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Coalitional Game Based Transactive Energy Management in Local Energy Communities

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***Abstract*—This paper focuses on transactive energy management in local energy communities and proposes a coalitional game model which considers the presence and utilization of flexible loads and the power output uncertainties from renewable energy sources. The superadditivity and balancedness of the proposed game model are proved rigorously by demonstrating that a nucleolus-based solution leads to a stable and fair payoff distribution scheme for all players. More specifically, the objective function is proved concave with its analytical expression, derived through an approximated piecewise linear function. The proposed allocation of realized payoffs is then proved to converge consistently to a nucleolus-based solution. It is demonstrated by numerical simulations that the grand coalition effectively increases the global payoff and the proposed allocation scheme contributes to peak shaving and valley filling in the overall load profile.**

***Index Terms*—Coalitional game model, Energy management, Local energy community.**

NOMENCLATURE

ESS	Energy storage systems
EV	Electric vehicle
LEC	Local energy community
RES	Renewable energy resource
TC	Transactive control
TE	Transactive energy

I. INTRODUCTION

WITH an ever-increasing penetration of renewable energy resource (RES)-based distributed generators, power distribution systems are undergoing a transition from passive to active networks. This transition poses significant challenges to the network and market operators, such as bidirectional power flow and even back-feeding at distribution substations [1].

At the present stage, the energy policies in many countries encourage self-consumption of solar power, which can reduce power transmission losses, shave load peaks, and cut operational costs [2]. Hence, the European Commission has recognized a crucial role for the local energy communities (LECs) in implementing local energy management. As such, a considerable number of LEC initiatives have started in several European countries over the past decade [3].

Traditionally, electricity prosumers can participate in electricity markets indirectly through retailers or aggregators. For an LEC, many options have been proposed for designing market mechanisms to perform local energy trading. For example, a wholesale and short-term local markets are integrated and simultaneously cleared in [2], which can facilitate the participation of the small-scale consumers in the local energy market. Similarly, a bilateral contract has been proposed in [4] for a market environment with real-time and forward frames to perform local energy trading in a peer-to-peer mode. A localized event-driven market has been demonstrated in [5] to provide extra choices for prosumers to implement peer-to-peer energy trading in an LEC. In addition, a Nash bargaining theory-based incentive mechanism has been employed in [6] and [7] for peer-to-peer energy trading and profit allocation in networked microgrids. Besides, some state-of-the-art distribution management techniques, such as micro-phasor measurement units and advanced metering infrastructure, have been used in [3] to facilitate local control and monitoring of the distribution system and to enable the implementation of various local energy trading mechanisms.

The transactive energy (TE) is commonly referred to as *a system of economic and control mechanisms that allows the dynamic balance of supply and demand, using value as a key operational parameter* [8]. Thus, a TE framework offers a platform for the small-scale generation and consumption units to automatically negotiate their actions with each other. This can be realized using advanced energy resource management and market algorithms. The existing publications show that the concepts of TE and transactive control (TC) have been commonly applied to achieve optimality in energy management for multi-microgrid systems [7], dispatch of thermostatically controlled loads [9], integration of RES to distribution networks [10], and charging scheduling of electric vehicles (EVs) [11].

Mechanism design is one of the most important factors in ensuring success in the application of TE and TC. Such mechanisms usually aim to set a price signal with incomplete information of players in the market environment. This will induce the players to individually make decisions, that are optimal not only to themselves but also for global goal maximization (e.g., social welfare). Ref. [12] has proposed a mechanism wherein the market-clearing strategies have been designed to motivate self-interested users to realize efficient system-wide

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TABLE I
COMPARISON OF THE CONSIDERED FACTORS IN COALITIONAL GAME METHODOLOGIES IN THE EXISTING LITERATURE AND THIS PAPER

Ref.	Uncertainty	Flexible loads	Value function of flexible loads
[14]	✓	×	×
[15]	✓	×	×
[16]	✓	×	×
[17]	×	✓	×
[18]	×	✓	✓
[19]	✓	×	×
This paper	✓	✓	✓

energy allocation. Alternatively, a double-auction market has been constructed in [9] to implement the TC approach over a variety of controllable devices in a commercial building for demand response. Yet, the research on implementing TE or TC at the distribution level is still at an early stage and systems operating fully under the TE framework are largely missing.

In addition to the mechanism design, the game theory has also been widely employed in the framework of TE and TC, and can specifically model strategic interactions among rational decision-makers. The game theory can be broadly divided into non-cooperative and cooperative games [13]. The work about market mechanisms at the distribution level or in an LEC in the abovementioned studies is more or less based on the non-cooperative game theory. On the other hand, the cooperative game has also been applied to many fields of the power system. For example, an algorithm has been proposed in [14] to allow microgrids to cooperate and self-organize a partition autonomously. However, the operational constraints of flexible loads as well as the pertinent utility functions are not considered. On the other hand, the coalitional game theory has been applied in [15] and [16] respectively to solve the optimal bidding strategies of aggregated wind units, as well as investing and operating strategies of energy storage systems (ESSs). However, these studies have not considered the operational constraints of flexible loads. Similarly, the value obtained from flexible loads has been neglected in the value function of coalitional game model in [17] and [18]. Moreover, these studies have employed the forecasted values of the output power of the RESs, which is a challenge to the practicality of the obtained solutions. Also, their pertinent effectiveness and accuracy is questionable. Alternatively, [19] has focused on small-scale local energy trading based on the asymptotic Shapley value but has not considered the controllable resources within the smart grid. Table I presents a comparison among the key features considered in the existing literature and compares them against the proposal of this paper.

In the above backdrop, this paper proposes a coalitional game based TE management for LECs, which not only resolves the conflict of interest among different households, but also distributes the total payoff to each participant in a stable and efficient way. The contributions are threefold:

1) The proposed technique integrates the flexible loads into a coalitional game model with the value function, to model demand response initiatives and offer rigorous proof of superadditivity and balancedness of the pertinent game model. This is new and different from similar works such as [14] to [21].

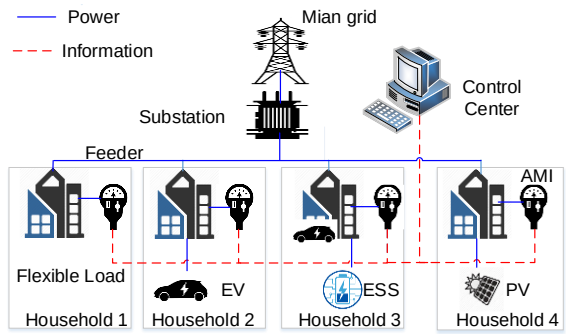

Fig. 1. Schematic illustration of an LEC with flexible loads, operating under an energy management paradigm.

2) The uncertainties in the output power of RESs have been considered, and the objective function has been proved concave with an analytical expression, derived through an approximated piecewise linear function.
3) The core of the proposed coalition game model has been rigorously proved nonempty, where a nucleolus-based solution leads to a stable and fair payoff distribution schehme for all players. On this basis, an allocation scheme for the realized payoffs has been then proved budget balanced and strongly consistent with the nucleolus-based solution.

The remainder of the paper is organized as follows: Section II presents the TE management framework. The background of coalitional game theory is explicitly presented in Section III. Section IV presents the game model for the proposed problem while Section V reformulates the proposed game model into an optimization framework. The performance of the proposed model is evaluated through numerical analyses in Section VI while the scalability of the proposed method is discussed in Section VII. Finally, the key findings and highlights of the research have been summarized. Five ,appendices at the end of the paper provide proofs for the lemmas used in the paper.

II. PROBLEM FORMULATION

As depicted in Fig. 1, several households, equipped with RESs, ESSs, and EVs are providing flexible load services within an assumed LEC. For clarity, the concept of prosumers is used to represent the participants in the LEC. $\mathbf{N}$ denotes the set of prosumers while the cardinality of $\mathbf{N}$ is shown by N, i.e., $N = |\mathbf{N}|$. Similarly, $\mathbf{N}_S$ denotes the set of prosumers with RESs while the cardinality of $\mathbf{N}_S$ is represented by N_S, i.e., $N_S = |\mathbf{N}_S|$. Obviously, $\mathbf{N}_S \subseteq \mathbf{N}$. Let $\mathbf{N}' \subseteq \mathbf{N}$ denote arbitrary subsets of prosumers and thereby $\mathbf{N}'_S$ represents the collection of prosumers with RESs in the group of $\mathbf{N}'$. Note that vector, matrix, and set are denoted by boldface font characters while the index, scalar, and element of set are represented by italic letters hereinafter except noted.

A. Bilateral Energy Trading

In the proposed model, it is assumed that a control center, (e.g., a neutral system operator) is implementing bilateral energy trading with the utility, on behalf of the prosumers, to sell the excess electricity or purchase during a deficiency. Also, it is assumed that prosumers in an LEC can communicate with

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the control center using a two-way communication infrastructure, shown schematically in Fig. 1.

Assumption-1: The prosumers have *a zero-marginal cost* for renewable energy production.

For simplicity, in this bilateral energy trading framework, the payoff function J_t at time t for a coalition $\mathbf{N}'$ can be expressed as

$$J_t(\mathbf{s}_{\mathbf{N}'},\mathbf{b}_{\mathbf{N}'}) = \lambda_S\left[\sum_{i\in\mathbf{N}'_S} s_{i,t} - \sum_{j\in\mathbf{N}'} b_{j,t}\right]^+ - \lambda_B\left[\sum_{j\in\mathbf{N}'} b_{j,t} - \sum_{i\in\mathbf{N}'_S} s_{i,t}\right]^+ + \sum_{j\in\mathbf{N}_B^F} U_{j,t}^f\left(b_{j,t}^f\right) + \sum_{j\in\mathbf{N}_B^{ES}} U_{j,t}^{ES}\left(b_{j,t}^{ES}\right) \quad (1)$$

where $\mathbf{s}_{\mathbf{N}'}$ and $\mathbf{b}_{\mathbf{N}'}$ respectively denote the matrices of the output power of the RESs and the consumed power of prosumers in coalition $\mathbf{N}'$; $s_{i,t}$ denotes the output power of the RES of prosumer-*i* while $b_{j,t}$ denotes the power consumed by prosumer-*j* at time *t*; λ_S / λ_B represents the selling/buying electricity price for the households in the LEC while $U_{j,t}^f\left(b_{j,t}^f\right)$ and $U_{j,t}^{ES}\left(b_{j,t}^{ES}\right)$ are respectively the value (utility) functions of flexible loads and ESS in prosumer-*j*. $b_{j,t}^f$ is the power for flexible load-*j* at time *t*; $b_{j,t}^{ES}$ is the charging power for ESS-*j* at time *t* where a negative value represents power discharging; $\mathbf{N}_B^F$ and $\mathbf{N}_B^{ES}$ respectively denote the collection of flexible loads and ESSs. $[x]^+ = \max(x,0)$. This payoff function consists of the net revenue brought by trading electricity with the utility (the first and second terms) and the customers' value (the third term). Note that load power **b** is a matrix containing decision variables while the RES output power matrix **s** contains random variables. Thus, the payoff function defined in (1) is also random as a result of its dependence on the RES random output power **s**. Thus, the expected payoff function Ψ is defined as

$$\Psi(\mathbf{s}_{\mathbf{N}'},\mathbf{b}_{\mathbf{N}'}) = \max \mathbb{E}\sum_{t\in\mathbf{T}} J_t(\mathbf{s}_{\mathbf{N}'},\mathbf{b}_{\mathbf{N}'}) \quad (2)$$

where **T** denotes the set of optimization horizon.

B. Agent Model

The prosumers in an LEC broadly comprise three categories of controllable resources, i.e., flexible loads, ESSs and EVs, as discussed below:

1) Flexible Loads

For any flexible load-*j*, $j\in\mathbf{N}_B^F$, the following constraints are valid:

$$b_{j,t}^{\min} \le b_{j,t}^f \le b_{j,t}^{\max} \quad \forall t\in\mathbf{T} \quad (3)$$

$$-R_j^D \le b_{j,t}^f - b_{j,t-1}^f \le R_j^U \quad \forall t\in\mathbf{T}\setminus\{1\} \quad (4)$$

$$\Delta T\sum_{t\in\mathbf{T}} b_{j,t}^f = E_j^F \quad (5)$$

where $b_{j,t}^{\min} / b_{j,t}^{\max}$ is the lower/upper limit of the power of flexible load-*j* at time *t*; R_j^U / R_j^D is the ramping up/down limit of the flexible load-*j* while E_j^F represents the power consumption requirement of the flexible load-*j* during the optimization horizon.

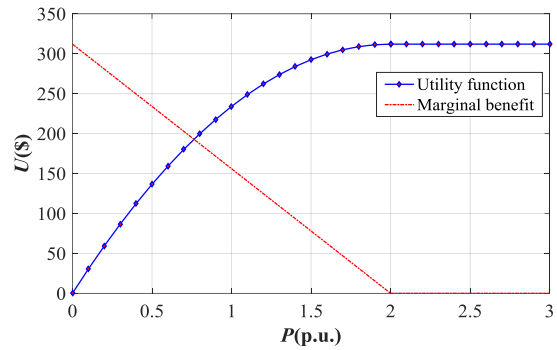

Fig. 2. A sample value (utility) function for flexible loads.

The consumers are always interested in consuming more power to reach to their maximum satisfaction levels. However, this gradually gets saturated concerning human preferences. Thus, most economics-based value functions are non-decreasing and quasi-concave [22]. In this paper a concave quadratic value function is considered, which corresponds to a linear decreasing marginal benefit, as shown in Fig 2. For load-*j*, $j\in\mathbf{N}_B^F$, at time *t*, $t\in\mathbf{T}$, the value function is in the form of

$$U_{j,t}^f\left(b_{j,t}^f\right) = \begin{cases} \zeta_j b_{j,t}^f - \upsilon_j (b_{j,t}^f)^2 & 0 \le b_{j,t}^f \le \zeta_j / 2\upsilon_j \\ (\zeta_j)^2 / 4\upsilon_j & b_{j,t}^f \ge \zeta_j / 2\upsilon_j \end{cases} \quad (6)$$

where $\upsilon_j > 0, \zeta_j \ge 0$.

In addition, some prosumers can provide some load reduction services, which can be taken as a negative load behind the meters. For simplicity, a reducible load can be modeled like flexible loads excluding constraint (5). Furthermore, the fixed loads can be considered as negative suppliers in the proposed model.

2) ESS

For ESS-*j*, $j\in\mathbf{N}_B^E$, the constraints can be formulated as

$$b_j^{\min} \le b_{j,t}^{ES} \le b_j^{\max} \quad \forall t\in\mathbf{T} \quad (7)$$

$$SOC_{j,t+1} = SOC_t + ([b_{j,t}^{ES}]^+ \eta_j^{ch} / E_j^B - [-b_{j,t}^{ES}]^+ / E_j^B \eta_j^{dch}) \forall t\in\mathbf{T}\setminus\{T\} \quad (8)$$

$$SOC^{\min} \le SOC_{j,t} \le SOC^{\max} \quad \forall t\in\mathbf{T} \quad (9)$$

$$SOC_{j,0} = SOC_{j,T} \quad (10)$$

in which SOC_t is the state of charge for ESS-*j* while $SOC_{j,0}$ is its initial state of charge; $\eta_j^{ch} / \eta_j^{dch}$ is the charging/discharging efficiency; $SOC^{\min} / SOC^{\max}$ is the lower/upper limit of state of charge and E_j^B is the energy capacity of ESS-*j*.

Note that an ESS will suffer a certain degradation cost after being charged and discharged multiple times. Considering the degradation cost, a payoff function can be defined for the ESS as [18]

$$U_{j,t}^{ES}\left(b_{j,t}^{ES}\right) = -\lambda_d \left|b_{j,t}^{ES}\right| \quad (11)$$

where $\lambda_d \ge 0$ represents the cost for the storage degradation when charging/discharging one unit of energy. Obviously, the payoff function of the ESS is a concave function. The opera-

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tional constraints for the EVs are assumed like those of the ESSs and are not repeated.

Assumption-2: Most local participants in a LEC are geographically close to each other and more likely connected to the same distribution feeder.

Based on *Assumption-2*, the power flow constraints of the distribution networks are not considered in the proposed model; however, a future study can be dedicated on evaluating the impacts of congestion and network loss on the proposed model.

III. Coalitional Game Theory

First, a simple version of a coalitional game is introduced in this Section. Then, a balanced game model, along with the solution concept of nucleolus, are presented and discussed.

A. Coalitions and Solution Concepts

Let $\mathbf{N}$: = {1, 2, …, N} denote a finite set of players. Each subset $\mathbf{N}'$ of the players $\mathbf{N}$ can form a *coalition* and $\mathbf{N}$ is the *grand coalition*. The set of all possible coalitions is defined as the power set $2^{\mathbf{N}}$ of $\mathbf{N}$.

A coalitional game is defined by an ordered pair ($\mathbf{N}$, v) where v: $2^{\mathbf{N}} \to \mathbb{R}$ is the *value function* that maps each coalition $\mathbf{N}'$ to a real number $v(\mathbf{N}')$. Note that, value function v can be interpreted as revenue or cost, and thereby, ($\mathbf{N}$, v) can be accordingly called the revenue game or the cost game. The revenue game is utilized in the proposed model of this paper. Since the value function of a coalition includes the revenue from trading energy with the utility and operation cost of flexible loads, i.e., a certain amount of money, it thus can be distributed in any manner among the coalition members, which implies that the energy management problem in a LEC can be modeled as a transferable payoff game.

Definition-1 (Superadditivity): A coalitional game ($\mathbf{N}$, v) is *superadditive* if the value of a union of disjoint coalitions is no less than the sum of the coalitions' separate values, i.e., $v(\mathbf{N}_1') + v(\mathbf{N}_2') \le v(\mathbf{N}_1' \cup \mathbf{N}_2')$ whenever $\mathbf{N}_1'$, $\mathbf{N}_2' \subseteq 2^{\mathbf{N}}$ respect $\mathbf{N}_1' \cap \mathbf{N}_2' = \varnothing$.

Obviously, forming the grand coalition is the optimal decision for payoff maximization when the value function is superadditive. After calculating the payoff of the grand coalition, assigning the calculated payoff to each player is the concept of the cooperative game's solution.

Definition-2 (Imputation): $\mathbf{x}$ denotes the payoff allocation, of which the entry x_n represents the payment to player n. An imputation is defined as a payoff allocation $\mathbf{x}$ for the grand coalition only if it simultaneously respects the following properties:

- A payoff allocation $\mathbf{x}$ is efficient if $\sum_{n\in\mathbf{N}} x_n = v(\mathbf{N})$.
- A payoff allocation is individually rational if $x_n \ge v(\{n\}), \forall n \in \mathbf{N}$.

Thus, the set of all imputations for the proposed game in this paper is defined as

$$\mathbf{I} := \{\mathbf{x} \mid \sum_{n\in\mathbf{N}} x_n = v(\mathbf{N}),\ x_n \ge v(\{n\}), \forall n \in \mathbf{N}\} \tag{12}$$

However, an imputation still cannot guarantee the grand coalition stable as some players could achieve higher payoffs by forming smaller coalitions. Thus, a fundamental solution concept has been proposed for coalitional game, namely the *core*.

Definition-3 (Core): The core $\mathbf{C}$ is defined as the set under which no coalition has a value greater than the sum of its members' payoffs, i.e.,

$$\mathbf{C} := \{\mathbf{x} \mid \sum_{n\in\mathbf{N}} x_n = v(\mathbf{N}),\ \sum_{n\in\mathbf{N}'} x_n \ge v(\mathbf{N}'), \forall \mathbf{N}' \subseteq 2^{\mathbf{N}}\} \tag{13}$$

Obviously, no coalition has incentive to leave the grand coalition and receives a larger payoff if the payoff allocation $\mathbf{x}$ belongs to the core $\mathbf{C}$, namely stable.

B. Balanced Games and Nucleolus

The concept of the balanced game along with nucleolus can now be introduced to guarantee that the core of a coalitional game model is nonempty. First, two definitions are provided about the balanced map and game, and thereby, the *Bondareva-Shapley theorem*:

Definition-4 (Balanced Map): A map $\alpha : 2^{\mathbf{N}} \to [0,1]$ is called balanced if $\sum_{\mathbf{N}'\in\mathbf{N}} \alpha(\mathbf{N}')\mathbf{1}^{\mathbf{N}'} = \mathbf{1}^{\mathbf{N}}$ for any $n \in \mathbf{N}$, where $\mathbf{1}^{\mathbf{N}'}$ is a $N \times 1$ vector of $\mathbf{N}'$, given by $\mathbf{1}_n^{\mathbf{N}'} = 1$ if $n \in \mathbf{N}'$ and 0 otherwise.

Definition-5 (Balanced Game): A game ($\mathbf{N}$,v) is balanced if $\sum_{\mathbf{N}'\in\mathbf{N}} \alpha(\mathbf{N}')v(\mathbf{N}') \le v(\mathbf{N}')$ for any balanced map α.

Theorem-1 (Bondareva-Shapley Theorem [13]): A coalitional game with transferable payoff has a nonempty core if and only if it is balanced.

A coalition's dissatisfaction with respect to the imputation $\mathbf{x}$ is measured by its *excess* defined as

$$e(\mathbf{x}, \mathbf{N}') = v(\mathbf{N}') - \sum_{n\in\mathbf{N}'} x_n \tag{14}$$

where $\mathbf{e}(\mathbf{x})$ denotes a (2^N-1)-dimensional vector, the excess vector whose entries represent the excesses for all coalitions arranged in non-increasing order. $\Theta = \{\mathbf{e}(\mathbf{x}) \mid \forall \mathbf{x} \in \mathbf{I}\}$ denotes the set of excess vectors with respect to each imputation $\mathbf{x} \in \mathbf{I}$. The elements of Θ are arranged in lexicographic order. The definition of the nucleolus is given as:

Definition-6 (Nucleolus): The nucleolus of the game ($\mathbf{N}$, v) is the lexicographically minimal imputation. The nucleolus belongs to the core and minimizes dissatisfactions of all players.

IV. Coalitional Game Formulation for an LEC

For any coalition $\mathbf{N}' \in 2^{\mathbf{N}}$, the corresponding value function and constraints can be written as

$$\begin{cases} \Psi(\mathbf{s}_{\mathbf{N}'}, \mathbf{b}_{\mathbf{N}'}) = \max \mathbb{E} \sum_{t\in\mathbf{T}} J_t(\mathbf{s}_{\mathbf{N}'}, \mathbf{b}_{\mathbf{N}'}) \\ s.t.\ \ (3)-(5), (7)-(10) \end{cases} \tag{15}$$

Assumption-3: The price for purchasing energy from the utility is assumed higher than the feed-in tariff, i.e., $\lambda_B \ge \lambda_S$.

Note that *Assumption-3* is reasonable in a rational electricity market, especially for those countries with a relatively high penetration of RESs, such as Germany. Such an assumption is

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made in [17], [19] and [23]. Thus, the price of purchasing energy from the utility is usually higher than the feed-in tariff in Germany and the UK [24]. However, many public authorities fund renewable energy suppliers for promoting the widespread adoption of renewable generation sources. At this time, the feed-in tariff would be higher, and households with RES intend to sell their electricity to the utility for achieving higher revenues instead of participating in the coalitional games.

Lemma-1: The proposed coalitional game, defined in (15), is superadditive based on *Assumption-3*. The proof of this lemma is given in Appendix-A.

Lemma-1 shows that the coalitional strategy always yields a net increase in the collective payoffs. Moreover, the larger the coalition, the greater the improvement in the total expected payoff value. This indicates that forming the grand coalition is the optimal choice for all players. However, this property still cannot guarantee the existence of a stable payoff allocation. Thus, it is necessary to examine whether the game model is balanced, and the core is nonempty. To this end, first, the following lemma is given:

Lemma-2: For $\mathbf{N}' \in 2^{\mathbf{N}}$ and scalar $\theta \in [0,1]$, the value function Ψ has the following property:

$$\theta\Psi(\mathbf{s}_{\mathbf{N}'}, \mathbf{b}_{\mathbf{N}'}) \le \Psi(\theta\mathbf{s}_{\mathbf{N}'}, \theta\mathbf{b}_{\mathbf{N}'}) \tag{16}$$

with a proof provided in Appendix-B. Based on this lemma, the following lemma can be obtained:

Lemma-3: The coalition game of (15) for an LEC is balanced. Hence, it has a nonempty core. Appendix-C presents the proof of this lemma. Based on this lemma, the proposed game mode always has a stable payoff allocation.

V. Implementation of Energy Management Model

In this section, the transformation of the coalitional game model into a tractable optimization model with the a closed-form analytical expression is first discussed, so as to approximate the value function therein. Then, a nucleolus-based allocation scheme is proposed for the realized payoff, which is stable and fair for the grand coalition.

A. Optimization Model

The value function for coalition $\mathbf{N}'$ can be written as (2). Now, a closed-form analytical expression can be derived for the expected payoff. To this end, let us define

$$g(\beta) = \mathbb{E}\lambda_S\left[\sum_{i\in\mathbf{N}'_S} s_{i,t} - \sum_{j\in\mathbf{N}'} b_{j,t}\right]^+ - \mathbb{E}\lambda_B\left[\sum_{j\in\mathbf{N}'} b_{j,t} - \sum_{i\in\mathbf{N}'_S} s_{i,t}\right]^+ \tag{17}$$

where $\beta = \sum_{j\in\mathbf{N}'} b_{j,t}$. Also, let us define $\varsigma = \sum_{i\in\mathbf{N}'_S} s_{i,t}$ while $\rho(\varsigma)$ represents the probability density function of $\sum_{i\in\mathbf{N}'_S} s_{i,t}$ with a mean value of μ.

Lemma-4: Given $\rho(\varsigma)$ as first-order differentiable, $g(\beta)$ is concave based on *Assumption-3*. This is proved in Appendix-D.

Since $g(\beta)$ is concave, it can be approximated by the piecewise-linear function, as shown in Fig. 3. Based on *Lemma-4*, $g(\beta)$ can be written as

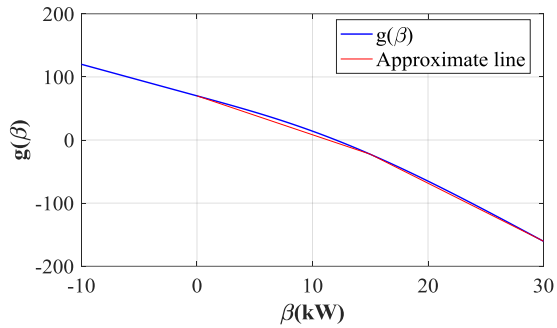

Fig. 3. Approximation of $g(\beta)$.

$$g(\beta) = \begin{cases} \lambda_S\mu_\beta - \lambda_S\beta + (\lambda_S - \lambda_B)\int_{-\infty}^{\beta}(\beta-\varsigma)\rho(\varsigma)\mathrm{d}\varsigma & \beta \ge 0 \\ \lambda_S(\mu-\beta) & \textit{otherwise} \end{cases} \tag{18}$$

Eq. (18) can be rewritten as

$$g(\beta) = \lambda_S\mu_\beta - \lambda_S\beta + (\lambda_S - \lambda_B)\big(\Phi_1(\beta) - \Phi_2(\beta)\big) \tag{19}$$

when $\beta \ge 0$, while

$$\Phi_1(\beta) = (\lambda_S - \lambda_B)\beta\int_{-\infty}^{\beta}\rho(\varsigma)\mathrm{d}\varsigma = (\lambda_S - \lambda_B)\beta\Pr(\varsigma \le \beta) \tag{20}$$

$$\Phi_2(\beta) = (\lambda_S - \lambda_B)\int_{-\infty}^{\beta}\varsigma\rho(\varsigma)\mathrm{d}\varsigma \tag{21}$$

If the distribution function is unavailable, the corresponding historical data can be used. Assume that the historical data of the output power of RES can be interpreted as $\boldsymbol{\chi} = \{\chi_1, \chi_2, ..., \chi_M\}$ and let us define $\boldsymbol{\chi}_\beta = \{\chi_i \mid \chi_i \le \beta, i = 1,...,M\}$, then (20) and (21) can be redrafted as

$$\Phi_1(\beta) = \frac{(\lambda_S - \lambda_B)\beta}{M}\left|\boldsymbol{\chi}_\beta\right| \tag{22}$$

$$\Phi_2(\beta) = (\lambda_S - \lambda_B)\int_{-\infty}^{\beta}\varsigma\rho(\varsigma)\mathrm{d}\varsigma = \frac{\lambda_S - \lambda_B}{M}\sum_{\chi_i\in\boldsymbol{\chi}_\beta}\chi_i \tag{23}$$

To visualize and expedite the approximation process, the Rayleigh distribution [23] is used here to capture the uncertainties in the output power of RESs. Thus, a closed-form analytical expression of (17) can be obtained. For simplicity, it is assumed that ς is independently distributed in a Rayleigh distribution with a mean value of μ across time slots $t \in \mathbf{T}$. Then, the probability density function of ς can be defined as

$$\rho(\varsigma) = \frac{\pi\varsigma}{2\mu^2}e^{\left(-\frac{\pi\varsigma^2}{4\mu^2}\right)}, \varsigma \ge 0 \tag{24}$$

where μ is the forecasted value of the output power of RES.

Lemma-5: The closed-form analytical expression for $g(\beta)$ can be presented as

$$g(\beta) = \begin{cases} \lambda_S\mu - \lambda_B\beta + \mu(\lambda_B - \lambda_S)\,\mathrm{erf}\left(\frac{\sqrt{\pi}}{2\mu}\beta\right) & \beta \ge 0 \\ \lambda_S(\mu-\beta) & \textit{otherwise} \end{cases} \tag{25}$$

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where $\mathrm{erf}(x) = (1/\sqrt{\pi})\int_{-x}^{x} e^{-t^2}\mathrm{d}t$. Appendix-E provides the proof of this lemma.

Thus, the piecewise linear function can be given by

$$g \le \varphi_k \beta + \gamma_k \quad \forall k \in \{1,...,K\} \tag{26}$$

where φ_k and γ_k are respectively the slope and the intercept of the kth segment linear value function.

Now the coalition payoff for coalition $\mathbf{N}'$ can be modeled as an optimization problem and stated as

$$\begin{cases} v(\mathbf{N}') = \max\limits_{\mathbf{b}_{\mathbf{N}'}} \sum_{t\in\mathbf{T}} [g_t + \sum_{j\in\mathbf{N}'} U_{j,t}(b_{j,t})] \\ s.t.\ (3)-(5),(7)-(10),(26) \end{cases} \tag{27}$$

where g_t represents the value of $g(\beta)$ at time t. Obviously, the problem in (27) is a linear programming model which pertains to some thoroughly formalized methodical tools and solution concepts and can be directly solved by an off-the-shelf solver.

B. Distribution of the Coalitional Payoff

In this part, the solution concept nucleolus is used to achieve a stable distribution of the total payoff. To solve the nucleolus of the proposed game model, a series of optimization programs are recursively defined as [26]

$$(O_1)\begin{cases} \min \varepsilon \\ s.t. \sum_{i\in\mathbf{N}'} x_i \ge v(\mathbf{N}') - \varepsilon \quad \forall \mathbf{N}' \in 2^{\mathbf{N}} \end{cases} \tag{28}$$

$$(O_m)\begin{cases} \min \varepsilon \\ s.t. \begin{cases} \sum_{i\in\mathbf{N}'} x_i \ge v(\mathbf{N}') - \varepsilon_1 \ \forall \mathbf{N}' \in \Pi_1 \\ \sum_{i\in\mathbf{N}'} x_i \ge v(\mathbf{N}') - \varepsilon_l \ \forall \mathbf{N}' \in \Pi_l \setminus \Pi_{l-1}, \forall l \in \{2,...,\mathrm{m}-1\} \\ \sum_{i\in\mathbf{N}'} x_i \ge v(\mathbf{N}') - \varepsilon \ \forall \mathbf{N}' \in 2^{\mathbf{N}} \setminus \Pi_l \end{cases} \end{cases} \tag{29}$$

where ε_l is the optimal solution to O_l and Π_l is the set of coalitions of which the inequalities are strictly binding in the optimal solution to O_l. Note that, there is a sequence of $o(2^{\mathbf{N}})$ linear programs that need to be solved in the worst scenario, which bring an exponential calculation burden for solving the nucleolus of game model. A detailed discussion is provided in Section VII about the computational complexity alleviation through altering the nucleolus with the imputation to minimize the worst-case excess.

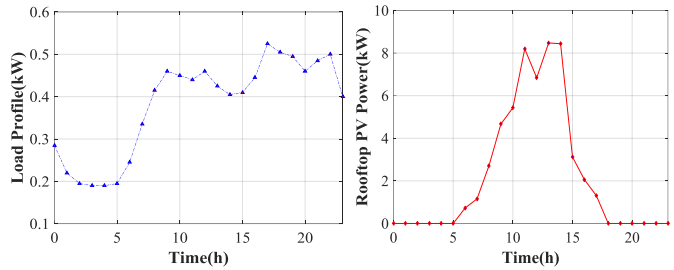
The realized profit for the grand coalition can be written in the form of

$$v^R(\mathbf{N}) = \sum_{t\in\mathbf{T}} J_t(\mathbf{s}_{\mathbf{N}}^*, \mathbf{b}_{\mathbf{N}}^*) \tag{30}$$

where $\mathbf{s}_{\mathbf{N}}^*$ is the realized value of the random variable $\mathbf{s}_{\mathbf{N}}$. The realized payoff can be allocated by

$$y_n = \sigma_n v^R(\mathbf{N}) \quad \forall n \in \mathbf{N} \tag{31}$$

where $\sigma_n = \dfrac{x_n^*}{\sum_{n\in\mathbf{N}} x_n^*}$ and y_n and x_n^* are the allocation of realized and expected payoff for player n, respectively. Given that the output power of RESs is independent and identically distributed across days, it is easy to verify this allocation mechanism is budget balanced and strongly consistent with the nucleolus $\mathbf{x}^*$ according to the strong law of large numbers. The detailed proof of this is available in [15].



(a) (b)

Fig. 4. Assumed profile of the (a) fixed load, (b) output power of the PV.

TABLE II
HOUSEHOLD SETTING

H1	H2	H3	H4	H5	H6	H7	H8	H9
{1}	{2}	{3}	{4}	{3, 4}	{2, 3}	{1, 4}	{2, 4}	{1, 2, 3}

1-Flexible load, 2-PV, 3-ESS, 4-EV

TABLE III
ESS PARAMETER VALUES

$b^{\min}$ (kW)	$b^{\max}$ (kW)	E_B (kWh)	$SOC^{\max}$	$SOC^{\min}$	SOC_0	η^{ch}	η^{dch}
– 3	3	10	0.95	0.1	0.15	0.95	0.95

VI. PERFORMANCE EVALUATION

To illustrate the performance of the proposed model, several numerical analyses have been conducted, a few of which discussed below. All studies are conducted in MATLAB® R2014a on a laptop with an Intel Core (i7, 2.80GHz) and 16GB memory.

Let us consider an LEC, in the form of Fig. 1, consisting of 9 households, equipped randomly with some controllable devices, as shown in Table II. All the households are assumed to have a fixed load profile, with the data taken from typical UK domestic load profiles [27], as shown in Fig. 4a.

The considered RESs are rooftop PVs that represent a dominant RES in a low voltage distribution system. The output power profile of the PV is taken from [1], and is depicted in Fig. 4b. In these studies, the Rayleigh distribution is used to capture the uncertainties in the output power of the PVs.

The selected EV model is BMW i3 with a capacity of 33 kWh, and the assumed rate of a residential charger is 3 kW [28]. Besides, it is assumed that the EV is plugged into the LEC at 18:00, and the departure time is fixed at 08:00. The EVs' initial and departure time state of charge are respectively set as 45 and 95% [28]. For simplicity, the ESSs in the LEC share the same parameters, given in Table III.

The number of segment linear functions is set as 11, i.e., K = 11. The coefficients of υ and ζ for the value function of the flexible load are respectively assumed as 0.025 and 0.32 while λ_d is thought to be 0.5 p/kWh [18]. The utility's electricity price is taken from UK's TIDE tariff scheme: 4.99 p/kWh for 23:00 to 06:00, 24.99 p/kWh for 16:00 to 19:00, and 11.99 p/kWh for the rest of the time. The UK domestic feed-in tariff of 4.85 p/kWh is also taken as the selling electricity price [29].

The performance of the LEC with and without the proposed cooperation is illustrated in Fig. 5. It can be seen from this figure that each household in the LEC achieves more payoffs

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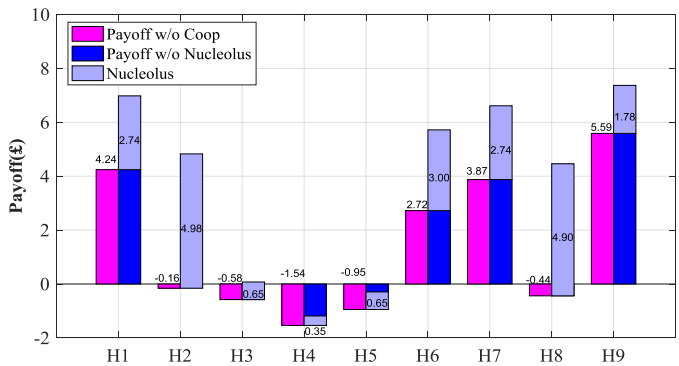

Fig. 5. Comparison between payoff with and without cooperation.

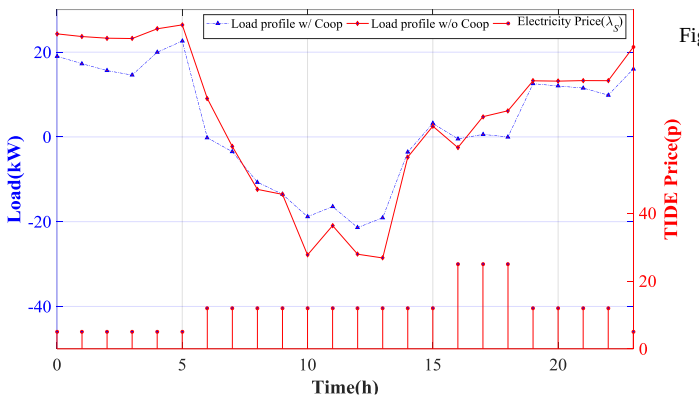

Fig. 6. The total load profile for the assumed 9 players in the LEC.

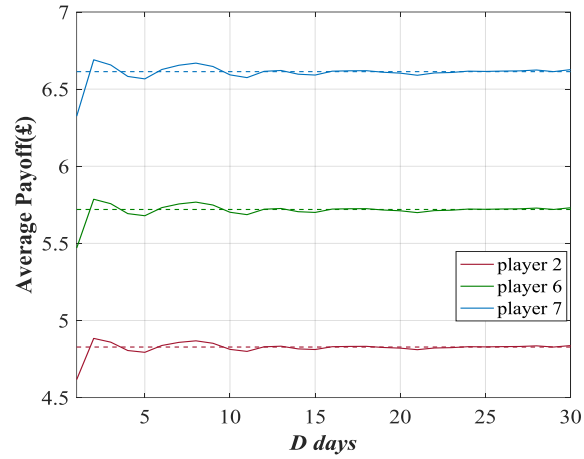

Fig. 7. Average allocation of realized payoffs for *D* days.

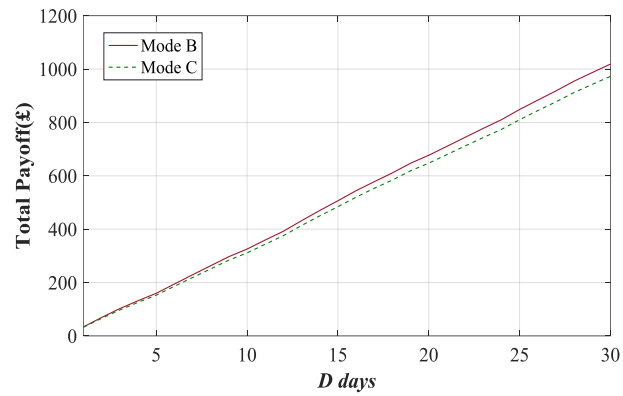

Fig. 8. Comparison between the total payoff of mode-B and C.

under the proposed technique (referred to as mode-B) compared with trading with the utility without cooperation (refer red to as mode-A). Specifically, the payoffs for prosumers with PVs experience a remarkable increase. This is mainly because some households can be supplied by the PVs, and thus, the corresponding coalition avoid the higher cost of purchasing electricity from the utility. Also, the prosumers with ESS or flexible load can shift their load profiles and adjust their consumption patterns to reduce the operational costs, which is represented by the increase in the payoffs. Note that, the payoffs of prosumers with PV or ESS is negative when there is no cooperation since they must purchase electricity from the utility to supply their fixed loads.

As is shown in Fig. 6, the proposed model can effectively transfer the load from the price peak periods (i.e., 16:00 to 18:00) to low price periods (i.e., 23:00 to 06:00). Compared with mode-A, the proposed model can also reduce the reverse power flow to a certain degree, especially during the periods of 10:00 to 13:00 when there is a remarkable increase in the output power of PVs.

A. Allocation of Realized Payoff

To examine whether the allocation scheme of the realized payoff is strongly consistent with the nucleolus $\mathbf{x}^*$ payoff, an evaluation index is defined as

$$\bar{y}_n = \frac{1}{D}\sum_{d=1}^{D} y_n^d \tag{32}$$

where $\bar{y}_n$ represents the time-average value of the realized payoffs obtained by player *n* for *D* days. As is shown in Fig. 7, the average payoffs of the three representative players rapidly converge to the expected values of the optimal payoff allocation (i.e., nucleolus $\mathbf{x}^*$) within 30 days. In this figure, it was assumed that the output power forecasts of the PVs are set at the same value across *D* days. The above conclusion can be further generalized even without assuming the same forecasts for the PV power, such that the total payoff obtained by player *n* for one year or more will surely converge to the sum of its expected values of allocations, i.e., strong consistency with respect to the nucleolus.

B. Impact of Uncertainty

To evaluate the impact of uncertainty in the output power of the RESs, let us consider a case in which the uncertainties in the output power of the PVs are neglected and the corresponding forecasted values are adopted. This is referred to as mode-C. A comparison has been carried out between the total realized payoff of the proposed model (i.e., mode-B) and that of mode-C for 30 days. The study shows that the grand coalition always suffers from loss in total accumulated realized payoff when neglecting the randomness in the output power of the RESs, as is shown in Fig. 8. This is valid because the expected value of the payoff has been maximized by the proposed model, which effectively mitigates the risk of loss when the actual output power of the RESs deviates from the forecasted values.

C. Evaluation of Flexible Loads

To evaluate the impact of flexible loads, another study has been conducted in which the flexible loads are set as fixed

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TABLE IV
COMPARISON BETWEEN PAYOFF WITH AND WITHOUT FLEXIBLE LOADS.

Household	H1	H7	H9	Total
Payoff in mode-B (£)	6.98	6.61	7.37	34.54
Payoff in mode-D (£)	6.22	5.85	7.01	33.50

loads while consuming the same amount of electricity during the optimization horizon (referred to as mode-D) and the other parameters remain the same as those in mode-B. Then, the difference between payoffs of the households with flexible loads in mode-B (i.e., H1, H7 and H9) and those of mode-D are investigated. As is shown in Table IV, households H1, H2 and H3 all gain remarkable increases in payoffs versus those with fixed loads. More importantly, the grand coalition also experiences an increase in the total payoff when flexible loads are considered. This is reasonable because the grand coalition can avoid peak prices through reducing the power of flexible loads and instead increase their power at lower-price periods.

D. Nucleolus and Shapley Value

In addition to the nucleolus, the Shapley value is also a solution concept which can uniquely distribute the total payoff generated by the grand coalition and is given by

$$\phi_n = \sum_{\mathbf{N}' \in 2^{\mathbf{N}}, n \in \mathbf{N}'} \frac{(|\mathbf{N}'|-1)!(N-|\mathbf{N}'|)!}{N!}[v(\mathbf{N}')-v(\mathbf{N}' \setminus \{n\})] \quad (33)$$

where ϕ_n is the Shapley value of player n. The Shapley value belongs to the core if the coalitional game is convex. However, the proposed model is not a convex game model. Several studies have been conducted to show that the Shapley value is not in the core. The largest excess value among all the coalitions is selected to evaluate the position of Shapley value.

As shown in Fig. 9, the largest excesses always exceed zero regardless of the number of players in the grand coalition. Fig. 10 depicts the excess of all the coalitions in a 12-player game, which shows that most of the coalitions share approximately equal values in nucleolus excess and Shapley excess. It is to be noted that, there are still some data points exceeding zero with respect to Shapley excess but not with respect to the nucleolus excess.

E. Sensitivity Analysis

A sensitivity analysis has been performed to evaluate the impact of the feed-in tariff on the payoffs of the players in the grand coalition. Five cases are studied in which the feed-in tariff of λ_S is respectively set as {1, 2, 3, 4, 4.85} p/kWh while keeping the other parameters the same as those in mode-B. Fig. 11 demonstrates the results of this study. As seen from this figure, the PVs' payoff without nucleolus increases considering a higher feed-in tariff which could offer more profit to the prosumers with PVs. Besides, the payoff allocated to prosumer with PVs, i.e., the nucleolus, also increases significantly since the PVs' marginal contribution to the grand coalition also rises with respect to a higher feed-in tariff.

VII. SCALABILITY

As the number of optimization problems in energy management model or in solution process of nucleolus increases

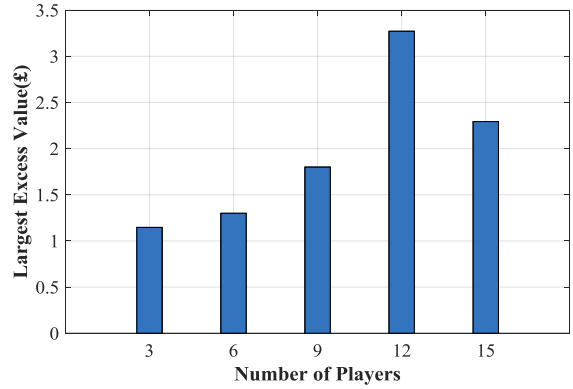

Fig. 9. Largest excess value of the Shapley value with respect to the number of players.

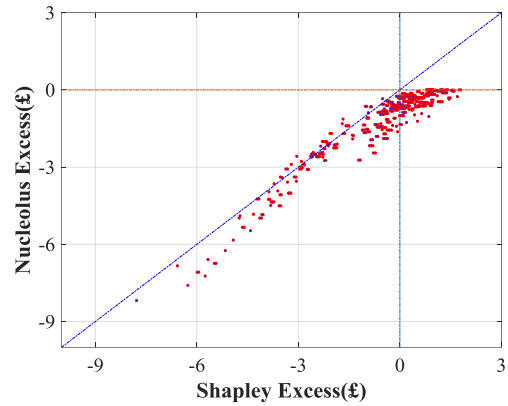

Fig. 10. Comparison of nucleolus excess and Shapley excess for all coalitions in a 12-player coalitional game.

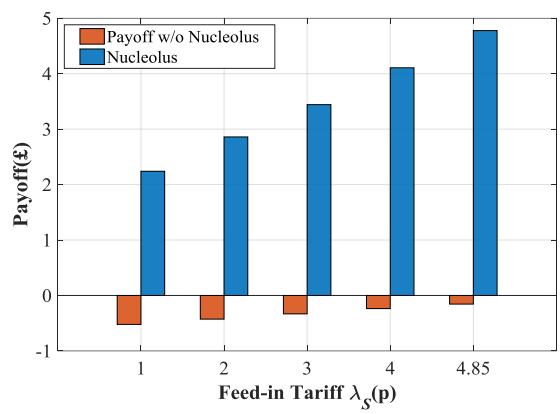

Fig. 11. Impact of feed-in tariff on payoff and nucleolus for the prosumers with PVs.

exponentially with respect to the number in grand coalition, the scalability of the proposed method can be limited. In addition, the dimension of the optimization problems in solving nucleolus, as shown in (28) and (29), is proportional to the number of all possible coalitions. Although the Shapley value can provide a closed-form and axiomatic solution for allocating the payoff of grand coalition, the Shapley value does not belong to the core of the proposed model, as discussed in Section VI.D.

The imputation that minimizes the worst-case excess for all the coalitions is first proposed in [15] where it is used to allocate the payoff avoiding the possible exponential calculation

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TABLE V
COMPUTATIONAL TIME FOR TWO MODES

Number of players	3	6	9	12	15
Nucleolus(s)	0.69	1.30	8.82	449.6	7958.7
Imputation minimizing the worst-case excess(s)	0.36	0.37	0.39	0.51	1.02

burden. As such, it has also been used in the payoff allocation scheme in the proposed method, as

$$\begin{cases} \min\limits_{x,\varepsilon} \max\limits_{\mathbf{N}'\in 2^{\mathbf{N}}} \varepsilon(x,\mathbf{N}') \\ s.t. \sum_{i\in\mathbf{N}'} x_i = v(\mathbf{N}) \end{cases} \tag{34}$$

and can be further formulated as a single linear program, in the form of

$$\begin{cases} \min\limits_{x,\varepsilon} \\ s.t. \sum_{i\in\mathbf{N}'} x_i \ge v(\mathbf{N}') - \varepsilon \quad \forall \mathbf{N}' \in 2^{\mathbf{N}} \\ \quad \sum_{i\in\mathbf{N}} x_i = v(\mathbf{N}) \end{cases} \tag{35}$$

To demonstrate the scalability of the proposed technique, let us assume that the number of players increases from 3 to 15 in steps of 3 while the remaining parameters are the same as those in mode-B. The results of this study are provided in Table V and shows that the computational time for nucleolus increases exponentially with respect to the number of players while the imputation minimizing the worst-case excess can be achieved at a high speed regardless of the player number.

VIII. CONCLUSION

A coalition game theory-based energy management problem is proposed in this paper for local energy communities. Based on some reasonable assumptions, the superadditivity and balancedness of the proposed game model is proved rigorously. A nucleolus-based solution is achieved to distribute the total payoff among all players in a fair and stable manner. Through extensive numerical analyses, it is demonstrated that the grand coalition effectively increases the global payoff and the proposed allocation scheme contributes to peak shaving and valley filling. Besides, it is also shown that the allocation scheme converges rapidly to the nucleolus for the realized payoff.

The realized profit may be deviated from the expected profit and accordingly some players with RESs would rather leave the coalition when they are dissatisfied with their allocated profits during a long time period. As such, our future research effort will potentially focus on designing a new value function for the coalition and then distributing the realized profit in a reasonable and stable way. Also, there is a loss of the overall profit for the players in the long term if the expected profit of a coalition is not maximized. Thus, a new settlement mechanism for improving the long-term profit of this coalition will also be investigated.

APPENDIX

The proof of Lemma-1 to 5, introduced in Section IV and V are respectively presented below:

A. Proof of Lemma-1

Let $\mathbf{N}_1'$ and $\mathbf{N}_2'$ denote two disjoint subsets of $\mathbf{N}$ and $\Omega_{\mathbf{N}'} = \{\mathbf{b}_{\mathbf{N}'} \mid \text{const.}(3)-(5),(7)-(10)\}$. Let us define

$$\mathbf{b}_{\mathbf{N}_1'}^* := \operatorname*{argmax}_{\mathbf{b}_{\mathbf{N}_1'}\in\Omega_{\mathbf{N}_1'}} \mathbb{E}\sum_{t\in\mathbf{T}} J_t(\mathbf{s}_{\mathbf{N}_1'}, \mathbf{b}_{\mathbf{N}_1'}) \tag{A1}$$

$$\mathbf{b}_{\mathbf{N}_2'}^* := \operatorname*{argmax}_{\mathbf{b}_{\mathbf{N}_2'}\in\Omega_{\mathbf{N}_2'}} \mathbb{E}\sum_{t\in\mathbf{T}} J_t(\mathbf{s}_{\mathbf{N}_2'}, \mathbf{b}_{\mathbf{N}_2'}) \tag{A2}$$

$$v(\mathbf{N}_1') + v(\mathbf{N}_2') = \mathbb{E}\sum_{t\in\mathbf{T}} J_t(\mathbf{s}_{\mathbf{N}_1'}, \mathbf{b}_{\mathbf{N}_1'}^*) + \mathbb{E}\sum_{t\in\mathbf{T}} J_t(\mathbf{s}_{\mathbf{N}_2'}, \mathbf{b}_{\mathbf{N}_2'}^*) \tag{A3}$$

For convenience of notation, let us consider a positive homogeneous function in the form of $f(x) = \lambda_S[x]^+ - \lambda_B[-x]^+$. Given, α an arbitrary nonnegative scalar and $[x]^+$ positive homogeneous, yields

$$\begin{aligned} f(\alpha x) &= \lambda_S[\alpha x]^+ - \lambda_B[-\alpha x]^+ = \alpha\lambda_S[x]^+ - \alpha\lambda_B[-x]^+ \\ &= \alpha f(x) \end{aligned} \tag{A4}$$

demonstrating that $f(x)$ is a positive homogeneous function. Then,

$$\begin{aligned} & J_t(\mathbf{s}_{\mathbf{N}_1'} \cup \mathbf{s}_{\mathbf{N}_2'}, \mathbf{b}_{\mathbf{N}_1'}^* \cup \mathbf{b}_{\mathbf{N}_2'}^*) - J_t(\mathbf{s}_{\mathbf{N}_1'}, \mathbf{b}_{\mathbf{N}_1'}^*) - J_t(\mathbf{s}_{\mathbf{N}_2'}, \mathbf{b}_{\mathbf{N}_2'}^*) \\ &= f\Big(\sum_{i\in\mathbf{N}_1'\cup\mathbf{N}_2'} s_{i,t} - \sum_{j\in\mathbf{N}_1'\cup\mathbf{N}_2'} b_{j,t}^*\Big) - \\ &\qquad f\Big(\sum_{i\in\mathbf{N}_1'} s_{i,t} - \sum_{j\in\mathbf{N}_1'} b_{j,t}^*\Big) - f\Big(\sum_{i\in\mathbf{N}_2'} s_{i,t} - \sum_{j\in\mathbf{N}_2'} b_{j,t}^*\Big) \end{aligned} \tag{A5}$$

Based on *Assumption-3*, $f(x)$ is a concave and positive homogeneous function. Thus, for any time slot t, it attains

$$J_t(\mathbf{s}_{\mathbf{N}_1'}, \mathbf{b}_{\mathbf{N}_1'}^*) + J_t(\mathbf{s}_{\mathbf{N}_2'}, \mathbf{b}_{\mathbf{N}_2'}^*) \le J_t(\mathbf{s}_{\mathbf{N}_1'} \cup \mathbf{s}_{\mathbf{N}_2'}, \mathbf{b}_{\mathbf{N}_1'}^* \cup \mathbf{b}_{\mathbf{N}_2'}^*) \tag{A6}$$

Taking expectations on both sides, we have

$$\mathbb{E}J_t(\mathbf{s}_{\mathbf{N}_1'}, \mathbf{b}_{\mathbf{N}_1'}^*) + \mathbb{E}J_t(\mathbf{s}_{\mathbf{N}_2'}, \mathbf{b}_{\mathbf{N}_2'}^*) \le \mathbb{E}J_t(\mathbf{s}_{\mathbf{N}_1'} \cup \mathbf{s}_{\mathbf{N}_2'}, \mathbf{b}_{\mathbf{N}_1'}^* \cup \mathbf{b}_{\mathbf{N}_2'}^*) \tag{A7}$$

Readers can also distinguish different cases of signs of $\big(\sum_{i\in\mathbf{N}_1'} s_{i,t} - \sum_{j\in\mathbf{N}_1'} b_{j,t}^*\big)$ and $\big(\sum_{i\in\mathbf{N}_2'} s_{i,t} - \sum_{j\in\mathbf{N}_2'} b_{j,t}^*\big)$ to draw the same conclusion. Besides, it is easy to get

$$\begin{aligned} & \mathbb{E}\sum_{t\in\mathbf{T}} J_t(\mathbf{s}_{\mathbf{N}_1'} \cup \mathbf{s}_{\mathbf{N}_2'}, \mathbf{b}_{\mathbf{N}_1'}^* \cup \mathbf{b}_{\mathbf{N}_2'}^*) \\ &\qquad \le \max_{\mathbf{b}_{\mathbf{N}_1'}, \mathbf{b}_{\mathbf{N}_2'}} \mathbb{E}\sum_{t\in\mathbf{T}} J_t(\mathbf{s}_{\mathbf{N}_1'} \cup \mathbf{s}_{\mathbf{N}_2'}, \mathbf{b}_{\mathbf{N}_1'} \cup \mathbf{b}_{\mathbf{N}_2'}) \end{aligned} \tag{A8}$$

Therefore, it attains

$$\begin{aligned} & \mathbb{E}\sum_{t\in\mathbf{T}} [J_t(\mathbf{s}_{\mathbf{N}_1'}, \mathbf{b}_{\mathbf{N}_1'}^*) + J_t(\mathbf{s}_{\mathbf{N}_2'}, \mathbf{b}_{\mathbf{N}_2'}^*)] \\ &\qquad \le \max_{\mathbf{b}_{\mathbf{N}_1'}, \mathbf{b}_{\mathbf{N}_2'}} \mathbb{E}\sum_{t\in\mathbf{T}} J_t(\mathbf{s}_{\mathbf{N}_1'} \cup \mathbf{s}_{\mathbf{N}_2'}, \mathbf{b}_{\mathbf{N}_1'} \cup \mathbf{b}_{\mathbf{N}_2'}) \end{aligned} \tag{A9}$$

or equivalently

$$v(\mathbf{N}_1') + v(\mathbf{N}_2') \le v(\mathbf{N}_1' \cup \mathbf{N}_2') \tag{A10}$$

which proves the superadditivity of (15).

B. Proof of Lemma-2

As $\Omega_{\mathbf{N}'} = \{\mathbf{b}_{\mathbf{N}'} \mid \text{const.}(3)-(5),(7)-(10)\}$ is affine, $\Omega_{\mathbf{N}'} = \{\theta\mathbf{b}_{\mathbf{N}'} \mid \text{const.}(3)-(5),(7)-(10)\}$

$$\begin{aligned} & \Psi(\theta\mathbf{s}_{\mathbf{N}'}, \theta\mathbf{b}_{\mathbf{N}'}) \\ &= \max_{\mathbf{b}_{\mathbf{N}'}\in\Omega_{\mathbf{N}'}} \mathbb{E}\sum_{t\in\mathbf{T}} J_t(\theta\mathbf{s}_{\mathbf{N}'}, \theta\mathbf{b}_{\mathbf{N}'}) \end{aligned}$$

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$$= \max_{\mathbf{b}_{\mathbf{N}'} \in \Omega_{\mathbf{N}'}} \mathbb{E} \sum_{t \in \mathbf{T}} [f_t(\theta \sum_{i \in \mathbf{N}'} s_{i,t} - \theta \sum_{j \in \mathbf{N}} b_{j,t}) + \sum_{j \in \mathbf{N}'_f} U^f_{j,t}(\theta b^f_{j,t}) + \sum_{j \in \mathbf{N}'_{ES}} U^{ES}_{j,t}(\theta b^{ES}_{j,t})] \quad \text{(B1)}$$

where $\mathbf{N}'_f$ and $\mathbf{N}'_{ES}$ represent the collections of households with respectively flexible loads and ESSs. For brevity, the value function of flexible loads, shown in (6), are studied:

1) If $0 \le b_{j,t} \le \zeta_j / 2\upsilon_j$, then

$$\begin{aligned}
&\max_{\mathbf{b}_{\mathbf{N}'} \in \Omega_{\mathbf{N}'}} \mathbb{E} \sum_{t \in \mathbf{T}} [f_t(\theta \sum_{i \in \mathbf{N}'} s_{i,t} - \theta \sum_{j \in \mathbf{N}} b_{j,t}) + \sum_{j \in \mathbf{N}'_f} U^f_{j,t}(\theta b^f_{j,t})] \\
&\ge \max_{\mathbf{b}_{\mathbf{N}'} \in \Omega_{\mathbf{N}'}} \mathbb{E} \sum_{t \in \mathbf{T}} \{ f_t(\theta \sum_{i \in \mathbf{N}'} s_{i,t} - \theta \sum_{j \in \mathbf{N}} b_{j,t}) + \sum_{j \in \mathbf{N}'_f} [\theta \zeta_j b^f_{j,t} - \theta \upsilon_j (b^f_{j,t})^2]\} \\
&\ge \max_{\mathbf{b}_{\mathbf{N}'} \in \Omega_{\mathbf{N}'}} \mathbb{E} \sum_{t \in \mathbf{T}} \{ \theta f_t(\sum_{i \in \mathbf{N}'} s_{i,t} - \sum_{j \in \mathbf{N}} b_{j,t}) + \sum_{j \in \mathbf{N}'_f} \theta [\zeta_j b^f_{j,t} - \upsilon_j (b^f_{j,t})^2]\}
\end{aligned} \quad \text{(B2)}$$

2) If $b_{j,t} \ge \zeta_j / 2\upsilon_j$, then

$$\max_{\mathbf{b}_{\mathbf{N}'} \in \Omega_{\mathbf{N}'}} \mathbb{E} \sum_{j \in \mathbf{N}'_1} U^f_{j,t}(\theta b^f_{j,t}) = \max_{\mathbf{b}_{\mathbf{N}'} \in \Omega_{\mathbf{N}'}} \mathbb{E}[\theta \sum_{j \in \mathbf{N}'_1} U^f_{j,t}(b^f_{j,t})] \quad \text{(B3)}$$

For the ESS, it is easy to get

$$\begin{aligned}
\max_{\mathbf{b}_{\mathbf{N}'} \in \Omega_{\mathbf{N}'}} \mathbb{E} \sum_{j \in \mathbf{N}'_{ES}} U^{ES}_{j,t}(\theta b^{ES}_{j,t}) &= \max_{\mathbf{b}_{\mathbf{N}'} \in \Omega_{\mathbf{N}'}} \mathbb{E} \sum_{j \in \mathbf{N}'_{ES}} \lambda_d \left| \theta b^{ES}_{j,t} \right| \\
&= \max_{\mathbf{b}_{\mathbf{N}'} \in \Omega_{\mathbf{N}'}} \mathbb{E} [\theta \sum_{j \in \mathbf{N}'_{ES}} \lambda_d \left| b^{ES}_{j,t} \right|]
\end{aligned} \quad \text{(B4)}$$

Combining (B2), (B3) and (B4) yields

$$\begin{aligned}
&\Psi(\theta \mathbf{s}_{\mathbf{N}'}, \theta \mathbf{b}_{\mathbf{N}'}) \ge \\
&\max_{\mathbf{b}_{\mathbf{N}'} \in \Omega_{\mathbf{N}'}} \mathbb{E} \sum_{t \in \mathbf{T}} \{ \theta f_t(\sum_{i \in \mathbf{N}'} s_{i,t} - \sum_{j \in \mathbf{N}} b_{j,t}) + \theta \sum_{j \in \mathbf{N}'_f} U^f_{j,t}(b^f_{j,t}) + \theta \sum_{j \in \mathbf{N}'_{ES}} \lambda_d \left| b^{ES}_{j,t} \right| \} \\
&= \theta \max_{\mathbf{b}_{\mathbf{N}'} \in \Omega_{\mathbf{N}'}} \mathbb{E} \sum_{t \in \mathbf{T}} \{ f_t(\sum_{i \in \mathbf{N}'} s_{i,t} - \sum_{j \in \mathbf{N}} b_{j,t}) + \sum_{j \in \mathbf{N}'_f} U^f_{j,t}(b^f_{j,t}) + \sum_{j \in \mathbf{N}'_{ES}} \lambda_d \left| b^{ES}_{j,t} \right| \} \\
&= \theta \max_{\mathbf{b}_{\mathbf{N}'} \in \Omega_{\mathbf{N}'}} \mathbb{E} \sum_{t \in \mathbf{T}} \{ f_t(\sum_{i \in \mathbf{N}'} s_{i,t} - \sum_{j \in \mathbf{N}} b_{j,t}) + \sum_{j \in \mathbf{N}'_f} U^f_{j,t}(b^f_{j,t}) + \sum_{j \in \mathbf{N}'_{ES}} \lambda_d \left| b^{ES}_{j,t} \right| \} \\
&= \theta \max_{\mathbf{b}_{\mathbf{N}'} \in \Omega_{\mathbf{N}'}} \mathbb{E} \sum_{t \in \mathbf{T}} J_t(\mathbf{s}_{\mathbf{N}'}, \mathbf{b}_{\mathbf{N}'}) = \theta \Psi(\mathbf{s}_{\mathbf{N}'}, \mathbf{b}_{\mathbf{N}'})
\end{aligned} \quad \text{(B5)}$$

and the desired results are obtained.

By extension, if the consumers' value function $U_{j,t}(b_{j,t})$ is concave with nonnegative values, the same conclusion as the above can be attained, even when the value function is described by a multi-block function with a step-wise decreasing marginal benefit. To prove this, assume that the value function $U(b)$ be concave and $U(b) \ge 0$, thus

$$\begin{aligned}
U(\theta \bullet b) &= U(0 + \theta b) \\
&\ge (1-\theta) U(0) + \theta U(b) \\
&\ge \theta U(b) \qquad \text{by } U(0) \ge 0
\end{aligned}$$

This shows that $U(b)$ is a positive homogeneous value function.

C. Proof of Lemma-3

$$\begin{aligned}
&\sum_{\mathbf{N}' \in 2^{\mathbf{N}}} \alpha(\mathbf{N}') v(\mathbf{N}') \\
&= \sum_{\mathbf{N}' \in 2^{\mathbf{N}}} \alpha(\mathbf{N}') \Psi(\mathbf{s}_{\mathbf{N}'}, \mathbf{b}_{\mathbf{N}'}) \\
&\le \sum_{\mathbf{N}' \in 2^{\mathbf{N}}} \Psi(\alpha(\mathbf{N}') \mathbf{s}_{\mathbf{N}'}, \alpha(\mathbf{N}') \mathbf{b}_{\mathbf{N}'}) \qquad \text{by lemma-2} \\
&\le \Psi(\sum_{\mathbf{N}' \in 2^{\mathbf{N}}} \alpha(\mathbf{N}') \mathbf{s}_{\mathbf{N}'}, \sum_{\mathbf{N}' \in 2^{\mathbf{N}}} \alpha(\mathbf{N}') \mathbf{b}_{\mathbf{N}'}) \quad \text{by lemma-1} \\
&= \Psi(\sum_{n \in \mathbf{N}} \sum_{\mathbf{N}' \in 2^{\mathbf{N}}} \alpha(\mathbf{N}') \mathbf{1}^{\mathbf{N}'}_n \mathbf{s}_n, \sum_{n \in \mathbf{N}} \sum_{\mathbf{N}' \in 2^{\mathbf{N}}} \alpha(\mathbf{N}') \mathbf{1}^{\mathbf{N}'}_n \mathbf{b}_n) \\
&= \Psi(\sum_{n \in \mathbf{N}} \mathbf{s}_n, \sum_{n \in \mathbf{N}} \mathbf{b}_n) \quad \text{by the property of balanced map} \\
&= v(\mathbf{N})
\end{aligned}$$

Thus, the established game model $(\mathbf{N}, v)$ is balanced and thus the core is nonempty based on *Bondareva-Shapley Theorem*.

D. Proof of Lemma-4

Based on (15), we have

$$g(\beta) = \mathbb{E}\lambda_S \left[\sum_{i \in \mathbf{N}'_S} s_{i,t} - \sum_{j \in \mathbf{N}'_B} b_{j,t} \right]^+ - \mathbb{E}\lambda_B \left[\sum_{j \in \mathbf{N}'_B} b_{j,t} - \sum_{i \in \mathbf{N}'_S} s_{i,t} \right]^+ \quad \text{(D1)}$$

As some households contain ESSs, β may be negative when ESSs discharge. Thus, let us consider two cases:

1) if $\beta \ge 0$ then

$$\begin{aligned}
g(\beta) &= \lambda_S \int_\beta^{+\infty} (\varsigma - \beta) \rho(\varsigma) \mathrm{d}\varsigma - \lambda_B \int_{-\infty}^{\beta} (\beta - \varsigma) \rho(\varsigma) \mathrm{d}\varsigma \\
&= \lambda_S \int_{-\infty}^{+\infty} (\varsigma - \beta) \rho(\varsigma) \mathrm{d}\varsigma + (\lambda_S - \lambda_B) \int_{-\infty}^{\beta} (\beta - \varsigma) \rho(\varsigma) \mathrm{d}\varsigma \\
&= \lambda_S \mu_\beta - \lambda_S \beta + (\lambda_S - \lambda_B) \int_{-\infty}^{\beta} (\beta - \varsigma) \rho(\varsigma) \mathrm{d}\varsigma
\end{aligned} \quad \text{(D2)}$$

Thus, it attains

$$\begin{aligned}
g'(\beta) &= -\lambda_S + (\lambda_S - \lambda_B)(-\beta) \rho(\beta) \\
&\quad + (\lambda_S - \lambda_B) \int_{-\infty}^{\beta} \rho(\varsigma) \mathrm{d}\varsigma + (\lambda_S - \lambda_B) \beta \rho(\beta)
\end{aligned} \quad \text{(D3)}$$

$$\begin{aligned}
g''(\beta) &= -(\lambda_S - \lambda_B) \rho(\beta) + (\lambda_S - \lambda_B)(-\beta) \rho'(\beta) \\
&\quad + (\lambda_S - \lambda_B) \rho(\beta) + (\lambda_S - \lambda_B) \rho(\beta) + (\lambda_S - \lambda_B) \beta \rho'(\beta) \\
&= (\lambda_S - \lambda_B) \rho(\beta)
\end{aligned} \quad \text{(D4)}$$

2) if $\beta \le 0$ then

$$g(\beta) = \lambda_S \int_0^{+\infty} (\varsigma - \beta) \rho(\varsigma) \mathrm{d}\varsigma \quad \text{(D5)}$$

$$g'(\beta) = \lambda_S \int_0^{+\infty} (\varsigma - \beta) \rho(\varsigma) \mathrm{d}\varsigma = -\lambda_S \quad \text{(D6)}$$

$$g''(\beta) = 0 \quad \text{(D7)}$$

Based on *Assumption-3*, it is easy to get $g''(\beta) \le 0$ and $g(\beta)$ is a concave function with respect to β.

E. Proof of Lemma-5

Similar to Appendix-D, let us consider two cases to derive the closed-form analytical expression of (15), as:

(1) if $\beta \ge 0$ then

$$\begin{aligned}
g(\beta) &= \lambda_S \int_\beta^{+\infty} (\varsigma - \beta) \rho(\varsigma) \mathrm{d}\varsigma - \lambda_B \int_{-\infty}^{\beta} (\beta - \varsigma) \rho(\varsigma) \mathrm{d}\varsigma \\
&= \lambda_S \int_\beta^{+\infty} (\varsigma - \beta) \frac{\pi \varsigma}{2\mu^2} e^{(-\frac{\pi \varsigma^2}{4\mu^2})} \mathrm{d}\varsigma - \lambda_B \int_0^{\beta} (\beta - \varsigma) \frac{\pi \varsigma}{2\mu^2} e^{(-\frac{\pi \varsigma^2}{4\mu^2})} \mathrm{d}\varsigma \\
&= \lambda_S (\beta * e^{(-\frac{\pi \beta^2}{4\mu^2})} + \int_\beta^{+\infty} e^{(-\frac{\pi \varsigma^2}{4\mu^2})} \mathrm{d}\varsigma) - \lambda_S (\beta * e^{(-\frac{\pi \beta^2}{4\mu^2})}) -
\end{aligned}$$

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$$\lambda_B(-\beta * e^{(-\frac{\pi\beta^2}{4\mu^2})} + \beta) + \lambda_B(-\beta * e^{(-\frac{\pi\beta^2}{4\mu^2})} + \int_0^{\beta} e^{(-\frac{\pi\varsigma^2}{4\mu^2})} \mathrm{d}\varsigma)$$
$$= \lambda_S \int_{\beta}^{+\infty} e^{(-\frac{\pi\varsigma^2}{4\mu^2})} \mathrm{d}\varsigma + \lambda_B \int_0^{\beta} e^{(-\frac{\pi\varsigma^2}{4\mu^2})} \mathrm{d}\varsigma - \lambda_B \beta$$
$$= \lambda_S \mu - \lambda_B \beta + \mu(\lambda_B - \lambda_S)\,\mathrm{erf}(\frac{\sqrt{\pi}}{2\mu}\beta) \quad \text{(E1)}$$

(2) if $\beta \le 0$ then

$$g(\beta) = \lambda_S \int_0^{+\infty} (\varsigma - \beta)\rho(\varsigma)\mathrm{d}\varsigma = \lambda_S(\mu - \beta) \quad \text{(E2)}$$

Thus, it attains

$$g(\beta) = \begin{cases} \lambda_S \mu - \lambda_B \beta + \mu(\lambda_B - \lambda_S)\,\mathrm{erf}(\frac{\sqrt{\pi}}{2\mu}\beta) & \beta \ge 0 \\ \lambda_S(\mu - \beta) & otherwise \end{cases} \quad \text{(E3)}$$

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