profiles (-), modelled boundary layer thickness (*), predicted from (8) boundary layer thickness (- -) for case S200 in a shoaling position ($x = -1200$ m).

455 When the tsunami shoals, the wave front steepens and velocities increase,
 456 as seen in Figure 3a. Figure 9 shows the free-stream velocity (Figure 9a) and
 457 the velocity profiles (Figure 9b-e) of the now-shoaled wave at $x = -1200$
 458 m now corresponding to a water depth of $h = 6$ m. Included in the figure
 459 is also the modelled and predicted boundary layer thickness, similar to be-
 460 fore. During the first acceleration phase (Figure 9b) it can be seen that the
 461 initial acceleration is not that large but from $t = 336$ s to $t = 348$ s the
 462 wave accelerates from approximately 2 m/s to approximately 6 m/s. This
 463 corresponds to the arrival of the very steep wave front. Due to the very
 464 large acceleration, the boundary layer barely has time to grow, and at the
 465 peak velocity the boundary layer thickness is only around 1 m high. This
 466 development is well predicted by equation (8), shown as the dashed line.
 467 During the subsequent deceleration (Figure 9c) the boundary layer grows
 468 all the way to the free surface, which is also captured by (8). During the
 469 draw-down (Figure 9d) a new boundary layer forms. Here the water depth
 470 is almost constant, and the boundary layer eventually stretches all the way
 471 to the free surface. In the deceleration phase of the draw-down (Figure 9e),
 472 the boundary layer remains depth limited. Again an excellent agreement
 473 between the modelled and predicted boundary layer thickness is achieved.

Figure 10: (a) free-stream velocity signal where (:) and (o) marks the separation between the phases and times for the velocity profiles shown in (b)-(e) and (b)-(e) Modelled velocity profiles (-), modelled boundary layer thickness (*), predicted from (8) boundary layer thickness (- -) for case S200 at the shore ($x = 0$ m).

474 When the tsunami hits the coast it becomes even steeper than at the
 475 shoaling position, and the front of the tsunami is breaking as seen in Figure
 476 3b. Figure 10 shows the free-stream velocity (Figure 10a) and the velocity
 477 profiles (Figure 10b-e) at the original shoreline ($x = 0$ m). Included in the
 478 figure is also the modelled and predicted boundary layer thickness, similar to
 479 before. As a result of the increased steepness the initial acceleration phase
 480 is only approximately 1% of the period, where it accelerates from zero to 12
 481 m/s in just 8 s, as seen in Figure 10b. In this case the wave is running up like
 482 a breaking bore, and the boundary layers are essentially at all times limited
 483 by the water depth. This behaviour is not captured by (8), but as can be
 484 seen during the subsequent longer deceleration phase (Figure 10c), due to
 485 the high velocities, this expression quickly predicts that the boundary layer
 486 will extend all the way to the free-surface, similar to the modelled results.
 487 Similar to the shoaling position, Figure 10d shows that a new boundary
 488 layer is forming during the acceleration phase of the draw-down, and this
 489 boundary layer eventually reaches the free-surface. During the latter stages
 490 of this acceleration phase oscillations in the dashed line can be seen. At this
 491 stage boundary layers is depth limited and the oscillations are due to varying
 492 velocities in the top most cell which contain a mixture of air and water and
 493 might be slightly affected by air velocities. During the final deceleration
 494 stage (Figure 10e), the boundary layer remains limited by the water depth,
 495 which slowly declines towards zero.

496 It is worth noting that the velocity profiles during run-up and draw-down
 497 are very sheared. This is similar to the velocity profiles from O'Donoghue
 498 et al. (2010) for the run-up and draw-down of a bore in hydraulically rough
 499 conditions. The sheared velocity profiles might be important for impact
 500 forces by floating debris which depending on geometry and buoyancy will
 501 be affected very differently by this sheared flow.

502 In some studies logarithmic velocity profiles spanning the entire depth
 503 have been assumed, see e.g. Kuriyama et al. (2014) and Sugawara et al.
 504 (2014b). This assumption was partially backed up by Lacy et al. (2012),
 505 who showed a clear logarithmic profile beneath the 2010 Chilean tsunami
 506 (measured outside the coast of California), though this logarithmic pro-
 507 file only spanned a small fraction of the water depth. To demonstrate the
 508 appearance of the tsunami-induced boundary layers Figure 8a,b and Fig-
 509 ure 9a,b are re-plotted now using a logarithmic vertical scale in Figure 11.
 510 This shows the velocity profiles during the first acceleration and decelera-
 511 tion phase from case S200 at both the toe of the slope at $x = -2800$ m
 512 (Figure 11a,b) and in the shoaling position at $x = -1200$ m (Figure 11c,d).
 513 Figure 11 clearly illustrates that the computed tsunami-induced boundary

514 layers are indeed logarithmic, as nearly perfect straight lines near the bed
 515 are shown. The results, however, also indicate that the velocity profiles do
 516 not follow a logarithmic profile all the way to the free-surface, but rather a
 517 limited part of the water depth. It can likewise be seen that (8) depicted
 518 as a dashed line, gives a reasonable approximation for the region where the
 519 velocity profiles are logarithmic. This is typical for all the cases simulated.
 This indicates that assumed logarithmic velocity profiles could work reason-

Figure 11: Modelled velocity profiles (-) and predicted boundary layer thickness (8) (- -) for case S200 at (a),(b) $x = -2800$ m and (c),(d) $x = -1200$ m.

520
 521 ably for sediment transport purposes, provided that the limited thickness
 522 of the boundary layer is taken into account. For floating debris an assumed
 523 logarithmic profile will therefore probably not be a good assumption. Above
 524 the line given by (8) the velocity profiles might be uniform as was the case at
 525 the toe (Figure 11a,b) or they be further sheared as a consequence of break-
 526 ing, very steep wave front or shorter dispersive waves as in the shoaling
 527 position (Figure 11c,d).

528 3.3. Predicting the bed shear stress/friction velocity

529 With the good agreement between the modelled and predicted time vary-
 530 ing boundary layer thickness, it seems obvious to extend this approach to
 531 likewise predict the bed shear stress (or friction velocity), which is often
 532 regarded as the primary factor governing sediment transport. Williams and
 533 Fuhrman (2016) showed that standard wave friction factor expressions also
 534 applied reasonably to the tsunami-scale waves at peak flow. We therefore
 535 suggest the following simple engineering model to predict the friction veloc-
 536 ity beneath tsunami waves:

$$U_f(t) = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{f(t)}{2}} u_0(t) \quad (10)$$

537 where τ_0 is the bed shear stress, ρ is the water density and $f(t)$ is a time
 538 varying friction factor. Another variant of a time varying friction factor ap-
 539 proach has also been used by Gonzalez-Rodriguez and Madsen (2007), see
 540 Appendix A for more details. For hydraulically rough flows, the constant
 541 friction factor is traditionally given as a function of a/k_s . There exist many
 542 formulae for the friction factor as a function of a/k_s , see e.g. Fredsøe and
 543 Deigaard (1992), Nielsen (1992) or Fuhrman et al. (2013). In what follows,
 544 we will utilize the expression from Fuhrman et al. (2013), but again sub-
 545 stituting a with $a(t)$ from (9), leading to the following time-varying wave
 546 friction factor:

$$f_w(t) = \exp \left(5.5 \left(\max \left(\frac{a(t)}{k_s}, 1 \right) \right)^{-0.16} - 6.7 \right) \quad (11)$$

547 From this equation it can be seen that large orbital free stream motion will
 548 lead to smaller friction factors. The reason for this is that the boundary
 549 layer will have longer time to grow, and thus the velocity gradients become
 550 smaller. A max condition on $a(t)/k_s$ has been introduced to handle the
 551 singularity as well as limit the friction factor for very small values of $a(t)/k_s$.
 552 In addition to this, it is important to recognize (as just shown) that the
 553 tsunami boundary layer may extend all the way to the free surface, and at
 554 this stage it would be more current-like. Therefore, to also account for this
 555 possibility we define $f(t)$ generally as

$$f(t) = \max(f_w(t), f_c) \quad (12)$$

556 where f_c is the current friction factor given by the Colebrook-White formula

$$f_c = \frac{2}{(C - 2.5 \log(k_s/h))^2} \quad (13)$$

557 with $C = 8.5$. The change from the traditional coefficient $C = 6.5$ from
 558 Colebrook and White to $C = 8.5$ is because the present formulation is based
 559 on the free-stream velocity ($u_0 = u(y = \delta)$ i.e. at the top of the boundary
 560 layer) rather than the depth-averaged velocity. The above formulation en-
 561 sures a continuous transition to a current-based friction factor at positions
 562 where the boundary layer extends all the way to the free-surface. Note that
 563 the model described above may also be utilized in conjunction with depth-
 564 averaged velocities e.g from NLSW models, by instead setting $C = 6.5$.
 565 Additionally, for comparison, we will also estimate the friction velocity by
 566 what we will deem a standard approach for NLSW models. In the literature
 567 different approaches have been used. Kuriyama et al. (2014) and Cheng and
 568 Weiss (2013) e.g. used an assumed logarithmic profile spanning the entire
 569 depth and Yamashita et al. (2016) as well as Li et al. (2012) used a Man-
 570 ning approach. Here we will compare with the Manning approach, where
 571 the friction velocity for the present cases can alternatively be calculated as

$$U_f = \langle U \rangle n h^{-1/6} \sqrt{g} \quad (14)$$

572 where

$$\langle U \rangle = \frac{1}{h} \int_{y_b}^{\eta} u dy \quad (15)$$

573 is the depth-averaged velocity and n is the Manning coefficient, which in
 574 what follows has been calculated by

$$n = \frac{k_s^{1/6}}{8.1\sqrt{g}} = 0.0125 \frac{\text{s}}{\text{m}^{1/3}} \quad (16)$$

575 We will now compare the predicted time variation of the friction veloc-
 576 ity using both the new engineering approach and the Manning approach
 577 for cases N100 and S030. We will likewise compare the modelled and pre-
 578 dicted boundary layer thickness to illustrate the performance of (8) for two
 579 additional cases.

580 Figure 12 shows free-stream velocities (Figure 12a-c), surface elevations,
 581 modelled- and predicted boundary layer thickness (Figure 12d-f) as well
 582 as friction velocities using both the present engineering approach and the
 583 Manning approach (Figure 12g-i) for case N100 at the toe of the slope ($x =$

584 -1400 m), at a shoaling position ($x = -900$ m,) and at the shore ($x = 0$
585 m). For the Manning approach we have used the calculated free-stream
586 velocity rather than the depth-averaged velocity. The difference between the
587 depth-averaged and free-stream velocity is small, although using the depth-
588 averaged velocity resulted in slightly lower friction velocity predictions than
589 are presented here.

590 Figure 12g-i shows the friction velocity being lowest at the toe of slope
591 and highest at the shore. This is both due to an increased velocity as the
592 wave shoals (see the increasing free stream velocity in Figure 12a-c), but
593 also an increased asymmetry, resulting in smaller boundary layer thickness
594 and higher velocity gradients. The predicted friction velocity using both
595 the Manning approach (14) as well as the engineering model (10) follows
596 the shape of the modelled friction velocity. There is, however, a tendency
597 to under-predict the magnitude of the friction velocity using the Manning
598 approach at the toe of the slope and in the shoaling position. This can
599 be explained by the Manning approach not capturing the effect of the time
600 varying boundary layer thickness, which at these positions only spans a small
601 fraction of the water depth, when the front of the tsunami arrives (Figure
602 12g-h). This physical feature, by design, seems to be well captured by the
603 present approach based on (10). It can also be seen that (8), in general,
604 does a consistently good job of predicting the time varying boundary layer
605 thickness for case N100. The largest discrepancies are found at the shoaling
606 position during the second and third deceleration phases (Figure 12e, $t =$
607 $430-670$ s and $t = 900-950$ s).

608 Figure 13 shows free-stream velocities (Figure 13a-c), surface elevations
609 as well as modelled- and predicted boundary layer thickness (Figure 13d-f)
610 and modelled friction velocities as well as predicted friction velocities using
611 both the present engineering approach and the Manning approach (Figure
612 13g-i). These are shown for case S030 at the toe of the slope ($x = -420$ m),
613 at a shoaling position ($x = -180$ m,) and at the shore ($x = 0$ m). Similar to
614 before both the present engineering model and the Manning approach follow
615 the shape of the computed friction velocity (Figure 13g-i), but again there
616 is a tendency to under-predict the magnitude of the friction velocity using
617 the Manning approach at the toe of the slope and at the shoaling position
618 (Figure 13g,h). The proposed engineering model again captures this effect.
619 It is also worth noting, that in this case, where the wave is not breaking,
620 the boundary layer only stretches the entire depth during the deceleration
621 phase of the draw-down at the shoaling position (Figure 13e, $t = 550 - 800$
622 s) as well as the deceleration phase of the draw-down at the shore (Figure
623 13f, $t = 430 - 600$ s). This indicates that the tsunami-induced boundary

layer is very dependent on the run-up type. Finally, it can be seen from Figure 13d-f that the match between the modelled boundary layer thickness and that predicted by (8) is again reasonable.

To demonstrate further the performance of the proposed engineering model and the difference between this and the standard Manning approach, Figure 14 shows the computed and predicted friction velocity at both the toe of the slope, at a shoaling position and at the shore, for 13 of the 14 cases simulated in Part 1, sampled at 2 second intervals. The only case not included is case S030L, which ran up as a undular bore. In this case sometimes the near-bed flow reversed without the outer flow reversing which then caused $a(t) \rightarrow 0$. When the near-bed flow then re-reversed the flow some distance away from the bed had maintained its shape, thus creating a much larger boundary layer and in turn much lower friction velocities than those predicted by (10). It should be noted that, using a standard Manning approach for this case, would also result in poor predictions, as the depth-averaged velocities never changed sign, and thus this approach would at these instants predict the friction velocity with a wrong sign. This case was therefore omitted from Figure 14 as it would make comparison of the models for the remaining cases more difficult. The difficulty predicting this particular case is duly noted, however. From Figure 14 it can clearly be seen that the proposed model improves the prediction of the friction velocity significantly at the toe of the slope (Figure 14a,b) and at the shoaling position (Figure 14c,d), but remains very similar to the prediction using the Manning approach at the shore (Figure 14e,f). The greatest improvement is seen at the higher friction velocities, which correspond to the arrival of the steep wave front where the tsunami-induced boundary layer has not had significant time to grow. At the shore, where wave breaking occurs in many of the cases, the scatter is naturally larger for both methods.

The general way in which the present formulas (8)-(11) are derived and implemented, suggests that they may well also be applicable for shorter waves i.e. that they are not limited to predicting the boundary layer thickness and bed shear stresses from the free-stream velocity signal alone in the tsunami-induced flows considered above. That the developed approach is more generally valid is verified in more detail in Appendix A.

4. Conclusions

In this work results from CFD simulations of full scale tsunamis running up different constant sloped regions are presented, with special focus on tsunami-induced boundary layers, bed shear stresses and implications for

662 sediment transport. The results presented here are connected to Part 1
663 (Larsen and Fuhrman, 2018a), where the run-up process from the same full
664 scale simulations were discussed.

665 Spatial distributions of Shields and Rouse parameters have revealed that,
666 for the cases considered, significant sediment transport could be expected to
667 occur (Shields parameters of up to $O(10)$ - $O(100)$), and that this sediment
668 transport would in general be dominated by suspended sediment transport,
669 provided that the sediment corresponds to medium sand. Large Rouse pa-
670 rameters beneath the inundating tsunami front have indicated that the sedi-
671 ment in suspension would probably span the entirety of the boundary layer.
672 Furthermore, behind the tsunami front the lower Rouse parameters indi-
673 cate that suspended sediment transport would still be dominant, but that
674 it would likely be confined to the lower part of the boundary layer. Dur-
675 ing the draw-down both Shields and Rouse parameters likewise indicate
676 that substantial suspended sediment transport could be expected. This,
677 thus, questions the validity of one of the primary assumptions in many
678 inverse paleo-tsunami research models, namely that sediment will not get
679 re-suspended during draw-down.

680 Subsequently, the detailed physics of the tsunami-induced boundary lay-
681 ers have been discussed. The observed boundary layer thicknesses have
682 ranged from spanning a small proportion of the water depth to spanning the
683 entire depth. Especially in the non-breaking case considered, the boundary
684 layer rarely stretched the entire depth, at least from the original shoreline
685 and shoreward. In the cases running up as breaking bores, the boundary
686 layers stretched the entire water depth during run up, but during draw-down
687 a new boundary layer formed which was not depth limited.

688 It has been shown that the temporal variation of the boundary layer
689 thickness can be reasonably predicted using standard empirical engineering
690 formulas, but replacing the orbital amplitude with the cumulative distance
691 travelled by a free-stream particle following each flow reversal. The com-
692 puted velocity profiles beneath the tsunami have been shown to yield a
693 reasonable match with a logarithmic profile up to the predicted boundary
694 layer thickness.

695 Finally, it has been shown that the time-varying friction velocity could
696 be similarly well predicted by a simple engineering model based on a time-
697 varying friction factor, while also taking into account the time-varying bound-
698 ary layer thickness described above. This approach was shown to give better
699 estimates for the friction velocity beneath the steep tsunami wave front than
700 a standard Manning approach, which does not account for the time varying
701 boundary layer thickness.

702 The new approach is also shown to give accurate predictions of the bed
 703 shear stresses for much shorter waves across a wide range of a/k_s as well as
 704 both sinusoidal, asymmetric, skewed and irregular short waves in Appendix
 705 A. Both the engineering model for the time-varying boundary layer thick-
 706 ness and the friction velocity can easily be implemented in any NLSW or
 707 Boussinesq wave model, potentially improving their ability to predict sedi-
 708 ment transport beneath tsunamis (or much shorter waves). This can prove
 709 valuable in assessing the impact of tsunamis on coastal regions.

710 5. Acknowledgements

711 The authors acknowledge support from the European Union project
 712 ASTARTE–Assessment, Strategy And Risk Reduction for Tsunamis in Eu-
 713 rope, Grant no. 603839 (FP7-ENV-2013.6.4-3). The authors likewise ac-
 714 knowledge financial support from the Independent Research Fund Denmark
 715 project SWASH: Simulating WAVE Surf-zone Hydrodynamics and sea bed
 716 morphology, Grant no. 8022-00137B The authors finally acknowledge Do-
 717 minic A. van der A for providing the data used for comparison in Figure
 718 A.16 as well as Jian Yuan for providing the data used for comparison in
 719 Figure A.17.

720 Abreu, T., Michallet, H., Silva, P. A., Sancho, F., van der A, D. A., Ruessink,
 721 B. G., 2013. Bed shear stress under skewed and asymmetric oscillatory
 722 flows. *Coast. Eng.* 73, 1–10.

723 Apotsos, A., Buckley, M., Gelfenbaum, G., Jaffe, B., Vatvani, D., 2011a.
 724 Nearshore tsunami inundation model validation: Toward sediment trans-
 725 port applications. *Pure Appl. Geophys.* 168 (11), 2097–2119.

726 Apotsos, A., Gelfenbaum, G., Jaffe, B., 2011b. Process-based modeling of
 727 tsunami inundation and sediment transport. *J. Geophys. Res.* 116 (1),
 728 1–20.

729 Apotsos, A., Gelfenbaum, G., Jaffe, B., Watt, S., Peck, B., Buckley, M.,
 730 Stevens, A., 2011c. Tsunami inundation and sediment transport in a
 731 sediment-limited embayment on American Samoa. *Earth-Sci Rev.* 107 (1-
 732 2), 1–11.

733 Baykal, C., Sumer, B. M., Fuhrman, D. R., Jacobsen, N. G., Fredsøe, J.,
 734 2015. Numerical investigation of flow and scour around a vertical circular
 735 cylinder. *Phil. Trans. Roy. Soc. A* 373, article no. 20140104.

- 736 Bayraktar, D., Ahmad, J., Larsen, B. E., Carstensen, S., Fuhrman, D. R.,
737 2016. Experimental and numerical study of wave-induced backfilling be-
738 neath submarine pipelines. *Coast. Eng.* 118, 63–75.
- 739 Borrero, J. C., Synolakis, C. E., Fritz, H., 2006. Northern Sumatra field
740 survey after the December 2004 Great Sumatra earthquake and Indian
741 Ocean tsunami. *Earthquake Spectra* 22 (3), S93–S104.
- 742 Cheng, W., Weiss, R., 2013. On sediment extent and runup of tsunami
743 waves. *Earth and Planetary Science Letters* 362, 305–309.
- 744 Fredsøe, J., 1984. Turbulent wave boundary layer in wave-current motion.
745 *J. Hydraul. Eng. ASCE* 110, 1103–1120.
- 746 Fredsøe, J., Deigaard, R., 1992. *Mechanics of Coastal Sediment Transport*.
747 World Scientific, Singapore.
- 748 Fuhrman, D. R., Baykal, C., Sumer, B. M., Jacobsen, N. G., Fredsoe, J.,
749 2014. Numerical simulation of wave-induced scour and backfilling pro-
750 cesses beneath submarine pipelines. *Coast. Eng.* 94, 10–22.
- 751 Fuhrman, D. R., Schløer, S., Sterner, J., 2013. RANS-based simulation of
752 turbulent wave boundary layer and sheet-flow sediment transport pro-
753 cesses. *Coast. Eng.* 73, 151–166.
- 754 Gonzalez-Rodriguez, D., Madsen, O. S., 2007. Seabed shear stress and bed-
755 load transport due to asymmetric and skewed waves. *Coast. Eng.* 54,
756 914–929.
- 757 Gonzalez-Rodriguez, D., Madsen, O. S., 2011. Boundary-layer hydrodynam-
758 ics and bedload sediment transport in oscillating water tunnels. *J. Fluid*
759 *Mech.* 667, 48–84.
- 760 Jacobsen, N. G., Fuhrman, D. R., Fredsøe, J., 2012. A wave generation
761 toolbox for the open-source CFD library: OpenFOAM (R). *Int. J. Numer.*
762 *Meth. Fluids* 70, 1073–1088.
- 763 Jaffe, B. E., Gelfenbuam, G., 2007. A simple model for calculating tsunami
764 flow speed from tsunami deposits. *Sedimentary Geol.* 200 (3-4), 347–361.
- 765 Jensen, B. L., Sumer, B. M., Fredsøe, J., 1989. Turbulent oscillatory bound-
766 ary layers at high Reynolds numbers. *J. Fluid Mech.* 206, 265–297.

767 Johnson, J. P. L., Delbecq, K., Kim, W., Mohrig, D., 2016. Experimen-
768 tal tsunami deposits: Linking hydrodynamics to sediment entrainment,
769 advection lengths and downstream fining. *Geomorphology* 253, 478–490.

770 Kato, F., Noguchi, K., Suwa, Y., Sakagami, T., Sato, Y., 2012. Field survey
771 on tsunami-induced topographical change. *Journal of Japan Society of*
772 *Civil Engineers*, Ser. B3 68 (2), 174–179.

773 Kuriyama, Y., Takahashi, K., Yanagishima, S., Tomita, T., 2014. Beach
774 profile change at Hasaki, Japan caused by 5-m-high tsunami due to the
775 2011 off the Pacific coast of Tohoku earthquake. *Mar. Geol.* 355, 234–243.

776 Lacy, J. R., Rubin, D. M., Buscombe, D., 2012. Currents, drag, and sediment
777 transport induced by a tsunami. *J. Geophys. Res.- Oceans* 117 (9), article
778 no. C0928.

779 Larsen, B. E., 2018. Tsunami-seabed interactions. Ph.D. thesis, Technical
780 University of Denmark, Kgs. Lyngby, Denmark.

781 Larsen, B. E., Arbøll, L. K., Frigaard, S., Carstensen, S., Fuhrman, D. R.,
782 2018. Experimental study of tsunami-induced scour around a monopile
783 foundation. *Coast. Eng.* 138, 9–21.

784 Larsen, B. E., Fuhrman, D. R., 2018a. Full-scale CFD simulation of tsuma-
785 nis. Part 1: Model validation and run-up. Submitted. Referred to as Part
786 1.

787 Larsen, B. E., Fuhrman, D. R., 2018b. On the over-production of turbu-
788 lence beneath surface waves in Reynolds-averaged Navier-Stokes models.
789 *J. Fluid Mech.* 853, 419–460.

790 Larsen, B. E., Fuhrman, D. R., Baykal, C., Sumer, B. M., 2017. Tsunami-
791 induced scour around monopile foundations. *Coast. Eng.* 129, 36–49.

792 Larsen, B. E., Fuhrman, D. R., Sumer, B. M., 2016. Simulation of wave-
793 plus-current scour beneath submarine pipelines. *J. Waterw. Port C-ASCE*
794 142 (5), article no. 04016003.

795 Li, L., Qiu, Q., Huang, Z., 2012. Numerical modeling of the morpholog-
796 ical change in Lhok Nga, West Banda Aceh, during the 2004 Indian
797 Ocean tsunami: understanding tsunami deposits using a forward mod-
798 eling method. *Nat. Haz.* 64 (2), 1549–1574.

799 Lu, S., Liu, H., Deng, X., 2018. An experimental study of the run-up pro-
800 cess of breaking bores generated by dam-break under dry- and wet-bed
801 conditions. *J. Earthquake and Tsunami* 12 (2), 1840005.

802 Madsen, P. A., Fuhrman, D. R., 2008. Run-up of tsunamis and long waves
803 in terms of surf-similarity. *Coast. Eng.* 55, 209–223.

804 Madsen, P. A., Schäffer, H. A., 2010. Analytical solutions for tsunami runup
805 on a plane beach. *J. Fluid Mech.* 645, 27–57.

806 Moore, A. L., McAdoo, B. G., Ruffman, A., 2007. Landward fining from mul-
807 tiple sources in a sand sheet deposited by the 1929 Grand Banks tsunami,
808 Newfoundland. *Sedimentary Geol.* 200 (3-4), 336–346.

809 Nielsen, P., 1992. *Coastal Bottom Boundary Layers and Sediment Trans-*
810 *port.* World Scientific, Singapore.

811 Nielsen, P., 2002. Shear stress and sediment transport calculations for swash
812 zone modelling. *Coast. Eng.* 45 (1), 53–60.

813 Nielsen, P., 2006. Sheet flow sediment transport under waves with accelera-
814 tion skewness and boundary layer streaming. *Coast. Eng.* 53 (9), 749–758.

815 Nielsen, P., Callaghan, D. P., 2003. Shear stress and sediment transport
816 calculations for sheet flow under waves. *Coast. Eng.* 47, 347–354.

817 O’Donoghue, T., Pokrajac, D., Hondebrink, L. J., 2010. Laboratory and nu-
818 merical study of dambreak-generated swash on impermeable slopes. *Coast.*
819 *Eng.* 57 (5), 513–530.

820 Ontowirjo, B., Paris, R., Mano, A., 2013. Modeling of coastal erosion and
821 sediment deposition during the 2004 Indian Ocean tsunami in Lhok Nga,
822 Sumatra, Indonesia. *Nat. Haz.* 65 (3), 1967–1979.

823 Paris, R., Wassmer, P., Sartohadi, J., Lavigne, F., Barthomeuf, B., Des-
824 gages, E., Granher, D., Baumert, P., Vautier, F., Brunstein, D., Gomez,
825 C., 2009. Tsunamis as geomorphic crises: Lessons from the December
826 26, 2004 tsunami in Lhok Nga, West Banda Aceh (Sumatra, Indonesia).
827 *Geomorphology* 104 (1-2), 59–72.

828 Soulsby, R. L., Smith, D. E., Ruffman, A., 2007. Reconstructing tsunami
829 run-up from sedimentary characteristics - a simple mathematical model.
830 *Coastal Sediments ’07 - Proceedings of 6th International Symposium on*
831 *Coastal Engineering and Science of Coastal Sediment Processes.*

832 Sugawara, D., Goto, K., Jaffe, B. E., 2014a. Numerical models of tsunami
833 sediment transport - current understanding and future directions. *Mar.*
834 *Geol.* 352, 295–320.

835 Sugawara, D., Takahashi, T., Imamura, F., 2014b. Sediment transport due
836 to the 2011 Tohoku-Oki tsunami at Sendai: Results from numerical mod-
837 eling. *Mar. Geol.* 358, 18–37.

838 Sumer, B. M., Fredsøe, J., 2002. *The Mechanics of Scour in the Marine*
839 *Environment*. World Scientific, Singapore.

840 Suppasri, A., Muhari, A., Ranasinghe, P., Mas, E., Shuto, N., Imamura, F.,
841 Koshimura, S., 2012. Damage and reconstruction after the 2004 Indian
842 Ocean tsunami and the 2011 Great East Japan tsunami. *J.l Nat. Disaster*
843 *Sc.* 34 (1), 19–39.

844 Tanaka, H., Adityawan, M. B., Mitobe, Y., Widiyanto, W., 2016. A new
845 computation method of bottom shear stress under tsunami waves. *J.*
846 *Coast. Res.* 1 (75), 1247–1251.

847 Terrile, E., Reniers, A. J. H. M., Stive, M. J. F., 2009. Acceleration and
848 skewness effects on the instantaneous bed-shear stresses in shoaling waves.
849 *J. Waterw. Port C-ASCE* 135 (5), 228–234.

850 Udo, K., Takeda, Y., Tanaka, H., 2016. Coastal morphology change before
851 and after 2011 off the Pacific coast of Tohoku earthquake tsunami at
852 Rikuzen-Takata coast. *Coast. Eng. J.* 58 (4), 1640016.

853 van der A, D. A., O'Donoghue, T., Davies, A. G., Ribberink, J. S., 2011. Ex-
854 perimental study of the turbulent boundary layer in acceleration-skewed
855 oscillatory flow. *J. Fluid Mech.* 684, 251–283.

856 Wilcox, D. C., 2006. *Turbulence Modeling for CFD*, 3rd Edition. DCW
857 Industries, Inc., La Canada, California.

858 Williams, I. A., Fuhrman, D. R., 2016. Numerical simulation of tsunami-
859 scale wave boundary layers. *Coast. Eng.* 110, 17–31.

860 Yamashita, K., Sugawara, D., Takahashi, T., Imamura, F., Saito, Y., Imato,
861 Y., Kai, T., Uehara, H., Kato, T., Nakata, K., Saka, R., Nishikawa, A.,
862 2016. Numerical simulations of large-scale sediment transport caused by
863 the 2011 Tohoku earthquake tsunami in Hirota Bay, Southern Sanriku
864 Coast. *Coast. Eng. J.* 58 (4), article no.1640015.

- 865 Yoshii, T., Tanaka, S., Matsuyama, M., 2017. Tsunami deposits in a super-
866 large wave flume. *Mar. Geol.* 391, 98–107.
- 867 Yoshii, T., Tanaka, S., Matsuyama, M., 2018. Tsunami inundation, sediment
868 transport, and deposition process of tsunami deposits on coastal lowland
869 inferred from the tsunami sand transport laboratory experiment (tstle).
870 *Mar. Geol.* 400, 107–118.
- 871 Yuan, J., Dash, S. M., 2017. Experimental investigation of turbulent wave
872 boundary layers under irregular coastal waves. *Coast. Eng.* 128, 22–36.
- 873 Yuan, J., Madsen, O. S., 2015. Experimental and theoretical study of wave-
874 current turbulent boundary layers. *J. Fluid Mech.* 765, 480–523.

Figure 12: Time series of free-stream velocities (a)-(c), surface elevation and boundary layer thickness (d)-(f) and friction velocities (g)-(i) for case N100 at the toe of the slope ($x = -1400$ m), at one shoaling position ($x = -900$ m) and at the shore ($x = 0$ m). Root mean square errors (RMSE) with modelled U_f : $\text{RMSE}_{\text{Predicted}, \text{toe}} = 0.004$ m/s, $\text{RMSE}_{\text{Manning}, \text{toe}} = 0.013$ m/s, $\text{RMSE}_{\text{Predicted}, \text{shoaling}} = 0.006$ m/s, $\text{RMSE}_{\text{Manning}, \text{shoaling}} = 0.016$ m/s, $\text{RMSE}_{\text{Predicted}, \text{shore}} = 0.020$ m/s, $\text{RMSE}_{\text{Manning}, \text{shore}} = 0.023$ m/s.

Figure 13: Time series of free-stream velocities (a)-(c), surface elevation and boundary layer thickness (d)-(f) and friction velocities (g)-(i) for case S030 at the toe of the slope ($x = -420$ m), at one shoaling position ($x = -180$ m) and at the shore ($x = 0$ m). RMSE with modelled U_f : $\text{RMSE}_{\text{Predicted},\text{toe}} = 0.006$ m/s, $\text{RMSE}_{\text{Manning},\text{toe}} = 0.011$ m/s, $\text{RMSE}_{\text{Predicted},\text{shoaling}} = 0.006$ m/s, $\text{RMSE}_{\text{Manning},\text{shoaling}} = 0.013$ m/s, $\text{RMSE}_{\text{Predicted},\text{shore}} = 0.013$ m/s, $\text{RMSE}_{\text{Manning},\text{shore}} = 0.016$ m/s.

Figure 14: Comparison of modelled and predicted friction velocities using both the new engineering model (10) and the Manning approach (14). (-) marks the line of perfect agreement.

875 **Appendix A. General applicability of the engineering model**

876 We have argued in the main text that the general way in which the en-
877 gineering approach for predicting bed shear stresses from the free-stream
878 velocity alone has been derived and formulated might make it suitable also
879 for shorter waves. The performance of the engineering model will be demon-
880 strated in this section, but first a small introduction to the literature involv-
881 ing the prediction of time varying bed shear stress will be given.

882 One of the key factors in predicting sediment transport is accurately
883 predicting the bed shear stress. This is by no means an easy task, and
884 has been the focus of much research. The task is particularly difficult in
885 the surf and swash zone where the free stream velocity signal can become
886 skewed or asymmetric. In a skewed velocity signal the maximum velocity is
887 higher than the minimum. This results in higher onshore bed shear stresses.
888 In an asymmetric signal the acceleration is skewed meaning that maximum
889 acceleration is higher than the minimum. This means that the boundary
890 layer has less time to grow during the acceleration phase, and as a result
891 the velocity gradients (and thus the bed shear stresses) are higher during
892 the positive half cycle than the negative. These two phenomena cannot be
893 captured by the standard constant friction factor approach

894 One of the earliest attempts to predict the time varying shear stress
895 was made by Fredsøe (1984) who used the momentum integral approach
896 together with an assumed logarithmic velocity profile. This approach can
897 handle the problems mentioned above, but involves solving a differential
898 equation, which contains singularities during flow reversal, and obtaining a
899 robust solution for this is not trivial. Another approach to predict the time
900 varying shear stress beneath asymmetric waves was made by Nielsen (1992).
901 He proposed a formula which included both the free stream velocity and the
902 acceleration of the free stream velocity. This was done to take into account
903 the effect of favourable pressure gradients in asymmetric waves. This ap-
904 proach was, however, built on a laminar boundary layer, and in Nielsen
905 (2002) the approach was extended to turbulent flows. In this paper Nielsen
906 argued for a phase shift of around 45° between the bed shear stress and the
907 free stream velocity. The reason for this phase shift was that it gave bet-
908 ter correspondence with experimental vertical sediment fluxes. Nielsen and
909 Callaghan (2003) further extended this approach to also take into account
910 streaming effects. The model was further calibrated by Nielsen (2006), but
911 overall the approach was similar, i.e to calculate the bed shear stress from
912 free stream velocity and the free stream acceleration accounting for a phase
913 shift, but keeping a constant friction factor. The approach initiated by the

914 various works by Nielsen and colleagues was further extended by Terrile
915 et al. (2009) who included a waveform parameter and by generalizing the
916 velocity signals as a sum of harmonics. Finally Abreu et al. (2013) extended
917 this by including an analytical expression capable of generating both skewed
918 and asymmetric waves.

919 Another approach was taken by Gonzalez-Rodriguez and Madsen (2007).
920 In their approach the friction factor was time varying. Their friction factor
921 was first estimated for both the crest and the trough of the wave and then
922 interpolated linearly for the entire period. Additionally, a function for the
923 phase shift between the free-stream velocity signal and the bed shear stress
924 was utilized. This model, in general, compared well against a RANS model.

925 Gonzalez-Rodriguez and Madsen (2011) developed an analytical model
926 of the turbulent boundary-layer by using a time-varying eddy viscosity and
927 gave analytical expressions to compute the bed shear stresses for asymmet-
928 ric and skewed waves, both in the absence or presence of an imposed cur-
929 rent. For this approach the free stream velocity signal was decomposed into
930 its first two harmonics and the analysis provided the first three harmonics
931 of the bed shear stress. Decomposing the free-stream velocity signal into
932 Fourier components was also done in the semi-analytical study of Yuan and
933 Madsen (2015) who extended the work by Gonzalez-Rodriguez and Madsen
934 (2011). They no longer assumed first-and second-harmonic contributions to
935 the function $f(t)$ multiplied by the mean eddy viscosity profile to be small
936 and showed that this gave better predictions for forward leaning waves.

937 The above studies have shown promising results, but also have some lim-
938 itations. All mentioned studies, except Fredsøe (1984), Gonzalez-Rodriguez
939 and Madsen (2011) and Yuan and Madsen (2015), require a priori knowl-
940 edge of the period and phase shift. This is not the case in the studies
941 by Gonzalez-Rodriguez and Madsen (2011) and Yuan and Madsen (2015).
942 These require a Fourier decomposition of the entire signal, however. This
943 means that none of the studies above can be straight-forwardly used on
944 an arbitrary instantaneous velocity signal, which could show up e.g. in a
945 potential (wave resolving) flow model. Furthermore, it is also questionable
946 how some of these models would handle the situation where the boundary
947 layer at some point extends all the way to the free surface as could occur
948 in a tsunami. Finally, all of the models (except for the momentum integral
949 approach) require some kind of periodicity, and it is therefore questionable
950 how they would behave in a situation e.g. where the flow is accelerated from
951 zero to a fixed maximum value.

952 The engineering approach for predicting the friction velocity, presented
953 in this paper, again based on (9)-(13), is designed to handle all of the chal-

Study	a/k_s	Re	Shape	Signal	T [s]	U_m [m/s]
Jensen et al. (1989)	3700	6×10^6	sinusoidal	regular	9.82	2
van der A et al. (2011)	90	1×10^6	acceleration-skewed	regular	7	1.1
Yuan and Dash (2017)	300*	1×10^6 *	velocity + acceleration-skewed	irregular	6.4	1.1*

Table A.2: Key parameters for the studies used for comparison with the engineering model.

* since these studies were irregular $U_{rms} = \sqrt{2}std(u_0)$ has been used instead of U_m .

954 lenges mentioned above. In what follows we will now compare the present
955 model to the well-known experiments of Jensen et al. (1989) which were
956 based on a sinusoidal oscillatory flow, the experiments of van der A et al.
957 (2011) involving acceleration-skewed flows and finally the experiments of
958 Yuan and Dash (2017) which involved random amplitude modulated (with
959 constant period), velocity and- acceleration-skewed flows. These three have
960 been chosen to demonstrate the performance of the engineering approach
961 for free stream signals typical of sinusoidal, velocity-skewed, acceleration-
962 skewed and random modulated waves waves, as well as to demonstrate the
963 performance across a wide range of a/k_s . These will hence complement the
964 applications on the long tsunami waves in the main text. A summary of the
965 cases to be considered is given in Table A.2.

966 First the model will be compared to the well known test 13 of Jensen et al.
967 (1989). The experiments were conducted in an oscillatory tunnel facility.
968 The free stream velocity signal was sinusoidal with a period $T = 9.72$ s,
969 maximum free-stream velocity $U_m = 2$ m/s and a roughness $k_s = 0.84$
970 mm, see Table A.2. Figure A.15 shows the experimental friction velocities
971 as well as that predicted with (9)-(13) based only on the measured free-
972 stream velocity u_0 as input. Included is also the predicted friction velocity
973 using a constant friction factor expression from Fuhrman et al. (2013) in
974 (10) i.e. substituting $a(t)$ with a in (11). It can be seen that the predicted
975 friction velocity matches the experimental well. It can also be seen that
976 the predictions are reasonably in phase with the measurements, whereas the
977 constant friction factor approach is in phase with the free-stream velocity
978 signal i.e. the phase shift between the free-stream velocity signal and the bed
979 shear stress seem to be well captured by only taking into account the effect of
980 the time varying boundary layer thickness and not considering acceleration
981 effects. Near flow reversal there will also be a phase shift due to the adverse
982 pressure gradient turning the near flow before the free-stream. This phase
983 shift will be not be captured with the present approach, but is not considered
984 particularly important, since this is only significant near flow reversal, i.e.
985 when the velocities are small.

Figure A.15: Comparison of experimentally measured friction velocities (test 13 of Jensen et al. (1989)) with those predicted using (10) and using a constant f_w .

986 Having demonstrated the performance of the engineering model for si-
 987 nusoidal flows it will now be compared to experimental results involving
 988 acceleration-skewed flows from van der A et al. (2011). These experiments
 989 were conducted in an oscillating flow tunnel. The waves in all three cases
 990 had a period $T = 7$ s, maximum free-stream velocity $U_m \approx 1.1$ m/s and
 991 $k_s = 13.79$ mm, with varying degrees of acceleration skewness, quantified
 992 by:

$$\beta = \frac{\dot{u}_{0max}}{\dot{u}_{0max} - \dot{u}_{0min}} \quad (\text{A.1})$$

993 where $\dot{u}_0$ is the free-stream acceleration. Specifically the model will be
 994 compared to their cases S5071012g, S6071012g and S7571012g having $\beta =$
 995 0.5, 0.6 and 0.75 respectively (note that $\beta = 0.5$ correspond to a sinusoidally
 996 varying flow).

997 Figure A.16 shows the experimental and predicted time series of the
 998 friction velocity using both (10) and a constant friction factor approach
 999 similarly to before. Here it can be seen that the present model captures
 1000 well the measured friction velocity and is again able to reasonably capture
 1001 the phase shift between the friction velocity and the free-stream velocity. In
 1002 contrast, the predictions using a fixed f_w are in phase with the free-stream
 1003 velocity signal, and do not capture the increased bed shear stress associated

Figure A.16: Comparison of experimentally measured friction velocities beneath acceleration skewed free-stream flows (van der A et al., 2011) with those predicted using (10) and using a constant f_w .

1004 with the smaller boundary layer thickness.

1005 Finally, the present engineering model will be compared to experiments
 1006 from Yuan and Dash (2017). These experiments were performed in an
 1007 oscillating water tunnel. The velocity signals consisted of random ampli-

1008 tude modulated flows with a constant period of $T = 6.4$ s and $u_{rms} =$
 1009 $\sqrt{2}\text{std}(u_0) = 1.1$ m/s. These flows were in one of the experiments velocity-
 1010 skewed, and in another acceleration-skewed. The bed had a roughness
 1011 $k_s = 3.7$ mm. Figure A.17 shows the comparison between the measured
 1012 and predicted friction velocity for both irregular velocity-skewed oscillatory
 1013 flows (Figure A.17a) and irregular acceleration-skewed oscillatory flows (Fig-
 1014 ure A.17b). The predicted friction velocity has again been found using both
 1015 (10) and a constant friction factor approach, using the measured u_0 as input.
 1016 In general it can be seen that the engineering model, with no adjustment,
 1017 predicts well the friction velocity for the irregular oscillatory flows. There
 1018 is a small underestimation of the friction velocity beneath the peak, but
 1019 this underestimation is of the same order of magnitude as the scatter in the
 1020 experiments on which (11) is built (see Fuhrman et al., 2013). Similar to
 1021 the previous cases, it seems that the phase shift is reasonably captured by
 1022 incorporating the effect of the time varying boundary layer thickness alone.
 1023 In comparison, the results using a constant friction factor approach seem to
 1024 capture as well as (10) the magnitude of the friction velocity of the largest
 1025 waves, but underestimate the magnitude of the smaller waves, and does not
 1026 capture the phase shift.

1027 The present engineering model for the friction velocity has thus been
 1028 shown to be able to accurately predict the time variation of the friction
 1029 velocity for a wide range of situations, again based only on the free-stream
 1030 velocity signal. These involve the long unsteady tsunami-induced flows cov-
 1031 ered in the main text, as well as a range of flows that would typically occur
 1032 beneath sinusoidal, velocity-skewed regular short waves as well as velocity-
 1033 skewed and acceleration-skewed irregular short wave signals. As discussed
 1034 in the main text, the simple way the approach is formulated enables it to be
 1035 implemented in potential flow models, possibly improving their predictions
 1036 of sediment transport. In the present formulations, the wave friction factor
 1037 expression from Fuhrman et al. (2013) has been used, though the method
 1038 may easily be adjusted to use any other friction factor formulations from
 1039 the literature.

Figure A.17: Comparison of experimentally measured friction velocities beneath (a) velocity-skewed irregular free-stream flows and (b) acceleration-skewed irregular free-stream flows (Yuan and Dash, 2017) with those predicted using (10) and using a constant f_w .